

**SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY**  
**DEPARTMENT OF ELECTRONICS AND COMMUNICATION**  
**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

1. Introduction
2. Problem of radio transmission
  - 2.1 Solution
  - 2.2 Remedy
3. Psuedo Noise Sequence
  - 3.1 Properties of PN Sequence
    - 3.1.1. Balance property
    - 3.1.2. Run length property
    - 3.1.3. Auto correlation property
  - 3.2 Range of PN Sequence Lengths
4. Notion of Spread Spectrum
5. Direct – Sequence Spread Spectrum with coherent binary Phase shift Keying
  - 5.1 Signal Space Dimensionality and Processing Gain
  - 5.2 Probability of error
  - 5.3 Antijam Characteristics
6. Frequency – Hop Spread Spectrum
  - 6.1 Slow frequency hopping
  - 6.2 Fast frequency hopping
  - 6.3 FHSS Performance Considerations
7. Code Division Multiple Access (CDMA)

**SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY**  
**DEPARTMENT OF ELECTRONICS AND COMMUNICATION**  
**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

**1. Introduction**

Initially developed for military applications during II world war, that was less sensitive to intentional interference or jamming by third parties. Spread spectrum technology has blossomed into one of the fundamental building blocks in current and next-generation wireless systems.

**2. Problem of radio transmission**

Narrow band can be wiped out due to interference. To disrupt the communication, the adversary needs to do two things,

- (a) to detect that a transmission is taking place and
- (b) to transmit a jamming signal which is designed to confuse the receiver.

**2.1 Solution**

A spread spectrum system is therefore designed to make these tasks as difficult as possible.

**Firstly**, the transmitted signal should be difficult to detect by an adversary/jammer, i.e., the signal should have a low probability of intercept (LPI).

**Secondly**, the signal should be difficult to disturb with a jamming signal, i.e., the transmitted signal should possess an anti-jamming (AJ) property

**2.2 Remedy**

spread the narrow band signal into a broad band to protect against interference

In a digital communication system the primary resources are **Bandwidth** and **Power**. The study of digital communication system deals with efficient utilization of these two resources, but there are situations where it is necessary to sacrifice their efficient utilization in order to meet certain other design objectives.

For example to provide a form of secure communication (i.e. the transmitted signal is not easily detected or recognized by unwanted listeners) the bandwidth of the transmitted signal is increased in excess of the minimum bandwidth necessary to transmit it. This requirement is catered by a technique known as "**Spread Spectrum Modulation**".

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**COURSE MATERIAL**

Subject Name : Digital Communications

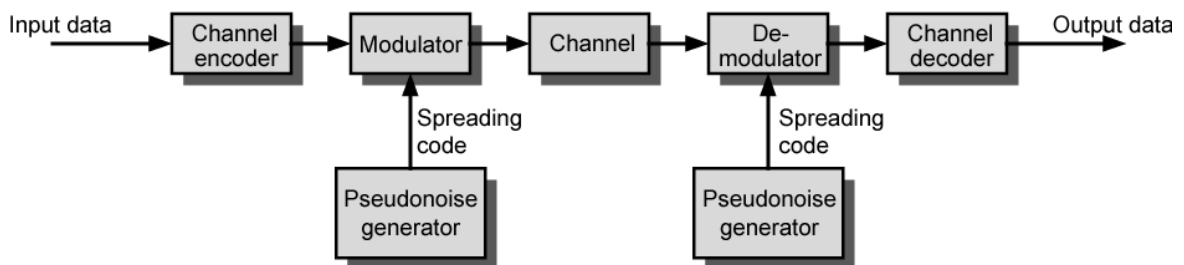
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**UNIT V- SPREAD SPECTRUM MODULATION**

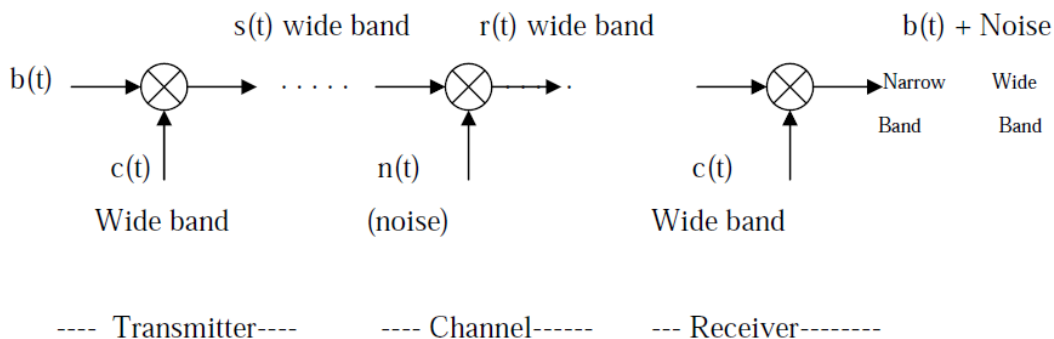
The primary advantage of a Spread – Spectrum communication system is its ability to reject ‘**Interference**’ whether it be the unintentional or the intentional interference.

The definition of Spread – Spectrum modulation may be stated in two parts.

1. Spread Spectrum is a mean of transmission in which the data sequence occupies a BW (Bandwidth) in excess of the minimum BW necessary to transmit it.
2. The Spectrum Spreading is accomplished before transmission through the use of a code that is independent of the data sequence. The Same code is used in the receiver to despread the received signal so that the original data sequence may be recovered.



**Fig. Block diagram for spread spectrum communication**



**Fig: Spread spectrum technique.**

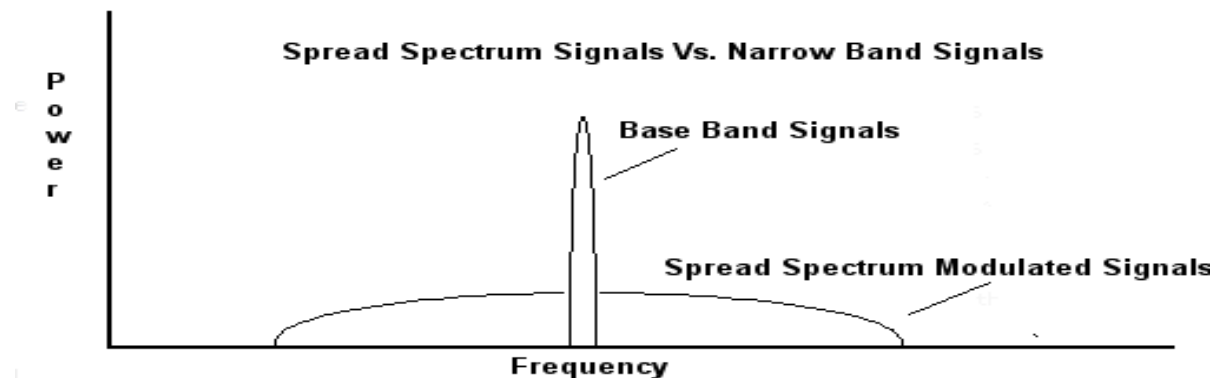
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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

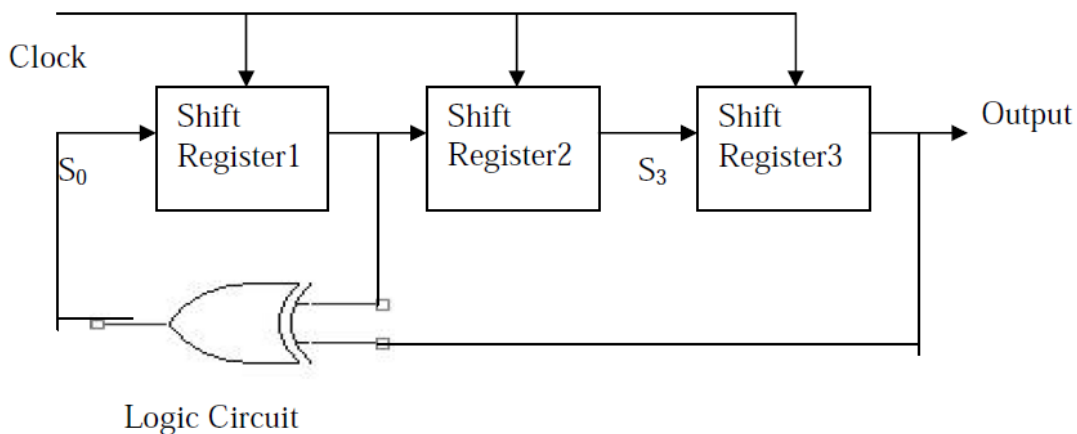
$b(t)$  = Data Sequence to be transmitted (Narrow Band);  
 $c(t)$  = Wide Band code ;  
 $s(t) = c(t) * b(t)$  – (wide Band)



**Fig: Spectrum of signal before & after spreading**

**3. PSUEDO-NOISE SEQUENCE:**

Generation of PN sequence:



**Fig: Maximum-length sequence generator for n=3**

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

A feedback shift register is said to be Linear when the feedback logic consists of entirely mod-2-adders (Ex-or gates). In such a case, the zero state is not permitted. The period of a PN sequence produced by a linear feedback shift register with 'n' flip flops cannot exceed  $2^n - 1$ .

