# SATHYABAMA UNIVERSITY

## FACULTY OF BUSINESS ADMINISTRATION

**Subject Title: Resource Management Techniques** **Subject Code: SMEX1017**

**Course: B.E (Common to all Engineering Branches)**

### UNIT – 5 – QUEUING THEORY, GAME THEORY AND REPLACEMENT MODEL

### QUEUING THEORY

Queuing theory concerns the mathematical study of queues or waiting lines (seen in banks, post offices, hospitals, airports etc.). The formation of waiting lines usually occurs whenever the current demand for a service exceeds the current capacity to provide that service.

In general, the customer's arrival and his or her service time are not known in advance and not predictable accurately. Otherwise, the operation of the service facility could be scheduled in a manner that would eliminate waiting completely. Both arrival and departure phenomena are random and this necessitates mathematical modeling of queuing systems to alleviate waiting which is costly. It involves excessive costs to provide too much service and not providing enough service capacity will create long waiting lines at times. Thus, there has to be a balance between these two. Excessive waiting is also costly due to various reasons like social cost, the cost of lost customers etc. Hence, it is the objective of the industry, to have an economic balance between the cost of service and the cost associated with waiting for that service.

The first queuing theory problem was considered by Erlang in 1908 who looked at how large a telephone exchange needed to be in order to keep to a reasonable value the number of telephone calls not connected because the exchange was busy (lost calls). Within ten years he had developed a (complex) formula to solve the problem.

The objective of the waiting line model is to minimize the cost of idle time & the cost of waiting time.

**IDLE TIME COST:**

If an organization operates with many facilities and the demand from customers is very low, then the facilities are idle and the cost involved due to the idleness of the facilities is the *idle time cost*. The cost of idle service facilities is the payment to be made to the services for the period for which they remain idle.

**WAITING TIME COST:**

If an organization operates with few facilities and the demand from customer is high and hence the customer will wait in queue. This may lead to dissatisfaction of customers, which leads to *waiting time cost*. The cost of waiting generally includes the indirect cost of lost business.

In terms of the analysis of queuing situations the types of questions in which we are interested includes:

- How long does a customer expect to wait in the queue before they are served, and how long will they have to wait before the service is complete?
- What is the probability of a customer having to wait longer than a given time interval before they are served?
- What is the average length of the queue?
- What is the probability that the queue will exceed a certain length?
- What is the expected utilization of the server and the expected time period during which he will be fully occupied (remember servers cost us money so we need to keep them busy). In fact if we can assign costs to factors such as customer waiting time and server idle time then we can investigate how to design a system at minimum total cost.

These are questions that need to be answered so that management can evaluate alternatives in an attempt to control/improve the situation. Some of the problems that are often investigated in practice are:

- Is it worthwhile to invest effort in reducing the service time?
- How many servers should be employed?
- Should priorities for certain types of customers be introduced?
- Is the waiting area for customers adequate?

In order to get answers to the above questions there are *two* basic approaches:

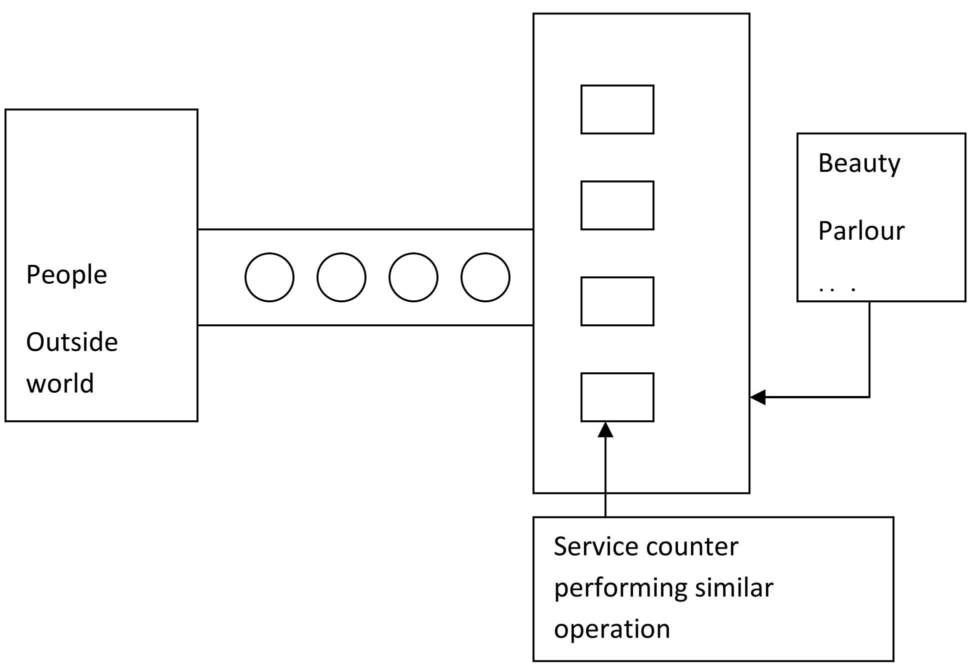
- analytic methods or queuing theory (formula based); and
- Simulation (computer based).

