UNIT IV KINEMATICS AND PATH PLANNING

Forward Kinematics – Denavit Hartenberg Representation - Inverse Kinematics – Geometric approach.

ROBOT KINEMATICS

Robot kinematics applies geometry to the study of the movement of multi-degree of freedom kinematic chains that form the structure of robotic systems. The emphasis on geometry means that the links of the robot are modeled as rigid bodies and its joints are assumed to provide pure rotation or translation.

Robot kinematics studies the relationship between the dimensions and connectivity of kinematic chains and the position, velocity and acceleration of each of the links in the robotic system, in order to plan and control movement and to compute actuator forces and torques. The relationship between mass and inertia properties, motion, and the associated forces and torques is studied as part of robot dynamics. The robot kinematics concepts related to both open and closed kinematics chains. Forward kinematics is distinguished from inverse kinematics.

SERIAL MANIPULATOR:

Serial manipulators are the most common industrial robots. They are designed as a series of links connected by motor-actuated joints that extend from a base to an end-effector. Often they have an anthropomorphic arm structure described as having a "shoulder", an "elbow", and a "wrist". Serial robots usually have six joints, because it requires at least six degrees of freedom to place a manipulated object in an arbitrary position and orientation in the workspace of the robot. A popular application for serial robots in today's industry is the pick-and-place assembly robot, called a SCARA robot, which has four degrees of freedom.

Fig 4.1 SCARA robot

STRUCTURE:

In its most general form, a serial robot consists of a number of rigid links connected with joints. Simplicity considerations in manufacturing and control have led to robots with only revolute or prismatic joints and orthogonal, parallel and/or intersecting joint axes the inverse kinematics of serial manipulators with six revolute joints, and with three consecutive joints intersecting, can be solved in closed-form, i.e. analytically this result had a tremendous influence on the design of industrial robots.

The main advantage of a serial manipulator is a large workspace with respect to the size of the robot and the floor space it occupies. The main disadvantages of these robots are:

- The low stiffness inherent to an open kinematic structure,
- Errors are accumulated and amplified from link to link,
- The fact that they have to carry and move the large weight of most of the actuators, and
- The relatively low effective load that they can manipulate.

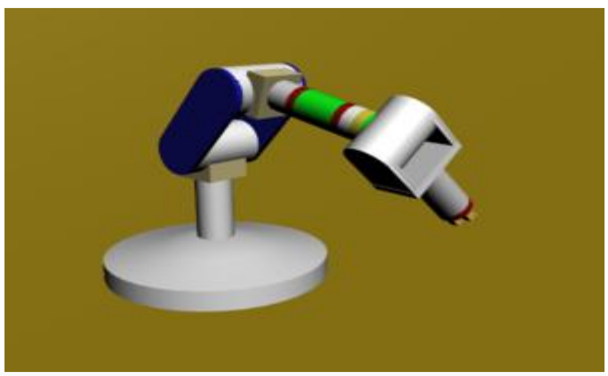


Fig.4.2 Serial manipulator with six DOF in a kinematic chain

PARALLEL MANIPULATOR:

A parallel manipulator is a mechanical system that uses several computer-controlled serial chains to support a single platform, or end-effector. Perhaps, the best known parallel manipulator is formed from six linear actuators that support a movable base for devices such as flight simulators. This device is called a Stewart platform or the Gough-Stewart platform in recognition of the engineers who first designed and used them.

Also known as parallel robots, or generalized Stewart platforms (in the Stewart platform, the actuators are paired together on both the basis and the platform), these systems are articulated robots that use similar mechanisms for the movement of either the robot on its base, or one or more manipulator arms. Their 'parallel' distinction, as opposed to a serial manipulator, is that the end effector (or 'hand') of this linkage (or 'arm') is connected to its base by a number of (usually three or six) separate and independent linkages working in parallel. 'Parallel' is used here in the computer science sense, rather than the geometrical; these linkages act together, but it is not implied that they are aligned as parallel lines; here *parallel* means that the position of the end point of each linkage is independent of the position of the other linkages.

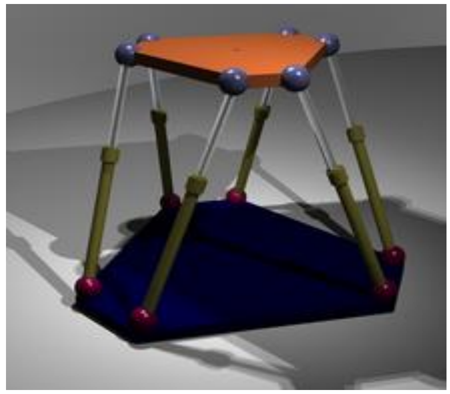


Fig: 4.3 Abstract render of a Hexapod platform (Stewart Platform)

Forward Kinematics:

It is used to determine where the robot's hand, if all joint variables are known)

Inverse Kinematics:

It is used to calculate what each joint variable, if we desire that the hand be located at a particular point.

ROBOTS AS MECHANISMS

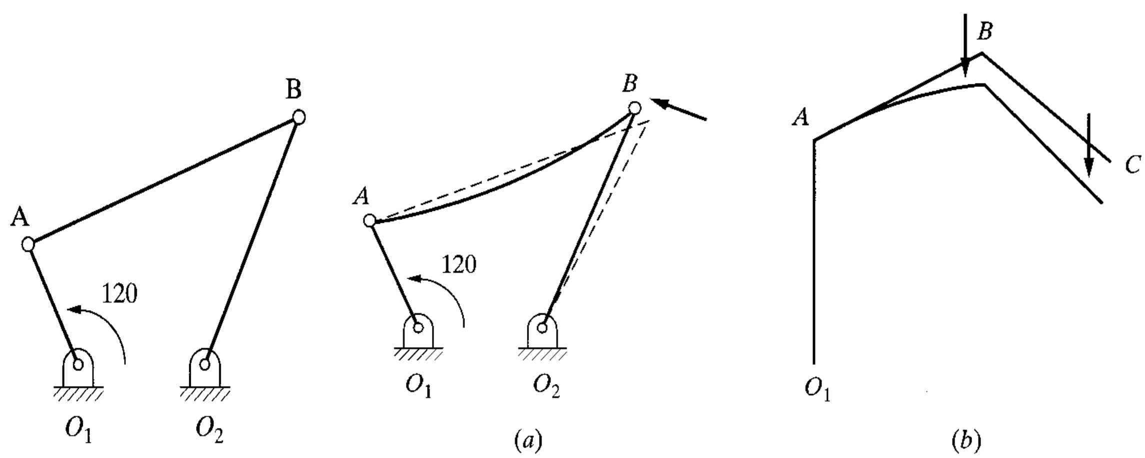
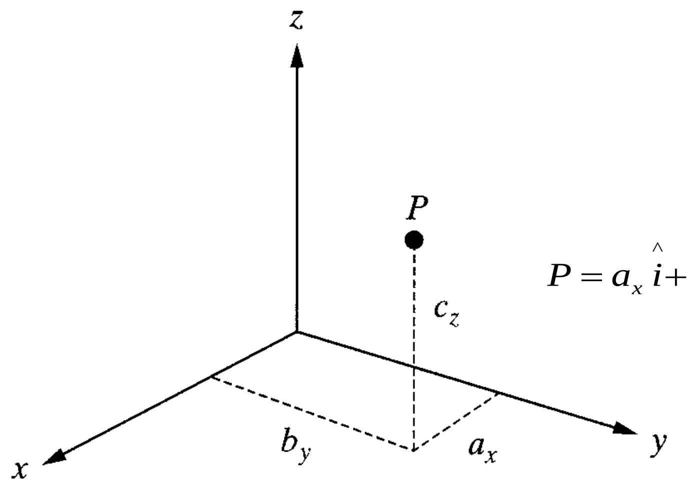


