SATHYABAMA UNIVERSITY

SECX1048 FUNDAMENTALS OF FUZZY LOGIC AND ARTIFICIAL NEURAL NETWORK

UNIT 1 INTRODUCTION TO FUZZY LOGIC

Introduction:

The classical set theory is built on the fundamental concept of “set” of which an individual is either a member or not a member. A sharp, crisp, and unambiguous distinction exists between a member and a nonmember for any well-defined “set” of entities in this theory, and there is a very precise and clear boundary to indicate if an entity belongs to the set.

Namely, in the classical set theory, it is not allowed that an element is in a set and not in the set at the same time. Thus, many real-world application problems cannot be described and handled by the classical set theory, including all those involving elements with only partial membership of a set. On the contrary, fuzzy set theory accepts partial memberships, and, therefore, in a sense generalizes the classical set theory to some extent.

Fuzzy logic is an extension of Boolean logic by Lot Zadeh in 1965 based on the mathematical theory of fuzzy sets, which is a generalization of the classical set theory. By introducing the notion of degree in the verification of a condition, thus enabling a condition to be in a state other than true or false, fuzzy logic provides a very valuable flexibility for reasoning, which makes it possible to take into account inaccuracies and uncertainties. In order to introduce the concept of fuzzy sets, we first review the elementary set theory of classical mathematics. It will be seen that the fuzzy set theory is a very natural extension of the classical set theory, and is also a rigorous mathematical notion.

Basic Concepts of Fuzzy Sets:

Fuzzy logic:

Fuzzy logic is defined as a Multivalued Logic with various degrees of values for its member elements.

Fuzzy logic is based on "degrees of truth" than the (1 or 0) Boolean logic on which the modern computer is based.

Classical set:

A classical set is defined by crisp boundaries; there is no uncertainty or vagueness in the prescription or location of the boundaries of the set.

Operations on Classical Sets

Let A and B be two subsets on the universe X. Operations are shown below

Union	$A \cup B = \{x \vert x \in A \text{ or } x \in B\}$	…. (1)
Intersection	$A \cap B = \{x \vert x \in A \text{ and } x \in B\}$	…. (2)
Complement	$A = \{x \vert x /\in A, x \in X\}$	…. (3)
Difference	$A \vert B = \{x \vert x \in A \text{ and } x /\in B\}$	…. (4)

Properties of Classical (Crisp) Sets

Commutativity	$A \cup B = B \cup A$	
	$A \cap B = B \cap A.$	…….. (5)
Associativity	$A \cup (B \cup C) = (A \cup B) \cup C$	
	$A \cap (B \cap C) = (A \cap B) \cap C$	…….. (6)
Distributivity	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	
	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	…….. (7)
Idempotency	$A \cup A = A$	
	$A \cap A = A$	…….. (8)
Identity	$A \cup \varnothing = A$	
	$A \cap X = A$	
	$A \cap \varnothing = \varnothing.$	…….. (9)

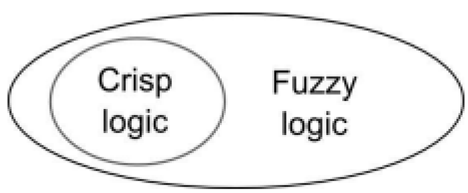
$A \cup X = X.$

Transitivity   If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$ …….. (10)

Involution   ……. (11)

Fuzzy sets:

A fuzzy set is a set with a smooth boundary. Fuzzy logic is based on the theory of fuzzy sets, which is a generalization of the classical set theory. Saying that the theory of fuzzy sets is a generalization of the classical set theory means that the latter is a special case of fuzzy sets theory. To make a metaphor in set theory speaking, the classical set theory is a subset of the theory of fuzzy sets,



"The classical set theory is a subset of the theory of fuzzy sets"

A fuzzy set, is defined as a set containing elements that have varying degrees of membership values in the range of zero to one.

