UNIT 3 MEASUREMENT OF RESISTANCE, INDUCTANCE AND CAPACITANCE

Low Resistance: Kelvin's double bridge - Medium Resistance: Voltmeter Ammeter method – Substitution method - Wheatstone bridge method. High Resistance: Megger - Direct deflection method - Megohm bridge method, Loss of Charge method - Earth resistance measurement. Introduction to A.C bridges Sources and Detectors in A.C. bridges. Measurement of Self Inductance: Maxwell's bridge - Hay's bridge, and Anderson's bridge. Measurement of Mutual Inductance: Heaviside M.I bridge - Measurement of Capacitance: Schering's bridge – De Sauty's bridge, Measurement of frequency using Wien's bridge.

CLASSIFICATION OF RESISTANCES

For the purposes of measurements, the resistances are classified into three major groups based on their numerical range of values as under:

• Low resistance (0 to 1 ohm)

• Medium resistance (1 to 100 kilo-ohm) and

• High resistance (>100 kilo-ohm)

Accordingly, the resistances can be measured by various ways, depending on their range of values, as under:

1. Low resistance (0 to 1 ohm): AV Method, Kelvin Double Bridge, potentiometer, doctor ohmmeter, etc.

2. Medium resistance (1 to 100 kilo-ohm): AV method, wheat stone's bridge, substitution method, etc.

3. High resistance (>100 kilo-ohm): AV method, Fall of potential method, Megger, loss of charge method, substitution method, bridge method, etc.

LOW RESISTANCE

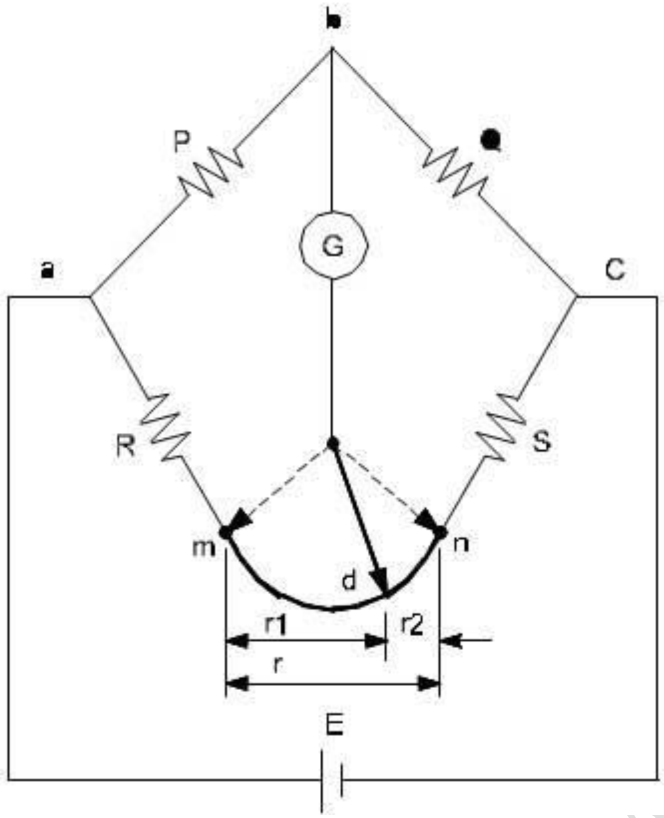
KELVIN DOUBLE BRIDGE

The Kelvin double bridge is one of the best devices available for the precise measurement of low resistances. It is the modification of wheatstone bridge by which the errors due to contact resistance and lead resistances are eliminated. This bridge is named double bridge because it contains a second set of ratio arms. An interesting variation of the Wheatstone bridge is the Kelvin Double bridge, used for measuring very low resistances (typically less than 1/10 of an ohm)

THEORY

Consider the bridge circuit shown in figure below. Here ‘r’ represents the resistance of the lead that connects the unknown resistance ‘R’ to standard resistance ‘S’. Two galvanometer connections indicated by dotted lines are possible. The connection may be either to point 'm' or to point 'n'. When the galvanometer is connected to point ‘m’ the resistance ‘ r’ of the connecting leads is added to the standard resistance ‘S’ resulting in indication of too low an indication for unknown resistance ‘R’. When the connection made to point the resistance ‘r’ is added to the unknown resistance resulting in indication of too high a value for ‘R’.

Suppose that instead of using point ‘m’ which gives a low result or ‘n’ which makes the result High, we make the galvanometer connection to any intermediate point‘d’ as shown by full line. If at point‘d’ the resistance ‘r’ is divided into two parts r1, r2 such that $r1/r2 = P/Q$



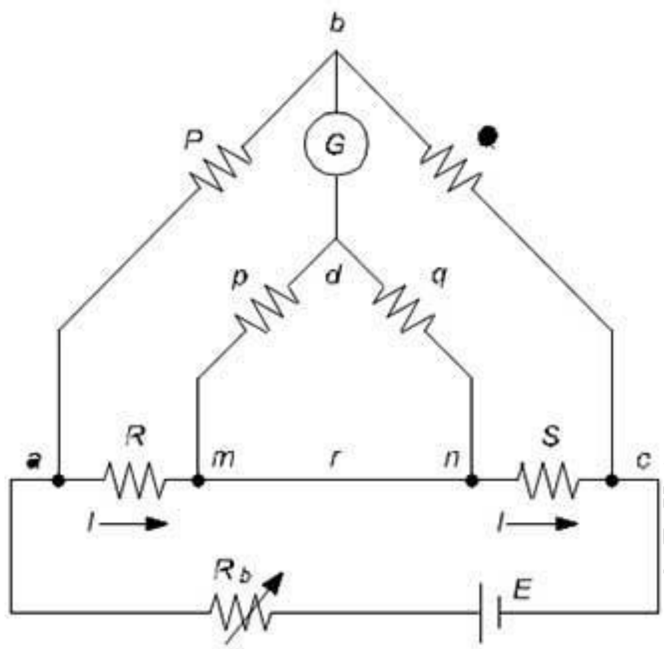
Then the presence of r the resistance of connecting leads causes no error in the result. We have,1

$$R + r_1 = \frac{P}{Q}(s + r_2) \; but \; \frac{r_1}{r_2} = \frac{P}{Q}$$

$$or \; \frac{r_1}{r_1 + r_2} = \frac{P}{P+Q} \; or \; r_1 = \frac{P}{P+Q} r \; as \; r_1 + r_2 = r \; and \; r_2 = \frac{Q}{P+Q} r$$

We can write eqn above as $\left(R + \frac{P}{P+Q}\right) = \frac{P}{Q}\left(S + \frac{Q}{P+Q}\right) \; or \; R + \frac{P}{Q} S$

Therefore we conclude that making the galvanometer connection as at C, the resistance of leads does not affect the result. The process described above is obviously not a practical way of achieving the desired result, as there would certainly be a trouble in determining the correct point for galvanometer connection. It does however suggest the simple modification that two actual resistance units of correct ratio be connected between points ‘m’ and ‘n’ the galvanometer be connected to the junction of the resistors. This is the actual Kelvin bridge arrangement which is shown in figure below.



The Kelvin double bridge incorporates the idea of a second set of ratio arms, hence the name of double bridge- and the use of four terminal resistors for the low resistance arms. Figure shows the schematic diagram of the Kelvin Bridge. The first of ratio arms is P and Q. The second set of ratio arms, p and q is used to connect the galvanometer to a point ‘d’ at the appropriate potential between points ‘m’ and ‘n’ to eliminate the effect of connecting lead of resistance ‘r’ between the known resistance ‘R’ and the standard resistance ‘S’. The ratio p /q is made equal to P/Q. Under balance conditions there is no current through the galvanometer, which means that the voltage drop between a and b, E is equal to the voltage drop Ed between a and b.

$$Now\ E_{ab}\ \frac{P}{P+Q}\ E_{ac}\ and\ E_{ac} = 1\left[R + S + \frac{(p+q)\,r}{p+q+r}\right]$$

$$and\ \ E_{amd} = 1\left[R + \frac{P}{p+q}\left\{\frac{(p + +q)\,r}{p+q+r}\right\}\right] = 1\left[R + \frac{pr}{p+q+r}\right]$$

For zero galvanometer deflection, E = E,

$$or\ \frac{P}{P+Q}\,1\left[R + S + \frac{(p+q)\,r}{p+q+r}\right] = 1\left[R + \frac{pr}{p+q+r}\right]$$

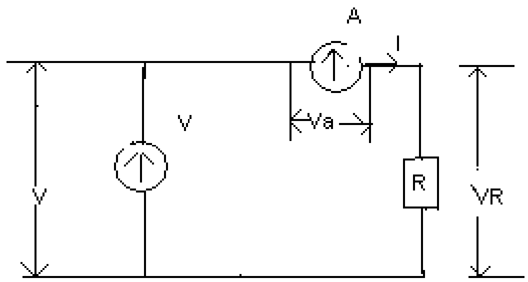
$$or\ R + \frac{P}{Q}\,S + \frac{qr}{p+q+r}\left[\frac{p}{q} - \frac{p}{q}\right]$$

Now if p/q = p/q becomes R = P / Q *S Above equation is the usual working equation for the Kelvin double bridge. It indicates that the resistance of connecting lead ‘r’ has no effect on the measurement provided that the two sets of ratio arms have equal ratios. The above equation is useful however as it shows the error that is introduced in case the ratios are not exactly equal. It is indicated that it is desirable to keep ‘r’ as small as possible in order to minimize the errors in case there is a difference between ratios P / Q and p/q. In a typical Kelvin bridge, the range of resistance calculated is 0.1S to 1.0S.

MEDIUM RESISTANCE

VOLTMETER AMMETER METHOD

In this method ammeter reads the true value of the current through the resistance but voltmeter does not measure the true voltage across the resistance. Voltmeter reads sum of voltage across ammeter and resistance Ra=resistance of ammeter Rm= measured value



Derivation: Now, Rm =V/I =(Va+VR) / I = Va / I + VR/ I =Ra +R

True value of resistance, R =Rm -Ra =Rm - Rm.Ra / Rm

=Rm (1-Ra/Rm)

Or, Rm = R +Ra

Thus measured value is higher than true value. True value is equal to measured value only if the ammeter resistance Ra is zero.

Relative error, ε r = (Rm-R) / R =Ra/R

If the value of resistance under measurement is large as compared to internal resistance of ammeter, the error in measurement would be small.