Fig 4.4 a one-degree-of-freedom closed-loop (a) Closed-loop versus (b) open-loop mechanism Four-bar mechanism

MATRIX REPRESENTATION

Representation of a Point in Space

A point *P* in space: 3 coordinate relative to a reference frame

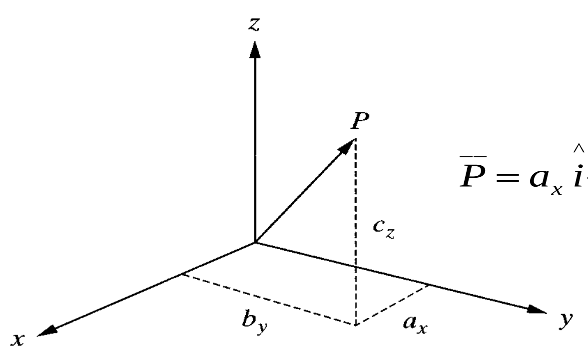


$$P = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

Fig. 4.5 Representation of a point in space

Representation of a Vector in Space

A Vector *P* in space: 3 coordinates of its tail and of its head



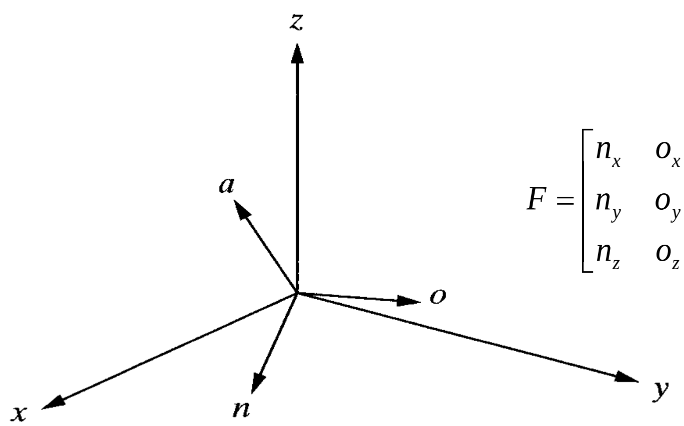
$$\bar{P} = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

Fig. 4.6 Representation of a vector in space

$$\bar{P} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Representation of a Frame at the Origin of a Fixed-Reference Frame

Each Unit Vector is mutually perpendicular: normal, orientation, approach vector



$$F = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}$$

Fig. 4.7 Representation of a frame at the origin of the reference frame

Representation of a Frame in a Fixed Reference Frame

Each Unit Vector is mutually perpendicular: normal, orientation, approach vector

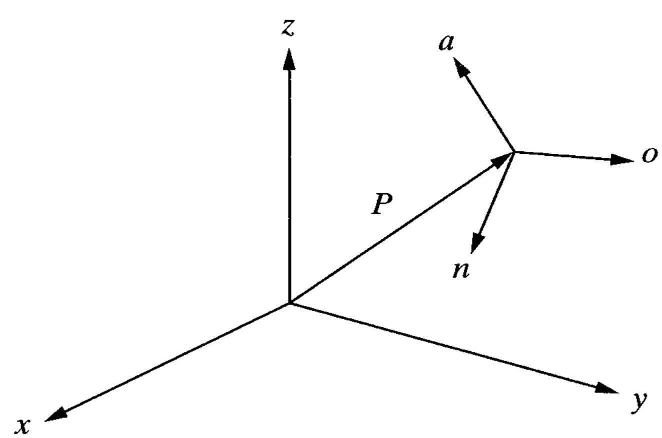


Fig. 4.8 Representation of a frame in a frame

$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Rigid Body

An object can be represented in space by attaching a frame to it and representing the frame in space.

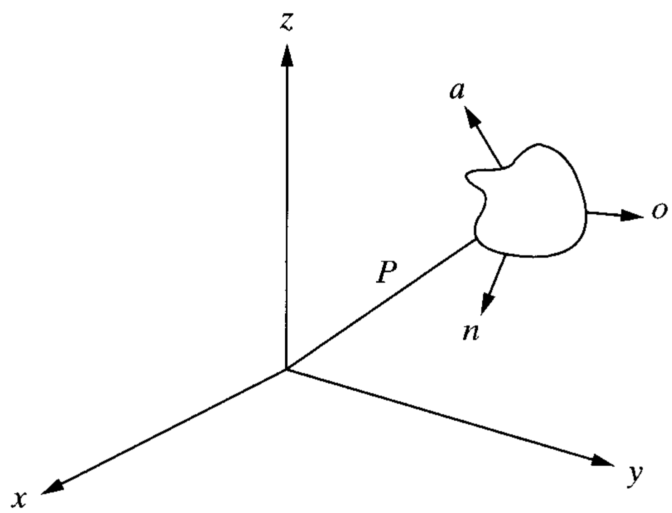


Fig. 4.9 Representation of an object in space

$$F_{object} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>HOMOGENEOUS TRANSFORMATION MATRICES</u>

Transformation matrices must be in square form. It is much easier to calculate the inverse of square matrices. To multiply two matrices, their dimensions must match.

