

SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF ELECTRONICS AND COMMUNICATION

COURSE MATERIAL

Subject Name : Digital Communications

Subject Code : SEC1313

UNIT III - DIGITAL MODULATION TECHNIQUES

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 - 2.1 Generation and Detection of ASK Signals
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1. Introduction

Given a binary source that emits symbols 0 and 1, the modulation process involves switching or keying the amplitude, phase, or frequency of a sinusoidal carrier wave between a pair of possible values in accordance with symbols 0 and 1. To be more specific, consider the sinusoidal carrier

$$c(t) = A_c \cos(2\pi f_c t + \phi_c) \quad \text{-----(1)}$$

Where A_c is the carrier amplitude, f_c is the carrier frequency, and ϕ_c is the carrier phase. Given these three parameters of the carrier(t), we may now identify three distinct forms of binary modulation:

1. Binary amplitude shift-keying (BASK), in which the carrier frequency and carrier phase are both maintained constant, while the carrier amplitude is keyed between the two possible values used to represent symbols 0 and 1.

2. Binary phase-shift keying (BPSK), in which the carrier amplitude and carrier frequency are both maintained constant, while the carrier phase is keyed between the two possible values (e.g., 0° and 180°) used to represent symbols 0 and 1.

3. Binary frequency-shift keying (BFSK), in which the carrier amplitude and carrier phase are both maintained constant, while the carrier frequency is keyed between the two possible values used to represent symbols 0 and 1.

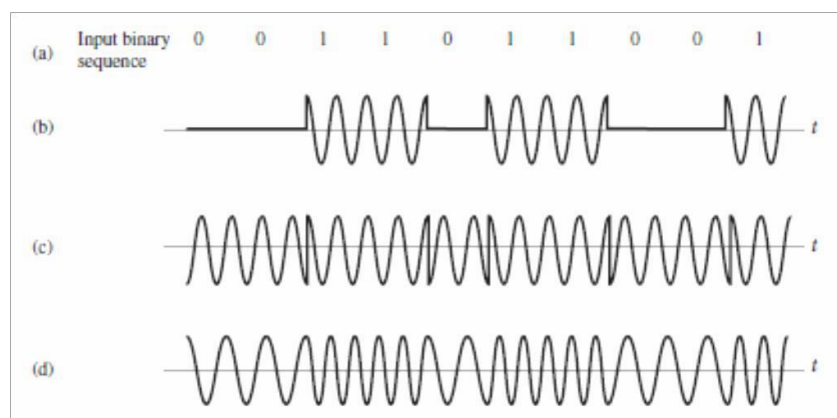


Fig. 3 Types of Binary signaling Information

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The three basic forms of signaling binary information. (a) Binary data stream. (b) Amplitude-shift keying. (c) Phase-shift keying. (d) Frequency-shift keying with continuous phase.

In the digital communications literature, the usual practice is to assume that the carrier $c(t)$ has unit energy measured over one symbol (bit) duration.

$$A_c = \sqrt{\frac{2}{T_b}} \quad \text{-----(2)}$$

Where T_b is the bit duration. Using the terminology of Eq. (2), we may thus express the Carrier $c(t)$ in the equivalent form

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \phi_c) \quad \text{-----(3)}$$

The spectrum of a digitally modulated wave, exemplified by BASK, BPSK and BFSK, is centered on the carrier frequency f_c implicitly or explicitly. Moreover, as with analog modulation, it is normal practice to assume that the carrier frequency f_c is large compared with the "bandwidth" of the incoming binary data stream that acts as the modulating signal. This band-pass assumption has certain implications, as discussed next. To be specific, consider a linear modulation scheme for which the modulated wave is defined by

$$s(t) = b(t)c(t) \quad \text{-----(4)}$$

where $b(t)$ denotes an incoming binary wave. Then, setting the carrier phase ϕ_c for convenience of presentation, we may use Eq. (3) to express the modulated wave as

$$s(t) = \sqrt{\frac{2}{T_b}} b(t) \cos(2\pi f_c t) \quad \text{-----(5)}$$

Under the assumption where $f_c \gg w$ is the bandwidth of the binary wave there will be no spectral

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overlap in the generation of $s(t)$ (i.e., the spectral content of the modulated wave for positive frequencies is essentially separated from its spectral content for negative frequencies). Another implication of the band-pass assumption is that we may express the transmitted signal energy per bit as

$$\begin{aligned} E_b &= \int_0^{T_b} |s(t)|^2 dt \\ &= \frac{2}{T_b} \int_0^{T_b} |b(t)|^2 \cos^2(2\pi f_c t) dt \end{aligned} \quad \text{-----(6)}$$

Using the trigonometric identity

$$\cos^2(2\pi f_c t) = \frac{1}{2} [1 + \cos(4\pi f_c t)]$$

we may rewrite Eq. (6) as

$$E_b = \frac{1}{T_b} \int_0^{T_b} |b(t)|^2 dt + \frac{1}{T_b} \int_0^{T_b} |b(t)|^2 \cos(4\pi f_c t) dt \quad \text{-----(7)}$$

The band-pass assumption implies that $|b(t)|^2$ is essentially constant over one complete cycle of the sinusoidal wave $\cos(4\pi f_c t)$ which, in turn, means that

$$\int_0^{T_b} |b(t)|^2 \cos(4\pi f_c t) dt \approx 0$$

Accordingly, we may approximate Eq. (7) as

$$E_b \approx \frac{1}{T_b} \int_0^{T_b} |b(t)|^2 dt \quad \text{-----(8)}$$

In words, for linear digital modulation schemes governed by Eq. (5), the transmitted signal

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energy (on a per bit basis) is a scaled version of the energy in the incoming binary wave responsible for modulating the sinusoidal carrier.

2. BINARY AMPLITUDE-SHIFT KEYING

Binary amplitude-shift keying (BASK) is one of the earliest forms of digital modulation used in radio telegraphy at the beginning of the twentieth century. To formally describe BASK, consider a binary data stream $b(t)$ which is of the ON-OFF signaling variety. That is, $b(t)$ is defined by

$$b(t) = \begin{cases} \sqrt{E_b}, & \text{for binary symbol 1} \\ 0, & \text{for binary symbol 0} \end{cases}$$

Then, multiplying $b(t)$ by the sinusoidal carrier wave of Eq. (3) with the phase set ϕ_c equal to zero for convenience of presentation, we get the BASK wave

$$s(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), & \text{for symbol 1} \\ 0, & \text{for symbol 0} \end{cases} \text{-----}(9)$$

The carrier frequency f_c may have an arbitrary value, consistent with transmitting the modulated signal anywhere in the electromagnetic radio spectrum, so long as it satisfies the band-pass assumption. When a bit duration is occupied by symbol 1, the transmitted signal energy E_b is When the bit duration is occupied by symbol 0, the transmitted signal energy is zero. On this basis, we may express the average transmitted signal energy as

$$E_{av} = \frac{E_b}{2}$$

-----(10)

2.1 GENERATION AND DETECTION OF ASK SIGNALS

From Eqs. (8) and (9), we readily see that a BASK signal is readily generated by using a product modulator with two inputs. One input, the ON-OFF signal of Eq. (8), is the modulating

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signal. The sinusoidal carrier wave supplies the other input.

