SEC1313 - DIGITAL COMMUNICATION

UNIT 1 SAMPLING AND QUANTIZATION **9 Hrs.**

Review of sampling process -Natural Sampling-Flat Sampling - Aliasing - Signal Reconstruction-Quantization - Uniform & non-uniform quantization - quantization noise Bandwidth -Noise trade off-PCM- Noise considerations in PCM- differential pulse code modulation - Delta modulation -Linear prediction - Adaptive Delta Modulation.

1.1 Introduction

Digital Communication Systems are designed for transmitting digital information using digital modulation schemes.

1.1.1 Basic Elements of a Digital Communication System

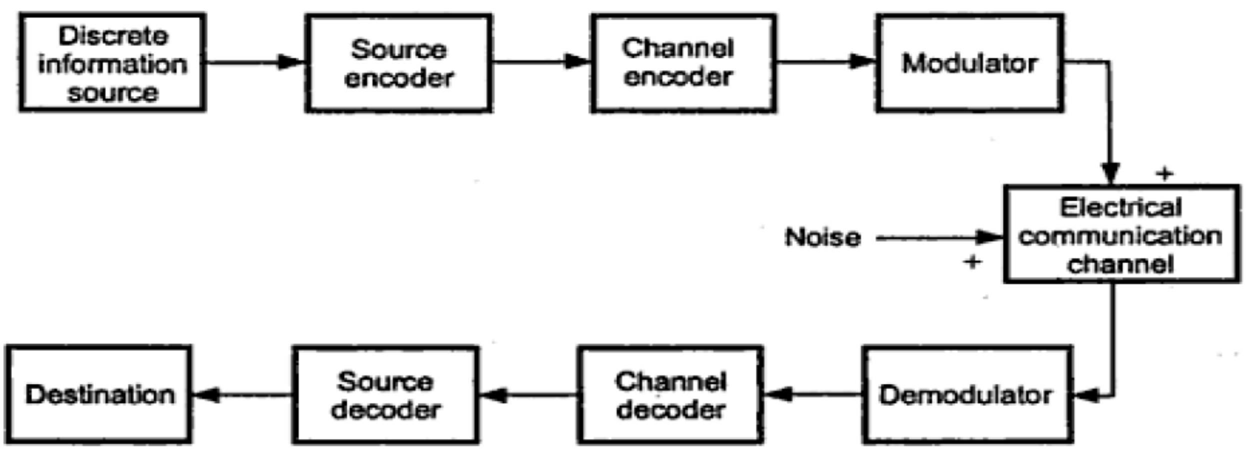


Fig. 1.1 Basic diqital communication system

Information Source

The information source generates the message signal to be transmitted. In case of analog communication, the information source is analog. In case of digital communication, the information source is digital. The analog signal can be converted to discrete signal by sampling and quantization.

The examples of discrete information sources are data from computers, teletype etc.

Source Encoder / Decoder

The Source encoder converts the input symbol sequence into a binary sequence of 0's and 1's.

The important parameters of a source encoder are **block size, code word lengths, average data rate and the efficiency.**

At the receiver, the source decoder converts the binary output of the channel decoder into a symbol sequence.

Aim of the source coding is to remove the redundancy in the transmitting information, so that bandwidth required for transmission is minimized.

Channel Encoder / Decoder

Error control is accomplished by the channel coding operation that consists of systematically adding extra bits to the output of the source coder. These extra bits do not convey any information but helps the receiver to detect and / or correct some of the errors in the information bearing bits.

The Channel decoder recovers the information bearing bits from the coded binary stream. Error detection and possible correction is also performed by the channel decoder.

The important parameters of coder / decoder are **Method of coding, efficiency, error control capabilities and complexity** of the circuit.

Modulator

The Modulator converts the input bit stream into an electrical waveform suitable for transmission over the communication channel. Modulator can be effectively used to minimize the effects of channel noise, to match the frequency spectrum of transmitted signal with channel characteristics, to provide the capability to multiplex many signals.

Demodulator

The extraction of the message from the information bearing waveform produced by the modulation is accomplished by the demodulator. The output of the demodulator is bit stream. The important parameter is the method of demodulation.

Channel

The Channel provides the electrical connection between the source and destination. The different channels are: Pair of wires, Coaxial cable, Optical fiber, Radio channel, Satellite channel or combination of any of these.

Noise

Noise is an error or undesired random disturbance of a useful information signal. The noise is a summation of unwanted or disturbing energy from natural and sometimes man-made sources.

1.2 Sampling process

1.2.1 Representation of CT signals by its samples

- A CT signal cannot be processed in the digital processor or computer.
- To enable digital transmission of CT signals.

Fig. 1.2 shows the CT signal and its sampled DT signal. In this figure observe that the CT signal is sampled at t = 0, T_s, $2T_s$, $3T_s$, ... and so on.

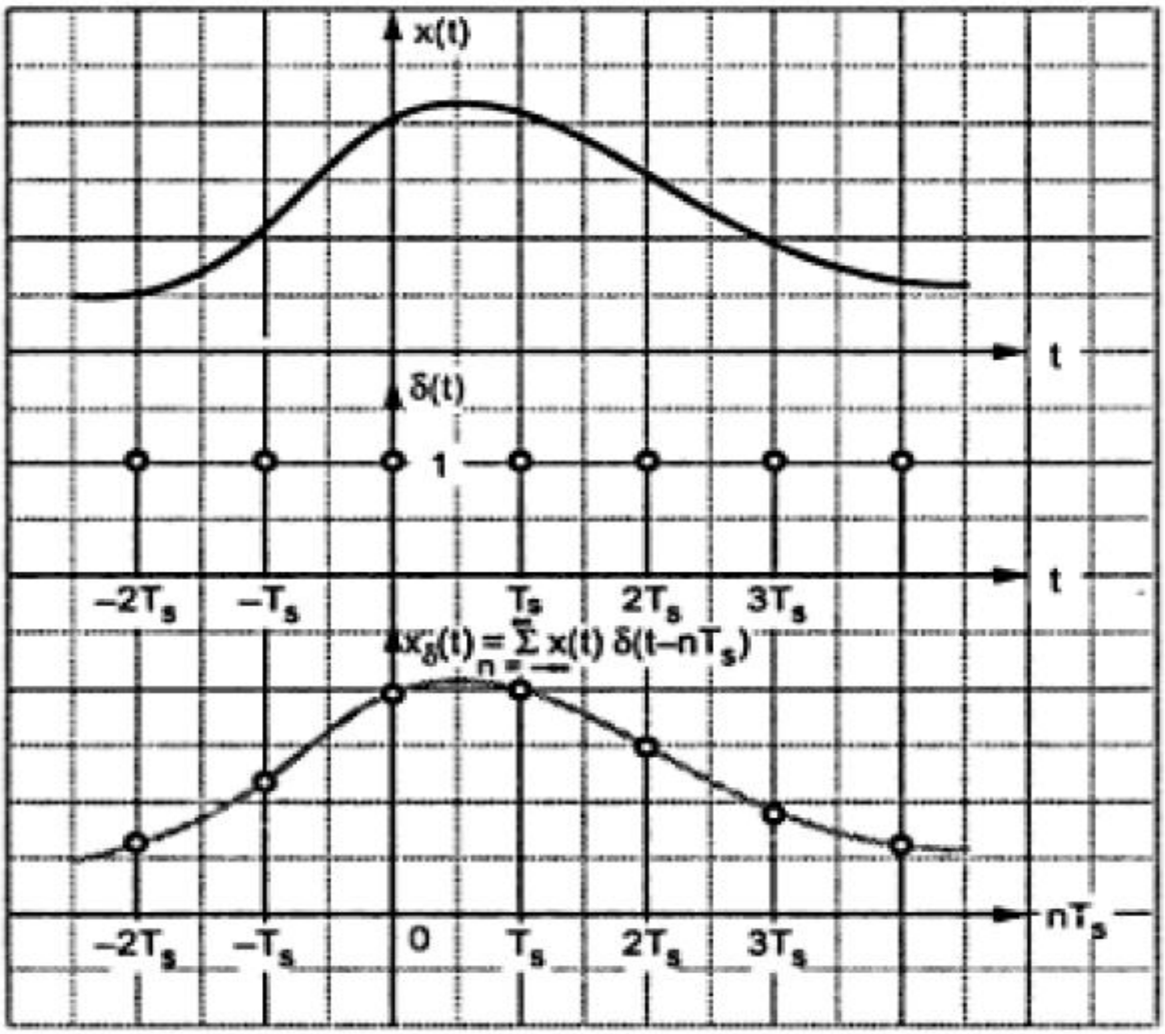


Fig. 1.2 CT and its DT signal

- Here sampling theorem gives the criteria for spacing 'T_s' between two successive samples.
- The samples $x_\delta(t)$ must represent all the information contained in x(t).

The sampled signal $x_\delta(t)$ is called discrete time (DT) signal. It is analyzed with the help of DTFT and z-transform.

1.3 Statement of sampling theorem

1) *A band limited signal of finite energy, which has no frequency components higher than W Hertz, is completely described by specifying the values of the signal at instants of time separated by* $\frac{1}{2W}$ *seconds and*

2) *A band limited signal of finite energy, which has no frequency components higher than W Hertz, may be completely recovered from the knowledge of its samples taken at the rate of 2W samples per second.*

The first part of above statement tells about sampling of the signal and second part tells about reconstruction of the signal. Above statement can be combined and stated alternately as follows :

A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is twice of the highest frequency content of the signal. i.e.,

$$f_s \geq 2W$$

Here f_s is the sampling frequency and

W is the higher frequency content

1.3.1 Proof of sampling theorem

There are two parts : (I) Representation of $x(t)$ in terms of its samples

(II) Reconstruction of $x(t)$ from its samples.

Part I : Representation of $x(t)$ in its samples $x(nT_s)$

Step 1 : Define $x_\delta(t)$

Step 2 : Fourier transform of $x_\delta(t)$ i.e. $X_\delta(f)$

Step 3 : Relation between $X(f)$ and $X_\delta(f)$

Step 4 : Relation between $x(t)$ and $x(nT_s)$

Step 1 : Define $x_\delta(t)$

Refer Fig. 1.2. The sampled signal $x_\delta(t)$ is given as,

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s) \qquad \ldots (1.1)$$

Here observe that $x_\delta(t)$ is the product of x_δ and impulse train $\delta(t)$ as shown in Fig. 1.2. In the above equation $\delta(t-nT_s)$ indicates the samples placed at $\pm T_s$, $\pm 2T_s$, $\pm 3T_s$ ... and so on.

Step 2 : FT of $x_\delta(t)$ i.e. $X_\delta(f)$

Taking FT of equation (1.1).

$$X_\delta(f) = \text{FT}\left\{\sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s)\right\}$$

$$= \text{FT}\ \{\text{Product of } x(t) \text{ and impulse train}\}$$

We know that FT of product in time domain becomes convolution in frequency domain. i.e.,

$$X_\delta(f) = \text{FT}\ \{x(t)\} * FT\ \{\delta(t-nT_s)\} \qquad \ldots(1.2)$$

By definitions, $x(t) \xleftrightarrow{FT} X(f)$ and

$$\delta(t-nT_s) \xleftrightarrow{FT} f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

Hence equation (1.2) becomes,

$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

Since convolution is linear,

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f-nf_s)$$

$$= f_s \sum_{n=-\infty}^{\infty} X(f-nf_s) \qquad \textit{By shifting property of impulse function}$$

$$= \cdots f_s\, X(f-2f_s) + f_s\, X(f-f_s) + f_s\, X(f) + f_s\, X(f-f_s) + f_s\, X(f-2f_s) + \cdots$$

Comments

(i) The RHS of above equation shows that X(f) is placed at $\pm f_s, \pm 2f_s, \pm 3f_s, \cdots$

(ii) This means X(f) is periodic in f_s.

(iii) If sampling frequency is $f_s = 2W$, then the spectrums $X(f)$ just touch each other.

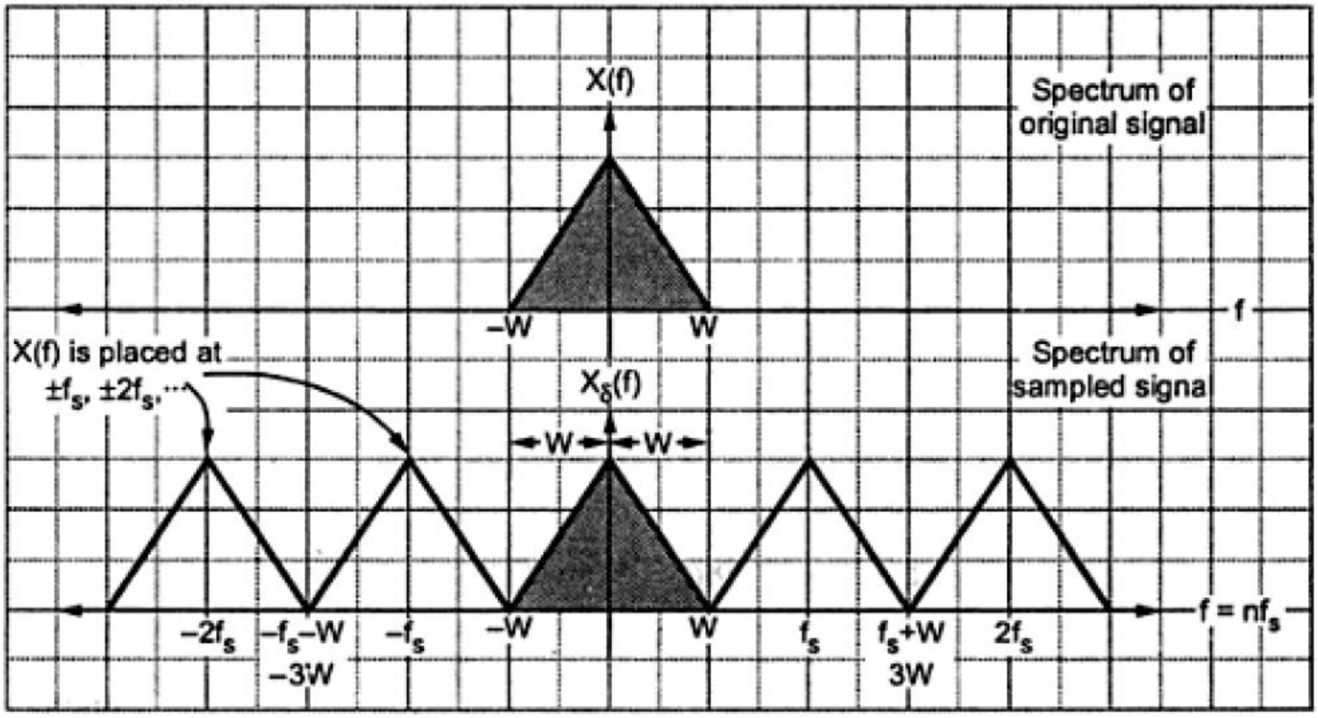


Fig. 1.3 Spectrum of original signal and sampled signal

Step 3 : Relation between $X(f)$ and $X_\delta(f)$

Important assumption : Let us assume that $f_s = 2W$, then as per above diagram.

$$X_\delta(f) = f_s X(f) \qquad \text{for } -W \le f \le W \text{ and } f_s = 2W$$

or
$$X(f) = \frac{1}{f_s} X_\delta(f) \qquad \dots (1.3)$$

Step 4 : Relation between $x(t)$ and $x(nT_s)$

DTFT is,
$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$\therefore$
$$X(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j2\pi f n} \qquad \dots (1.4)$$

In above equation 'f' is the frequency of DT signal. If we replace $X(f)$ by $X_\delta(f)$, then 'f' becomes frequency of CT signal. i.e.,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j2\pi \frac{f}{f_s} n}$$

In above equation 'f' is frequency of CT signal. And $\frac{f}{f_s}$ = Frequency of DT signal in equation (1.4). Since $x(n) = x(nT_s)$, i.e. samples of $x(t)$, then we have,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s} \quad \text{since } \frac{1}{f_s} = T_s$$

Putting above expression in equation (1.3),

$$X(f) = \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s}$$

Inverse Fourier Transform (IFT) of above equation gives $x(t)$ i.e.,

$$x(t) = IFT\left\{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s}\right\} \qquad \dots (1.5)$$

Comments :

i) Here $x(t)$ is represented completely in terms of $x(nT_s)$.

ii) Above equation holds for $f_s = 2W$. This means if the samples are taken at the rate of 2W or higher, $x(t)$ is completely represented by its samples.

iii) First part of the sampling theorem is proved by above two comments.