Advantage: Easy, simple, rough method.

Disadvantage:

- At full scale error may be around 0-1%.
- Errors sometimes considerably high.

SUBSTITUTION METHOD

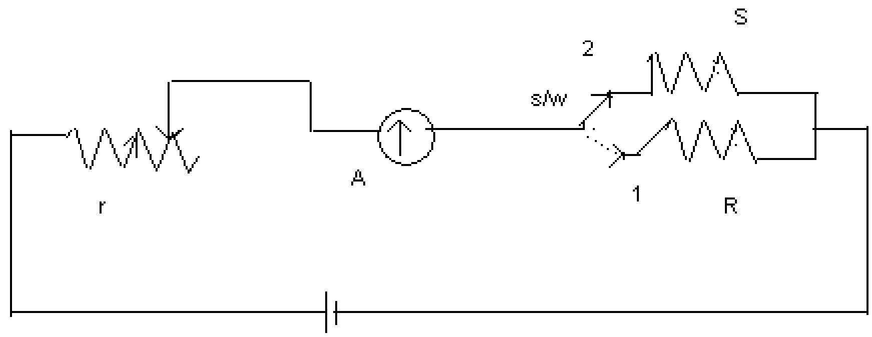
A --- Ammeter

r---regulating resistance

S---standard variable resistance

R--- Unknown resistance

S/W ---switch for putting R and S alternatively into circuit.



Procedure:

Case 1: Resistance R in the circuit .

- S/W is set to position 1.
- This brings R into the circuit.
- r is adjusted to give ammeter the chosen scale mark.

Case 2: Resistance S in the circuit.

- Change S/W to position 2.
- This brings S in to the circuit.
- Adjust S to give same chosen scale mark by ammeter.
- The substitution for one resistance by another has left current unaltered.
- The value of S gives the value of R.

Advantage:

- More accurate than ammeter voltmeter method as no error as the case of ammeter voltmeter method.
- Many applications in bridge method.
- Used in high frequency ac measurement.

Disadvantage:

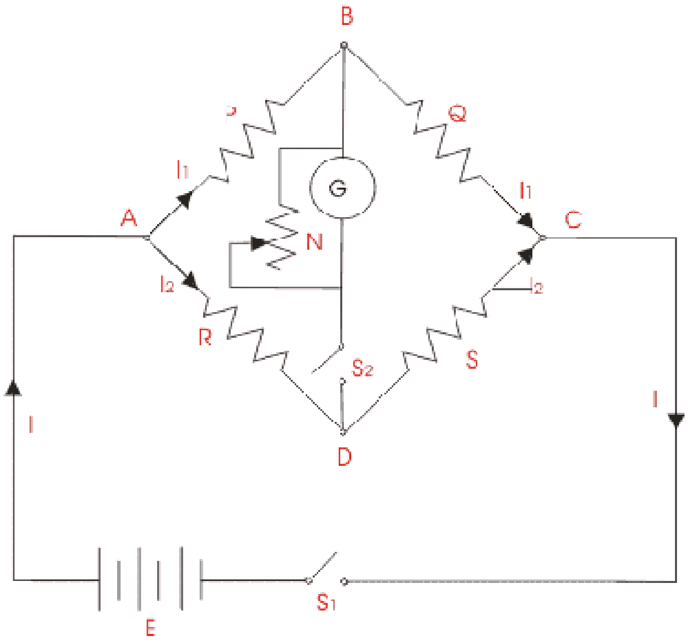
- Accuracy depends upon constancy of battery emf and resistance of circuit (excluding R & S)
- Also depends upon accuracy of measurement of S.

WHEATSTONE BRIDGE

For measuring accurately any electrical resistance Wheatstone bridge is widely used. There are two known resistors, one variable resistor and one unknown resistor connected in bridge form as shown below. By adjusting the variable resistor the current through the Galvanometer is made zero. When the current through the galvanometer becomes zero, the ratio of two known resistors is exactly equal to the ratio of adjusted value of variable resistance and the value of unknown resistance. In this way the value of unknown electrical resistance can easily be measured by using a Wheatstone Bridge.

THEORY

The general arrangement of Wheatstone bridge circuit is shown in the figure below. It is a four arms bridge circuit where arm AB, BC, CD and AD are consisting of electrical resistances P, Q, S and R respectively. Among these resistances P and Q are known fixed electrical resistances and these two arms are referred as ratio arms. An accurate and sensitive Galvanometer is connected between the terminals B and D through a switch S2. The voltage source of this Wheatstone bridge is connected to the terminals A and C via a switch S1 as shown. A variable resistor S is connected between point C and D.



Wheatstone Bridge

The potential at point D can be varied by adjusting the value of variable resistor. Suppose current I1 and current I2 are flowing through the paths ABC and ADC respectively. If we vary the electrical resistance value of arm CD the value of current I2 will also be varied as the voltage across A and C is fixed. If we continue to adjust the variable resistance one situation may comes when voltage drop across the resistor S that is I2.S is becomes exactly equal to voltage drop across resistor Q that is I1.Q. Thus the potential at point B becomes equal to the potential at point D hence potential difference between these two points is zero hence current through galvanometer is nil. Then the deflection in the galvanometer is nil when the switch S2 is closed.

Now, from **Wheatstone bridge circuit**

$$\boxed{\text{current } I_1 = \frac{V}{P + Q}}$$

And

$$\boxed{\text{current } I_2 = \frac{V}{R + S}}$$

Now potential of point B in respect of point C is nothing but the voltage drop across the resistor Q and this is

$$\boxed{I_1.Q = \frac{V.Q}{P+Q} \text{-----------------} (i)}$$

Again potential of point D in respect of point C is nothing but the voltage drop across the resistor S and this is

$$\boxed{I_2.S = \frac{V.S}{R+S} \text{-----------------} (ii)}$$

Equating, equations (i) and (ii) we get,

$$\boxed{\begin{array}{l} \frac{V.Q}{P+Q} = \frac{V.S}{R+S} \Rightarrow \frac{Q}{P+Q} = \frac{S}{R+S} \\ \Rightarrow \frac{P+Q}{Q} = \frac{R+S}{S} \Rightarrow \frac{P}{Q} + 1 = \frac{R}{S} + 1 \Rightarrow \frac{P}{Q} = \frac{R}{S} \\ \Rightarrow R = SX\frac{P}{Q} \end{array}}$$

MEASUREMENT OF HIGH RESISTANCE

There are different methods that can be employed for the measurement of high resistances. Some of the important methods are as follows.

i) Direct deflection method

ii) Loss of charge method

iii) Megohm bridge

iv) Megger

i) Direct Deflection Method

In this method, a high resistance (more than 1000 Cl) and very sensitive moving coil galvanometer is connected in series with the resistances to be measured along with supply voltage as shown in the Figure 1.

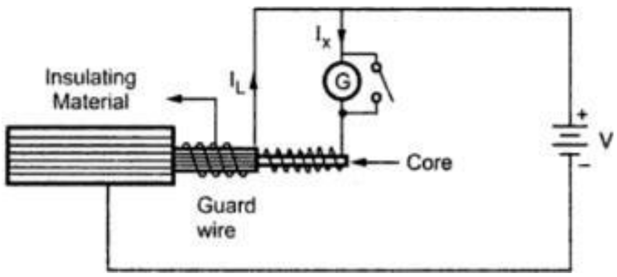


Figure.1

The direct deflection method is used for very high resistances such as insulation resistances cables. A sensitive galvanometer is used in place of micro ammeter.

The Figure 1shows this method making use of with and without Guard wire at the terminal A . The Galvanometer G measures the current Ig between the conductor and the metal sheath. The leakage current IL is actually carried out by the guard wire which ultimately do not flow through the galvanometer and thereby eliminating the source of error.

Cables without the metal sheaths can be tested in similar way but the cable is immersed in water tank first except its end where connections are made. The cable should be immersed for at least 24 hours in slightly alkaline water at a room temperature (approx..20˚) which will provide return path for the current as in Figure 2.

The readings obtained give the Volume resistance of the conductor.

The insulation resistance of the cable is given by,

$$R = V/I_R$$

In some cases, the deflection of the galvanometer is observed and its scale is afterwards calibrated by replacing the insulation by a standard high resistance (usually 1MΩ), the galvanometer shunt being varied, as required to give a deflection on the same order as before.

In tests on cable the galvanometer should be short-circuited before applying the voltage. The short-circuiting connection is removed only after sufficient time is elapsed so that charging and absorption currents cases to flow. The galvanometer should be well shunted during the early stages of measurement, and it is normally desirable to influence a protective series resistance (of several megaohm) in the galvanometer circuit. The value of this resistance should be subtracted from the observed resistance value in order to determine the true resistance. A high voltage battery of 500V emf is required and its emf should remain constant throughout the test.

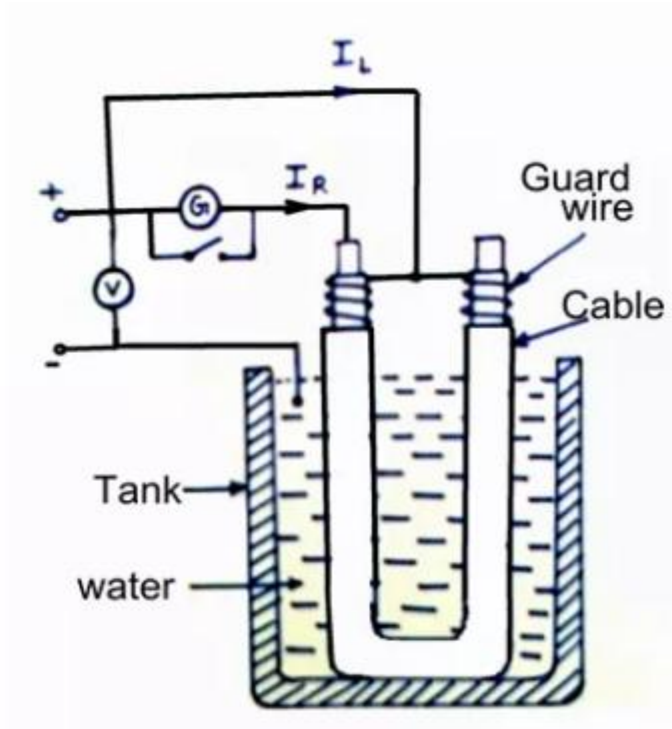


Figure.2

ii) Loss of charge method

In loss of charge method unknown resistance is connected in parallel with capacitor and electrostatic voltmeter. The capacitor is initially charged to some suitable voltage by means of a battery of voltage V and then allowed to discharge through the resistance. The terminal voltage is observed during discharge and it is given by,