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

----- (11)

2.2 SPECTRAL ANALYSIS OF BASK

Consider a binary data stream that consists of a square wave, the amplitude of which alternates between the constant levels and zero every T_b seconds. The square wave is centered on the origin for convenience of the presentation. The objective of the experiment is twofold:

- To investigate the effect of varying the carrier frequency on the power spectrum of the BASK signal assuming that the square wave is fixed. Recall that the power spectrum of a signal (expressed in decibels) is defined as 10 times the logarithm (to base 10) of the squared magnitude (amplitude) spectrum of the signal.
- To investigate the effect of varying the frequency of the square wave on the spectrum of the BASK signal, assuming that the sinusoidal carrier wave is fixed. For the purpose of computer evaluation, we set the carrier frequency $f_c = n/T_b$ where n is an integer.
- This choice of the carrier frequency f_c permits the simulation of a band-pass system on a digital computer without requiring $f_c \gg 1/T_b$ the only restriction on the choice is to make sure that spectral overlap is avoided.

3. BINARY PHASE-SHIFT KEYING (BPSK)

In the simplest form of phase-shift keying known as binary phase-shift keying (BPSK), the pair of signals $s_1(t)$ and $s_2(t)$ used to represent symbols 1 and 0, respectively, are defined by

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), & \text{for symbol 1 corresponding to } i = 1 \\ \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), & \text{for symbol 0 corresponding to } i = 2 \end{cases}$$

----- (11)

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Where with T_b denoting the bit duration E_b and denoting the transmitted signal energy per bit; see the waveform of Fig. 7.1 (c) for a representation example of BPSK. A pair of sinusoidal waves, $s_1(t)$ and $s_2(t)$ which differ only in a relative phase-shift of π radians as defined in Eq. (7.12), are referred to as antipodal signals. BPSK differs from BASK in an important

respect: the envelope of the modulated signal $s(t)$ is maintained constant at the value $\sqrt{E_b/T_b}$ for all time t . This property, which follows directly from Eq. (7.12), has two important consequences:

1. The transmitted energy per bit, E_b is constant; equivalently, the average transmitted power is constant.
2. Demodulation of BPSK cannot be performed using envelope detection; rather, we have to look to coherent detection as described next.

3.1 GENERATION OF BPSK SIGNALS

To generate the BPSK signal, we build on the fact that the BPSK signal is a special case of DSB-SC modulation. Specifically, we use a product modulator consisting of two components

- (i) Non-return-to-zero level encoder, whereby the input binary data sequence is encoded in polar form with symbols 1 and 0 represented by the constant-amplitude levels: $+1$ and -1 , respectively.

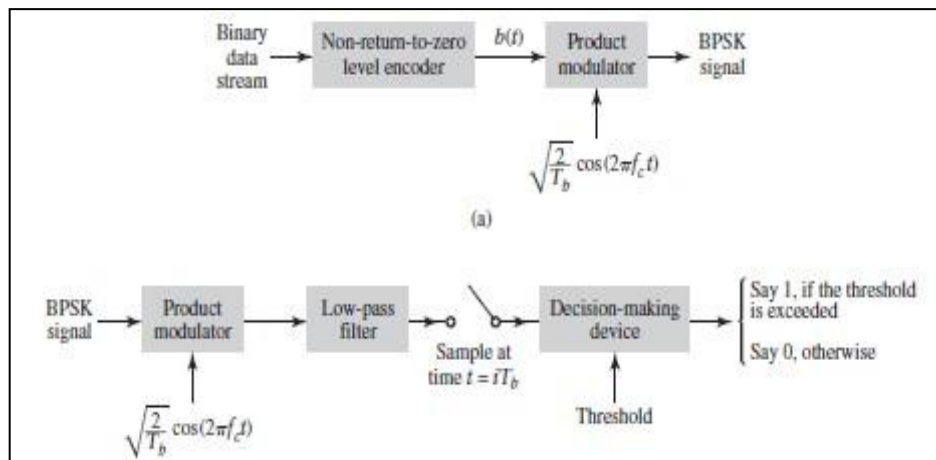


Fig 3.1 (a) BPSK modulator. (b) Coherent detector for BPSK; for the sampler, integer $i = 0, 1, 2, \dots$

- (ii) Product modulator, which multiplies the level-encoded binary wave by the sinusoidal carrier $c(t)$ of amplitude $\sqrt{E_b/T_b}$ to produce the BPSK signal. The timing pulses used to generate the level-

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encoded binary wave and the sinusoidal carrier wave are usually, but not necessarily, extracted from a common master clock.

3.2 COHERENT DETECTION OF BPSK SIGNALS

To detect the original binary sequence of 1s and 0s, the BPSK signal $x(t)$ at the channel output is applied to a receiver that consists of four sections, as depicted in Fig. 3.1(b)

- (i) Product modulator, which is also supplied with a locally generated reference signal that is a replica of the carrier wave $c(t)$.
- (ii) Low-pass filter, designed to remove the double-frequency components of the product modulator output and pass the zero-frequency components.
- (iii) Sampler, which uniformly samples the output of the low-pass filter at $t=iT_b$ where i is the local clock governing the operation of the sampler is synchronized with the clock responsible for bit-timing in the transmitter.
- (iv) Decision-making device, which compares the sampled value of the low-pass filter's output to an externally supplied threshold, every T_b seconds. If the threshold is exceeded, the device decides in favor of symbol 1; otherwise, it decides in favour of symbol 0.

The BPSK receiver described in Fig. 3.1 is said to be coherent in the sense that the sinusoidal reference signal applied to the product modulator in the demodulator is synchronous in phase (and, of course, frequency) with the carrier wave used in the modulator. In addition to synchrony with respect to carrier phase, the receiver also has an accurate knowledge of the interval occupied by each binary symbol.

The operation of the coherent BPSK receiver in Fig. 3.4(b) follows a procedure similar to that described for the demodulation of a double-sideband suppressed-carrier (DSBSC) modulated wave with a couple of important additions: sampler and decision-making device. The rationale for this similarity builds on what we have already stated: BPSK is simply another form of DSB-SC modulation. However, an issue that needs particular attention is how to design the low-pass filter in Fig. 3.4(b)

3.3 SPECTRAL ANALYSIS OF BPSK

As with the experiment on BASK, consider a binary data stream that consists of a square wave, the amplitude of which alternates between every T_b seconds. The square wave is centered on the origin. The objectives of this second experiment are similar to those of Computer Experiment I on BASK:

- (i) To evaluate the effect of varying the carrier frequency on the power spectrum of the

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BPSK signal, for a fixed square modulating wave.