Representation of a Pure Translation

- A transformation is defined as making a movement in space.
- A pure translation.
- A pure rotation about an axis.
- A combination of translation or rotations

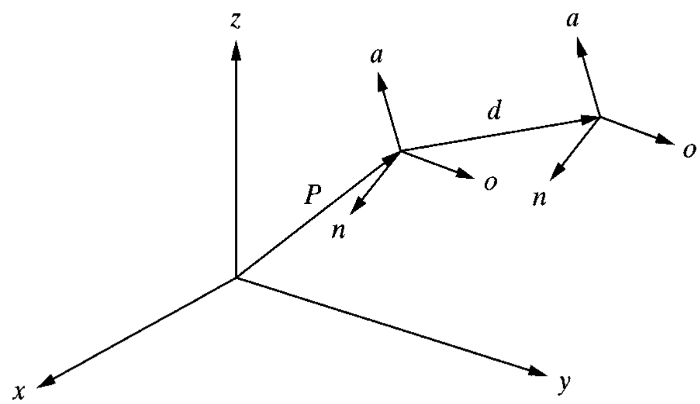


Fig. 4.10 Representation of a pure translation in space

$$T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation of a Pure Rotation about an Axis

Assumption: The frame is at the origin of the reference frame and parallel to it.

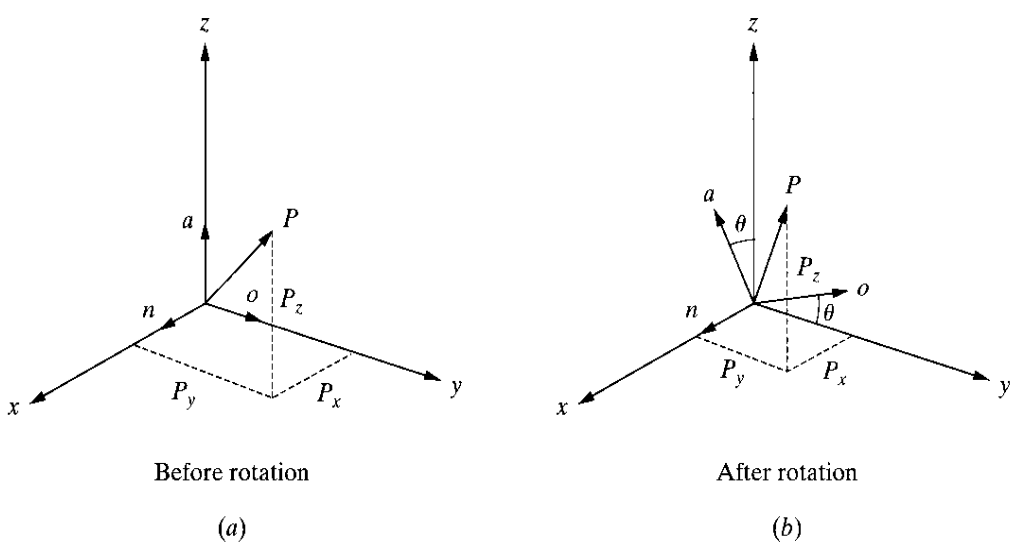


Fig. 4.11 Coordinates of a point in a rotating frame before and after rotation

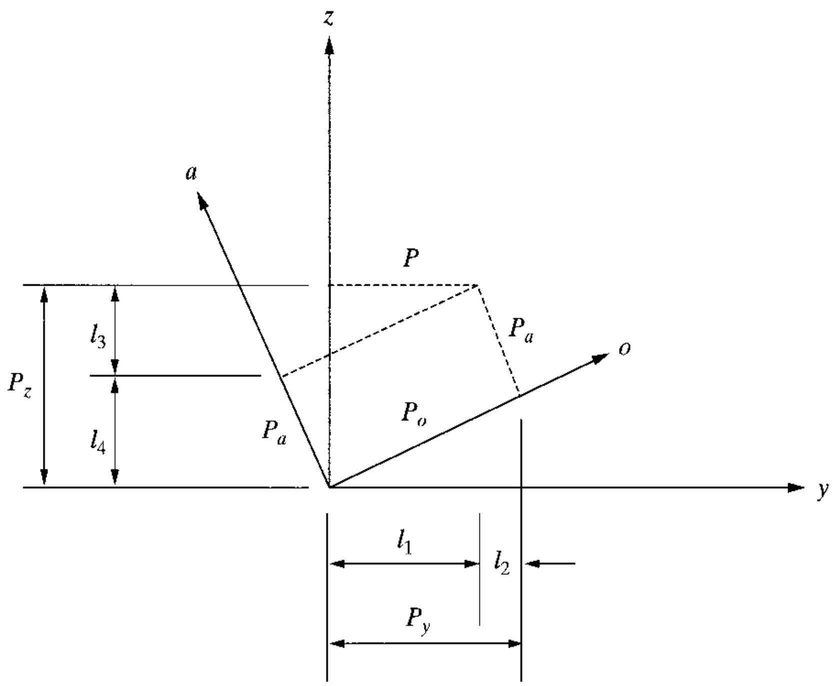


Fig. 4.12 Coordinates of a point relative to the reference

Representation of Combined Transformations

A number of successive translations and rotations

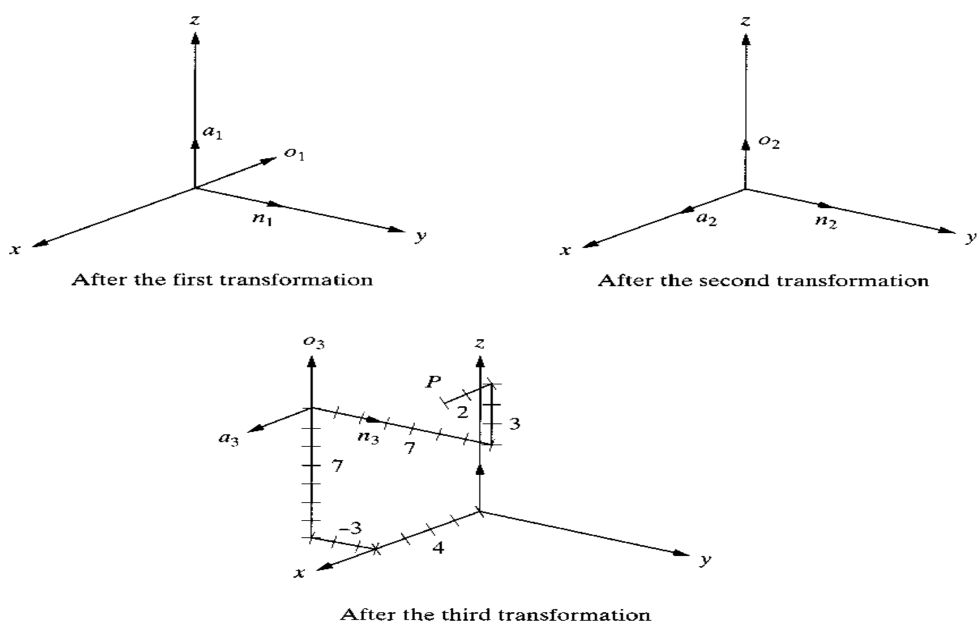


Fig. 4.13 Effects of three successive transformations

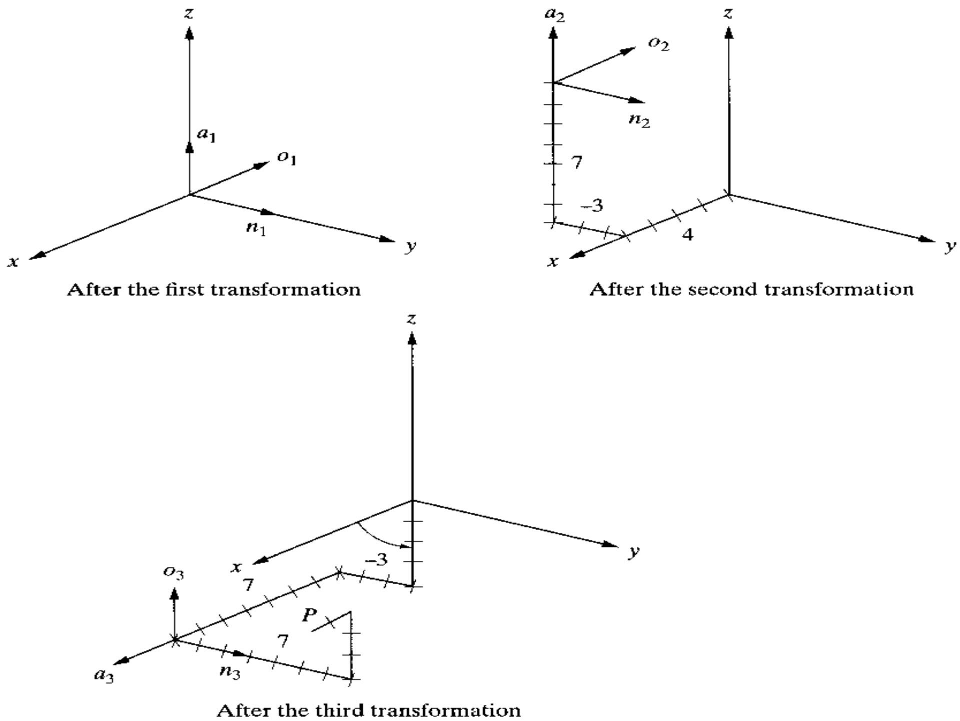


Fig 4.14 changing the order of transformations will change the final result

Transformations Relative to the Rotating Frame

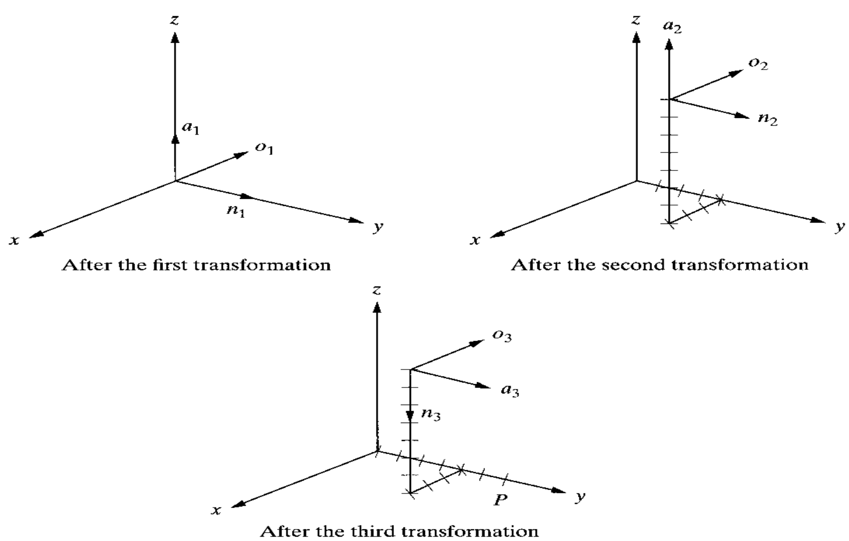


Fig. 4.15 Transformations relative to the current frames

KINEMATICS EQUATIONS:

A fundamental tool in robot kinematics is the kinematics equations of the kinematic chains that form the robot. These non-linear equations are used to map the joint parameters to the configuration of the robot system. Kinematics equations are also used in biomechanics of the skeleton and computer animation of articulated characters.

Forward kinematics uses the kinematic equations of a robot to compute the position of the end-effector from specified values for the joint parameters. The reverse process that computes the joint parameters that achieve a specified position of the end-effector is known as inverse kinematics. The dimensions of the robot and its kinematics equations define the volume of space reachable by the robot, known as its workspace.