When the period is exactly  $2^n - 1$ , the PN sequence is called a '**maximum length sequence**' or '**m-sequence**'.

**Example1:** Consider the linear feedback shift register shown in above figure

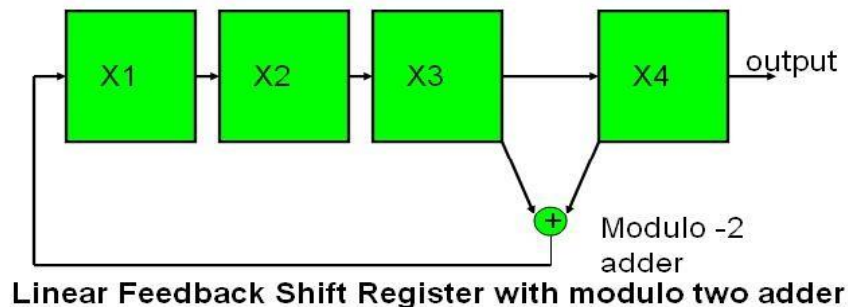
Involve three flip-flops. The input  $s_0$  is equal to the mod-2 sum of  $S_1$  and  $S_3$ . If the initial state of the shift register is 100. Then the succession of states will be as follows.

100,110,011,011,101,010,001,100 . . . . .

The output sequence (output  $S_3$ ) is therefore. 00111010 . . . . . Which repeats itself with period  $2^3 - 1 = 7$  ( $n=3$ ). Maximal length codes are commonly used PN codes In binary shift register, the maximum length sequence is

$$N = 2^m - 1$$

chips, where **m** is the number of stages of flip-flops in the shift register.



At each clock pulse

- Contents of register shifts one bit right.

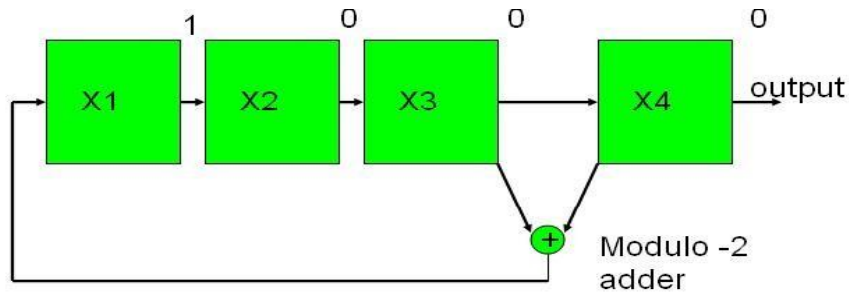
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Subject Name : Digital Communications

Subject Code : SEC1313

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- Contents of required stages are modulo 2 added and fed back to input.



**Fig: Initial stages of Shift registers 1000**

Let initial status of shift register be 1000

1	0	0	0
0	1	0	0
0	0	1	0
1	0	0	1
1	1	0	0
0	1	1	0
1	0	1	1
0	1	0	1
1	0	1	0
1	1	0	1
1	1	1	0
1	1	1	1
0	1	1	1
0	0	1	1
0	0	0	1
1	0	0	0

- We can see for shift Register of length  $m=4$ .  
 .At each clock the change in state of flip-flop is shown.

- Feed back function is modulo two of  $X_3$  and  $X_4$ .

- After 15 clock pulses the sequence repeats.  
 Output sequence is

0 0 0 1 0 0 1 1 0 1 0 1 1 1 1

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**DEPARTMENT OF ELECTRONICS AND COMMUNICATION**  
**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

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**3.1 Properties of PN Sequence**

Randomness of PN sequence is tested by following properties

1. Balance property
2. Run length property
3. Autocorrelation property

**3.1.1. Balance property**

In each Period of the sequence , number of binary ones differ from binary zeros by at most one digit.

Consider output of shift register 0 0 0 1 0 0 1 1 0 1 0 1 1 1 1  
Seven zeros and eight ones -meets balance condition.

**3.1.2. Run length property**

Among the runs of ones and zeros in each period, it is desirable that about one half the runs of each type are of length 1, one-fourth are of length 2 and one-eighth are of length 3 and so-on.

Consider output of shift register

Number of runs =8

<u>0 0 0</u>	<u>1</u>	<u>0 0</u>	<u>1 1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>1 1 1 1</u>
3	1	2	2	1	1	1	4

**3.1.3. Auto correlation property**

Auto correlation function of a maximal length sequence is periodic and binary valued. Autocorrelation sequence of binary sequence in polar format is given by

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

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$$R_c(k) = \frac{1}{N} \sum_{n=1}^N c_n c_{n-k}$$

Where N is length or the period of the sequence, k is the lag of auto correlation function.

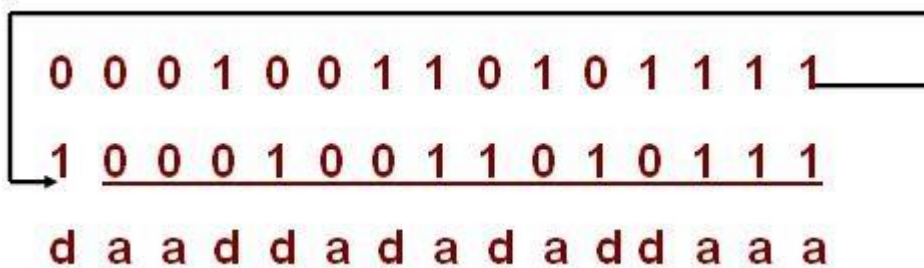
$$R_c(k) = \begin{cases} 1 & \text{if } k = 1N \\ -\frac{1}{N} & \text{if } k \neq 1N \end{cases}$$

Where 1 is any Integer. We can also state the auto correlation function is

$$R_c(k) = \frac{1}{N}$$

{ No. of agreements – No. of disagreements in comparison of one full period }

Consider output of shift register for l=1



$$R_c(k) = \frac{1}{15} (7 - 8) = -\frac{1}{15}$$

**Yields PN autocorrelation as**

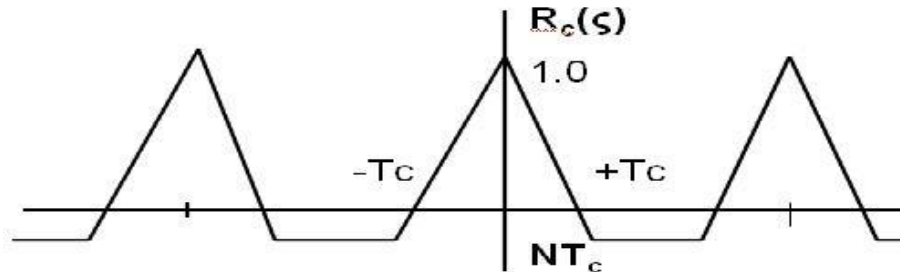


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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

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**3.2 Range of PN Sequence Lengths**

Length Of Shift Register, m	PN Sequence Length,
7	127
8	255
9	511
10	1023
11	2047
12	4095
13	8191
17	131071
19	524287

**4. Notion of Spread Spectrum**

An important attribute of Spread Spectrum modulation is that it can provide protection against externally generated interfering signals with finite power. Protection against jamming (interfering) waveforms is provided by purposely making the information – bearing signal occupy a BW far in excess of the minimum BW necessary to transmit it. This has the effect of making the transmitted signal a noise like

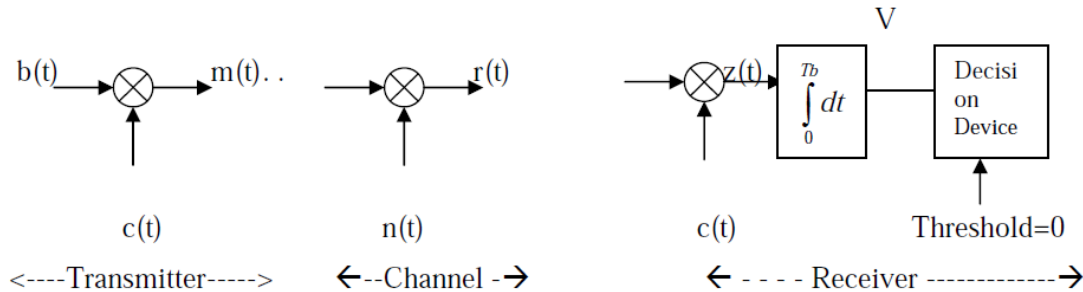
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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

appearance so as to blend into the background. Therefore Spread Spectrum is a method of 'camouflaging' the information – bearing signal.



