SEC1313 - DIGITAL COMMUNICATION

UNIT 2 BASEBAND PULSE TRANSMISSION **9 Hrs.**

Base band transmission - Wave form representation of binary digits -Matched Filter- Error Rate due to noise -- Nyquist's criterion for Distortionless Base band Binary Transmission- Inter symbol Interference - Ideal Nyquist channel - Raised cosine channels- Correlative level coding - Baseband M-ary PAM transmission- Equalization – Eye patterns- Companding - A law and μ law- correlation receiver.

2.1 Introduction

- **Definition of baseband transmission** : When the signal is transmitted over the channel, without any modulation, it is called baseband transmission.
- **Problems occurred in baseband transmission :** One of the major problem occurred in baseband transmission is intersymbol interference. This interference takes place due to dispersive nature of the channel.
- **Corrective measures to minimize errors in baseband transmission :** Nyquist criterion gives a condition for distortionless baseband transmission. It is possible to reduce the effect of intersymbol interference with the help of raised cosine spectrum.
- Correlative level coding is also used to minimize effects of intersymbol interference. It allows higher signaling rate on low bandwidth channel.
- Equalizers are used to compensate for distortion introduced in the channel.

2.2 Waveform representation of binary digits

It was shown how analog waveforms are transformed into binary digits via the use of PCM. There is nothing "physical" about the digits resulting from this process. Digits are just abstractions—a way to describe the message information. Thus, we need something physical that will represent or "carry" the digits.

We will represent the binary digits with electrical pulses in order to transmit them through a baseband channel. Such a representation is shown in Figure 2.21. Codeword time slots are shown in Figure 2.21a, where the codeword is a 4-bit representation of each quantized sample. In Figure 2.21b, each binary one is represented by a pulse and each binary zero is represented by the absence of a pulse.

Thus a sequence of electrical pulses having the pattern shown in Figure 2.21b can be used to transmit the information in the PCM bit stream, and hence the information in the quantized samples of a message.

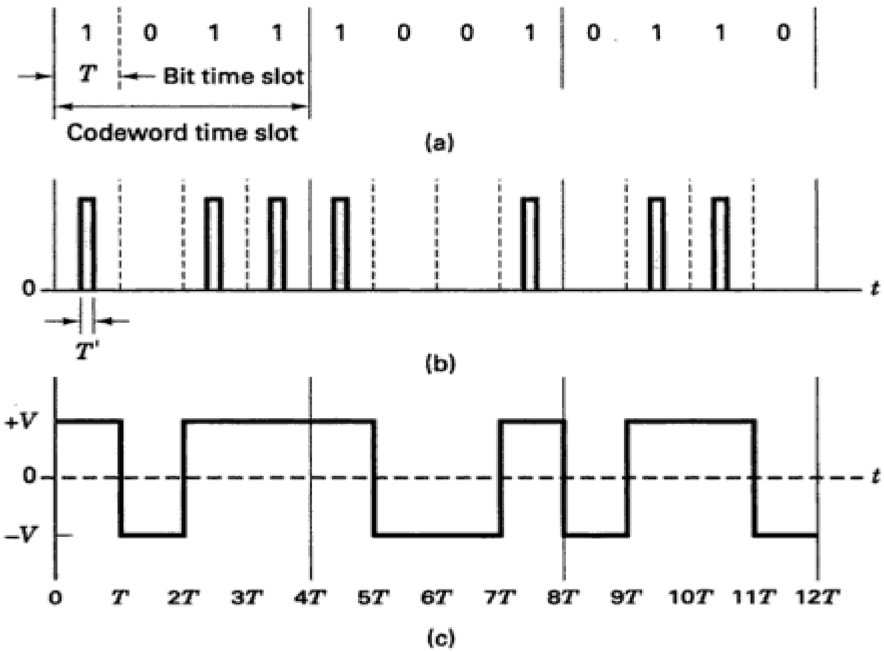


Figure 2.21 Example of waveform representation of binary digits. (a) PCM sequence. (b) Pulse representation of PCM. (c) Pulse waveform (transition between two levels).

At the receiver, a determination must be made as to the presence or absence of a pulse in each bit time slot. It will be shown in Section 2.9 that the likelihood of correctly detecting the presence of a pulse is a function of the received pulse energy (or area under the pulse). Thus there is an advantage in making the pulse width T' in Figure 2.21b as wide as possible. If we increase the pulse width to the maximum possible (equal to the bit time T), we have the waveform shown in Figure 2.21c. Rather than describe this waveform as a sequence of present or absent pulses, we can describe it as a sequence of transitions between two levels. When the waveform occupies the upper voltage level it represents a binary one; when it occupies the lower voltage level it represents a binary zero.

2.3 Matched Filter

Definition

- The matched filter is used for detection of signals in baseband and passband transmission.
- It is called matched filter since its impulse response is *matched* to the shape of input signal.

- **Requirements of Matched Filter**
    (i) Signal to noise ratio of the receiver must be improved.
    (ii) The signal must be checked at the instant in bit period, when signal to noise ratio is maximum.
    (iii) The error probability should be minimum.

A basic problem that often arises in the study of communication systems is that of *detecting* a pulse transmitted over a channel that is corrupted by channel noise (i.e., additive noise at the front end of the receiver). For the purpose of the discussion presented in this section, we assume that the major source of system limitation is the channel noise.

Consider then the receiver model shown in Figure 4.1, involving a linear time-invariant filter of impulse response $h(t)$. The filter input $x(t)$ consists of a pulse signal $g(t)$ corrupted by additive channel noise $w(t)$, as shown by

$$x(t) = g(t) + w(t), \qquad 0 \leq t \leq T \tag{4.1}$$

where T is an arbitrary observation interval. The pulse signal $g(t)$ may represent a binary symbol 1 or 0 in a digital communication system. The $w(t)$ is the sample function of a white noise process of zero mean and power spectral density $N_0/2$. It is assumed that the receiver has knowledge of the waveform of the pulse signal $g(t)$. The source of uncertainty lies in the noise $w(t)$. The function of the receiver is to detect the pulse signal $g(t)$ in an optimum manner, given the received signal $x(t)$. To satisfy this requirement, we have to optimize the design of the filter so as to minimize the effects of noise at the filter output in some statistical sense, and thereby enhance the detection of the pulse signal $g(t)$.

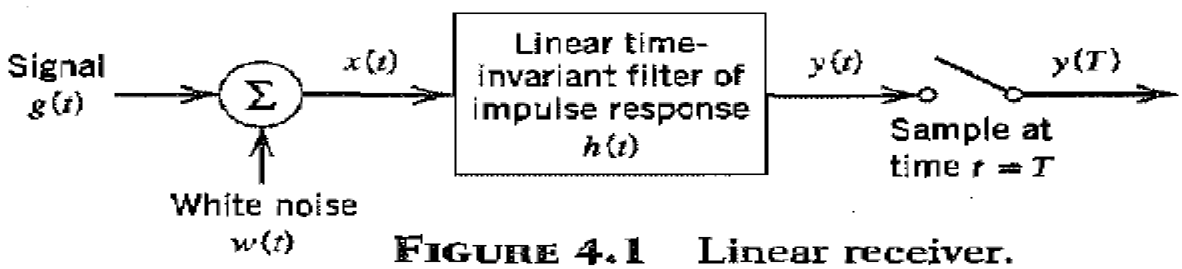


FIGURE 4.1 **Linear receiver.**

Since the filter is linear, the resulting output $y(t)$ may be expressed as

$$y(t) = g_o(t) + n(t) \tag{4.2}$$

where $g_o(t)$ and $n(t)$ are produced by the signal and noise components of the input $x(t)$, respectively. A simple way of describing the requirement that the output signal component $g_o(t)$ be considerably greater than the output noise component $n(t)$ is to have the filter make the instantaneous power in the output signal $g_o(t)$, measured at time $t = T$, as large as possible compared with the average power of the output noise $n(t)$. This is equivalent to maximizing the *peak pulse signal-to-noise ratio*, defined as

$$\eta = \frac{|g_o(T)|^2}{E[n^2(t)]} \tag{4.3}$$

where $|g_o(T)|^2$ is the instantaneous power in the output signal, E is the statistical expectation operator, and $E[n^2(t)]$ is a measure of the average output noise power. The requirement is to specify the impulse response $h(t)$ of the filter such that the output signal-to-noise ratio in Equation (4.3) is maximized.

Let $G(f)$ denote the Fourier transform of the known signal $g(t)$, and $H(f)$ denote the frequency response of the filter. Then the Fourier transform of the output signal $g_o(t)$ is equal to $H(f)G(f)$, and $g_o(t)$ is itself given by the inverse Fourier transform

$$g_o(t) = \int_{-\infty}^{\infty} H(f)G(f) \exp(j2\pi ft)\, df \tag{4.4}$$

Hence, when the filter output is sampled at time $t = T$, we have (in the absence of channel noise)

$$|g_o(T)|^2 = \left| \int_{-\infty}^{\infty} H(f)G(f) \exp(j2\pi fT)\, df \right|^2 \tag{4.5}$$

Consider next the effect on the filter output due to the noise $w(t)$ acting alone. The power spectral density $S_N(f)$ of the output noise $n(t)$ is equal to the power spectral density of the input noise $w(t)$ times the squared magnitude response $|H(f)|^2$ (see Section 1.7). Since $w(t)$ is white with constant power spectral density $N_0/2$, it follows that

$$S_N(f) = \frac{N_0}{2} |H(f)|^2 \tag{4.6}$$

The average power of the output noise $n(t)$ is therefore

$$\begin{aligned} E[n^2(t)] &= \int_{-\infty}^{\infty} S_N(f)\, df \\ &= \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df \end{aligned} \tag{4.7}$$

Thus substituting Equations (4.5) and (4.7) into (4.3), we may rewrite the expression for the peak pulse signal-to-noise ratio as

$$\eta = \frac{\left| \int_{-\infty}^{\infty} H(f)G(f) \exp(j2\pi fT)\, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df} \tag{4.8}$$

Our problem is to find, for a given $G(f)$, the particular form of the frequency response $H(f)$ of the filter that makes η a maximum. To find the solution to this optimization problem, we apply a mathematical result known as Schwarz's inequality to the numerator of Equation (4.8).

A derivation of *Schwarz's inequality* is given in Chapter 5. For now it suffices to say that if we have two complex functions $\phi_1(x)$ and $\phi_2(x)$ in the real variable x, satisfying the conditions

$$\int_{-\infty}^{\infty} |\phi_1(x)|^2 \, dx < \infty$$

and

$$\int_{-\infty}^{\infty} |\phi_2(x)|^2 \, dx < \infty$$

then we may write

$$\left| \int_{-\infty}^{\infty} \phi_1(x)\phi_2(x) \, dx \right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(x)|^2 \, dx \int_{-\infty}^{\infty} |\phi_2(x)|^2 \, dx \tag{4.9}$$

The equality in (4.9) holds if, and only if, we have

$$\phi_1(x) = k\phi_2^*(x) \tag{4.10}$$

where k is an arbitrary constant, and the asterisk denotes complex conjugation.

Returning to the problem at hand, we readily see that by invoking Schwarz's inequality (4.9), and setting $\phi_1(x) = H(f)$ and $\phi_2(x) = G(f)\exp(j\pi fT)$, the numerator in Equation (4.8) may be rewritten as

$$\left| \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT) \, df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 \, df \int_{-\infty}^{\infty} |G(f)|^2 \, df \tag{4.11}$$

Using this relation in Equation (4.8), we may redefine the peak pulse signal-to-noise ratio as

$$\eta \leq \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 \, df \tag{4.12}$$

The right-hand side of this relation does not depend on the frequency response $H(f)$ of the filter but only on the signal energy and the noise power spectral density. Consequently, the peak pulse signal-to-noise ratio η will be a maximum when $H(f)$ is chosen so that the equality holds; that is,

$$\eta_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 \, df \tag{4.13}$$

Correspondingly, $H(f)$ assumes its optimum value denoted by $H_{opt}(f)$. To find this optimum value we use Equation (4.10), which, for the situation at hand, yields

$$H_{opt}(f) = kG^*(f)\exp(-j2\pi fT) \tag{4.14}$$

where $G^*(f)$ is the complex conjugate of the Fourier transform of the input signal $g(t)$, and k is a scaling factor of appropriate dimensions. This relation states that, except for the factor $k \exp(-j2\pi fT)$, the frequency response of the optimum filter is the same as the complex conjugate of the Fourier transform of the input signal.

