FACULTY OF ELECTRICAL AND ELECTRONICS
SUBJECT: SIGNALS AND SYSTEMS **SUB CODE: SEC1208**

UNIT I CLASSIFICATION OF SIGNALS

10 hrs.

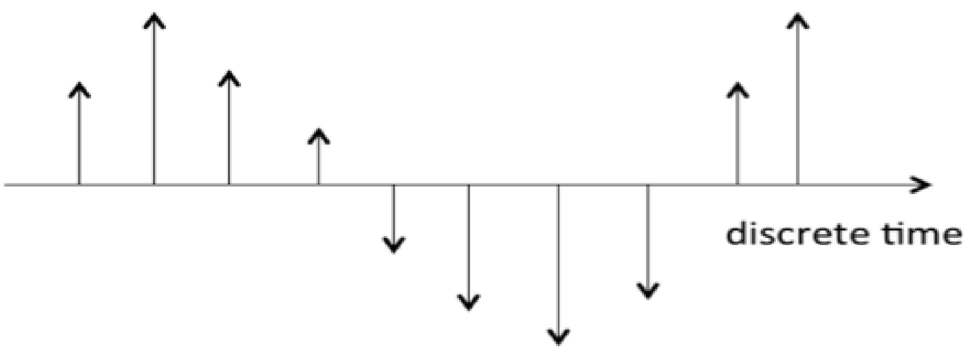
Continuous time signals (CT signals) and Discrete time signals (DT signals) – Step, Ramp, Pulse, Impulse, Exponential – Classification of CT and DT signals – Periodic, aperiodic and Random signals – Real and Complex signals – Energy and power signals.

Signal: Signals are represented mathematically as functions of one or more independent variables. It mainly focuses attention on signals involving a single independent variable. For convenience, this will generally refer to the independent variable as time**.** It is defined as physical quantities that carry information and changes with respect to time.
Ex: voice, television picture, telegraph.

Continuous Time signal – If the signal is defined over continuous-time, then the signal is a continuous-time signal.

Ex: Sinusoidal signal, Voice signal, Rectangular pulse function

Discrete Time signal – If the time t can only take discrete values, such as $t=kT_s$ is called Discrete Time signal

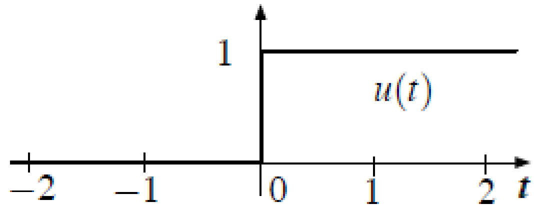


Unit Step Signal:

The Unit Step Signal u(t) is defined as

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

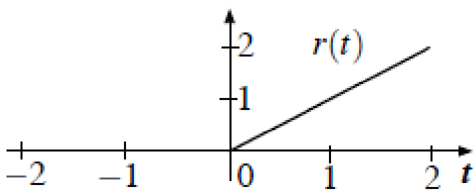
Graphically it is given by



Ramp Signal:

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Graphically it is given by



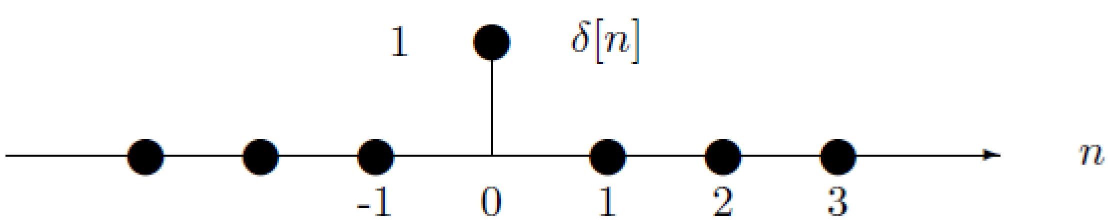
Pulse Signal:

A signal is having constant amplitude over a particular interval and for the remaining interval the amplitude is zero.

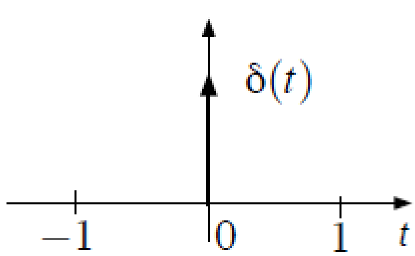
Impulse Signal:

$$\delta[n] \equiv \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

Impulse Signal DT representation



Impulse Signal CT representation



Exponential Signal:

Exponential signal is of two types. These two type of signals are real exponential signal and complex exponential signalwhich are given below.

