TRANSMISSION LINE THEORY

1.1 Introduction:

Electrical signal containing information can be transmitted from one point in the system to another point in the system physically separated by a certain distance through different media like

1)Free space: The propagation is from a source (radio transmitter) to a load (radio receiver) where wave propagation is carried out through free space. The separation between the source and the load could be very large e.g. thousands of kms as in cellular communication or small (few kms) as in microwave communication within a city. This type of wave propagation is called unguided wave propagation.

2)Transmission Lines (connecting wires): In some applications the information has to be conveyed from one point to the other through connecting wires

E.g.: (i) Telephone lines from a telephone subscriber to the telephone exchange or vice versa

ii) Connection between a radio transmitter and its antenna or a connection between an antenna and a radio receiver

This type of propagation is called guided wave propagation because the electromagnetic wave is guided between the wires. These wires are called transmission lines. The waves are guided between boundaries of the transmission lines.

Thus, a transmission line is a conductive method of guiding electrical energy from one place to another. It is employed not only to transmit energy, but also as circuit elements like inductors, capacitors, resonant circuits, filters, transformers and even insulators at very high frequencies. Also used as measuring devices and as an aid to obtain impedance matching.

The nature of transmission line and its performance depends upon

1. The amount of power to be transmitted and
2. The frequencies involved

Hence the transmission lines are categorized as

Power lines: used for transmission of large quantities of power over a fixed frequency

Communication lines: Used for transmission of small quantities of power over a band of frequencies

1.2 Types of transmission lines:

1.2.1 An open wire line (or) parallel wire type:

These lines are the parallel conductors open to air hence called open wire lines. The conductors are separated by air as the dielectric are rigidly supported by cross arms put at a certain height from ground by means of galvanized iron poles or any other structure.

E.g. The telephone lines and the electrical power transmission lines

Electrical energy propagating through these lines set up electric fields between line conductors. These fields are at right angles to each other and to the direction of propagation which is along the length of the line. This type of energy transmission is known as transverse electromagnetic mode of propagation (TEM).

Advantage: Less capacitance compared to underground cable

Disadvantage:
1. Initial cost is high due to the requirement of telephone posts and towers
2. Affected by atmospheric conditions like wind, air, ice…. maintenance is difficult.
3. Possibility of shorting due to flying objects and birds.

1.2.2 Coaxial type:

The radiation loss in an open two wire line is made negligible by employing a circular cylindrical conductor of smaller diameter placed coaxially inside another hollow conductor of larger diameter and of certain thickness. The resulting structure is called coaxial line. Here the inner tube is separated and insulated from the outer conductor by a dielectric medium which may be solid or gaseous.

The two conductors are at two different potentials. The fields re entirely confined to the space between the two conductors. No fields exist outside the outer conductor and similarly no external radiation can penetrate the outer tube and propagate inside.

Here the source is connected to the inner conductor of the coaxial line. At the other end load ZL is connected to the inner conductor. The other end of source, load and the outer conductor of the coaxial line are all connected to the ground, hence the voltage between the inner conductor and the ground are different. Therefore, coax is an unbalanced line.

The mode of wave propagation inside the coax is TEM mode. The electromagnetic wave is guided between the two tubes and the outer conductor acts as a shield to prevent leakage of signal from inside to outside and vice versa.

Coax are of three types

Flexible Coax: Use copper braided outer conductor, a thin center conductor and a low loss solid or foam polyethylene (PE) dielectric.

Semi-rigid cables: Have solid dielectric and use thin outer conductor so that it could be bent for convenience while laying cables.

Rigid cables: Have solid dielectric made of Teflon (PTFE). The space inside the tubes is essentially air with PTFE separating plugs at regular intervals.

The coax can be used up to 3 GHz for transmitting large signal powers. Beyond this frequency the transmission of electromagnetic waves along the coax becomes difficult due to losses that occur in the solid dielectric needed to support the conductors, losses in conductors due to skin effect.

1.2.3 Waveguide:

A transmission line consisting of a suitable shaped hollow conductor which may be filled with a dielectric material and is used to guide the EM waves of UHF propagated along its length, is called a waveguide.

The walls of the waveguide are made of brass, copper or aluminum. During the propagation, the waves suffer multiple reflections at the walls of the guide and the resulting distribution associated with the wave causes the transmission mode.

Since the electric and magnetic fields are confined to the space within the guides, there is no loss of power through radiation. Generally, the medium inside the waveguide is air there is no loss of power due to dielectrics. However, there is some power loss in the walls of the waveguide, but the loss is very small. The loss in a waveguide will be less than in coax.

Inside a waveguide, several modes of electromagnetic waves can propagate. The TEM mode does not exist, but either transverse electric (TE) or transverse magnetic (TM) modes can exist depending upon the mode of coupling the signal to the waveguide from the microwave source.

Advantages:

Higher power handling capability

A simpler mechanical structure which reduces the fabrication cost

Lower attenuation per unit length Disadvantage:

Larger cross-sectional dimensions and a lower usable bandwidth than in a coax

Again, the transmission line can be divided into two classes:

i) A balance transmission line is one where two signal wires are used to propagate electromagnetic waves relative to some fixed potential, usually assumed to be ground. E.g. A flat twin wire

ii) In an unbalanced line, one conductor forms the signal side, while the other is the ground

E.g. coax since the shield wire is always connected with a ground point

1.3 Constants of a transmission line:

Primary constants:

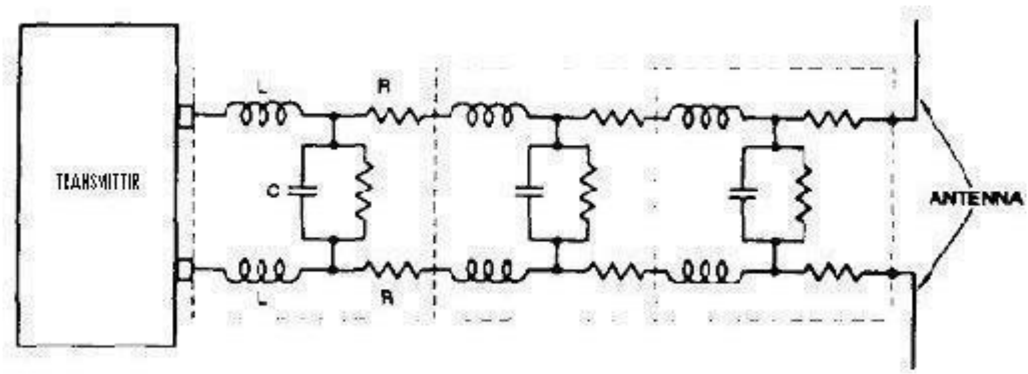

Fig:1 Transmission line equivalent circuit

A transmission line has the properties of inductance, capacitance, and resistance just as the more conventional circuits have. Usually, however, the constants in conventional circuits are lumped into a single device or component. For example, a coil of wire has the property of inductance. When a certain amount of inductance is needed in a circuit, a coil of the proper dimensions is inserted.

The inductance of the circuit is lumped into the one component. Two metal plates separated by a small space, can be used to supply the required capacitance for a circuit. In such a case, most of the capacitance of the circuit is lumped into this one component. Similarly, a fixed resistor can be used to supply a certain value of circuit resistance as a lumped sum. Ideally, a transmission line would also have its constants of inductance, capacitance, and resistance lumped together, as shown in figure. 1 unfortunately, this is not the case. Transmission line constants are as described in the following paragraphs.

Transmission line constants, called distributed constants, are spread along the entire length of the transmission line and cannot be distinguished separately. The amount of inductance, capacitance, and resistance depends on the length of the line, the size of the conducting wires, the spacing between the wires, and the dielectric (air or insulating medium) between the wires.

Resistance of a Transmission Line:

The transmission line shown has electrical resistance along its length depending upon its cross-sectional area. This resistance is usually expressed in ohms per unit length

Inductance of a Transmission Line:

When current flows through a wire, magnetic lines of force are set up around the wire. As the current increases and decreases in amplitude, the field around the wire expands and collapses accordingly. The energy produced by the magnetic lines of force collapsing back into the wire tends to keep the current flowing in the same direction. This represents a certain amount of inductance, which is expressed in micro henrys per unit length.

Capacitance of a Transmission Line:

Capacitance also exists between the transmission line wires. The two parallel wires act as plates of a capacitor and that the air between them acts as a dielectric. The capacitance between the wires is usually expressed in Pico farads per unit length. This electric field between the wires is similar to the field that exists between the two plates of a capacitor.

Secondary constants of a transmission line:

i)Characteristic impedance (Z_0):

If the line is infinitely long then the ratio $\frac{V}{I}$ will always produce a constant impedance referred as Characteristic impedance.
ii) Propagation constant (p):

It is the measure of the signal in terms of line attenuation per unit length and phase shift per unit length.

1.4 Transmission Line Equations:

A circuit with distributed parameter requires a method of analysis somewhat different from that employed in circuits of lumped constants. Since a voltage drop occurs across each series increment of a line, the voltage applied to each increment of shunt admittance is a variable and thus the shunted current is a variable along the line.

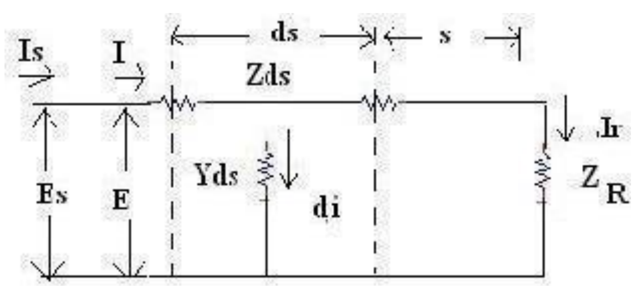

Fig:2 Transmission line with sending and receiving end

Hence the line current around the loop is not a constant, as is assumed in lumped constant circuits, but varies from point to point along the line. Differential circuit equations that describes that action will be written for the steady state, from which general circuit equation will be defined as follows.

R- series resistance, ohms per unit length of line (includes both wires) L= series inductance, henrys per unit length of line

C- capacitance between conductors, faradays per unit length of line

G- shunt leakage conductance between conductors, mhos per unit length of line

ωL-series reactance, ohms per unit length of line $Z = R+j\omega L$

ωC- series susceptance, mhos per unit length of line Y = G+jωC

S -distance to the point of observation, measured from the receiving end of the line

I -Current in the line at any point

E-V-voltage between conductors at any poin

l - length

Variation in voltage is given by $-\frac{dV}{dx} = I(R + j\omega L)$ (1.1)

Variation in current is given by $-\frac{dI}{dx} = V(G + j\omega C)$ (1.2)

The solution to the above equation is given b

$V(x) = ae^{-px} + be^{px}$ (1.3)

$I(x) = ce^{-px} + de^{px}$ (1.4)

The above equations suggest that the line will contain two waves, one travelling in the positive ‘x’ direction (e^(-px)). These are called incident waves which decay exponentially. The other term represents a wave propagating in negative ‘x’ direction. These are known as reflected wave.

An alternative solution to hyperbolic function is given by

$e^{px} = coshpx + sinhpx$ (1.5)

$e^{-px} = coshpx - sinhpx$ (1.6)

The solution of the above equation is given by

$V = a(coshpx + sinhpx) + b(coshpx - sinhpx) = (a + b)coshpx + (a - b)sinhpx$

$V = Acoshpx + Bsinhpx$ (1.7)

Similarly

$I = c\,[\,coshpx + sinhpx\,] + d\,[\,coshpx - sinhpx\,] = (c + d)coshpx + (c - d)sinhpx$

$I = Ccoshpx + Dsinhpx$ (1.8)

The four constants A, B, C, and D can be simplified to two constants A & B by

$-\frac{d}{dx}[Acoshpx + Bsinhpx] = I(R + j\omega L)$ (1.9)

$= \sqrt{\frac{(G+j\omega C)}{(R+j\omega L)}}[Acoshpx + Bsinhpx]$

$I = -\frac{1}{Z_0}[Acoshpx + Bsinhpx]$ (1.10)

By applying boundary conditions at sending end $V=V_s$, $I=I_s$ and x=0

We get V_s=A and B=-I_sZ_0

Therefore $V = V_s coshpx - I_s Z_0 sinhpx$ (1.11)

$I = I_s coshpx - \frac{V_s}{Z_0} sinhpx$ (1.12)

1.5 Infinite Line:

Long lines are called as infinite line. A signal fed into a line of infinite length could not reach the far end in a finite time. If a line of finite length is considered, then all power fed into it will be absorbed. As we move away from the input terminals towards load, the current and voltage become zero at an infinite distance. Hence the transmission line analysis begins with an infinite time in order to separate input conditions from output conditions.