Equation (4.14) specifies the optimum filter in the frequency domain. To characterize it in the time domain, we take the inverse Fourier transform of $H_{opt}(f)$ in Equation (4.14) to obtain the impulse response of the optimum filter as

$$h_{opt}(t) = k \int_{-\infty}^{\infty} G^*(f) \exp[-j2\pi f(T - t)]\, df \tag{4.15}$$

Since for a real signal $g(t)$ we have $G^*(f) = G(-f)$, we may rewrite Equation (4.15) as

$$\begin{aligned} h_{opt}(t) &= k \int_{-\infty}^{\infty} G(-f) \exp[-j2\pi f(T - t)]\, df \\ &= k \int_{-\infty}^{\infty} G(f) \exp\, [j2\pi f(T - t)]\, df \\ &= kg(T - t) \end{aligned} \tag{4.16}$$

Equation (4.16) shows that the impulse response of the optimum filter, except for the scaling factor k, is a time-reversed and delayed version of the input signal $g(t)$; that is, it is "matched" to the input signal. A linear time-invariant filter defined in this way is called a matched filter. Note that in deriving the matched filter the only assumption we have made about the input noise $w(t)$ is that it is stationary and white with zero mean and power spectral density $N_0/2$. In other words, no assumption was made on the statistics of the channel noise $w(t)$.

2.2.1 Properties of matched filter

an impulse response that is a time-reversed and delayed version of the input $g(t)$, as shown by

$$h_{opt}(t) = kg(T - t)$$

a frequency response is the complex conjugate of the Fourier transform of the input $g(t)$, as shown by

$$H_{opt}(f) = kG^*(f) \exp(-j2\pi fT)$$

The peak pulse signal-to-noise ratio of a matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter input.

Therefore, the peak pulse signal-to-noise ratio has the maximum value

$$\eta_{max} = \frac{(kE)^2}{(k^2 N_0 E/2)} = \frac{2E}{N_0} \tag{4.20}$$

From Equation (4.20) we see that dependence on the waveform of the input $g(t)$ has been completely removed by the matched filter.

2.3 Error Rate due to noise

Now that we are equipped with the matched filter as the optimum detector of a known pulse in additive white noise, we are ready to derive a formula for the error rate in such a system due to noise.

To proceed with the analysis, consider a binary PCM system based on *polar non-return-to-zero (NRZ) signaling*. In this form of signaling, symbols 1 and 0 are represented by positive and negative rectangular pulses of equal amplitude and equal duration. The channel noise is modeled as *additive white Gaussian noise* $w(t)$ of zero mean and power spectral density $N_0/2$; the Gaussian assumption is needed for later calculations. In the signaling interval $0 \leq t \leq T_b$, the received signal is thus written as follows:

$$x(t) = \begin{cases} +A + w(t), & \text{symbol 1 was sent} \\ -A + w(t), & \text{symbol 0 was sent} \end{cases} \tag{4.21}$$

where T_b is the *bit duration*, and A is the *transmitted pulse amplitude*. It is assumed that the receiver has acquired knowledge of the starting and ending times of each transmitted pulse; in other words, the receiver has prior knowledge of the pulse shape, but not its polarity. Given the noisy signal $x(t)$, the receiver is required to make a decision in each signaling interval as to whether the transmitted symbol is a 1 or a 0.

The structure of the receiver used to perform this decision-making process is shown in Figure 4.4. It consists of a matched filter followed by a sampler, and then finally a

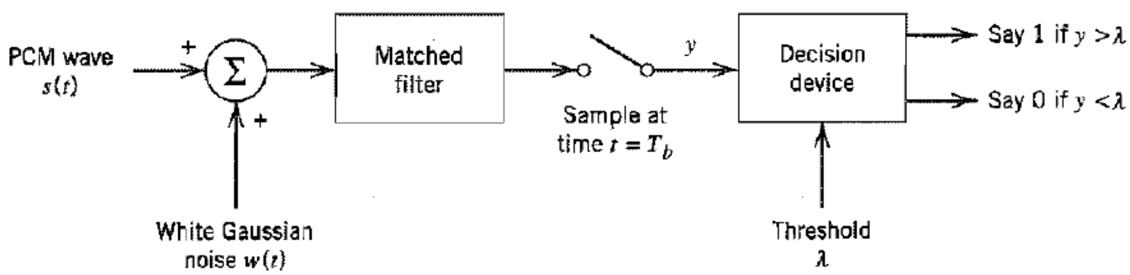


FIGURE 4.4 Receiver for baseband transmission of binary-encoded PCM wave using polar NRZ signaling.

decision device. The filter is matched to a rectangular pulse of amplitude A and duration T_b, exploiting the bit-timing information available to the receiver. The resulting matched filter output is sampled at the end of each signaling interval. The presence of channel noise $w(t)$ adds randomness to the matched filter output.

Let y denote the sample value obtained at the end of a signaling interval. The sample value y is compared to a preset *threshold* λ in the decision device. If the threshold is exceeded, the receiver makes a decision in favor of symbol 1; if not, a decision is made in favor of symbol 0. We adopt the convention that when the sample value y is exactly equal to the threshold λ, the receiver just makes a guess as to which symbol was transmitted; such a decision is the same as that obtained by flipping a fair coin, the outcome of which will not alter the average probability of error.

There are two possible kinds of error to be considered:

1. Symbol 1 is chosen when a 0 was actually transmitted; we refer to this error as an *error of the first kind.*
2. Symbol 0 is chosen when a 1 was actually transmitted; we refer to this error as an *error of the second kind.*

To determine the average probability of error, we consider these two situations separately.

Suppose that symbol 0 was sent. Then, according to Equation (4.21), the received signal is

$$x(t) = -A + w(t), \qquad 0 \le t \le T_b \tag{4.22}$$

Correspondingly, the matched filter output, sampled at time $t = T_b$, is given by (in light of Example 4.1 with kAT_b set equal to unity for convenience of presentation)

$$\begin{aligned} y &= \int_0^{T_b} x(t)\, dt \\ &= -A + \frac{1}{T_b}\int_0^{T_b} w(t)\, dt \end{aligned} \tag{4.23}$$

which represents the sample value of a random variable Y. By virtue of the fact that the noise $w(t)$ is white and Gaussian, we may characterize the random variable Y as follows:

- The random variable Y is Gaussian distributed with a mean of $-A$.
- The variance of the random variable Y is

$$\begin{aligned} \sigma_Y^2 &= E[(Y + A)^2] \\ &= \frac{1}{T_b^2} E\left[\int_0^{T_b}\int_0^{T_b} w(t)w(u)\, dt\, du\right] \\ &= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} E[w(t)w(u)]\, dt\, du \\ &= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} R_W(t, u)\, dt\, du \end{aligned} \tag{4.24}$$

where $R_W(t, u)$ is the autocorrelation function of the white noise $w(t)$. Since $w(t)$ is white with a power spectral density $N_0/2$, we have

$$R_W(t, u) = \frac{N_0}{2}\,\delta(t - u) \tag{4.25}$$

where $\delta(t - u)$ is a time-shifted delta function. Hence, substituting Equation (4.25) into (4.24) yields

$$\sigma_Y^2 = \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b}\frac{N_0}{2}\,\delta(t-u)\,dt\,du = \frac{N_0}{2T_b} \tag{4.26}$$

where we have used the sifting property of the delta function and the fact that its area is unity. The conditional probability density function of the random variable Y, given that symbol 0 was sent, is therefore

$$f_Y(y|0) = \frac{1}{\sqrt{\pi N_0/T_b}}\exp\left(-\frac{(y+A)^2}{N_0/T_b}\right) \tag{4.27}$$

This function is plotted in Figure 4.5(*a*). Let p_{10} denote the *conditional probability of error, given that symbol 0 was sent*. This probability is defined by the shaded area under the curve of $f_Y(y|0)$ from the threshold λ to infinity, which corresponds to the range of values assumed by y for a decision in favor of symbol 1. In the absence of noise, the matched filter output y sampled at time $t = T_b$ is equal to $-A$. When noise is present, y occasionally assumes a value greater than λ, in which case an error is made. The probability of this error, conditional on sending symbol 0, is defined by

$$\begin{aligned} p_{10} &= P(y > \lambda | \text{symbol 0 was sent}) \\ &= \int_\lambda^\infty f_Y(y|0)\,dy \\ &= \frac{1}{\sqrt{\pi N_0/T_b}}\int_\lambda^\infty \exp\left(-\frac{(y+A)^2}{N_0/T_b}\right)dy \end{aligned} \tag{4.28}$$

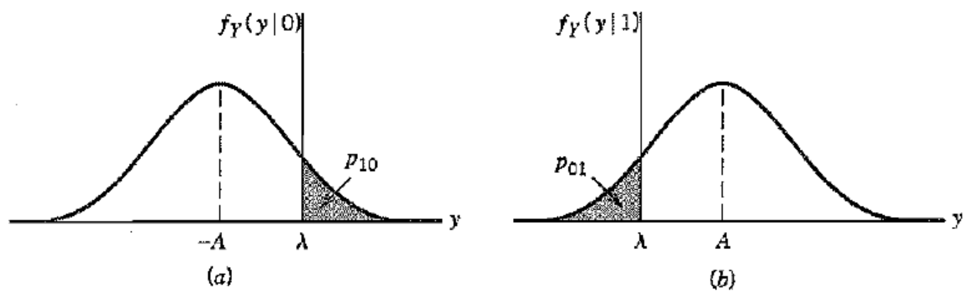


FIGURE 4.5 Noise analysis of PCM system. (*a*) Probability density function of random variable Y at matched filter output when 0 is transmitted. (*b*) Probability density function of *Y* when 1 is transmitted.

At this point in the discussion we digress briefly and introduce the definition of the so-called *complementary error function*:[3]

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}}\int_u^\infty \exp(-z^2)\,dz \tag{4.29}$$

which is closely related to the Gaussian distribution. For large positive values of u, we have the following *upper bound* on the complementary error function:

$$\operatorname{erfc}(u) < \frac{\exp(-u^2)}{\sqrt{\pi}u} \tag{4.30}$$

To reformulate the conditional probability of error p_{10} in terms of the complementary error function, we first define a new variable

$$z = \frac{y + A}{\sqrt{N_0/T_b}}$$

Accordingly, we may rewrite Equation (4.28) in the compact form

$$\begin{aligned} p_{10} &= \frac{1}{\sqrt{\pi}} \int_{(A+\lambda)/\sqrt{N_0/T_b}}^{\infty} \exp(-z^2)\, dz \\ &= \frac{1}{2} \operatorname{erfc}\left(\frac{A + \lambda}{\sqrt{N_0/T_b}}\right) \end{aligned} \tag{4.31}$$

Similarly you can derive

$$\begin{aligned} p_{01} &= \frac{1}{\sqrt{\pi}} \int_{(A-\lambda)/\sqrt{N_0/T_b}}^{\infty} \exp(-z^2)\, dz \\ &= \frac{1}{2} \operatorname{erfc}\left(\frac{A - \lambda}{\sqrt{N_0/T_b}}\right) \end{aligned} \tag{4.34}$$

Let p_0 and p_1 denote the *a priori* probabilities of transmitting symbols 0 and 1, respectively. Hence, the *average probability of symbol error* P_e in the receiver is given by

$$\begin{aligned} P_e &= p_0 p_{10} + p_1 p_{01} \\ &= \frac{p_0}{2} \operatorname{erfc}\left(\frac{A + \lambda}{\sqrt{N_0/T_b}}\right) + \frac{p_1}{2} \operatorname{erfc}\left(\frac{A - \lambda}{\sqrt{N_0/T_b}}\right) \end{aligned} \tag{4.35}$$

From Equation (4.35) we see that P_e is in fact a function of the threshold λ, which immediately suggests the need for formulating an *optimum threshold* that minimizes P_e. The optimum threshold is given as

$$\lambda_{\text{opt}} = \frac{N_0}{4AT_b} \log\left(\frac{p_0}{p_1}\right) \tag{4.37}$$

For the special case when symbols 1 and 0 are equiprobable, we have

$$p_1 = p_0 = \frac{1}{2}$$

in which case Equation (4.37) reduces to

$$\lambda_{\text{opt}} = 0$$

This result is intuitively satisfying as it states that, for the transmission of equiprobable binary symbols, we should choose the threshold at the midpoint between the pulse heights $-A$ and $+A$ representing the two symbols 0 and 1. Note that for this special case we also have

$$p_{01} = p_{10}$$

A channel for which the conditional probabilities of error p_{01} and p_{10} are equal is said to be *binary symmetric*. Correspondingly, the average probability of symbol error in Equation (4.35) reduces to

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{A}{\sqrt{N_0/T_b}}\right) \tag{4.38}$$

Now the *transmitted signal energy per bit* is defined by

$$E_b = A^2 T_b \tag{4.39}$$

Accordingly, we may finally formulate the average probability of symbol error for the receiver in Figure 4.4 as

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{4.40}$$

which shows that *the average probability of symbol error in a binary symmetric channel depends solely on E_b/N_0, the ratio of the transmitted signal energy per bit to the noise spectral density.*

This important result is further illustrated in Figure 4.6 where the average probability of symbol error P_e is plotted versus the dimensionless ratio E_b/N_0. In particular, we see that P_e decreases very rapidly as the ratio E_b/N_0 is increased, so that eventually a very "small increase" in transmitted signal energy will make the reception of binary pulses almost error free, as discussed previously in Section 3.8. Note, however, that in practical terms the increase in signal energy has to be viewed in the context of the bias; for example, a 3-dB increase in E_b/N_0 is much easier to implement when E_b has a small value than when its value is orders of magnitude larger.