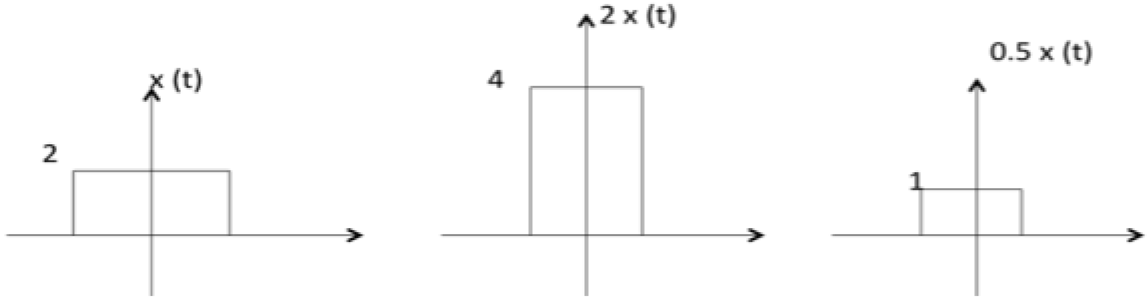
Real Exponential Signal: A real exponential signal is defined as $x(t)=Ae^{\sigma t}$

Complex exponential Signal: The complex exponential signal is given by $x(t)=Ae^{st}$ where $s=\sigma+j\omega$

Basic Operations on signals:

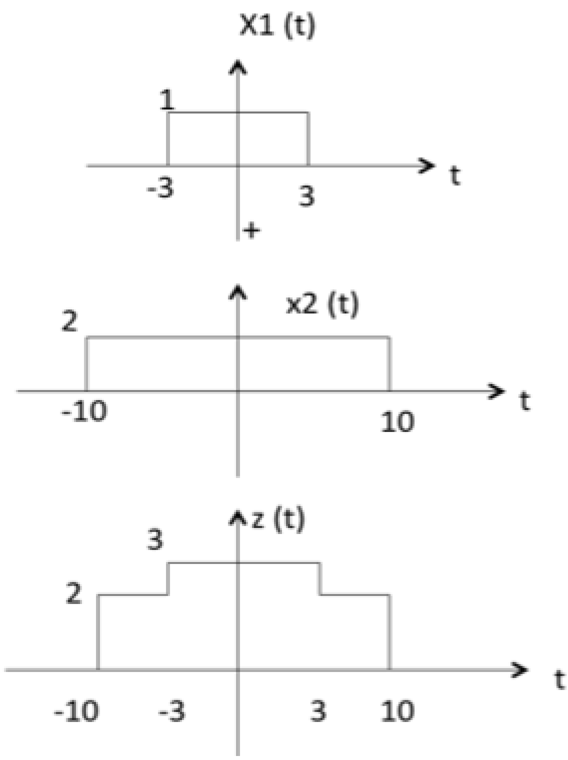
Several basic operations by which new signals are formed from given signals are familiar from the algebra and calculus of functions.

1. Amplitude Scaling :y(t)= a x(t), where a is a real (or possibly complex) constant. C x(t) is a amplitude scaled version of x(t) whose amplitude is scaled by a factor C.