An AC voltage is applied at a distance x from the sending end point of the infinite line. Current at any point at a distance x from the sending end is

At sending end, x=0, I=I_{si} , I_{si}= c+d

At the receiving end, x=∞ I=0, c=0

In this either c=0 or ∞=0, but ∞ cannot be equal to zero, so only possibility is c=0 and d= Isi

Therefore I= I_{si} (1.13)

At the receiving end, x=∞, V=0 a=0, since e∞ ≠ 0 and b= V_{si}

Therefore V= V_{si} (1.14)

Equations (1.13) & (1.14) are infinite line equations

1.6 Other units of a transmission line:

WaveIength:

The distance the wave travels along the line while the phase angle is changed through 2π radians is called wavelength.

λ =2π/ ß (1.15)

The change of 2π in phase angle represents one cycle in time and occurs in a distance of one wavelength

λ= v/f (1.16)

Velocity:

V_P= f λ (1.17)

This is the velocity of propagation along the line based on the observation of the change in the phase angle along the line. It is measured in miles/second if ß is in radians per meter.

Group Velocity:

If the transmission line is such that different frequencies travel with different velocities, then the line or the medium is said to be dispersive. In that cases, signals are propagated with a velocity known as group velocity. It is less than phase velocity.

1.7 Distortions:

Wave-form Distortion:

In general, α is a function of frequency. All the frequencies transmitted on a line will then not be attenuated equally. A complex applied voltage, such as voice voltage containing many frequencies, will not have all frequencies transmitted with equal attenuation, and the received for will be identical with the input waveform at the sending end. This variation is known as frequency distortion.

Phase Distortion:

It is apparent that ω and β do not both involve frequency in same manner and that the velocity of propagation will in general be some function of frequency.

All the frequencies applied to a transmission line will not have the same time of transmission, some frequencies delayed more than the others. For an applied voice voltage waves the received waves will not be identical with the input wave form at the receiving end, since some components will be delayed more than those of the other frequencies. This phenomenon is known as delay or phase distortion.

Frequency distortion is reduced in the transmission of high-quality radio broadcast over wire line by use of equalizers at line terminals. These circuits are networks whose frequency and phase characteristics are adjusted to be inverse to those of the lines, resulting in an over all uniform frequency response over the desired frequency band.

Delay distortion is relatively minor importance to voice and music transmission because of the characteristics of ear. It can be very series in circuits intended for picture transmission, and applications of the co axial cable have been made to overcome the difficulty.

In such cables the internal inductance is low at high frequencies because of skin effect, the resistance is small because of the large conductors, and capacitance is small because of the use of air dielectric with a minimum spacer.

1.8 DISTORTIONLESS LINE:

If a transmission line is free from frequency distortion and phase distortion then a line is said to be lo less or distortion less line. In order to avoid distortions in the signal transmission, the attenuation constant α, the phase velocity Vp must be made independent of frequency and the phase shift constant β must be with multiples of angular frequency.

Conditions for a distortion less line:

The distortion less condition can be attained in two methods

i) by making use of propagation constant calculations

ii) From the expressions of attenuation constant and phase shift constant

Condition for minimum attenuation:

To calculate the value of L, the other line parameter R, C, G & ω are considered as constant, only L may be varied, solving and reducing we ge

$$\frac{R}{L} = \frac{G}{C} \quad (1.18)$$

If only L is variable, the attenuation is minimum when

$$L = \frac{RC}{G} \text{ H/Km} \quad (1.19)$$

In practice L is normally less than the desired value

$$C = \frac{LG}{R} \text{ F/Km} \quad (1.20)$$

When R=0 and G=0, the attenuation constant is zero.

1.9 Telephone cable:

Telephone cable find wide application in the field of communication. Hence it is desirable to investigate their behavior as a special case of transmission line. Telephone cable is formed by two wires, insulated from each other by a layer of oil impregnated paper and then twisted in pairs.

Structural view of a Telephone wire

Fig:3 Structure of telephone cable

A large number of such pairs are combined inside a protective lead or plastic sheath to form one underground cable. such transmission lines are called as telephone cable.

Sectional view of a telephone cable

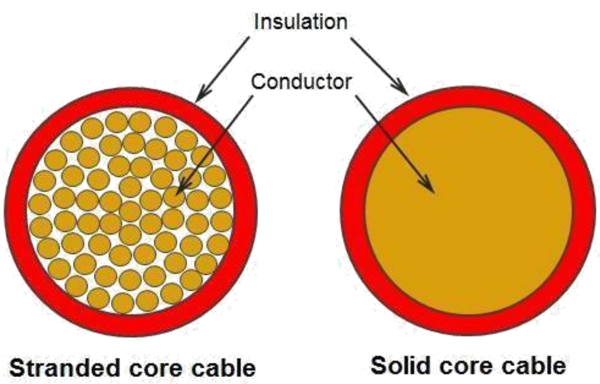

Fig:4: Sectional view of telephone cable

Small circle representing a cross sectional view of a twisted pair.The telephone cables are primarily designed to transmit low voltages such a construction leads to abnormally low inductance (negligible inductance) &abnormally low conductance (negligible conductance) at audio frequencies (because of the proximity of the conductors and the presence of solid insulation.

Thus, the reasonable assumptions, over the range of frequencies used in telephonic communication are $\omega L << R$ & $G << \omega C$

ω-Angular frequency

The propagation constant of a loaded line is different from the already existing transmission line.The new propagation constant of a loaded line is identified with campbell's formula. This is done by considering a

1.General transmission line analysis(or)

2. By adapting the notation of filter circuits.

1.10 Campbell's Equation

An analysis for the performance of a line loaded at uniform intervals can be obtained by considering a symmetrical section of line from the center of one loading coil to the center of the next, where the loading coil of the inductance is Z_c.

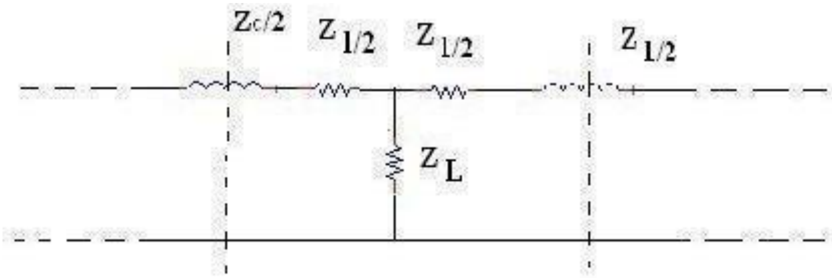

Fig:5 T-network equivalent circuit

The section line may be replaced with an equivalent T section having symmetrical series arms. Adopting the notation of filter circuits one of these series arms is called $Z_1/2$ and

$$\frac{Z_1}{2} = \frac{Z_0}{Sin\ h\gamma d[cosh\gamma d - 1]} \tag{1.21}$$

An equation relating that γ and the circuit element of a T section was already derived, which may be applied to the loaded T section as so that the above equation reduces to

$$\cos h\gamma' d = Z_c \frac{\sin h\gamma d}{2Z_0} + \cos h\,\gamma d \tag{1.22}$$

This expression is known as CampbeII's Equation and permits the determination value for γ of a line section consisting partially of lumped distributed elements. Campbell's equation makes possible the calculation of the effects of loading coils in reducing attenuation and distortion on lines.

1.11 General equation for a line with any termination

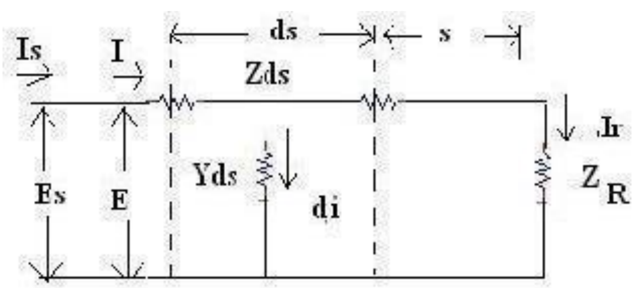

Fig:6 Transmission line

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L- series inductance, henrys per unit length of line

C- capacitance between conductors, faradays per unit length of line G= shunt leakage conductance between conductors, mhos per unit length Of line

ωL - series reactance, ohms per unit length of line Z = R+jωL

ωC -series susceptance, mhos per unit length of line Y = G+jωC

S -distance to the point of observation, measured from the receiving end of the line I = Current in the line at any point

E- voltage between conductors at any point l = length of line

E = a + + b –

E_R = A+B

I_R = C+D

By solving we get,

$$V_s = V_R coshpx + I_R Z_0 sinhpx \quad (1.23)$$

$$I_s = I_R coshpx + \frac{V_R}{Z_0} sinhpx \quad (1.24)$$

1.12 Reflection:

When the transmission line is not correctly terminated, the travelling electromagnetic wave from generator at the sending end is reflected completely or partially at the termination. The phenomenon of setting up of a reflected wave at the load due to improper termination or due to impedance irregularity in a line called reflection.

Reflection Coefficient:

The ratio of amplitudes of reflected to incident voltage components at the receiving end of a line is called the reflection coefficient by K.

$$k = \frac{Z_R - Z_0}{Z_R + Z_0} \quad (1.25)$$

1.14 Input Impedance:

The ratio of the voltage applied to the current flowing will give the input impedance of the transmission line. It is also known as characteristic impedance of the line for infinite line.

Input impedance of open circuited and short-circuited line:

As limited cases it is convenient to consider lines terminated in open circuit or short circuit, that is with $Z_R = \infty$ or $Z_R = 0$. The input impedance of a line of length l.

The open circuited condition is given by,

$Z_{oc} = \frac{V_s}{I_s} = Z_o Cothpl$ (1.26)

Short circuited condition is given by,

$Z_{oc} = \frac{V_s}{I_s} = Z_o tanhpl$ (1.27)

By multiplying the above two equations it can be seen that

$Z_0 = \sqrt{Z_{oc}} Z_{sc}$ (1.28)

PARAMETERS OF OPENWIRE LINE AT RADIO FREQUENCY:

2.1Approximations of a transmission line used at radio frequencies:

(i) At radio frequencies the line has considerable skin effect, so that almost the entire current may be assumed to flow through the outer surface of the conductor. Thus the internal inductance of the wires may be considered to be zero. $L_i = 0$

(ii) At VHF, the inductive reactance is comparatively large with the series resistance R.

(ii) The lines are well constructed, so that shunt conductance G may be considered to be zero at radio frequencies. G = 0

The analysis is made in either of two ways, depending on whether R is merely small with respect to ω L or R is small, the line is considered completely negligible compared with ω L.

If R is small, the line is considered one of small dissipation, and this concept is useful when lines are employed as circuit elements or where resonance properties are involved. If losses were neglected then infinite current or voltages would appear in calculations, and physical reality would not be achieved.

In applications where losses may be neglected, as in transmission of power at high efficiency, R may be considered as negligible, and the line as one of zero dissipation. These methods will be studied separately.

From the above approximations, the line parameters of open wire line at radiofrequency:

(i) Loop inductance for open wire line: Consider an open wire line having two circular conductors parallel to each other.
Let a-be radius of the conductor and d be the spacing between the two conductors. In general, the self inductance of the parallel wire line system is

$$L = \left(\mu_r + 9.21 \log\frac{d}{a}\right) 10^{-7} \quad \text{H/m} \tag{2.1}$$

Where μ_r is the relative permeability of the conducting material for non-magnetic material
But at high frequency due to skin effect, the internal inductance of the open wire line is $L_i=0$

Hence the self inductance at RF is $L = \left(9.21 \log\frac{d}{a}\right) 10^{-7}$ H/m (2.2)

(ii) Shunt capacitance:

The value of capacitance of a line is not affected by a skin effect or frequency, hence given as $C = \frac{\pi\varepsilon d}{ln\frac{d}{a}}$ (2.3)

(iii) Loop resistance:

At radio frequency due to appreciable skin effect, the current flows over the surface of the conductor in a thin layer with a resultant reduction in effective cross section area or an increase in resistance of the conductor

$$R_{ac} = \frac{R_{dc}}{2\pi f \mu} \tag{2.4}$$

(iv) Shunt conductance: since the lines are well constructed that is filled with solid dielectric between the conductors there is no shunt conductance. G=0

2.2 Parameters of coaxial lines at RF:

At radio frequency due to skin effect, the current flows on the outer surface of the inner conductor and the inner surface of the outer conductor, which eliminates flux linkages due to internal conductor flux and the

i)Loop inductance: For the co-axial line

$$L = \frac{\mu_o}{2\pi} \log \frac{b}{a} \quad H/m \tag{2.5}$$

Where a - radius of inner solid conductor
b-inner surface radius of the outer hollow tubular conductor
c-the outer surface radius
μ_0- relative permeability of the dielectric material
μ_o- absolute permeability

(ii)Shunt capacitance: The capacitance of a co-axial line is not affected by frequency, except the relative permittivity of the dielectric, so that

$$C = \frac{2\pi\varepsilon d}{ln\frac{b}{a}} \quad \text{F/m} \tag{2.6}$$

(iii)Shunt conductance: The shunt losses of air dielectric lines are zero, but many co axial lines employ solid dielectric materials and the conducting losses are measured in terms of the power factor of the material or in terms of dissipation factor. For a good dielectric with small power factor angles the approximations is $G << \omega C$, which makes shunt conductance, G=0.