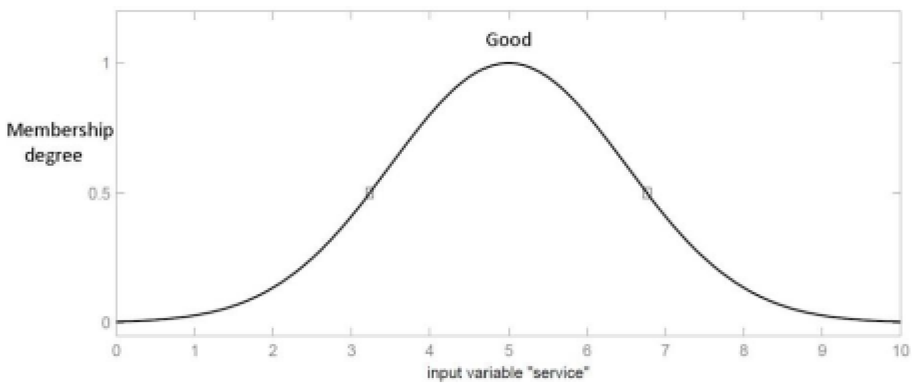
Fuzzy Set Operations

Union $\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x).$ …. (15)

Intersection $\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x).$ …. (16)

Complement $\mu_{\overline{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x).$ …. (17)

Fuzzy logic is based on fuzzy set theory, which is a generalization of the classical set theory. The classical sets are also called clear sets, classical logic is also known as Boolean logic or binary.



Membership function characterizing the subset of 'good' quality of service.

The figure 3 shows the membership function chosen to characterize the subset of 'good' quality of service. Definition 1. Let X be a set. A fuzzy subset A of X is characterized by a membership func-tion. fa : X ! [0; 1].

• Input 1: quality of service. Subsets: poor, good and excellent.

• Input 2: quality of food. Subsets: awful and delicious.

Output: tip amount. Subsets: low, medium and high.

The shape of the membership function is chosen arbitrarily by following the advice of the expert or by statistical studies: sigmoid, hyperbolic, tangent, exponential, Gaussian or any other form can be used.

A fuzzy set is defined by a functions that maps objects in a domain of concern into their membership value in a set.

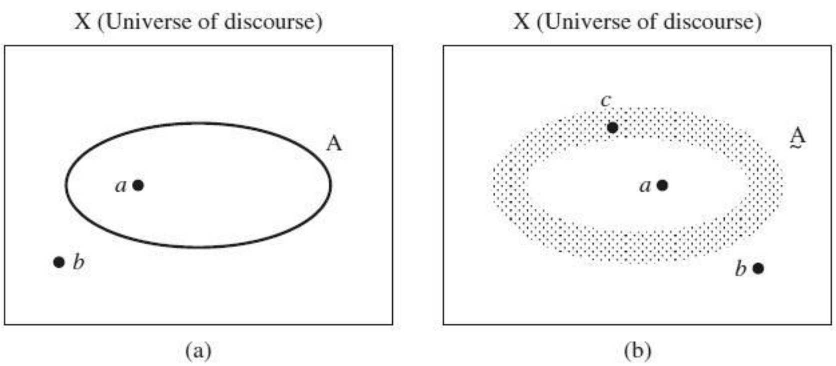
Such a function is called the *membership function*.

A fuzzy set, then, is a set containing elements that have varying degrees of membership in the set. This idea is in contrast with classical, or crisp, sets because members of a crisp set would not be members unless their membership is full, or complete, in that set (i.e., their membership is assigned a value of 1). Elements in a fuzzy set, because their membership need not be complete, can also be members of other fuzzy sets on the same universe.

Classical Set Vs Fuzzy Set:

Classical set:

A classical set is defined by crisp boundaries. there is no uncertainty in the prescription or location of the boundaries of the set.



Diagrams for (a) crisp set boundary and (b) fuzzy set boundary.

Universe of discourse: Define a universe of discourse, X, as a collection of objects all having the same characteristics.

Examples:

The clock speeds of computer CPUs

The operating currents of an electronic motor

The operating temperature of a heat pump (in degrees Celsius)

The total number of elements in a universe X is called its cardinal number, denoted nx, where x is a label for individual elements in the universe.

Collections of elements within a universe are called sets, and collections of elements within sets are called subsets.We define the null set, ∅, as the set containing no elements, and the whole set, X, as the set of all elements in the universe.

Fuzzy Set:

A fuzzy set is prescribed by vague or ambiguous properties; hence its boundaries are ambiguously specified.

Fuzzy set theory permits the gradual assessment of the membership of elements in a set, described with the aid of a membership function valued in the real unit [0,1].