2. Amplitude Shift: y(t)= x(t)+ b, where b is a real (or possibly complex) constant

3. Signal Addition: $y(t)= x_1(t) + x_2(t)$



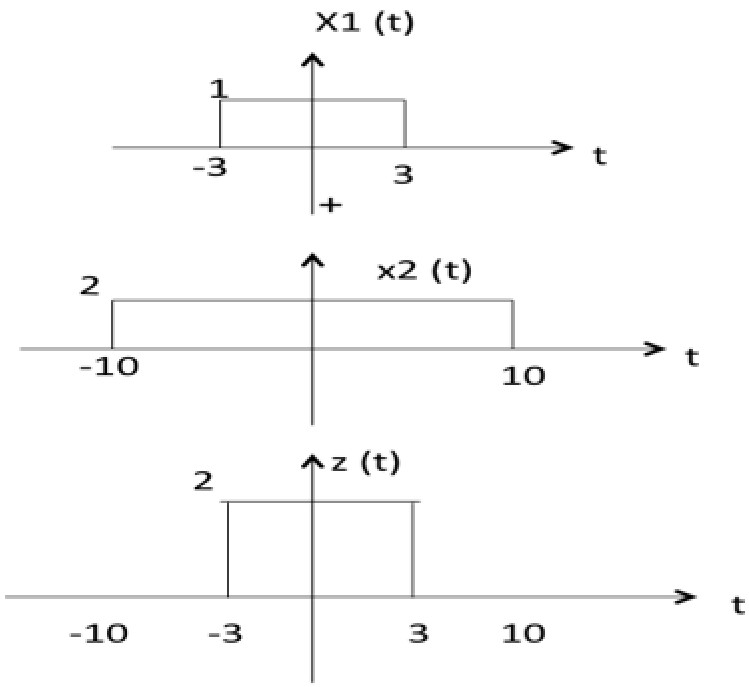
As seen from the diagram above,

-10 < t < -3 amplitude of z(t) = x1(t) + x2(t) = 0 + 2 = 2

-3 < t < 3 amplitude of z(t) = x1(t) + x2(t) = 1 + 2 = 3

3 < t < 10 amplitude of z(t) = x1(t) + x2(t) = 0 + 2 = 2

4. Signal Multiplication: $y(t) = x_1(t). x_2(t)$



As seen from the diagram above,

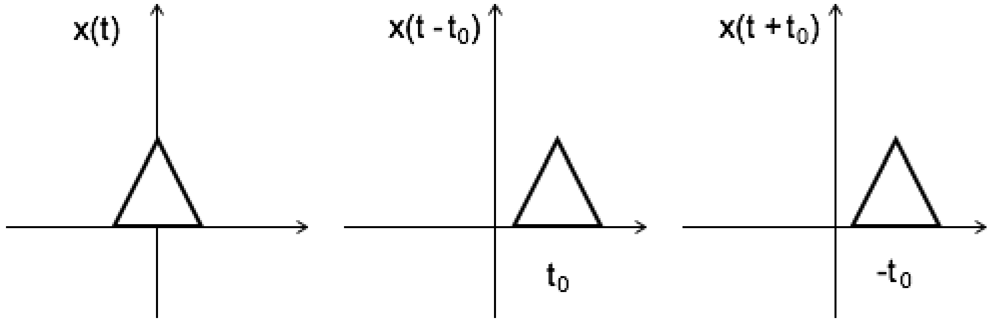
-10 < t < -3 amplitude of z (t) = x1(t) ×x2(t) = 0 ×2 = 0

-3 < t < 3 amplitude of z (t) = x1(t) ×x2(t) = 1 ×2 = 2

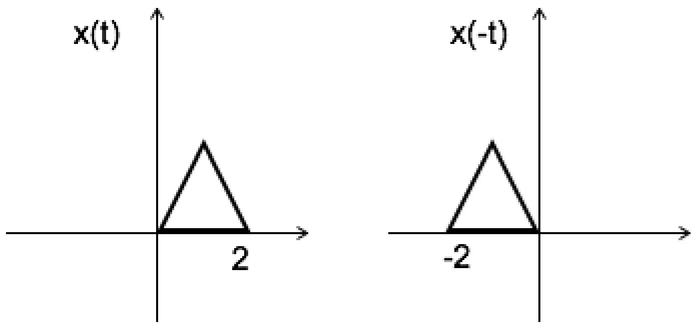
3 < t < 10 amplitude of z (t) = x1(t) × x2(t) = 0 × 2 = 0

5. Time Shift**:** If x(t) is a continuous function, the time-shifted signal is defined as $y(t) = x(t - t_0)$. If $t_0 > 0$, the signal is shifted to the right, and if $t_0 < 0$, the signal is shifted to the left.

$x(t \pm t_0)$ is time shifted version of the signal x(t).
$x(t + t_0)$ →→ negative shift
$x(t - t_0)$ →→ positive shift



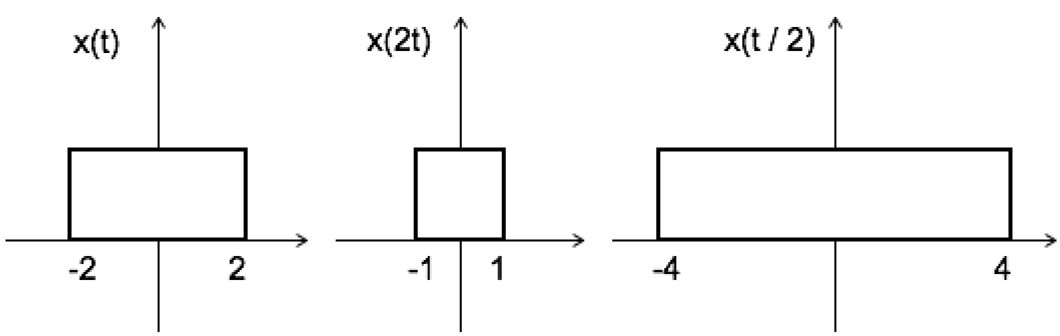
6. Time Reversal: If x(t) is a continuous function, the time-reversed signal is defined as y(t) = x(-t). x(-t) is the time reversal of the signal x(t).