V= v exp (-t/CR)

V/v = exp (-t/CR)

Or insulation resistance is given by,

R= t/(C ln V/v)

= 0.4343 t / (C log V/v)

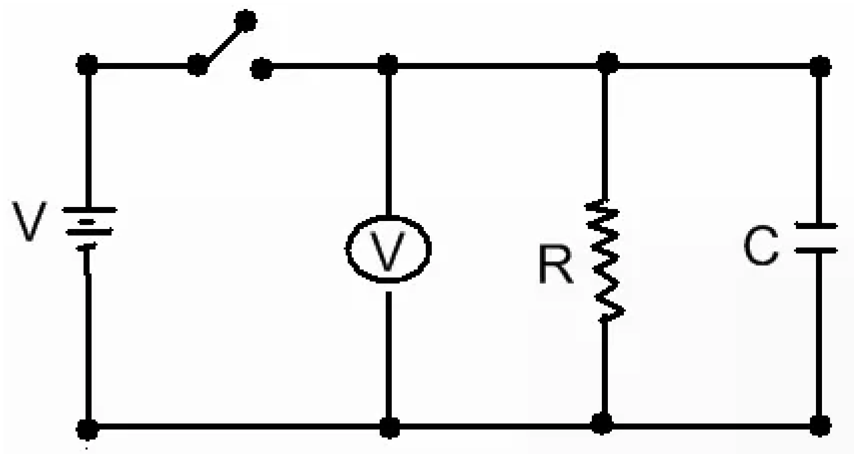


Figure.3

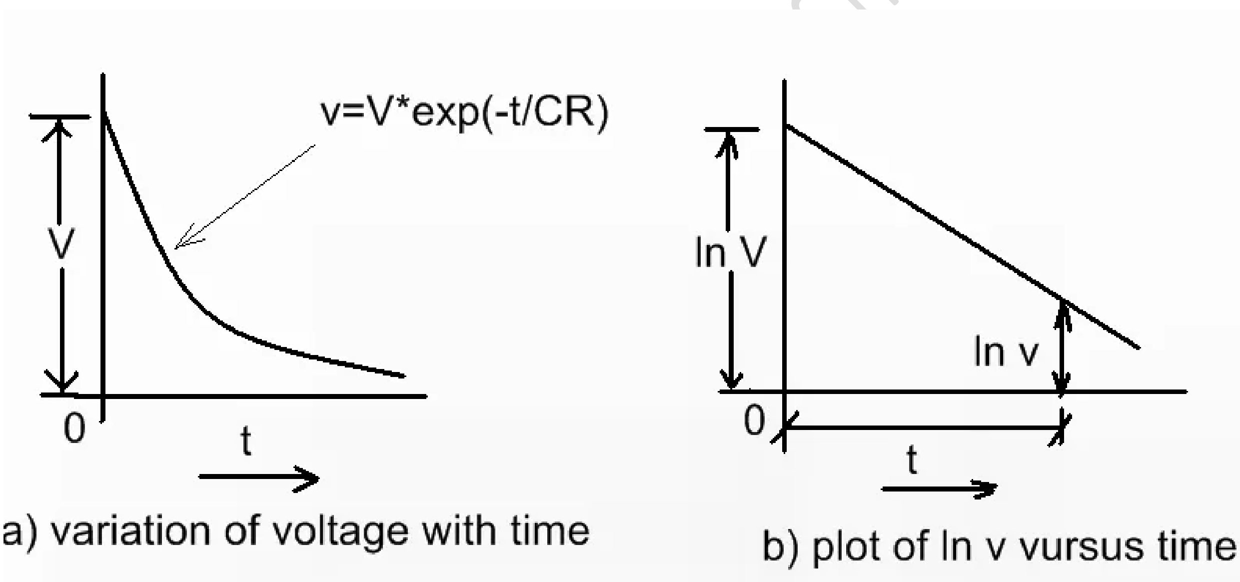


Figure.4

From above equation it follows that if V, v, C and t are known the value of R can be computed.

If the resistance R is very large the time for an appreciable fall in voltage is very large and thus this process may become time consuming. Also the voltage-time curve will thus be very flat and unless great care is taken in measuring voltages at the beginning and at the end of time t, a serious error may be made in the ratio V/v causing the considerable corresponding error in the measured value of R. more accurate results may be obtained by change in the voltage V-v directly and calling this change as e, the expression for R becomes:

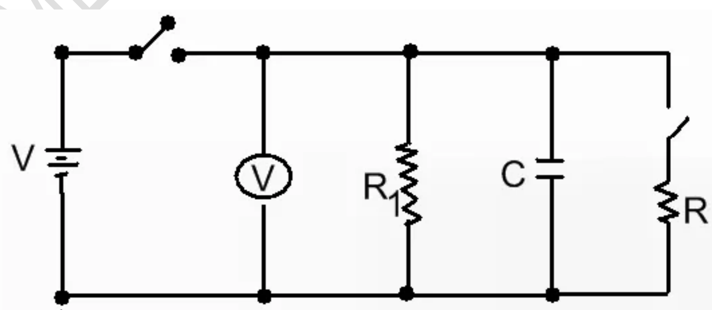
$$R = \frac{0.4343\ t}{C\ log_{10} \frac{V}{V-e}}$$

This change in voltage may be measured by a galvanometer.

However, from the experimental point of view, it may be advisable to determine the time t from the discharge curve of the capacitor by plotting curve of log v against time t. this curve is linear as shown in second figure and thus determination of time t from this curve for the voltage to fall from V to v yields more accurate results.

Loss of charge method is applicable to some high resistances, but it requires a capacitor of very high leakage resistance as high as resistance being measured. The method is very attractive if the resistance being measured is the leakage resistance of a capacitor as in this case auxiliary R and C units are not required.

Actually in this method, the true value of resistance is not measured, since it is assumed that the value of resistance of electrostatic voltmeter and the leakage resistance of the capacitor have infinite value. But in practice corrections must be applied to take into consideration the above two resistances. Let R1 be the leakage resistance of the capacitor. Also R' be the equivalent resistance of the parallel resistances R and R1.



Loss of charge method
considering effects of leakage resistance of capacitor

Figure 5

Then discharge equation of capacitor gives,

R’=0.4343 t / (C log V/v)

The test is then repeated with the unknown resistance R disconnected and the capacitor discharging through R1. The value of R1 obtained from this second test and substituted into the expression,

R’=(R R1) / (R+R1)

In order to get value of R,

The leakage resistance of the voltmeter, unless very high resistance should also be taken into consideration.

iii) Megohm bridge

Megohm bridge is another important method for measurement of high resistances. It has one three terminal high resistance located in one arm of the bridge.

Figure shows the very high resistance with terminals A and B, and a guard terminal, which is put on the insulation. So it forms a three terminal resistance.

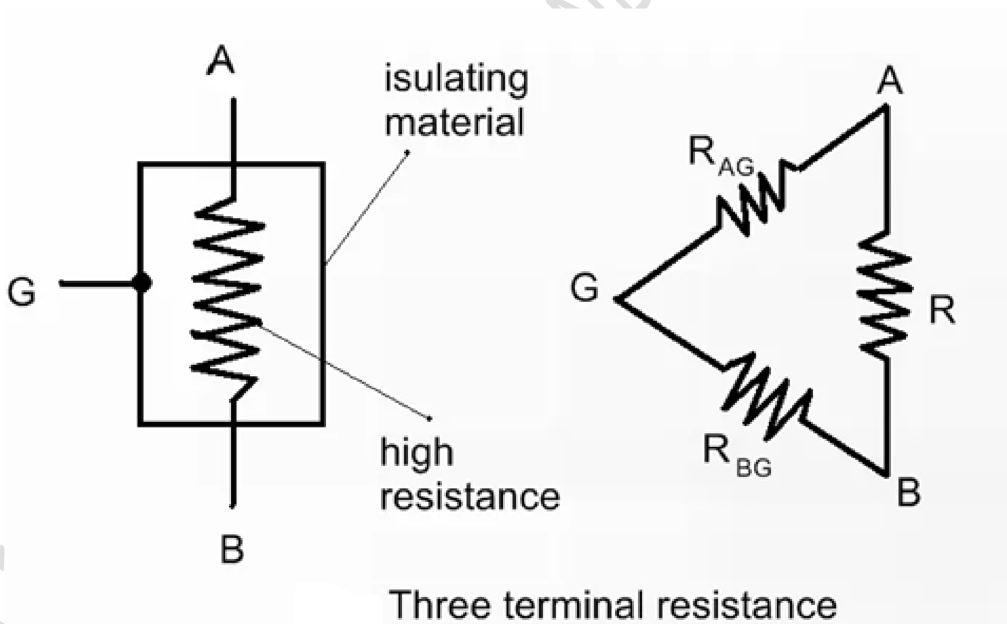


Figure 6

Let us consider take the hypothetical case of a 100 Mohm resistance and assume that this resistance is measured by an ordinary Wheatstone bridge. It is clear that Wheatstone will measure a resistance of 100*200/(100+200)=67Mohm instead of 100Mohm thus the error is 33 percent.