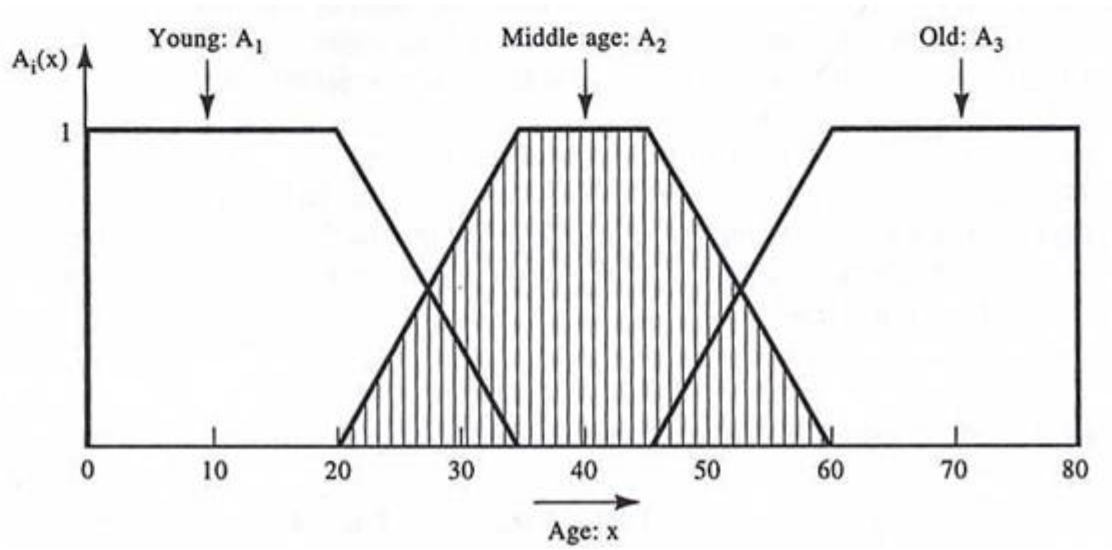
Examples:

Words like young,tall,good or high are fuzzy.

There is no single quantitative value which defines the young term.

For some people, age 25 is young, and for others ,age 35 is young.

The concept young has no boundary.



Fuzzy set Theory:

Fuzzy set theory is an extension of classical set theory where elements have varying degrees of membership.A logic is based on two truth values, TRUE and FALSE.Fuzzy logic uses the whole interval between 0(TRUE) and 1(FALSE).

Fuzzy logic is derived from fuzzy set theory dealing with reasoning that is approximate rather than precisely deduced from classical predicate logic.

Fuzzy set theory defines Fuzzy Operators on Fuzzy Sets. Fuzzy logic is capable of handling inherently imprecise concepts.

Properties of Fuzzy set:

Two special properties of set operations are known as the excluded middle axioms and De Morgan's principles. These properties are enumerated here for two sets A and B. The excluded middle axioms are very important because these are the only set operations described here that are not valid for both classical sets and fuzzy sets. There are two excluded middle axioms. The first, called the axiom of the excluded middle, deals with the union of a set A and its complement; the second, called the axiom of contradiction, represents the intersection of a set A and its complement.

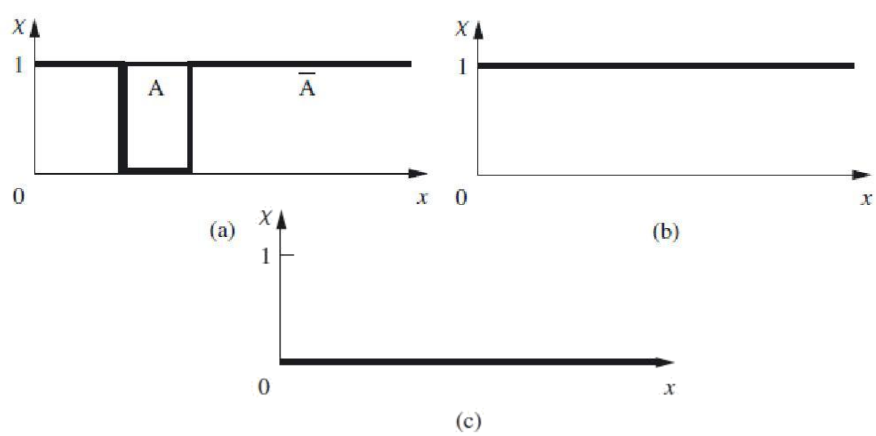
All Properties of classical sets also hold for fuzzy sets, except for the excluded middle and contradiction axioms.