Part II: Reconstruction of x(t) from its samples

Step 1 : Take inverse Fourier transform of $X(f)$ which is in terms of $X_\delta(f)$.

Step 2 : Show that $x(t)$ is obtained back with the help of interpolation function

Step 1 : The IFT of equation (1.5) becomes,

$$x(t) = \int_{-\infty}^{\infty} \left\{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s}\right\} e^{j2\pi f t}\, df$$

Here the integration can be taken from $-W \le f \le W$. Since $X(f) = \frac{1}{f_s} X_\delta(f)$ for $-W \le f \le W$. (See Fig. 1.3).

$$\therefore \quad x(t) = \int_{-W}^{W} \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s} \cdot e^{j2\pi f t}\, df$$

Interchanging the order of summation and integration,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \int_{-W}^{W} e^{j2\pi f(t-nT_s)}\, df$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \cdot \left[\frac{e^{j2\pi f(t-nT_s)}}{j2\pi(t-nT_s)} \right]_{-W}^{W}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \left\{ \frac{e^{j2\pi W(t-nT_s)} - e^{-j2\pi W(t-nT_s)}}{j2\pi(t-nT_s)} \right\}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \cdot \frac{\sin 2\pi W(t-nT_s)}{\pi(t-nT_s)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \pi(2Wt - 2WnT_s)}{\pi(f_s t - f_s nT_s)}$$

Here $f_s = 2W$, hence $T_s = \frac{1}{f_s} = \frac{1}{2W}$. Simplifying above equation,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \pi(2Wt-n)}{\pi(2Wt-n)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \operatorname{sinc}(2Wt-n) \qquad \text{since } \frac{\sin \pi\theta}{\pi\theta} = \operatorname{sinc}\theta \qquad \ldots(1.6)$$

Step 2 : Let us interpret the above equation. Expanding we get,

$$x(t) = \cdots + x(-2T_s)\operatorname{sinc}(2Wt+2) + x(-T_s)\operatorname{sinc}(2Wt+1) + x(0)\operatorname{sinc}(2Wt) + x(T_s)\operatorname{sinc}(2Wt-1) + \cdots$$

Comments :

i) The samples $x(nT_s)$ are weighted by sinc functions.

ii) The sinc function is the interpolating function. Fig. 1.4 shows, how $x(t)$ is interpolated.

Step 3 : Reconstruction of $x(t)$ by lowpass filter

When the interpolated signal of equation (1.6) is passed through the lowpass filter of bandwidth $-W \le f \le W$, then the reconstructed waveform shown in above Fig. 1.4 (b) is obtained. The individual sinc functions are interpolated to get smooth $x(t)$.

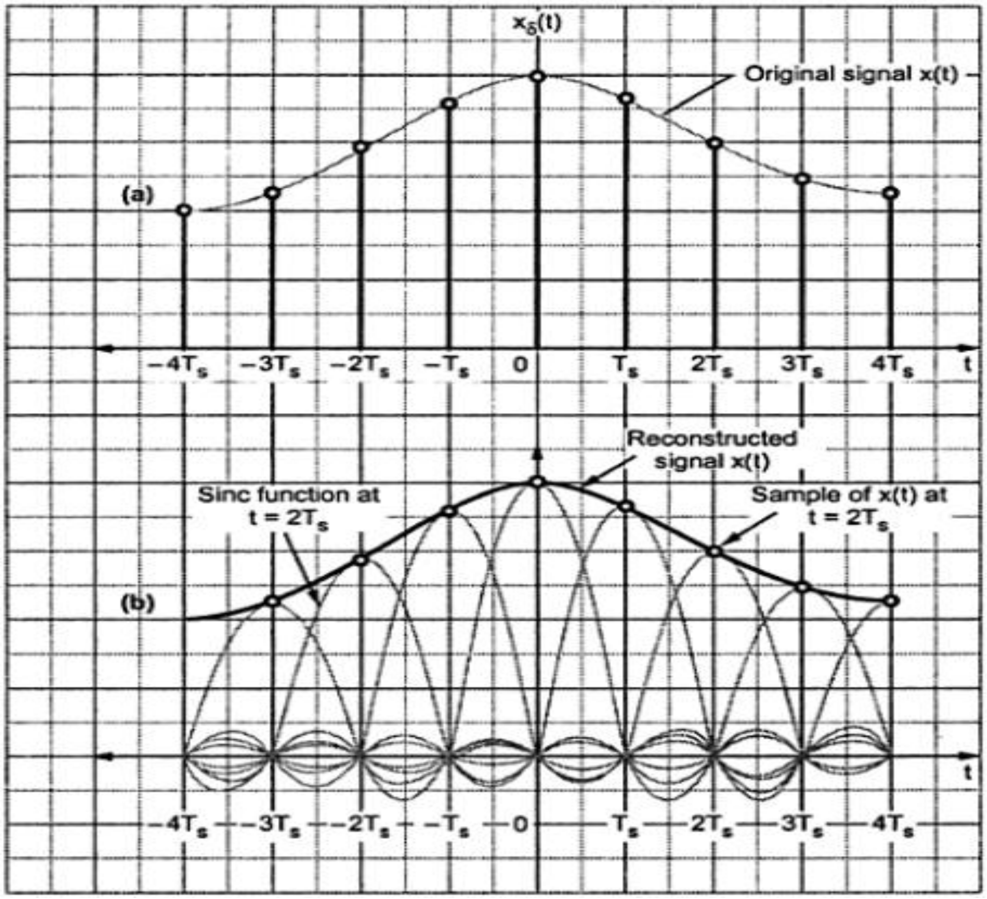


Fig. 1.4 (a) Sampled version of signal x(t)
(b) Reconstruction of x(t) from its samples

1.3.2 Aliasing (Effect of undersampling)

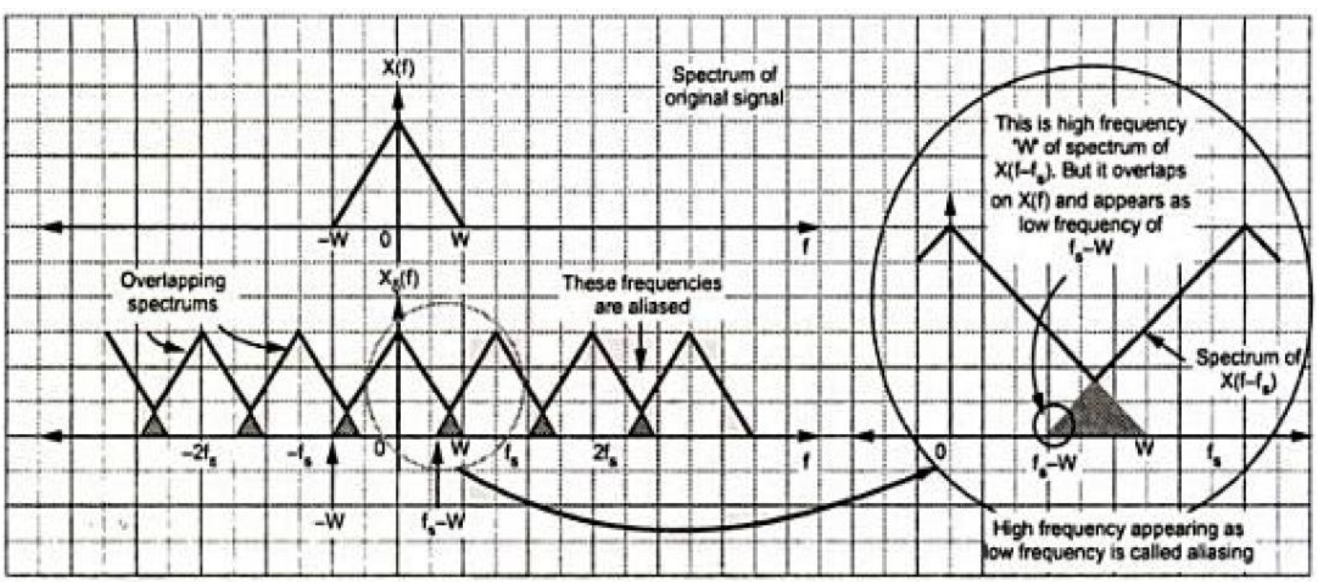


Fig. 1.5 Effect of undersampling or Aliasing

Comments :

i) The spectrums located at $X(f), X(f-f_s), X(f-2f_s), \ldots$ overlap on each other.

ii) Consider the spectrums of $X(f)$ and $X(f-f_s)$ shown as magnified in above figure. The frequencies from $(f_s - W)$ to W are overlapping in these spectrums.

iii) The high frequencies near 'ω' in $X(f-f_s)$ overlap with low frequencies $(f_s - W)$ in $X(f)$.

1.3.2.1 Definition of aliasing:

When the high frequency interferes with low frequency and appears as low frequency, then the phenomenon is called aliasing.

1.3.2.2 Effects of aliasing:

i) Since high and low frequencies interfere with each other, distortion is generated.
ii) The data is lost and it cannot be recovered.

1.3.2.3 Different ways to avoid aliasing

Aliasing can be avoided by two methods

i) Sampling rate $f_s \geq 2W$.
ii) Strictly bandlimit the signal to ' W '.

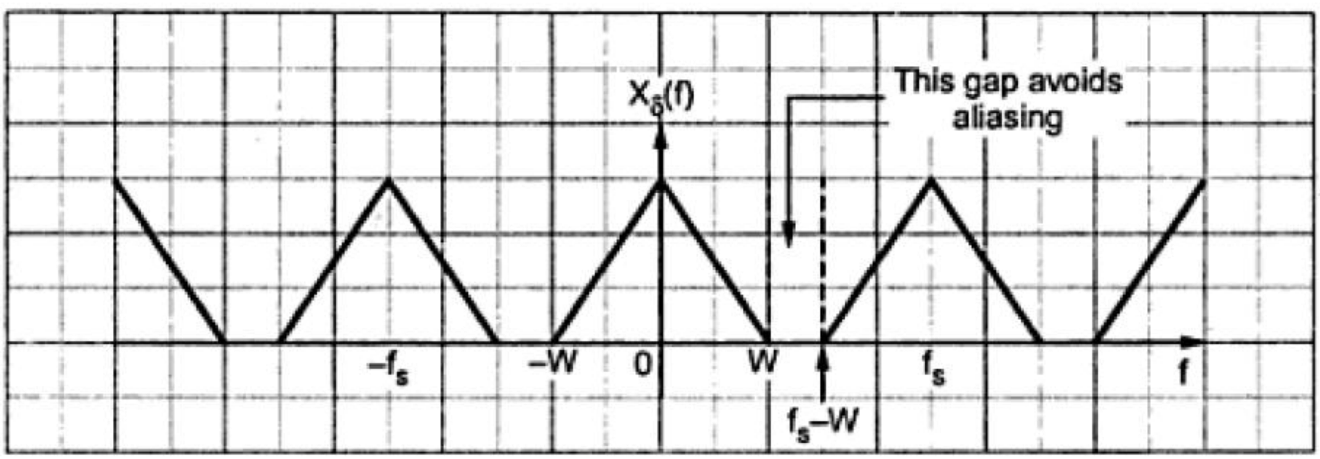


Fig. 1.6 $f_s \geq 2W$ avoids aliasing by creating a bandgap

i) Sampling rate $f_s \geq 2W$

When the sampling rate is made higher than 2W, then the spectrums will not overlap and there will be sufficient gap between the individual spectrums. This is shown in Fig. 1.6.

ii) Bandlimiting the signal

The sampling rate is, f_s = 2W. Ideally speaking there should be no aliasing. But there can be few components higher than 2W. These components create aliasing. Hence a lowpass filter is used before sampling the signals as shown in Fig. 1.7. Thus the output of lowpass filter is strictly bandlimited and there are no frequency components higher than 'W'. Then there will be no aliasing.

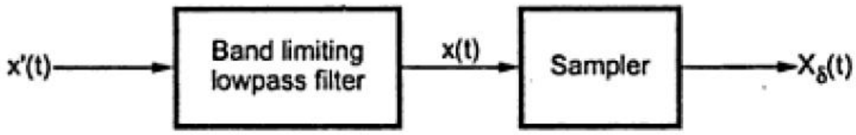


Fig. 1.7 Bandlimiting the signal. The bandlimiting LPF is called prealins filter

1.3.3 Nyquist Rate and Nyquist Interval

Nyquist rate : When the sampling rate becomes exactly equal to '2W' samples/sec, for a given bandwidth of W Hertz, then it is called Nyquist rate.

Nyquist interval : It is the time interval between any two adjacent samples when sampling rate is Nyquist rate.

$$\text{Nyquist rate} = 2W \text{ Hz} \quad \text{... (1.7)}$$

$$\text{Nyquist interval} = \frac{1}{2W} \text{ seconds} \quad \text{... (1.8)}$$

1.3.4 Sampling Theorem in Frequency Domain

Statement

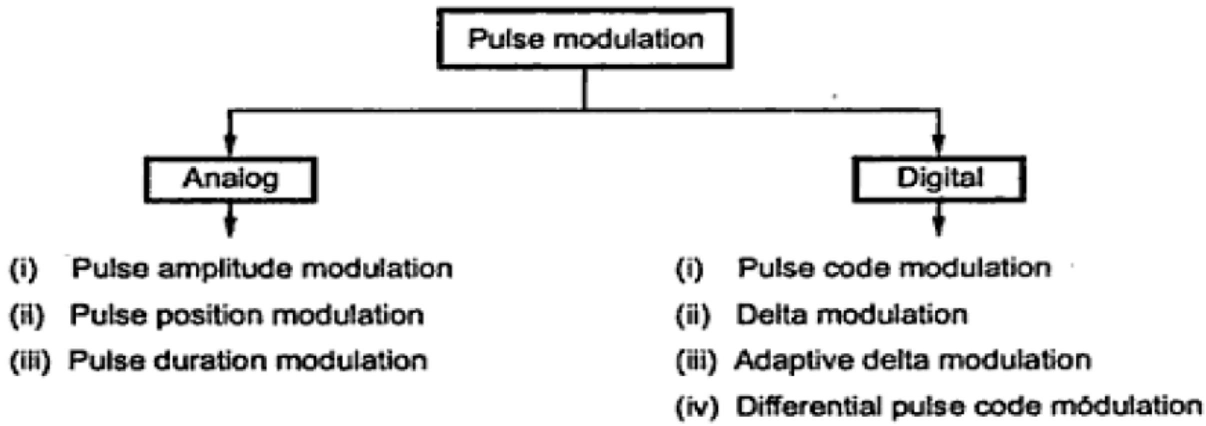
We have seen that if the bandlimited signal is sampled at the rate of ($f_s > 2W$) in time domain, then it can be fully recovered from its samples. This is sampling theorem in time domain. A dual of this also exists and it is called sampling theorem in frequency domain. It states that,

"A timelimited signal which is zero for $|t| > T$ is uniquely determined by the samples of its frequency spectrum at intervals less than $\frac{1}{2T}$ Hertz apart".