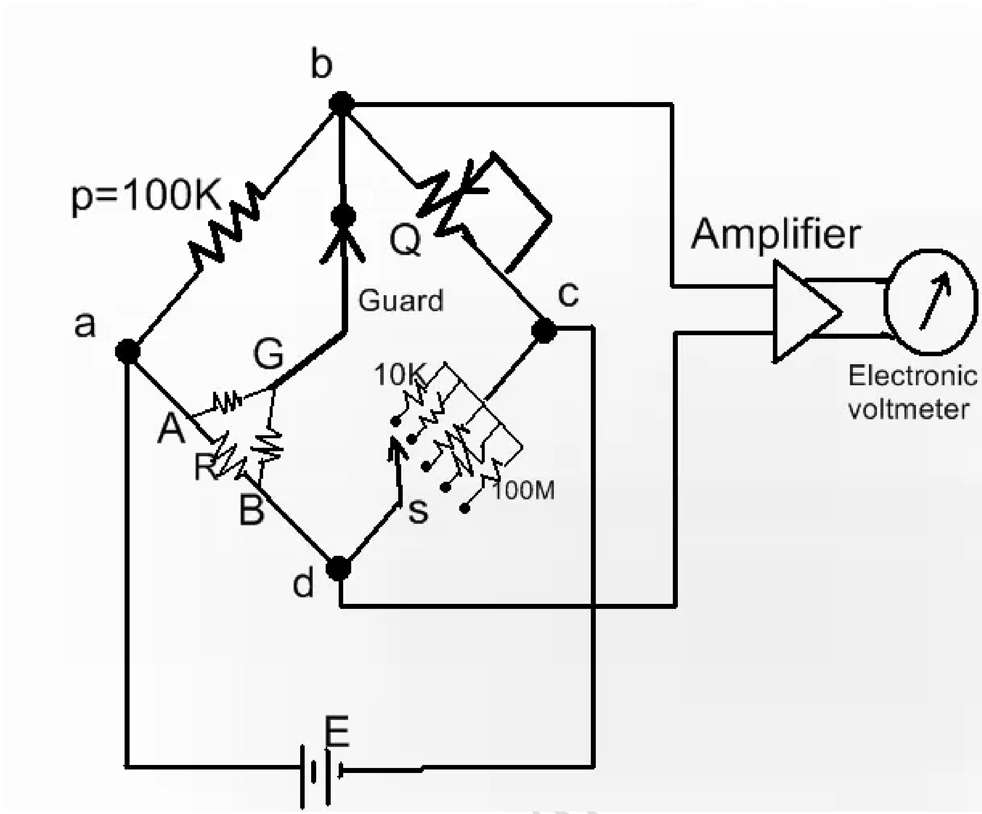


Figure 7

However if the same resistance is measured by a modified Wheatstone bridge as shown in fig b)with the guard connection G connected as indicated, the error in measurement will be redused and this modified Wheatstone bridge is called megohm bridge.

The arrangement of above figure illustrated the operation of Megohm Bridge.

Figure shows the circuit of the completely elf-contained Megohm Bridge which includes power supplies, bridge members, amplifiers, and indicating instrument. It has range from 0.1MΩ to 10^6MΩ. The accuracy is within 3% for the lower part of the range to possible 10% above 10000MΩ.

Sensitivity of balancing against high resistance is obtained by using an adjustable high voltage supplies of 500V or 1000V and the use of a sensitive null indicating arrangement such as a high gain amplifier with an electronic voltmeter or a C.R.O. The dial on Q is calibrated 1-10-100-1000 MΩ, with main decade 1-10 occupying grater part of the dial space. Since unknown resistance R=PS/Q, the arm Q is made, tapered, so that the dial calibration is approximately logarithmic in the main decade, 1-10. Arm S give five multipliers, 0.1,1,10,100 and 1000.

iv) Megger

1) **Deflecting & Control coil** : Connected parallel to the generator, mounted at right angle to each other and maintain polarities in such a way to produced torque in opposite direction.

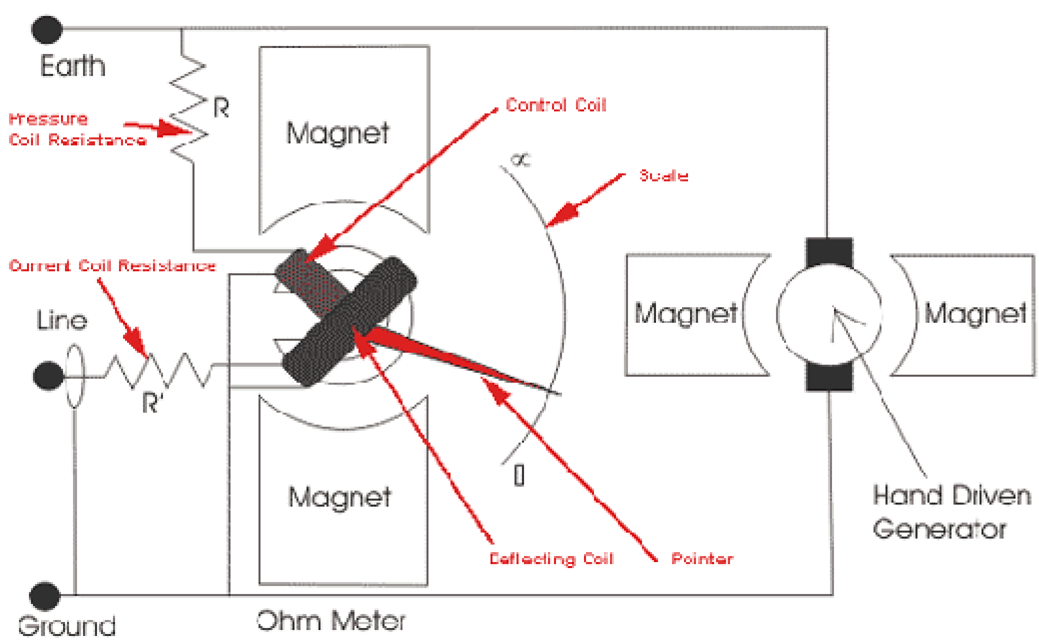
2) **Permanent Magnets**: Produce magnetic field to deflect pointer with North-South pole magnet.

3) **Pointer** : One end of the pointer connected with coil another end deflects on scale from infinity to zero.

4) **Scale** : A scale is provided in front-top of the megger from range 'zero' to 'infinity', enable us to read the value.

5) **D.C generator or Battery connection** : Testing voltage is produced by hand operated D.C generator for manual operated Megger. Battery / electronic voltage charger is provided for automatic type Megger for same purpose.

6) **Pressure coil resistance and Current coil resistance** : Protect instrument from any damage because of low external electrical resistance under test.



Working Principle of Megger

• Voltage for testing produced by hand operated Megger by rotation of crank in case of hand operated type, a battery is used for electronic tester.

• 500 Volt DC is sufficient for performing test on equipment range up to 440 Volts.

• 1000V to 5000V is used for testing for high voltage electrical systems.

• Deflecting coil or current coil connected in series and allows flowing the electric current taken by the circuit being tested.

• The control coil also known as pressure coil is connected across the circuit. Current limiting resistor (CCR & PCR) connected in series with control & deflecting coil to protect damage in case of very low resistance in external circuit.

• In hand operated megger electromagnetic induction effect is used to produce the test voltage i.e. armature arranges to move in permanent magnetic field or vice versa.

• Where as in electronic type megger battery are used to produce the testing voltage.

• As the voltage increases in external circuit the deflection of pointer increases and deflection of pointer decreases with a increases of current.

• Hence, resultant torque is directly proportional to voltage & inversely proportional to current.

• When electrical circuit being tested is open, torque due to voltage coil will be maximum & pointer shows 'infinity' means no shorting throughout the circuit and has maximum resistance within the circuit under test.

• If there is short circuit pointer shows 'zero', which means 'NO' resistance within circuit being tested.

Work philosophy based on ohm-meter or ratio-meter. The deflection torque is produced with megger tester due to the magnetic field produced by voltage & current, similarly like 'Ohm's Law' Torque of the megger varies in ration with V/I, (Ohm's Law :- V=IR or R=V/I). Electrical resistance to be measured is connected across the generator & in series with deflecting coil. Produced torque shall be in opposite direction if current supplied to the coil.

1. High resistance = No current :- No current shall flow through deflecting coil, if resistance is very high i.e. infinity position of pointer

2. Small resistance = High current :- If circuit measures small resistance allows a high electric current to pass through deflecting coil, i.e. produced torque make the pointer to set at 'ZERO'.

3. Intermediate resistance = varied current :- If measured resistance is intermediate, produced torque align or set the pointer between the range of 'ZERO to INIFINITY'

MEASUREMENT OF SELF INDUCTANCE

MAXWELL'S INDUCTANCE BRIDGE

The choke for which R1 and L1 have to measure connected between the points 'A' and 'B'. In this method the unknown inductance is measured by comparing it with the standard inductance.

L2 is adjusted, until the detector indicates zero current.

Let R1= unknown resistance

L1= unknown

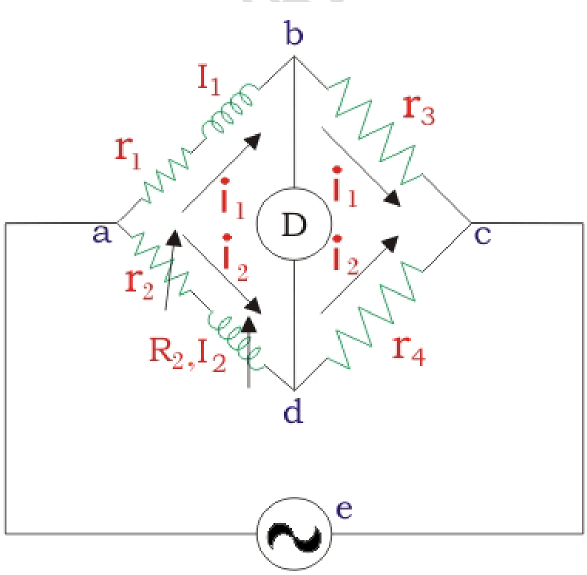
L2 is adjusted, until the detector indicates zero current.

Let R1= unknown resistance

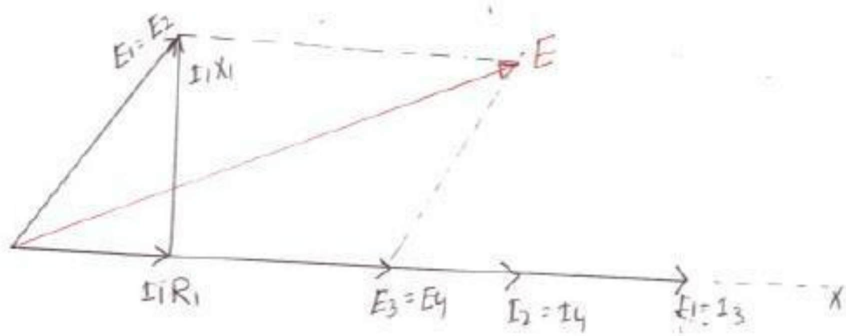
L1= unknown inductance of the choke.