Fuzzy sets can overlap. A set and its complement can overlap. The excluded middle axioms, extended for fuzzy sets, are expressed as

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X. \qquad \text{…. (18)}$$

$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \emptyset. \qquad \text{…. (19)}$$

Venn diagrams comparing the excluded middle axioms for classical (crisp) sets and fuzzy sets are shown in the below diagrams.



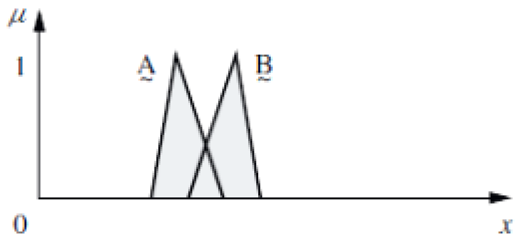
(a) Crisp set A and its complement; (b) Fuzzy $\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X$; and (c) crisp $A \cap \overline{A} \neq \emptyset$

Axiom of the excluded middle A ∪ A = X.

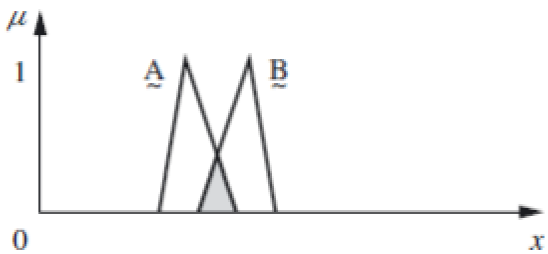
Axiom of the contradiction A ∩ A = Ø.

Fuzzy Logic Operation on Fuzzy Sets:

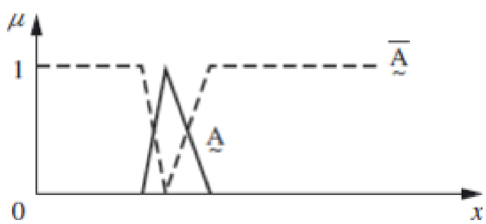
Venn diagrams for these operations, extended to consider fuzzy sets, are shown in the diagrams. The operations given in the equations are known as the standard fuzzy operations.



Union of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$.



Intersection of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$.



Complement of fuzzy set A

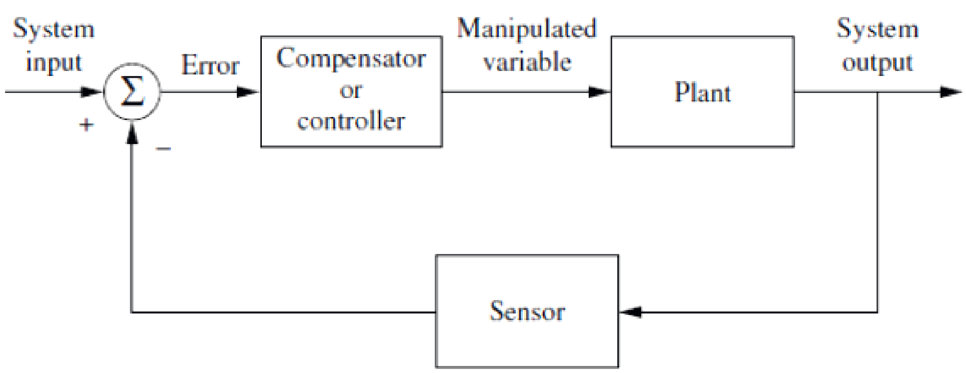
All other operations on classical sets also hold for fuzzy sets, except for the excluded middle axioms. These two axioms do not hold for fuzzy sets since they do not form part of the basic axiomatic structure of fuzzy sets. Since fuzzy sets can overlap, a set and its complement can also overlap. The excludedmiddle axioms, extended for fuzzy sets, are expressed as:

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X.$$
$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \emptyset.$$

Fuzzy Logic Control Principles:

Control systems abound in our everyday life; perhaps we do not see them as such, because some of them are larger than what a single individual can deal with, but they are ubiquitous. For example, economic systems are large, global systems that can be controlled; ecosystems are large, amorphous, and long-term systems that can be controlled.

The general form of a closed-loop control system is illustrated.