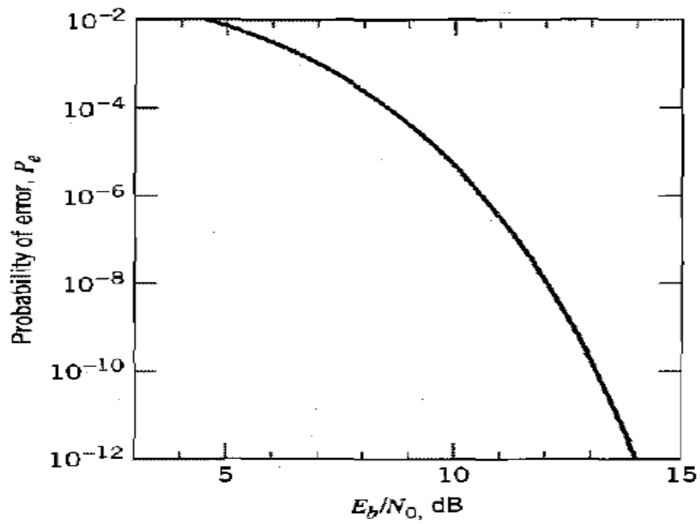


FIGURE 4.6 Probability of error in a PCM receiver.

2.4 Inter symbol Interference (ISI)

The residual effects due to the occurrence of pulses before and after the sampling instant is called inter symbol interference (ISI).

The presence of outputs due to other bits (symbols) interfere with the output of required bit (symbol). This effect is called Intersymbol Interference (ISI).

Parameters used to reduce ISI

The ISI can be reduced by proper design of pulse spectrum $G(f)$, transmit filter $H_T(f)$, receive filter $H_R(f)$ and the channel $H_C(f)$. We will discuss some of these issues in subsequent sections.

Consider then a *baseband binary PAM system*, a generic form of which is shown in Figure 4.7. The incoming binary sequence $\{b_k\}$ consists of symbols 1 and 0, each of duration T_b. The *pulse-amplitude modulator* modifies this binary sequence into a new sequence of short pulses (approximating a unit impulse), whose amplitude a_k is represented in the polar form

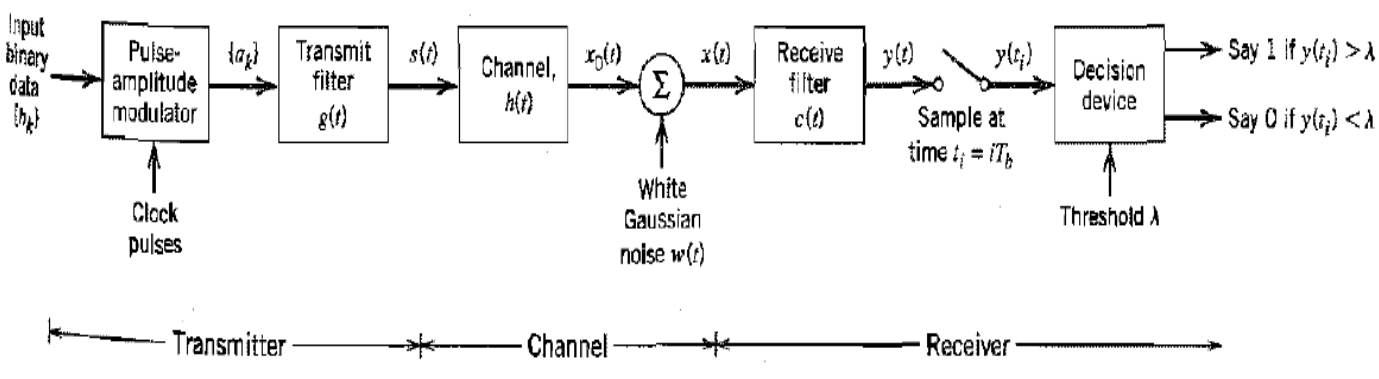


FIGURE 4.7 Baseband binary data transmission system.

$$a_k = \begin{cases} +1 & \text{if symbol } b_k \text{ is } 1 \\ -1 & \text{if symbol } b_k \text{ is } 0 \end{cases} \tag{4.42}$$

The sequence of short pulses so produced is applied to a *transmit filter* of impulse response $g(t)$, producing the transmitted signal

$$s(t) = \sum_k a_k g(t - kT_b) \tag{4.43}$$

The signal $s(t)$ is modified as a result of transmission through the *channel* of impulse response $h(t)$. In addition, the channel adds random noise to the signal at the receiver input. The noisy signal $x(t)$ is then passed through a *receive filter* of impulse response $c(t)$. The resulting filter output $y(t)$ is sampled *synchronously* with the transmitter, with the sampling instants being determined by a *clock* or *timing signal* that is usually extracted from the receive filter output. Finally, the sequence of samples thus obtained is used to reconstruct the original data sequence by means of a *decision device.* Specifically, the amplitude of each sample is compared to a *threshold* λ. If the threshold λ is exceeded, a decision is made in favor of symbol 1. If the threshold λ is not exceeded, a decision is made in favor of symbol 0. If the sample amplitude equals the threshold exactly, the flip of a fair coin will determine which symbol was transmitted (i.e., the receiver simply makes a random guess).

The receive filter output is written as

$$y(t) = \mu \sum_k a_k p(t - kT_b) + n(t) \tag{4.44}$$

where μ is a scaling factor, and the pulse $p(t)$ is to be defined. To be precise, an arbitrary time delay t_0 should be included in the argument of the pulse $p(t - kT_b)$ in Equation (4.44) to represent the effect of transmission delay through the system. To simplify the exposition, we have put this delay equal to zero in Equation (4.44) without loss of generality.

The scaled pulse $\mu p(t)$ is obtained by a double convolution involving the impulse response $g(t)$ of the transmit filter, the impulse response $h(t)$ of the channel, and the impulse response $c(t)$ of the receive filter, as shown by

$$\mu p(t) = g(t) \bigstar h(t) \bigstar c(t) \tag{4.45}$$

where the star denotes convolution. We assume that the pulse $p(t)$ is *normalized* by setting

$$p(0) = 1 \tag{4.46}$$

which justifies the use of μ as a scaling factor to account for amplitude changes incurred in the course of signal transmission through the system.

Since convolution in the time domain is transformed into multiplication in the frequency domain, we may use the Fourier transform to change Equation (4.45) into the equivalent form

$$\mu P(f) = G(f)H(f)C(f) \tag{4.47}$$

where $P(f)$, $G(f)$, $H(f)$, and $C(f)$ are the Fourier transforms of $p(t)$, $g(t)$, $h(t)$, and $c(t)$, respectively.

Finally, the term $n(t)$ in Equation (4.44) is the noise produced at the output of the receive filter due to the channel noise $w(t)$. It is customary to model $w(t)$ as a white Gaussian noise of zero mean.

The receive filter output $y(t)$ is sampled at time $t_i = iT_b$ (with i taking on integer values), yielding [in light of Equation (4.46)]

$$\begin{aligned} y(t_i) &= \mu \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b] + n(t_i) \\ &= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b] + n(t_i) \end{aligned} \tag{4.48}$$

In Equation (4.48), the first term μa_i represents the contribution of the ith transmitted bit. The second term represents the residual effect of all other transmitted bits on the decoding of the ith bit; this residual effect due to the occurrence of pulses before and after the sampling instant t_i is called intersymbol interference (ISI). The last term $n(t_i)$ represents the noise sample at time t_i.

In the absence of both ISI and noise, we observe from Equation (4.48) that

$$y(t_i) = \mu a_i$$

which shows that, under these ideal conditions, the ith transmitted bit is decoded correctly.

2.5 Nyquist's criterion for Distortionless Base band Binary Transmission

2.5.1 Nyquist Pulse Shaping Criterion

Time Domain Criterion

From equation 4.48 we know that the second term (summation) must be zero to eliminate effect of ISI. This is possible if the received pulse p(t) is controlled such that,

$$p[(i-k)T_b] = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{for } i \neq k \end{cases} \quad \text{... (2.8.1)}$$

If p(t) satisfies the above condition, then we get a signal which is free from ISI. i.e.,

$$y(t_i) = \mu A_i \qquad \text{from equation 2.7.10}$$

Hence equation 2.8.1 gives the condition for perfect reception in absence of noise. Equation 2.8.1 is the condition in time domain. This condition gives more useful criteria in frequency domain.

Criterion in Frequency Domain

- Let $p(nT_b)$ represent the impulses at which p(t) is sampled for decision. These samples are taken at the rate of T_b. Fourier spectrum of these impulses is given as

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - nf_b) \qquad \text{... (2.8.2)}$$

This means the spectrums of p(t) are periodic with period f_b. Here note that the sampling frequency (instants) is f_b. Here $P_\delta(f)$ represents the spectrum of $p(nT_b)$, and P(f) is the spectrum of p(t).

- We can think of $p(nT_b)$ as the infinite length of impulses with period T_b, which are weighted with amplitudes of p(t). i.e.,

$$p_\delta(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\delta(t - nT_b) \qquad \text{... (2.8.3)}$$

- Fourier transform of $p_\delta(t)$ becomes,

$$P_\delta(f) = \int_{-\infty}^{\infty} p_\delta(t) e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} p(nT_b)\delta(t - nT_b) \right] e^{-j2\pi ft} dt$$

- Let n = i – k in above equation,

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} p[(i-k)T_b]\delta[t-(i-k)T_b] e^{-j2\pi ft} dt$$

- Now let us apply the condition of equation 2.8.1 to above equation,

$$P_\delta(f) = \begin{cases} \int_{-\infty}^{\infty} p(0)\delta(t) e^{-j2\pi ft} dt & \text{for } i = k \\ \int_{-\infty}^{\infty} 0 \ \delta(t) e^{-j2\pi ft} dt & \text{for } i \neq k \end{cases}$$

$$\therefore \quad P_\delta(f) = \int_{-\infty}^{\infty} p(0)\delta(t)e^{-j2\pi ft}\,dt \quad for\ i = k$$

$$= p(0)\int_{-\infty}^{\infty} \delta(t)e^{-j2\pi ft}\,dt$$

An integration in above equation is the fourier transform of $\delta(t)$, which is 1. Hence,

$$P_\delta(f) = p(0) \quad for\ i = k \qquad \text{... (2.8.4)}$$

$$= 1 \qquad \text{by normalization of p(0).}$$

- Hence equation 2.8.2 becomes (with $P_\delta(f) = 1$),

$$1 = f_b \sum_{n=-\infty}^{\infty} P(f - nf_b)$$

or

$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = \frac{1}{f_b} \qquad \text{... (2.8.5)}$$

Since $\frac{1}{f_b} = T_b$,

$$\boxed{\sum_{n=-\infty}^{\infty} P(f - nf_b) = T_b} \qquad \text{... (2.8.6)}$$

This is the frequency domain condition for zero ISI. Above equation is called Nyquist pulse shaping criterion for baseband transmission.

2.5.2 Ideal Nyquist channel

The simplest way of satisfying Equation 2.8.6 is to specify the frequency function $P(f)$ to be in the form of a *rectangular function*, as shown by

$$P(f) = \begin{cases} \frac{1}{2W}, & -W < f < W \\ 0, & |f| > W \end{cases}$$
$$= \frac{1}{2W}\,\text{rect}\left(\frac{f}{2W}\right) \qquad (4.54)$$

where $\text{rect}(f)$ stands for a *rectangular function* of unit amplitude and unit support centered on $f = 0$, and the overall system bandwidth W is defined by

$$W = \frac{R_b}{2} = \frac{1}{2T_b} \qquad (4.55)$$

According to the solution described by Equations (4.54) and (4.55), no frequencies of absolute value exceeding half the bit rate are needed. Hence, from Fourier-transform pair 2 of Table A6.3 we find that a signal waveform that produces zero intersymbol interference is defined by the *sinc function*:

$$p(t) = \frac{\sin(2\pi Wt)}{2\pi Wt} = \text{sinc}(2Wt) \tag{4.56}$$

The special value of the bit rate $R_b = 2W$ is called the *Nyquist rate*, and W is itself called the *Nyquist bandwidth*. Correspondingly, the ideal baseband pulse transmission system described by Equation (4.54) in the frequency domain or, equivalently, Equation (4.56) in the time domain, is called the *ideal Nyquist channel*.