(ii) To evaluate the effect of varying modulation frequency on the power spectrum of the BPSK signal for a fixed carrier frequency.

BASK and BPSK signals occupy the same transmission bandwidth—namely, $2/T_b$ —which defines the width of the main lobe of the sinc-shaped power spectra. The BASK spectrum includes a carrier component, whereas this component is absent from the BPSK spectrum. With this observation we are merely restating the fact that BASK is an example of amplitude modulation, whereas BPSK is an example of double sideband-suppressed carrier modulation.

4. QUADRIPHASE-SHIFT KEYING

An important goal of digital communication is the efficient utilization of channel bandwidth. This goal is attained by a bandwidth-conserving modulation scheme known as quadriphase-shift keying.

In quadriphase-shift keying (QPSK), as with BPSK, information carried by the transmitted signal is contained in the phase of a sinusoidal carrier. In particular, the phase of the sinusoidal carrier takes on one of four equally spaced values, such as $\pi/4, 3\pi/4, 5\pi/4,$ and $7\pi/4$. For this set of values, we define the transmitted signal as

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos \left[2\pi f_c t + (2i-1)\frac{\pi}{4} \right], & 0 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases} \quad \text{-----(12)}$$

Where $i=1,2,3,4$; E is the transmitted signal energy per symbol and T is the symbol duration. Each one of the four equally spaced phase values corresponds to a unique pair of bits called a dibit. For example, we may choose the foregoing set of phase values to represent the Gray encoded set of dibits: 10, 00, 01, and 11. In this form of encoding, we see that only a single bit is changed from one dibit to the next. Note that the symbol duration (i.e., the duration of each dibit) is twice the bit duration, as shown by

$$T = 2T_b \quad \text{-----(13)}$$

Using a well-known trigonometric identity, we may recast the transmitted signal in the interval in the expanded form

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$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t)$$

-----(14)

where $i=1,2,3,4$. Based on the expanded form of Eq. (14), we can make some important observations:

- In reality, the QPSK signal consists of the sum of two BPSK signals.
- One BPSK signal, represented by the first term

$$\sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t),$$

defines the product of modulating a binary wave by the sinusoidal carrier π which has unit energy over the symbol duration T . We also recognize that

$$\sqrt{E} \cos\left[(2i-1)\frac{\pi}{4}\right] = \begin{cases} \sqrt{E/2} & \text{for } i = 1, 4 \\ -\sqrt{E/2} & \text{for } i = 2, 3 \end{cases}$$

-----(15)

We therefore see that this binary wave has an amplitude equal to

- The other BPSK signal, represented by the second term defines the product of modulating a different binary wave by the sinusoidal carrier π , which also has unit energy per symbol. This time, we recognize that

$$-\sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t),$$

$$-\sqrt{E} \sin\left[(2i-1)\frac{\pi}{4}\right] = \begin{cases} -\sqrt{E/2} & \text{for } i = 1, 2 \\ \sqrt{E/2} & \text{for } i = 3, 4 \end{cases}$$

-----(16)

We therefore see that this second binary wave also has an amplitude equal to albeit in a different way with respect to the index i .

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- The two binary waves defined in Eqs. (15) and (16) share a common value for the symbol duration—namely, T .

- The two sinusoidal carrier waves identified under points 2 and 3 are in phase quadrature with respect to each other. Moreover, they both have unit energy per symbol duration. We may therefore state that these two carrier waves constitute an orthonormal pair of basis functions.

- For each possible value of the index i , Eqs. (15) and (16) identify the corresponding dibit, as outlined in Table 1. This table also includes other related entries pertaining to the phase of the QPSK signal, and the amplitudes of the two binary waves identified under points 2 and 3.

Table:3.1 Relation between index i and identity of corresponding dibit and other related matters

Index i	Phase of QPSK signal (radians)	Amplitudes of constituent binary waves		Input dibit $0 < t < T$
		Binary wave 1 $a_1(t)$	Binary wave 2 $a_2(t)$	
1	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	10
2	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	00
3	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	01
4	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	11

4.1 GENERATION OF QPSK SIGNALS

Generation and coherent detection of QPSK signals, as described here:

To generate the QPSK signal, the incoming binary data stream is first converted into polar form by a non-return-to-zero level encoder; the encoder output is denoted by $b(t)$.

Symbols 1 and 0 are thereby represented by $\bar{1}$ and $\bar{0}$ where $E_b = E/2$. The resulting binary wave is next divided by means of a demultiplexer (consisting of a serial- to-parallel converter) into two separate binary waves consisting of the odd- and even-numbered input bits of $b(t)$. These two binary waves, referred to as the demultiplexed components of the input binary wave, are denoted by $a_1(t)$ and $a_2(t)$. In any signalling interval, the amplitudes of $a_1(t)$ and $a_2(t)$ are determined in accordance with columns 3 and 4 of Table 3.1, depending on the particular dibit that is being transmitted. The demultiplexed binary waves $a_1(t)$ and $a_2(t)$ are used to modulate the pair of quadrature carriers—namely, $\cos \pi t$ and $\sin \pi t$. Finally, the two BPSK signals are subtracted to produce the desired QPSK signals, as depicted in Fig 3.2 (a).

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4.2 COHERENT DETECTION OF QPSK SIGNALS

The QPSK receiver consists of an in-phase (I)-channel and quadrature (Q)-channel with a common input, as depicted in Fig.(b). Each channel is itself made up of a product modulator, low-pass filter, sampler, and decision-making device. Under ideal conditions, For the two synchronous samplers, integer i = the I- and Q-channels of the receiver, respectively, recover the demultiplexed components $a_1(t)$ and $a_2(t)$ responsible for modulating the orthogonal pair of carriers in the transmitter.

Accordingly, by applying the outputs of these two channels to a multiplexer (consisting of a parallel-to-serial converter), the receiver recovers the original binary sequence. The design of the QPSK receiver builds on the strategy described for the coherent BPSK receiver. Specifically, each of the two low-pass filters in the coherent QPSK receiver of Fig.3.2 (b) must be assigned a bandwidth equal to or greater than the reciprocal of the symbol duration T for satisfactory operation of the receiver.