The reason for there being two approaches (instead of just one) is that analytic methods are only available for relatively simple queuing systems. Complex queuing systems are almost always analysed using Simulation

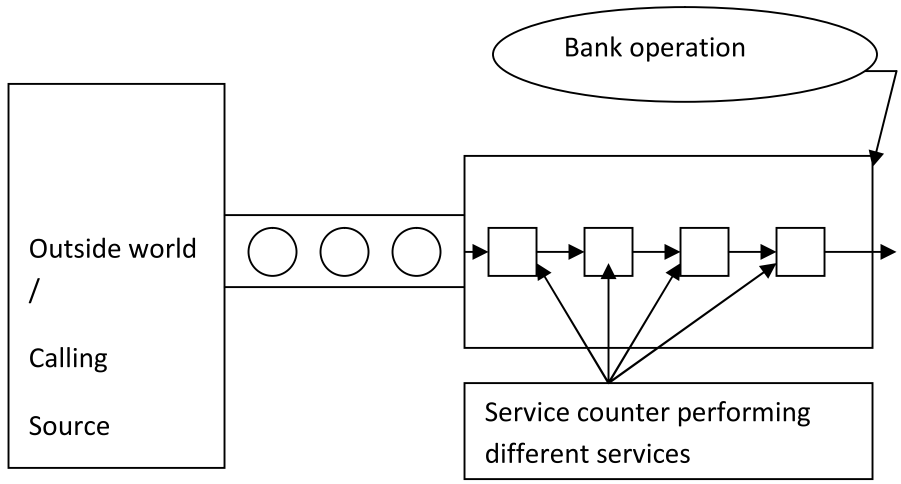
**<u>TYPE OF QUEUE</u>**

a) Parallel queues. b) Sequential queues.

**PARALLEL QUEUES:** If there is more than one server performing the same function, then queues are parallel.



**SEQUENTIAL QUEUES :** If there is one server performing one particular function or many servers performing sequential operations then the queue will be sequential.




a. **Limited Queue:**

In some facilities, only a limited number of customers are allowed in the system and new arriving customers are not allowed to join the system unless the number below less the limiting value. (Number of appointments in hospitals)

b. **Unlimited Queue:**

In some facilities, there is no limit to the number of customer allowed in the system. (Entertainment centers).

a. **Infinite queue:** If the customer who arrives and forms the queue from a very large population the queue is referred to as infinite queue.

b. **Finite Queue:** if the customer who arrives and forms the queue from a small population then the queue is referred to as finite queue.

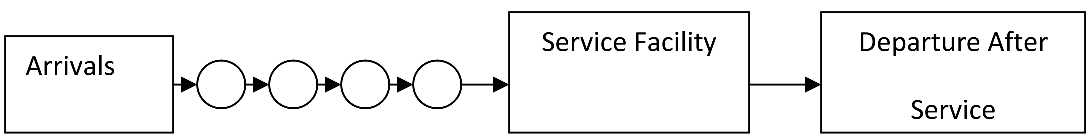
**DEFINITION:**

1. **The customer:** The arriving unit that requires some service to be provided.
2. **Server**: A server is one who provides the necessary service to the arrived customer.
3. **Queue (Waiting line):** The number of customers, waiting to be serviced. **The queue does not include the customer being serviced.**
4. **Service channel**: The process or system, which performs the service to the customer.

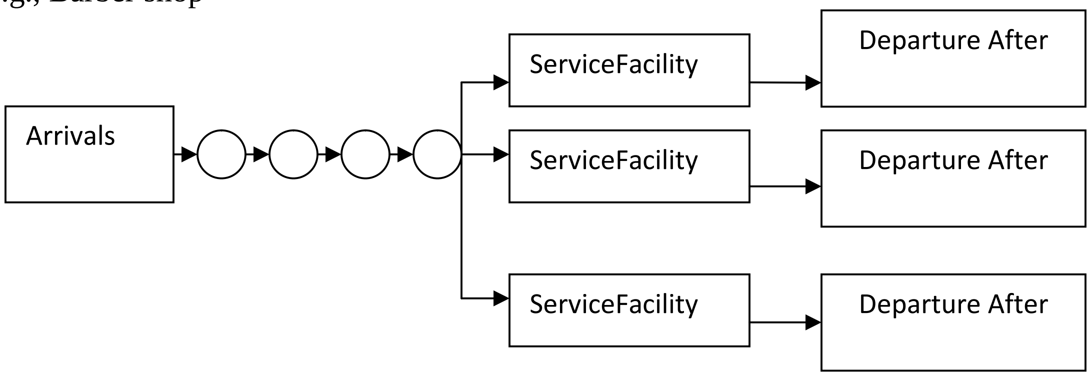
Based on the number of servers available.

4A. **Single Channel:** If there is a single service station, customer arrivals from a single line to be serviced then the channel is said to Single Channel Model or Single Server Model.

**Eg.** Doctor’s clinic



4B. **Multiple Channel Waiting Line Model:** If there are more than one service station to handle customer who arrive then it is called Multiple Channel Model. Symbol “c” is used.
E.g., Barber shop