Constants for lines of zero dissipation:

(i)Internal inductance, due to skin effect $L_i = 0$
(ii)The inductive reactance is comparatively large with loop resistance
(iii)Shunt conductance G = 0

Low dissipation line: These lines are used where resonance properties are involved. Example transmission line acts as simple resistor, inductance, capacitor.

If the loop resistance R is negligible, then the line is termed as zero dissipation lines or lossless lines. Example: transmission of power at high efficiency is done through zero dissipation lines only. That is the transmission line which is used to transfer the power signal between the power amplifier and the antenna section at transmit end.

2.3 Representation of a radio frequency lines:

(i)The primary line constants are series loop inductance 'L' and shunt capacitance 'C'.

(ii)Secondary line constants: for low dissipation lines

(a)Characteristic impedance:

(b)Propagation constant

2.4 Voltages and currents on a dissipation less line:

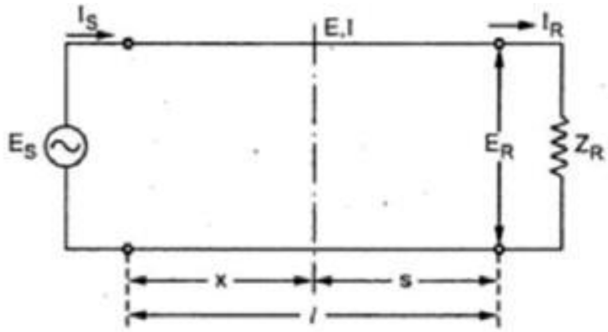


Fig:1 Transmission line of length l

The voltage at any distance s is measured from the receiving end of a line, terminated by the impedance Z_R, is given by E_s.

After grouping and simplifying, the voltage on a line is represented as,

$E_s = E_R\cos\beta s + jI_R R_0 \sin\beta s$ (2.7)

Similarly,

The current on the line is given by,

$I_s = I_R\cos\beta s + jE_R/R_0 \sin\beta s$ (2.8)

2.5 Standing waves:

When the transmission line is not matched with its load i.e., load impedance is not equal to the characteristic impedance ($Z_R = Z_0$), the energy delivered to the load is reflected back to the source. The combination of incident and reflected waves give rise to the standing waves.

The measurement of standing waves on a transmission line yields information about equipment operating conditions. Maximum power is absorbed by the load when $Z_L = Z_0$. If a line has no standing waves, the termination for that line is correct and maximum power transfer takes place.

Voltage Standing-Wave Ratio:

$$VSWR = \frac{V_{max}}{V_{min}} \quad (2.9)$$

The ratio of maximum voltage to minimum voltage on a line is called the voltage standing-wave ratio (VSWR). Therefore: The vertical lines in the formula indicate that the enclosed quantities are absolute and that the two values are taken without regard to polarity, Depending on the nature of the standing waves, the numerical value of VSWR ranges from a value of 1 ($Z_L = Z_0$, no standing waves) to an infinite value for theoretically complete reflection.

Since there is always a small loss on a line, the minimum voltage is never zero and the VSWR is always some finite value. However, if the VSWR is to be a useful quantity the power losses along the line must be small in comparison to the transmitted power voltage. Since power is proportional to the square of the voltage, the ratio of the square of the maximum and minimum voltages is called the power standing- wave ratio. In a sense, the name is misleading because the power along a transmission line does not vary.

Current Standing-Wave Ratio:

The ratio of maximum to minimum current along a transmission line is called current standing- wave ratio (ISWR). Therefore: This ratio is the same as that for voltages. It can be used where measurements are made with loops that sample the magnetic field along a line. It gives the same results as VSWR measurements.

Reflection coefficient:

Reflection coefficient is the ratio of the reflected wave amplitude and incident wave amplitude at the receiving end of the line. It is denoted as K, which is having magnitude and phase values.

Voltage reflection coefficient:

$$K_{VRC} = \frac{Z_R - Z_0}{Z_R + Z_0} \quad (2.10)$$

Current reflection coefficient:

$$K_{CRC} = \frac{Z_R - Z_0}{Z_R + Z_0} \quad (2.11)$$

Reflection loss:

If the line is not terminated by its characteristic impedance, then reflection occurs which in turn increases the power loss, which is termed as reflection loss.

$$F_l = 20 \frac{\log_{10} \sqrt{Z_0 Z_R}}{\left(\frac{Z_0 + Z_R}{2}\right)} \quad (2.12)$$

F_r is representing a reflection factor which defines a level of mismatch between two impedances.

F_r in terms of K is given by,

$$F_r = \frac{2\sqrt{Z_0 Z_R}}{Z_0 + Z_R} \quad (2.13)$$

Relation between SWR and reflection coefficient:

$$S = \frac{1+K}{1-K} \quad (2.14)$$

Reflection coefficient in terms of SWR is given by

$$K = \frac{S-1}{S+1} \tag{2.15}$$

2.6 Practical types of Transmission line:

1.Coaxial cable:

Coaxial lines confine the electromagnetic wave to the area inside the cable, between the center conductor and the shield. The transmission of energy in the line occurs totally through the dielectric inside the cable between the conductors. Coaxial lines can therefore be bent and twisted (subject to limits) without negative effects, and they can be strapped to conductive supports without inducing unwanted currents in them. In radio-frequency applications up to a few gigahertz, the wave propagates in the transverse electric and magnetic mode (TEM) only, which means that the electric and magnetic fields are both perpendicular to the direction of propagation (the electric field is radial, and the magnetic field is circumferential). However, at frequencies for which the wavelength (in the dielectric) is significantly shorter than the circumference of the cable, transverse electric (TE) and transverse magnetic (TM) waveguide modes can also propagate. The most common use for coaxial cables is for television and other signals with bandwidth of multiple megahertz. In the middle 20th century they carried long distance telephone connections.

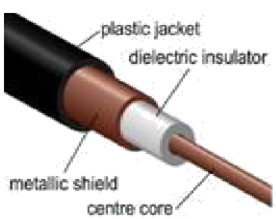


Fig:2 Co-axial able

2.Microstrip line:

A microstrip circuit uses a thin flat conductor which is parallel to a ground plane. Microstrip can be made by having a strip of copper on one side of a printed circuit board (PCB) or ceramic substrate while the other side is a continuous ground plane. The width of the strip, the thickness of the insulating layer (PCB or ceramic) and the dielectric constant of the insulating layer determine the characteristic impedance. Microstrip is an open structure whereas coaxial cable is a closed structure.

3.Stripline :

A strip line circuit uses a flat strip of metal which is sandwiched between two parallel ground planes. The insulating material of the substrate forms a dielectric. The width of the strip, the thickness of the substrate and the relative permittivity of the substrate determine the characteristic impedance of the strip which is a transmission line.

Stripline is a conductor sandwiched by dielectric between a pair of ground planes.In practice, "classic" stripline is usually made by etching circuitry on a substrate that has a groundplane on the opposite face, then adhesively attaching a second substrate (which is metalized on only one surface) on top to achieve the second ground plane. Stripline is most often a "soft-board" technology, but using low-temperature co-fired ceramics (LTCC), ceramic stripline circuits are also possible.

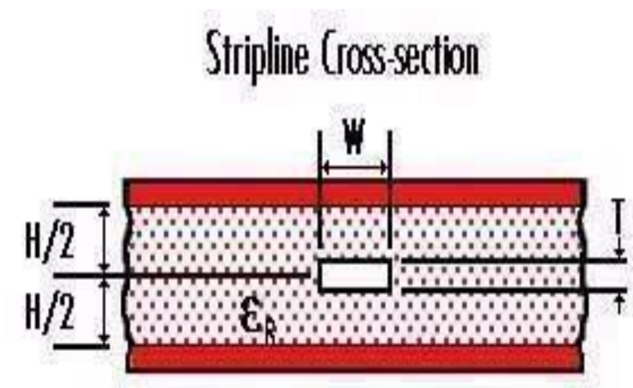


Fig:3 cross sectional view of stripline

Other variants of the stripline are offset strip line and suspended air stripline (SAS).

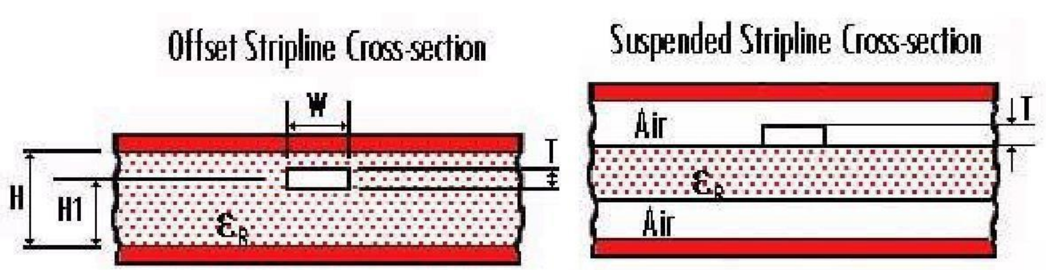


Fig:4: Off set and suspended stripline

For stripline and offset stripline, because all of the fields are constrained to the same dielectric, the effective dielectric constant is equal to the relative dielectric constant of the chosen dielectric material. For suspended stripline, you will have to calculate the effective dielectric constant, but if it is "mostly air", the effective dielectric constant will be close to 1.

Advantages and disadvantages of stripline :

Stripline filters and couplers always offer better bandwidth than their counterparts in microstrip, and the roll off of stripline BPFs can be quite symmetric (unlike microstrip). Stripline has no lower cut-off frequency (like waveguide does).

Another advantage of stripline is that fantastic isolation between adjacent traces can be achieved (as opposed to microstrip). The best isolation results when a picket fence of vias surrounds each transmission line, spaced at less than 1/4 wavelength. Stripline can be used to route RF signals across each other quite easily when offset stripline is used.

Disadvantages of stripline are two: first, it is much harder (and more expensive) to fabricate than microstrip, some old guys would even say it's a lost art. Lumped element and active components either have to be buried between the ground planes or transitions to microstrip must be employed as needed to get the components onto the top of the board.

The second disadvantage of stripline is that because of the second ground plane, the strip widths are much narrower for a given impedance (such as 50 ohms) and board thickness than for microstrip

3. Balanced lines:

A balanced line is a transmission line consisting of two conductors of the same type, and equal impedance to ground and other circuits. There are many formats of balanced lines, amongst the most common are twisted pair, star quad and twin-lead.

4. Twisted pair:

Twisted pairs are commonly used for terrestrial telephone communications. In such cables, many pairs are grouped together in a single cable, from two to several thousand. The format is also used for data network distribution inside buildings, but in this case the cable used is more expensive with much tighter controlled parameters and either two or four pairs per cable.

5. Single-wire line:

Unbalanced lines were formerly much used for telegraph transmission, but this form of communication has now fallen into disuse. Cables are similar to twisted pair in that many cores are bundled into the same cable but only one conductor is provided per circuit and there is no twisting. All the circuits on the same route use a common path for the return current (earth return). There is a power transmission version of single-wire earth return in use in many locations.

6.Waveguide:

Waveguides are rectangular or circular metallic tubes inside which an electromagnetic wave is propagated and is confined by the tube. Waveguides are not capable of transmitting the transverse electromagnetic mode found in copper lines and must use some other mode. Consequently, they cannot be directly connected to cable and a mechanism for launching the waveguide mode must be provided at the interface.

6.Microwave transmission line:

Microwave transmission is the transmission of information or energy by electromagnetic waves whose wavelengths are measured in small numbers of centimetre; these are called microwaves.This part of the radio spectrum ranges across frequencies of roughly 1.0 gigahertz (GHz) to 30 GHz. These correspond to wavelengths from 30 centimeters down to 0.1 cm.