A closed-loop control system.

Control systems are sometimes divided into two classes. If the objective of the control system is to maintain a physical variable at some constant value in the presence of disturbances, the system is called a regulatory type of control, or a regulator. The second class of control systems is set point tracking controllers. In this scheme of control, a physical variable is required to follow or track some desired time function. An example of this type of system is an automatic aircraft landing system, in which the aircraft follows a “ramp”to the desired touchdown point.

Assumptions in a Fuzzy Control System Design:

A number of assumptions are implicit in a fuzzy control system design. Six basic assumptions are commonly made whenever a fuzzy rule based control policy is selected.

The plant is observable and controllable: state, input, and output variables are usually available for observation and measurement or computation. There exists a body of knowledge comprising a set of linguistic rules, engineering common sense, intuition, or a set of input–

output measurements data from which rules can be extracted. A solution exists. The control engineer is looking for a “good enough” solution, not necessarily the optimum one. The controller will be designed within an acceptable range of precision. The problems of stability and optimality are not addressed explicitly; such issues are still open problems in fuzzy controller design.

FUZZY LOGIC CONTROL SYSTEM (FLCS)

Control system is a set of hardware component which regulates or alters or modifies the behaviour of the system. Fuzzy control system uses approximation so that the nonlinearity, data or knowledge incompleteness is reduced.

Assumptions in FLC a FLCS Design:

To design a FLCS we must take into consideration the following points:

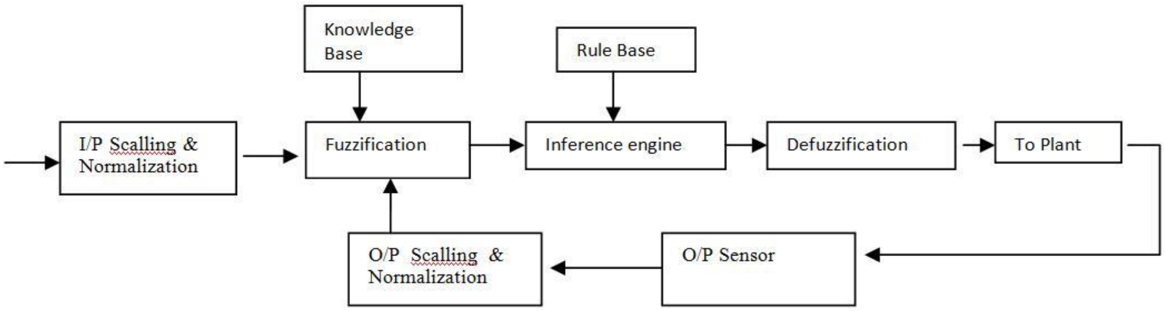
- ✓ The plant is observable and controllable
- ✓ There exists a set of knowledge about that process from which the rules can be framed
- ✓ There exists a solution
- ✓ The control engineer is looking for “Good enough solution” and not an optimum one

- ✓ Controller can be designed within an acceptable range of precision

Steps involved in FLCS:

- ✓ Identify the variables
- ✓ By assigning appropriate membership function convert these parameters in fuzzy sub sets.
- ✓ Assign fuzzy relationship to input states and fuzzy output states
- ✓ Use fuzzy approximate reasoning to infer the outcomes
- ✓ Aggregate the outcomes recommended by each rule

- ✓ Apply Defuzzification to form a crisp output



Fuzzy logic Control system Block Diagram

FUZZY RELATIONS

Fuzzy relations is used to map elements of one universe, say X, to those of another universe, say Y, with the help of Cartesian product .The "strength" of the relation measured with a membership function having "degrees" of strength of the relation on the unit interval [0,1]. Hence, a fuzzy relation $\underset{\sim}{R}$ is a mapping from the Cartesian space X × Y to the interval [0,1], where the strength of the mapping is expressed by the membership function $\mu_{\underset{\sim}{R}}(x, y)$.

Cardinality of Fuzzy Relations

Cardinality of fuzzy sets is infinity; the cardinality of a fuzzy relation between two or more universes is also infinity.