There are two broad classes of robots and associated kinematics equations serial manipulators and parallel manipulators. Other types of systems with specialized kinematics equations are air, land, and submersible mobile robots, hyper-redundant, or snake, robots and humanoid robots.

DENAVIT-HARTENBERG PARAMETERS:

The Denavit–Hartenberg parameters (also called DH parameters) are the four parameters associated with a particular convention for attaching reference frames to the links of a spatial kinematic chain, or robot manipulator.

Denavit-Hartenberg convention:

A commonly used convention for selecting frames of reference in robotics applications is the **Denavit and Hartenberg (D–H) convention**. In this convention, coordinate frames are attached to the joints between two links such that one transformation is associated with the joint, [Z], and the second is associated with the link [X]. The coordinate transformations along a serial robot consisting of *n* links form the kinematics equations of the robot,

$$[T] = [Z_1][X_1][Z_2][X_2]\ldots[X_{n-1}][Z_n],$$

Where, [T] is the transformation locating the end-link.

In order to determine the coordinate transformations [Z] and [X], the joints connecting the links are modeled as either hinged or sliding joints, each of which have a unique line S in space that forms the joint axis and define the relative movement of the two links. A typical serial robot is characterized by a sequence of six lines S_i, $i=1,...,6$, one for each joint in the robot. For each sequence of lines S_i and S_{i+1}, there is a common normal line $A_{i,i+1}$. The system of six joint axes S_i and five common normal lines $A_{i,i+1}$ form the kinematic skeleton of the typical six degree of freedom serial robot. Denavit and Hartenberg introduced the convention that Z coordinate axes are assigned to the joint axes S_i and X coordinate axes are assigned to the common normal's $A_{i,i+1}$.