Microwaves are widely used for point-to-point communications because their small wavelength allows conveniently-sized antennas to direct them in narrow beams, which can be pointed directly at the receiving antenna. This allows nearby microwave equipment to use the same frequencies without interfering with each other, as lower frequency radio waves do. Another advantage is that the high frequency of microwaves gives the microwave band a very large information-carrying capacity; the microwave band has a bandwidth 30 times that of all the rest of the radio spectrum below it. A disadvantage is that microwaves are limited to line of sight propagation; they cannot pass around hills or mountains as lower frequency radio waves can.

Microwave radio transmission is commonly used in point-to-point Communication systems on the surface of the Earth, in satellite communications, and in deep space radio communications. Other parts of the microwave radio band are used for radars, radio navigation systems, sensor systems, and radio astronomy.

The next higher part of the radio electromagnetic spectrum, where the frequencies are above 30 GHz and below 100 GHz, are called "millimeter waves" because their wavelengths are conveniently measured in millimeters, and their wavelengths range from 10 mm down to 3.0 mm Radio waves in this band are usually strongly attenuated by the Earthly atmosphere and particles contained in it, especially during wet weather. Also, in wide band of frequencies around 60 GHz, the radio waves are strongly attenuated by molecular oxygen in the atmosphere. The electronic technologies needed in the millimeter wave band are also much more difficult to utilize than those of the microwave band

7. Super conducting transmission line:

The obvious advantage of superconducting transmission lines is they have no resistive losses in the bulk. If superconducting transmission lines had no other sources of power dissipation, the choice between types of transmission lines would be easy. We would simply calculate the cost of conventional power lines and subtract the cost of the power that is dissipated in transporting the electricity.

Real superconducting cables have other sources of loss which must also be factored in. There are a number of major sources of losses in superconducting transmission lines, many of them fundamentally different from those in conventional transmission lines. There are a number of relatively small losses due to need to cool the line. No cooling system is perfectly efficient, so there is some loss of liquid nitrogen needed to cool the line. Furthermore, there are losses due to the imperfect efficiency of the liquid nitrogen pumping system itself, as well as hydraulic losses due to the flow friction in the circulating liquid nitrogen.

Similar to conventional transmission lines, superconducting transmission lines also have shield and dielectric losses, which can be calculated using the same physical models. Unlike conventional lines, superconducting transmission lines have conductor AC losses. There is no generally accepted physical model to describe these losses, so much of the data is empirical. There are also losses due to imperfect thermal insulation of the superconducting cable. The result is a thermal leak between the cold liquid nitrogen and the warm surroundings. The losses can be reduced but not eliminated by creating a vacuum between the superconducting cable and the thermal insulator. Finally, there are small losses due to joints and terminations of cables.

A High Temperature Superconducting (HTS) power cable is a wire-based device that carries large amounts of electrical current. There are two types of HTS cables.

Warm Dielectric Cable:

The warm dielectric cable configuration features a conductor made from HTS wires wound around a flexible hollow core (figure 1). Liquid nitrogen flows through the core, cooling the HTS wire to the zero resistance state. The conductor is surrounded by conventional dielectric insulation. The efficiency of this design reduces losses.

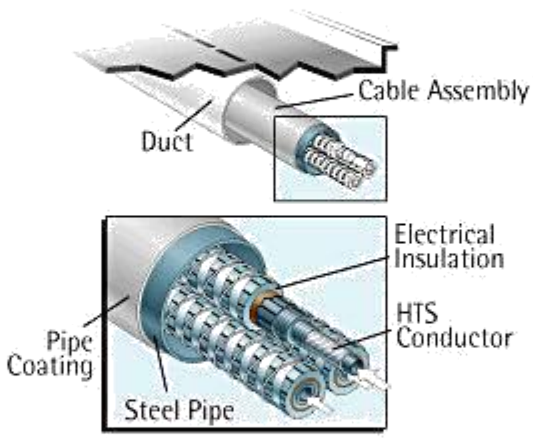


Fig:5 Construction of a warm dielectric HTS cable.

Cryogenic Dielectric Cable :

The cryogenic dielectric is a coaxial configuration comprising an HTS conductor cooled by liquid nitrogen flowing through a flexible hollow core and an HTS return conductor, cooled by circulating liquid nitrogen. This represents an enhancement to the warm dielectric design, providing even greater ampacity, further reducing losses and entirely eliminating the need for dielectric fluids.

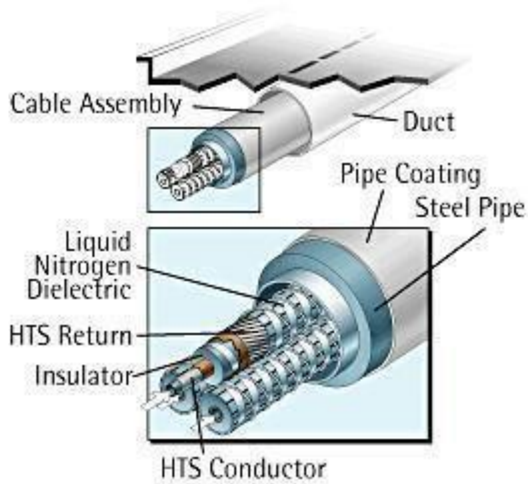


Fig:6 Construction of a cryogenic dielectric HTS cable.

HTS transmission cables would be used for power transmission and distribution in urban areas throughout the United States and the world.

Advantages:

- Can meet increasing power demands in urban areas via retrofit applications carrying two to five times more power than conventional cable.
- Eliminates need to acquire new rights of way.
- Replaces overhead transmission lines when environmental and other concerns prohibit their installation
- Enhanced overall system efficiency due to exceptionally low losses
- Increased utility system operating flexibility
- Reduced electricity costs

With an estimated 80,000 miles of existing underground cable throughout the world, High Temperature Superconducting (HTS) cables will provide enormous benefits to a utility industry that is poised for growth and is faced with an ever-rising demand for electricity and tightening constraints on siting flexibility.

Conventional underground power transmission cables are utilised to transmit large amounts of power to congested urban areas. Conventional (copper-based) cables are capable of transmitting power (40 to 600 MVA) at high voltages (40 to 345 kV) through integrated underground duct systems. Existing duct systems limit the size of the conventional cables and the amount of power that can be transmitted through them.

MATCHING, MEASUREMENTS AND INTERFERENCE

3.1Types of Transmission Line sections:

The Transmission line is characterised in terms of its length in wavelength. These are

(i) The one eighth wavelength line

(ii) The Half wave line

(iii) The quarter wave line

(i) The one-eighth wavelength line:

The length of line is one-eight wavelength long, the input impedance of such a line with $l = \lambda/8$ is

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}\right] \quad (3.1)$$

Here $L=\lambda/8$; $\beta=2\Pi/\lambda$ after substitution

$$|Z_{in}(\lambda/8)| = R_0 \quad (3.2)$$

(ii) The quarter wave line:

The input impedance of a quarter line is Zin ($\lambda/4$), is

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}\right] \quad (3.3)$$

Here $Z_0=R_0$, $\beta=2\Pi/\lambda$, $l=\lambda/4$, solving we get

$$Z_{in} = \frac{R_o^2}{Z_R} \quad (3.4)$$

(iii) The Half Wave line:

$$Z_{in}(\lambda/2) = Z_R \quad (3.5)$$

3.2 Impedance Matching:

A transmission line is acting as a connecting links between a transmitter and an antenna or between an antenna and a receiver, which affects the efficiency of power transfer .According to maximum power transfer theorem, when the impedance of one is the complex conjugate of the other, the maximum power is absorbed by the load. Different types of impedance matching are (i)stub matching

(ii)tapered section

(iii)quarter wave transformer matching

The Smith Chart diagram is a

(i)polar impedance diagram

(ii)It consists of 2 sets of circles

(a) Constant resistance circles

(b) Constant reactance Circles

Applications:

1.This chart is applicable to the analysis of lossless line as well as lossy line.

2.It can be used as impedance and admittance diagram

3.Determination of input impedance

4.Conversion of impedance to admittance.

5.To measure a standing wave pattern directly

6.From this, the magnitudes of the reflection coefficient, reflected power, transmitted power and the load impedance can be calculated from it.

Characteristics of the smith chart:

1.The constant K and constant X loci forms, two families of orthogonal circles in the chart.

2.The upper half of the diagram represents +jX

3.The lower half of the diagram represents –jX

4.For admittance the constant r circles become constant g circle and the constant x circles become constant susceptance b circles.

5.The distance around the smith chart once is one-half wavelength($\lambda/2$).

6.At a point of $Z_{min}=1/\rho$, there is a V_{min} on the line

7.At a point of $Z_{max}=\rho$, there is a V_{max} on the line

8.The horizontal radius to the right of the chart center corresponds to V_{max}, I_{min}, Z_{max} and ρ(SWR)

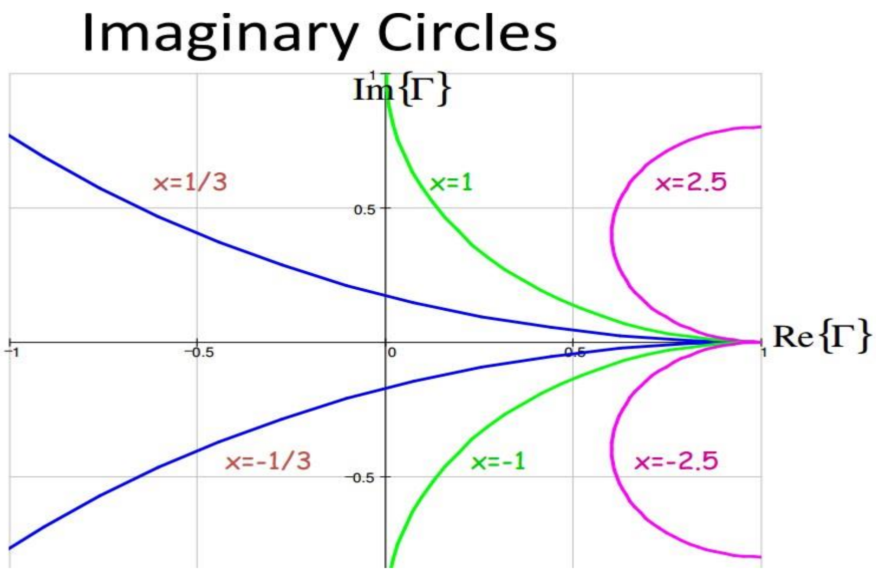
9. The horizontal radius to the left of the chart center corresponds to V_{min}, I_{max}, Z_{min} and

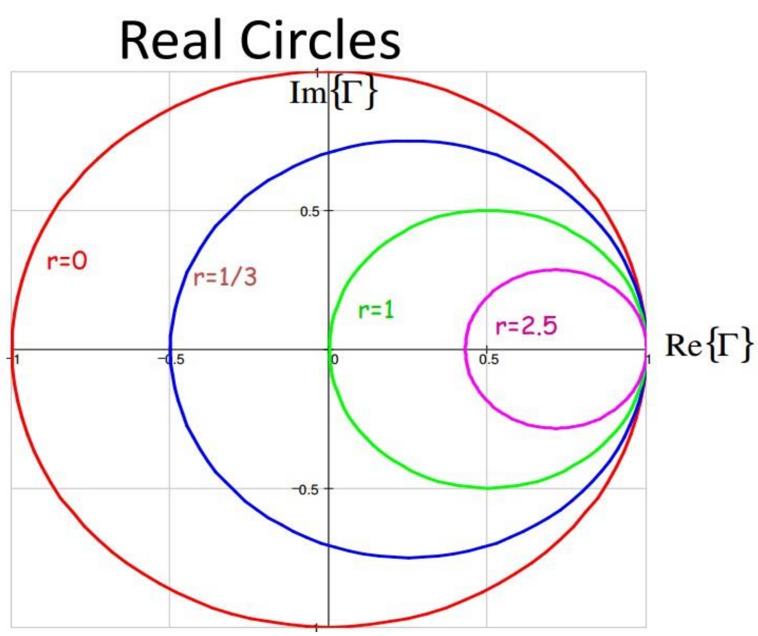
$1/\rho$.

10.Since the Normalized admittance Y is a reciprocal of the normalized impedance, the corresponding quantities in the admittance chart are 180 degree out of phase with those in the impedance chart.