Let  $\{ b_k \}$  denotes a binary data sequence.

$\{ c_k \}$  denotes a PN sequence.

$b(t)$  and  $c(t)$  denotes their NRZ polar representation respectively.

The desired modulation is achieved by applying the data signal  $b(t)$  and PN signal  $c(t)$  to a product modulator or multiplier. If the message signal  $b(t)$  is narrowband and the PN sequence signal  $c(t)$  is wide band, the product signal  $m(t)$  is also wide band. The PN sequence performs the role of a **'Spreading Code'**.

For base band transmission, the product signal  $m(t)$  represents the transmitted signal. Therefore  $m(t) = c(t).b(t)$

The received signal  $r(t)$  consists of the transmitted signal  $m(t)$  plus an additive interference noise  $n(t)$ , Hence

$$r(t) = m(t) + n(t)$$

$$= c(t).b(t) + n(t)$$

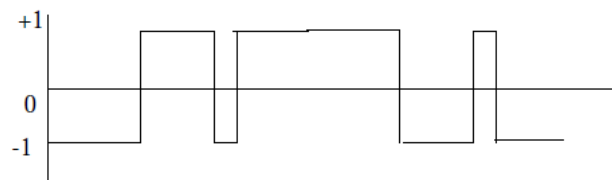
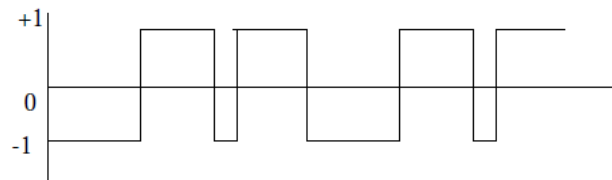
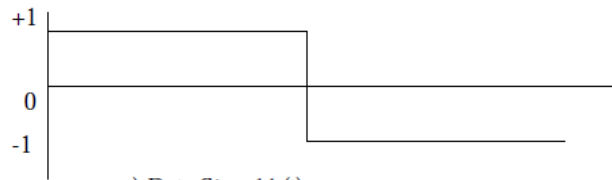
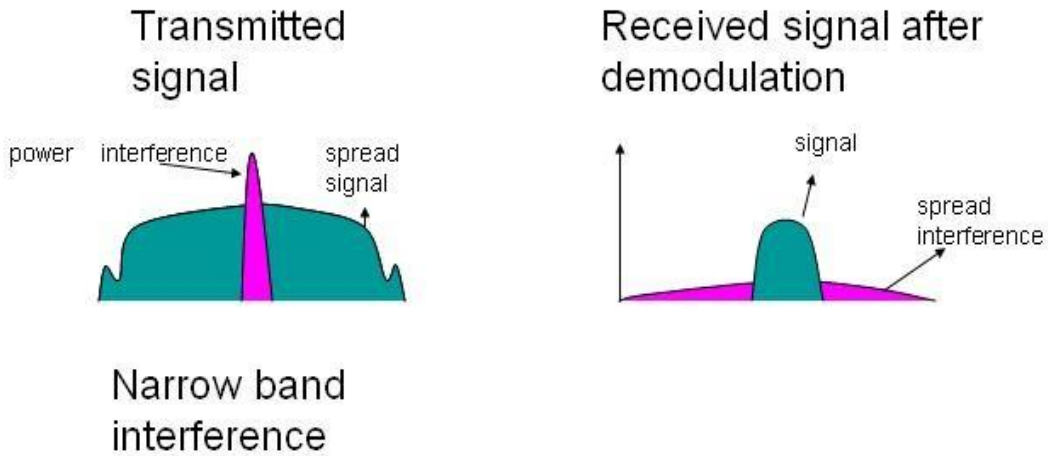
To recover the original message signal  $b(t)$ , the received signal  $r(t)$  is applied to a demodulator that consists of a multiplier followed by an integrator and a decision device. The multiplier is supplied with a locally generated PN sequence that is exact replica of that used in the transmitter.

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**



The multiplier output is given by

$$Z(t) = r(t) \cdot c(t)$$

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

$$= [b(t) * c(t) + n(t)] c(t) = c^2(t).b(t) + c(t).n(t)$$

The data signal  $b(t)$  is multiplied twice by the PN signal  $c(t)$ , where as unwanted signal  $n(t)$  is multiplied only once. But  $c^2(t) = 1$ , hence the above equation reduces to

$$Z(t) = b(t) + c(t).n(t)$$

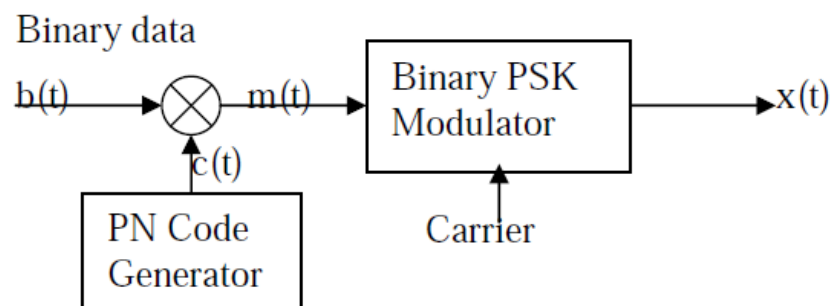
Now the data component  $b(t)$  is narrowband, where as the spurious component  $c(t)n(t)$  is wide band. Hence by applying the multiplier output to a base band (low pass) filter most of the power in the spurious component  $c(t)n(t)$  is filtered out. Thus the effect of the interference  $n(t)$  is thus significantly reduced at the receiver output.

The integration is carried out for the bit interval  $0 \leq t \leq T_b$  to provide the sample value  $V$ . Finally, a decision is made by the receiver.

If  $V > \text{Threshold Value '0'}$ , say binary symbol '1' If  $V < \text{Threshold Value '0'}$ , say binary symbol '0'

**5. Direct – Sequence Spread Spectrum with coherent binary Phase shift**

**Keying:-**



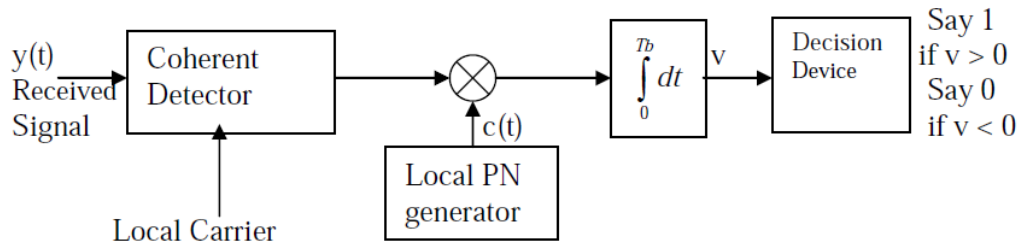
a) Transmitter

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Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**



b) Receiver

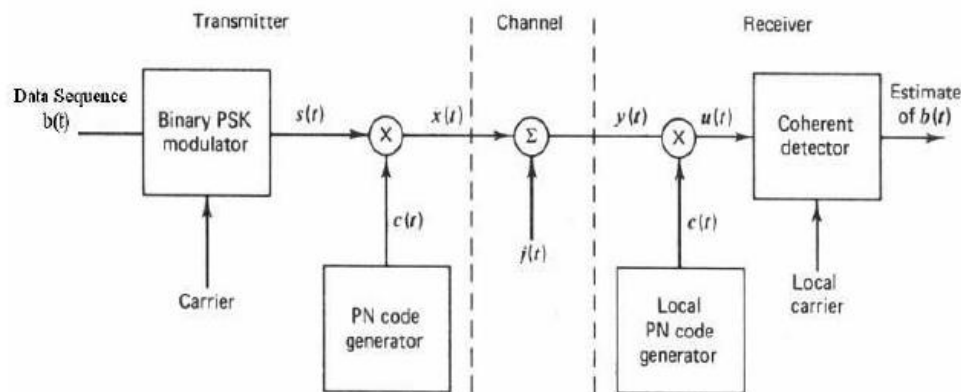


Fig: model of direct – sequence spread binary PSK system(alternative form)

To provide band pass transmission, the base band data sequence is multiplied by a Carrier by means of shift keying. Normally binary phase shift keying (PSK) is used because of its advantages. The transmitter first converts the incoming binary data sequence  $\{b_k\}$  into an NRZ waveform  $b(t)$ , which is followed by two stages of modulation.