Operations on Fuzzy Relations

Let $\underset{\sim}{R}$ and $\underset{\sim}{S}$ be fuzzy relations on the Cartesian space X × Y. Then the following operations apply for the membership values for various set operations

$$\underset{\sim}{R} \subset \underset{\sim}{S} \Rightarrow \mu_{\underset{\sim}{R}}(x, y) \le \mu_{\underset{\sim}{S}}(x, y).$$

Let R and S be fuzzy relations on the Cartesian space X × Y. Then the following operations apply for the membership values for various set operations (these are similar to the same operations on crisp sets

Union $\quad \mu_{\underset{\sim}{R} \cup \underset{\sim}{S}}(x, y) = \max(\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)).$

Intersection $\quad \mu_{\underset{\sim}{R} \cap \underset{\sim}{S}}(x, y) = \min(\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(x, y)).$

Complement $\quad \mu_{\overline{\underset{\sim}{R}}}(x, y) = 1 - \mu_{\underset{\sim}{R}}(x, y).$

Containment $\quad \underset{\sim}{R} \subset \underset{\sim}{S} \Rightarrow \mu_{\underset{\sim}{R}}(x, y) \le \mu_{\underset{\sim}{S}}(x, y).$

Properties of Fuzzy Relations:

Just as for crisp relations, the properties of commutativity, associativity, distributivity, involution, and idempotency all hold for fuzzy relations. Moreover, De Morgan's principles hold for fuzzy relations just as they do for crisp (classical) relations, and the null relation, O, and the complete relation, E, are analogous to the null set and the whole set in set-theoretic form, respectively. Fuzzy relations are not constrained, as is the case for fuzzy sets in general, by the excluded middle axioms. Since a fuzzy relation R also a fuzzy set, there is overlap between a relation and its complement; hence,

The excluded middle axioms for fuzzy relations do not result, in general, in the null relation, O, or the complete relation, E.

$$\underset{\sim}{R} \cup \overline{\underset{\sim}{R}} \neq \mathbf{E}. \quad \text{..........(24)}$$

$$\underset{\sim}{R} \cap \overline{\underset{\sim}{R}} \neq \mathbf{O}. \quad \text{.............(25)}$$

From the above equations, the excluded middle axioms for fuzzy relations do not result, in general, in the null relation, O, or the complete relation, E.

Fuzzy Cartesian product and Composition

Let $\underset{\sim}{A}$ be a fuzzy set on universe X and $\underset{\sim}{B}$ be a fuzzy set on universe Y, then the Cartesian product between fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ will result in fuzzy relation $\underset{\sim}{R}$, which is given as

$$\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R} \subset X \times Y, \quad \text{..........(26)}$$

Where the fuzzy relation $\underset{\sim}{R}$ has membership function

$$\mu_{\underset{\sim}{R}}(x, y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x, y) = \min(\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)). \quad \text{........(27)}$$

The Cartesian product defined by $\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R}$ is implemented in the same way as the cross product of two vectors. Cartesian product is not the same as the arithmetic product. Cartesian product employs the idea of pairing of elements among sets. For example, for a fuzzy set (vector) $\underset{\sim}{A}$ that has four elements, for a fuzzy set (vector) $\underset{\sim}{B}$ that has five elements, the

resulting fuzzy relation $\underset{\sim}{R}$ will be represented by a matrix of size 4 × 5, that is, $\underset{\sim}{R}$ will have four rows and five columns.

Fuzzy composition can be defined as of crisp relations. Suppose $\underset{\sim}{R}$ is a fuzzy relation on the Cartesian space X × Y, $\underset{\sim}{S}$ is a fuzzy relation on Y × Z, and $\underset{\sim}{T}$ is a fuzzy relation on X × Z, then fuzzy max–min composition is defined in the following manner:

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S}, \qquad ....(28)$$

$$\mu_{\underset{\sim}{T}}(x, z) = \bigvee_{y \in Y} (\mu_{\underset{\sim}{R}}(x, y) \wedge \mu_{\underset{\sim}{S}}(y, z)), \qquad ....(29)$$

and fuzzy max–product composition is defined in terms of the membership function theoretic notation as

$$\mu_{\underset{\sim}{T}}(x, z) = \bigvee_{y \in Y} (\mu_{\underset{\sim}{R}}(x, y) \bullet \mu_{\underset{\sim}{S}}(y, z)). \qquad ....(30)$$

It should be noted out that neither crisp nor fuzzy compositions are commutative in general so

$$\underset{\sim}{R} \circ \underset{\sim}{S} \neq \underset{\sim}{S} \circ \underset{\sim}{R}.$$

Different types of composition are (1) MAX-MIN (2) MAX –PRODUCT (3) MAX-MAX (4) MIN-MIN (5) MIN-MAX etc., Compositions provides more information which reduces the impreciseness present in the problem.