1.4 Introduction to Pulse Modulation techniques

- **There are three types of modulation**
  - **(i) Amplitude modulation**
  - **(ii) Angle modulation**
  - **(iii) Pulse modulation**

- Pulse modulation can be further classified as,
  - (i) Pulse analog modulation
  - (ii) Pulse digital modulation
- The above two techniques can be further classified as,



1.5 Pulse Amplitude Modulation (PAM)

- **Definition** : The amplitude of the pulse change according to amplitude of modulation signal at the sampling instant.
- **Types of PAM** : Depending upon the shape of the pulse of PAM, there are three types of PAM :

(i) Ideally or instantaneously sampled PAM.

(ii) Naturally sampled PAM.

(iii) Flat top sampled PAM.

1.5.1 Natural Sampling or Chopper Sampling

- **Basic Principle**

In natural sampling the pulse has a finite width τ. Natural sampling is some times called chopper sampling because the waveform of the sampled signal appears to be chopped off from the original signal waveform.

- **Explanation**

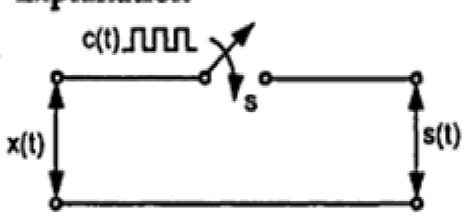


Fig. 1.8 Natural sampler

Let us consider an analog continuous time signal $x(t)$ to be sampled at the rate of f_s Hz and f_s is higher than Nyquist rate such that sampling theorem is satisfied. A sampled signal

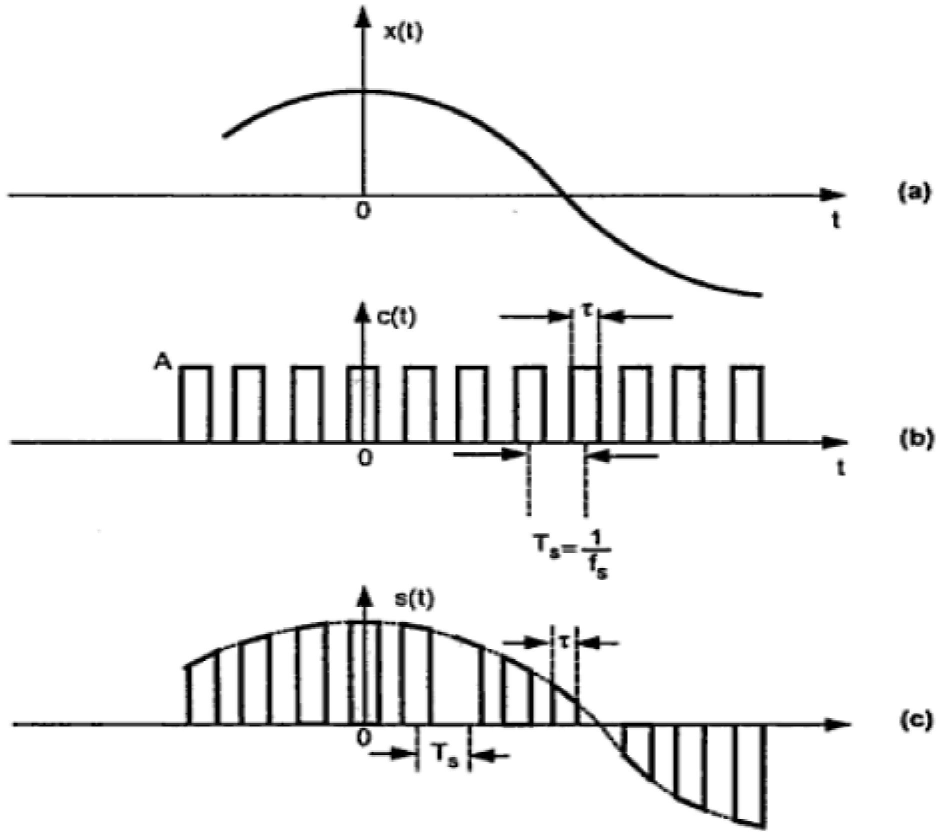


Fig. 1.9 **(a) Continuous time signal $x(t)$**
(b) Sampling function waveform i.e. periodic pulse train
(c) Naturally sampled signal waveform $s(t)$

$s(t)$ is obtained by multiplication of a sampling function and signal $x(t)$. Sampling function $c(t)$ is a train of periodic pulses of width τ and frequency equal to f_s Hz. Fig. 1.8 shows a functional diagram of natural sampler. When $c(t)$ goes high, a switch 's' is closed. Therefore,

$$s(t) = x(t) \quad \text{when} \quad c(t) = A$$

$$s(t) = 0 \quad \text{when} \quad c(t) = 0$$

Here A is amplitude of $c(t)$.

- The waveforms of $x(t)$, $c(t)$ and $s(t)$ are shown in Fig. 1.9 (a), 1.9 (b) and 1.9 (c) respectively. Signal $s(t)$ can also be defined mathematically as,

$$s(t) = c(t) \cdot x(t) \qquad \text{... (1.9)}$$

Here, $c(t)$ is the periodic train of pulses of width τ and frequency f_s.

Spectrum of Naturally Sampled Signal

- Exponential Fourier Series for a periodic waveform is given as

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nt/T_0}$$

For the periodic pulse train of $c(t)$ we have,

$$T_0 = T_s = \frac{1}{f_s} = \text{Period of } c(t).$$

$\therefore$ or $$f_0 = f_s = \frac{1}{T_0} = \frac{1}{T_s} = \text{Frequency of } c(t).$$

$\therefore$ Above equation will be, [with $x(t) = c(t)$],

$$c(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_s nt} \quad \text{Putting } \frac{1}{T_0} = f_s \quad \cdots (1.10)$$

$c(t)$ is a rectangular pulse train. C_n for this waveform is given as :

$$C_n = \frac{TA}{T_0} sinc(f_n T)$$

Here T = Pulse width = τ

and f_n = Harmonic frequency. Here $f_n = n f_s$ or $f_n = \frac{n}{T_0} = nf_0$

$\therefore$ $$C_n = \frac{\tau A}{T_s} sinc(f_n \tau) \quad \cdots (1.11)$$

$\therefore$ Fourier series for periodic pulse train will be written from equation 1.10 and equation 1.11 as,

$$c(t) = \sum_{n=-\infty}^{\infty} \frac{\tau A}{T_s} sinc(f_n \tau) e^{j2\pi f_s nt} \quad \cdots (1.12)$$

On putting the value of $c(t)$ in equation 1.9 we get,

$$s(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} sinc(f_n \tau) e^{j2\pi f_s nt} \cdot x(t) \quad \ldots (1.12 (a))$$

This equation represents naturally sampled signals.

Now Fourier transform of $s(t)$ is obtained by definition of FT as,

$$S(f) = FT\{s(t)\}$$

$$= \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} sinc(f_n \tau)\, FT\left\{e^{j2\pi f_s n t} \cdot x(t)\right\} \quad \text{... (1.13)}$$

We know from frequency shifting property of Fourier transform that,

$$e^{j2\pi f_s n t}\, x(t) \leftrightarrow X(f - f_s n) \quad \text{... (1.14)}$$

$$S(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} sinc(f_n \tau)\, X(f - f_s n) \quad \text{... (1.15)}$$

We know that $f_n = nf_s$ i.e. harmonic frequency

∴ Above equation becomes,

Spectrum of Naturally Sampled Signal : $S(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} sinc(nf_s \tau)\, X(f - nf_s)$ ... (1.16)

Comments :

(i) $X(f)$ are periodic in f_s and are weighed by the sinc function. Fig. 1.10 (a) shows some arbitrary spectra for $x(t)$ and corresponding spectrum $S(f)$ is shown in Fig. 1.10 (b).

(ii) Thus the spectrum of naturally sampled signal is weighted by sinc function.

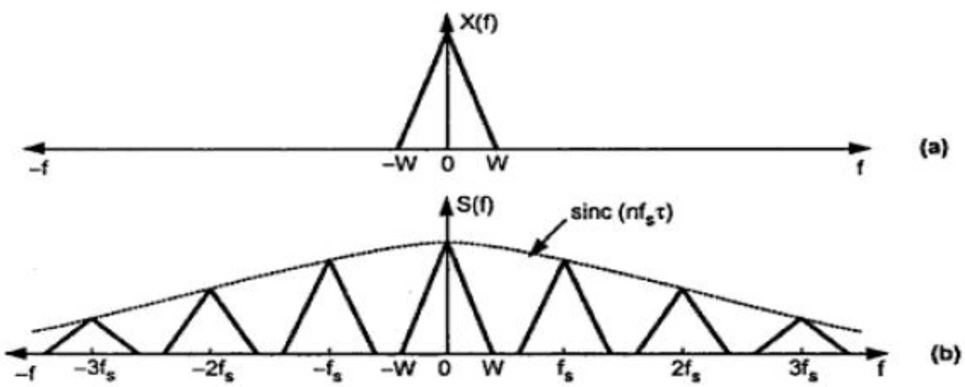


Fig. 1.10 (a) Spectrum of continuous time signal $x(t)$
(b) Spectrum of naturally sampled signal

1.5.2 Flat Top sampling or Rectangular Pulse Sampling

Basic Principle

This is also a practically possible sampling method. Natural sampling is little complex, but it is very easy to get flat top samples. The top of the samples remains constant and equal to instantaneous value of baseband signal $x(t)$ at the start of sampling. The duration of each sample is τ and sampling rate is equal to $f_s = \frac{1}{T_s}$.

Generation of flat top samples

Fig. 1.11 (a) shows the functional diagram of sample and hold circuit generating flat top samples and Fig. 1.11 (b) shows waveforms.

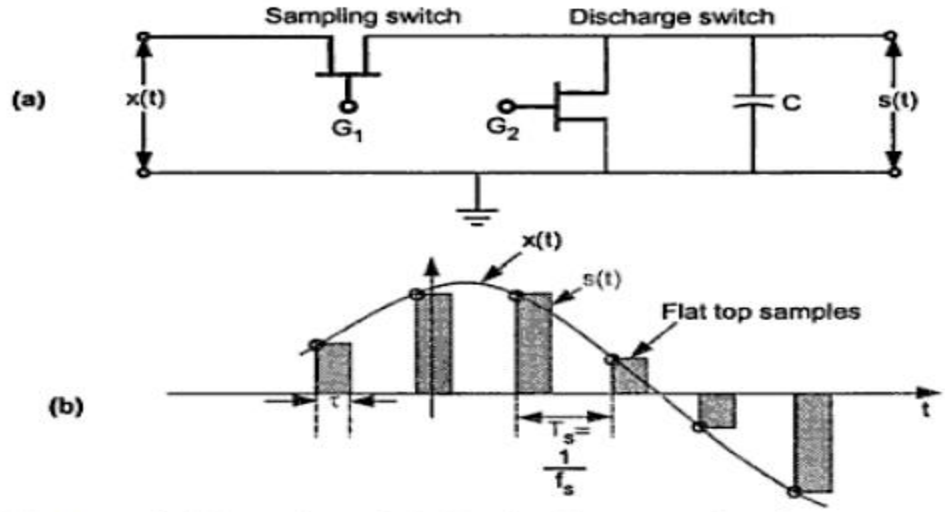


Fig. 1.11 (a) Sample and hold circuit generating flat top sampling
(b) Waveforms of flat top sampling

Normally the width of the pulse in flat top sampling and natural sampling is increased as far as possible to reduce the transmission bandwidth.

Explanation of Flat top Sampled PAM

Here we can see from Fig. 1.11 (b) that only starting edge of the pulse represents instantaneous value of the baseband signal $x(t)$. The flat top pulse of $s(t)$ is mathematically equivalent to the convolution of instantaneous sample and pulse $h(t)$ as shown in Fig. 1.12 .

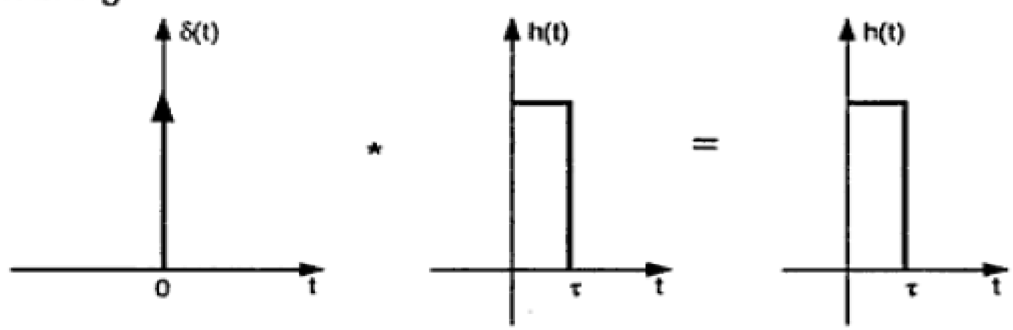


Fig. 1.12 Convolution of any function with delta function is equal to that function

- That is width of the pulse in $s(t)$ is determined by width of $h(t)$, and sampling instant is determined by delta function. In the waveforms shown in Fig. 1.11 (b), the starting edge of pulse represents the point where baseband signal is sampled and width is determined by function $h(t)$. Therefore $s(t)$ will be given as,

$$s(t) = x_\delta(t) * h(t) \quad \cdots (1.17)$$

The meaning of this equation is further explained by Fig. 1.13.

By the replication property of delta function we know that

$$x(t) * \delta(t) = x(t) \quad \cdots (1.18)$$

This is explained in Fig. 1.12 also. The same property is used to obtain flat top samples.

- The delta function in equation 1.18 is instantaneously sampled signal $x_\delta(t)$, and function $h(t)$ is convolved with $x_\delta(t)$. Clearly observe that we are not directly applying equation 1.18 here, but we are using it similarly. In equation 1.18, $\delta(t)$ is constant amplitude delta function. But in Fig. 1.13 (b), $x_\delta(t)$ is varying amplitude train of impulses. Therefore on convolution of $x_\delta(t)$ and $h(t)$ we get a pulse whose duration is equal to $h(t)$ only but amplitude is defined by $x_\delta(t)$.