This convention allows the definition of the movement of links around a common joint axis S_i by the screw displacement,

$$[Z_i] = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Where ϑ_i is the rotation around and d_i is the slide along the Z axis---either of the parameters can be constants depending on the structure of the robot. Under this convention the dimensions of each link in the serial chain are defined by the screw displacement around the common normal $A_{i,i+1}$ from the joint S_i to S_{i+1}, which is given by

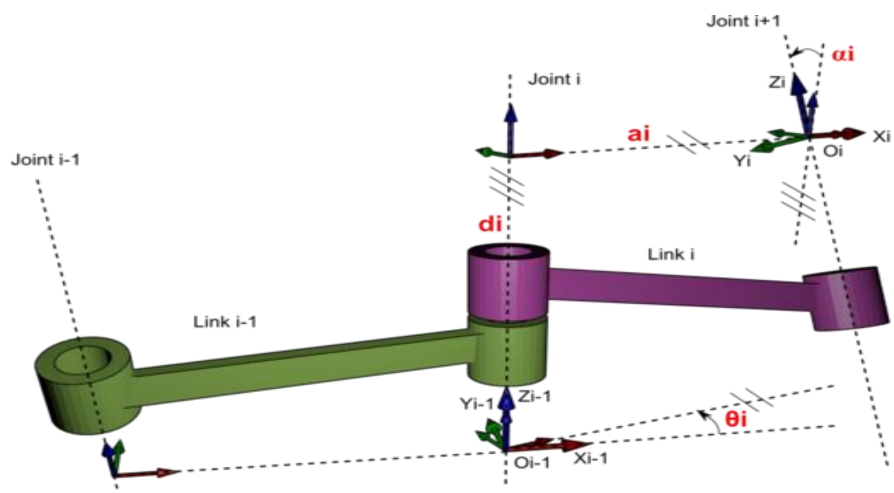
$$[X_i] = \begin{bmatrix} 1 & 0 & 0 & r_{i,i+1} \\ 0 & \cos\alpha_{i,i+1} & -\sin\alpha_{i,i+1} & 0 \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Where $\alpha_{i,i+1}$ and $r_{i,i+1}$ define the physical dimensions of the link in terms of the angle measured around and distance measured along the X axis.

In summary, the reference frames are laid out as follows:

- the z-axis is in the direction of the joint axis
- the x-axis is parallel to the common normal: $x_n = z_{n-1} \times z_n$ If there is no unique common normal (parallel z axes), then d (below) is a free parameter. The direction of x_n is from z_{n-1} to z_n, as shown in the video below.
- the y-axis follows from the x- and z-axis by choosing it to be a right-handed coordinate system.

<u>Four parameters</u>



The four parameters of classic DH convention are θ_i , d_i, a_i , α_i. With those four parameters, we can translate the coordinates from $O_{i-1}X_{i-1}Y_{i-1}Z_{i-1}$ to $O_iX_iY_iZ_i$.

The transformation the following four parameters known as D–H parameters:

d: offset along previous ***z*** to the common normal

θ: angle about previous ***z***, from old **x** to new **x**

r: length of the common normal. Assuming a revolute joint, this is the radius about previous z.

α: angle about common normal, from old ***z*** axis to new ***z*** axis

There is some choice in frame layout as to whether the previous ***x*** axis or the next **x** points along the common normal. The latter system allows branching chains more efficiently, as multiple frames can all point away from their common ancestor, but in the alternative layout the ancestor can only point toward one successor. Thus the commonly used notation places each down-chain ***x*** axis collinear with the common normal, yielding the transformation calculations shown below.

We can note constraints on the relationships between the axes:

- x_n-axis is perpendicular to both the z_{n-1} and z_n axes
- x_n-axis intersects both z_{n-1} and z_n axes
- Origin of joint n is at the intersection of x_n and z_n
- y_n completes a right-handed reference frame based on x_n and z_n

Denavit-Hartenberg Matrix:

It is common to separate a screw displacement into the product of a pure translation along a line and a pure rotation about the line,[5][6] so that

$$[Z_i] = \mathrm{Trans}_{Z_i}(d_i)\,\mathrm{Rot}_{Z_i}(\theta_i),$$

And,

$$[X_i] = \mathrm{Trans}_{X_i}(r_{i,i+1})\,\mathrm{Rot}_{X_i}(\alpha_{i,i+1}).$$

Using this notation, each link can be described by a coordinate transformation from the previous coordinate system to the next coordinate system.

$${}^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n)$$

Note that this is the product of two screw displacements, the matrices associated with these operations are:

$$\operatorname{Trans}_{z_{n-1}}(d_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\operatorname{Rot}_{z_{n-1}}(\theta_n) = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\operatorname{Trans}_{x_n}(r_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & r_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\operatorname{Rot}_{x_n}(\alpha_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

This gives:

$${}^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Where R is the 3×3 sub matrix describing rotation and T is the 3×1 sub matrix describing translation.

<u>DENAVIT-HARTENBERG REPRESENTATION OF FORWARD KINEMATIC EQUATIONS OF ROBOT</u>:

Denavit-Hartenberg Representation:

1. Simple way of modeling robot links and joints for any robot configuration, regardless of its sequence or complexity.
2. Transformations in any coordinates are possible.
3. Any possible combinations of joints and links and all-revolute articulated robots can be represented

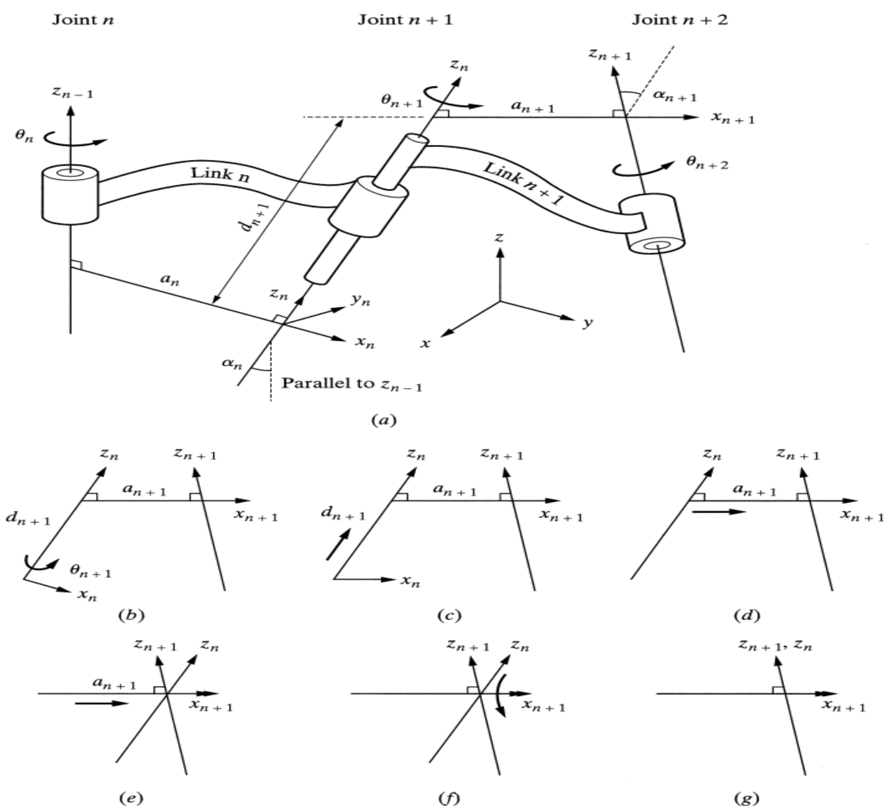


Fig 4.16 a D-H representation of a general-purpose joint-link combination

<u>DENAVIT-HARTENBERG REPRESENTATION PROCEDURES</u>:

Start point:

- Assign joint number *n* to the first shown joint.
- Assign a local reference frame for each and every joint before or after these joints.
- *Y*-axis does not used in D-H representation.