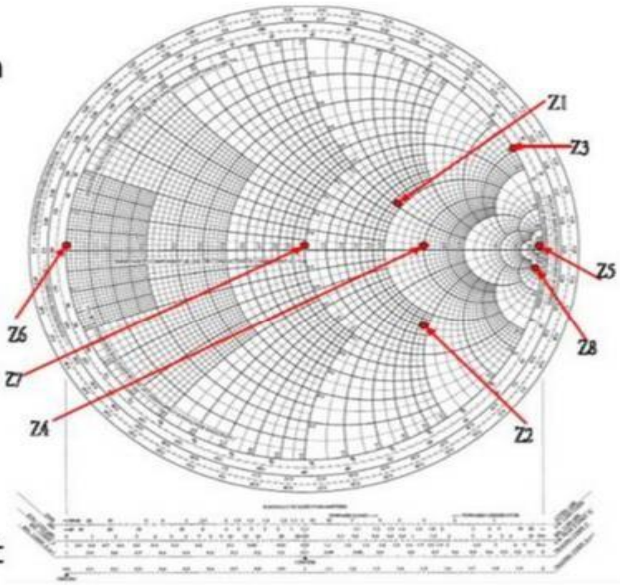
11.The normalised impedance or admittance is repeated for every half wavelength of distance.

12.The distances are given in wavelength toward the generator and also toward the load.


Imaginary Circles
Im{Γ}
Re{Γ}
x=1/3
x=1
x=2.5
x=-1/3
x=-1
x=-2.5
0.5
-0.5
-1
0
1


Real Circles
Im{Γ}
Re{Γ}
r=0
r=1/3
r=1
r=2.5
0.5
-0.5
-1
0
1

- Impedance divided by lin impedance (50 Ohms)
  - Z1 = 100 + j50
  - Z2 = 75 -j100
  - Z3 = j200
  - Z4 = 150
  - Z5 = infinity (an open circuit)
  - Z6 = 0 (a short circuit)
  - Z7 = 50
  - Z8 = 184 -j900



- Then, normalize and plot The points are plotted as follows:
  - z1 = 2 + j
  - z2 = 1.5 -j2
  - z3 = j4
  - z4 = 3
  - z5 = infinity
  - z6 = 0
  - z7 = 1
  - z8 = 3.68 -j18S

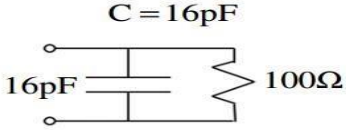
Matching Example

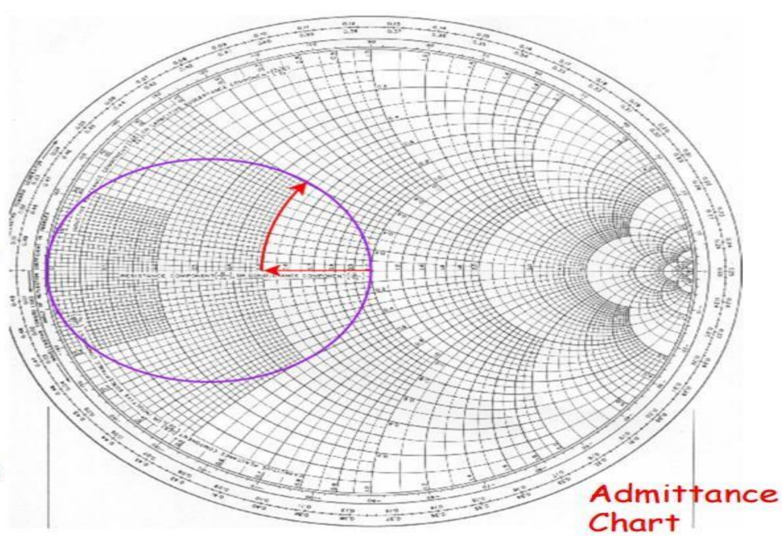
- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance jb = j0.5

$$jb = j0.5$$

$$\frac{j0.5}{50\Omega} = j2\pi(100\text{MHz})C$$

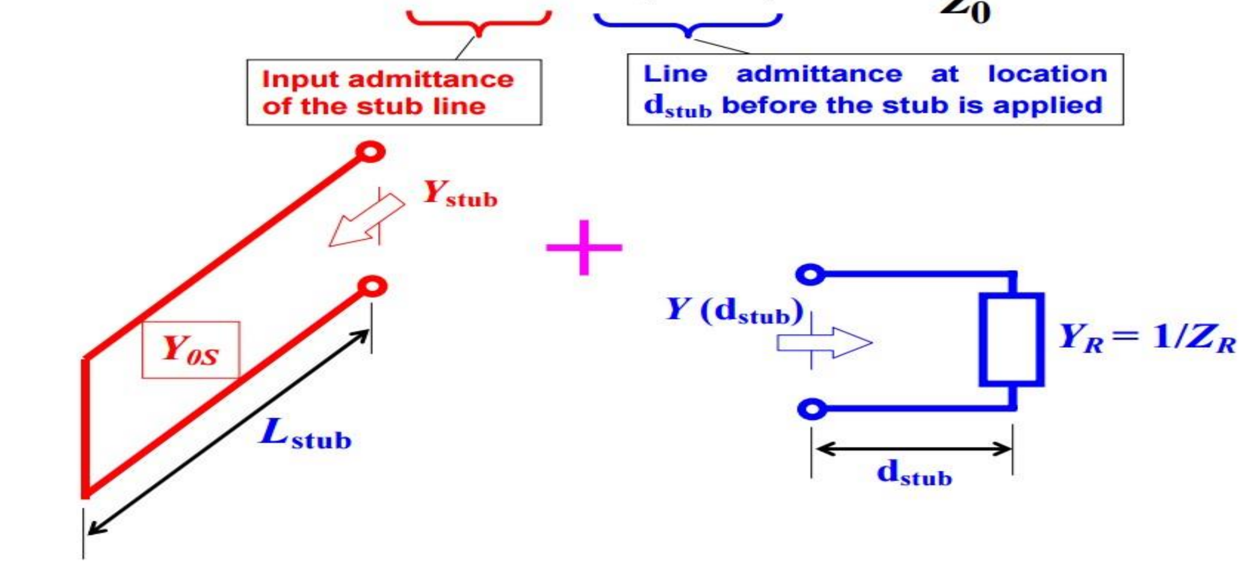
$$C = 16\text{pF}$$



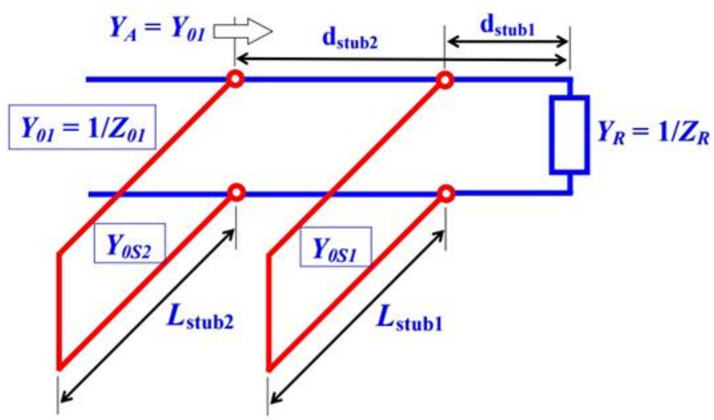


For proper impedance match:

$$Y_A = Y_{stub} + Y(d_{stub}) = Y_0 = \frac{1}{Z_0}$$



Double stub impedance matching:



Design parameters:

(i) The length of the first stub line
(ii) The length of the second stub line
Here the stubs are inserted at predetermined locations. The drawback is that a certain range of load admittances cannot be matched once the stub locations are fixed.

Smith Chart

Find

1) Reflection coefficient at load

$$z_R = 0.3 - j0.4 \Rightarrow \Gamma_R = 0.6e^{j227°}$$

2) SWR on the line

SWR=4.0

3) d_{min}

$$d_{min} = (0.5 - 0.435)\lambda = 0.065\lambda$$

4) Line impedance at 0.05λ to the left

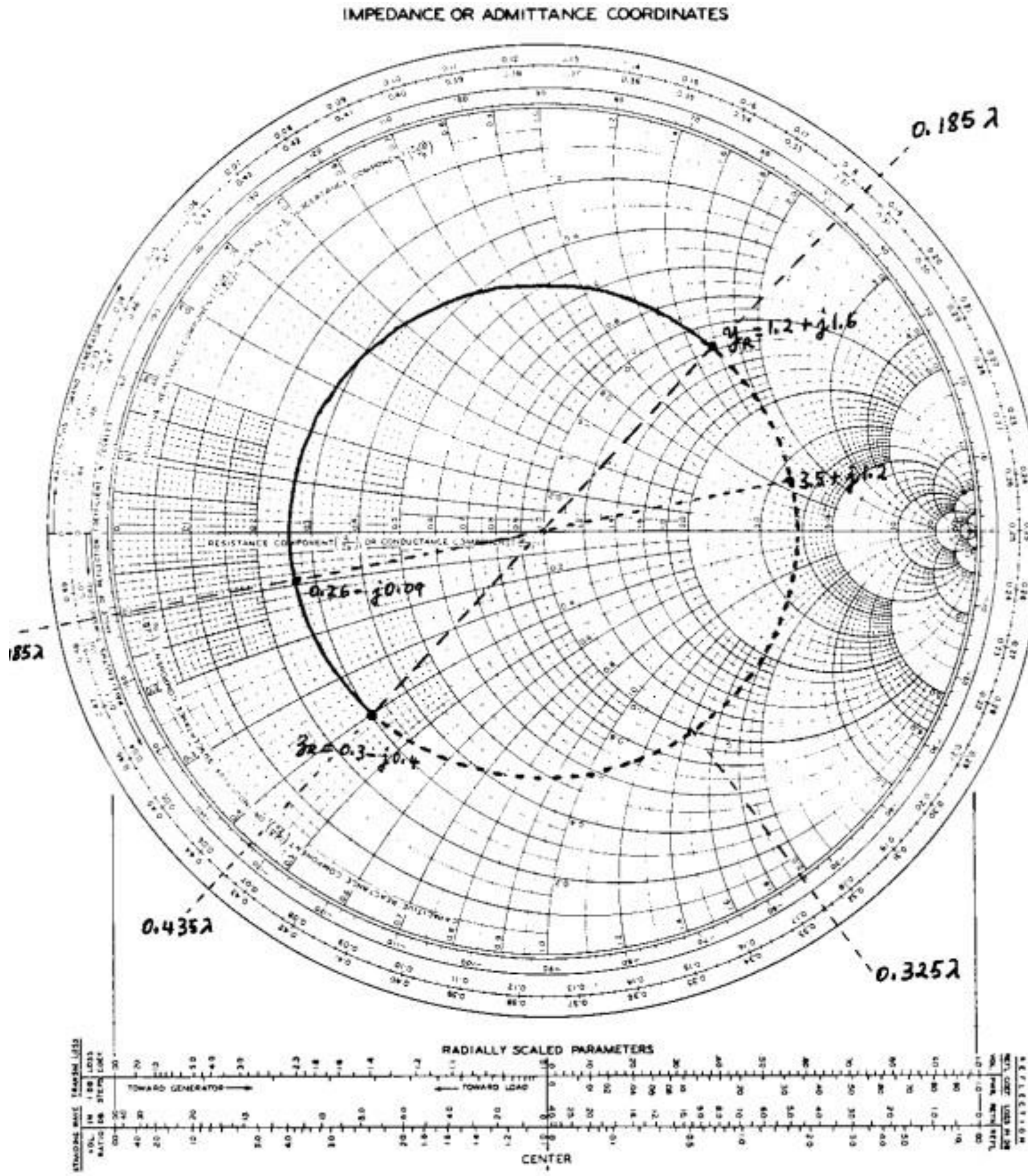
$$50(0.26 - j0.09) = 13 - j4.5\Omega$$

5) Line admittance at 0.05λ

$$(3.5 + j1.2)/50 = 0.068 + j0.025\,\mho$$

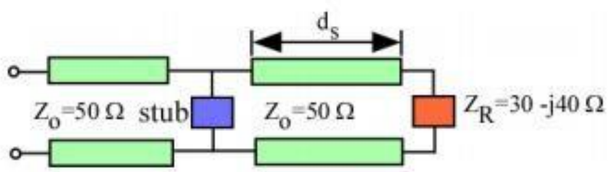
6) Location nearest to load where Real[y]=1

$$0.14\lambda = 0.325\lambda - j0.185\lambda = 0.14\lambda$$



IMPEDANCE OR ADMITTANCE COORDINATES
0.185λ
y_R = 1.2 + j1.6
3.5 + j1.2
0.26 - j0.09
z_R = 0.3 - j0.4
.185λ
0.435λ
0.325λ
RADIALLY SCALED PARAMETERS
TOWARD GENERATOR
TOWARD LOAD
CENTER