Fuzzification:

Process of converting a crisp value into fuzzy .Example if we have a variable TEMPERATURE = 35 ·C then this is converted into MAXTEMP, MINTEMP etc in the range of Zero to one by assigning membership functions.

Types of Fuzzification

- ✓ Inference
- ✓ Intuition
- ✓ Rank ordering
- ✓ Using GA
- ✓ Using ANN
- ✓ Inductive reasoning
- ✓ Meta rules
- ✓ Fuzzy statistics

These are some methods used to generate membership values and there by used to convert a crisp value into fuzzy.

FUZZY RULES

In a FLS, a rule base is constructed to control the output variable. A fuzzy rule based system consists of simple IF-THE with a condition and a conclusion. Sample fuzzy rule for an air conditioner system is given below.

IF Temp = Too Cold THEN Command is HEAT

Table 1 shows the matrix representation of the fuzzy rules for the above said FLS. Row contains the values that current room temperature can take, column is the values for target temperature, and each cell is the resulting command. For instance, if temperature is cold and target is warm then command is heat.

temperature/target	too-cold	cold	warm	hot	too-hot
too-cold	no-change	heat	heat	heat	heat
cold	cool	no-change	heat	heat	heat
warm	cool	cool	no-change	heat	heat
hot	cool	cool	cool	no-change	heat
too-hot	cool	cool	cool	cool	no-change

Table 1: Fuzzy matrix example

A fuzzy rule is defined as a conditional statement in the form:

IF *x is A*

THEN *y is B*

where x and y are linguistic variables; A and B are linguistic values determined by fuzzy sets on the universe of discourse X and Y, respectively.

In the field of artificial intelligence (machine intelligence), there are various ways to represent knowledge. Perhaps the most common way to represent human knowledge is to form it into natural language expressions of the type IF premise (antecedent), THEN conclusion (consequent). The form is commonly referred to as the IF–THEN rule-based form; this form is generally referred to as the deductive form.

It typically expresses an inference such that if we know a fact (premise, hypothesis, antecedent), then we can infer, or derive, another fact called a conclusion (consequent).This form of knowledge representation, characterized as shallow knowledge, is quite appropriate in the context of linguistics because it expresses human empirical and heuristic knowledge in our own language of communication.

It does not, however, capture the deeper forms of knowledge usually associated with intuition, structure, function, and behavior of the objects around us simply because these latter forms of knowledge are not readily reduced to linguistic phrases or representations; this deeper form, is referred to as inductive.

The fuzzy rule-based system is most useful in modeling some complex systems that can be observed by humans because they make use of linguistic variables as their antecedents and consequents; as described here these linguistic variables can be naturally represented by fuzzy sets and logical connectives of these sets.

DEFUZZIFICATION:

Defuzzification is the process of producing a quantifiable value. Fuzzy values can't be given to machines since they understand only two valued logic. Hence these linguistic values have to be converted into machine understandable two valued logic. Those techniques used to convert fuzzy into classical values are known as Defuzzification methods. It is the process of conversion of a fuzzy quantity into crisp quantity. Various Defuzzification methods are listed as:

- ✓ Centroid method
- ✓ Weighted average method
- ✓ Mean-max membership method
- ✓ Center of sums method
- ✓ Center of largest area method
- ✓ First(or last) of maxima method etc

Any of the above method can be used based on the level of intelligent control required.

QUESTIONS FOR PRACTICE

PART A

1. Differentiate between crisp and fuzzy logic
2. Define fuzzy set?
3. Define membership function?
4. Write about fuzzy relation?
5. What is fuzzification?
6. Define defuzzification
7. Write any two properties of fuzzy set?
8. What are Demorgan's laws?
9. What is the advantage of fuzzy logic?
10. What is FLC?

PART B

1. Discuss in detail about the fuzzy set theory?
2. Differentiate crisp set theory with fuzzy set using examples?
3. Discuss in detail about the Fuzzy logic controller?
4. Write in detail about the fuzzy relations and fuzzy rules?
5. Explain about fuzzy logic operation on fuzzy sets?