Figures 4.8*a* and 4.8*b* show plots of $P(f)$ and $p(t)$, respectively. In Figure 4.8*a*, the normalized form of the frequency function $P(f)$ is plotted for positive and negative frequencies. In Figure 4.8*b*, we have also included the signaling intervals and the corresponding centered sampling instants. The function $p(t)$ can be regarded as the impulse response of an ideal low-pass filter with passband magnitude response $1/2W$ and bandwidth W. The function $p(t)$ has its peak value at the origin and goes through zero at integer multiples of the bit duration T_b. It is apparent that if the received waveform $y(t)$ is sampled at the

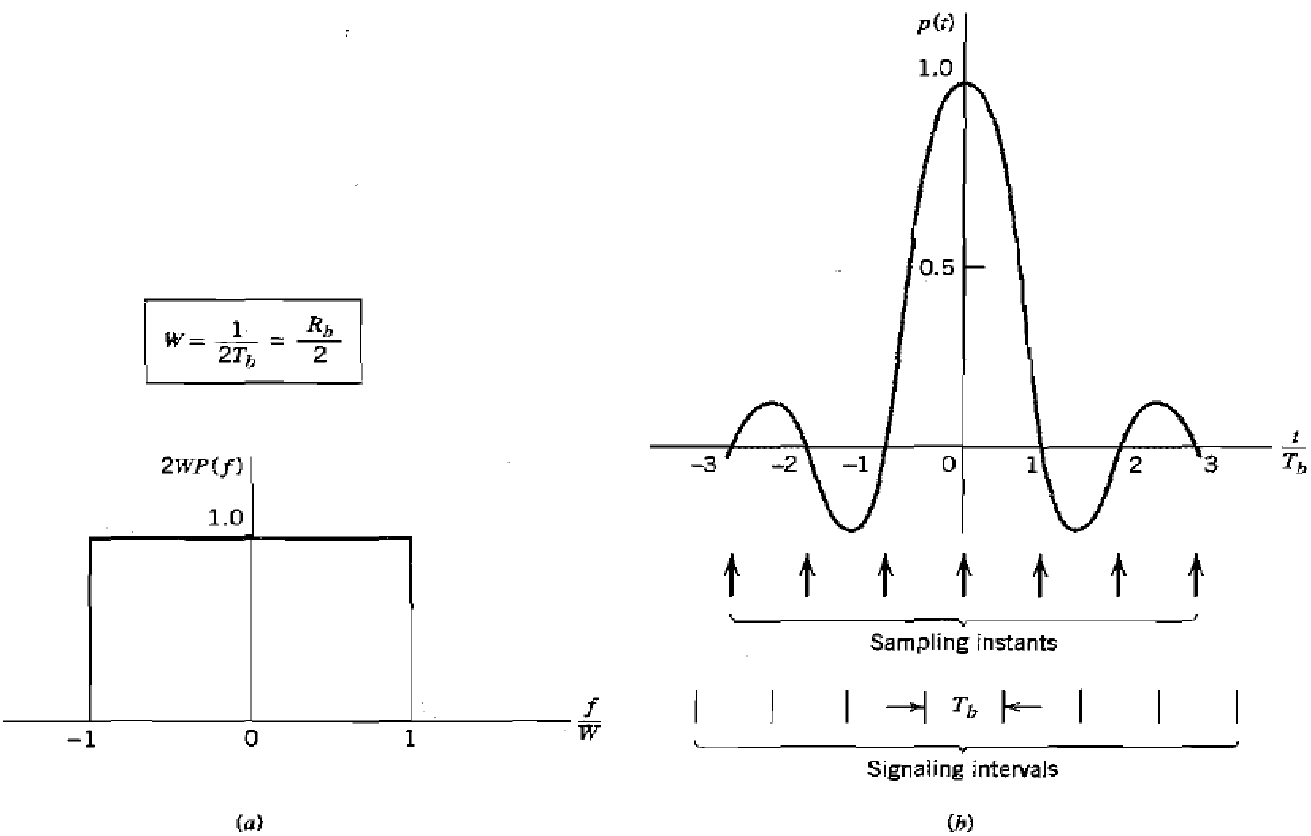


FIGURE 4.8 (*a*) Ideal magnitude response. (*b*) Ideal basic pulse shape.

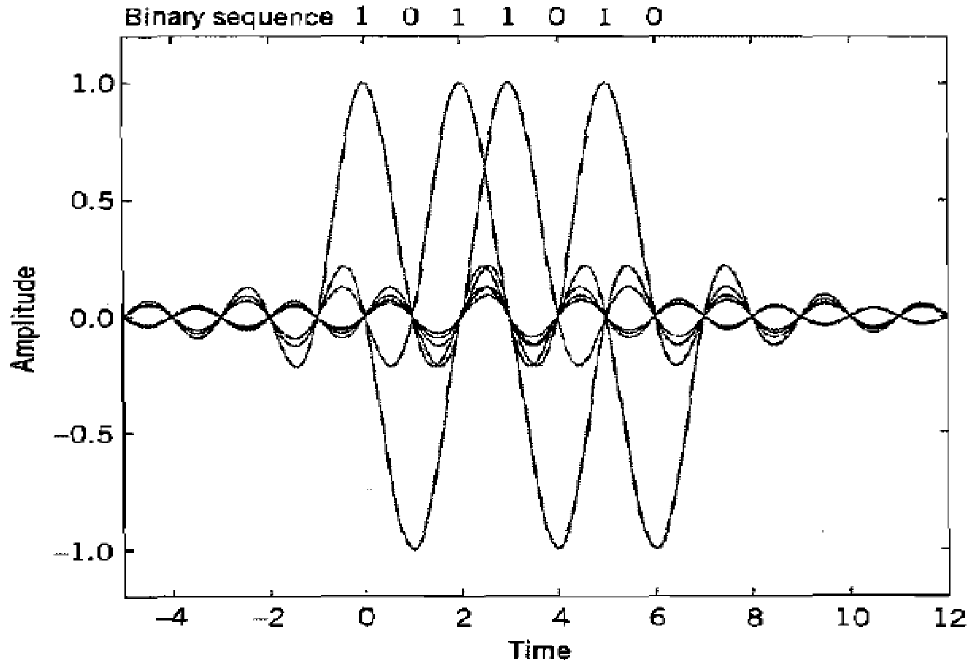


FIGURE 4.9 A series of sinc pulses corresponding to the sequence 1011010.

instants of time $t = 0, \pm T_b, \pm 2T_b, \cdots$, then the pulses defined by $\mu p(t - iT_b)$ with arbitrary amplitude μ and index $i = 0, \pm 1, \pm 2, \cdots$, will not interfere with each other. This condition is illustrated in Figure 4.9 for the binary sequence 1011010.

Although the use of the ideal Nyquist channel does indeed achieve economy in bandwidth in that it solves the problem of zero intersymbol interference with the minimum bandwidth possible

2.5.3 Raised cosine channels

We may overcome the practical difficulties encountered with the ideal Nyquist channel by extending the bandwidth from the minimum value $W = R_b/2$ to an adjustable value between W and $2W$. We now specify the overall frequency response $P(f)$ to satisfy a condition more elaborate than that for the ideal Nyquist channel; specifically, we retain three terms of Equation (4.53) and restrict the frequency band of interest to $[-W, W]$, as shown by

$$P(f) + P(f - 2W) + P(f + 2W) = \frac{1}{2W}, \qquad -W \leq f \leq W \qquad (4.59)$$

We may devise several band-limited functions that satisfy Equation (4.59). A particular form of $P(f)$ that embodies many desirable features is provided by a *raised cosine spectrum*. This frequency response consists of a *flat* portion and a *rolloff* portion that has a sinusoidal form, as follows:

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 \le |f| < f_1 \\ \dfrac{1}{4W}\left\{1 - \sin\left[\dfrac{\pi(|f| - W)}{2W - 2f_1}\right]\right\}, & f_1 \le |f| < 2W - f_1 \\ 0, & |f| \ge 2W - f_1 \end{cases} \tag{4.60}$$

The frequency parameter f_1 and bandwidth W are related by

$$\alpha = 1 - \frac{f_1}{W} \tag{4.61}$$

The parameter α is called the *rolloff factor*; it indicates the *excess bandwidth* over the ideal solution, W. Specifically, the transmission bandwidth B_T is defined by

$$\begin{aligned} B_T &= 2W - f_1 \\ &= W(1 + \alpha) \end{aligned}$$

The frequency response $P(f)$, normalized by multiplying it by $2W$, is plotted in Figure 4.10*a* for three values of α, namely, 0, 0.5, and 1. We see that for $\alpha = 0.5$ or 1, the

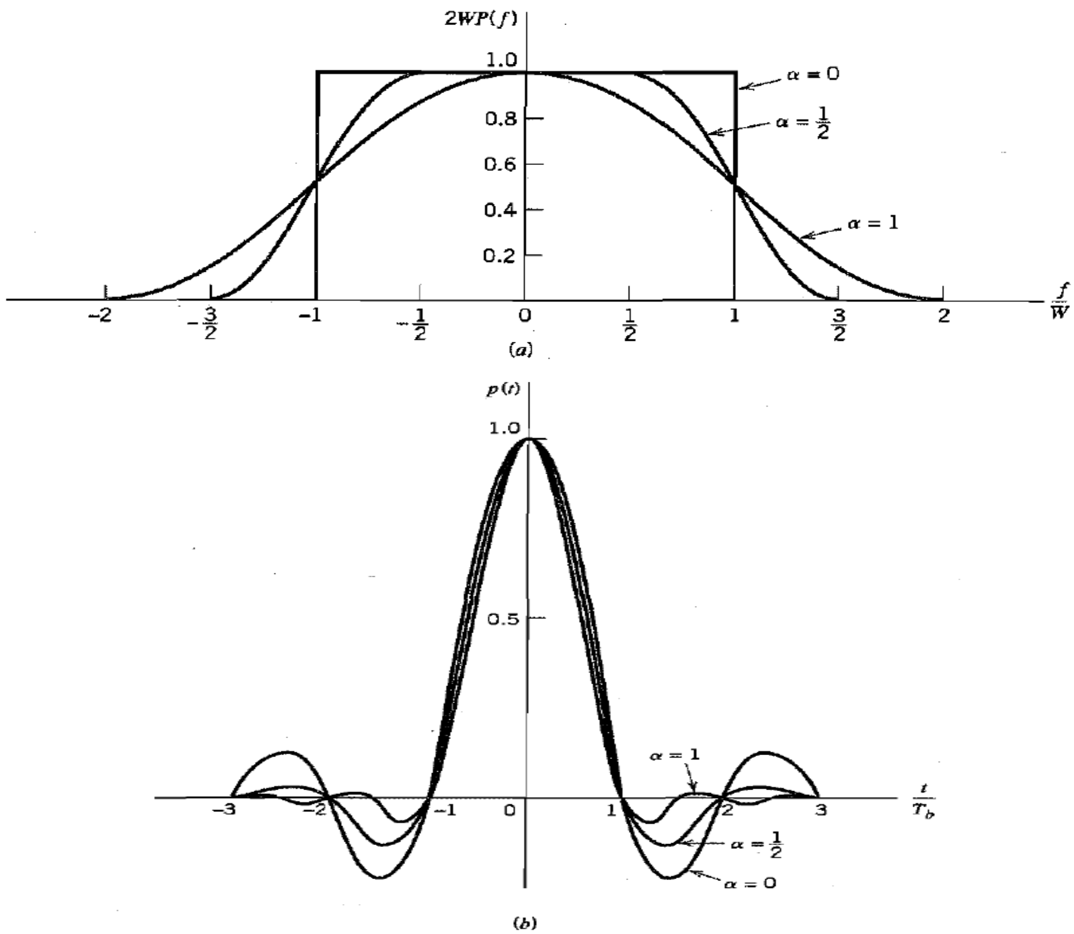


FIGURE 4.10 Responses for different rolloff factors. (*a*) Frequency response. (*b*) Time response.

function $P(f)$ cuts off gradually as compared with the ideal Nyquist channel (i.e., $\alpha = 0$) and is therefore easier to implement in practice. Also the function $P(f)$ exhibits odd symmetry with respect to the Nyquist bandwidth W, making it possible to satisfy the condition of Equation (4.59).