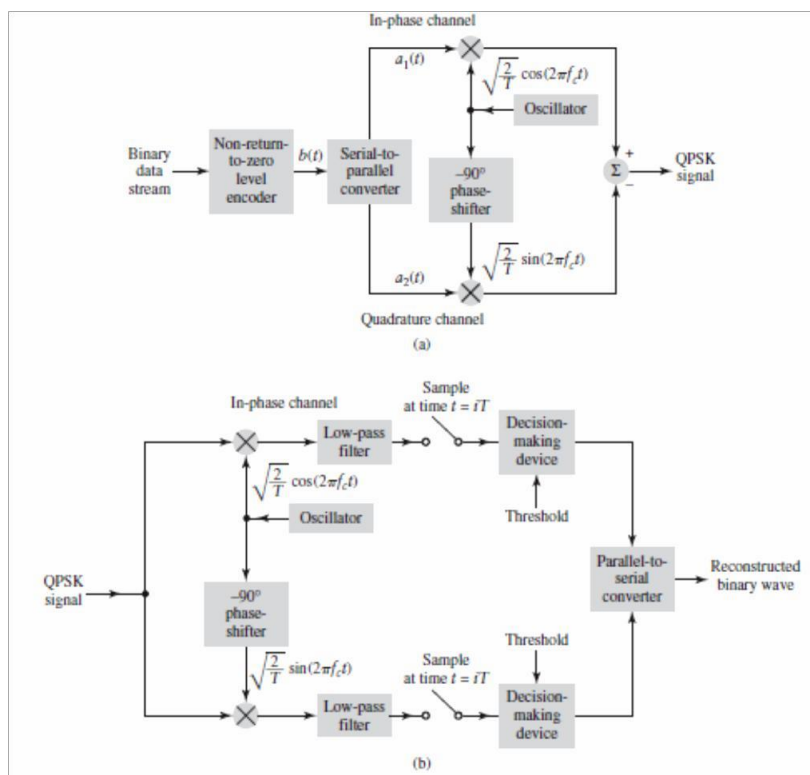


Fig 3.2 Block diagrams of (a) QPSK transmitter and (b) coherent QPSK receiver;

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5. BINARY FREQUENCY-SHIFT KEYING (BFSK)

In the simplest form of frequency-shift keying known as binary frequency-shift keying (BFSK), symbols 0 and 1 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount. A typical pair of sinusoidal waves is described by

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t), & \text{for symbol 1 corresponding to } i = 1 \\ \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t), & \text{for symbol 0 corresponding to } i = 2 \end{cases} \quad \text{-----(17)}$$

Where E_b is the transmitted signal energy per bit. When the frequencies f_1 and f_2 are chosen in such a way that they differ from each other by an amount equal to the reciprocal of the bit duration T_b , the BFSK signal is referred to as Sunde's BFSK after its originator. This modulated signal is a continuous-phase signal in the sense that phase continuity is always maintained, including the inter-bit switching times.

Waveform Figure plots the waveform of Sunde's BFSK produced by the input binary sequence 0011011001 for a bit duration Part (a) of the figure displays the waveform of the input sequence, and part (b) displays the corresponding waveform of the BFSK signal. The latter part of the figure clearly displays the phase-continuous property of Sunde's BFSK.

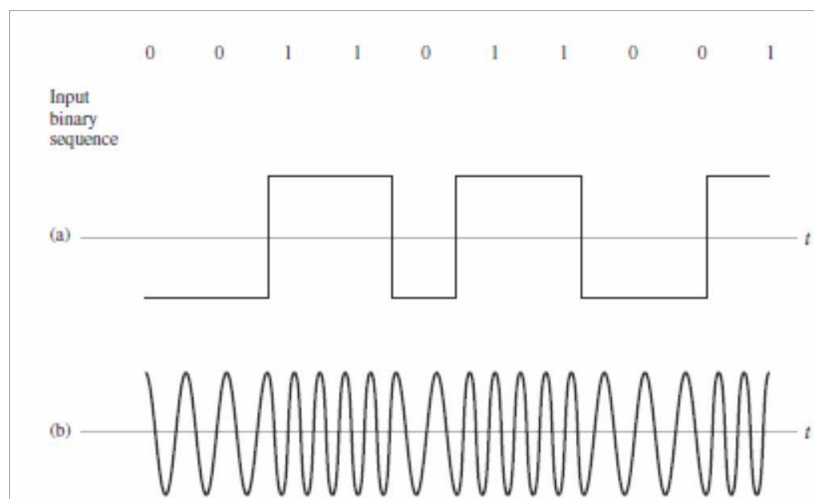


Fig. 3.4 (a) Binary sequence and its non-return-to-zero level-encoded waveform.(b) Sunde's BFSK signal.

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6. MINIMUM-SHIFT KEYING

In MSK, the overall frequency excursion f from binary symbol 1 to symbol 0, or vice versa, is one half the bit rate, as shown by

$$\begin{aligned}\delta f &= f_1 - f_2 \\ &= \frac{1}{2T_b}\end{aligned}$$

------(18)

The unmodulated carrier frequency is the arithmetic mean of the two transmitted frequencies f_1 and f_2 that is,

$$f_c = \frac{1}{2}(f_1 + f_2)$$

------(19)

Expressing f_1 and f_2 in terms of the carrier frequency f_c and overall frequency excursion we have

$$\begin{aligned}f_1 &= f_c + \frac{\delta f}{2}, & \text{for symbol 1} \\ f_2 &= f_c - \frac{\delta f}{2}, & \text{for symbol 0}\end{aligned}$$

------(20) and (21)

Accordingly, we formally define the MSK signal as the angle-modulated wave

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + \theta(t)]$$

------(22)

Where $\theta(t)$ is the phase of the MSK signal. In particular, when frequency f_1 is transmitted, corresponding to symbol 1, we find from Eqs. (20) and (22) that the phase $\theta(t)$ assumes the value

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$$\begin{aligned}\theta(t) &= 2\pi\left(\frac{\delta f}{2}\right)t \\ &= \frac{\pi t}{2T_b}, \quad \text{for symbol 1}\end{aligned}\quad \text{-----(23)}$$

In words, this means that at time $t=T_b$, the transmission of symbol 1 increases the phase of the MSK signal $s(t)$ by π radians. By the same token, when frequency f_2 is transmitted, corresponding to symbol 0, we find from Eqs. (21) and (22) that the phase assumes the value

$$\begin{aligned}\theta(t) &= 2\pi\left(-\frac{\delta f}{2}\right)t \\ &= -\frac{\pi t}{2T_b}, \quad \text{for symbol 0}\end{aligned}\quad \text{-----(24)}$$

This means that at time $t=T_b$ the transmission of symbol 0 decreases the phase of $s(t)$ by $\pi/2$ radians.

The phase changes described in Eqs. 23 and 24) for MSK are radically different from their corresponding counterparts for Sunde's BFSK. Specifically, the phase of Sunde's BFSK signal changes by $-\pi$ radians at the termination of the interval representing symbol 0, and $+\pi$ radians for symbol 1. However, the changes $-\pi$ and $+\pi$ are exactly the same, modulo 2π .

This observation, in effect, means that Sunde's BFSK has no memory; in other words, knowing which particular change occurred in the previous bit interval provides no help in the current bit interval. In contrast, we have a completely different situation in the case of MSK by virtue of the different ways in which the transmissions of symbols 1 and 0 affect the phase as shown in Eqs. (23) and (24). Note also that the overall frequency excursion in MSK is the minimum frequency spacing between symbols 0 and 1 that allows their FSK representations to be coherently orthogonal, hence the terminology "minimum-shift keying."