5. **Arrival rate:** The rate at which the customers arrive to be serviced. It is denoted by λ. λ indicates take average number of customer arrivals per time period.
6. **Service rate:** The rate at which the customers are actually serviced. It in indicated by µ. µ indicates the mean value of customer serviced per time period.
7. **Infinite queue:** If the customers who arrive and form the queue from a very large population the queue is referred to as infinite queue.
8. **Priority:** This refers to method of deciding as to which customer will be serviced. Priority is said to occur when an arriving customer is chosen for service ahead of some other customer already in the queue.
9. **Expected number in the queue**“**Lq**”: This is average or mean number of customer waiting to be serviced. This is indicated by “Lq”.
10. **Expected number in system Ls**.**:** This is average or mean number of customer either waiting to be serviced or being serviced. This is denoted by Ls.
11. **Expected time in queue Wq”.:** This is the expected or mean time a customer spends waiting in the queue. This is denoted by “Wq”.
12. **The Expected time in the system “Ws’:** This is the expected time or mean time customers spends for waiting in the queue and for being serviced. This is denoted by “Ws’.
13. **Expected number in a non-empty queue:** Expected number of customer waiting in the line excluding those times when the line is empty.
14. **System utilization or traffic intensity**: This is ratio between arrival and service rate.
15. **Customer Behaviour:** The customer generally behaves in 4 ways:
   a) **Balking:** A customer may leave the queue, if there is no waiting space or he has no time to wait.
   b) **Reneging:** A customer may leave the queue due to impatience
   c) **Priorities:** Customers are served before others regardless of their arrival
   d) **Jockeying**: Customers may jump from one waiting line to another.
16. **Transient and Steady State**:

A system is said to be in Transient state when its operating characteristics are dependent on time.

A system is said to be in Steady state when its operating characteristics are not dependent on time

**CHARACTERISTICS OF QUEUING MODELS:**

1. Input or arrival (inter –arrival) distribution.
2. Output or Departure (Service) distribution.
3. Service channel
4. Service discipline.
5. Maximum number of customers allowed in the system.
6. Calling source or Population.

1. **ARRIVAL DISTRIBUTION:**

It represents the rate in which the customer arrives at the system.

Arrival rate/interval rate:

► Arrival rate is the rate at which the customers arrive to be serviced per unit of time.

► Inter-arrival time is the time gap between two arrivals.

► Arrival may be separated

1) By **equal** interval of time
2) By **unequal** interval of time which is **definitely known.**
3) Arrival may be **unequal** interval of time whose **probability is known.**

► Arrival rate may be

1. Deterministic (D)
2. Probabilistic
    a. Normal (N)
    b. Binomial (B)
    c. Poisson (M/N)
    d. Beta (β)
    e. Gama (g)
    f. Erlongian (Eh)

**The typical assumption is that arrival rate is randomly distributed according to Poisson distribution it is denoted by $^{a}$. $^{a}$ indicates average number of customer arrival per time period.**

2. **SERVICE OR DEPARTURE DISTRIBUTON:**

It represents the pattern in which the customer leaves the system. Service rate at which the customer are actually serviced. It indicated by μ. μ indicates the mean value of service per time period. Interdeparture is the rate time between two departures.

Service time may be

► **Constant.**

► **Variable** with **definitely known** probability.

► **Variable** with **known probability**.

**Service Rate Or Departure Rate may be:**

1. Deterministic
2. Probabilistic.
    a. Normal (N)
    b. Binomial (B)
    c. Poisson (M/N)
    d. Beat (β)

e. Gama (g)
f. Erlongian (Ek)
g. Exponential (M/N)
The typical assumption used is that service rate is randomly distributed according to exponential distribution. Service rate at which the customer are actually serviced. It indicated by $\mu$. $\mu$ indicates the mean value of service per time period.

3. **SERVICE CHANNELS:**

The process or system, which is performing the service to the customer.

Based on the number of channels**:**

**Single channel**

If there is a single service station and customer arrive and from a single line to be serviced, the channel is said to single channel. **Single Channel – 1.**

**Multiple channel**

If there is more than one service station to handle customer who arrive, then it is called multiple channel model. **Multiple Channel - C.**

**4.SERVICE DISCIPLINE:** Service discipline or order of service is the rule by which customer are selected from the queue for service.

**FIFO:** First In First Out – Customer are served in the order of their arrival. Eg. Ticket counter, railway station, banks.

**LIFO:** Last In First Out **–** Items arriving last come out first.

**Priority**: is said to occur when a arriving customer is chosen ahead of some other customer for service in the queue.

**SIRO:** Service in random order

**Here the common service discipline “First Come, First Served”.**

5. **MAXIMUM NUMBER OF CUSTOMER ALLOWED IN THE SYSTEM:**

Maximum number of customer in the system can be either finite or finite.

a. **Limited Queue:**

In some facilities, only a limited number of customers are allowed in the system and new arriving customers are not allowed to join the system unless the number below less the limiting value. (Number of appointments in hospitals)

b. **Unlimited Queue:**

In some facilities, there is no limit to the number of customer allowed in the system. (Entertainment centers).

6. **POPULATION:**

The arrival pattern of the customer depends upon the source, which generates them.

a. **Finite population (<40):**

If there are a few numbers of potential customers the calling source is finite.

b. **Infinite calling source or population:**

If there are large numbers of potential customer, it is usually said to be infinite.

**KENDALL'S NOTATION:** a/b/c; d/e/f.

Where, a – Arrival rate.

b – Service rate.

c – Number of service s 1 or c.

d – Service discipline (FIFO)

e - Number of persons allowed in the queue (N or∞)

f - Number of people in the calling source (∞ or N)

1. **M/M/1, FIFO/∞/∞:**

Means Poisson arrival rate, Exponential service rate/one server /FIFO service discipline/Unlimited queues & Unlimited queue in the calling source.

2. **M/M/C, FIFO/∞/∞:**

Poisson arrival rate, Exponential service rate, more than one server, FIFO service discipline Unlimited queues and unlimited persons in the calling source.