The first stage consists of a multiplier with data signal  $b(t)$  and the PN signal  $c(t)$  as inputs. The output of multiplier is  $m(t)$  is a wideband signal. Thus a narrow – band data sequence is transformed into a noise like wide band signal.

The second stage consists of a binary Phase Shift Keying (PSK) modulator. Which converts base band signal  $m(t)$  into band pass signal  $x(t)$ . The transmitted signal  $x(t)$  is thus a direct – sequence spread binary PSK signal. The phase modulation  $\theta(t)$  of  $x(t)$  has one of the two values '0' and ' $\pi$ ' ( $180^\circ$ ) depending upon the polarity of the message signal  $b(t)$  and PN signal  $c(t)$  at time  $t$ .

Polarity of PN & Polarity of PN signal both +, + or - - Phase '0'

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

Polarity of PN & Polarity of PN signal both +, - or - + Phase 'π'

		Polarity of data sequence b(t)	
		+	-
Polarity of PN sequence C(t)	+	<b>0</b>	$\pi$
	-	$\pi$	<b>0</b>

The receiver consists of two stages of demodulation.

In the first stage the received signal  $y(t)$  and a locally generated carrier are applied to a coherent detector (a product modulator followed by a low pass filter), Which converts band pass signal into base band signal.

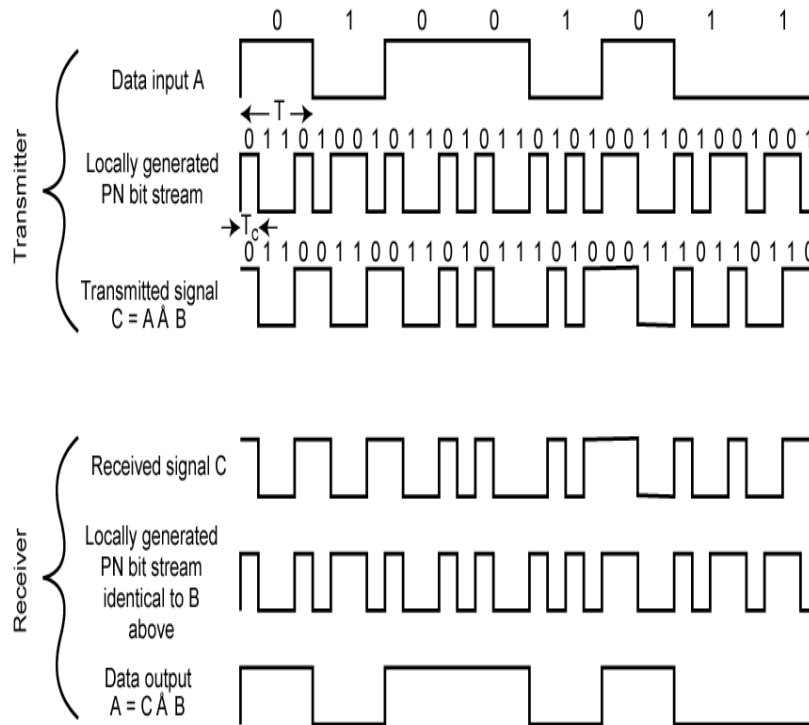
The second stage of demodulation performs Spectrum desreading by multiplying the output of low-pass filter by a locally generated replica of the PN signal  $c(t)$ , followed by integration over a bit interval  $T_b$  and finally a decision device is used to get binary sequence.

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**



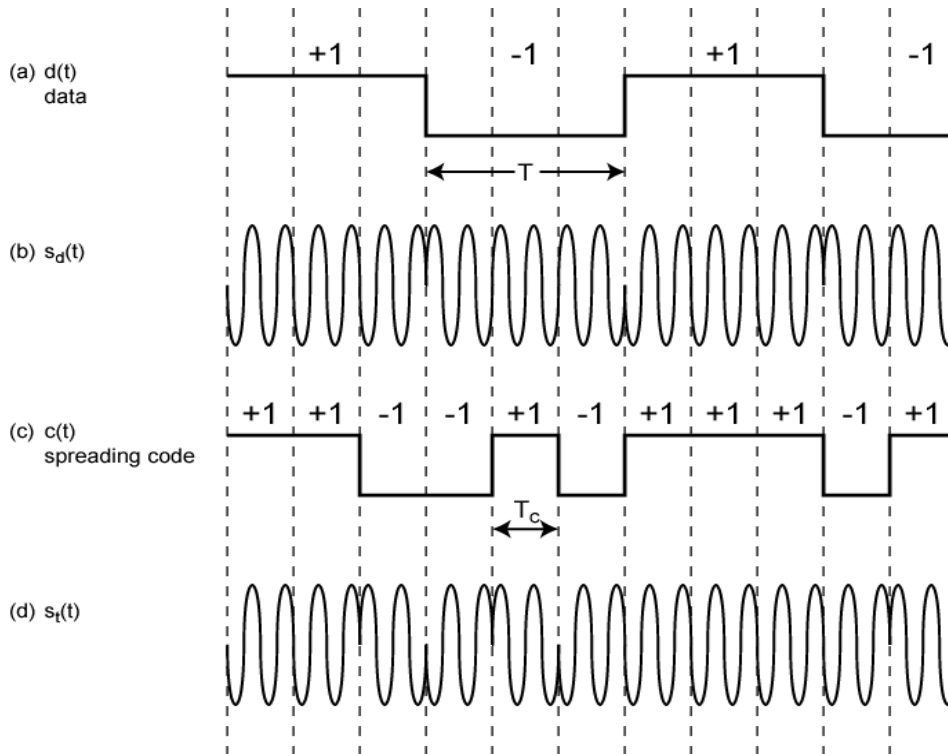
**Fig : Direct Sequence Spread Spectrum Example**

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

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**Fig : Direct Sequence Spread Spectrum Using BPSK Example**

### 5.1 Signal Space Dimensionality and Processing Gain

- Fundamental issue in SS systems is how much protection spreading can provide against interference.
- SS technique distribute low dimensional signal into large dimensional signal space (hide the signal).
- Jammer has only one option; to jam the entire space with fixed total power or to jam portion of signal space with large power.

Consider set of orthonormal basis functions;



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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

$$\varphi_k(t) = \begin{cases} \sqrt{\frac{2}{T_c}} \cos(2\pi f_c t) & kT_c \leq t \leq (k+1)T_c \\ 0 & \text{Otherwise} \end{cases}$$

$$\tilde{\varphi}_k(t) = \begin{cases} \sqrt{\frac{2}{T_c}} \sin(2\pi f_c t) & kT_c \leq t \leq (k+1)T_c \\ 0 & \text{Otherwise} \end{cases} \quad k = 0, 1, \dots, N-1$$

Where  $T_c$  is chip duration,  $N$  is number of chips per bit.

Transmitted signal  $x(t)$  for the interval of an information bit is

$$x(t) = c(t)s(t)$$

$$\varphi_k(t) = \pm \sqrt{\frac{2}{T_c}} c(t) \cos(2\pi f_c t)$$

$$\varphi_k(t) = \pm \sqrt{\frac{E_b}{N}} \sum_{k=0}^{N-1} c_k \varphi_k(t) \quad 0 \leq t \leq T_b$$

where,  $E_b$  is signal energy per bit.