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UNIT III - DIGITAL MODULATION TECHNIQUES

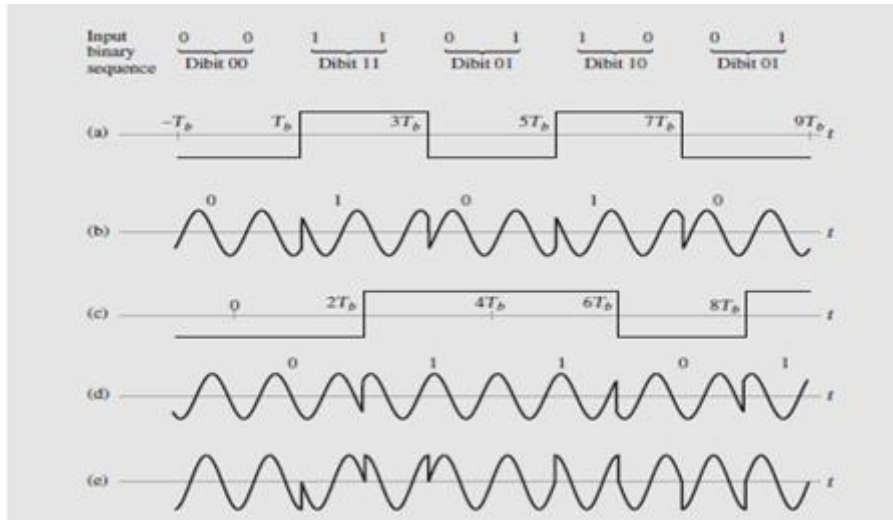


Figure 3.6 OQPSK signal components: (a) modulating signal for in phase component (b) modulated waveform of in phase component (c) modulating signal for quadrature component (d) modulated waveform of quadrature component (e) waveform of OQPSK signal obtained by subtracting d from b.

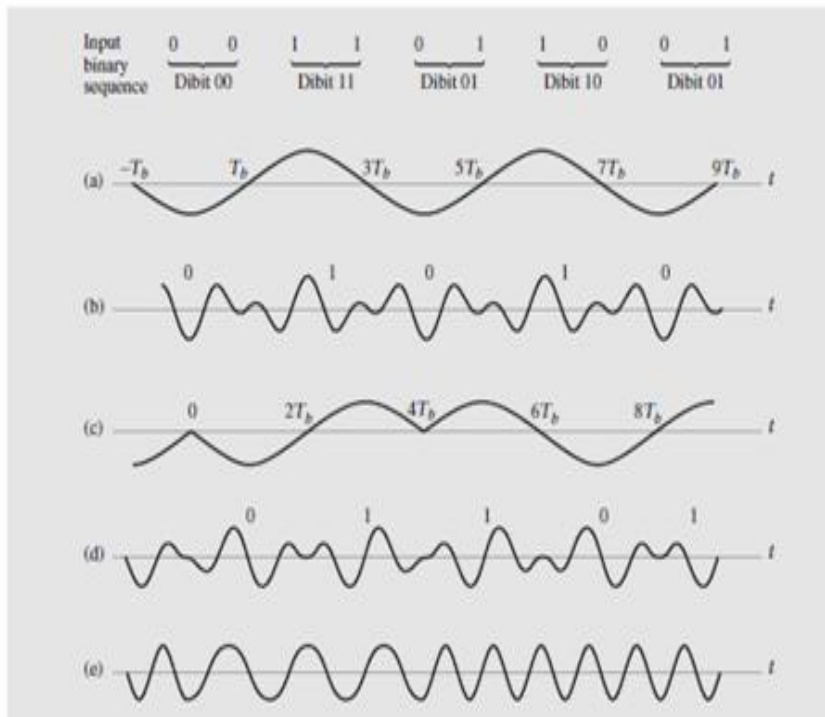


Figure 3.7 MSK signal components (a) modulating signal for in phase component (b) modulated waveform of in phase component (c) modulating signal for quadrature component (d) modulated waveform of quadrature component (e) waveform of MSK signal obtained by subtracting d from b.

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6.1 FORMULATION OF MINIMUM-SHIFT KEYING

To proceed with the formulation, we refer back to Eq. (22), and use a well-known trigonometric identity to expand the angle-modulated wave (i.e., MSK signal) $s(t)$ as

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\theta(t)) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin(\theta(t)) \sin(2\pi f_c t) \quad \text{-----(25)}$$

In light of this equation, we make two identifications:

(i) $S_I(t) =$ is the in-phase (I) component associated with the carrier
 π -----(26)

(ii) $S_Q(t) =$ is the quadrature (Q) component associated with the 90°-
Phase shifted carrier.-----(27)

To highlight the bearing of the incoming binary data stream on the composition of $s_1(t)$ and $s_2(t)$, we reformulate them respectively as follows:

$$s_1(t) = a_1(t) \cos(2\pi f_0 t) \quad \text{-----(28) and}$$

$$s_2(t) = a_2(t) \sin(2\pi f_0 t) \quad \text{-----(29)}$$

The $a_1(t)$ and $a_2(t)$ are two binary waves that are extracted from the incoming binary data stream through demultiplexing and offsetting, in a manner similar to OQPSK. As such, they take on the value +1 or -1 in symbol (i.e., dibit) intervals of duration $T=2T_b$, where T_b is the bit duration of the incoming binary data stream. The two data signals $a_1(t)$ and $a_2(t)$ are respectively weighted by the sinusoidal functions $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$, where the frequency f_0 is to be determined. To define f_0 , we use Eqs. (28) and (29) to reconstruct the original anglemodulated wave $s(t)$ in terms of the data signals $a_1(t)$ and $a_2(t)$, In so doing, we obtain

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$$\begin{aligned}\theta(t) &= -\tan^{-1}\left[\frac{s_Q(t)}{s_I(t)}\right] \\ &= -\tan^{-1}\left[\frac{a_2(t)}{a_1(t)}\tan(2\pi f_0 t)\right] \text{-----(30)}\end{aligned}$$

on the basis of which we recognize two possible scenarios that can arise:

1. $a_1(t)=a_2(t)$, This scenario arises when two successive binary symbols (constituting a dibit) in the incoming data stream are the same (i.e., both are 0s or 1s); hence, Eq. (30) reduces to

$$\begin{aligned}\theta(t) &= -\tan^{-1}[\tan(2\pi f_0 t)] \\ &= -2\pi f_0 t \text{-----(31)}\end{aligned}$$

2. $a_2(t)=-a_1(t)$, This second scenario arises when two successive binary symbols (i.e., dibits) in the incoming data stream are different; hence, Eq. (7.31) reduces to

$$\begin{aligned}\theta(t) &= -\tan^{-1}[-\tan(2\pi f_0 t)] \\ &= 2\pi f_0 t \text{-----(32)}\end{aligned}$$

Equations (31) and (32) are respectively of similar mathematical forms as Eqs. (24) and (23). Accordingly, we may now formally define

$$f_0 = \frac{1}{4T_b} \text{-----(33)}$$

To sum up, given a non-return-to-zero level encoded binary wave $b(t)$ of prescribed bit T_b duration and a sinusoidal carrier wave of frequency f_c , we may formulate the MSK signal by proceeding as follows:

1. Use the given binary wave $b(t)$ to construct the binary demultiplexed-offset waves $a_1(t)$ and $a_2(t)$
2. Use Eq. (33) to determine the frequency f_0 .