3. **M/M/I, FIFO/N/∞:**

Means Poisson arrival rate, Exponential service rate, One server, FIFO, Limited queue & Unlimited population.

**SINGLE CHANNEL /MULTIPLE CHANNELPOPULATION MODEL:**

1. Find an expression for probability of n customer in the system at time (Pn) in terms of λ and μ
2. Find an expression for probability of zero customers in the system at time t.(Po)
3. Having known Pn, find out the expected number of units in the Queue (Lq)
4. Find out the expected number of units in the system (Ls)
5. Expected waiting time in system (Ws)
6. Expected waiting time queue (Wq)

**SOLUTION PROCESS**

1. Determine what quantities you need to know.
2. Identify the server
3. Identify the queued items
4. Identify the queuing model
5. Determine the service time
6. Determine the arrival rate
7. Calculate ρ
8. Calculate the desired values

- **arrival process:**
  - how customers arrive e.g. singly or in groups (batch or bulk arrivals)
  - how the arrivals are distributed in time (e.g. what is the probability distribution of time between successive arrivals (the ***interarrival time distribution***))
  - whether there is a finite population of customers or (effectively) an infinite number

    The simplest arrival process is one where we have completely regular arrivals (i.e. the same constant time interval between successive arrivals). A Poisson stream of arrivals corresponds to arrivals at random. In a Poisson stream successive customers arrive after intervals which independently are exponentially distributed.

The Poisson stream is important as it is a convenient mathematical model of many real life queuing systems and is described by a single parameter - the average arrival rate. Other important arrival processes are scheduled arrivals; batch arrivals; and time dependent arrival rates (i.e. the arrival rate varies according to the time of day).

- **service mechanism:**
    - a description of the resources needed for service to begin
    - how long the service will take (the ***service time distribution***)
    - the number of servers available
    - whether the servers are in series (each server has a separate queue) or in parallel (one queue for all servers)
    - whether preemption is allowed (a server can stop processing a customer to deal with another "emergency" customer)

    Assuming that the service times for customers are independent and do not depend upon the arrival process is common. Another common assumption about service times is that they are exponentially distributed.
- **queue characteristics:**

    The **most common,** and apparently fair queue discipline is the FCFS rule (first come,
    first served) or FIFO (first in first out) discipline. LCFS (last come, first served) and **SIR0** (service in random order) may also arise in practical situations
    Do we have :
    - balking (customers deciding not to join the queue if it is too long)
    - reneging (customers leave the queue if they have waited too long for service)
    - jockeying (customers switch between queues if they think they will get served faster by so doing)
    - a queue of finite capacity or (effectively) of infinite capacity

# GAME THEORY

A competitive situation in business can be treated similar to a **game**. There are two or more players and each player uses a strategy to out play the opponent.
A strategy is an action plan adopted by a player in-order to counter the other player.In our of game theory we have two players namely Player A and Player B.
The basic objective would be that
Player A – plays to **Maximize profit** (offensive) - Maxi (min) criteria
Player B – plays to **Minimize losses** (defensive) - Mini (max) criteria
The Maxi (Min) criteria is that – Maximum profit out of minimum possibilities
The Mini (max) criteria is that – Minimze losses out of maximum possibilities.
Game theory helps in finding out the best course of action for a firm in view of the anticipated counter-moves from the competing organizations.

## Characteristics of a game

A competitive situation is a competitive game if the following properties hold good
1. The number of competitors is finite, say N.
2. A finite set of possible courses of action is available to each of the N competitors.
3. A play of the game results when each competitor selects a course of action from the set of courses available to him. In game theory we make an important assumption that all the players select their courses of action simultaneously. As a result,no competitor will be in a position to know the choices of his competitors.
4. The outcome of a play consists of the particular courses of action chosen by the individual players. Each outcome leads to a set of payments, one to each player, which may be either positive, or negative, or zero.

## TERMINOLOGIES

**Zero Sum game** because the Gain of A – Loss of B = 0. In other words, the gain of Player A is the Loss of Player B.

**Pure strategy** If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to minimize the gain Therefore the pure strategy is a decision rule always to select a particular course of action.

**Mixed strategy** If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus, the mixed strategy is a selection among pure strategies with fixed probabilities.

**Optimal strategy** The strategy that puts the player in the most preferred position irrespective of the strategy of his opponents is called an optimal strategy Any deviation from this strategy would reduce his payoff
**Saddle Point** : If the Maxi (min) of A = Mini (max) of B then it is known as the Saddle Point Saddle point is the number, which is lowest in its row and highest in its column.When minimax

value is equal to maximin value , the game is said to have saddle point. It is the cell in the payoff matrix which satisfies minimax to maximin value

**Value of the Game** : It is the average wining per play over a long no. of plays.It is the expected pay off when all the players adopt their optimum strategies .If the value of game is zero it is said to be a fair game , If the value of game is not zero it is said to be a unfair game .
In all problems relating to game theory, first look for saddle point, then check out for rule of dominance and see if you can reduce the matrix.

**Rule of Dominance:**
The dominance and modified dominance principles and their applications for reducing the size of a game with or without a saddle point. If every value of one strategy of A is lesser than that of the other strategy of A,Then A will play the strategy with greater values and remove the strategy with the lesser payoff values.
If every value of one strategy of B is greater than that of other strategy of B , B will play the lesser value strategy and remove the strategy with higher payoff values.