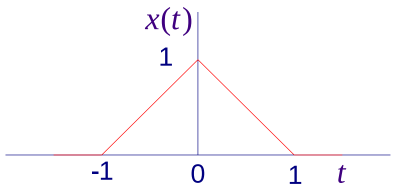
7. Time Scaling: If x(t) is a continuous function, a time-scale version of this signal is defined as y(t) = x(at). If a>1, the signal y(t) is a compressed version of x(t), i.e., the time interval is compressed to $\frac{1}{a}$. If 0<a< 1, the signal y(t) is a stretched version of x(t), i.e., the time interval is stretched by $\frac{1}{a}$. When operating on signals, the time-shifting operation must be performed first, and then the time-scaling operation is performed. x(At) is time scaled version of the signal x(t). where A is always positive.

   |A| > 1 →→ Compression of the signal

   |A| < 1 →→ Expansion of the signal



1. A triangular pulse signal x(t) is depicted below.



Sketch each of the following signals:

(a) $x(3t)$

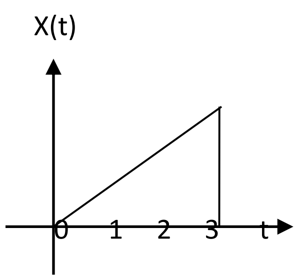
(b) $x(3t+2)$

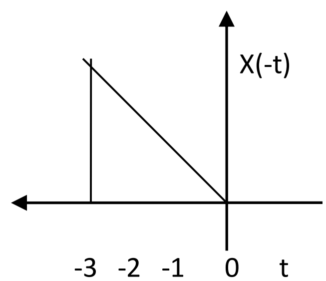
(c) $x(-2t-1)$

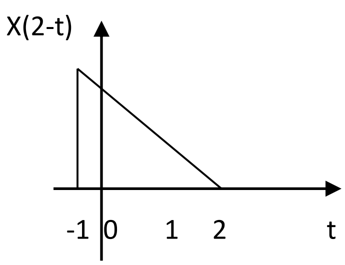
(d) $x(0.5t-1)$

2. Draw the waveform x(-t) and x(2-t) of the signal x(t) = t   0≤t≤3

    0   t>3







Classification of DT and CT Signals:

1. Even and Odd signal
2. Deterministic and Random Signal
3. Periodic and Aperiodic signal
4. Energy and Power signal

Even and Odd Signal:

An even signal is any signal 'x' such that x(t) = x(-t). Odd signal is a signal 'x' for which x(t) = -x(-t).

The even and odd parts of a signal x(t) are
The even and odd parts of a signal are given by

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

Here $x_e(t)$ denotes the even part of signal x(t) and $x_o(t)$ denotes the odd part of signal x(t).

Deterministic Signal:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signals can be modeled exactly by a mathematical formula are known as deterministic signals.

Random (or) Nondeterministic Signals:

Nondeterministic signals and events are either random or irregular. Random signals are also called non deterministic signals are those signals that take random values at any given time and must be characterized statistically.Random signals, on the other hand, cannot be described by a mathematical equation they are modeled in probabilistic terms.

Periodic signal:

A CT signal x(t) is said to be periodic if it satisfies the following property: x(t)=x(t+T) at all time t, where T=Fundamental Time Interval (T=2π/ω)

Ex:

1. x(t)=sin(4πt). It is periodic with period of 1/2
2. x(t)=cos(3πt). It is periodic with period of 2/3

Aperiodic Signal:

A CT signal x(t) is said to be periodic if it satisfies the following property: x(t)≠x(t+T) at all time t, where T=Fundamental Time Interval

Energy Signal:

The Energy in the signal is defined as :

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt .$$

Power Signal:

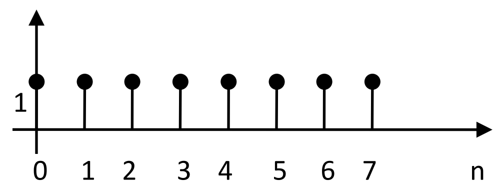
The Power in the signal is defined as

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

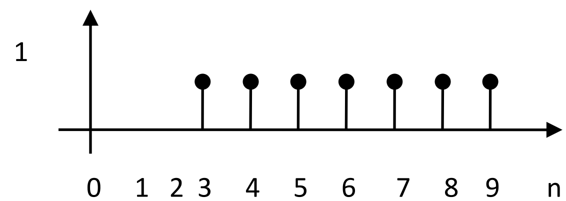
If 0<E<∞ then the signal x(t) is called as Energy signal. However there are signals where this condition is not satisfied. For such signals we consider the power. If 0<P<∞ then the signal is called a power signal. Note that the power for an energy signal is zero (P=0) and that the energy for a power signal is infinite (E=∞). Some signals are neither energy nor power signals.