PN Code sequence  $\{c_0, c_1, \dots, c_{N-1}\}$  with  $c_k = \pm 1$ , Transmitted signal  $x(t)$  is therefore  $N$  dimensional and requires  $N$  orthonormal functions to represent it.  $j(t)$  represent interfering signal (jammer). As said jammer tries to place all its available energy in exactly same  $N$  dimension signal space. But jammer has no knowledge of signal phase. Hence tries to place equal energy in two phase coordinates that is cosine and sine. As per that jammer can be represented as

$$j(t) = \sum_{k=0}^{N-1} j_k \varphi_k(t) + \sum_{k=0}^{N-1} \tilde{j}_k \tilde{\varphi}_k(t) \quad 0 \leq t \leq T_b$$

Where

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

$$j_k = \int_0^{T_b} j(t) \varphi_k(t) dt \quad k = 0, 1, \dots, N-1$$

$$\tilde{j}_k = \int_0^{T_b} j(t) \tilde{\varphi}_k(t) dt \quad k = 0, 1, \dots, N-1$$

Thus  $j(t)$  is  $2N$  dimensional, twice the dimension as that of  $x(t)$ .

Average interference power of  $j(t)$

$$J = \frac{1}{T_b} \int_0^{T_b} j^2(t) dt = \frac{1}{T_b} \sum_{k=0}^{N-1} j_k^2 + \frac{1}{T_b} \sum_{k=0}^{N-1} \tilde{j}_k^2$$

as jammer places equal energy in two phase coordinates , hence

$$\sum_{k=0}^{N-1} j_k^2 = \sum_{k=0}^{N-1} \tilde{j}_k^2$$

$$J = \frac{2}{T_b} \sum_{k=0}^{N-1} j_k^2$$

To evaluate system performance we calculate SNR at input and output of **DS/BPSK** receiver. The coherent receiver input is  $u(t) = s(t) + c(t)j(t)$  and using this  $u(t)$ , output at coherent receiver

$$V = \sqrt{\frac{2}{T_b}} \int_0^{T_b} u(t) \cos(2\pi f_c t) dt = V_s + V_{cj}$$

Where  $v_s$  is despread component of BPSK and  $v_{cj}$  of spread interference.

$$V_s = \sqrt{\frac{2}{T_b}} \int_0^{T_b} s(t) \cos(2\pi f_c t) dt$$

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**DEPARTMENT OF ELECTRONICS AND COMMUNICATION**  
**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

$$V_{cj} = \sqrt{\frac{2}{T_b}} \int_0^{T_b} c(t)j(t) \cos(2\pi f_c t) dt$$

Consider despread BPSK signal s(t)

$$s(t) = \pm \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) dt \quad 0 \leq t \leq T_b$$

Where + sign is for symbol 1  
 - sign for symbol 0.

If carrier frequency is integer multiple of  $1 / T_b$ , we have  $V_s = \pm \sqrt{E_b}$

Consider spread interference component  $v_{cj}$ , here  $c(t)$  is considered in sequence form  $\{c_0, c_1, \dots, c_{N-1}\}$

$$V_{cj} = \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} C_k \int_0^{T_b} j(t) \varphi_k(t) dt = \sqrt{\frac{T_c}{T_b}} \sum_{k=0}^{N-1} C_k j_k$$

With  $C_k$  treated as independent identical random variables with both symbols having equal probabilities

$$P(C_k = 1) = P(C_k = -1) = \frac{1}{2}$$

Expected value of Random variable  $v_{cj}$  is zero, for fixed  $k$  we have

$$E[c_k j_k | j_k] = j_k P(C_k = 1) - p(C_k = -1) = \frac{1}{2} j_k - \frac{1}{2} j_k = 0$$

And Variance

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

$$\text{Var} [V_{c_j}|j] = \frac{1}{N} \sum_{k=0}^{N-1} j_k^2 = \frac{JT_c}{2}$$

Spread factor **N = T<sub>b</sub>/T<sub>c</sub>**

Output signal to noise ratio is

$$(SNR)_o = \frac{2E_b}{JT_c}$$

The average signal power at receiver input is E<sub>b</sub>/T<sub>b</sub> hence input SNR

$$(SNR)_i = \frac{E_b/T_b}{J}$$
$$(SNR)_o = \frac{2T_b}{T_c} (SNR)_i$$

Expressing SNR in decibels

$$10 \log_{10}(SNR)_o = 10 \log_{10}(SNR)_i + 3 + 10 \log_{10}(PG), dB$$

Where  $PG = \frac{T_b}{T_c}$

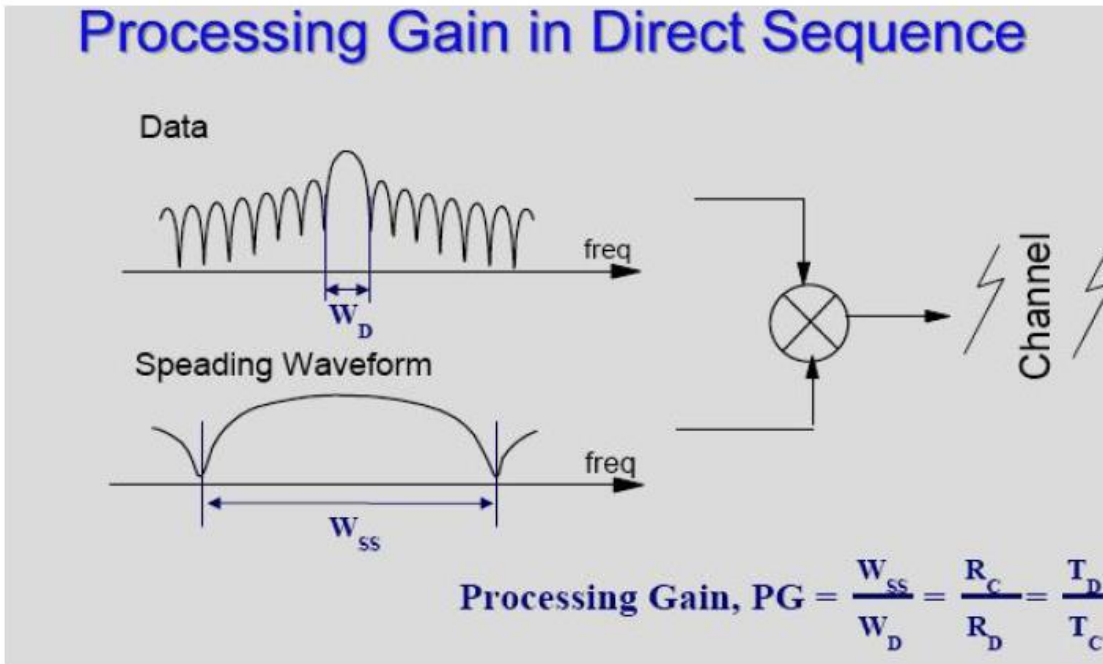
3db term on right side accounts for gain in SNR due to coherent detection. Last term accounts for gain in SNR by use of spread spectrum. **PG is called Processing Gain.**

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**DEPARTMENT OF ELECTRONICS AND COMMUNICATION**  
**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**



1. Bit rate of binary data entering the transmitter input is  $R_b = \frac{1}{T_b}$

2. The bandwidth of PN sequence  $c(t)$ , of main lobe is  $W_C$

$$W_C = \frac{1}{T_C}$$

$$PG = \frac{W_C}{R_b}$$

**5.2 Probability of error**

To calculate probability of error, we consider output component  $v$  of coherent detector as sample value of random variable

$$V = \pm \sqrt{E_b + V_{cj}}$$

$E_b$  is signal energy per bit and  $V_{cj}$  is noise component

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

Decision rule is, if detector output exceeds a threshold of zero volts; received bit is symbol 1 else decision is favored for zero.

- Average probability of error  $P_e$  is nothing but conditional probability which depends on random variable  $V_{cj}$ .
- As a result receiver makes decision in favor of symbol 1 when symbol 0 transmitted and vice versa
- Random variable  $V_{cj}$  is sum of  $N$  such random variables. Hence for Large  $N$  it can assume Gaussian distribution .
- As mean and variance has already been discussed , zero mean and variance  $JT_c/2$

**Probability of error can be calculated from simple formula for DS/BPSK system**

$$P_e \cong \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{JT_c}} \right)$$

**5.3 Antijam Characteristics**

Consider error probability of BPSK

$$P_e = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right)$$

Comparing both probabilities;

$$\frac{N_0}{2} = \frac{JT_c}{2}$$

Since bit energy  $E_b = PT_b$  ,  $P$ = average signal power.

We can express bit energy to noise density ratio as

$$\frac{E_b}{N_0} = \left( \frac{T_b}{T_c} \right) \left( \frac{P}{J} \right)$$

Or

$$\frac{J}{P} = \frac{PG}{E_b/N_0}$$

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

The ratio **J/P** is termed jamming margin. Jamming Margin is expressed in decibels as

$$(\text{jamming margin})_{dB} = (\text{Processing gain})_{dB} - 10 \log_{10} \left( \frac{E_b}{N_0} \right)_{min}$$

Where  $\left( \frac{E_b}{N_0} \right)$  is minimum bit energy to noise ration needed to support a prescribed average probability of error.