**Dominance rule for the row**
If all the elements in a particular row is lower than or equal to all the elements in another row, then the row with the lower items are said to be dominated by row with higher ones, Then the row with lower elements will be eliminated.

**Dominance rule for the column**
If all the elements in a particular column is higher than or equal to all the elements in another column, then the column with the higher items are said to be dominated by column with lower ones, Then the column with higher elements will be eliminated.

**Modified Dominance Rule**
In few cases, if the given strategy is inferior to the average of two or more pure strategies, then the inferior strategy is deleted from the pay-off matrix and the size of the matrix is reduced considerably. In other words , if a given row has lower elements than the elements of average of two rows then particular row can be eliminated. Similarly if a given column has higher elements than the elements of average of two columns then particular column can be eliminated. Average row/column cannot be eliminated under any circumstances.. This type of dominance property is known as the modified dominance property

**Graphical Method**
If one of the players, play only two strategies or if the game can be reduced such that one of the players play only two strategies. Then the game can be solved by the graphical method.
In case the pay-off matrix is of higher order (say m x n ), then we try to reduce as much as possible using dominance and modified dominance ,f we get a pay-off matrix of order 2 x n or n x 2 we try to reduce the size of the pay-off matrix to that of order 2 x 2 with the graphical method so that the value of game could be obtained
.

**Managerial Applications of the Theory of Games**

The techniques of game theory can be effectively applied to various managerial problems as detailed below:

1. Analysis of the market strategies of a business organization in the long run.
2. Evaluation of the responses of the consumers to a new product.
3. Resolving the conflict between two groups in a business organization.
4. Decision making on the techniques to increase market share.
5. Material procurement process.
6. Decision making for transportation problem.
7. Evaluation of the distribution system.
8. Evaluation of the location of the facilities.
9. Examination of new business ventures and
10. Competitive economic environment

# REPLACEMENT MODEL

If any equipment or machine is used for a long period of time, due to wear and tear, the item tends to worsen. A remedial action to bring the item or equipment to the original level is desired. Then the need for replacement becomes necessary. This may be due physical impairment, due to normal wear and tear, obsolescence etc. The resale value of the item goes on diminishing with the passage of time.
The depreciation of the original equipment is a factor, which is responsible not to favor replacement because the capital is being spread over a long time leading to a lower average cost. Thus there exists an economic trade-off between increasing and decreasing cost functions. We strike a balance between the two opposing costs with the aim of obtaining a minimum cost.

Replacement model aims at identifying the **time** at which the assets must be replaced in order to minimize the cost.

## REASONS FOR REPLACEMENT OF EQUIPMENT:

1. Physical impairment or malfunctioning of various parts refers to
   - The physical condition of the equipment itself
   - Leads to a decline in the value of service rendered by the equipment
   - Increasing operating cost of the equipment
   - Increased maintenance cost of the equipment
   - Or a combination of the above.
2. Obsolescence of the equipment, caused due to improvement in the existing tools and machinery mainly when the technology becomes advanced.
3. When there is sudden failure or breakdown.

## REPLACEMENT MODELS:

- **Assets that fails Gradually:**
  Certain assets wear and tear as they are used. The efficiency of the assets decline with time. The maintenance cost keeps increasing as the years pass by eg. Machinery, automobiles, etc.
  1. Gradual failure without taking time value of money into consideration
  2. Gradual failure taking time value of money into consideration

- **Assets which fail suddenly**
  Certain assets fail suddenly and have to be replaced from time to time eg. bulbs.
  1. Individual Replacement policy (IRP)
  2. Group Replacement policy (GRP)

### I. Gradual failure without taking time value of money into consideration

As mentioned earlier the equipments, machineries and vehicles undergo wear and tear with the passage of time. The cost of operation and the maintenance are bound to increase year by year. A stage may be reached that the maintenance cost amounts prohibitively large that it is better and economical to replace the equipment with a new one. We also take into account the salvage value of the items in assessing the appropriate or opportune time to replace the item. We assume that the details regarding the costs of operation, maintenance and the salvage value of the item are already known

**Procedure for replacement of an asset that fails gradually (without considering Time value of money):**

a) Note down the years
b) Note down the running cost ‘R’ (Running cost or operating cost or Maintenance cost or other expenses)
c) Calculate Cumulative the running cost ‘∑R’
d) Note down the capital cost ‘C’
e) Note down the scrap or resale value ‘S’
f) Calculate Depreciation = Capital Cost – Resale value
g) Find the Total Cost
   Total Cost = Cumulative Running cost + Depreciation
h) Find the average cost
   Average cost = Total cost/No. of corresponding year
i) Replacement decision: Average cost is minimum (Average cost will decrease and reach minimum, later it will increase)

| year | Running cost | Cumulative running cost | Capital cost | Salvage value or Resale value | Depn. = Capital cost – salvage value | Total cost= Cumulative running cost + Depreciation | Average annual cost $P_n$ = Total cost / no. of corresponding year |
|---|---|---|---|---|---|---|---|
| n | $R_n$ | $\sum R_n$ | C | $S_n$ | C - $S_n$ | $\sum R_n + C - S_n$ | $(\sum R_n + C - S_n)/n$ |
| 1 | 2 | 3 | 4 | 5 | 6 (4-5) | 7 (3+6) | 8 (7/1) |
| | | | | | | | |

### II. Gradual failure taking time value of money into consideration

In the previous section we did not take the interest for the money invested, the running costs and resale value. If the effect of time value of money is to be taken into account, the analysis must be based on an equivalent cost. This is done with the present value or present worth analysis.