L2= known standard inductance

R1,R2,R4= known resistances



Maxwell Induction Bridge



Phasor diagram of maxwell bridge

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$

$$(R_1 + jwL_1)R_4 = (R_2 + jwL_2)R_3$$

$$R_1R_4 + jwL_1R_4 = R_2R_3 + jwL_2R_3$$

Comparing real part,

$$R_1R_4 = R_2R_3$$

$$\therefore R_1 = \frac{R_2R_3}{R_4}$$

Comparing the imaginary parts,

$$wL_1R_4 = wL_2R_3$$

$$L_1 = \frac{L_2R_3}{R_4}$$

Q-factor of choke, $Q = \dfrac{WL_1}{R_1} = \dfrac{WL_2R_3R_4}{R_4R_2R_3}$

$$Q = \frac{WL_2}{R_2}$$

Advantage

Expression for R1 and L1 are simple.

Equations area simple

They do not depend on the frequency (as w is cancelled) R1 and L1 are independent of each other

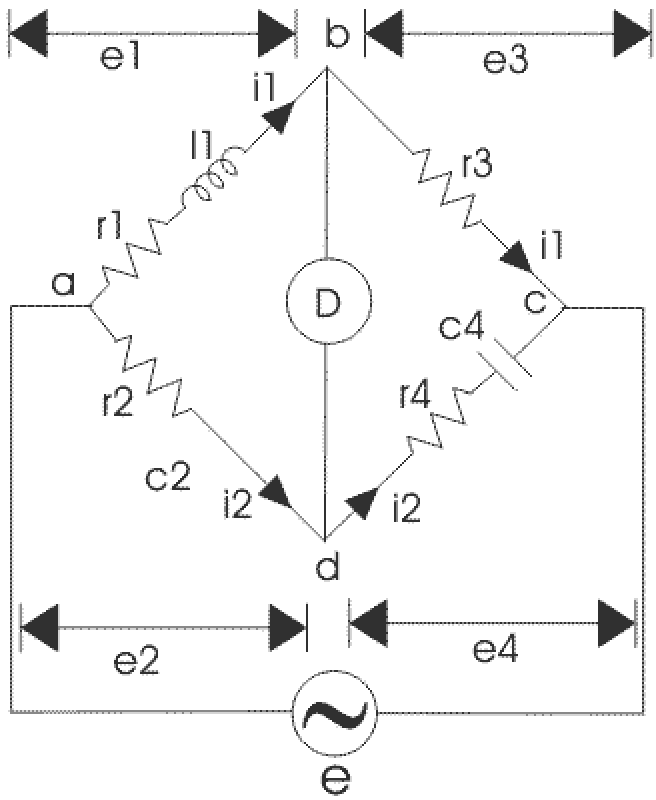
Disadvantage

Variable inductor is costly.

Variable inductor is bulky

HAY'S BRIDGE

Hay's bridge is modified Maxwell bridge, now question arises here in our mind that where we need to do modification. In order to to understand this, let us consider the connection diagram given



In this bridge the electrical resistance is connected in series with the standard capacitor. Here l1 is unknown inductor connected in series with resistance r1. c4 is standard capacitor and r2, r3, r4 are pure electrical resistance forming other arms of the bridge.

From the theory of ac bridge we can write at balance point,

$$z_1 \cdot z_4 = z_3 \cdot z_2 \cdots\cdots\cdots(1)$$

$$Here,\ z_1 = r_1 + j \cdot \omega l_1$$

$$z_2 = r_2$$

$$z_3 = r_3$$

$$z_4 = r_4 - \frac{j}{\omega c_4}$$

Substituting the values of z1, z2, z3 and z4 in equation

$$(r_1 + j\omega l_1) \cdot (r_4 - \frac{j}{\omega c_4}) = r_2 \cdot r_3$$

$$\frac{r_1 \cdot r_4 + l_1}{c_4} = r_2 \cdot r_3 \cdots\cdots\cdots(2)$$

$$and\ l_1 = \frac{r_1}{\omega^2 \cdot r_4 \cdot c_4} \cdots\cdots\cdots(3)$$

On solving equation (2) and (3), we have,

$$l_1 = \frac{r_2 \cdot r_3 \cdot c_4}{1 + \omega^2 \cdot c_4^2 \cdot r_4^2} \cdots\cdots\cdots(4)$$

$$r_1 = \frac{\omega^2 \cdot r_2 \cdot r_3 \cdot r_4 \cdot c_4^2}{1 + \omega^2 \cdot c_4^2 \cdot r_4^2} \cdots\cdots\cdots(5)$$

(1) we get,

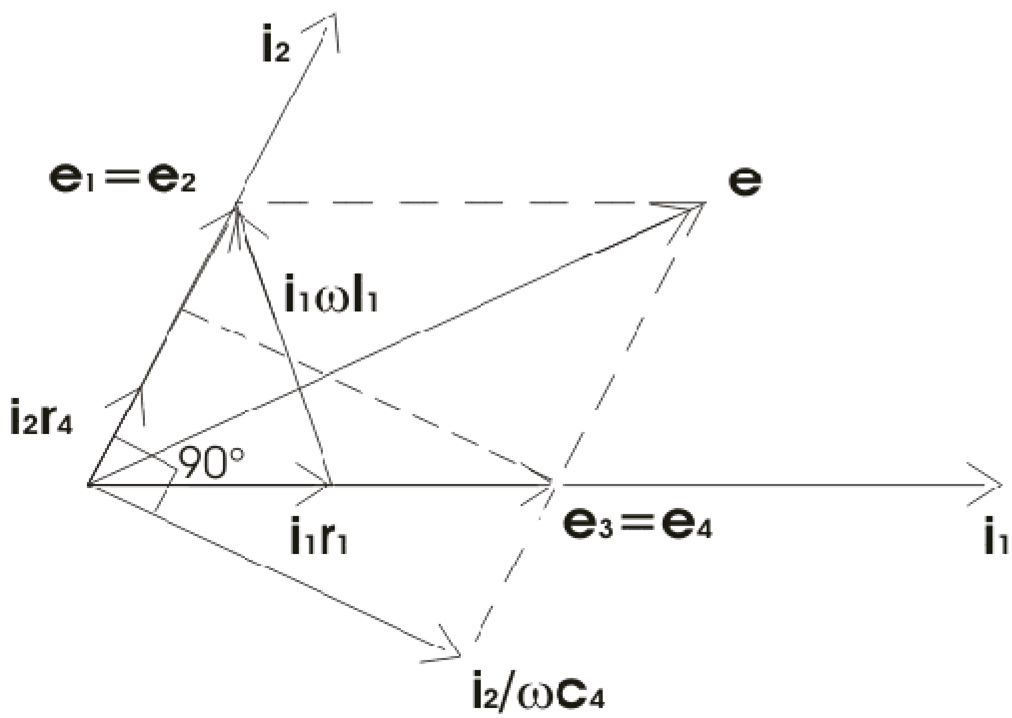
(2) Now, Q factor of a coil is given by $Q = \frac{\omega l_1}{r_1} = \frac{1}{\omega \cdot c_4 \cdot r_4}$

(3) The equations (4) and (5) are dependent on the source frequency hence, in order to find the accurate value of l1 and r1 we should know the correct value of source frequency. Let us rewrite the expression for l1,

$$l_1 = \frac{r_2 \cdot r_3 \cdot c_4}{1 + \omega^2 \cdot c_4^2 \cdot r_4^2} = \frac{r_2 \cdot r_3 \cdot c_4}{1 + \frac{1}{Q^2}}$$

(4) Now if we substitute Q >10 then 1/Q2 = 1 / 100 and hence we can neglect this value, thus neglecting 1/Q2 we get r2r3c4 which is same as we have obtained in Maxwell bridge hence Hay's bridge circuit is most suitable for high inductor measurement. Let us know more about Hay's bridge circuit diagram of Hay's bridge that will be very useful in understanding the Hay's bridge phasor diagram. A meter is connected between points b and d of the bridge. The arm ab consists

of resistance r1 and inductor, l1 (total drop across this is e1) and arm ad consists of pure resistance r2 (total drop across this is e2). The arm bc consists of pure resistance making a drop of e3 while the arm cd consists of resistor r4 and a capacitor making the total drop of e4. Now let us draw phasor diagram of Hay's bridge, at null point e1 must be equal to e2 and also e3 must be equal to e4 as the current flow through bd is zero. Let us take i1 as the reference axis and thus current i2 leads by i1 by some angle (as shown in Hay's bridge phasor diagram below) because a capacitor is connected in branch cd making current i2 lead by i1. Let us mark e1 and e2 and the resultant of e1 and e2 of course equal to e. The phase difference between the voltage drop across the electrical resistance r4 and >capacitor c4 is 90° (measured in degrees) is clearly shown in the phasor diagram of Hay's Bridge.



Phasor diagram for hays bridge

- $\dot{E}1 = I_1 R_1 + jI_1 X_1$
- $\dot{E} = \dot{E}1 + \dot{E}3$
- $\dot{E}_4 = \dot{I}_4 R_4 + \dfrac{I_4}{jwC_4}$
- $\dot{E}3 = I_3 R_3$

$$Z_4 = R_4 + \frac{1}{jwC_4} = \frac{1 + jwR_4C_4}{jwC_4}$$

Comparing imaginary part

At balance condition, $Z_1Z_4 = Z_3Z_2$

$$(R_1 + jwL_1)\left(\frac{1 + jwR_4C_4}{jwC_4}\right) = R_2R_3$$

$$(R_1 + jwL_1)(1 + jwR_4C_4) = jwR_2C_4R_3$$

$$R_1 + jwC_4R_4R_1 + jwL_1 + j^2w^2L_1C_4R_4 = jwC_4R_2R_3$$

$$(R_1 - w^2L_1C_4R_4) + j(wC_4R_4R_1 + wL_1) = jwC_4R_2R_3$$

Comparing the real term,

$$R_1 - w^2L_1C_4R_4 = 0$$

$$wC_4R_4R_1 + wL_1 = wC_4R_2R_3$$

$$C_4R_4R_1 + L_1 = C_4R_2R_3$$

$$L_1 = C_4R_2R_3 - C_4R_4R_1$$

Substituting the value of R_1

$$L_1 = C_4R_2R_3 - C_4R_4 \times w^2L_1C_4R_4$$

$$L_1 = C_4R_2R_3 - w^2L_1C_4{}^2R_4{}^2$$

$$L_1(1 + w^2L_1C_4{}^2R_4{}^2) = C_4R_2R_3$$

$$L_1 = \frac{C_4R_2R_3}{1 + w^2L_1C_4{}^2R_4{}^2}$$

Substituting the value of L_1

$$R_1 = \frac{w^2C_4{}^2R_2R_3R_4}{1 + w^2C_4{}^2R_4{}^2}$$

$$Q = \frac{wL_1}{R_1} = \frac{w \times C_4R_2R_3}{1 + w^2C_4{}^2R_4{}^2} \times \frac{1 + w^2C_4{}^2R_4{}^2}{w^2C_4{}^2R_4R_2R_3}$$

$$Q = \frac{1}{wC_4R_4}$$

Advantage

Fixed capacitor is cheaper than variable capacitor.