From equation 1.1 $x_\delta(t)$ is given as,

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t-nT_s) \quad \cdots (1.19)$$

∴ From equation 1.17 we can write the convolution as,

$$s(t) = x_\delta(t) * h(t)$$

i.e.,

$$= \int_{-\infty}^{\infty} x_\delta(u)\, h(t-u)\, du$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(u-nT_s)\, h(t-u)\, du \quad \text{From equation (1.19)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \delta(u-nT_s)\, h(t-u)\, du \quad \cdots (1.20)$$

Fr

From the sifting property of delta function we know that

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0) = f(t_0) \quad \cdots (1.21)$$

Using this equation we can write equation 1.20 as,

$$s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t-nT_s) \quad \cdots (1.22)$$

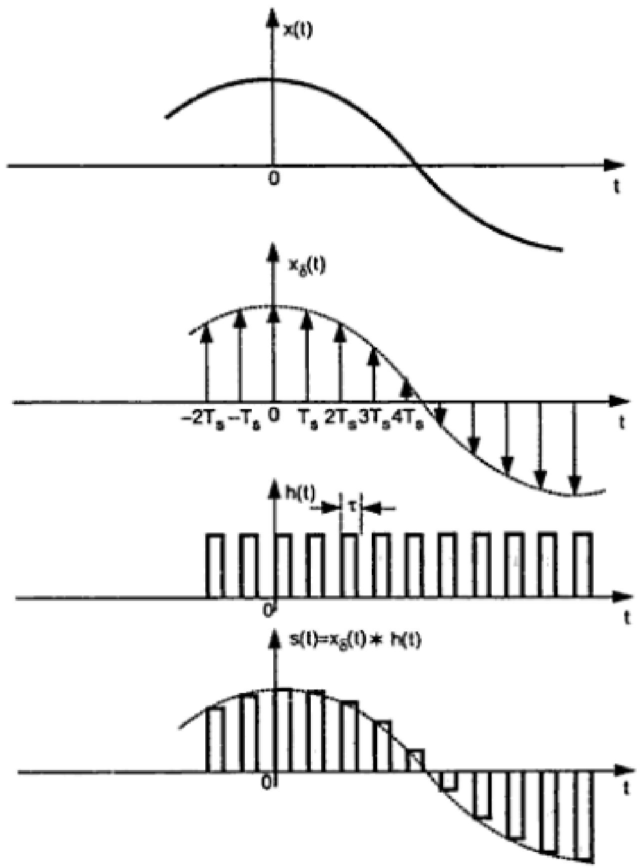


Fig. 1.13 **(a) Baseband signal $x(t)$**
(b) Instantaneously sampled signal $x_\delta(t)$
(c) Constant pulse width function $h(t)$
(d) Flat top sampled signal $s(t)$ obtained through convolution of $h(t)$ and $x_\delta(t)$

- This equation represents value of $s(t)$ in terms of sampled value $x(nT_s)$ and function $h(t-nT_s)$ for flat top sampled signal.

we also know from equation 1.17 that,

$$s(t) = x_\delta(t) * h(t)$$

By taking Fourier transform of both sides of above equation,

$$S(f) = X_\delta(f)\,H(f) \qquad \ldots (1.23)$$

Convolution in time domain is converted to multiplication in frequency domain.

$X_\delta(f)$ is given as,

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s) \qquad \ldots (1.24)$$

$\therefore$ Equation 1.23 becomes,

Spectrum of Flat Top Sampled Signal : $S(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)\,H(f) \qquad \ldots (1.25)$

This equation represents the spectrum of flat top sampled signal.

The spectrum of a rectangular pulse is given as,

$$H(f) = \tau \, sinc(f\tau) \, e^{-j\pi f\tau} \qquad \because A = 1 \qquad \text{....(1.26)}$$

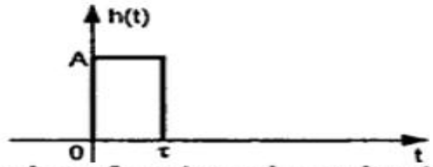


Fig. 1.14 (a) One pulse of rectangular pulse train

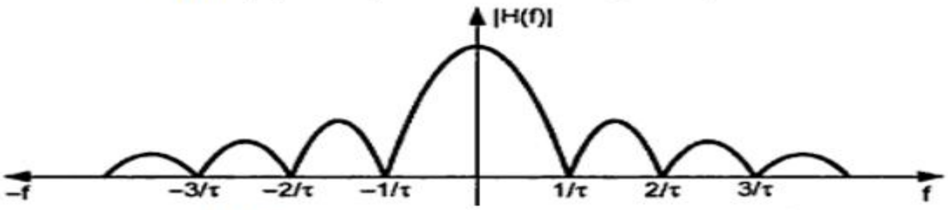
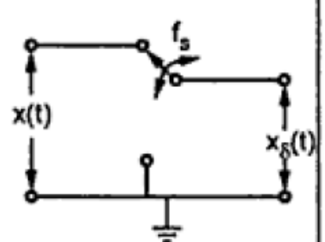
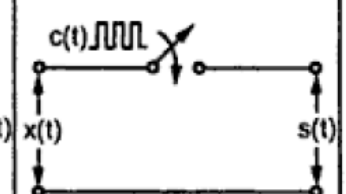
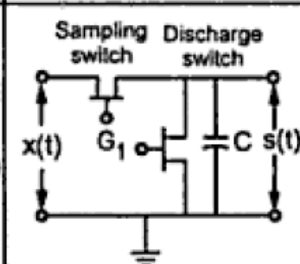
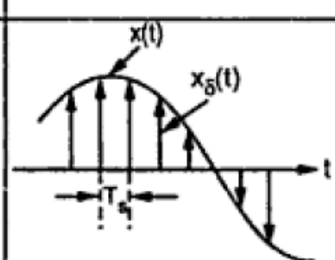
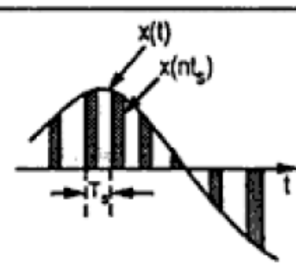
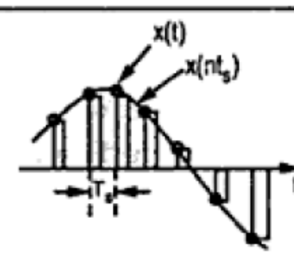


Fig. 1.14 (b) Spectrum of the pulse of Fig. (a)

1.6 Aperture Effect

In flat top sampling, due to the lengthening of the sample, amplitude distortion as well as a delay of T/2 was introduced. This distortion is referred to as Aperture effect.

1.7 Comparison of various sampling techniques

Sr. No.	Parameter of comparison	Ideal or instantaneous sampling	Natural sampling	Flat top sampling
1	Principle of sampling	It uses multiplication by an impulse function	It uses chopping principle	It uses sample and hold circuit
2	Circuit of sampler			
3	Waveforms			
4	Realizability	This is not practically possible method	This method is used practically	This method is used practically

5	Sampling rate	Sampling rate tends to infinity	Sampling rate satisfies Nyquist criteria	Sampling rate satisfies Nyquist criteria
6	Noise interference	Noise interference is maximum	Noise interference is minimum	Noise interference is maximum
7	Time domain representation	$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s)$	$s(t) = \frac{\tau A}{T_s}\sum_{n=-\infty}^{\infty} x(t)\,sinc(nf_s\tau)\,e^{j2\pi nf_s t}$	$s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)h(t-nT_s)$
8	Frequency domain representation	$X_\delta(f) = f_s\sum_{n=-\infty}^{\infty} X(f-nf_s)$	$S(f) = \frac{\tau A}{T_s}\sum_{n=-\infty}^{\infty} sinc(nf_s\tau)X(f-nf_s)$	$S(f) = f_s\sum_{n=-\infty}^{\infty} X(f-nf_s)H(f)$

Example : *The spectrum of signal x(t) is shown below. This signal is sampled at the Nyquist rate with a periodic train of rectangular pulses of duration 50/3 milliseconds. Find the spectrum of the sampled signal for frequencies upto 50 Hz giving relevant expression.*

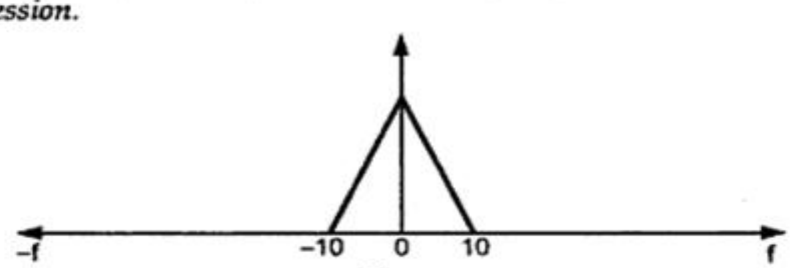


Fig.

Solution : It is clear from Fig. that the signal is bandlimited to 10 Hz.

$$\therefore \quad W = 10 \text{ Hz}$$

$$\therefore \quad \text{Nyquist rate} = 2\times W = 2\times 10 = 20\,Hz$$

Since the signal is sampled at Nyquist rate, the sampling frequency will be,

$$f_s = 20 \text{ Hz}$$

Rectangular pulses are used for sampling. That is flat top sampling is used. The spectrum of flat top sampled signal is given by equation 1.25 as,

$$S(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)H(f) \qquad \cdots (1.27)$$

Value of $H(f)$ is given by equation 1.26 as,

$$H(f) = \tau\, sinc(f\tau)\, e^{-j\pi f\tau} \qquad \cdots (1.28)$$

Here τ is the width of the rectangular pulse used for sampling. The given value of rectangular sampling pulse is 50/3 milliseconds. i.e.,

$$\tau = \frac{50}{3} \times 10^{-3}$$

or $$\tau = \frac{0.05}{3} \text{ seconds}$$

Putting the value of τ in equation 1.28 we get,

$$H(f) = \frac{0.05}{3} sinc\left(\frac{0.05f}{3}\right) e^{-j\,0.05\pi f/3}$$

Put this value of $H(f)$ and f_s in equation 1.25

$$S(f) = 20 \sum_{n=-\infty}^{\infty} X(f-20n) \times \frac{0.05}{3} sinc\left(\frac{0.05f}{3}\right) e^{-j\,0.05\pi f/3} \qquad (\text{since } f_s = 20)$$

$$S(f) = \frac{1}{3} \sum_{n=-3}^{3} X(f-20n) \times sinc\left(\frac{0.05f}{3}\right) e^{-j\,0.05\pi f/3}$$

This expression gives the spectrum up to 60 Hz (since $n = \pm 3$) for the sampled signal.

Example : *A flat top sampling system samples a signal of maximum 1 Hz with 2.5 Hz sampling frequency. The duration of the pulse is 0.2 seconds. Calculate the amplitude distortion due to aperture effect at highest signal frequency. Also find out the equalization characteristic.*

Solution : It is given that

Sampling frequency $f_s = 2.5$ Hz

Maximum signal frequency $f_{max} = 1$ Hz

Pulse width $\tau = 0.2$ sec.

By equation 1.26 the aperture effect is given by a transfer function $H(f)$ as,

$$H(f) = \tau\, sinc(f\tau)\, e^{-j\pi f \tau}$$

The magnitude of the above equation is given as,

$$|H(f)| = \tau\, sinc(f\tau) \qquad \text{... (1.29)}$$

$$|H(f)| = 0.2\, sinc(f \times 0.2)$$

Aperture effect at highest frequency will be obtained by putting $f = f_{max} = 1\,Hz$ in above equation i.e.,

$$|H(1)| = 0.2 \text{ sinc } (0.2) = 0.18709$$

or $$|H(1)| = 18.70\% \qquad \text{... (Ans)}$$

From equation the equalizer characteristic is given as,

$$H_{eq}(f) = \frac{k}{\tau \, sinc(f\tau)}$$

Putting $\tau = 0.2$ second and assuming $k = 1$, the above equation will be,

$$H_{eq}(f) = \frac{1}{0.2 \, sinc(0.2 f)} \quad \cdots(1.30)$$

This equation is the plot of $H_{eq}(f)\, Vs\, f$ and it represents the equalization characteristic to overcome aperture effect.

1.8 Transmission Bandwidth of PAM signal

The pulse duration 'τ' is supposed to be very very small compared to time period T_s between the two samples. If the maximum frequency in the signal $x(t)$ is 'W' then by sampling theorem, f_s should be higher than Nyquist rate or,

$$f_s \geq 2W \text{ or}$$

$$T_s \leq \frac{1}{2W} \quad \text{since } f_s = \frac{1}{T_s}$$

and

$$t << T_s \leq \frac{1}{2W} \quad \dots (1.4.29)$$

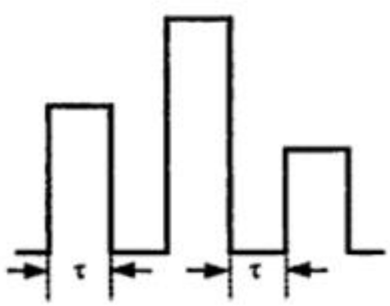


Fig. 1.4.11 Maximum frequency of PAM signal

If ON and OFF time of the pulse is same, then frequency of the PAM pulse becomes,

$$f = \frac{1}{\tau + \tau} = \frac{1}{2\tau} \quad \dots(1.4.30)$$

Thus Fig. 1.4.11 shows that if ON and OFF times of PAM signal are same, then maximum frequency of

PAM signal is given by equation 1.4.30 i.e.,

$$f_{max} = \frac{1}{2\tau} \quad \text{... (1.4.31)}$$

$\therefore$ Bandwidth required for transmission of PAM signal will be equal to maximum frequency f_{max} given by above equation. This bandwidth gives adequate pulse resolution i.e.,

$$B_T \geq f_{max}$$

$$\therefore \quad B_T \geq \frac{1}{2\tau} \quad \text{... (1.4.32)}$$

Since $\tau << \frac{1}{2W}$ $B_T \geq \frac{1}{2\tau} >> W$ i.e.,

Transmission bandwidth of PAM signal : $B_T >> W$	... (1.4.33)

Thus the transmission bandwidth B_T of PAM signal is very very large compared to highest frequency in the signal $x(t)$.

PAM signal is given by equation 1.4.30 i.e.,

$$f_{max} = \frac{1}{2\tau} \quad \text{... (1.4.31)}$$

$\therefore$ Bandwidth required for transmission of PAM signal will be equal to maximum frequency f_{max} given by above equation. This bandwidth gives adequate pulse resolution i.e.,

$$B_T \geq f_{max}$$

$$\therefore \quad B_T \geq \frac{1}{2\tau} \quad \text{... (1.4.32)}$$

1.9 Disadvantages of PAM

1. As we have seen just now, the bandwidth needed for transmission of PAM signal is very very large compared to its maximum frequency content.
2. The amplitude of PAM pulses varies according to modulating signal. Therefore interference of noise is maximum for the PAM signal and this noise cannot be removed very easily.
3. Since amplitude of PAM signal varies, this also varies the peak power required by the transmitter with modulating signal.

1.10 Uniform Quantization (Linear Quantization)

We know that input sample value is quantized to nearest digital level. This quantization can be uniform or nonuniform. In uniform quantization, the quantization step or difference between two quantization levels remains constant over the complete amplitude range. Depending upon the transfer characteristic there are three types of uniform or linear quantizers as discussed next.