For example, suppose the interest rate is given as 10% and Rs. 100 today would amount to Rs. 110 after a year's time. In other words the expenditure of Rs. 110 in year's time is equivalent to Rs. 100 today. Likewise one rupee a year from now is equivalent to (1.1)-1 rupees today and one-rupee in '*n*' years from now is equivalent to (1.1)-n rupees today. This quantity (1.1)-n is called the present value or present worth of one rupee spent 'n' years from now

**Procedure for replacement of an asset that fails gradually (with considering Time value of money):**

**Assumption:**

i. Maintenance cost will be calculated at the beginning of the year
ii. Resale value at the end of the year

**Procedure**:

a) Note down the years
b) Note down the running cost 'R' (Running cost or operating cost or Maintenance cost or other expenses)
c) Write the present value factor at the beginning for running cost
d) Calculate present value for Running cost
e) Calculate Cumulative the running cost '$\sum$ R'
f) Note down the capital cost 'C'
g) Note down the scrap or resale value 'S'
h) Write the present value factor at the end of the year and also calculate present value for salvage or scrap or resale value.
i) Calculate Depreciation = Capital Cost – Resale value
j) Find the Total Cost = Cumulative Running cost + Depreciation
k) Calculate annuity factor (Cumulative present value factor at the beginning)
l) Find the Average cost = Total cost / Annuity
m) Replacement decision: Average cost is minimum (Average cost will decrease and reach minimum, later it will increase)

| Year n | $R_n$ | $Pv^{n-1}$ | $R_nPv^{n-1}$ | $\sum R_nPv^{n-1}$ | C | $S_n$ | $Pv^n$ | $S_nPv^n$ | C - $S_nPv^n$ | $\sum R_nv^{n-1}$ + C -$S_nPv^n$ | $\sum PV^{n-1}$ | $W_n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4(2*3) | 5 | 6 | 7 | 8 | 9(7*8) | 10 | 11(5+10) | 12 | 13 |
| | | | | | | | | | | | | |

## ITEMS THAT FAIL COMPLETELY AND SUDDENLY

There is another type of problem where we consider the items that fail completely. The item fails such that the loss is sudden and complete. Common examples are the electric bulbs, transistors and replacement of items, which follow sudden failure mechanism.

### I. INDIVIDUAL REPLACEMENT POLICY (IRP):

Under this strategy equipments or facilities break down at various times. Each breakdown can be remedied as it occurs by replacement or repair of the faulty unit.
Examples: Vacuum tubes, transistors

**Calculation of Individual Replacement Policy (IRP):**

$$\textbf{Average life of an item} = \sum_{i-1}^{n} i * P_i$$

**Pi** denotes Probability of failure during that week
**i** denotes no. of weeks

$$\text{No. of failures} = \frac{\text{Total no. of items}}{\text{Average life of an item}}$$

Total IRP Cost = No. of failures * IRP cost

## II. GROUP REPLACEMENT

As per this strategy, an optimal group replacement period '*P*' is determined and common preventive replacement is carried out as follows.

(a) Replacement an item if it fails before the optimum period '*P*'.

(b) Replace all the items every optimum period of '*P*' irrespective of the life of individual item. Examples: Bulbs, Tubes, and Switches.

Among the three strategies that may be adopted, the third one namely the group replacement policy turns out to be economical if items are supplied cheap when purchased in bulk quantities. With this policy, all items are replaced at certain fixed intervals.

**Procedure for Group Replacement Policy (GRP):**

1. Write down the weeks
2. Write down the individual probability of failure during that week
3. Calculate No. of failures:
   $N_0$ - No. of items at the beginning
   $N_1$ - No. of failure during $1^{st}$ week ($N_0P_1$)
   $N_2$ - No. of failure during $2^{nd}$ week ($N_0P_2 + N_1P_1$)
   $N_3$ - No. of failure during $3^{rd}$ week ($N_0P_3 + N_1P_2 + N_2P_1$)
4. Calculate cumulative failures
5. Calculate IRP Cost = Cumulative no. of failures * IRP cost
6. Calculate and write down GRP Cost = Total items * GRP Cost
7. Calculate Total Cost = IRP Cost + GRP Cost
8. Calculate Average cost = Total cost / no. of corresponding year

# QUESTION BANK

## QUEUING THEORY

**M/M/1, FIFO/∞/∞:**

**SINGLE CHANNEL/INFINITE POPULATION**

Arrival Rate: Poisson
Service Rate: Exponential
No of Channels: Single
Service Discipline: FIFO
Queue Discipline: Infinite
Population: Infinite

1. Consider a self-service store with one cashier. Assume Poisson arrival and exponential service times. Suppose 9 customers arrive on an average for every 5 minutes and the cashier can service 10 in 5 minutes. Find the average number of customer in the system and average time a customer spends in the store.

2. In a public telephone booth, the arrivals are on an average 15 per hour. A call on the average takes 3 minutes. If there are just one phone (Poisson arrivals and exponential service), find the expected number of customer in the booth and the idle time of the booth.

**M/M/1, FIFO/N/∞:**

**SINGLE CHANNEL/FINITE POPULATION**

Arrival Rate: Poisson
Service Rate: Exponential
No of Channels: Single
Service Discipline: FIFO
Queue Discipline: finite
Population: Infinite

3. At a one-man barbershop, the customer arrives according to Poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. There are only 5 seats available for waiting of the customer and customer do not wait if they find no seat available. Find the average number of customer in the system, average queue length and the average time a customer spends in the barbershop. Also find the idle time of the barber.