The time response $p(t)$ is the inverse Fourier transform of the frequency response $P(f)$. Hence, using the $P(f)$ defined in Equation (4.60), we obtain the result (see Problem 4.13)

$$p(t) = (\text{sinc}(2Wt))\left(\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2W^2t^2}\right) \tag{4.62}$$

which is plotted in Figure 4.10b for $\alpha = 0$, 0.5, and 1.

2.6 Correlative level coding

By adding inter symbol interference to the transmitted signal in a controlled manner, it is possible to achieve a signaling rate equal to the Nyquist rate of 2W symbols per second in a channel of bandwidth W Hertz. Such schemes are called correlative- level coding or partial-response signaling schemes.

- Correlative level coding allows the signaling rate of $2B_0$ in the channel of bandwidth B_0. This is made physically possible by allowing ISI in the transmitted signal in controlled manner. This ISI is known to the receiver. Hence effects of ISI are eliminated at the receiver.
- The correlative coding is implemented by duobinary signaling and modified duobinary signaling.

2.6.1 Duobinary signalling

The basic idea of correlative-level coding will now be illustrated by considering the specific example of *duobinary signaling*, where "duo" implies doubling of the transmission capacity of a straight binary system. This particular form of correlative-level coding is also called *class I partial response.*

Consider a binary input sequence $\{b_k\}$ consisting of uncorrelated binary symbols 1 and 0, each having duration T_b. As before, this sequence is applied to a pulse-amplitude modulator producing a two-level sequence of short pulses (approximating a unit impulse), whose amplitude a_k is defined by

$$a_k = \begin{cases} +1 & \text{if symbol } b_k \text{ is } 1 \\ -1 & \text{if symbol } b_k \text{ is } 0 \end{cases} \tag{4.65}$$

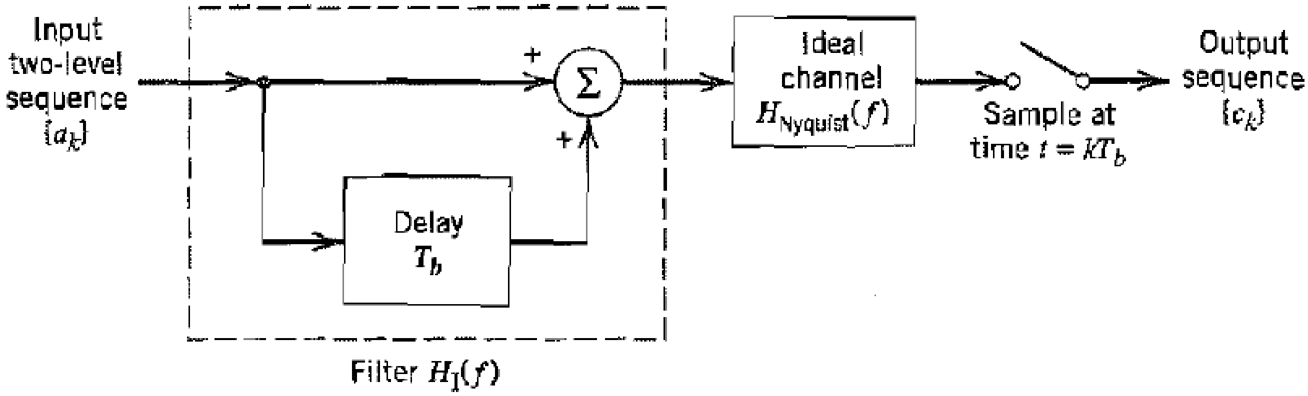


FIGURE 4.11 Duobinary signaling scheme.

When this sequence is applied to a *duobinary encoder*, it is converted into a *three-level output*, namely, -2, 0, and $+2$. To produce this transformation, we may use the scheme shown in Figure 4.11. The two-level sequence $\{a_k\}$ is first passed through a simple filter involving a single delay element and summer. For every unit impulse applied to the input of this filter, we get two unit impulses spaced T_b seconds apart at the filter output. We may therefore express the duobinary coder output c_k as the sum of the present input pulse a_k and its previous value a_{k-1}, as shown by

$$c_k = a_k + a_{k-1} \tag{4.66}$$

One of the effects of the transformation described by Equation (4.66) is to change the input sequence $\{a_k\}$ of uncorrelated two-level pulses into a sequence $\{c_k\}$ of correlated three-level pulses. This correlation between the adjacent pulses may be viewed as introducing intersymbol interference into the transmitted signal in an artificial manner. However, the intersymbol interference so introduced is under the designer's control, which is the basis of correlative coding.

An ideal delay element, producing a delay of T_b seconds, has the frequency response $\exp(-j2\pi f T_b)$, so that the frequency response of the simple delay-line filter in Figure 4.11 is $1 + \exp(-j2\pi f T_b)$. Hence, the overall frequency response of this filter connected in cascade with an ideal Nyquist channel is

$$\begin{aligned} H_I(f) &= H_{\text{Nyquist}}(f)[1 + \exp(-j2\pi f T_b)] \\ &= H_{\text{Nyquist}}(f)[\exp(j\pi f T_b) + \exp(-j\pi f T_b)]\exp(-j\pi f T_b) \\ &= 2H_{\text{Nyquist}}(f)\cos(\pi f T_b)\exp(-j\pi f T_b) \end{aligned} \tag{4.67}$$

where the subscript I in $H_I(f)$ indicates the pertinent class of partial response. For an ideal Nyquist channel of bandwidth $W = 1/2T_b$, we have (ignoring the scaling factor T_b)

$$H_{\text{Nyquist}}(f) = \begin{cases} 1, & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases} \tag{4.68}$$

Thus the overall frequency response of the duobinary signaling scheme has the form of a half-cycle cosine function, as shown by

$$H_I(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b), & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases} \tag{4.69}$$

for which the magnitude response and phase response are as shown in Figures 4.12*a* and 4.12*b*, respectively. An advantage of this frequency response is that it can be easily approximated, in practice, by virtue of the fact that there is continuity at the band edges.

From the first line in Equation (4.67) and the definition of $H_{\text{Nyquist}}(f)$ in Equation (4.68), we find that the impulse response corresponding to the frequency response $H_I(f)$ consists of two sinc (Nyquist) pulses that are time-displaced by T_b seconds with respect to each other, as shown by (except for a scaling factor)

$$\begin{aligned} h_I(t) &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t - T_b)/T_b]}{\pi(t - T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t - T_b)/T_b} \\ &= \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)} \end{aligned} \tag{4.70}$$

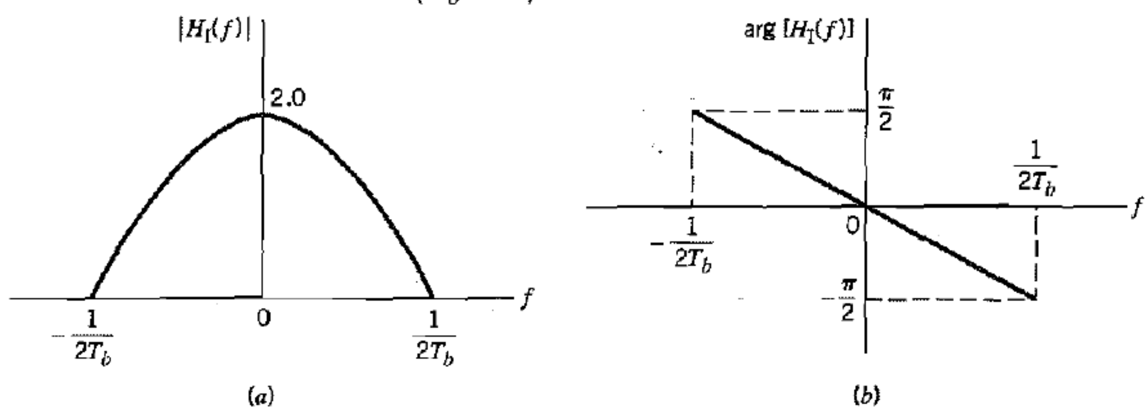


FIGURE 4.12 Frequency response of the duobinary conversion filter. (*a*) Magnitude response. (*b*) Phase response.

The impulse response $h_I(t)$ is plotted in Figure 4.13, where we see that it has only *two* distinguishable values at the sampling instants. The form of $h_I(t)$ shown here explains why we also refer to this type of correlative coding as partial-response signaling. The response to an input pulse is spread over more than one signaling interval; stated in another way, the response in any signaling interval is "partial." Note also that the tails of $h_I(t)$ decay as $1/|t|^2$, which is a faster rate of decay than the $1/|t|$ encountered in the ideal Nyquist channel.

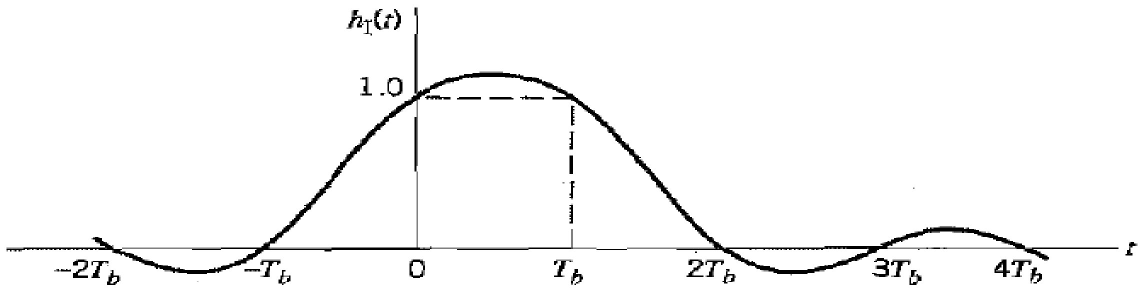


FIGURE 4.13 Impulse response of the duobinary conversion filter.

2.6.2 Modified Duobinary Signalling

In the duobinary signaling technique the frequency response $H(f)$, and consequently the power spectral density of the transmitted pulse, is nonzero at the origin. This is considered to be an undesirable feature in some applications, since many communications channels cannot transmit a DC component. We may correct for this deficiency by using the *class IV partial response* or *modified duobinary* technique, which involves a correlation span of two binary digits. This special form of correlation is achieved by subtracting amplitude-modulated pulses spaced $2T_b$ seconds apart, as indicated in the block diagram of Figure

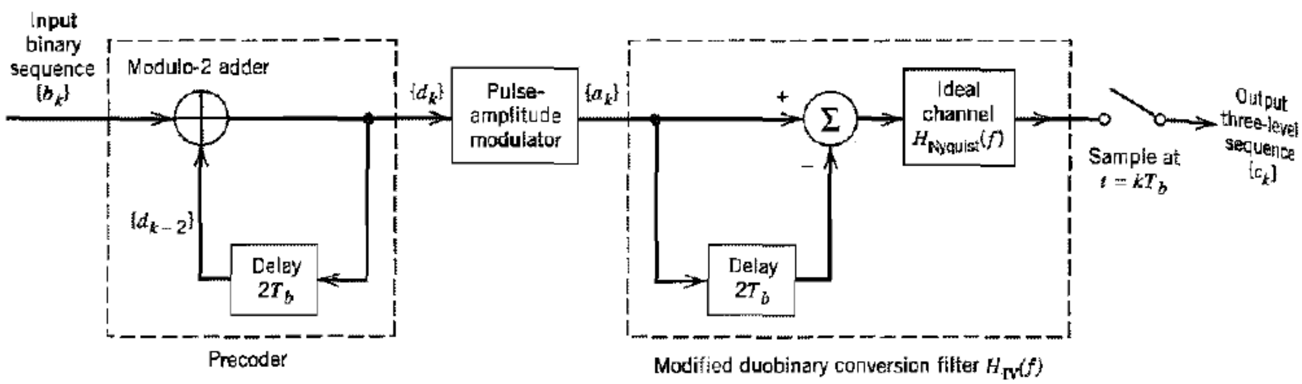


FIGURE 4.16 Modified duobinary signaling scheme.

4.16. The precoder involves a delay of $2T_b$ seconds. The output of the modified duobinary conversion filter is related to the input two-level sequence $\{a_k\}$ at the pulse-amplitude modulator output as follows:

$$c_k = a_k - a_{k-2} \tag{4.77}$$

Here, again, we find that a three-level signal is generated. With $a_k = \pm 1$, we find that c_k takes on one of three values: $+2$, 0, and -2.