1.10.1 Midtread Quantizer

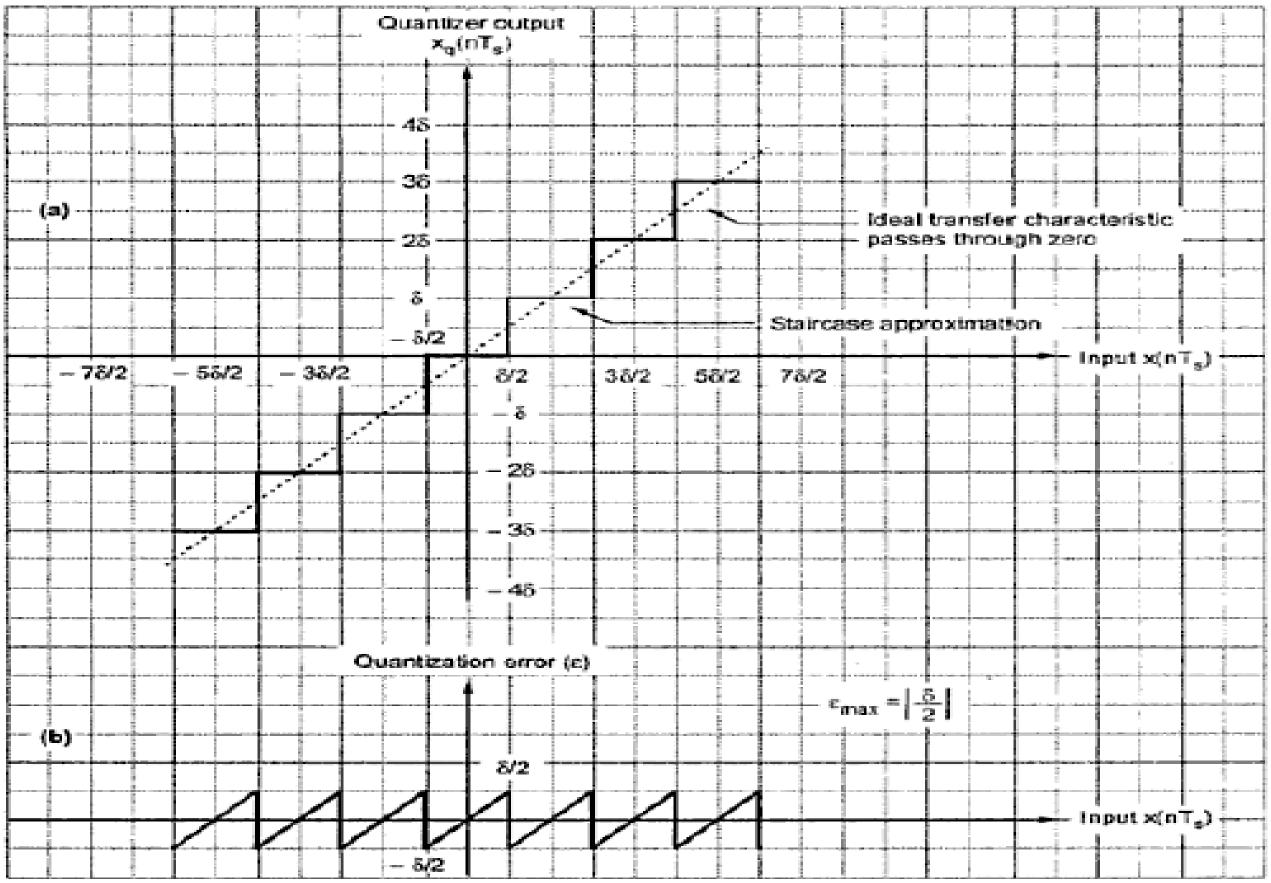


Fig. 1.8.3 (a) Quantization characteristic of midtread quantizer (b) Quantization error

The transfer characteristic of the midtread quantizer is shown in Fig. 1.8.3.

As shown in this figure, when an input is between – δ/2 and + δ/2 then the quantizer output is zero. i.e.,

For $\quad -\delta/2 \leq x(nT_s) < \delta/2 ; \quad x_q(nT_s) = 0$

Here 'δ' is the step size of the quantizer.

for $\quad \delta/2 \leq x(nT_s) < 3\delta/2 ; \quad x_q(nT_s) = \delta$

Similarly other levels are assigned. It is called midtread because quantizer output is zero when $x(nT_s)$ is zero. Fig.1.8.3 (b) shows the quantization error of midtread quantizer. Quantization error is given as,

$$\varepsilon = x_q(nT_s) - x(nT_s) \quad \text{... (1.8.7)}$$

In Fig. 1.8.3 (b) observe that when $x(nT_s) = 0$, $x_q(nT_s) = 0$. Hence quantization error is zero at origin. When $x(nT_s) = \delta/2$, quantizer output is zero just before this level. Hence error is $\delta/2$ near this level. From Fig. 1.8.3 (b) it is clear that,

$$-\delta/2 \le \varepsilon \le \delta/2 \quad \text{... (1.8.8)}$$

Thus quantization error lies between $-\delta/2$ and $+\delta/2$. And maximum quantization error is, maximum quantization error, $\varepsilon_{max} = \left|\frac{\delta}{2}\right|$ ... (1.8.9)

1.10.2 Midriser Quantizer

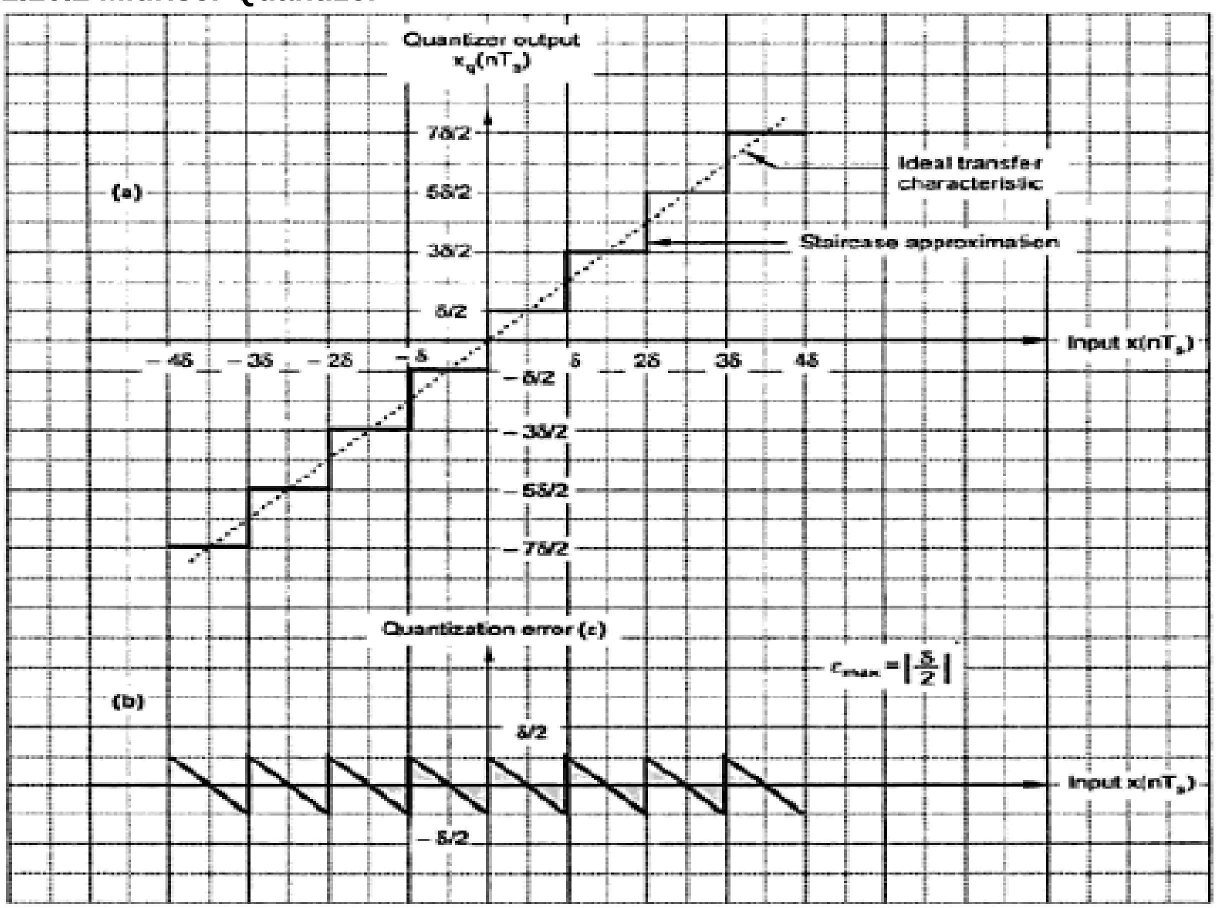


Fig. 1.8.4 (a) Transfer characteristic of midriser quantizer (b) Quantization error

The transfer characteristic of the midriser quantizer is shown in Fig. 1.8.4.

In Fig. 1.8.4 observe that, when an input is between 0 and δ, the output is $\delta/2$. Similarly when an input is between 0 and $-\delta$, the output is $-\delta/2$. i.e.,

$$\text{For} \quad 0 \leq x(nT_s) < \delta ; \quad x_q(nT_s) = \delta/2$$

$$-\delta \leq x(nT_s) < 0 ; \quad x_q(nT_s) = -\delta/2$$

Similarly when an input is between 3 δ and 4 δ, the output is 7 δ/2. This is called midriser quantizer because its output is either + δ/2 or – δ/2 when input is zero.

Fig. 1.8.4 (b) shows the quantization error in midriser quantization. When input $x(nT_s) = 0$, the quantizer will assign the level of δ/2. Hence quantization error will be,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

$$= \delta/2 - 0 = \delta/2$$

Thus the quantization error lies between – δ/2 and + δ/2. i.e.,

$$-\delta/2 \leq \varepsilon \leq \delta/2 \qquad \ldots (1.8.10)$$

And the maximum quantization error is,

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right| \qquad \ldots (1.8.11)$$

1.10.3 Biased Quantizer

The midriser and midtread quantizers are rounding quantizers. But biased quantizer is truncation quantizer. This is clear from above diagram. When input is between 0 and δ, the output is zero. i.e.,

$$\text{for} \quad 0 \leq x(nT_s) < \delta ; \quad x_q(nT_s) = 0$$

$$\text{Similarly, for} \quad -\delta \leq x(nT_s) < 0 ; \quad x_q(nT_s) = -\delta$$

Fig. 1.8.5 shows quantization error. When input is δ, output is zero. Hence quantization error is,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

$$= 0 - \delta = -\delta$$

Thus the quantization error lies between 0 and – δ. i.e.,

$$-\delta \leq \varepsilon \leq 0 \qquad \ldots (1.8.12)$$

And the maximum quantization error is,

$$\varepsilon_{max} = |\delta| \qquad \ldots (1.8.13)$$

Fig. 1.8.5 shows the transfer characteristic of biased uniform quantizer.

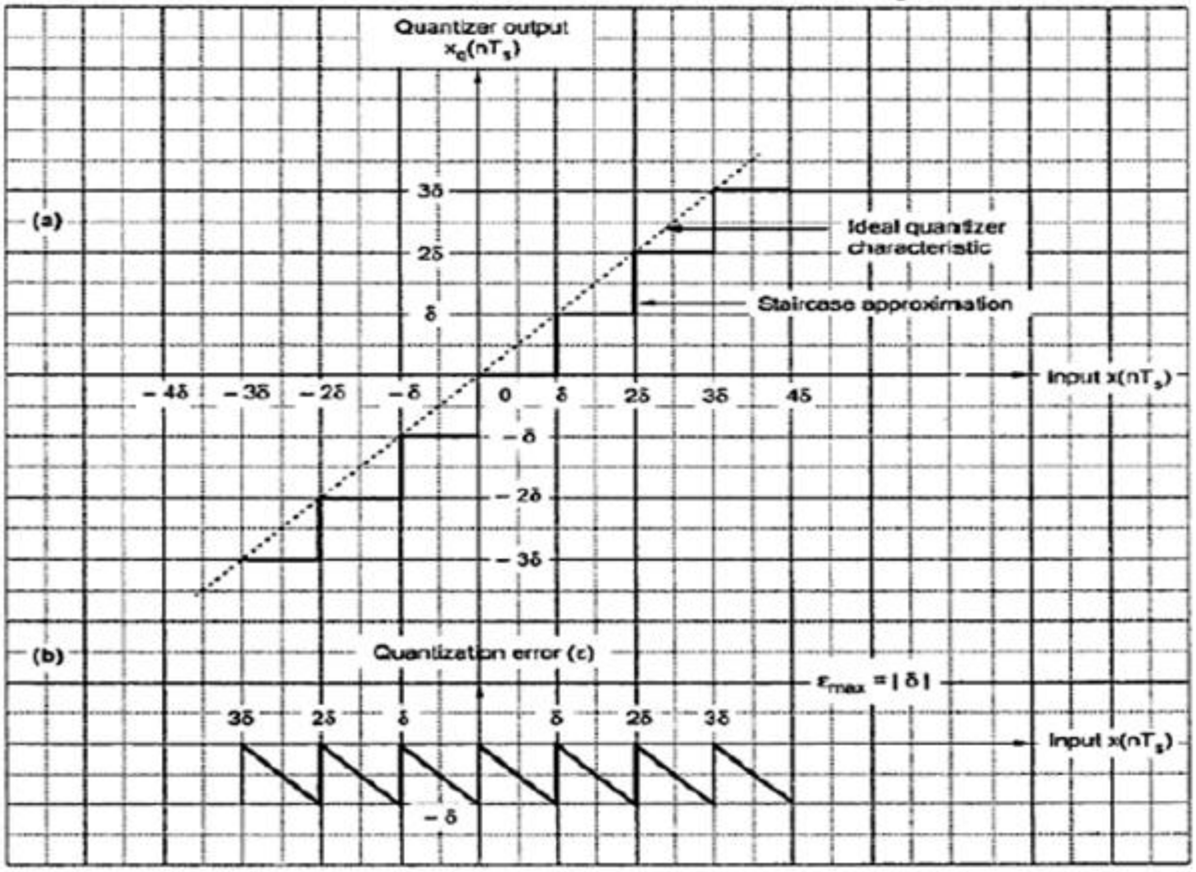


Fig. 1.8.5 (a) Biased quantizer transfer characteristic
(b) Quantization error

1.11 Non-uniform Quantization

In nonuniform quantization, the step size is not fixed. It varies according to certain law or as per input signal amplitude. Fig. 1.8.8 shows the transfer characteristic and error in nonuniform quantization.

In this figure observe that step size is small at low input signal levels. Hence quantization error is also small at these inputs. Therefore signal to quantization noise power ratio is improved at low signal levels. Stepsize is higher at high input levels. Hence signal to noise power ratio remains almost same throughout the dynamic range of quantizer.