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3. Use Eqs. (28) and (29) to determine the in-phase component $s_1(t)$ and quadrature component $s_Q(t)$ respectively from which the MSK signal $s(t)$ follows.

7. NONCOHERENT DIGITAL MODULATION SCHEMES

Coherent receivers, exemplified by the schemes shown in Figs. 3.4(b) and 3.5(b), require knowledge of the carrier wave's phase reference to establish synchronism with their respective transmitters. However, in some communication environments, it is either impractical or too expensive to phase-synchronize a receiver to its transmitter. In situations of this kind, we resort to the use of noncoherent detection by abandoning the use of phase synchronization between the receiver and its transmitter, knowing that when we do so the receiver performance is degraded in the presence of channel noise..

7.1 NONCOHERENT DETECTION OF BASK SIGNAL

The generation of BASK signals involves the use of a single sinusoidal carrier of frequency f_c for symbol 1 and switching off the transmission for symbol 0. Now, the system designer would have knowledge of two system parameters:

- The carrier frequency f_c
- The transmission bandwidth, which is determined by the bit duration T_b

It is therefore natural to make use of these known parameters in designing the noncoherent receiver for BASK. Specifically, the receiver consists of a band-pass filter, followed by an envelope detector, then a sampler, and finally a decision-making device, as depicted in Fig. 3.6. The band-pass filter is designed to have a mid-band frequency equal to the carrier frequency f_c and a bandwidth equal to the transmission bandwidth of the BASK signal. Moreover, it is assumed that the intersymbol interference (ISI) produced by the filter is negligible, which, in turn, requires that the rise time and decay time of the response of the filter to a rectangular pulse be short compared to the bit duration T_b . Under these conditions, we find that in response to the incoming BASK signal (assumed to be noise-free), the band-pass filter produces a pulsed sinusoid for symbol 1 and, ideally, no output for symbol 0. Next, the envelope detector traces the envelope of the filtered version of the BASK signal.

Finally, the decision-making device working in conjunction with the sampler, regenerates the original binary data stream by comparing the sampled envelope-detector output against a preset threshold every T_b seconds; this operation assumes the availability of bit-timing in the receiver. If the threshold is exceeded at time $t=iT_b, i=0$, the receiver decides in favor of symbol 1; otherwise, it decides in favor of symbol 0. In the absence of channel noise and channel distortion, the receiver output (on a bit-by-bit basis) would be an exact replica of the original binary data stream applied to the transmitter, subject to the abovementioned assumptions on the band-pass filter.

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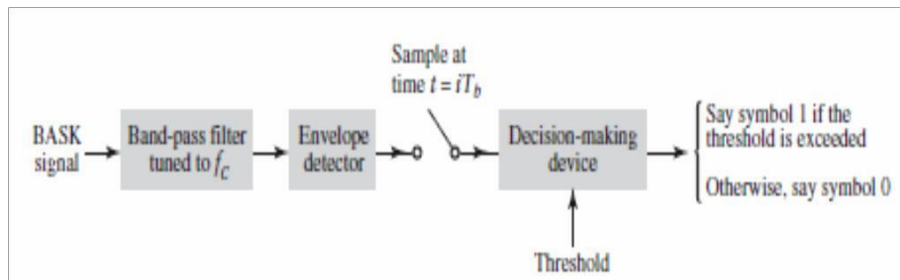


Figure 3.8 Noncoherent BASK receiver; the integer i for the sampler equals 0,

7.2 NONCOHERENT DETECTION OF BFSK SIGNALS

In the case of BFSK, the transmissions of symbols 1 and 0 are represented by two carrier waves of frequencies f_1 and f_2 respectively, with adequate spacing between them. In light of this characterization, we may build on the noncoherent detection of BASK by formulating the noncoherent BFSK receiver of Fig. 3.5. The receiver consists of two paths, one dealing with frequency f_1 (i.e., symbol 1) and the other dealing with frequency f_2 (i.e., symbol 0)

- Path 1 uses a band-pass filter of mid-band frequency f_1 . The filtered version of the incoming BFSK signal is envelope-detected and then sampled at time $t=iT_b, i=0, 1, \dots$, to produce the output v_1 .
- Path 2 uses a band-pass filter of mid-band frequency f_2 . As with path 2, the filtered version of the BFSK signal is envelope-detected and then sampled at time $t=iT_b, i=0, 1, \dots$, to produce the output v_2 .

The two band-pass filters have the same bandwidth, equal to the transmission bandwidth of the BFSK signal. Moreover, as pointed out in dealing with BASK, the intersymbol interference produced by the filters is assumed to be negligible. The outputs of the two paths, v_1 and v_2 are applied to a comparator, where decisions on the composition of the BFSK signal are repeated every T_b seconds. Here again, the availability of bit timing is assumed in the receiver. Recognizing that the upper path corresponds to symbol 1 and the lower path corresponds to symbol 0, the comparator decides in favour of symbol 1 if v_1 is greater than v_2 at the specified bit-timing instant; otherwise, the decision is made in favor of symbol 0. In a noise-free environment and no channel distortion, the receiver output (on a bit-by-bit basis) would again be a replica of the original binary data stream applied to the transmitter input.

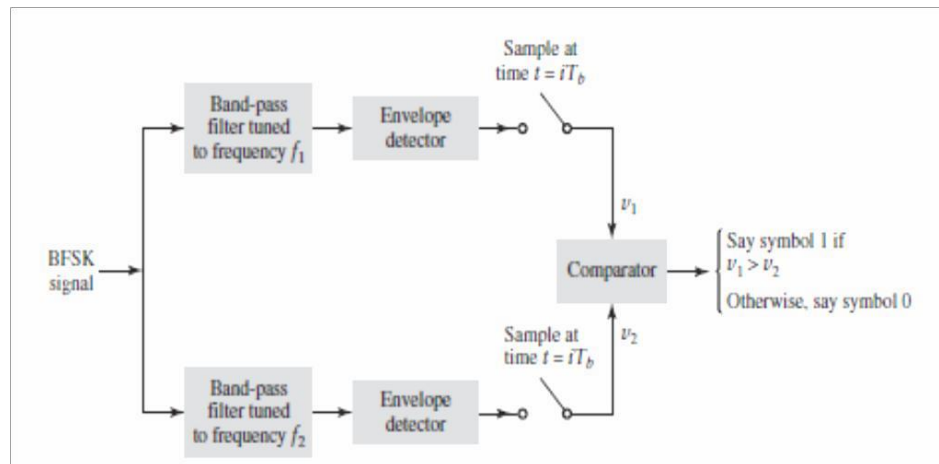


Figure 3.9 Noncoherent BFSK receiver; the two samplers operate synchronously, with $i=0, 1, 2, \dots$

7.3 DIFFERENTIAL PHASE-SHIFT KEYING

From the above discussion, we see that both amplitude-shift keying and frequency-shift keying lend themselves naturally to noncoherent detection whenever it is impractical to maintain carrier-phase synchronization of the receiver to the transmitter. But in the case of phase-shift keying, we cannot have noncoherent detection in the traditional sense because the term “noncoherent” means having to do without carrier-phase information. To get around this difficulty, we employ a “pseudo PSK” technique known as differential phase-shift keying (DPSK), which, in a loose sense, does permit the use of noncoherent detection.