The overall frequency response of the delay-line filter connected in cascade with an ideal Nyquist channel, as in Figure 4.16, is given by

$$\begin{aligned} H_{IV}(f) &= H_{Nyquist}(f)[1 - \exp(-j4\pi f T_b)] \\ &= 2jH_{Nyquist}(f)\sin(2\pi f T_b)\exp(-j2\pi f T_b) \end{aligned} \tag{4.78}$$

where the subscript IV in $H_{IV}(f)$ indicates the pertinent class of partial response and $H_{Nyquist}(f)$ is as defined in Equation (4.68). We therefore have an overall frequency response in the form of a half-cycle sine function, as shown by

$$H_{IV}(f) = \begin{cases} 2j \sin(2\pi f T_b) \exp(-j2\pi f T_b), & |f| \leq 1/2T_b \\ 0, & \text{elsewhere} \end{cases} \tag{4.79}$$

The corresponding magnitude response and phase response of the modified duobinary coder are shown in Figures 4.17*a* and 4.17*b*, respectively. A useful feature of the modified duobinary coder is the fact that its output has no DC component. Note also that this

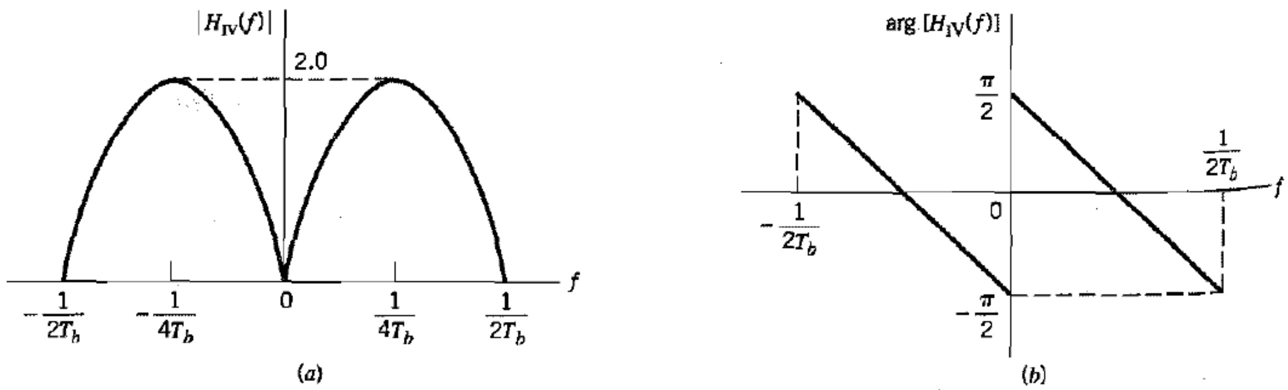


FIGURE 4.17 Frequency response of the modified duobinary conversion filter. (*a*) Magnitude response. (*b*) Phase response.

second form of correlative-level coding exhibits the same continuity at the band edges as in duobinary signaling.

From the first line of Equation (4.78) and the definition of $H_{Nyquist}(f)$ in Equation (4.68), we find that the impulse response of the modified duobinary coder consists of two sinc (Nyquist) pulses that are time-displaced by $2T_b$ seconds with respect to each other, as shown by (except for a scaling factor)

$$\begin{aligned} h_{IV}(t) &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi(t-2T_b)/T_b)]}{\pi(t-2T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-2T_b)/T_b} \\ &= \frac{2T_b^2 \sin(\pi t/T_b)}{\pi t(2T_b - t)} \end{aligned} \tag{4.80}$$

This impulse response is plotted in Figure 4.18, which shows that it has *three* distinguishable levels at the sampling instants. Note also that, as with duobinary signaling, the tails of $h_{IV}(t)$ for the modified duobinary signaling decay as $1/|t|^2$.

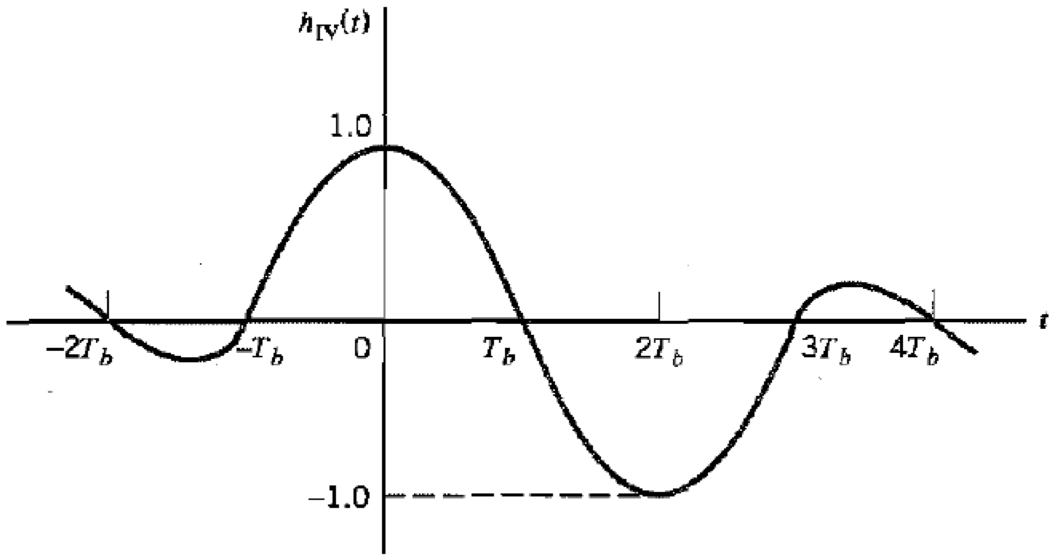


FIGURE 4.18 Impulse response of the modified duobinary conversion filter.

2.6.3 Generalized form of correlative level coding (Partial response signaling)

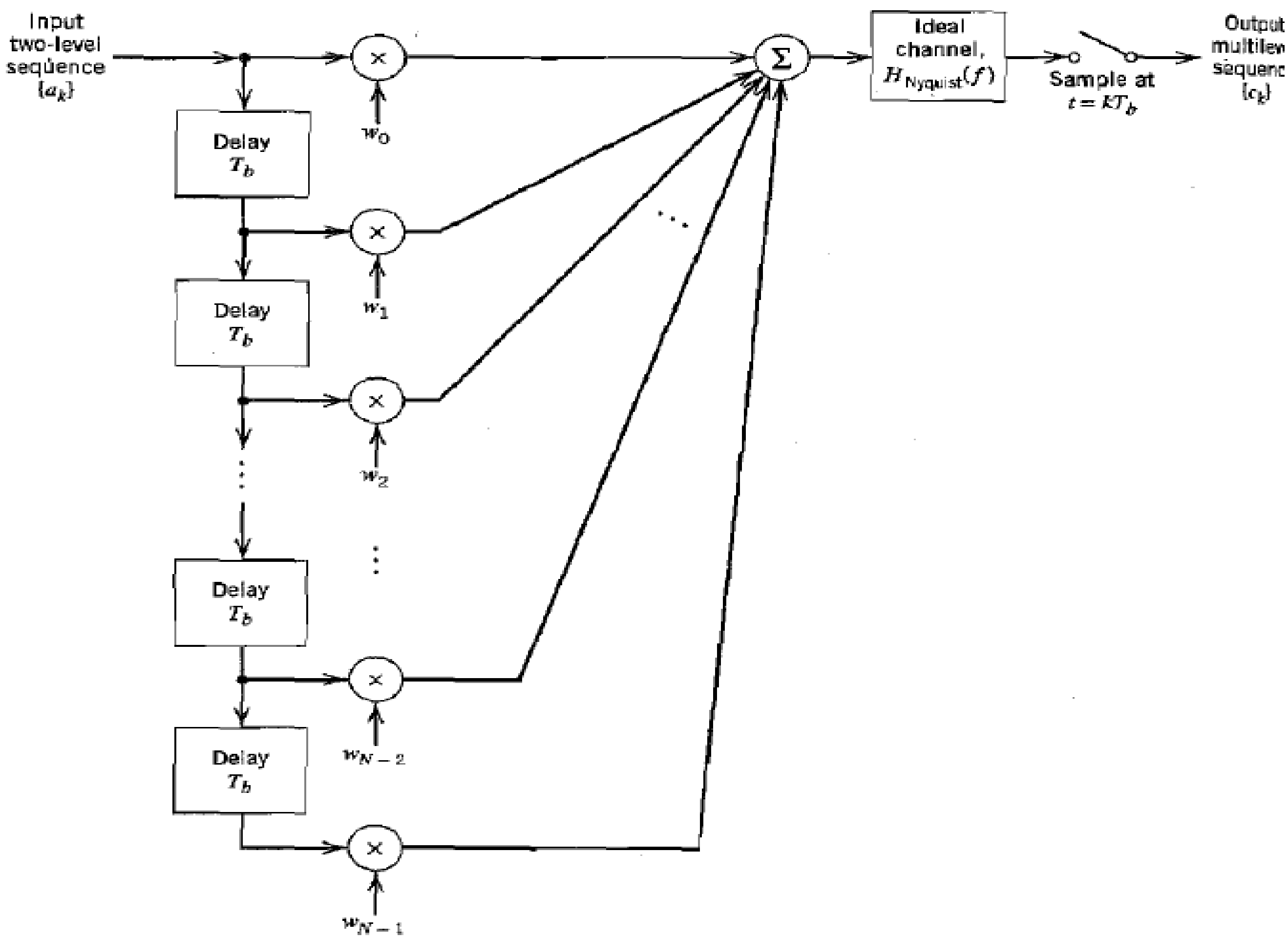


FIGURE 4.19 Generalized correlative coding scheme.

The duobinary and modified duobinary techniques have correlation spans of 1 binary digit and 2 binary digits, respectively. It is a straightforward matter to generalize these two techniques to other schemes, which are known collectively as *correlative-level coding* or *partial-response signaling* schemes. This generalization is shown in Figure 4.19, where $H_{\text{Nyquist}}(f)$ is defined in Equation (4.68). It involves the use of a tapped-delay-line filter with tap-weights $w_0, w_1, \cdots, w_{N-1}$. Specifically, different classes of partial-response signaling schemes may be achieved by using a weighted linear combination of N ideal Nyquist (sinc) pulses, as shown by

$$h(t) = \sum_{n=0}^{N-1} w_n \operatorname{sinc}\left(\frac{t}{T_b} - n\right) \qquad (4.83)$$

An appropriate choice of the tap-weights in Equation (4.83) results in a variety of spectral shapes designed to suit individual applications. Table 4.2 presents the specific details of five different classes of partial-response signaling schemes.