This bridge is best suitable for measuring high value of Q-factor

Disadvantage

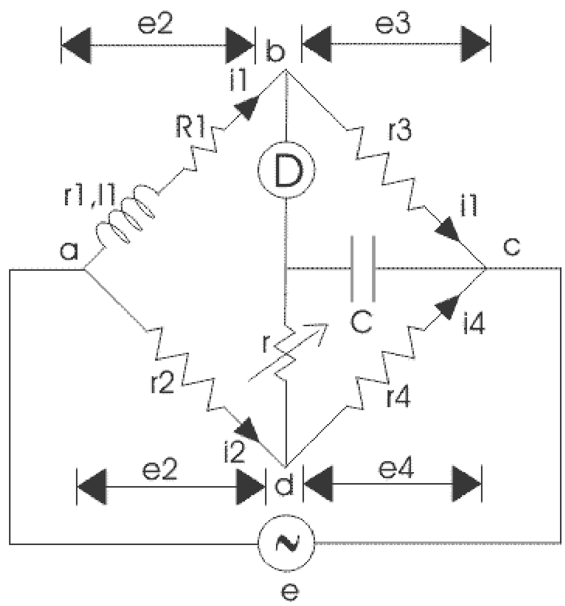
Equations of L1and R1 are complicated.

Measurement of R1 and L1 require the value of frequency

This bridge cannot be used for measuring low Q - factor

ANDERSON'S BRIDGE

Let us understand why there is need of Anderson's bridge though we have Maxwell bridge and Hay's bridge to measure quality factor of the circuit. The main disadvantage of using Hay's bridge and Maxwell bridge is that, they are unsuitable of measuring the low quality factor. However Hay's bridge and Maxwell bridge are suitable for measuring accurately high and medium quality factor respectively. So, there is need of bridge which can measure low quality factor and this bridge is modified Maxwell's bridge and known as Anderson's bridge. Actually this bridge is the modified Maxwell inductor capacitance bridge. In this bridge double balance can obtained by fixing the value of capacitance and changing the value of electrical resistance only. It is well known for its accuracy of measuring inductor from few micro Henry to several Henry. The unknown value of self inductor is measured by method of comparison of known value of electrical resistance and capacitance. Let us consider the actual circuit diagram of Anderson's



bridge

In this circuit the unknown inductor is connected between the point a and b with electrical resistance r1 (which is pure resistive). The arms bc, cd and da consist of resistances r3, r4 and r2 respectively which are purely resistive. A standard capacitor is connected in series with variable electrical resistance r and this combination is connected in parallel with cd. A supply is connected between b and e. Now let us derive the expression for l1 and r1: At balance point, we have the following relations that holds good and they are: i1 = i3 and i2 = ic + i4

Now equating voltages drops we get,

$$i_1 \cdot r_3 = \frac{i_c}{j\omega C} \cdots\cdots\cdots (1)$$

$$i_1 \cdot (R_1 + r_1 + j \cdot \omega \cdot L_1) = i_2 \cdot r_2 + i_c \cdot r \cdots\cdots\cdots (2)$$

$$i_c\left(r + \frac{1}{j \cdot \omega C}\right) = (i_2 - i_c) \cdot r_4 \cdots\cdots\cdots (3)$$

\

Putting the value of ic in above equations, we get

$$i_1 \cdot (r_1 + R_1 + j \cdot \omega \cdot l_1) = i_2 \cdot r_2 + j \cdot i_1 \cdot \omega \cdot C \cdot r_3 \cdot r$$

$$\Rightarrow i_1 \cdot (r + r_1 + j \cdot \omega \cdot l_1 - j\omega C \cdot r_3 \cdot r) = i_2 \cdot r_2 \cdots\cdots\cdots (4)$$

$$\text{or we have } i_1 \cdot (j \cdot \omega \cdot C \cdot r_3 \cdot r + j \cdot \omega \cdot C \cdot r_3 \cdot r_4 + r_3) = i_2 \cdot r_4 \cdot r_2 \cdots\cdots\cdots (5)$$

On equating (4) and (5) and separating the real and imaginary parts we have,

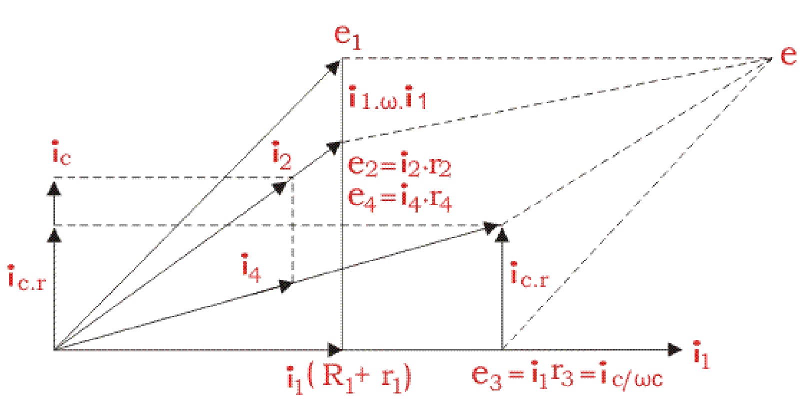
$$r_1 = \frac{r_2 \cdot r_3}{r_4} - R_1 \cdot r_2 \cdots\cdots\cdots (6)$$

$$\text{and } l_1 = \frac{C \cdot r_3}{r_4}(r \cdot (r_4 + r_2) + r_2 \cdot r_4) \cdot r_2 \cdots\cdots\cdots (7)$$

The above equation (7) obtained is more complex that we have obtained in Maxwell bridge. On observing the above equations we can easily say that to obtain convergence of balance more easily, one should make alternate adjustments of r1 and r in Anderson's bridge. Now let us look how we can obtain the value of unknown inductor experimentally. At first set the signal generator frequency at audible range. Now adjust r1 and r such that phones gives a minimum sound. Measure the values of r1 and r (obtained after these adjustments) with the help of multimeter. Use the formula that we have derived above in order to find out the value of unknown inductance. The experiment can be repeated with the different value of standard capacitor.

Phasor Diagram of Anderson's Bridge

Let us mark the voltage drops across ab, bc, cd and ad as e1, e2, e3 and e4 as shown



Phasor Diagram for Andersons

in figure above.

Here in the phasor diagram of Anderson's bridge, we have taken i1 as reference axis. Now ic is perpendicular to i1 as capacitive load is connected at ec, i4 and i2 are lead by some angle as shown in figure. Now the sum of all the resultant voltage drops i.e. e1, e2, e3 and e4 is equal to e, which is shown in phasor diagram. As shown in the phasor diagram of Anderson's bridge the resultant of voltages drop i1

(R1 + r1) and i1.ω.l1 (which is shown perpendicular to i1) is e1. e2 is given by i2.r2 which makes angle 'A' with the reference axis. Similarly, e4 can be obtained by voltage drop i4.r4 which is making angle 'B' with reference axis.

Advantages of Anderson's Bridge

1. It is very easy to obtain the balance point in Anderson's bridge as compared to Maxwell bridge in case of low quality factor coils.
2. There is no need of variable standard capacitor is required instead of thin a fixed value capacitor is used.
3. This bridge also gives accurate result for determination of capacitance in terms of inductance.

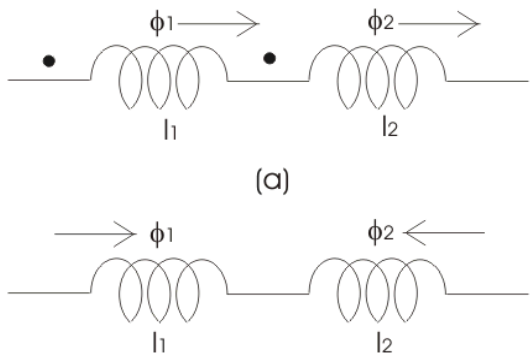
Disadvantages of Anderson's Bridge

1. The equations obtained for inductor in this bridge is more complex as complex as compared to Maxwell's bridge.
2. The addition of capacitor junction increases complexity as well as difficulty of shielding the bridge.

MEASUREMENT OF MUTUAL INDUCTANCE

HEAVISIDE BRIDGE

Let us consider two coils connected in series as shown in figure given below.



Such that the magnetic fields are additive, the resultant inductor of these two can be calculated as

$L_x = L_1 + L_2 + 2M \cdots\cdots\cdots\cdots(1)$

Where, L_1 is the self inductor of first coil, L_2 is the self inductor of second coil, M is the mutual inductor of these two coils. Now if the connections of any one of the coils is reversed then we have

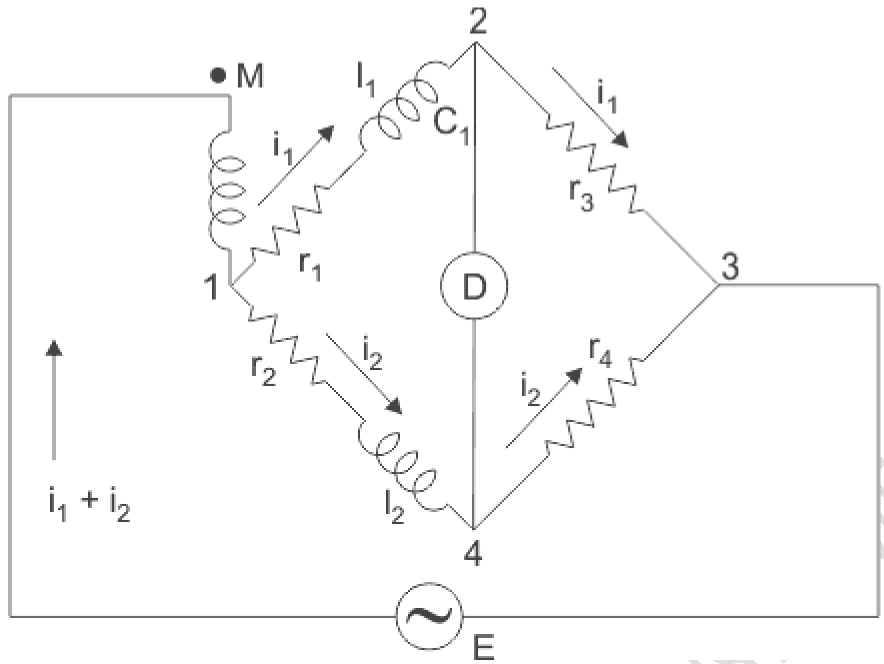
$L_y = L_1 + L_2 - 2M \cdots\cdots\cdots\cdots(2)$

On solving these two equations we have

$$M = \frac{L_x - L_y}{4}$$

Thus the mutual inductor of the two coils connected in series is given by one-fourth of the difference between the measured value of self inductor when taking the direction of field in the same direction and value of self inductor when the direction of field is reversed.