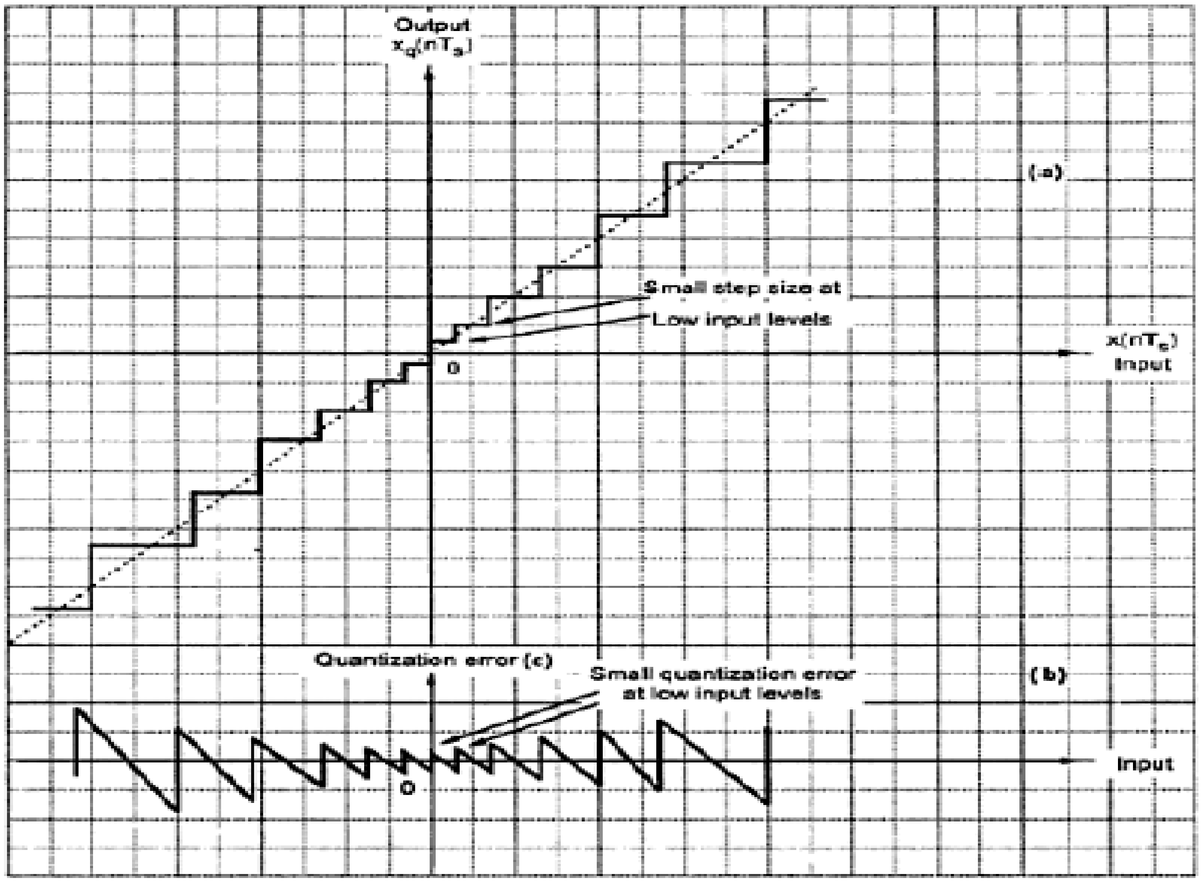


Fig. 1.8.8 (a) Nonuniform quantization transfer characteristic
(b) Quantization error

1.12 Bandwidth – Noise trade off

The noise analysis of PPM and FM have similar results as follows :

i) For both systems, the figure of merit is proportional to square of the ratio $\left(\frac{B_T}{W}\right)$.

ii) As the signal to noise ratio is reduced, both the systems exhibit threshold effect.

- With digital pulse modulation, the better noise performance than square law can be obtained.
- The digital pulse modulation such as pulse code modulation gives negligible noise effect by increasing the average power in binary PCM signal.
- With PCM, the bandwidth noise trade-off can be related by exponential law.

1.13 Pulse Code Modulation (PCM)

1.13.1 PCM Generator

The pulse code modulator technique samples the input signal $x(t)$ at frequency $f_s \geq 2W$. This sampled 'Variable amplitude' pulse is then digitized by the analog to digital converter. The parallel bits obtained are converted to a serial bit stream. Fig.1.8.1 shows the PCM generator.

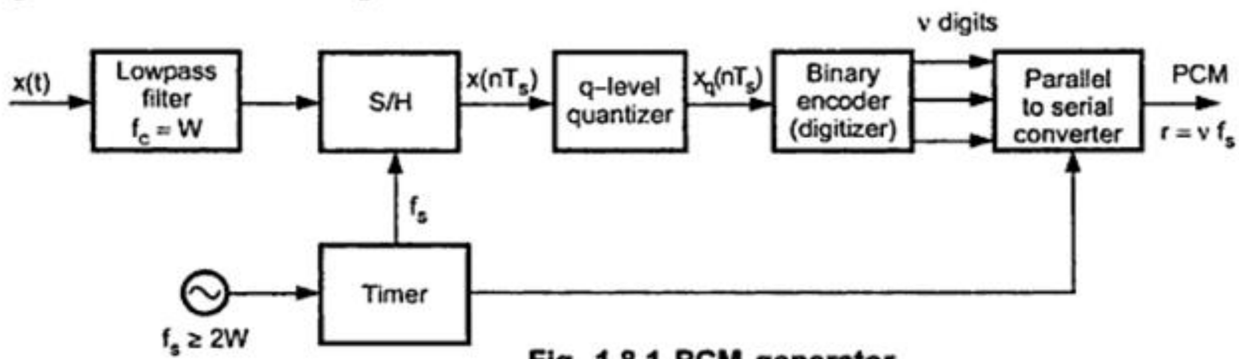


Fig. 1.8.1 PCM generator

In the PCM generator of above figure, the signal $x(t)$ is first passed through the lowpass filter of cutoff frequency 'W' Hz. This lowpass filter blocks all the frequency components above 'W' Hz. Thus $x(t)$ is bandlimited to 'W' Hz. The sample and hold circuit then samples this signal at the rate of f_s. Sampling frequency f_s is selected sufficiently above Nyquist rate to avoid aliasing i.e.,

$$f_s \geq 2W$$

In Fig. 1.8.1 output of sample and hold is called $x(nT_s)$. This $x(nT_s)$ is discrete in time and continuous in amplitude. A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called *quantization error*. Thus output of quantizer is a digital level called $x_q(nT_s)$.

Now coming back to our discussion of PCM generation, the quantized signal level $x_q(nT_s)$ is given to binary encoder. This encoder converts input signal to 'v' digits binary word. Thus $x_q(nT_s)$ is converted to 'V' binary bits. The encoder is also called digitizer.

It is not possible to transmit each bit of the binary word separately on transmission line. Therefore '*v*' binary digits are converted to serial bit stream to generate single baseband signal. In a parallel to serial converter, normally a shift register does this job. The output of PCM generator is thus a single baseband signal of binary bits.

An oscillator generates the clocks for sample and hold an parallel to serial converter. In the pulse code modulation generator discussed above ; sample and hold, quantizer and encoder combinely form an analog to digital converter.

1.13.2 PCM Receiver

Fig. 1.8.2 (a) shows the block diagram of PCM receiver and Fig. 1.8.2 (b) shows the reconstructed signal. The regenerator at the start of PCM receiver reshapes the pulses and removes the noise. This signal is then converted to parallel digital words for each sample.

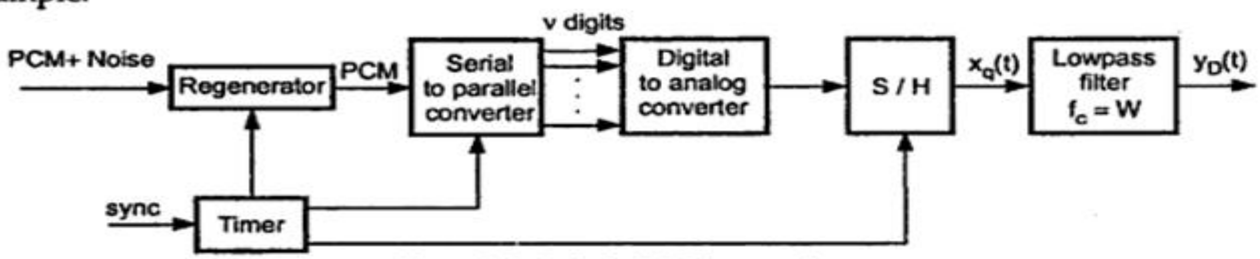


Fig. 1.8.2 (a) PCM receiver

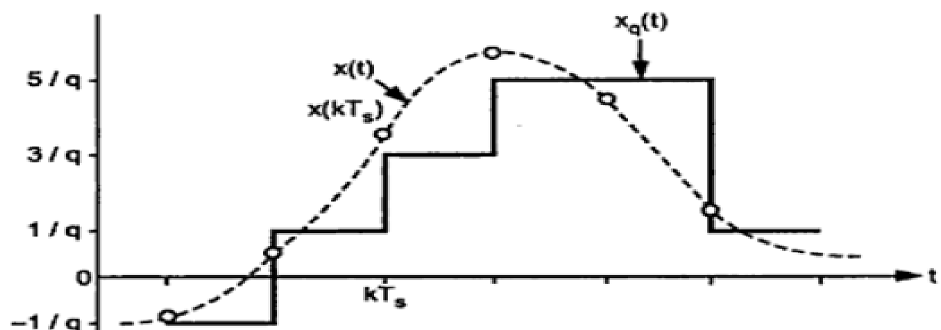


Fig. 1.8.2 (b) Reconstructed waveform

The digital word is converted to its analog value $x_q(t)$ along with sample and hold. This signal, at the output of S/H is passed through lowpass reconstruction filter to get $y_D(t)$. As shown in reconstructed signal of Fig. 1.8.2 (b), it is impossible to reconstruct exact original signal $x(t)$ because of permanent quantization error introduced during quantization at the transmitter. This quantization error can be reduced by increasing the binary levels. This is equivalent to increasing binary digits (bits) per sample. But increasing bits 'v' increases the signaling rate as well as transmission bandwidth as we have seen in equation 1.8.3 and equation 1.8.6. Therefore the choice of these parameters is made, such that noise due to quantization error (called as quantization noise) is in tolerable limits.

1.13.3 Transmission Bandwidth in PCM

Let the quantizer use 'v' number of binary digits to represent each level. Then the number of levels that can be represented by 'v' digits will be,

$$q = 2^v \quad \text{... (1.8.1)}$$

Here 'q' represents total number of digital levels of q-level quantizer.

For example if $v = 3$ bits, then total number of levels will be,

$$q = 2^3 = 8 \text{ levels}$$

Each sample is converted to 'v' binary bits. i.e. Number of bits per sample = v

We know that, Number of samples per second = f_s

$\therefore$ Number of bits per second is given by,

(Number of bits per second) = (Number of bits per samples) × (Number of samples per second)

$$= v \text{ bits per sample} \times f_s \text{ samples per second} \quad \text{... (1.8.2)}$$

The number of bits per second is also called signaling rate of PCM and is denoted by 'r' i.e.,

$$\boxed{\textbf{Signaling rate in PCM : } r = v f_s} \quad \text{... (1.8.3)}$$

Here $f_s \geq 2W$.

Bandwidth needed for PCM transmission will be given by half of the signaling rate i.e.,

Transmission Bandwidth of PCM :

$$B_T \geq \frac{1}{2} r \quad \text{... (1.8.4)}$$

$$B_T \geq \frac{1}{2} v f_s \quad \text{Since } f_s \geq 2W \quad \text{... (1.8.5)}$$

$$B_T \geq v W \quad \text{... (1.8.6)}$$

1.14 Noise considerations in PCM

The performance of a PCM system is influenced by two major sources of noise:

1. Channel noise, which is introduced anywhere between the transmitter output and the receiver input. Channel noise is always present, once the equipment is switched on.
2. Quantization noise, which is introduced in the transmitter and is carried all the way

along to the receiver output. Unlike channel noise, quantization noise is signal dependent in the sense that it disappears when the message signal is switched off.

The main effect of channel noise is to introduce *bit errors* into the received signal. In the case of a binary PCM system, the presence of a bit error causes symbol 1 to be mistaken for symbol 0, or vice versa.

Clearly, the more frequently bit errors occur, the more dissimilar the receiver output becomes compared to the original message signal.

The fidelity of information transmission by PCM in the presence of channel noise may be measured in terms of the *average probability of symbol error,* which is defined as the probability that the reconstructed symbol at the receiver output differs from the transmitted binary symbol, on the average.

The average probability of symbol error, also referred to as the *bit error rate* (BER), assumes that all the bits in the original binary wave are of equal importance.

To optimize system performance in the presence of channel noise, we need to minimize the average probability of symbol error.

For this evaluation, it is customary to model the channel noise as additive, white, and Gaussian.

The effect of channel noise can be made practically negligible by ensuring the use of an adequate signal energy-to-noise density ratio through the provision of short-enough spacing between the regenerative repeaters in the PCM system.

Quantization noise is essentially under the designer's control. It can be made negligibly small through the use of an adequate number of representation levels in the quantizer and the selection of a companding strategy matched to the characteristics of the type of message signal being transmitted.

1.15 Advantages and Limitations of PCM

Advantages of PCM

(i) Effect of channel noise and interference is reduced.

(ii) PCM permits regeneration of pulses along the transmission path. This reduces noise interference.

(iii) The bandwidth and signal to noise ratio are related by exponential law.

(iv) Multiplexing of various PCM signals is easily possible.

(v) Encryption or decryption can be easily incorporated for security purpose.

Limitations of PCM

(i) PCM systems are complex compared to analog pulse modulation methods.

(ii) The channel bandwidth is also increased because of digital coding of analog pulses.

1.16 Differential Pulse Code Modulation

1.16.1 Redundant Information in PCM

The samples of a signal are highly corrected with each other. This is because any signal does not change fast. That is its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with little difference. When these samples are encoded by standard PCM system, the resulting encoded signal contains redundant information. Fig. 1.11.1 illustrates this.

1.16.2 Principle of DPCM

If this redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced. This type of digital pulse modulation scheme is called Differential Pulse Code Modulation.

1.16.3 DPCM Transmitter

The differential pulse code modulation works on the principle of prediction. The value of the present sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual sample value. Fig. 1.11.2 shows the transmitter of Differential Pulse Code Modulation (DPCM) system. The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator finds out the difference between the actual sample value $x(nT_s)$ and predicted sample value $\hat{x}(nT_s)$. This is called error and it is denoted by $e(nT_s)$. It can be defined as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{... (1.11.1)}$$

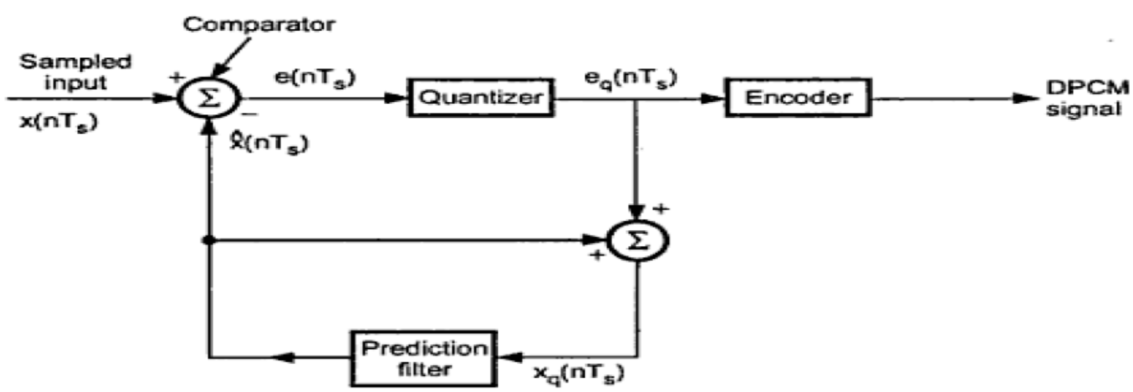


Fig. 1.11.2 Differential pulse code modulation transmitter

Thus error is the difference between unquantized input sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. The predicted value is produced by using a prediction filter. The quantizer output signal $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called $x_q(nT_s)$. This makes the prediction more and more close to the actual sampled signal. We can see that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small number of bits. Thus number of bits per sample are reduced in DPCM.