1. Draw the signal x(n) = u(n) – u(n-3)

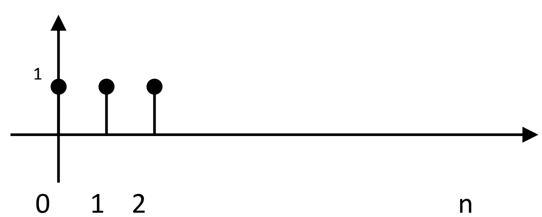
u(n)



u(n-3)



X(n)=u(n)-u(n-3)



2. What is the total energy of the discrete time signal x(n) which takes the value of unity at n= -1,0,1?

   Energy of the signal is given as,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-1}^{1} |x(n)|^2$$

$$= |x(-1)|^2 + |x(0)|^2 + |x(1)|^2 = 3$$

3. Determine if the following signals are Energy signals, Power signals, or neither, and evaluate E and P for each signal $a(t) = 3\sin(2\pi t),\ -\infty < t < \infty$,

$$E_a = \int_{-\infty}^{\infty} |a(t)|^2\ dt = \int_{-\infty}^{\infty} |3\sin(2\pi t)|^2\ dt$$

$$= 9\int_{-\infty}^{\infty} \frac{1}{2}\left[1-\cos(4\pi t)\right]dt$$

$$= 9\int_{-\infty}^{\infty} \frac{1}{2}dt - 9\int_{-\infty}^{\infty} \cos(4\pi t)dt$$

$$= \infty \quad \text{J}$$

$$P_a = \frac{1}{1}\int_{0}^{1} |a(t)|^2\ dt = \int_{0}^{1} |3\sin(2\pi t)|^2\ dt$$

$$= 9\int_{0}^{1} \frac{1}{2}\left[1-\cos(4\pi t)\right]dt$$

$$= 9\int_{0}^{0} \frac{1}{2}dt - 9\int_{0}^{1} \cos(4\pi t)dt$$

$$= \frac{9}{2} - \left[\frac{9}{4\pi}\sin(4\pi t)\right]_0^1$$

$$= \frac{9}{2} \quad \text{W}$$

So, the energy of that signal is infinite and its average power is finite (9/2). This means that it is a power signal as expected. It is a power signal.

Real and Complex signals:

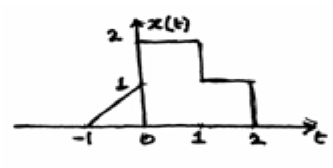
Exponential signal is of two types. These two type of signals are real exponential signal and complex exponential signalwhich are given below.

Real Exponential Signal: A real exponential signal is defined as $x(t)=Ae^{\sigma t}$

Complex exponential Signal:The complex exponential signal is given by $x(t)=Ae^{st}$ where $s=\sigma+j\omega$

PART - A

1. For the signal shown in Fig. 1, find x (2t + 3).



2. Sketch the following signals
   i)x(t)=4(t+3)

   ii)x(t)=-2r(t)
3. Define continuous time complex exponential signal.
4. Define unit impulse and unit step signal.
5. State the relationship between step, ramp and delta function (CT).
6. Define even and odd signal?
7. Determine whether the following signal is energy or power? $x(t) = e^{-2t}\,u(t)$
8. Find the fundamental period of the given signal x(n)=sin((6nπ/7)+1).
9. Check whether the discrete time signal sin3n is periodic.
10. Define a random signal.
11. Determine the power and RMS value of the following signals x(t)=10cos5tcos10t.
12. Determine whether the following signal is energy or power? x(n) = u(n)

PART – B

1. (i) Find the following period T of the following signal:
   X(n) = cos(nπ/2) – sin(nπ/8) + 3 cos{ (nπ/4) + (n/3) }

   (ii) Define and plot the following signals. Ramp, Step, Pulse, Impulse, Exponential signal

2. (i) What is the periodicity of the signal x(t) = sin 100πt + cos 150 πt?
   (ii) What are the basic continuous time signals? Draw any four Waveforms and write their equations.

3. (i) Determine the energy of the discrete time signal. $X(n) = \begin{cases} (1/2)^n, & n \geq 0 \\ 3^n, & n < 0 \end{cases}$