4. Consider a single server queuing system with poisson input and exponential service times. Suppose mean arrival rate is 3 units per hour and expected service time is 0.25 hours and the maximum calling units in the system is two. Calculate expected number in the system .

5. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. the line capacity is 9 trains Calculate the following:
   a) The probability that the yard is empty
   b) Average queue length

# GAME THEORY

**PURE STRATEGIES**

1. Solve the game whose payoff matrix is given below

| | B1 | B2 | B3 |
|---|---|---|---|
| A1 | -2 | 5 | -3 |
| A2 | 1 | 3 | 5 |
| A3 | -3 | -7 | 11 |

2.

| | B1 | B2 | B3 |
|---|---|---|---|
| A1 | 0 | -4 | -2 |
| A2 | 3 | -5 | 1 |
| A3 | -2 | -1 | 6 |
| A4 | 1 | 0 | 4 |

**MIXED STRATEGIES**

3. Solve the game whose payoff matrix is given below

| | B1 | B2 |
|---|---|---|
| A1 | 1 | 7 |
| A2 | 5 | 1 |

4.

| | B1 | B2 |
|---|---|---|
| A1 | 6 | -3 |
| A2 | -3 | 7 |

**MIXED STRATEGIES (DOMINANCE PRINCIPLE)/MODIFIED DOMINANCE**

5.

| | B1 | B2 | B3 |
|---|---|---|---|
| A1 | 2 | -2 | 4 |
| A2 | 6 | 1 | 12 |
| A3 | -3 | 2 | 10 |

6.

| | B1 | B2 | B3 |
|---|---|---|---|
| A1 | -3 | 7 | 4 |
| A2 | -2 | -2 | 5 |
| A3 | 3 | -2 | 5 |

7.

| | B1 | B2 | B3 | B4 | B5 | B6 |
|---|---|---|---|---|---|---|
| A1 | 4 | 2 | 0 | 2 | 1 | 1 |
| A2 | 4 | 3 | 1 | 3 | 2 | 2 |
| A3 | 4 | 3 | 7 | -5 | 1 | 2 |
| A4 | 4 | 3 | 4 | -1 | 2 | 2 |
| A5 | 4 | 3 | 3 | -2 | 2 | 2 |

8.

| | B1 | B2 | B3 | B4 |
|---|---|---|---|---|
| A1 | 3 | 2 | 4 | 0 |
| A2 | 3 | 4 | 2 | 4 |
| A3 | 4 | 2 | 4 | 0 |
| A4 | 0 | 4 | 0 | 8 |

9.

| | B1 | B2 | B3 |
|---|---|---|---|
| A1 | -1 | 2 | 8 |
| A2 | 7 | 5 | -1 |
| A3 | 6 | 0 | -12 |

10.

| | B1 | B2 | B3 |
|---|---|---|---|
| A1 | 19 | 20 | 23 |
| A2 | 10 | 5 | 9 |
| A3 | 21 | 14 | 10 |

**GRAPHICAL METHOD**

11.

| | B1 | B2 | B3 | B4 |
|---|---|---|---|---|
| A1 | 2 | 4 | -2 | 8 |
| A2 | 3 | 6 | 5 | -5 |

12.

| | B1 | B2 |
|---|---|---|
| A1 | 1 | -3 |
| A2 | 3 | 5 |
| A3 | -1 | 6 |
| A4 | 4 | 1 |
| A5 | 2 | 2 |
| A6 | -5 | 0 |

# REPLACEMENT

## REPLACEMENT OF ASSET THAT FAIL GRADUALLY (WITHOUT TIME VALUE)

1. The purchase price of an asset is ₹8000, maintenance cost and resale value are given as follows.

   | Year | M.C | R.V |
   |---|---|---|
   | 1 | 1000 | 4000 |
   | 2 | 1200 | 2000 |
   | 3 | 1700 | 1200 |
   | 4 | 2200 | 600 |
   | 5 | 2900 | 500 |
   | 6 | 3800 | 400 |

   Find out optimum year and cost for replacement.

2. Cost of machine is ₹7000, maintenance cost is given by equation. 1000x (n-1), resale value is 4000, 2000, 1200, 600,500,400 thereafter. Find out when to replace the asset.

3. Purchase cost ₹4100, scrap value 100. Installation ₹2000

   | Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
   |---|---|---|---|---|---|---|---|---|
   | M.C | 50 | 125 | 200 | 300 | 450 | 600 | 800 | 1200 |
   | OP.C | 50 | 125 | 200 | 300 | 450 | 600 | 800 | 800 |

4. There are 2 machines A and B. Machine A cost RS.45000, operating cost is Rs.1000 in first year and it increases by 10000 every year.
   Machine B cost Rs.50000, operating cost is Rs.2000 and it increases by RS.4000 every year. Prove if Machine A must be replaced by Machine B. If yes, when? Assume both machines do not have resale value.

## REPLACEMENT OF ASSET THAT FAILS GRADUALLY TAKING TIME VALUE INTO CONSIDERATION

5. Find out time of replacement if the maintenance cost is given by the equation 500(n-1). Discount rate is 15%. No resale value. The machinery cost RS. 5000.