However, one needs to have the two series coils on the same axis in order to get most accurate result. Let us consider the circuit of **Heaviside mutual inductor bridge**, given below



Main application of this bridge in industries is to measure the mutual inductor in terms of self inductance. Circuit of this bridge consists of four non inductive resistors r_1, r_2, r_3 and r_4 connected on arms 1-2, 2-3, 3-4 and 4-1 respectively. In series of this bridge circuit an unknown mutual inductor is connected. A voltage is applied to across terminals 1 and 3. At balance point electric current flows through 2-4 is zero hence the voltage drop across 2-3 is equal to voltage drop across 4-3. So by equating the voltage drops of 2-4 and 4-3 we have,

$$i_1 r_3 \; = \; i_2 r_4$$

Also we have,

$$(i_1 + i_2)(j\omega M) \; + \; i_1(r_1 + r_3 + j\omega l_1) \; = \; i_2(r_2 + r_4 + j\omega l_2)$$

$$Therefore, \; i_2 \frac{r_4}{r_3 + 1} j\omega M \; + \; i_2 \frac{r_2}{r_3}(r_1 + r_3 + j\omega l_1) \; = \; i_2(r_2 + r_4 + j\omega l_2)$$

$$or \; j\omega M \left(\frac{r_4}{r_3 + 1} \; + 1 \right) \; + \; \frac{r_4}{r_3} r_1 \; + \; r_4 \; + \; j\omega l_1 \frac{r_4}{r_3} \; = \; r_2 + r_4 + j\omega l_2$$

$$Thus, \; r_1 \; = \; r_2 \frac{r_3}{r_4}$$

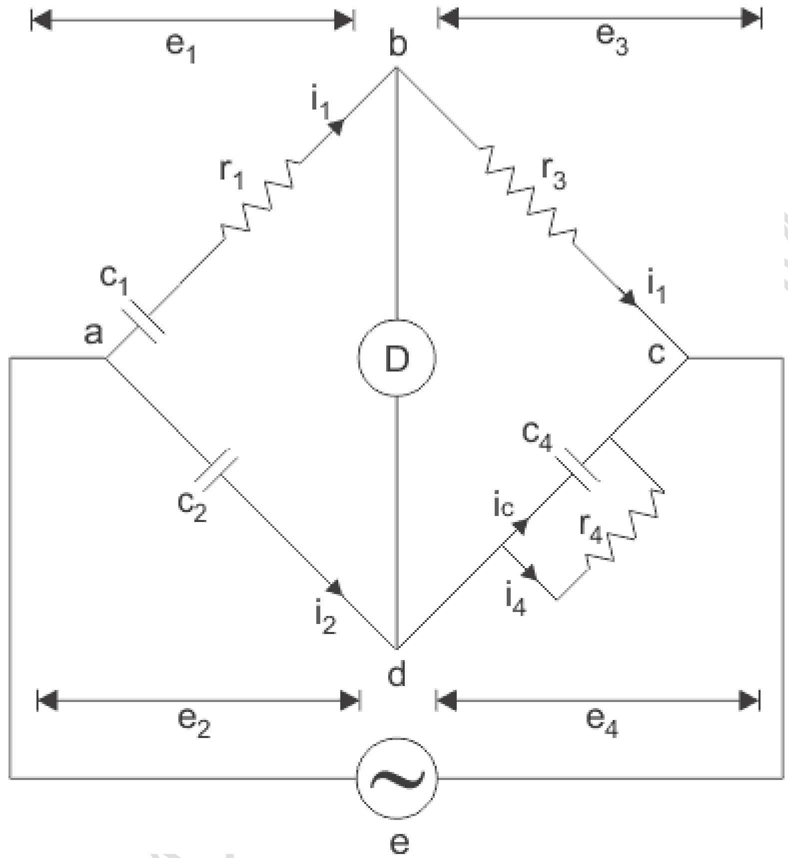
and mutual inductor is given by,

$$\frac{r_3 l_2 - r_4 l_1}{r_3 + r_4}$$

MEASUREMENT OF CAPACITANCE

SCHERING BRIDGE

This bridge is used to measure to the capacitance of the capacitor, dissipation factor and measurement of relative permittivity. Let us consider the circuit of Schering bridge



as shown

Here, c_1 is the unknown capacitance whose value is to be determined with series electrical resistance r_1. c_2 is a standard capacitor. c_4 is a variable capacitor. r_3 is a pure resistor (i.e. non inductive in nature). And r_4 is a variable non inductive resistor connected in parallel with variable capacitor c_4. Now the supply is given to the bridge between the points a and c. The detector is connected between b and d. From the theory of ac bridges we have at balance condition,

Substituting the values of z_1, z_2, z_3 and z_4 in the above equation, we get

$$\left(r_1 + \frac{1}{j\omega c_1}\right)\left(\frac{r_4}{1 + j\omega c_4 r_4}\right) = \frac{r_3}{j\omega c_2}$$

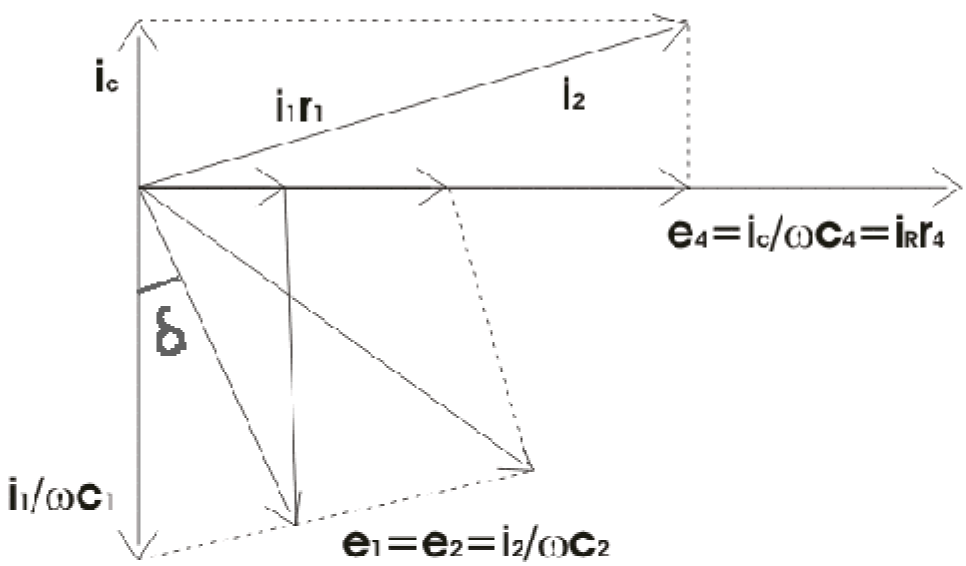
$$(r_1 + \frac{1}{j\omega c_1})r_4 = \frac{r_3}{j\omega c_2}(1 + j\omega c_4 r_4)$$

$$r_1 r_4 - \frac{j r_4}{\omega c_1} = -\frac{j r_3}{\omega c_2} + \frac{r_3 r_4 c_4}{c_2}$$

Equating the real and imaginary parts and the separating we get,

$$r_1 = \frac{r_3 c_4}{c_2}$$

$$c_1 = c_2 \frac{r_4}{r_3}$$

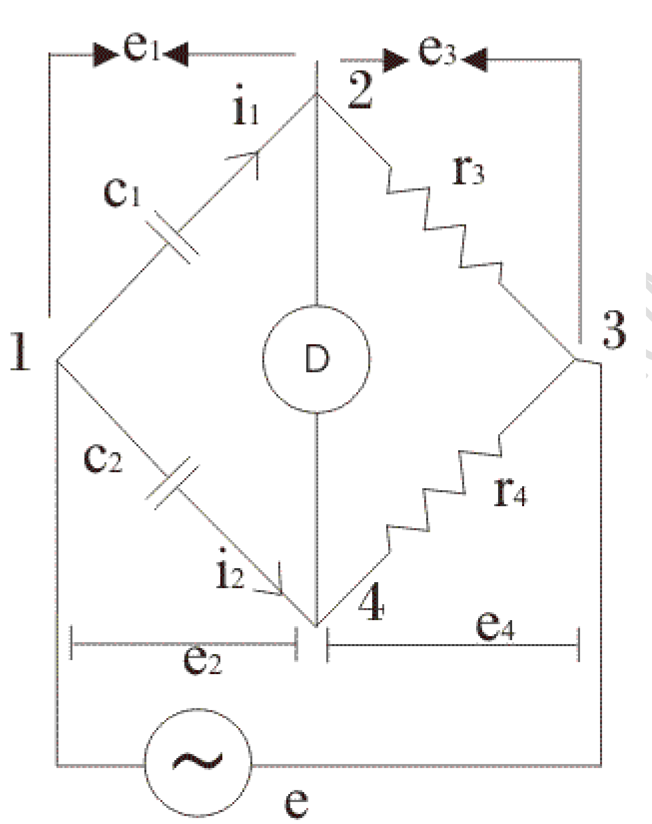


Let us consider the phasor diagram of the above Shering bridge circuit and mark the voltage drops across ab,bc,cd and ad as e_1, e_3,e_4 and e_2 respectively. From the above Schering bridge phasor diagram, we can calculate the value of tanδ which is also called the dissipation factor.

$$tan\delta = \omega c_1 r_1 = \omega \frac{c_2 r_4}{r3} \times \frac{r_3 c_4}{c_2} = \omega c_4 r_4$$

DE SAUTY'S BRIDGE

This bridge provide us the most suitable method for comparing the two values of capacitor if we neglect dielectric losses in the bridge circuit. The circuit of De Sauty's bridge is shown.