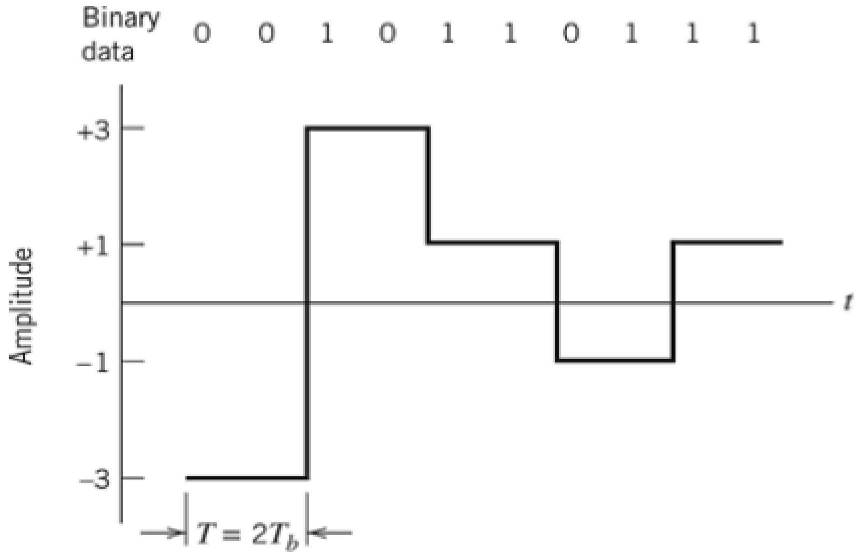
TABLE 4.2 ***Different classes of partial-response signaling schemes referring to Figure 4.19***

Type of Class	N	w_0	w_1	w_2	w_3	w_4	*Comments*
I	2	1	1				Duobinary coding
II	3	1	2	1			
III	3	2	1	−1			
IV	3	1	0	−1			Modified duobinary coding
V	5	−1	0	2	0	−1	

2.7 Baseband M- ary PAM transmission

- Up to now for binary systems the pulses have two possible amplitude levels.
- In a baseband M-ary PAM system, the pulse amplitude modulator produces M possible amplitude levels with M>2.
- In an M-ary system, the information source emits a sequence of symbols from an alphabet that consists of M symbols.
- Each amplitude level at the PAM modulator output corresponds to a distinct symbol.
- The symbol duration T is also called as the signaling rate of the system, which is expressed as symbols per second or bauds.
- Let's consider the following quaternary (M=4) system.
- The symbol rate is 1/ ($2T_b$), since each symbol consists of two bits.
- The symbol duration T of the M- ary system is related to the bit duration T_b of the equivalent binary PAM system as

$$T = T_b \log_2 M$$



Dibit	Amplitude
00	−3
01	−1
11	+1
10	+3

- For a given channel bandwidth, using M-ary PAM system, $\log_2 M$ times more information is transmitted than binary PAM system.
- The price we paid is the increased bit error rate compared binary PAM system.
- To achieve the same probability of error as the binary PAM system, the transmit power in M -ary PAM system must be increased.
- For M much larger than 2 and an average probability of symbol error small compared to 1, the transmitted power must be increased by a factor of $M^2 / \log_2 M$ compared to binary PAM system.
- The M-ary PAM transmitter and receiver is similar to the binary PAM transmitter and receiver.
- In transmitter, the M-ary pulse train is shaped by a transmit filter and transmitted through a channel which corrupts the signal with noise and ISI.
- The received signal is passed through a receive filter and sampled at an appropriate rate in synchronism with the transmitter.
- Each sample is compared with preset threshold values and a decision is made as to which symbol was transmitted.
- Obviously, in M-ary system there are M -1 threshold levels which makes the system complicated.
- The raised cosine pulse shape, which is ISI-free for binary signaling is also ISI-free for M -ary signaling.

2.8 Equalization

When the signal is passed through the channel, distortion is introduced in terms of (i) amplitude and (ii) delay. This distortion creates the problems of ISI. The detection of the signal also becomes difficult. This distortion can be compensated with the help of *equalizers*. Equalizers are basically filters, which correct the channel distortion. Fig. 7.7.1 shows channel and equalizer for correction of distortion.

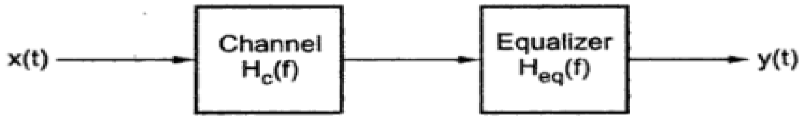


Fig. 7.7.1 Equalizer for correction of distortion introduced in the channel

In second chapter, we have derived a condition for distortionless transmission. The transfer function of distortionless system is given as,

$$H(f) = Ke^{-j2\pi f t_0}$$

The cascade connection of channel + equalizer shown in above figure will be distortionless if,

$$H_c(f) \cdot H_{eq}(f) = Ke^{-j2\pi f t_0}$$

Hence transfer function of the equalizer will be,

$$H_{eq}(f) = \frac{Ke^{-j2\pi f t_0}}{H_c(f)} \quad \text{... (7.7.1)}$$

The equation is difficult to realize directly, but approximations are available. It can be implemented with the help of tapped delay line filters.

2.8.1 Adaptive Equalization

Necessity :

Most of the channels are made up of individual links. For example, in the switched telephone network, the distortion induced depends upon

i) transmission characteristics of individual links and

ii) number of links in connnection

Hence, the fixed pair of transmit and receive filters will not serve the equalization problem completely. The transmission characteristics of the channel keep on changing. Hence *adaptive equalization* is used.

Basic Principle :

In adaptive equalization, the filters adapt themselves to the dispersive effects of the channel. That is the coefficients of the filters are changed continuously according to the received data. The filter coefficients are changed in such a way that the distortion in the data is reduced.

Types :

When an equalization is done at the transmitting side it is called prechannel equalization. This type of equalization requires feedback to know the amount of distortion in the received data. When an equalization is done at the receiving side, it is called postchannel equalization. In this case, no feedback is required. The equalizer is placed after the receiving filter in the receiver.

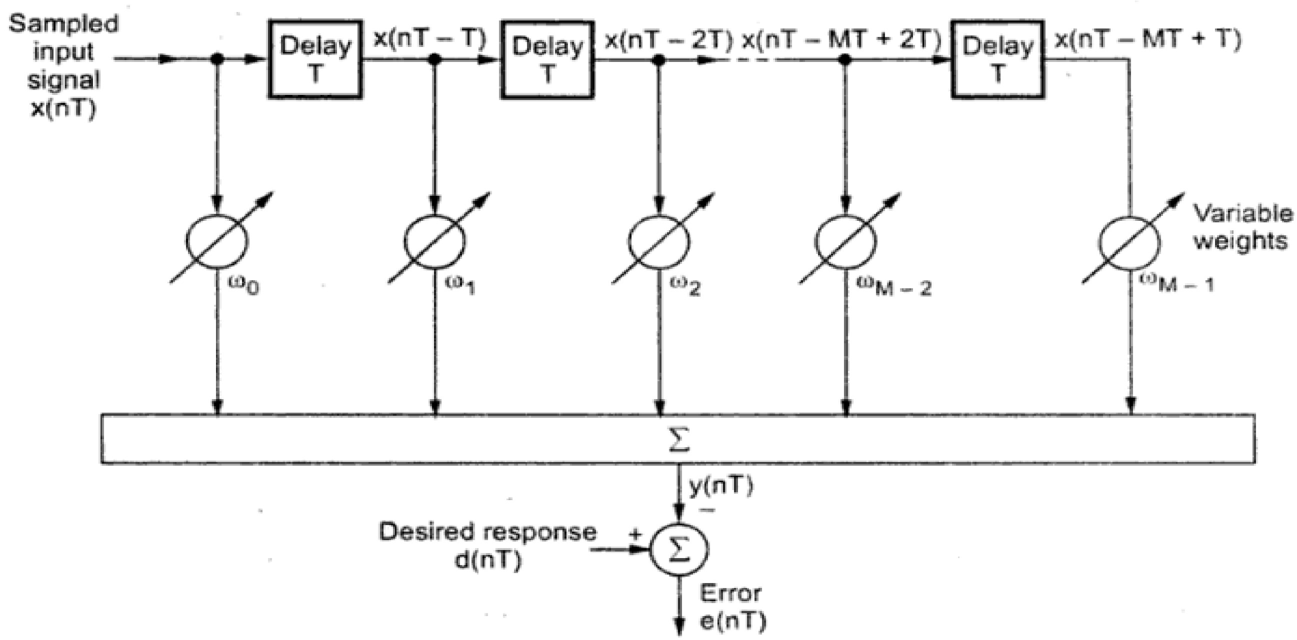


Fig. 7.7.3 Structure of adaptive equalizer

The adaptive equalizer shown in above figure is a tapped-delay-line fillter. It consists of set of delay elements and variable multipliers. The sequence $x(nT)$ is applied to the input of the adaptive filter. The output $y(nT)$ of the adaptive filter will be,

$$y(nT) = \sum_{i=0}^{M} w_i \, x(nT - iT) \qquad \ldots (7.7.1)$$

The weights w_i on the taps are basically adaptive filter coefficients. A known sequence $\{d(nT)\}$ is transmitted first. This sequence is known to the receiver. The response sequence $y(nT)$ is observed. As shown in Fig. 7.7.3, the error sequence between the two sequences is calculated. i.e.,

$$e(nT) = d(nT) - y(nT), \qquad n = 0, 1, \ldots N-1 \qquad \ldots (7.7.2)$$

Here note that if there is no distortion in the channel, then $d(nT)$ and $y(nT)$ will be exactly same producing zero error sequence. Then the weights of the filter i.e. w_i are changed recursively such that error $e(nT)$ is minimized. There are standard algorithms to change weights of the filter recursively.

Least Mean Square (LMS) Algorithm :

This is one of the algorithm to change the tap weights of the adaptive filter recursively. The tap weights are adapted by this algorithm as follows :

$$\hat{w}_i(nT + T) = \hat{w}_i(nT) + \mu \, e(nT) \, x(nT - iT) \qquad \ldots (7.7.3)$$

Here $i = 0, 1, \ldots\ldots M-1$

$\hat{w}_i(nT)$ is the present estimate for tap 'i' at time nT.

$\hat{w}_i\ (nT+T)$ is the updated estimate for tap 'i' at time nT

μ is the adaption constant

$x(nT-iT)$ is the filter input and

$e(nT)$ is the error signal.

The parameter μ controls the amount of correction applied to the old estimate to produce updated estimate. With the help equation 7.7.3, the tap weights are obtained in recursive manner. In this algorithm, initial tap weights are assumed zero.

2.9 Eye patterns

The quality of digital transmission systems are evaluated using the bit error rate. Degradation of quality occurs in each process modulation, transmission, and detection. The eye pattern is experimental method that contains all the information concerning the degradation of quality. Therefore, careful analysis of the eye pattern is important in analyzing the degradation mechanism.

Eye patterns can be observed using an oscilloscope. The received wave is applied to the vertical deflection plates of an oscilloscope and the saw tooth wave at a rate equal to transmitted symbol rate is applied to the horizontal deflection plates, resulting display is eye pattern as it resembles human eye.

- The Eye Pattern is used to study the effect of ISI in baseband digital transmission.
- When the sequence is transmitted over a baseband binary data transmission system of Fig. 2.11.1 the signal obtained at the output i.e. $y(t)$ is a continuous time signal as shown in Fig. 2.11.1. Ideally this signal should go high and low depending on the symbol that was transmitted. But because of the nature of transmission channel, the signal becomes continuous with increasing and decreasing amplitudes. Fig. 2.11.1(a) shows the binary sequence that is transmitted and Fig. 2.11.1 (b) shows the signal $y(t)$ obtained at the output. Fig. 2.11.1 (b) also shows various sampling instants t_1, t_2, t_3 .... etc. Thus based on the signal obtained over the period T_b between two sampling instants, decision is taken by the decision device. If we cut the signal $y(t)$ shown in Fig. 2.11.1 (b) in each interval (T_b) and place it over one another, then we obtain the diagram as shown in Fig. 2.11.1 (c). This diagram is called Eye pattern of the signal $y(t)$.

- The name 'eye' is given because it looks like an eye. This pattern can also be obtained on CRO if we apply $y(t)$ to one of the input channels and apply an external trigger signal of $1 / T_b$ Hz. This makes one sweep of beam equal to 'T_b' seconds. Therefore the pattern shown in Fig. 2.11.1 (c) will be obtained. When there are large number of bits of the sequence, then eye patterns will be as shown in Fig. 2.11.1 (d).