The quantizer output can be written as,

$$e_q(nT_s) = e(nT_s) + q(nT_s) \qquad \text{... (1.11.2)}$$

Here $q(nT_s)$ is the quantization error. As shown in Fig. 1.11.2, the prediction filter input $x_q(nT_s)$ is obtained by sum $\hat{x}(nT_s)$ and quantizer output i.e.,

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \qquad \text{... (1.11.3)}$$

Putting the value of $e_q(nT_s)$ from equation 1.11.2 in the above equation we get,

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \qquad \text{... (1.11.4)}$$

Equation 1.11.1 is written as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$\therefore \quad e(nT_s) + \hat{x}(nT_s) = x(nT_s) \qquad \text{... (1.11.5)}$$

$\therefore$ Putting the value of $e(nT_s) + \hat{x}(nT_s)$ from above equation into equation 1.11.4 we get,

$$x_q(nT_s) = x(nT_s) + q(nT_s) \qquad \text{... (1.11.6)}$$

Thus the quantized version of the signal $x_q(nT_s)$ is the sum of original sample value and quantization error $q(nT_s)$. The quantization error can be positive or negative. Thus equation 1.11.6 does not depend on the prediction filter characteristics.

1.16.4 Reconstruction of DPCM Signal

Fig. 1.11.3 shows the block diagram of DPCM receiver.

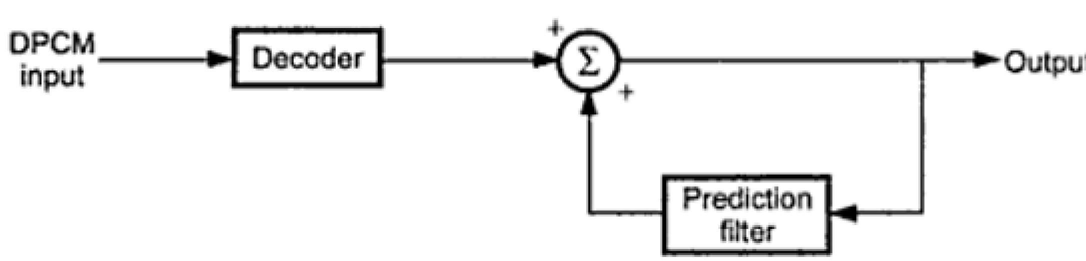


Fig. 1.11.3 DPCM receiver

The decoder first reconstructs the quantized error signal from incoming binary signal. The prediction filter output and quantized error signals are summed up to give the quantized version of the original signal. Thus the signal at the receiver differs from actual signal by quantization error $q(nT_s)$, which is introduced permanently in the reconstructed signal.

1.17 Delta Modulation

We have seen in PCM that, it transmits all the bits which are used to code the sample. Hence signaling rate and transmission channel bandwidth are large in PCM.To overcome this problem Delta Modulation is used.

1.17.1 Operating Principle of DM

Delta modulation transmits only one bit per sample. That is the present sample value is compared with the previous sample value and the indication,whether the amplitude is increased or decreased is sent. Input signal $x(t)$ is approximated to step signal by the delta modulator. This step size is fixed . The difference between the input signal $x(t)$ and staircase approximated signal confined to two levels, i.e. $+\delta$ and $-\delta$. If the difference is positive, then approximated signal is increased by one step i.e. 'δ'. If the difference is negative, then approximated signal is reduced by 'δ'. When the step is reduced, '0' is transmitted and if the step is increased, '1' is transmitted. Thus for each sample, only one binary bit is transmitted. Fig. 2.1.1 shows the analog signal $x(t)$ and its staircase approximated signal by the delta modulator.

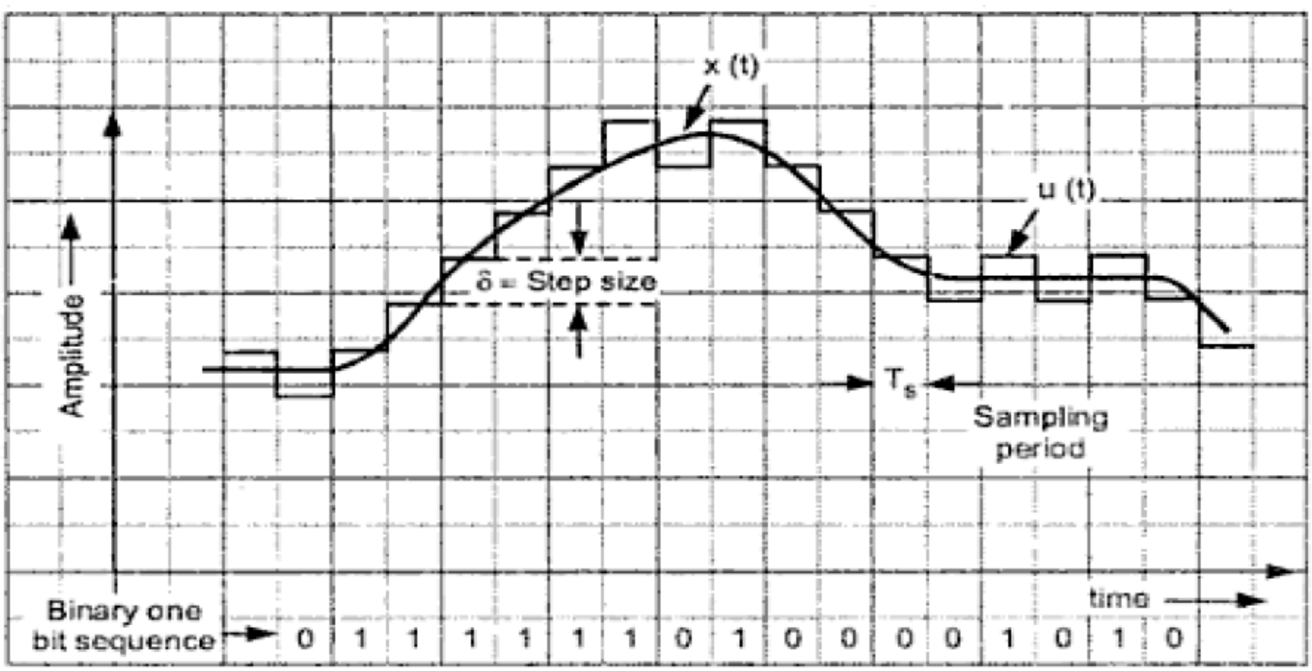


Fig. 2.1.1 Delta modulation waveform

1.17.2 DM Transmitter

Fig. 2.1.2 (a) shows the transmitter based on equations 2.1.3 to 2.1.5.

The principle of delta modulation can be explained by the following set of equations. The error between the sampled value of $x(t)$ and last approximated sample is given as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{... (2.1.1)}$$

Here, $e(nT_s)$ = Error at present sample

$x(nT_s)$ = Sampled signal of $x(t)$

$\hat{x}(nT_s)$ = Last sample approximation of the staircase waveform.

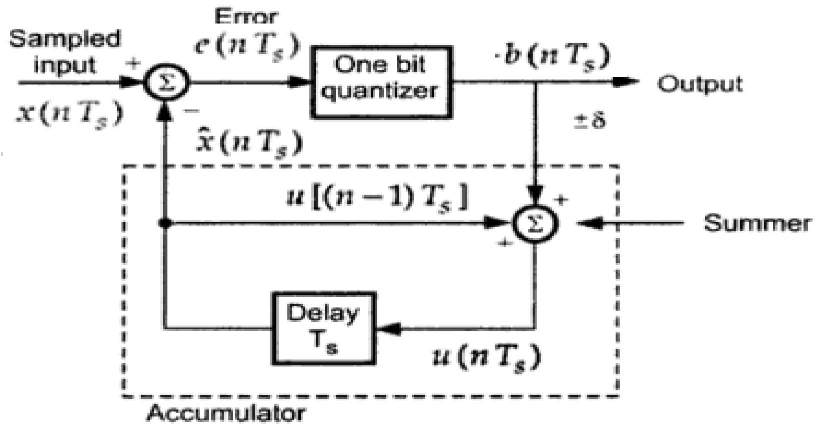


Fig. 2.1.2. a) Delta Modulation Transmitter

We can call $u(nT_s)$ as the present sample approximation of staircase output.

Then, $u[(n-1)T_s] = \hat{x}(nT_s)$ ... (2.1.2)

= Last sample approximation of staircase waveform.

Let the quantity $b(nT_s)$ be defined as,

$$b(nT_s) = \delta\, sgn[e(nT_s)] \quad \text{... (2.1.3)}$$

That is depending on the sign of error $e(nT_s)$ the sign of step size δ will be decided. In other words,

$$b(nT_s) = +\delta \quad \text{if} \quad x(nT_s) \geq \hat{x}(nT_s)$$
$$= -\delta \quad \text{if} \quad x(nT_s) < \hat{x}(nT_s) \quad \text{... (2.1.4)}$$

If $b(nT_s) = +\delta$; binary '1' is transmitted

and if $b(nT_s) = -\delta$; binary '0' is transmitted.

T_s = Sampling interval.

The summer in the accumulator adds quantizer output ($\pm\delta$) with the previous sample approximation. This gives present sample approximation. i.e.,

$$u(nT_s) = u(nT_s - T_s) + [\pm\delta] \quad \text{or}$$

$$= u[(n-1)T_s] + b(nT_s) \qquad \dots (2.1.5)$$

The previous sample approximation $u[(n-1)T_s]$ is restored by delaying one sample period T_s. The sampled input signal $x(nT_s)$ and staircase approximated signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$.

Depending on the sign of $e(nT_s)$ one bit quantizer produces an output step of $+\delta$ or $-\delta$. If the step size is $+\delta$, then binary '1' is transmitted and if it is $-\delta$, then binary '0' is transmitted.

1.17.3 DM Receiver

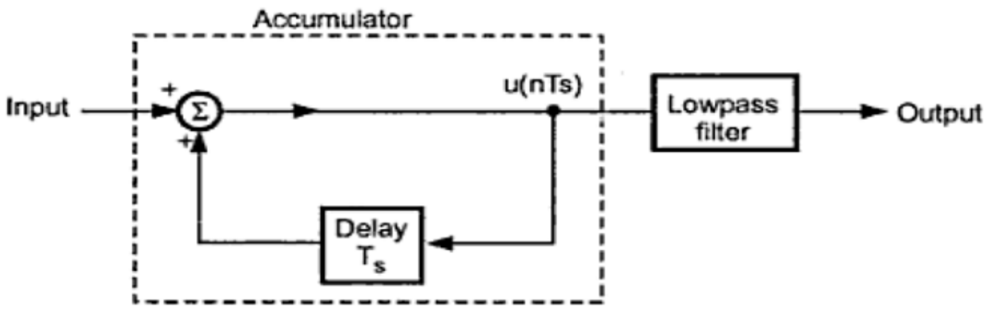


Fig. 2.1.2 (b) Delta modulation receiver

At the receiver shown in Fig. 2.1.2 (b), the accumulator and low-pass filter are used. The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s. It is then added to the input signal. If input is binary '1' then it adds $+\delta$ step to the previous output (which is delayed). If input is binary '0' then one step 'δ' is subtracted from the delayed signal. The low-pass filter has the cutoff frequency equal to highest frequency in $x(t)$. This filter smoothen the staircase signal to reconstruct $x(t)$.

1.17.4 Advantages and Disadvantages of Delta Modulation

Advantages of Delta Modulation

The delta modulation has following advantages over PCM,

1. Delta modulation transmits only one bit for one sample. Thus the signaling rate and transmission channel bandwidth is quite small for delta modulation.

2. The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter involved in delta modulation.

Disadvantages of Delta Modulation

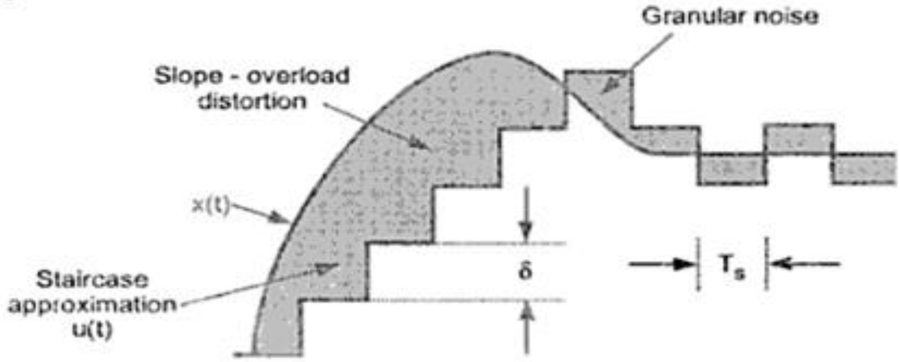


Fig. 2.2.1 Quantization errors in delta modulation

The delta modulation has two drawbacks -

Slope Overload Distortion (Startup Error)

This distortion arises because of the large dynamic range of the input signal.

As can be seen from Fig. 2.2.1 the rate of rise of input signal $x(t)$ is so high that the staircase signal cannot approximate it, the step size 'δ' becomes too small for staircase signal $u(t)$ to follow the steep segment of $x(t)$. Thus there is a large error between the staircase approximated signal and the original input signal $x(t)$. This error is called *slope overload distortion*. To reduce this error, the step size should be increased when slope of signal of $x(t)$ is high.

Since the step size of delta modulator remains fixed, its maximum or minimum slopes occur along straight lines. Therefore this modulator is also called Linear Delta Modulator (LDM).

Granular Noise (Hunting)

Granular noise occurs when the step size is too large compared to small variations in the input signal. That is for very small variations in the input signal, the staircase signal is changed by large amount (δ) because of large step size. Fig. 2.2.1 shows that when the input signal is almost flat, the staircase signal $u(t)$ keeps on oscillating by $\pm\delta$ around the signal. The error between the input and approximated signal is called *granular noise*. The solution to this problem is to make step size small.

Thus large step size is required to accommodate wide dynamic range of the input signal (to reduce slope overload distortion) and small steps are required to reduce granular noise. Adaptive delta modulation is the modification to overcome these errors.

1.18 Linear Prediction

Linear prediction is used for estimating the future samples of the signal from present and past input sample values. Prediction is said to be linear if the future sample value is linear combination of present and past input samples. The predicted sample value is given as,

$$\hat{x}(n) = \sum_{k=1}^{M} w_k x(n-k) \quad \dots (1.13.1)$$

Here $\hat{x}(n)$ is the predicted value of $x(n)$.