Procedures for assigning a local reference frame to each joint:

All joints are represented by a *z*-axis. (Right-hand rule for rotational joint, linear movement for prismatic joint)

- The common normal is one line mutually perpendicular to any two skew lines.
- Parallel *z*-axes joints make a infinite number of common normal.
- Intersecting *z*-axes of two successive joints make no common normal between them(Length is 0.).

Symbol Terminologies:

- θ : A rotation about the *z*-axis.
- *d* : The distance on the *z*-axis.
- *a* : The length of each common normal (Joint offset).
- α : The angle between two successive *z*-axes (Joint twist)

Only θ and *d* are joint variables

The necessary motions to transform from one reference frame to the next.

I) Rotate about the z_n-axis an able of θ_{n+1}. (Coplanar)

II) Translate along z_n-axis a distance of d_{n+1} to make x_n and x_{n+1} colinear.

III) Translate along the x_n-axis a distance of a_{n+1} to bring the origins of x_{n+1} together.

IV) Rotate z_n-axis about x_{n+1} axis an angle of α_{n+1} to align z_n-axis with z_{n+1}-axis.

Determine the value of each joint to place the arm at a desired position and orientation.

$$^{R}T_H = A_1A_2A_3A_4A_5A_6$$

$$= \begin{bmatrix} \begin{matrix} C_1(C_{234}C_5C_6 - S_{234}S_6) \\ -S_1S_5C_6 \end{matrix} & \begin{matrix} C_1(-C_{234}C_5C_6 - S_{234}C_6) \\ +S_1S_5C_6 \end{matrix} & C_1(C_{234}S_5) + S_1C_5 & C_1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ \begin{matrix} S_1(C_{234}C_5C_6 - S_{234}S_6) \\ +C_1S_5C_6 \end{matrix} & \begin{matrix} S_1(-C_{234}C_5C_6 - S_{234}C_6) \\ -C_1S_5C_6 \end{matrix} & S_1(C_{234}S_5) - C_1C_5 & S_1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ S_{234}C_5C_6 + C_{234}S_6 & -S_{234}C_5C_6 + C_{234}C_6 & S_{234}S_5 & S_{234}a_4 + S_{23}a_3 + S_2a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

THE INVERSE KINEMATIC SOLUTION OF ROBOT:

$$A_1^{-1}\times\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1^{-1}[RHS] = A_2A_3A_4A_5A_6$$

$$\begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ S_1 & -C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\times\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2A_3A_4A_5A_6$$

$$\theta_1 = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

$$\theta_2 = \tan^{-1}\frac{(C_3a_3 + a_2)(p_z - S_{234}a_4) - S_3a_3(p_xC_1 + p_yS_1 - C_{234}a_4)}{(C_3a_3 + a_2)(p_xC_1 + p_yS_1 - C_{234}a_4) + S_3a_3(P_z - S_{234}a_4)}$$

$$\theta_3 = \tan^{-1}\left(\frac{S_3}{C_3}\right)$$

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3$$

$$\theta_5 = \tan^{-1}\frac{C_{234}(C_1a_x + S_1a_y) + S_{234}a_z}{S_1a_x - C_1a_y}$$

$$\theta_6 = \tan^{-1}\frac{-S_{234}(C_1n_x + S_1n_y) + S_{234}n_z}{-S_{234}(C_1o_x + S_1o_y) + C_{234}o_z}$$

INVERSE KINEMATIC PROGRAM OF ROBOTS:

A robot has a predictable path on a straight line, or an unpredictable path on a straight line.

- A predictable path is necessary to recalculate joint variables. (Between 50 to 200 times a second)
- To make the robot follow a straight line, it is necessary to break the line into many small sections.
- All unnecessary computations should be eliminated.

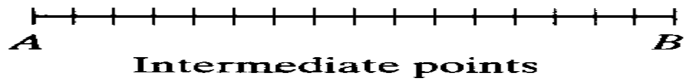


Fig. 4.17 Small sections of movement for straight-line motions

DEGENERACY AND DEXTERITY:

Degeneracy: The robot looses a degree of freedom and thus cannot perform as desired.

- When the robot's joints reach their physical limits, and as a result, cannot move any further.
- In the middle point of its workspace if the *z*-axes of two similar joints becomes co-linear.

Dexterity: The volume of points where one can position the robot as desired, but not orientate it.

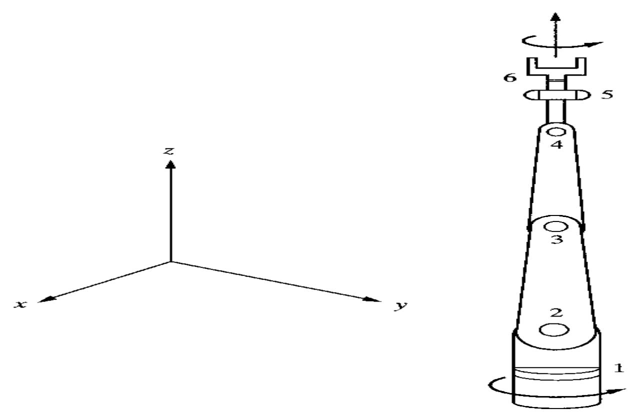


Fig. 4.18 An example of a robot in a degenerate position

THE FUNDAMENTAL PROBLEM WITH D-H REPRESENTATION:

Defect of D-H presentation: D-H cannot represent any motion about the *y*-axis, because all motions are about the *x*- and *z*-axis.