**Example1**

**A pseudo random sequence is generated using a feed back shift register of length  $m=4$ . The chip rate is 107 chips per second. Find the following**

- a) PN sequence length b) Chip duration of PN sequence c) PN sequence period**

**Solution**

- a) Length of PN sequence  $N = 2^m - 1 = 2^4 - 1 = 15$   
b) Chip duration  $T_c = 1/\text{chip rate} = 1/107 = 0.1\mu \text{ sec}$   
c) PN sequence period  $T = NT_c$   
 $= 15 \times 0.1\mu \text{ sec} = 1.5\mu \text{ sec}$

**Example2**

**A direct sequence spread binary phase shift keying system uses a feedback shift register of length 19 for the generation of PN sequence. Calculate the processing gain of the system.**

**Solution**

Given length of shift register =  $m = 19$

Therefore length of PN sequence  $N = 2^m - 1$   
 $= 2^{19} - 1$

Processing gain  $PG = T_b/T_c = N$  in db  $= 10 \log_{10} N = 10 \log_{10} (2^{19}) = 57\text{db}$

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**

**Example3**

A Spread spectrum communication system has the following parameters. Information bit duration  $T_b = 1.024$  msecs and PN chip duration of  $1\mu$ secs. The average probability of error of system is not to exceed  $10^{-5}$ . calculate a) Length of shift register b) Processing gain c) jamming margin

**Solution**

Processing gain  $PG = N = T_b/T_c = 1024$   
corresponding length of shift register  $m = 10$

In case of coherent BPSK For Probability of error  $10^{-5}$ . [Referring to error function table]  
 $E_b/N_0 = 10.8$

Therefore jamming margin

$$(\text{jamming margin})_{dB} = (\text{Processing gain})_{dB} - 10 \log_{10} \left( \frac{E_b}{N_0} \right)_{min}$$

$$(\text{jamming margin})_{dB} = 10 \log_{10} PG_{dB} - 10 \log_{10} \left( \frac{E_b}{N_0} \right)_{min}$$

$$(\text{jamming margin})_{dB} = 10 \log_{10} 1024 - 10 \log_{10} 10.8$$

$$(\text{jamming margin})_{dB} = 30.10 - 10.33 = 19.8 \text{ dB}$$

**6. Frequency – Hop Spread Spectrum:**

In a frequency – hop Spread – Spectrum technique, the spectrum of data modulated carrier is widened by changing the carrier frequency in a pseudo – random manner. The type of spread – spectrum in which the carrier hops randomly from one frequency to another is called **Frequency – Hop (FH) Spread Spectrum.**

Since frequency hopping does not covers the entire spread spectrum instantaneously. We are led to consider the rate at which the hop occurs. Depending upon this we have two types of frequency hop.



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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

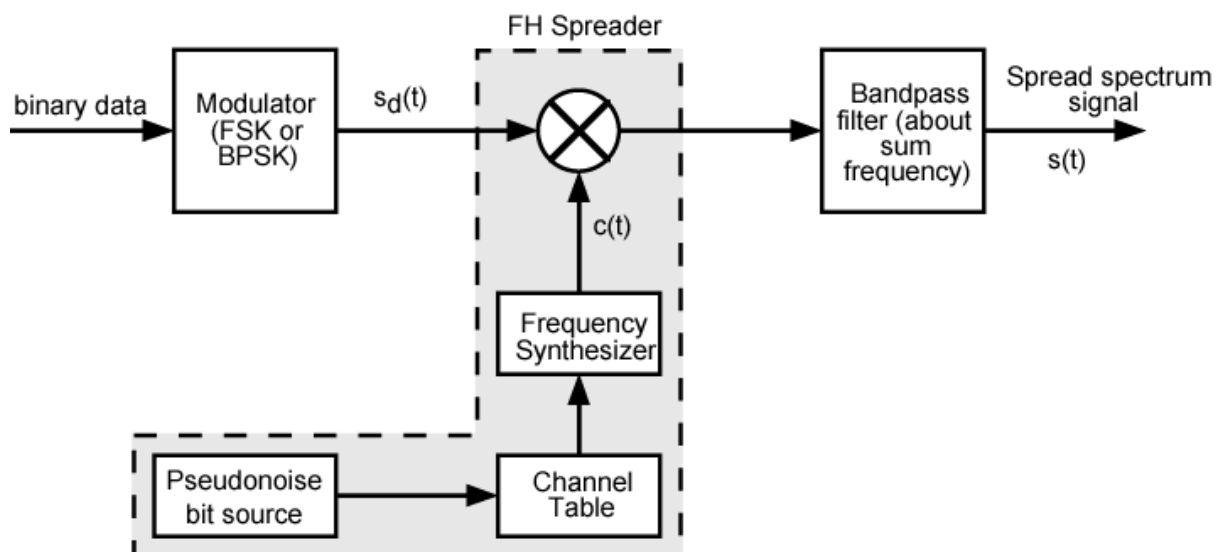
**UNIT V- SPREAD SPECTRUM MODULATION**

1. Slow frequency hopping:- In which the symbol rate  $R_S$  of the MFSK signal is an integer multiple of the hop rate  $R_H$ . That is several symbols are transmitted on each frequency hop.
2. Fast – Frequency hopping:- In which the hop rate  $R_H$  is an integral multiple of the MFSK symbol rate  $R_S$ . That is the carrier frequency will hop several times during the transmission of one symbol. A common modulation format for frequency hopping system is that of M- ary frequency – shift – keying (MFSK).

**6.1 Slow frequency hopping:-**

Fig Shows the block diagram of an FH / MFSK transmitter, which involves frequency modulation followed by mixing.

The incoming binary data are applied to an M-ary FSK modulator. The resulting modulated wave and the output from a digital frequency synthesizer are then applied to a mixer that consists of a multiplier followed by a band – pass filter. The filter is designed to select the sum frequency component resulting from the multiplication process as the transmitted signal. An 'k' bit segments of a PN sequence drive the frequency synthesizer, which enables the carrier frequency to hop over  $2^k$  distinct values. Since frequency synthesizers are unable to maintain phase coherence over successive hops, most frequency hops spread spectrum communication system use non coherent M-ary modulation system.



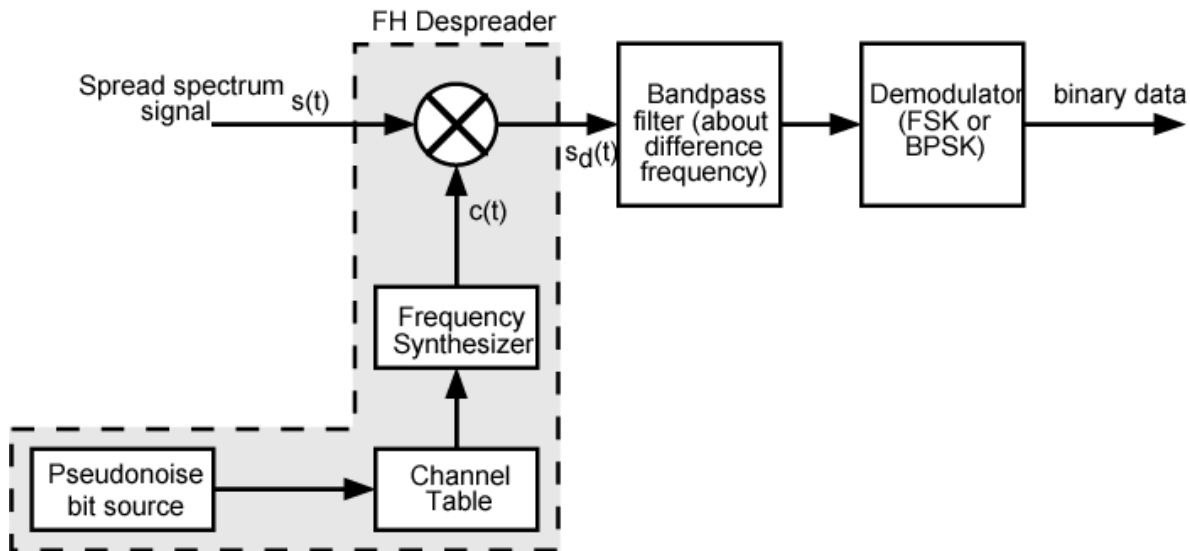
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Subject Name : Digital Communications

Subject Code : SEC1313

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**Fig :- Frequency hop spread transmitter**



**Fig :- Frequency hop spread receiver**

In the receiver the frequency hopping is first removed by mixing the received signal with the output of a local frequency synthesizer that is synchronized with the transmitter. The resulting output is then band pass filtered and subsequently processed by a non coherent M-ary FSK demodulator. To implement this M-ary detector, a bank of M non coherent matched filters, each of which is matched to one of the MFSK tones is used. By selecting the largest filtered output, the original transmitted signal is estimated.