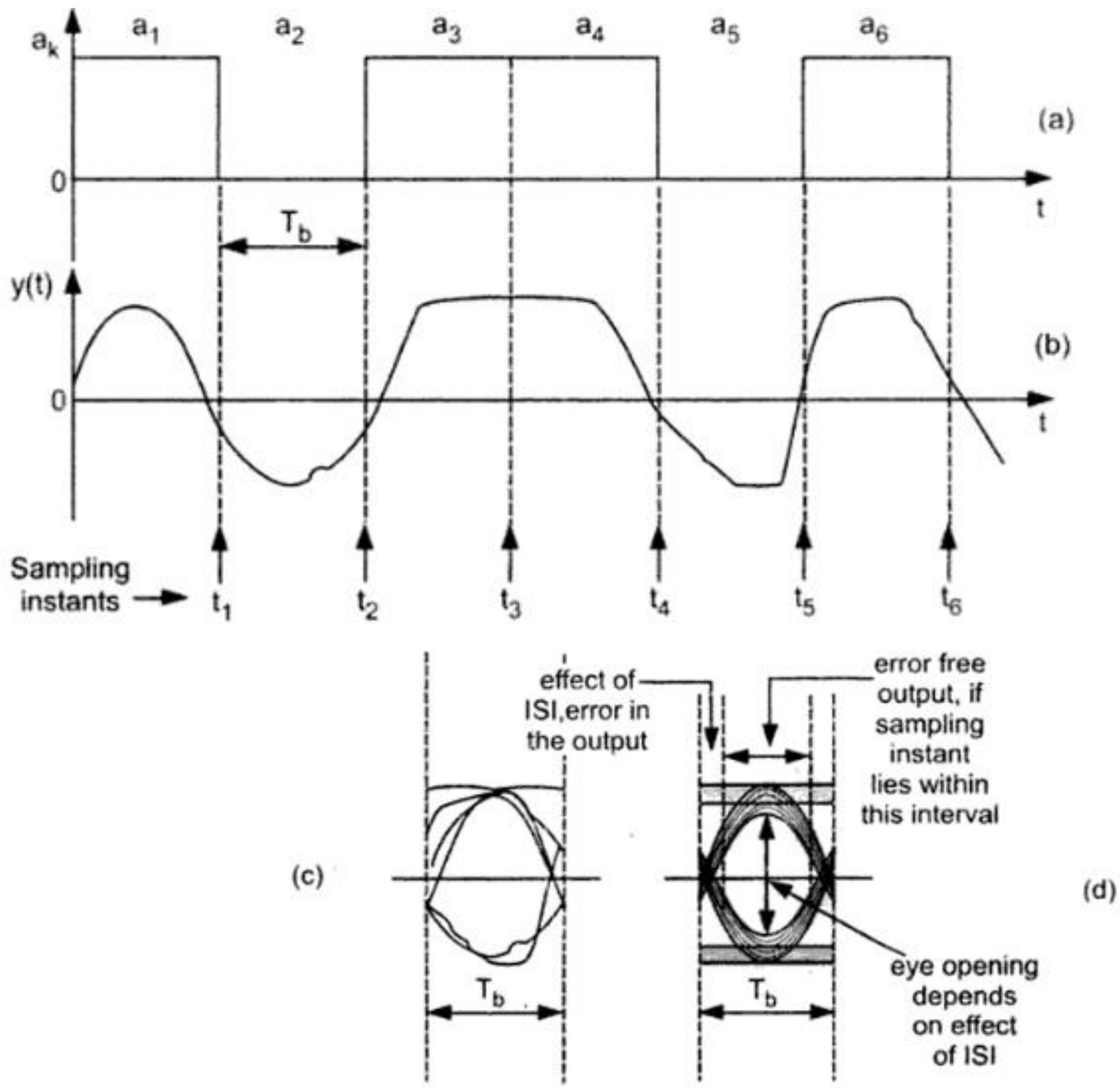


Fig. 2.11.1 (a) Binary sequence transmitted (b) Received signal by baseband transmission system (c) Eye pattern of signal in (b) (d) Eye pattern for large number of bits in waveform $y(t)$

2.9.1 Performance of data transmission system using eye pattern

- Various important conclusions can be derived from eye pattern. Fig. 2.11.2 shows various points related to eye pattern.

(i) The width of the eye opening defines the interval over which the received wave can be sampled without error from intersymbol interference. It is preferable to sample the instant at which eye is open widest. The instant is shown as best sampling time in Fig. 2.11.2.

(ii) The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied.

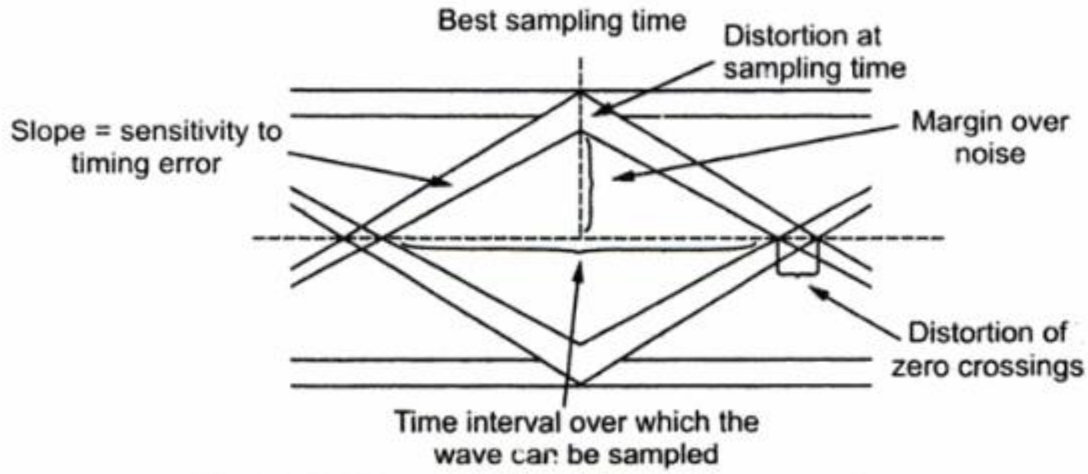


Fig. 2.11.2 Interpretation of the eye pattern

(iii) The height of the eye opening, at the specified sampling time, is called margin over the noise.

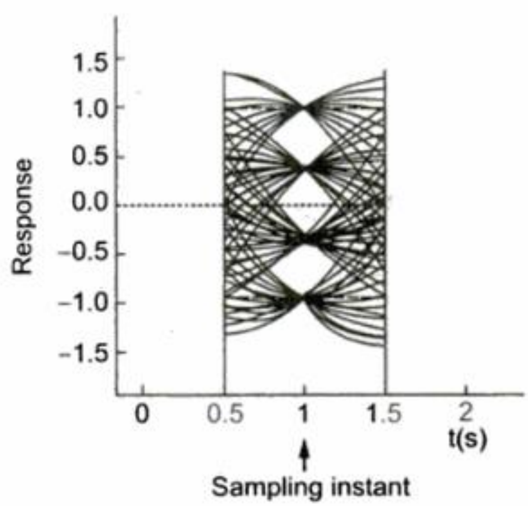


Fig. 2.11.3 Eye diagram for 4-level system

- As the effect of inter symbol interference increases, the eye opening reduces. If the eye is closed completely, then it is not possible to avoid errors in the output.
- All the above description is for two level (binary) system. If there are M-levels (M-ary system), then eye pattern contains (M-1) eye openings stacked vertically one upon the other. Fig. 2.11.3 shows the eye diagram for 4 level (M = 4) system. Therefore there are 3 eye openings.

2.10 Companding

Thus the compression of signal at transmitter and expansion at receiver is called combinely as *companding*. Fig. 1.8.9 shows compression and expansion curves.

As can be seen from Fig. 1.8.9, at the receiver, the signal is expanded exactly opposite to compression curve at transmitter to get original signal. A dotted line in the Fig. 1.8.9 shows uniform quantization. The compression and expansion is obtained by passing the signal through the amplifier having nonlinear transfer characteristic as shown in Fig. 1.8.9. That is nonlinear transfer characteristic means compression and expansion curves.

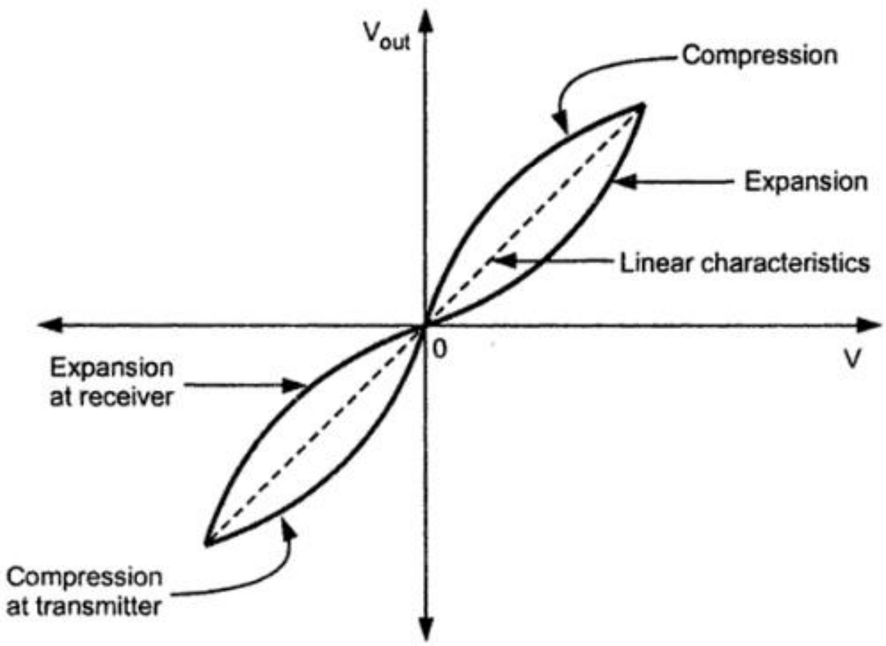


Fig. 1.8.9 Companding curves for PCM

2.10.1 μ law Companding

Normally for speech and music signals a μ - law compression is used. This compression is defined by the following equation,

$$Z(x) = (\text{Sgn } x) \frac{ln(1+\mu|x|)}{ln(1+\mu)} \quad |x| \leq 1 \qquad \dots (1.8.52)$$

Fig. 1.8.10 shows the variation of signal to noise ratio with respect to signal level without companding and with companding.

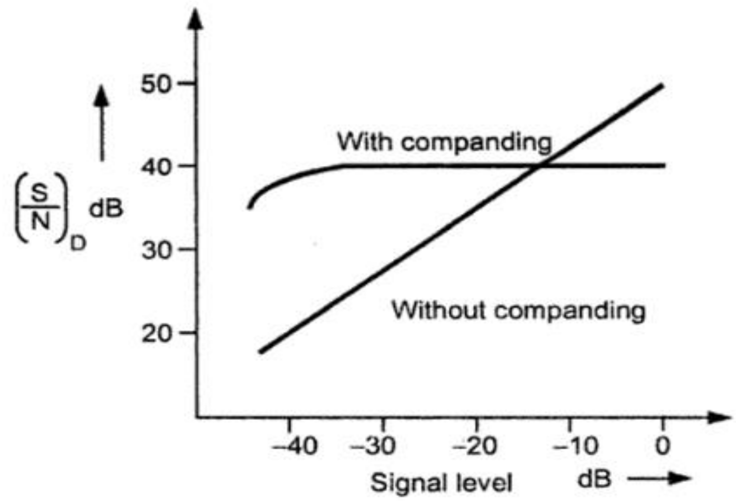


Fig. 1.8.10 PCM performance with μ - law companding

It can be observed from above figure that signal to noise ratio of PCM remains almost constant with companding.

2.10.2 A law Companding

The A law provides piecewise compressor characteristic. It has linear segment for low level inputs and logarithmic segment for high level inputs. It is defined as,

$$Z(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} & \text{for } 0 \le |x| \le \dfrac{1}{A} \\ \dfrac{1+\ln(A|x|)}{1+\ln A} & \text{for } \dfrac{1}{A} \le |x| \le 1 \end{cases} \quad \ldots (1.8.53)$$

When A = 1, we get uniform quantization. The practical value for A is 87.56. Both A-law and μ-law companding is used for PCM telephone systems.

2.11 Correlation Receiver

In this section we will study a little different type of receiver which is called *correlator*. Fig. 4.4.1 shows the block diagram of this correlator.

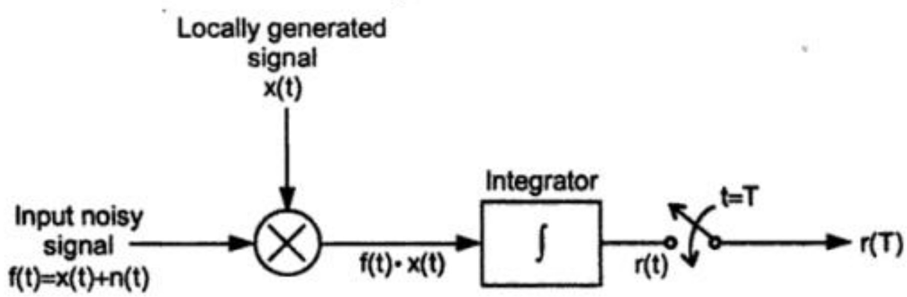


Fig. 4.4.1 Block diagram of the correlator

In the above figure $f(t)$ represents input noisy signal, i.e., $f(t) = x(t) + n(t)$. The signal $f(t)$ is multiplied to the locally generated replica of input signal $x(t)$. This result of multiplication $f(t) \cdot x(t)$ is integrated. The output of the integrator is sampled at $t = T$ (i.e. end of one symbol period). Then based on this sampled value, decision is made. This is how the correlator works. *It is called correlator* since *it correlates the received signal $f(t)$ with a stored replica of the known signal $x(t)$*. In the block diagram of above figure, the product $f(t)x(t)$ is integrated over one symbol period, i.e. T. Hence output $r(t)$ can be written as,

$$r(t) = \int_0^T f(t)\, x(t)\, dt$$

At $t = T$, the above equation will be,

$$\textbf{Output of correlator : } r(T) = \int_0^T f(t)\, x(t)\, dt \qquad \ldots (4.4.1)$$