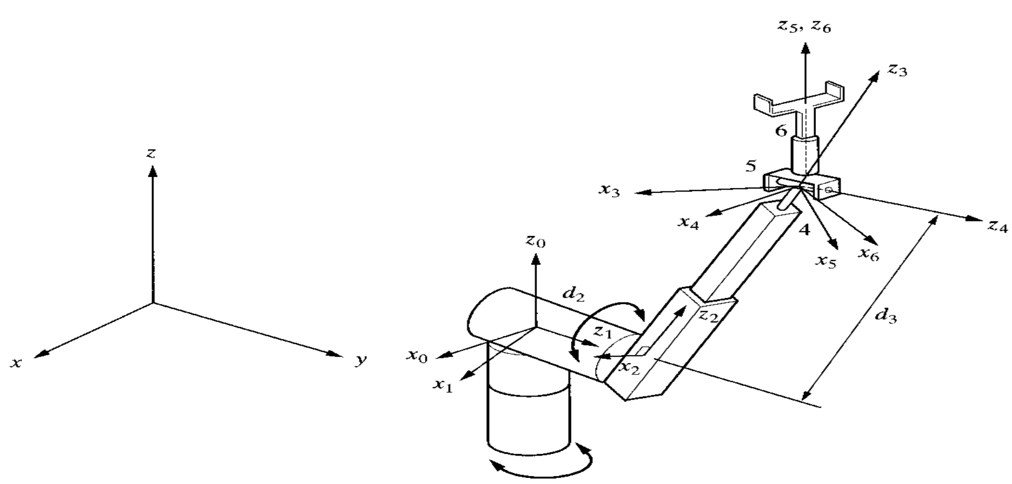


Fig. 4.19 the frames of the Stanford Arm.

#	θ	d	a	α
1	θ_1	0	0	-90
2	θ_2	d_1	0	90
3	0	d_1	0	0
4	θ_4	0	0	-90
5	θ_5	0	0	90
6	θ_6	0	0	0

Table 4.1 Parameters Table for the Stanford Arm

INVERSE OF TRANSFORMATION MATIRICES

Inverse of a matrix calculation steps:

- Calculate the determinant of the matrix.
- Transpose the matrix.
- Replace each element of the transposed matrix by its own minor (ad-joint matrix).
- Divide the converted matrix by the determinant.

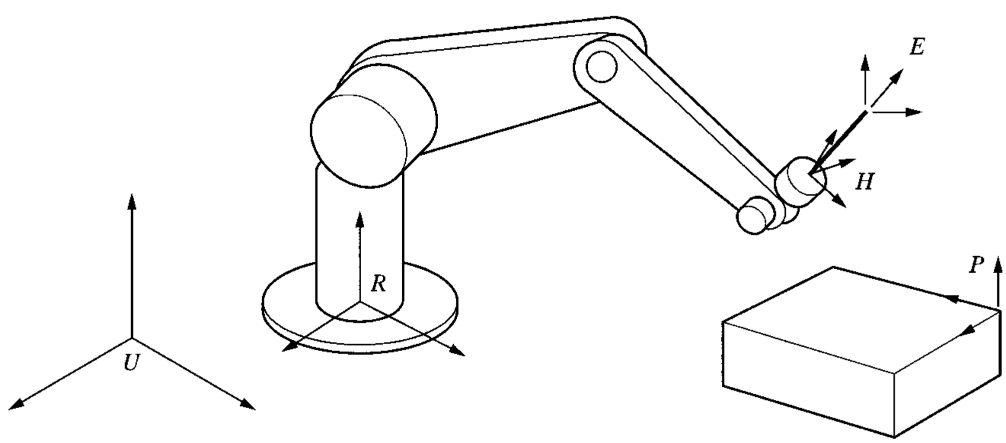


Fig 4.20 The Universe, robot, hand, part, and end effecter frames.

FORWARD AND INVERSE KINEMATICS OF ROBOTS:

Forward Kinematics Analysis:

- Calculating the position and orientation of the hand of the robot.
- If all robot joint variables are known, one can calculate where the robot is at any instant.

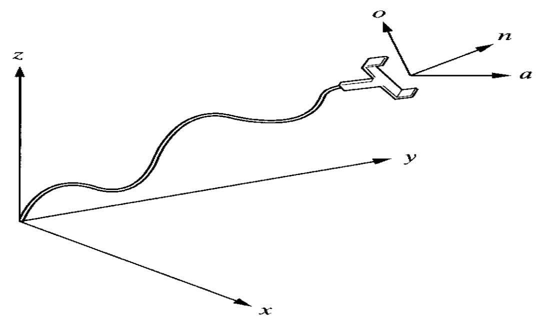


Fig. 4.21 The hand frame of the robot relative to the reference frame

Forward Kinematics and Inverse Kinematics equation for position analysis:

a) Cartesian (gantry, rectangular) coordinates.
b) Cylindrical coordinates.
c) Spherical coordinates.
d) Articulated (anthropomorphic, or all-revolute) coordinates

Forward and Inverse Kinematics Equations for Position

(a) Cartesian (Gantry, Rectangular) Coordinates: IBM 7565 robot

- All actuator is linear
- A gantry robot is a Cartesian robot

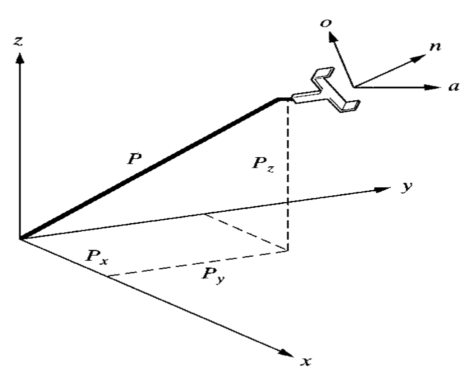
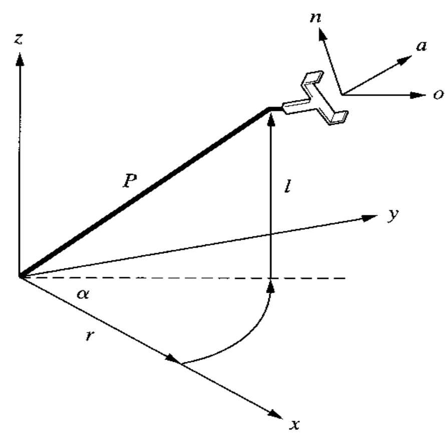


Fig. 4.22 Cartesian Coordinates

$$^{R}T_{P} = T_{cart} = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Cylindrical Coordinates: 2 Linear translations and 1 rotation

- translation of *r* along the *x*-axis
- rotation of α about the *z*-axis
- translation of *l* along the *z*-axis



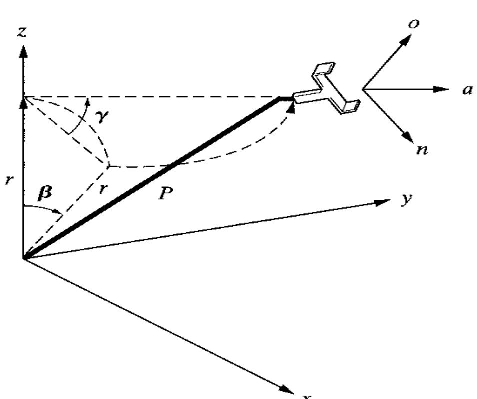
$$ {}^{R}T_{P} = T_{cyl}(r, \alpha, l) = \mathrm{Trans}(0,0,l)\mathrm{Rot}(z, \alpha)\mathrm{Trans}(r,0,0) $$

$$ {}^{R}T_{P} = T_{cyl} = \begin{bmatrix} C\alpha & -S\alpha & 0 & rC\alpha \\ S\alpha & C\alpha & 0 & rS\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Fig. 4.23 Cylindrical Coordinates