$x(n-1), x(n-2), \dots x(n-M)$ are the past input samples.

$w_1, w_2, w_3 \dots w_M$ are the set of multipliers, called filter coefficients.

Above equation is a linear equation. It shows that $\hat{x}(n)$ is linear combination of $x(n-1), x(n-2), \dots x(n-M)$. Hence it is linear prediction. Basically above equation is the linear convolution for discrete signals.

Let the actual sample value be $x(n)$. Then the difference between actual sample value and predicted sample value is called prediction error. It is denoted by e(n) i.e.

Prediction error, $$e(n) = x(n) - \hat{x}(n) \quad \dots (1.13.2)$$

The filter coefficients $w_1, w_2 \dots w_M$ should be selected such that the mean square value of error $\overline{e^2(n)}$ is minimized. Starting from this condition, an equation is derived which gives values of 'w_k' i.e.,

$$\sum_{j=1}^{M} w_j R(k-j) = R(k) \quad \text{and } k = 1, 2, \dots M \quad \dots (1.13.3)$$

Here 'w_j' are the prediction filter coefficients,

$$R(k-j) = \overline{x(n-k) \cdot x(n-j)}, \text{ is autocorrelation of } x(n)$$

and $$R(k) = \overline{x(n) \cdot x(n-k)}, \quad \text{is autocorrelation of } x(n)$$

Above equation gives 'M' simultaneous linear equations with 'M' values of unknowns. These equations can be solved to get $w_1, w_2, \dots w_M$ values.

1.18.1 Prediction Filter

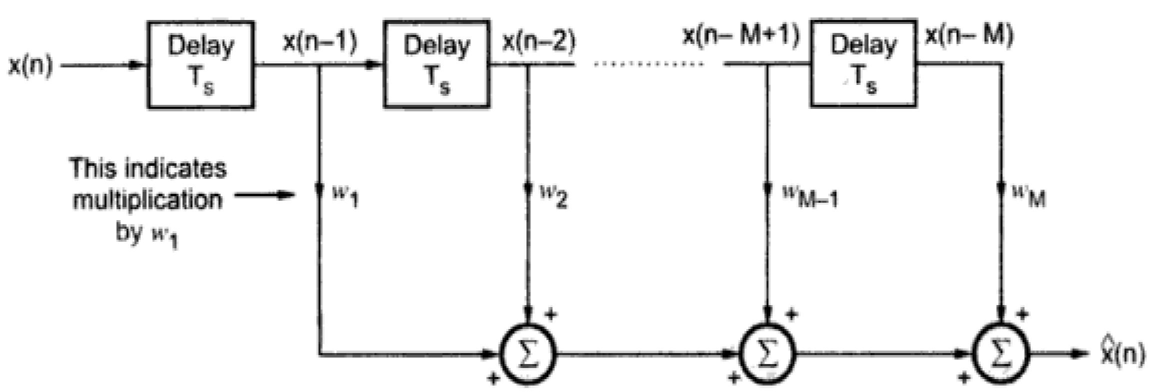


Fig. 1.13.1 Prediction filter

Implementation of linear predictor of equation 1.13.1 is also called prediction filter. Fig. 1.13.1 shows the block diagram of such filter. In this figure the input sample is delayed by one sampling duration (T_s). Hence we get $x(n-1)$, it is the previous input sample. Similarly if $x(n-1)$ is delayed by T_s we get $x(n-2)$. The past input samples are multiplied by $w_1, w_2, \ldots w_m$ and then added to give $\hat{x}$(n). It is called 'filter' since equation 1.13.1 represents linear convolution. And such convolution represents filtering. Here $x(n-1), x(n-2), \ldots x(n-m)$ are past inputs, $x(n)$ is present input. These input samples are used for filtering with the help of coefficients $w_1, w_2, \ldots w_M$. Such filter is of the type of finite impulse response digital filter. The filtering operation can be modified by changing the filter coefficients. Prediction filter is used earlier in DPCM.

Prediction error can also be calculated using prediction filter. Fig. 1.13.2 shows the implementation of equation 1.13.2. Consider equation 1.13.2,

$$e(n) = x(n) - \hat{x}(n)$$

Putting for $\hat{x}$(n) from equation 1.13.1,

$$e(n) = x(n) - \sum_{k=1}^{M} w_k x(n-k)$$

$$= x(n) - \{w_1 x(n-1) + w_2 x(n-2) + \ldots + w_M x(n-M)\}$$

$$= x(n) - w_1 x(n-1) - w_2 x(n-2) - \ldots - w_M x(n-M)$$

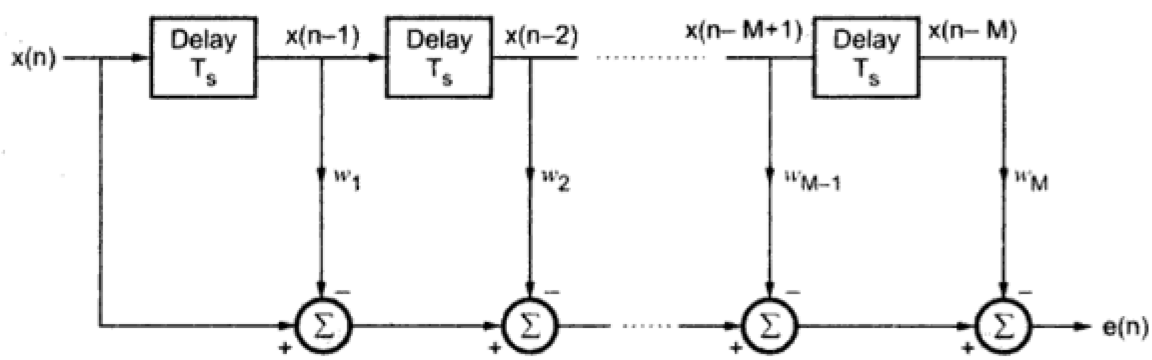


Fig. 1.13.2 Prediction filter used to calculate error e(n)

In the above diagram observe that $w_1\ x(n-1)$, $w_2 x(n-2)$, ... $w_M x(n-M)$ are subtracted from x(n). Hence output of prediction filter is error $\hat{e}(n)$.

1.19 Adaptive Delta Modulation

1.19.1 Operating Principle

To overcome the quantization errors due to slope overload and granular noise, the step size (δ) is made adaptive to variations in the input signal $x(t)$. Particularly in the steep segment of the signal $x(t)$, the step size is increased. When the input is varying slowly, the step size is reduced. Then the method is called *Adaptive Delta Modulation (ADM).*

The adaptive delta modulators can take continuous changes in step size or discrete changes in step size.

1.19.2 Transmitter and Receiver

Fig. 2.3.1 (a) shows the transmitter and 2.3.1 (b) shows receiver of adaptive delta modulator. The logic for step size control is added in the diagram. The step size increases or decreases according to certain rule depending on one bit quantizer output.

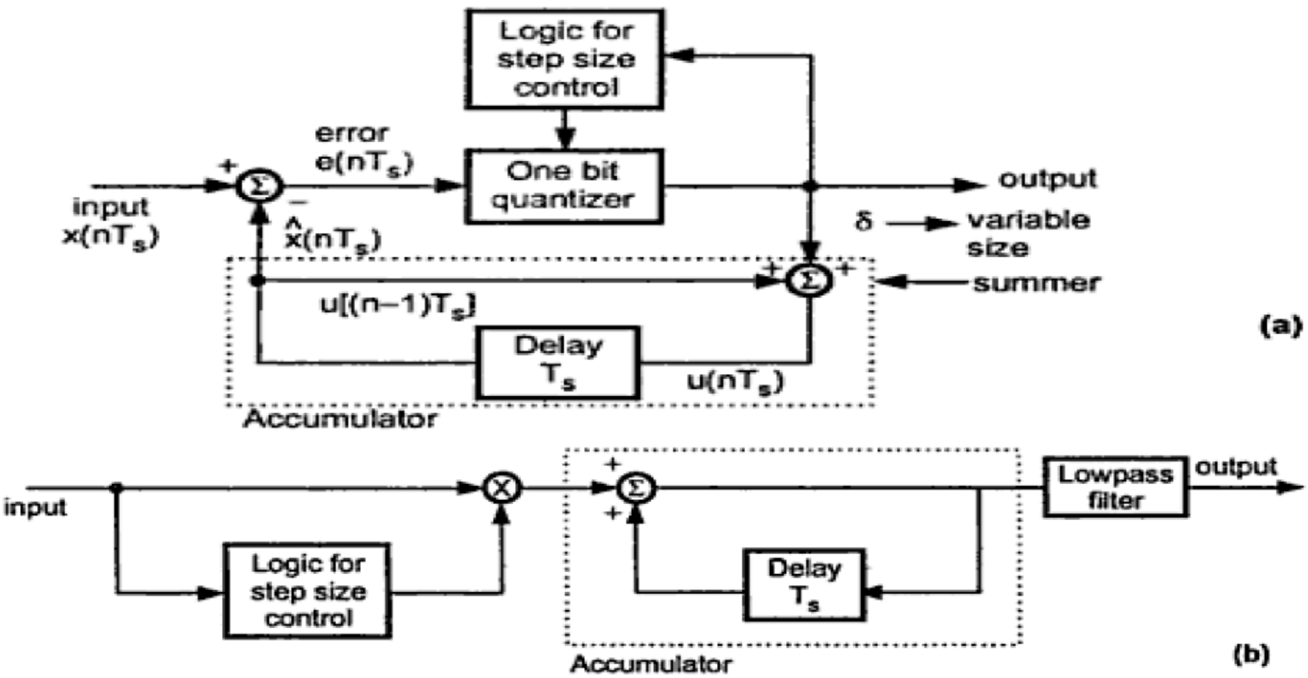


Fig. 2.3.1 Adaptive delta modulator (a) Transmitter (b) Receiver

For example if one bit quantizer output is high (1), then step size may be doubled for next sample. If one bit quantizer output is low, then step size may be reduced by one step. Fig. 2.3.2 shows the waveforms of adaptive delta modulator and sequence of bits transmitted.

In the receiver of adaptive delta modulator shown in Fig. 2.3.1 (b) the first part generates the step size from each incoming bit. Exactly the same process is followed as that in transmitter. The previous input and present input decides the step size. It is then given to an accumulator which builds up staircase waveform. The low-pass filter then smoothens out the staircase waveform to reconstruct the smooth signal.

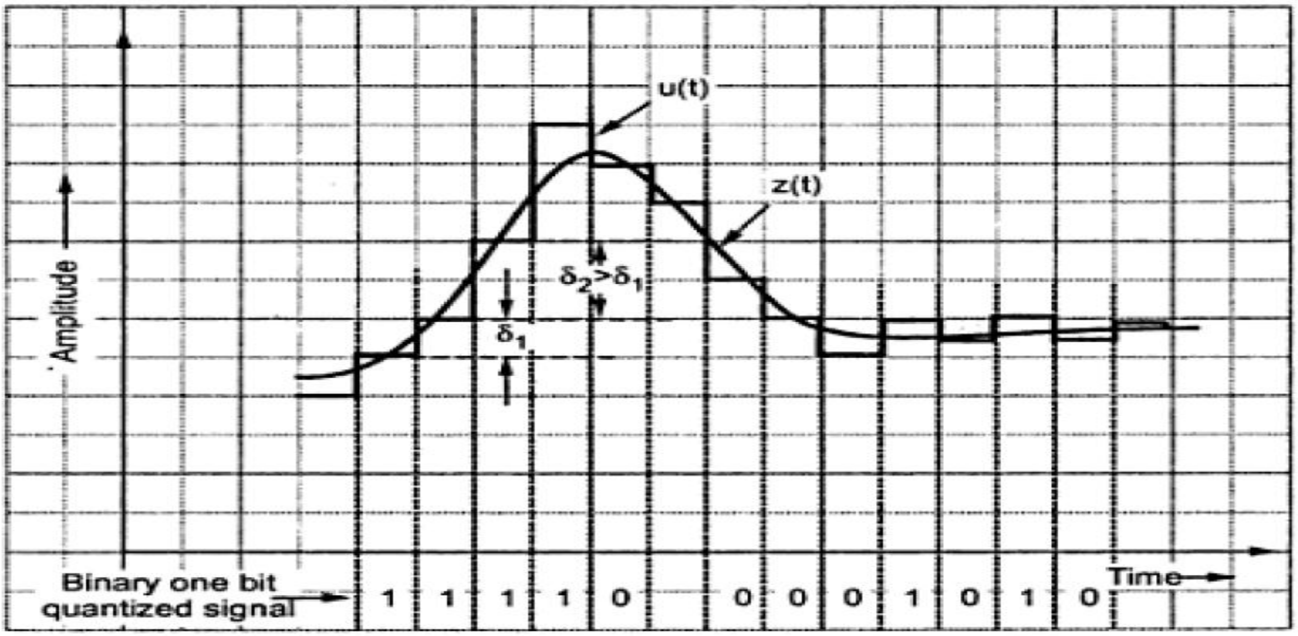


Fig. 2.3.2 Waveforms of adaptive delta modulation

1.19.3 Advantages of Adaptive Delta Modulation

Adaptive delta modulation has certain advantages over delta modulation. i.e.,

1. The signal to noise ratio is better than ordinary delta modulation because of the reduction in slope overload distortion and granular noise.
2. Because of the variable step size, the dynamic range of ADM is wide.
3. Utilization of bandwidth is better than delta modulation.

Plus other advantages of delta modulation are, only one bit per sample is required and simplicity of implementation of transmitter and receiver.

1.20 Comparision of Digital Pulse Modulation Methods

Sr. No.	Parameter	PCM	Delta modulation (DM)	Adaptive Delta Modulation (ADM)	Differential Pulse Code Modulation (DPCM)
1.	Number of bits	It can use 4, 8 or 16 bits per sample.	It uses only one bit for one sample.	Only one bit is used to encode one sample.	Bits can be more than one but are less than PCM.
2	Levels, step size	The number of levels depend on number of bits. Level size is fixed.	Step size is fixed and cannot be varied.	According to the signal variation, step size varies (Adapted).	Fixed number of levels are used.
3	Quantization error and distortion	Quantization error depends on number of levels used.	Slope overload distortion and granular noise is present.	Quantization error is present but other errors are absent.	Slope overload distortion and quantization noise is present.

4	Bandwidth of transmission channel	Highest bandwidth is required since number of bits are high.	Lowest bandwidth is required.	Lowest bandwidth is required.	Bandwidth required is lower than PCM.
5	Feedback.	There is no feedback in transmitter or receiver.	Feedback exists in transmitter.	Feedback exists.	Feedback exists.
6	Complexity of notation	System is complex.	Simple.	Simple.	Simple.
7.	Signal to noise ratio	Good.	Poor.	Better than DM.	Fair.
8.	Area of applications	Audio and video Telephony.	Speech and images.	Speech and images.	Speech and video.

Sr.No.	Parameter	PCM	DM	ADM	DPCM
9	Sampling rate kHz	8	64-128	48-64	8
10	Bits/sample	7 - 8	1	1	4-6
11	Bit rate	56-64	64-128	46-64	32-48