DPSK eliminates the need for a coherent reference signal at the receiver by combining two basic operations at the transmitter:

- Differential encoding of the input binary wave
- Phase-shift keying

It is because of this combination that we speak of “differential phase-shift keying.” In effect, to send symbol 0, we phase advance the current signal waveform by 180 degrees, and to send symbol 1 we leave the phase of the current signal waveform unchanged. Correspondingly, the receiver is equipped with a storage capability (i.e., memory) designed to measure the relative phase difference between the waveforms received during two successive bit intervals. Provided the unknown phase varies slowly (i.e., slow enough for it to be considered essentially constant over two bit intervals), the phase difference between waveforms received in two successive bit intervals will be essentially independent of θ .

7.4 GENERATION AND DETECTION OF DPSK SIGNALS

(i) Generation

The differential encoding process at the transmitter input starts with an arbitrary first bit, serving merely as reference. Let $\{d_k\}$ denote the differentially encoded sequence with this added

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reference bit. To generate this sequence, the transmitter performs the following two operations:

- If the incoming binary symbol $\{b_k\}$ is 1, then the symbol $\{d_k\}$ is unchanged with respect to the previous symbol d_{k-1} .
- If the incoming binary symbol $\{b_k\}$ is 0, then the symbol $\{d_k\}$ is changed with respect to the previous symbol d_{k-1} .

The differentially encoded sequence $\{d_k\}$, thus generated is used to phase-shift a sinusoidal carrier wave with phase angles 0 and π radians, representing symbols 1 and 0, respectively. The block diagram of the DPSK transmitter is shown in Fig. 3.5(a). It consists, in part, of a logic network and a one-bit delay element (acting as the memory unit) interconnected so as to convert the raw binary sequence $\{b_k\}$ into a differentially encoded sequence $\{d_k\}$. This sequence is amplitude-level encoded and then used to modulate a carrier wave of frequency f_c thereby producing the desired DPSK signal.

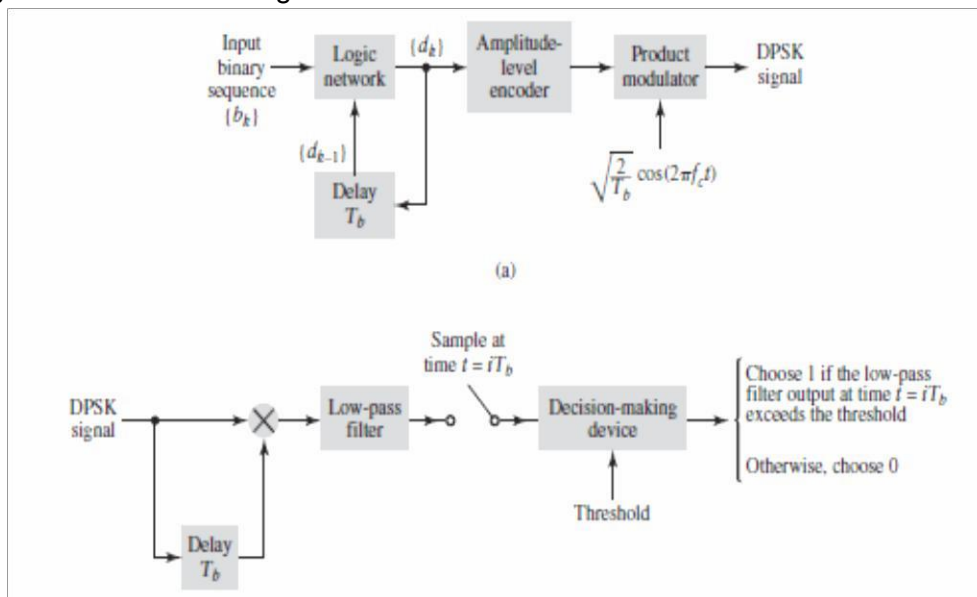


figure 3.11 Block diagrams for (a) DPSK transmitter and (b) DPSK receiver; for the sampler, integer $i=0$,

(ii) Detection

For the detection of DPSK signals, we take advantage of the fact that the phase-modulated pulses pertaining to two successive bits are identical except for a possible sign reversal. Hence, the incoming pulse is multiplied by the preceding pulse, which, in effect, means that the preceding pulse serves the purpose of a locally generated reference signal. On this basis, we may formulate the receiver of Fig. 3.8(b) for the detection of DPSK signals. Comparing the DPSK detector of Fig. 3.8(b) and the coherent BPSK detector of Fig. 3.3(b),

we see that the two receiver structures are similar except for the source of the locally generated reference signal. According to Fig. 3.8(b), the DPSK signal is detectable, given knowledge of the reference bit, which, as mentioned previously, is inserted at the very beginning of the

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incoming binary data stream. In particular, applying the sampled output of the low-pass filter to a decision-making device supplied with a prescribed threshold, detection of the DPSK signal is accomplished. If the threshold is exceeded, the receiver decides in favor of symbol 1; otherwise, the decision is made in favor of symbol 0. Here again, it is assumed that the receiver is supplied with bit-timing information for the sampler to work properly.

8. M-ARY DIGITAL MODULATION SCHEMES

By definition, in an M-ary digital modulation scheme, we send any one of M possible signals $s_1(t), s_2(t), \dots, s_M(t)$, during each signaling (symbol) interval of duration T. In almost all applications, $M=2^m$, where m is an integer. Under this condition, the symbol duration, $T=MT_b$, where T_b is the bit duration. M-ary modulation schemes are preferred over binary modulation schemes for transmitting digital data over band-pass channels when the requirement is to conserve bandwidth at the expense of both increased power and increased system complexity.

In practice, we rarely find a communication channel that has the exact bandwidth required for transmitting the output of an information-bearing source by means of binary modulation schemes. Thus, when the bandwidth of the channel is less than the required value, we resort to an M-ary modulation scheme for maximum bandwidth conservation.