(c) Spherical Coordinates: 1 linear translation and 2 rotations

- translation of *r* along the *z*-axis
- rotation of β about the *y*-axis
- rotation of γ along the *z*-axis



$$ {}^{R}T_{P} = T_{sph}(r, \beta, l) = \mathrm{Rot}(z, \gamma)\mathrm{Rot}(y, \beta)\mathrm{Trans}(0,0,\gamma) $$

$$ {}^{R}T_{P} = T_{sph} = \begin{bmatrix} C\beta \cdot C\gamma & -S\gamma & S\beta \cdot C\gamma & rS\beta \cdot C\gamma \\ C\beta \cdot S\gamma & C\gamma & S\beta \cdot S\gamma & rS\beta \cdot S\gamma \\ -S\beta & 0 & C\beta & rC\beta \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Fig. 4.24 Spherical Coordinates

(d) Articulated Coordinates: 3 rotations -> Denavit-Hartenberg representation

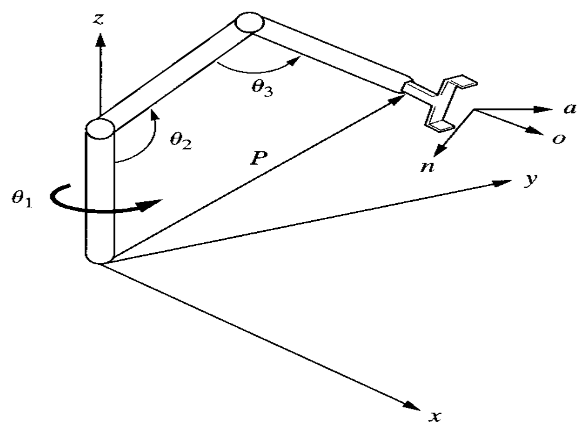


Fig. 4.25 Articulated Coordinates.

Forward and Inverse Kinematics Equations for Orientation

- ♦Roll, Pitch, Yaw (RPY) angles
- ♦ Euler angles
- ♦ Articulated joints

(a) Roll, Pitch, Yaw (RPY) Angles

- Roll: Rotation of about - axis (*z*-axis of the moving frame)
- Pitch: Rotation of about - axis (*y*-axis of the moving frame)
- Yaw: Rotation of about - axis (*x*-axis of the moving frame)

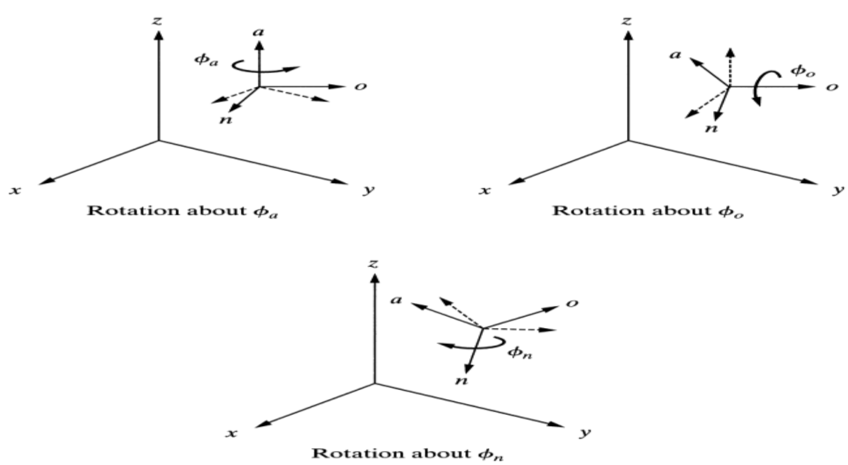


Fig. 4.26 RPY rotations about the current axes

(b) Euler Angles

- Rotation of about - axis (*z*-axis of the moving frame) followed by
- Rotation of about -axis (*y*-axis of the moving frame) followed by
- Rotation of about -axis (*z*-axis of the moving frame)

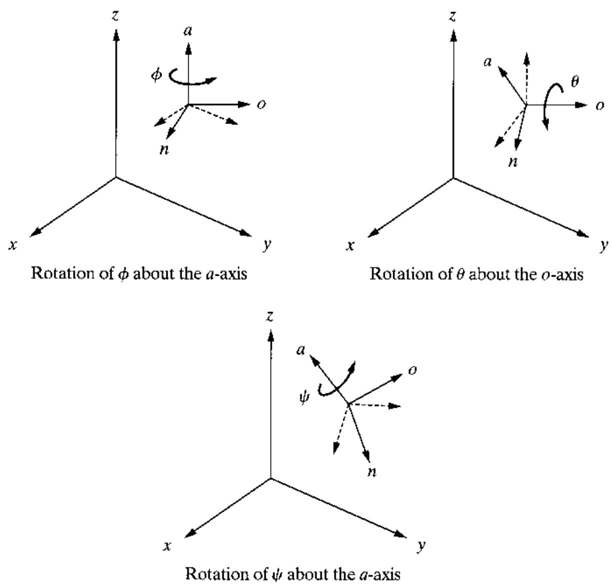


Fig. 4.27 Euler rotations about the current axes

Forward and Inverse Kinematics Equations for Orientation:

Assumption : Robot is made of a Cartesian and an RPY set of joints.

$$^{R}T_{H} = T_{cart}(P_x, P_y, P_z) \times RPY(\phi_a, \phi_o, \phi_n)$$

Assumption : Robot is made of a Spherical Coordinate and an Euler angle.

$$^{R}T_{H} = T_{sph}(r, \beta, \gamma) \times Euler(\phi, \theta, \psi)$$

Questions

PART-A

1. What are the methods to obtain the jacobian for a six –link manipulator with rotator joints?
2. Give an example of inverse kinematics
3. What is DH convention?
4. What is degeneracy and dexterity?
5. What is Euler's angle?

PART-B

1. For the point 3i+7j+5k,perform the following operations
   a) Rotate 30 degree about the x axis
   b) Rotate 45 degree about the y axis
   c) Rotate 90 degree about the z axis
   d) Rotate 30 degree about the x axis, then translate 6 along y axis
2. Explain the kinematics of an industrial manipulator.
3. Explain the homogenous transformation matrix approach in planning of Cartesian path trajectories.
4. Given the end of arm position in world space, derive the equation to find the joint angles of a two degree of freedom robot arm.