Battery is applied between terminals marked as 1 and 4. The arm 1-2 consists of capacitor c_1 (whose value is unknown) which carries current i_1 as shown, arm 2 - 4 consists of pure resistor (here pure resistor means we assuming it non inductive in nature), arm 3 - 4 also consists of pure resistor and arm 4 - 1 consists of standard capacitor whose value is already known to us. Let us derive the expression for capacitor c_1 in terms of standard capacitor and resistors.

At balance condition we have,

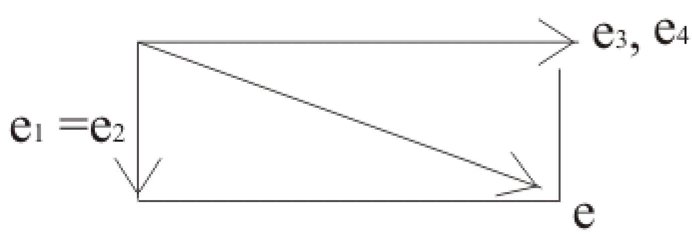
$$\frac{1}{j\omega c_1} \times r_4 = \frac{1}{j\omega c_2} \times r_3$$

It implies that the value of capacitor is given by the expression

$$c_1 = c_2 \times \frac{r_4}{r_3}$$

In order to obtain the balance point we must adjust the values of either r_3 or r_4 without disturbing any other element of the bridge. This is the most efficient method of comparing the two values of capacitor if all the dielectric losses are neglected from the circuit.

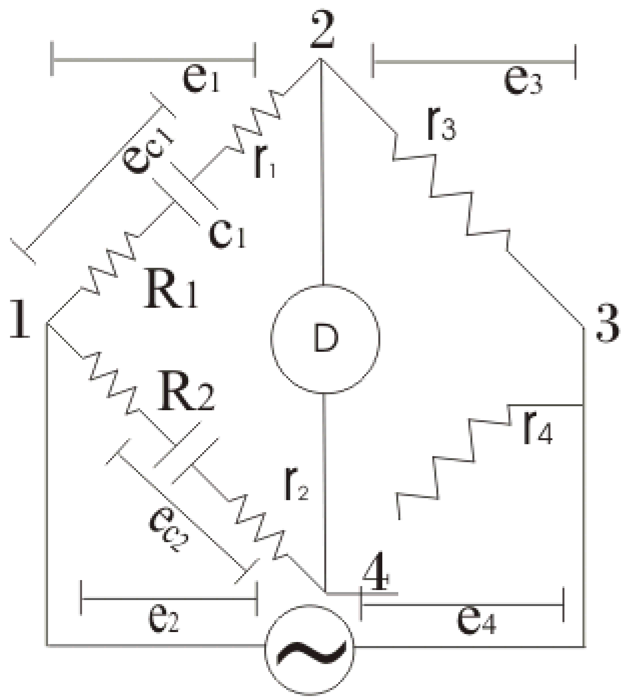
Phasor diagram of De Sauty bridge is shown below:



Phasor diagram

Now instead of some advantages like bridge is quite simple and provides easy calculations, there are some disadvantages of this bridge because this bridge give inaccurate results for imperfect capacitor (here imperfect means capacitors which not free from dielectric losses). Hence we can use this bridge only for comparing perfect capacitors. Here we interested in modify the De Sauty's bridge, we want to have such a kind of bridge that will gives us accurate results for imperfect capacitors also.

This modification is done by Grover. The modified circuit diagram is shown below:



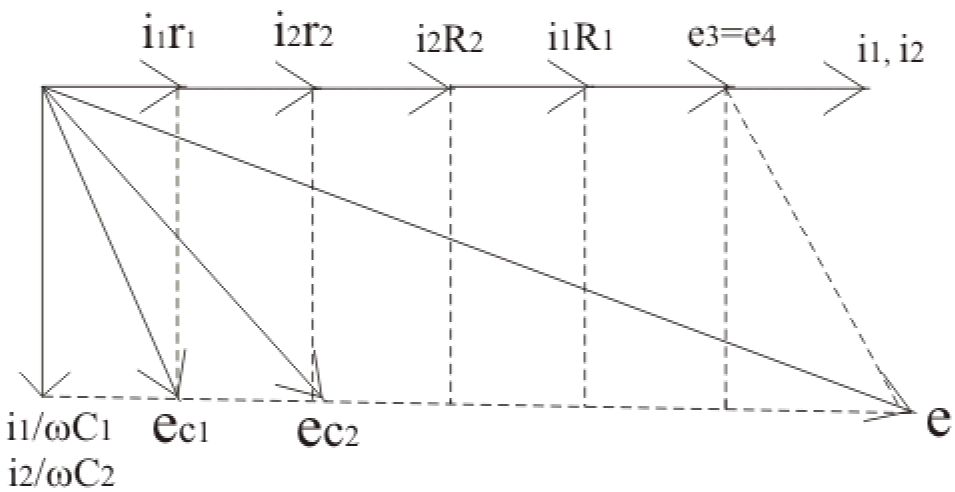
Here Grover has introduced electrical resistances r_1 and r_2 as shown in above on arms 1 - 2 and 4 - 1 respectively, in order to include the dielectric losses. Also he has connected resistances R_1 and R_2 respectively in the arms 1 - 2 and 4 - 1. Let us derive the expression capacitor c_1 whose value is unknown to us. Again we connected standard capacitor on the same arm 1 - 4 as we have done in De Sauty's bridge. At balance point on equating the voltage drops we have:

$$\left(R_1 + r_1 + \frac{1}{j\omega c_1}\right) r_4 \; = \; \left(R_2 + r_2 + \frac{1}{j\omega c_2}\right) r_3 \cdots\cdots\cdots (1)$$

On solving above equation we get:

$$\frac{c_1}{c_2} \; = \; \frac{R_2 + r_2}{R_1 + r_1} \; = \; r_4 r_3$$

This the required equation. By making the phasor diagram we can calculate dissipation factor. Phasor diagram for the above circuit is shown below



Let us mark δ_1 and δ_2 be phase angles of the capacitors c_1 and c_2 capacitors respectively. From the phasor diagram we have $\tan(\delta_1)$ = dissipation factor = $\omega c_1 r_1$ and similarly we have $\tan(\delta_2) = \omega c_2 r_2$. From equation (1) we have

$$c_2 r_2 - c_1 r_1 = c_1 R_1 - c_2 R_2$$

on multiplying ω both sides we have

$$\omega c_2 r_2 - \omega c_1 r_1 = \omega(c_1 R_1 - c_2 R_2)$$

$$But\ \frac{c_1}{c_2} = \frac{r_4}{r_3}$$

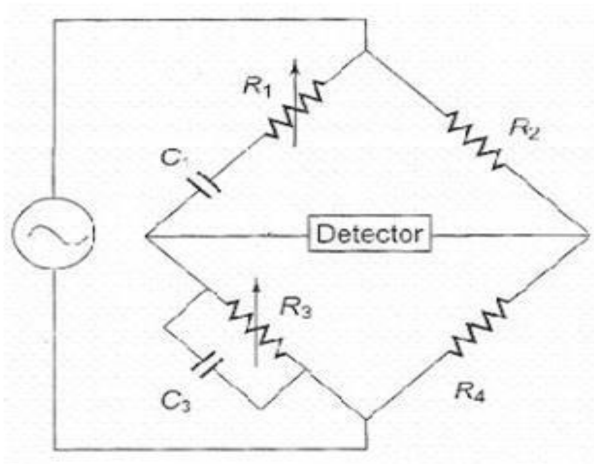
Therefore the final expression for the dissipation factor is written as

$$tan(\delta_1) - tan(\delta_2) = \omega c_2 \left(R_1 \frac{r_4}{r_3} - R_2\right)$$

Hence if dissipation factor for one capacitor is known. However this method is gives quite inaccurate results for dissipation factor.

WIEN'S BRIDGE

Circuit and derives the expression for the unknown element at balance, Wien Bridge has a series RC combination in one and a parallel combination in the adjoining arm. Wien's bridge is shown in fig. Its basic form is designed to measure frequency. It can also be used for the instrument of an unknown capacitor with great accuracy, The impedance of one arm is



$$Z_1 = R_1 - j/\omega C_1$$

The admittance of the parallel arm is

$$Y_3 = 1/R_3 + j\,\omega\,C_3$$

Using the bridge balance equation, we have

$$Z_1 Z_4 = Z_2 Z_3$$

Therefore $Z_1 Z_4 = Z_2/Y_3$, i.e. $Z_2 = Z_1 Z_4 Y_3$

$$R_2 = R_4 \left(R_1 - \frac{j}{\omega C_1}\right)\left(\frac{1}{R_3} + j\,\omega C_3\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} - \frac{j R_4}{\omega C_1 R_3} + j\,\omega\,C_3\,R_1\,R_4 + \frac{C_3 R_4}{C_1}$$

Equating the real and imaginary terms we have as,

$$R_2 = \left(\frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1}\right) - j\left(\frac{R_4}{\omega C_1 R_3} - \omega\,C_3\,R_1\,R_4\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} \quad \text{and} \quad \frac{R_4}{\omega C_1 R_3} - \omega\,C_3\,R_1\,R_4 = 0$$

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$$

$$\frac{1}{\omega C_1 R_3} = \omega C_3 R_1$$

$$\omega^2 = \frac{1}{C_1 R_1 R_3 C_3}$$

$$\omega = \frac{1}{\sqrt{C_1 R_1 C_3 R_3}}$$

$$\omega = 2 \pi f$$

$$f = \frac{1}{2\pi \sqrt{C_1 R_1 C_3 R_3}}$$

The bridge is used for measuring frequency in the audio range. Resistances R1 and R3 can be ganged together to have identical values. Capacitors C1 and C3 are normally of fixed values. The audio range is normally divided into 20 - 200 - 2 k - 20 kHz range In this case, the resistances can be used for range changing and capacitors, and C3 for fine frequency control within the range. The bridge can also be use for measuring capacitance. In that case, the frequency of operation must be known.

The bridge is also used in a harmonic distortion analyzer, as a Notch filter, an in audio frequency and radio frequency oscillators as a frequency determine element.

An accuracy of 0.5% - 1% can be readily obtained using this bridge. Because it is frequency sensitive, it is difficult to balance unless the waveform of the applied voltage is purely sinusoidal.