6. A lorry cost RS. 80000, running cost and salvage value are given. Use 10% discount rate.

   | Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
   |---|---|---|---|---|---|---|---|---|
   | R.C | 6000 | 7500 | 9000 | 12000 | 15000 | 20000 | 20000 | 30000 |
   | SV | 60000 | 40000 | 30000 | 25000 | 20000 | 2000 | 2000 | 2000 |

## ASSETS THAT FAIL SUDDENLY-INDIVIDUAL & GROUP REPLACEMENT POLICY

7. Find out whether to use IRP Or GRP given the following details

| Week | Cum. Prob |
|---|---|
| 1 | 0.07 |
| 2 | 0.15 |
| 3 | 0.25 |
| 4 | 0.45 |
| 5 | 0.75 |
| 6 | 0.9 |
| 7 | 1 |

8.

| Week | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| % of failure | 10 | 25 | 50 | 80 | 100 |
| IRP cost | Rs.2 | | | | |
| GRP cost | 0.50ps. | | | | |

9.

| Week | | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| Survival rate | | 100 | 97 | 90 | 70 | 30 | 15 | 0 |
| IRP cost | Rs.1 | | | | | | | |
| GRP cost | 0.35ps | | | | | | | |
| Assume 1000 | bulbs. | | | | | | | |

*Hint:* *When cumulative probability is given, convert it in to individual probability*
*When probability of failure is given in percentage, convert it into decimals*
*In case survival rate is given, calculate failure rate which is equal to 1- survival rate*

## UNIT – 5 – QUEUING THEORY, GAME THEORY AND REPLACEMENT MODEL

## MODEL QUESTION PAPER

### PART – A

1. What do you mean by a) Parallel queues. b) Sequential queues.
2. List the characterisitics of queing model.
3. Write the objectives waiting line model.
4. What is meant Game theory?
5. What do you mean by a) pay-off matrix b) saddle point c) pure strategy d) Mixed strategy e) Maximin principle f) Minimax principle
6. What do you mean by Dominance principle?
7. Write the rules of dominance principle?
8. What is meant by replacement?
9. What are the different methods of replacement of assets?
10. What do you mean by a) Individual replacement policy b) Group replacement policy?

## PART – B

11. At a one-man barbershop, the customer arrives according to Poisson process at an average rate of 2 per hour and they are served according to exponential distribution with an average service rate of 5 minutes. There are only 4 seats available for waiting of the customer and customer do not wait if they find no seat available. Find the average number of customer in the system, average queue length and the average time a customer spends in the barbershop. Also find the idle time of the barber.

**OR**

12. Solve the Game using Dominance principle

Player B

| Player A | B1 | B2 | B3 | B4 | B5 |
|---|---|---|---|---|---|
| A1 | 2 | 4 | 3 | 8 | 4 |
| A2 | 5 | 6 | 3 | 7 | 8 |
| A3 | 3 | 7 | 9 | 8 | 7 |
| A4 | 4 | 2 | 8 | 4 | 3 |

13. Consider a bank with one cashier. Assume Poisson arrival and exponential service times. Suppose 9 customers arrive on an average for every 5 minutes and the cashier can service 10 in 5 minutes. Find the average number of customer in the system and average time a customer spends in the bank.

**OR**

14. The cost of a new machine is Rs. 3000. Discounted factor is 10%. Find the Optimum period of replacement.

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Running cost | 500 | 600 | 800 | 1000 | 1300 | 1600 | 2000 |

15. Solve the Game using Dominance principle

Player B

| Player A | B1 | B2 | B3 | B4 |
|---|---|---|---|---|
| A1 | 3 | 2 | 4 | 0 |

| | | | | |
|---|---|---|---|---|
| A2 | 3 | 4 | 2 | 4 |
| A3 | 4 | 2 | 4 | 0 |
| A4 | 0 | 4 | 0 | 4 |

**OR**

16. There are 1000 bulbs. The following failure rates have been observed for a certain items.

| End of week: | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Prob. of failure: | 0.10 | 0.30 | 0.55 | 0.85 | 1.00 |

The cost of replacing an individual item is Rs 1.25. The decision is made to replace all items simultaneously and also replace individual items as they fail. The cost of group replacement is 50 Paise. Which is better individual replacement or group replacement?

17. Solve the Game using Graphical method.

| | B1 | B2 |
|---|---|---|
| A1 | 1 | -3 |
| A2 | 3 | 5 |
| A3 | -1 | 6 |
| A4 | 4 | 1 |
| A5 | 2 | 2 |
| A6 | -5 | 0 |

**OR**

18. The following failure rates have been observed for a certain type of transistors in a digital computer.

| End of week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Failure to date | .05 | .13 | .25 | .43 | .68 | .88 | .96 | 1 |

The cost of replacing an individual failed transistor is Rs1.25. The decision is made to replace all these transistors simultaneously at fixed intervals and to replace the individual transistor as they fail in service. If the cost of group replacement is 30 paise per transistor. What is the interval between group replacements? It is preferable over individual replacement policy?

19. Purchase cost of a machie is ₹4100, scrap value ₹100 and installation charges ₹2000

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Maintenance Cost | 50 | 125 | 200 | 300 | 450 | 600 | 800 | 1200 |
| Operating Cost | 50 | 125 | 200 | 300 | 450 | 600 | 800 | 800 |

Find the optimum period of replacement.

**OR**

20. Solve the game whose payoff matrix is given below

| | B1 | B2 |
|---|---|---|
| A1 | 7 | 5 |
| A2 | 3 | 2 |