Single Stub



Find location and length of stub

$z_R = Z_R/50 = 0.6\text{-}j0.8$

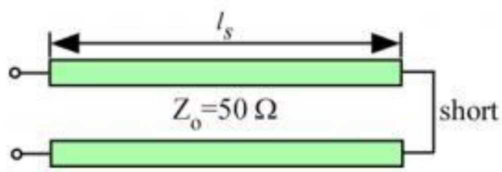
$y_R = 0.6+j0.8$

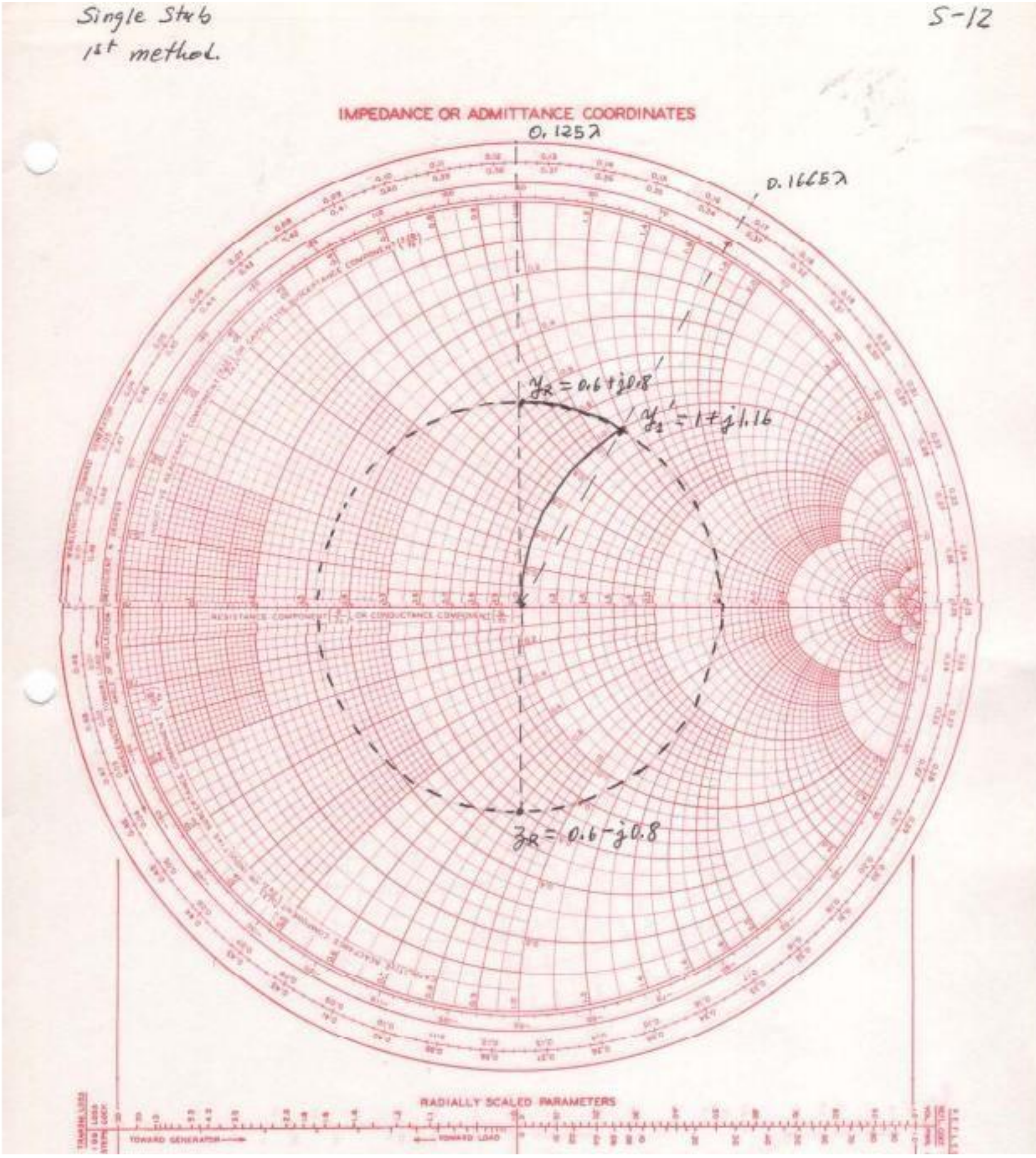
First Method

Rotate on constant VSWR circle from 0.125λ to 0.1665λ until intersection with unit conductance circle at $y_1' = 1 + j1.16$. Distance $d_s = (0.1665\lambda - 0.125\lambda) = 0.0415\lambda$ Move toward center of chart. Change in susceptance: j(0-0.16)=-j1.16

The length of the stub is such that

$$\frac{1}{tan\beta l_s} = -1.16 \text{ or } l_s = (0.363 - 0.25)\lambda = 0.113\lambda$$





3.3Phenomenon of Corona:

Electrical Transmission overhead line provides one of the most important property, which is a specific line voltage reveres occur. Transmission efficiency and therefore provides a degree of power in the Loss decreased. Moreover, because the line to get his prey life span can be much reduced. And bodies created by the induced current harmonics line charging current increases and the nearest line of the unwanted noise is telecommunication. Also, if the line is dirty or rough weather - rainy when the bodies of thousands of adverse effects. In line due to the need to design the appropriate safety system, which provides line protection from the harmful effects can be.

Effects of Corona:

(i) Conductor all side Purple - Glow (violet glow) is observed.

(ii)This hissing sound (hissing noise) generated by.

(iii)Provides a certain amount of power is wasted which can be measured using meters. The transmission efficiency decreases.

(iv)Provides for the weight of the gas line to the wire with the chemical reaction. The loss is received.

(v)Max is rough and dirty conductors' glow.

Factors affecting corona

i. Atmosphere. ii. Conductor size.

iii. Spacing between conductors. iv. Line Voltage.

The phenomenon of corona is accompanied by a hissing sound, production of ozone, power loss and radio interference. The higher the voltage is raised, the larger and higher the luminous envelope becomes, and greater are the sound, the power loss and the radio noise. If the applied voltage is increased to breakdown value, a flash-over will occur between the conductors due to the breakdown of air insulation.

The phenomenon of violet glow, hissing noise and production of ozone gas in an overhead transmission line is known as corona. If the conductors are polished and smooth, the corona glow will be uniform throughout the length of the conductors, otherwise the rough points will appear brighter. With d.c. voltage, there is difference in the appearance of the two wires. The positive wire has uniform glow about it, while the negative conductor has spotty glow.

Explanation of corona formation

Some ionization is always present in air due to cosmic rays, ultraviolet radiations and radioactivity. Therefore, under normal conditions, the air around the conductors contains some ionized particles (i.e., free electrons and +ve ions) and neutral molecules. When p.d. is applied between the conductors, potential gradient is set up in the air which will have maximum value at the conductor surfaces. Under the influence of potential gradient, the existing free electrons acquire greater velocities. The greater the applied voltage, the greater the potential gradient and more is the velocity of free electrons.

When the potential gradient at the conductor surface reaches about 30 kV per cm (max. value), the velocity acquired by the free electrons is sufficient to strike a neutral molecule with enough force to dislodge one or more electrons from it. This produces another ion and one or more free electrons, which is turn, are accelerated until they collide with other neutral molecules, thus producing other ions. Thus, the process of ionisation is cumulative, result of this ionization is that either corona is formed or spark takes place between the conductors.

Factors Affecting corona effect

The phenomenon of corona is affected by the physical state of the atmosphere as well as by the conditions of the line. The following are the factors upon which corona depends:

i) Atmosphere: As corona is formed due to ionization of air surrounding the conductors, therefore, it is affected by the physical state of atmosphere. In the stormy weather, the number of ions is more than normal and as such corona occurs at much less voltage as compared with fair weather.

(ii)Conductor size: The corona effect depends upon the shape and conditions of the conductors. The rough and irregular surface will give rise to more corona because unevenness of the surface decreases the value of breakdown voltage. Thus, a stranded conductor has irregular surface and hence gives rise to more corona that a solid conductor.

(iii)Spacing between conductors: If the spacing between the conductors is made very large as compared to their diameters, there may not be any corona effect. It is because larger distance between conductors reduces the electro-static stresses at the conductor surface, thus avoiding corona formation.

(iv)Line voltage: The line voltage greatly affects corona. If it is low, there is no change in the condition of air surrounding the conductors and hence no corona is formed. However, if the line voltage has such a value that electrostatic stresses developed at the conductor surface make the air around the conductor conducting, then corona is formed.

Advantages of Corona effect

(i)Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.

(ii)Corona reduces the effects of transients produced by surges.

Disadvantages of Corona effect

(i)Corona is accompanied by a loss of energy. This affects the transmission efficiency of the line.

(ii)Ozone is produced by corona and may cause corrosion of the conductor due to chemical action.

(iii)The current drawn by the line due to corona is non-sinusoidal and hence nonsinusoidal voltage drop occurs in the line. This may cause inductive interference with neighboring communication lines.

Methods of Reducing Corona Effect

It has been seen that intense corona effects are observed at a working voltage of 33 kV or above. Therefore, careful design should be made to avoid corona on the substations or bus-bars rated for 33kV and higher voltages otherwise highly ionized air may cause flash-over in the insulators or between the phases, causing considerable damage to the equipment. The corona effects can be reduced by the following methods.

(i)By increasing conductor size: By increasing conductor size, the voltage at which corona occurs is raised and hence corona effects are considerably reduced. This is one of the reasons that ACSR conductors which have a larger cross-sectional area are used in transmission lines.

(ii)By increasing conductor spacing: By increasing the spacing between conductors, the voltage at which corona occurs is raised and hence corona effects can be eliminated. However, spacing cannot be increased too much otherwise the cost of supporting structure (e.g., bigger cross arms and supports) may increase to a considerable extent.

4.1 Maxwell's Equation:

The waves guided or directed by the guided structures are called guided wave. In general wave equations are derived from Maxwell's equation. To obtain the solution of this problem it is essential to apply certain restrictions or boundary conditions to the Maxwell's equation.

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon} \qquad \text{(Gauss' Law)}$$

$$\nabla \cdot \mathbf{H} = 0 \qquad \text{(Gauss' Law for Magnetism)}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \text{(Faraday's Law)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad \text{(Ampere's Law)}$$

In Gauss' law, p_v is the volume electric charge density, J is the electric current density (in Amps/meter-squared),u is the permittivity and is the permeability.

The last two equations (Faraday's law and Ampere's law) are responsible for electromagnetic radiation. The curl operator represents the spatial variation of the fields, which are coupled to the time variation. When the E-field travels, it is altered in space, which gives rise to a time-varying magnetic field. A time-varying magnetic field then varies as a function of location (space), which gives rise to a time varying electric field. These equations wrap around each other in a sense, and give rise to a wave equation. These equations predict electromagnetic radiation .

Consider a parallel-plate waveguide of two perfectly conducting plates separated by a distance b and filled with a dielectric medium having constitutive parameter as shown in Fig. 1. The plates are assumed be infinite in extent in the X direction.

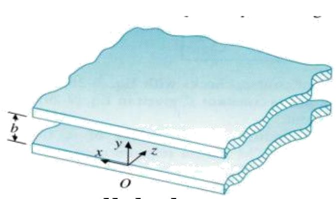


Fig.1 parallel-plate waveguide

a) Obtain the time-harmonic field expressions for TM modes in the guide.
b) Determine the cutoff frequency.

For TM modes, - Eq. becomes

$$\frac{d^2 E_z^o(y)}{dy^2} + h^2 E_z^o(y) = 0. \qquad (4.1)$$

The general solution for Eq. (4.1) : - Boundary conditions (The tangential component of the electric field must vanish on the surface of the perfectly conducting plates.) :

(i) At y=0, E_z=0

(ii)At y=b, E_z=0 - The value of the eigenvalue h :

$$h = \frac{n\pi}{b}, \qquad n = 1, 2, 3, \ldots .$$

Transverse electric (TE) wave has the magnetic field component in the direction of propagation, but no component of the electric field in the same direction. Hence the TE waves also known as M –waves or H-waves.