An individual FH / MFSK tone of shortest duration is referred as a chip. The chip rate  $R_C$  for an FH / MFSK system is defined by

$$R_C = \text{Max}(R_h, R_s)$$

Where  $R_h$  is the hop rate and  $R_s$  is Symbol Rate

In a slow rate frequency hopping multiple symbols are transmitted per hop. Hence each symbol of a slow FH / MFSK signal is a chip. The bit rate  $R_b$  of the incoming binary data. The symbol rate  $R_s$  of the MFSK signal, the chip rate  $R_C$  and the hop rate  $R_h$  are related by

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Subject Name : Digital Communications

Subject Code : SEC1313

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$$R_c = R_s = R_b / k \geq R_h$$

where  $k = \log_2 M$

**6.2 Fast frequency hopping:-**

A fast FH / MFSK system differs from a slow FH / MFSK system in that there are multiple hops per m-ary symbol. Hence in a fast FH / MFSK system each hop is a chip.

<b>Fast Frequency Hopping</b>	<b>Slow Frequency Hopping</b>
<b>Several frequency hops Per modulation</b>	<b>Several modulation symbols per hop</b>
<b>Shortest uninterrupted waveform in the system is that of hop</b>	<b>Shortest uninterrupted waveform in the system is that of data symbol</b>
<b>Chip duration =hop duration</b>	<b>Chip duration=bit duration.</b>

The following figure illustrates the variation of the frequency of a slow FH/MFSK signal with time for one complete period of the PN sequence. The period of the PN sequence is  $2^4 - 1 = 15$ .

The FH/MFSK signal has the following parameters:

Number of bits per MFSK symbol  $K = 2$ . Number of MFSK tones  $M = 2^K = 4$

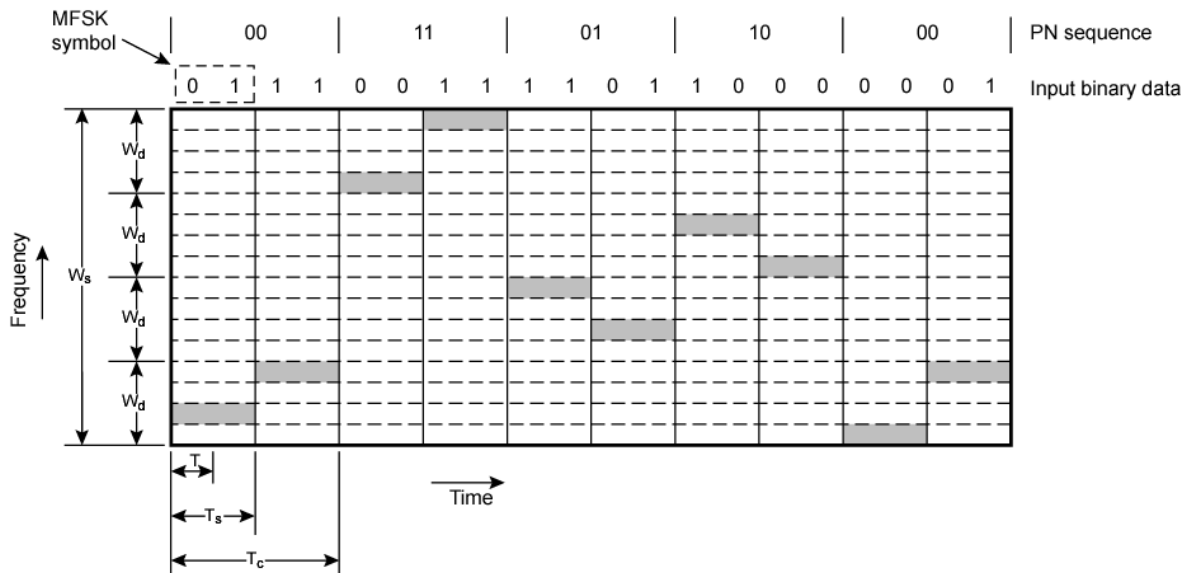
Length of PN segment per hop  $k = 3$ ; Total number of frequency hops  $2^k = 8$

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

**UNIT V- SPREAD SPECTRUM MODULATION**



**Fig. Slow frequency hopping**

The following figure illustrates the variation of the transmitted frequency of a fast FH/MFSK signal with time.

The signal has the following parameters:

Number of bits per MFSK symbol  $K = 2$ . Number of MFSK tones  $M = 2^K = 4$

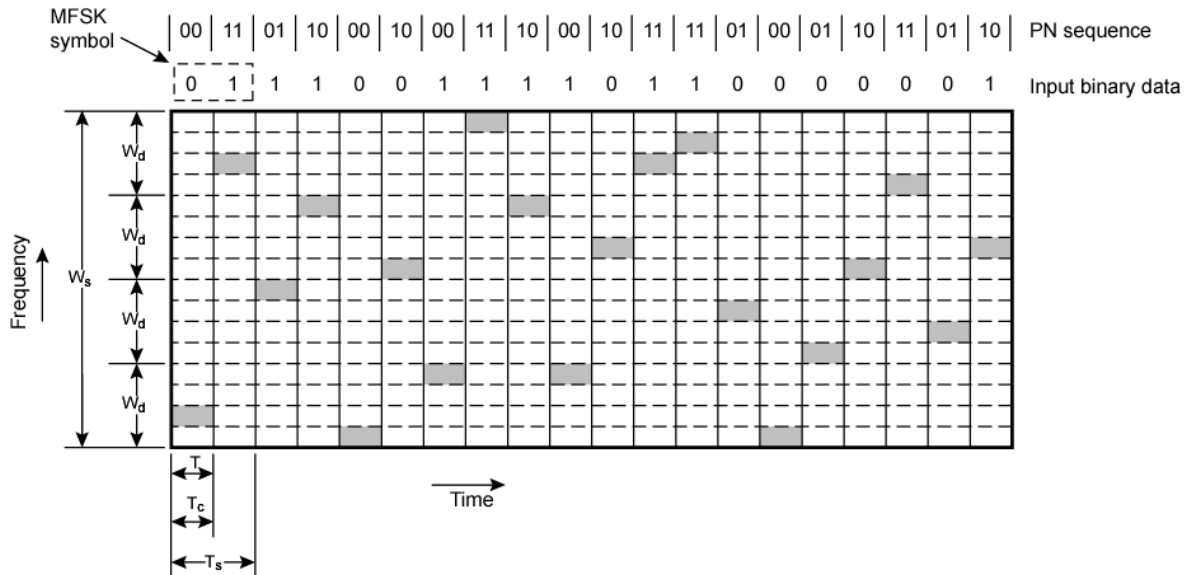
Length of PN segment per hop  $k = 3$ ; Total number of frequency hops  $2^k = 8$

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**COURSE MATERIAL**

Subject Name : Digital Communications

Subject Code : SEC1313

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**Fig. Fast frequency hopping**

**6.3 FHSS Performance Considerations:**

- Typically large number of frequencies used
  - Improved resistance to jamming

**7. Code Division Multiple Access (CDMA):**

- Multiplexing Technique used with spread spectrum
- Start with data signal rate  $D$ 
  - Called bit data rate
- Break each bit into  $k$  chips according to fixed pattern specific to each user
  - User's code
- New channel has chip data rate  $kD$  chips per second
- E.g.  $k=6$ , three users (A,B,C) communicating with base receiver R
- Code for A =  $\langle 1,-1,-1,1,-1,1 \rangle$
- Code for B =  $\langle 1,1,-1,-1,1,1 \rangle$
- Code for C =  $\langle 1,1,-1,1,1,-1 \rangle$