8.1 M-ARY PHASE-SHIFT KEYING

To illustrate the capability of M-ary modulation schemes for bandwidth conservation, consider first the transmission of information consisting of a binary sequence with bit duration T_b . If we were to transmit this information by means of binary PSK, for example, we would require a channel bandwidth that is inversely proportional to the bit duration T_b . However, if we take blocks of m bits to produce a symbol and use an M-ary PSK scheme $M=2^m$ with a symbol duration $T=mT_b$, then the bandwidth required is proportional to $1/(mT_b)$. This simple argument shows that the use of M-ary PSK provides a reduction in transmission bandwidth by a factor $m=\log_2 M$ over binary PSK. In M-ary PSK, the available phase of 2π radians is apportioned equally and in a discrete way among the M transmitted signals, as shown by the phase-modulated signal

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M} i\right), \quad \begin{array}{l} i = 0, 1, \dots, M-1 \\ 0 \leq t \leq T \end{array} \quad \text{-----(34)}$$

where E is the signal energy per symbol, and f_c is the carrier frequency. Using a well-known trigonometric identity, we may expand Eq. (34) as

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$$s_i(t) = \left[\sqrt{E} \cos\left(\frac{2\pi}{M}i\right) \right] \left[\sqrt{\frac{2}{T}} \cos(2\pi f_c t) \right] \\ - \left[\sqrt{E} \sin\left(\frac{2\pi}{M}i\right) \right] \left[\sqrt{\frac{2}{T}} \sin(2\pi f_c t) \right], \quad \begin{array}{l} i = 0, 1, \dots, M-1 \\ 0 \leq t \leq T \end{array}$$

-----(35)

The discrete coefficients $\sqrt{E} \cos\left(\frac{2\pi}{M}i\right)$ and $-\sqrt{E} \sin\left(\frac{2\pi}{M}i\right)$ are respectively referred to as the in-phase and quadrature components of the M-ary PSK signal $S_i(t)$. We now recognize that

$$\left\{ \left[\sqrt{E} \cos\left(\frac{2\pi}{M}i\right) \right]^2 + \left[\sqrt{E} \sin\left(\frac{2\pi}{M}i\right) \right]^2 \right\}^{1/2} = \sqrt{E}, \quad \text{for all } i$$

-----(36)

Accordingly, M-ary PSK modulation has the unique property that the in-phase and quadrature components of the modulated signal $S_i(t)$ are interrelated in such a way that the discrete envelope of the signal is constrained to remain constant at the value \sqrt{E} for all M.

8.2 SIGNAL-SPACE DIAGRAM

The result described in Eq. (36) combined with the fact that the in-phase and quadrature components of M-ary PSK are discrete, leads to an insightful geometric portrayal of M-ary PSK. To explain, suppose we construct a two-dimensional diagram with the horizontal and vertical axes respectively defined by the following pair of orthonormal functions:

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \leq t \leq T$$

-----(37)

And

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \quad 0 \leq t \leq T$$

-----(38)

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where the band-pass assumption implies orthogonality; the scaling factor-assures unit energy over the interval T for both $\phi_1(t)$ and $\phi_2(t)$. On this basis, we may represent the in-phase component $s_i(t)$ and quadrature component $q_i(t)$ for $i=0,1,\dots,M-1$, as a set of points in this two-dimensional diagram, as illustrated in Fig. 7.19 for $M = 8$. Such a diagram is referred to as a signal-space diagram.

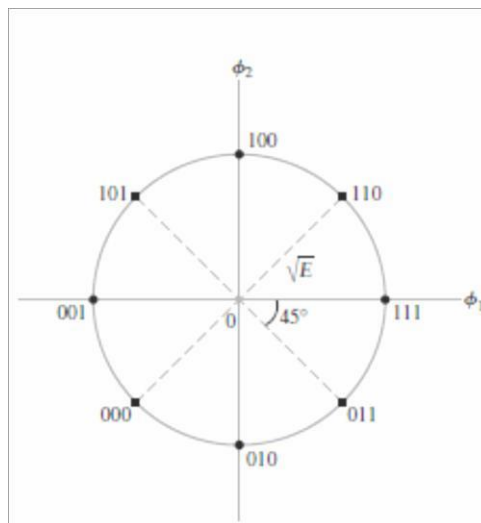


Fig 3.12 Signal-space diagram of 8-PSK.

Figure 3.9 leads us to make three

important observations:

1. M-ary PSK is described in geometric terms by a constellation of M signal points distributed uniformly on a circle of radius \sqrt{E} .
2. Each signal point in the figure corresponds to the signal $S_i(t)$ of Eq. (34) for a particular value of the index i .
3. The squared length from the origin to each signal point is equal to the signal energy E . In light of these observations, we may now formally state that the signal-space-diagram of Fig. 3.6 completely sums up the geometric description of M-ary PSK in an insightful manner. Note that the 3-bit sequences corresponding to the 8 signal points are Gray-encoded, with only a single bit changing as we move along the constellation in the figure from one signal point to an adjacent one.

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9. M-ARY FREQUENCY-SHIFT KEYING

However, when we consider the M-ary version of frequency-shift keying, the picture is quite different from that described for M-ary PSK or M-ary QAM. Specifically, in one form of M-ary FSK, the transmitted signals are defined for some fixed integer n as follows:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos \left[\frac{\pi}{T} (n + i)t \right], \quad \begin{array}{l} i = 0, 1, \dots, M - 1 \\ 0 \leq t \leq T \end{array} \quad \text{-----(40)}$$

The M transmitted signals are all of equal duration T and equal energy E. With the individual signal frequencies separated from each other by 1/2T hertz, the signals in Eq.(40) are orthogonal; that is, they satisfy the condition

$$\int_0^T s_i(t)s_j(t) dt = \begin{cases} E & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} \quad \text{-----(41)}$$

Like M-ary PSK, the envelope of M-ary FSK is constant for all M, which follows directly from Eq. (40). Hence, both of these M-ary modulation strategies can be used over nonlinear channels. On the other hand, M-ary QAM can only be used over linear channels because its discrete envelope varies with the index i (i.e., the particular signal point chosen for transmission).

To develop a geometric representation of M-ary FSK, we start with Eq. (40). In terms of the signals $S_i(t)$ defined therein, we introduce a complete set of orthonormal functions:

$$\phi_i(t) = \frac{1}{\sqrt{E}} s_i(t) \quad \begin{array}{l} i = 0, 1, \dots, M - 1 \\ 0 \leq t \leq T \end{array} \quad \text{-----(42)}$$

Unlike M-ary PSK and M-ary QAM, we now find that M-ary FSK is described by an M-dimensional signal-space diagram, where the number of signal points is equal to the number of coordinates. The visualization of such a diagram is difficult beyond M=3. Figure 7.22 illustrates the geometric representation of M-ary FSK for M = 3 it is the basis of a synchronous communication system.