Transverse magnetic (TM) wave has the electric field in the direction of propagation, but no component of the magnetic field in the same direction. Hence the TM waves are also called E-waves.

Transverse electromagnetic (TEM) wave: No field in the direction of propagation **Attenuation of parallel plane guides:** When the electromagnetic wave propagates through the wave guide, the amplitude of the fields or the signal strength of the wave decreases as the distance from the source increases. This is because when the wave strikes the walls of the guide, the loss in the power takes place the attenuation factor is denoted by α.

α =Power lost per unit length/2*power transmitted

Attenuation due to finite wall conductivity is inversely proportional to the square root of wall conductivity, but depends on the mode and the frequency in a complicated way.

Attenuation due to wall losses in rectangular copper waveguide: TE10 mode has the lowest attenuation in a rectangular waveguide. The attenuation constant increases rapidly toward infinity as the operating frequency approaches the cut off frequency.

4.1.1 Cut-off frequency:

The frequency at which wave motion ceases is called cut-off frequency

$$f_c = \frac{n}{2b\sqrt{\mu\epsilon}} \qquad \text{(Hz)},$$

(4.2)

$$Z_{TE} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} \qquad (\Omega),$$

4.1.2 Propagation Constant:

$$\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}. \tag{4.3}$$

4.1.3 Wave impedance:

It is the ratio of the component of the electric field to that of magnetic field. Wave impedance for TEM wave

$$Z_{\text{TEM}} = \frac{E_x^0}{H_y^0} = \frac{j\omega\mu}{\gamma_{\text{TEM}}} = \frac{\gamma_{\text{TEM}}}{j\omega\epsilon}, \tag{4.4}$$

Wave impedance for TM and TE wave

$$Z_{\text{TM}} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{\gamma}{j\omega\epsilon} \quad (\Omega). \tag{4.5}$$

4.1.4 Phase velocity:

It is the velocity at which energy propagates along a wave guide

$$u_p = \frac{\omega}{\beta} \quad \text{(m/s)}. \tag{4.6}$$

- The phase velocity and the wave impedance for TEM waves are independent of the frequency of the waves.
- TEM waves cannot exist in a single-conductor hollow (or dielectric-filled) waveguide of any shape.

Rectangular waveguides are the one of the earliest types of the transmission lines. They are used in many applications. A lot of components such as isolators, detectors, attenuators, couplers and slotted lines are available for various standard waveguide bands between 1 GHz to above 220 GHz.

A rectangular waveguide supports TM and TE modes but not TEM waves because we cannot define a unique voltage since there is only one conductor in a rectangular waveguide. The shape of a rectangular waveguide is as shown below. A material with permittivity e and permeability m fills the inside of the conductor. A rectangular waveguide cannot propagate below some certain frequency. This frequency is called the cut-off frequency

4.2 TM Modes:

Consider the shape of the rectangular waveguide above with dimensions a and b (assume a>b) and the parameters e and m. For TM waves $H_z = 0$ and E_z should be solved from equation for TM mode;

Ñ ^{2} xy Ez ^{0} + h^2 Ez ^{0} = 0

Since $E_z(x,y,z)$ = Ez ^{0} (x,y)e^{-gz}, we get the following equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+h^2\right)E_z^0(x,y)=0 \quad (4.7)$$

If we use the method of separation of variables, that is Ez ^{0} (x,y)=X(x).Y(y) we get,

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2}=\frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2}+h^2 \quad (4.8)$$

Since the right side contains x terms only and the left side contains y terms only, they are both equal to a constant. Calling that constant as kx ^{2} , we get;

$$\frac{d^2X(x)}{dx^2}+k_x^2X(x)=0 \quad (4.9)$$

$$\frac{d^2Y(y)}{dy^2}+k_y^2Y(y)=0 \quad (4.10)$$

Where ky ^{2} =h^2 -kx ^{2}

Now, we should solve for X and Y from the preceding equations. Also we have the boundary conditions of;

Ez ^{0} (0,y)=0;Ez ^{0} (a,y)=0

Ez ^{0} (x,0)=0; Ez ^{0} (x,b)=0

From all these, we conclude that

X(x) is in the form of sin $k_x x$, where k_x=mp/a, m=1,2,3,…Y(y) is in the form of sin $k_y y$, where k_y=np/b, n=1,2,3,…

So the solution for Ez ^{0} (x,y) is

$$E_z^0(x,y)=E_0\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\text{(V/m)} \quad (4.11)$$

From all these, we conclude that

X(x) is in the form of sin $k_x x$, where k_x=mp/a, m=1,2,3,…
Y(y) is in the form of sin $k_y y$, where k_y=np/b, n=1,2,3,…
So the solution for Ez ^{0} (x,y) is

$$E_z^0(x,y) = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \text{(V/m)} \quad (4.12)$$

From ky 2 =h2 -kx 2 , we have;

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad (4.13)$$

For TM waves, we have

$$H_x^0 = \frac{j w\varepsilon}{h^2}\frac{\partial E_z^0}{\partial y}$$

$$H_y^0 = -\frac{j w\varepsilon}{h^2}\frac{\partial E_z^0}{\partial x}$$

$$E_x^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial x}$$

$$E_y^0 = -\frac{\gamma}{h^2}\frac{\partial E_z^0}{\partial y} \quad (4.14)$$

From these equations, we get

$$E_x^0(x,y) = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \quad (4.15)$$

$$E_y^0(x,y) = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \quad (4.16)$$

$$H_x^0(x,y) = \frac{jw\varepsilon}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \quad (4.17)$$

$$H_y^0(x,y) = -\frac{jw\varepsilon}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \quad (4.18)$$

where

$$\gamma = j\beta = j\sqrt{w^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad (4.19)$$

Here, m and n represent possible modes and it is designated as the TMmn mode. m denotes the number of half cycle variations of the fields in the x-direction and n denotes the number of half cycle variations of the fields in the y-direction.

When we observe the above equations, we see that for TM modes in rectangular waveguides, neither m nor n can be zero. This is because of the fact that the field expressions are identically zero if either m or n is zero. Therefore, the lowest mode for rectangular waveguide TM mode is TM_{11} .

The cut-off frequency is at the point where g vanishes. Therefore

$$f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\,(Hz) \qquad (4.20)$$

Since l=u/f, we have the cut-off wavelength

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}\,(m)} \qquad (4.21)$$

At a given operating frequency f, only those frequencies, which have $f_c<f$ will propagate. The modes with $f<f_c$ will lead to an imaginary b which means that the field components will decay exponentially and will not propagate. Such modes are called cut-off or evanescent modes.

The mode with the lowest cut-off frequency is called the dominant mode. Since TM modes for rectangular waveguides start from TM_{11} mode, the dominant frequency is

$$(f_c)_{11} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}\,(Hz) \qquad (4.22)$$

The wave impedance is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for E_x and H_y (see the equations above);

$$Z_{TM} = \frac{E_x}{H_y} = \frac{\gamma}{jw\varepsilon} = \frac{j\beta}{jw\varepsilon} \Rightarrow Z_{TM} = \frac{\beta\eta}{k} \qquad (4.23)$$

The guide wavelength is defined as the distance between two equal phase planes along the waveguide and it is equal to

$$\boxed{\lambda_g = \frac{2\pi}{\beta}} > \frac{2\pi}{k} = \lambda \qquad (4.24)$$

which is thus greater than l, the wavelength of a plane wave in the filling medium. The phase velocity is

$$u_p = \frac{w}{\beta} > \frac{w}{k} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (4.25)$$

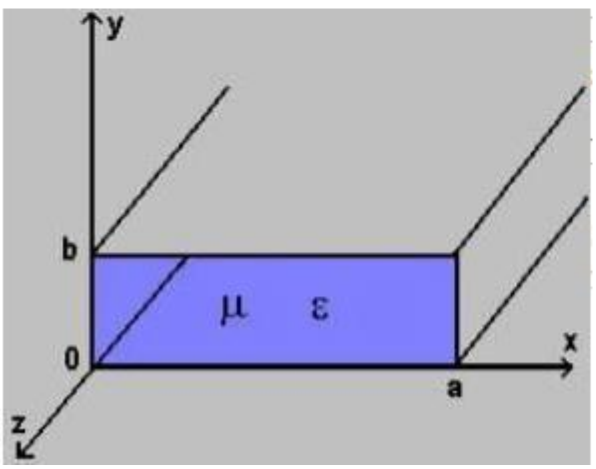
which is greater than the speed of light (plane wave) in the filling material.

Attenuation for propagating modes results when there are losses in the dielectric and in the imperfectly conducting guide walls. The attenuation constant due to the losses in the dielectric can be found as follows:

$$\gamma = j\beta = j\sqrt{k^2 - k_c^2} = jk\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu}\sqrt{\varepsilon + \frac{\sigma}{jw}}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \qquad (4.26)$$

4.3 TE Modes:

Consider again the rectangular waveguide below with dimensions a and b (assume a>b) and the parameters e and m.



For TE waves Ez = 0 and Hz should be solved from equation for TE mode;

$\tilde{N}^2_{xy} H_z + h^2 H_z = 0$ (4.27)

Since $H_z(x,y,z) = Hz^0 (x,y)e^{-gz}$, we get the following equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right)H_z^0(x,y) = 0 \qquad (4.28)$$

If we use the method of separation of variables, that is $Hz^0 (x,y)=X(x).Y(y)$ we get,

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + h^2 \qquad (4.29)$$

Since the right side contains x terms only and the left side contains y terms only, they are both equal to a constant. Calling that constant as kx^2, we get;

$$\frac{d^2X(x)}{dx^2}+k_x^2X(x)=0 \qquad (4.30)$$

$$\frac{d^2Y(y)}{dy^2}+k_y^2Y(y)=0 \qquad (4.31)$$

where $k_y{}^2 = h^2 - k_x{}^1$

Here, we must solve for X and Y from the preceding equations. Also, we have the following boundary conditions:

$$\frac{\partial H_z^0}{\partial x}=0(E_y=0) \text{ at } x=0$$

$$\frac{\partial H_z^0}{\partial x}=0(E_y=0) \text{ at } x=a$$

$$\frac{\partial H_z^0}{\partial y}=0(E_x=0) \text{ at } y=0$$

$$\frac{\partial H_z^0}{\partial y}=0(E_x=0) \text{ at } y=b$$

From all these, we get

$$H_z^0(x,y)=H_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \text{ (A/m)} \qquad (4.32)$$

From $k_y{}^2=h^2-k_x{}^2$, we have;

$$\boxed{h^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

For TE waves, we have

$$H_x^0=-\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial x} \qquad (4.33)$$

$$H_y^0=-\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial y} \qquad (4.34)$$

$$E_x^0=-\frac{jw\mu}{h^2}\frac{\partial H_z^0}{\partial y} \qquad (4.35)$$

$$E_y^0=-\frac{jw\mu}{h^2}\frac{\partial H_z^0}{\partial x} \qquad (4.36)$$

From these equations, we obtain

$$E_x^0(x,y)=\frac{jw\mu}{h^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \qquad (4.37)$$

$$E_y^0(x,y) = -\frac{jw\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \tag{4.38}$$

$$H_x^0(x,y) = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \tag{4.39}$$

$$H_y^0(x,y) = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \tag{4.40}$$

As explained before, m and n represent possible modes and it is shown as the TEmn mode. m denotes the number of half cycle variations of the fields in the x-direction and n denotes the number of half cycle variations of the fields in the y-direction.

The mode with the lowest cut-off frequency is called the dominant mode. Since TE10 mode is the minimum possible mode that gives nonzero field expressions for rectangular waveguides, it is the dominant mode of a rectangular waveguide with a>b and so the dominant frequency is

The wave impedance is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for Ex and Hy (see the equations above);

$$Z_{TE} = \frac{E_x}{H_y} = \frac{jw\mu}{\gamma} = \frac{jw\mu}{j\beta} \Rightarrow Z_{TE} = \frac{k\eta}{\beta} \tag{4.41}$$

The guide wavelength is defined as the distance between two equal phase planes along the waveguide and it is equal to

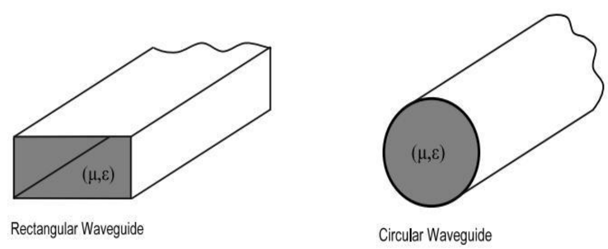
$$\boxed{\lambda_g = \frac{2\pi}{\beta}} > \frac{2\pi}{k} = \lambda \tag{4.42}$$

The attenuation constant due to the losses in the dielectric is obtained as follows:

$$\gamma = j\beta = j\sqrt{k^2 - k_c^2} = jk\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu}\sqrt{\varepsilon + \frac{\sigma}{jw}}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \tag{4.43}$$

GUIDED WAVES AND WAVEGUIDE THEORY:

Waveguides, like transmission lines, are structures used to guide electromagnetic waves from point to point. However, the fundamental characteristics of waveguide and transmission line waves (modes) are quite different. The differences in these modes result from the basic differences in geometry for a transmission line and a waveguide. Waveguides can be generally classified as either metal waveguides or dielectric waveguides. Metal waveguides normally take the form of an enclosed conducting metal pipe. The waves propagating inside the metal waveguide may be characterized by reflections from the conducting walls. The dielectric waveguide consists of dielectrics only and employs reflections from dielectric interfaces to propagate the electromagnetic wave along the waveguide.