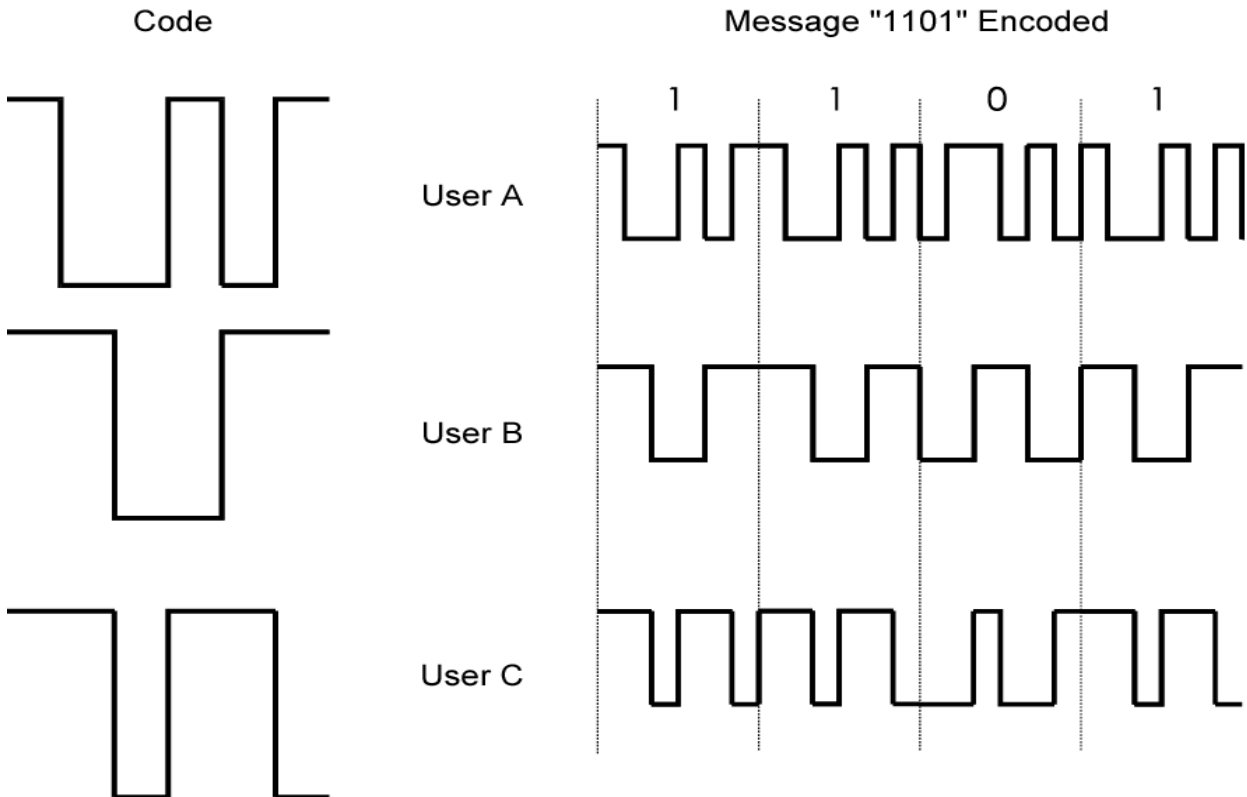
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**CDMA Example:**



- Consider A communicating with base
- Base knows A's code
- Assume communication already synchronized
- A wants to send a 1
  - Send chip pattern  $\langle 1, -1, -1, 1, -1, 1 \rangle$ 
    - A's code
- A wants to send 0
  - Send chip pattern  $\langle -1, 1, 1, -1, 1, -1 \rangle$ 
    - Complement of A's code
- Decoder ignores other sources when using A's code to decode
  - Orthogonal codes
  -

**CDMA for DSSS:**

- n users each using different orthogonal PN sequence

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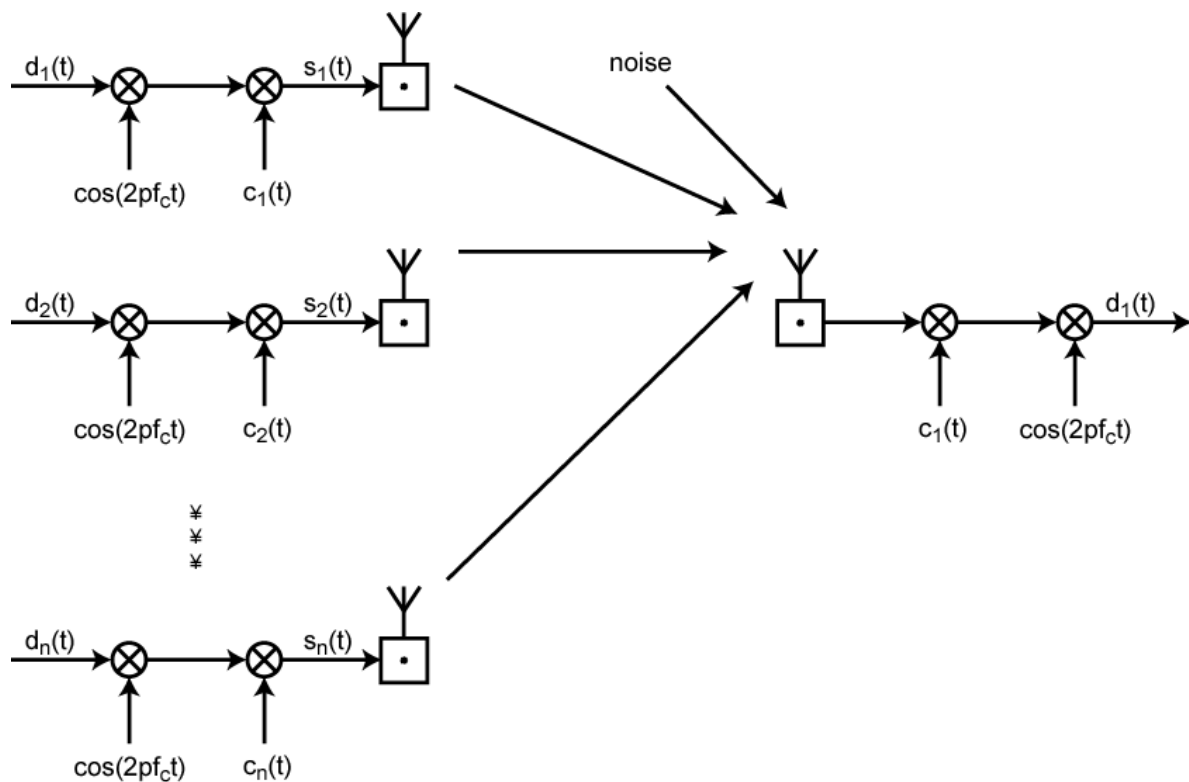
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**UNIT V- SPREAD SPECTRUM MODULATION**

- Modulate each users data stream
  - Using BPSK
- Multiply by spreading code of user

**CDMA in a DSSS Environment:**



**QUESTIONS FOR PRACTISE**

**Part A ( 2 Marks)**

1. Define constraint length in convolutional codes?
2. What is pseudo noise sequence?
3. What is direct sequence spread spectrum modulation

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Subject Code : SEC1313

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4. What is frequency hop spread spectrum modulation?
5. What is processing gain?
6. What is jamming margin ?
7. When is the PN sequence called as maximal length sequence?
8. What is meant by processing gain of DS spread spectrum system?
9. What is the period of the maximal length sequence generated using 3 bit shift register.
10. Define frequency hopping.
11. What are the Advantages of DS-SS system
12. What are the Disadvantages of DS-SS system.
13. What are the Advantages of FH-SS System
14. What are the Disadvantages of FH-SS System
15. Define synchronization in Spread Spectrum Systems
16. Comparison between DS-SS and FH-SS
17. What are the Application of Direct Sequence Spread Spectrum
18. State the balance property of random binary sequence.
19. Mention about the run property.
20. What is called jamming effect.
21. What is Anti jamming ?
22. What is slow and fast frequency hopping.
23. What is called multipath Interference?

**PART B (12 Marks)**

1. What is Spread Spectrum Techniques Explain in detail about Direct Sequence Spread Spectrum Techniques with necessary diagrams?
  - i. Concept of Spread Spectrum Techniques
  - ii. Block Diagram Representation.
  - iii. Waveform at all stages of the system.
  - iv. Derivation of processing Gain.
2. What is Frequency Hopping? Explain the different types of frequency hopping with necessary diagrams.
  - i. Concept of frequency hopping.
  - ii. Explanation of slow frequency hopping
  - iii. Explanation of Fast frequency hopping
  - iv. Block Diagrams and waveform