5.1 Rectangular Waveguides:

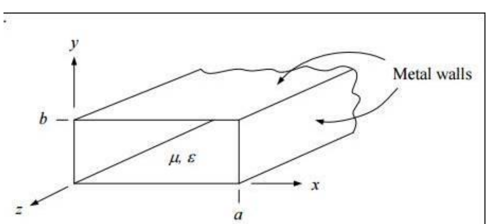


Fig 1: Rectangular Waveguide

The wave is propagating in z-direction as shown in figure.1.Consider a rectangular waveguide with $0 < x < a$, $0 < y < b$ and $a > b$.There are two types of waves in a hollow waveguide with only one conductor.

- Transverse electric waves (TE-waves). E = (Ex, Ey, 0) and H = (Hx, Hy, Hz).
- Transverse magnetic waves (TM-waves). E = (Ex, Ey, Ez) and H = (Hx, Hy, 0).

They need to satisfy the Maxwell's equations and the boundary conditions. The boundary conditions are that the tangential components of the electric field and the normal derivative of the tangential components of the magnetic field are zero at the boundaries.

5.1.1 TE-waves

We now try to find the electromagnetic fields for TE-waves, when E_z is zero. The electromagnetic fields are obtained from Hz. The equation to be solved is

$$\nabla^2 H_z + k^2 H_z = 0$$
$$\frac{\partial H_z}{\partial x}(0,y,z) = \frac{\partial H_z}{\partial x}(a,y,z) = \frac{\partial H_z}{\partial y}(x,a,z) = \frac{\partial H_z}{\partial y}(x,b,z) = 0 \quad (5.1)$$

where k = ω/c is the wave number. There are infinitely many solutions to this equation

$$H_{zmn}(x,y,z) = h_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (5.2)$$

The m, n values can take the values m = 0, 1, 2 . . . and n = 0, 1, 2 . . ., but (m, n) ≠ (0, 0). The Corresponding transverse electric and magnetic fields are obtained from Maxwell's equations. The spatial dependence of these components are

$$E_x \sim \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$
$$E_y \sim \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$
$$H_x \sim \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$
$$H_y \sim \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z} \quad (5.3)$$

Each of these components satisfies the Helmholtz equation and the boundary conditions. The electromagnetic field corresponding to (m, n) is called a TE_{mn} mode. Thus there are infinitely many TE_{mn} modes. The k_z is the z−component of the wave vector. For a given frequency it is given by

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad (5.4)$$

This means that for m and n values such that

$$k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 > 0 \quad (5.5)$$

$$f > \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

then k_z is real and the TE_{mn} mode is propagating.

For m and n values such that

$$k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 < 0 \quad (5.6)$$

$$E_x \sim \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

$$f < \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{5.7}$$

then k_z is imaginary and the TE_{mn} mode is a non-propagating mode.

Cut-off frequency:

For a TE_{mn} mode the cut-off frequency is the frequency for which $k_z = 0$. This means that the mode is in between its propagating and non-propagating stages. The cut off frequency for the TE_{mn} mode is

$$f_{cmn} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}. \tag{5.8}$$

5.1.2 The fundamental mode TE_{10} (or) Dominant Mode

The fundamental mode or Dominant mode of a waveguide is the mode that has the lowest cut-off frequency. For a rectangular waveguide it is the TE_{10} mode that is the fundamental mode. It has

$$f_{c10} = \frac{c}{2a}. \tag{5.9}$$

The electric field of the fundamental mode is

$$\boldsymbol{E} = E_0 \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z} \boldsymbol{e}_y \tag{5.10}$$

It is almost always the fundamental mode that is used in the waveguide. It is then crucial to make sure that the frequency is low enough such that only the fundamental mode can propagate. Otherwise there will be more than one mode in the waveguide and since the modes travel with different speeds, as will be seen below, one cannot control the phase of the wave.

$$\begin{aligned} &\nabla^2 E_z + k^2 E_z = 0 \\ &E_z(0,y,z) = E_z(a,y,z) = E_z(x,0,z) = E_z(x,b,z) = 0 \end{aligned} \tag{5.11}$$

where $k = \omega/c$ is the wave number. There are infinitely many solutions top this equation

$$E_{zmn}(x,y,z) = e_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

The m, n values can take the values m = 1, 2 . . . and n = 1, 2 . . .. The corresponding transverse electric and magnetic fields are obtained from Maxwell's equations. The spatial dependence of these components is the same as for the TE-waves

$$E_x \sim \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$$
$$E_y \sim \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$$
$$H_x \sim \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$$
$$H_y \sim \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$$

(5.12)

The electromagnetic field corresponding to (m, n) is called a TE_{mn} mode. Thus, there are infinitely many TE_{mn} modes.

For a given frequency k_z for the TE-modes is the same as for the TM-modes.

5.1.4 Cut-off frequency

For a TMmn mode the cut-off frequencies are the same as for the TE_{mn} modes, i.e.,

$$f_{cmn} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

(5.13)

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

(5.14)

5.1.5 Impossibility of TEM waves in Waveguides:

TEM modes can only exist in two-conductor waveguides such as two-wire transmission lines, co-axial lines, parallel-plate waveguides, etc, but not in single-conductor waveguides such as rectangular waveguides and circular waveguides. This is because either longitudinal field components or longitudinal currents are required to support the transverse magnetic field components Hx and Hy which form close loops in the transverse plane. There are no longitudinal currents (not longitudinal surface currents) inside hollow waveguides and hence hollow waveguides cannot support TEM modes. But they can support TE and TM modes

5.2 Wave Impedances:

For any transverse electromagnetic wave, the wave impedance (in ohms) is defined as being approximately equal to the ratio of the electric and magnetic fields, and converges as a function of frequency to the intrinsic impedance of the dielectric:

For TM mode

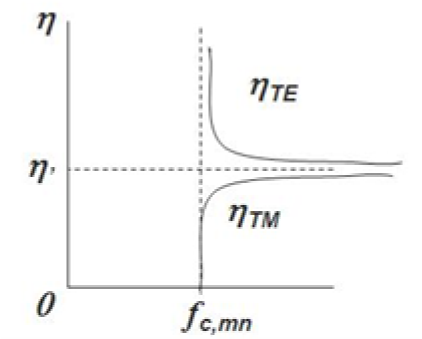
$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \eta'\sqrt{1-\left[\frac{f_c}{f}\right]^2}$$

(5.15)

For TE mode

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}} \qquad (5.16)$$

Wave impedance varies with frequency and mode as follows



5.3 Circular Waveguides:

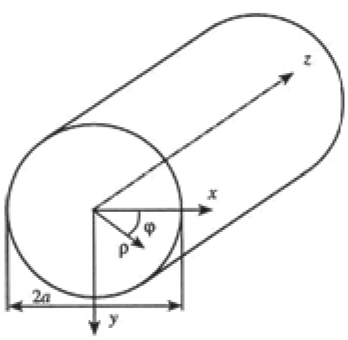


Fig.3 Circular Waveguide

The circular waveguide is occasionally used as an alternative to the rectangular waveguide. Like other wave guides constructed from a single, enclosed conductor, the circular waveguide supports transverse electric (TE) and transverse magnetic (TM) modes. These modes have a cutoff frequency, below which electromagnetic energy is severely attenuated. Wave Propagates in z-direction. There are two sets of modes, TE and TM modes, which can propagate in a cylindrical waveguide.

5.3.1 TE mode

For TE waves in a cylindrical waveguide, E_z=0 and $H_z \neq 0$, all other field components can be expressed in terms of H_z. The Maxwell's equations can be expanded in the cylindrical coordinate as

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} \pm j\beta_z E_\phi = -j\omega\mu H_\rho \qquad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} \pm j\beta_z H_\phi = j\omega\varepsilon E_\rho$$

$$\mp j\beta_z E_\rho - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \qquad \mp j\beta_z H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega\varepsilon E_\phi$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \overline{E}_\phi\right) - \frac{1}{\rho}\frac{\partial E_\rho}{\partial \phi} = -j\omega\mu H_z \qquad \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho H_\phi\right) - \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} = j\omega\varepsilon E_z$$

(5.17)

The ρ and φ components can be expressed in terms of E_z and H_z as follows

$$E_\rho = \frac{1}{\beta_z^2 - \beta^2}\left[\pm j\beta_z \frac{\partial E_z}{\partial \rho} + \frac{j\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

$$E_\phi = -\frac{1}{\beta_z^2 - \beta^2}\left[\mp j\frac{\beta_z}{\rho}\frac{\partial E_z}{\partial \phi} + j\omega\mu\frac{\partial H_z}{\partial \rho}\right]$$

$$H_\rho = -\frac{1}{\beta_z^2 - \beta^2}\left[\frac{j\omega\varepsilon}{\rho}\frac{\partial E_z}{\partial \phi} \mp j\beta_z \frac{\partial H_z}{\partial \rho}\right]$$

$$H_\phi = \frac{1}{\beta_z^2 - \beta^2}\left[j\omega\varepsilon\frac{\partial E_z}{\partial \rho} \pm j\frac{\beta_z}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

(5.18)

5.3.2 TM Mode

The derivation for TM mode is the same except that we are solving for Ez. We can therefore write

$$E_z(\rho, \phi, z) = [A\sin(\nu\phi) + B\cos(\nu\phi)]\, J_\nu(k_c\rho)e^{-j\beta z}$$

(5.19)

The boundary condition in this case is as follows

$$E_z(a, \phi, z) = 0 \text{ or } J_\nu(k_c a) = 0.$$

(5.20)

This leads to

$$k_c a = p_{\nu n} \qquad \rightarrow \qquad k_c = \frac{p_{\nu n}}{a}$$

(5.21)

5.4 Intrinsic Wave Impedance (η):

Intrinsic Wave Impedance in Circular Waveguide for TE mode is given as follows

$$\eta_{TE} = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1-\frac{f_c^2}{f^2}}} = \frac{\eta_0}{\sqrt{1-\frac{f_c^2}{f^2}}}$$

(5.22)

$$\boxed{\eta_{TE} = \frac{\eta_0}{\sqrt{1 - \frac{f_c^2}{f^2}}}} \tag{5.23}$$

Where η_o is the intrinsic impedance of a uniform plane wave in a lossless dielectric medium

$$\eta_0 = \sqrt{\frac{\mu}{\varepsilon}} \tag{5.24}$$

Intrinsic Wave Impedance in Circular Waveguide for TM mode is given as follows

$$\eta_{TM} = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r} = \frac{\beta}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - \frac{f_c^2}{f^2}} = \eta_0\sqrt{1 - \frac{f_c^2}{f^2}} \tag{5.25}$$

5.5 Power Flow in Waveguides:

Power Flow in a Rectangular Waveguide (TE10) The time-average Poynting vector for the TE10 mode in a rectangular waveguide is given by

$$\langle \mathbf{P} \rangle = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] = \hat{\mathbf{z}}\frac{|E_o|^2}{2}\frac{\beta_z}{\omega\mu}\sin^2\frac{\pi x}{a}$$

$$\langle \text{Power} \rangle = \int_0^a\int_0^b \frac{|E_o|^2}{2}\frac{\beta_z}{\omega\mu}\sin^2\frac{\pi x}{a}\,dxdy$$

$$\langle \text{Power} \rangle = \frac{|E_o|^2}{4}\frac{\beta_z ab}{\omega\mu} = \frac{|E_o|^2 ab}{4\eta_{gTE_{10}}} \tag{5.26}$$

Therefore, the time-average power flow in a waveguide is proportional to its cross-section area.