**SUBJECT NAME: ENGINEERING MATHEMATICS II**

**(COMMON TO BIO GROUPS)**

**SUBJECT CODE: SMT1106**

**COURSE MATERIAL**

**UNIT-III INTEGRAL CALCULUS**

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**Standard results**

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

2. $\int \frac{1}{x}\, dx = \log x + c$

3. $\int e^x dx = e^x + c$

4. $\int \sin x\, dx = -\cos x + c$

5. $\int \cos x\, dx = \sin x + c$

6. $\int \tan x\, dx = \log \sec x + c$

7. $\int \cot x\, dx = \log \sin x + c$

8. $\int \sec x \tan x\, dx = \sec x + c$

9. $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

10. $\int \sec^2 x\, dx = \tan x + c$

11. $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

12. $\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + c$

13. $\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + c$

14. $\int \frac{1}{x\sqrt{x^2-1}}\, dx = \sec^{-1} x + c$

**Problems**

**1. Evaluate** $\int \left( x + \frac{1}{x} \right)^3 dx$

**Solution:**

$$\int \left( x + \frac{1}{x} \right)^3 dx = \int \left( x^3 + \frac{1}{x^3} + 3x + \frac{3}{x} \right) dx$$

$$= \int \left( x^3 + x^{-3} + 3x + 3x^{-1} \right) dx$$

$$= \int x^3 dx + \int x^{-3} dx + 3\int x\ dx + 3\int x^{-1} dx$$

$$= \frac{x^4}{4} + \frac{x^{-2}}{-2} + \frac{3x^2}{2} + 3\log x + c$$

$$= \frac{x^4}{4} - \frac{1}{2x^2} + \frac{3x^2}{2} + 3\log x + c$$

**2. Find** $\int \frac{1}{\sin^2 x \cos^2 x} dx$

**Solution:**

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \sec^2 x\ dx + \int \operatorname{cosec}^2 x\ dx$$

$$= \tan x - \cot x + c$$

**3. Evaluate** $\int \frac{1}{1 + \cos x} dx$

**Solution:**

$$\int \frac{1}{1+\cos x} dx = \int \frac{1}{1+\cos x} \times \frac{1-\cos x}{1-\cos x} dx$$

$$= \int \frac{1-\cos x}{1-\cos^2 x} dx = \int \operatorname{cosec}^2 x\ dx - \int \operatorname{cosec} x \cot x\ dx$$

$$= -\cot x + \operatorname{cosec} x + c$$

**Standard results**

i. $\int (ax+b)^n \, dx = \dfrac{(ax+b)^{n+1}}{a\,(n+1)} + c \quad if \quad n \neq -1$

ii. $\int \dfrac{1}{ax+b} \, dx = \dfrac{1}{a} \log(ax+b) + c$

iii. $\int e^{ax+b} dx = \dfrac{e^{ax+b}}{a} + c$

iv. $\int \sin(ax+b) \, dx = \dfrac{-\cos(ax+b)}{a} + c$

v. $\int \cos(ax+b) \, dx = \dfrac{\sin(ax+b)}{a} + c$

vi. $\int \tan(ax+b) \, dx = \dfrac{\log[\sec(ax+b)]}{a} + c$

vii. $\int \cot(ax+b) \, dx = \dfrac{\log[\sin(ax+b)]}{a} + c$

viii. $\int \sec(ax+b) \tan(ax+b) \, dx = \dfrac{\sec(ax+b)}{a} + c$

ix. $\int \cos ec(ax+b) \cot(ax+b) \, dx = \dfrac{-\cos ec(ax+b)}{a} + c$

x. $\int \sec^2(ax+b) \, dx = \dfrac{\tan(ax+b)}{a} + c$

xi. $\int \cos ec^2(ax+b) \, dx = \dfrac{-\cot(ax+b)}{a} + c$

xii. $\int \dfrac{1}{a^2 - x^2} \, dx = \dfrac{1}{2a} \log\left(\dfrac{a+x}{a-x}\right) + c$

xiii. $\int \dfrac{1}{x^2 - a^2} \, dx = \dfrac{1}{2a} \log\left(\dfrac{x-a}{x+a}\right) + c$

xiv. $\int \dfrac{1}{a^2 + x^2} \, dx = \dfrac{1}{a} \tan^{-1}\left(\dfrac{x}{a}\right) + c$

xv. $\int \dfrac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c$

xvi. $\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \log\left(x + \sqrt{x^2 - a^2}\right) + c$

xvii. $\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \log\left(x + \sqrt{x^2 + a^2}\right) + c$

xviii. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

xix. $\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left(x + \sqrt{a^2 + x^2}\right) + c$

xx. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left(x + \sqrt{x^2 - a^2}\right) + c$

**Problems:**

**1. Evaluate**

**i.** $\int \sin^3 x \cos^2 x\, dx$ **ii.** $\int \frac{\sin x}{\sin(x+a)}\, dx$

**Solution:**

i. We have $\int \sin^3 x \cos^2 x\, dx = \int \sin^2 x \cos^2 x (\sin x)\, dx$

$$= \int \left(1 - \cos^2 x\right) \cos^2 x\, (\sin x)\, dx$$

*Put* $t = \cos x$ *so that* $dt = -\sin x\, dx$

*Therefore,* $\int \sin^2 x \cos^2 x\, (\sin x)\, dx = -\int \left(1 - t^2\right) t^2 dt$

$$= -\int \left(t^2 - t^4\right) dt = -\left(\frac{t^3}{3} - \frac{t^5}{5}\right) + c$$

$$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c$$

ii. *Put* $x + a = t$. *Then* $dx = dt$. *Therefore*

$$\int \frac{\sin x}{\sin(x+a)}\, dx = \int \frac{\sin(t-a)}{\sin t}\, dt$$

$$= \int \frac{\sin t \cos a - \cos t \sin a}{\sin t}\, dt$$

$$= \cos a\int dt - \sin a\int \cot t\ dt$$

$$= (\cos a)t - (\sin a)(\log \sin t) + c$$

$$= (\cos a)(x+a) - (\sin a)[\log \sin (x+a)] + c$$

$$= x \cos a + a \cos a - (\sin a)[\log \sin (x+a)] + c$$

$$\therefore \int \frac{\sin x}{\sin(x+a)} dx = x \cos a - \sin a \log[\sin (x+a)] + c_1$$

where $c_1 = a \cos a + c$ is another arbitrary constant.

**2 . Evaluate** $\int \frac{x^3 dx}{(x^2+1)^3}$

**Solution:**

Put $x^2 + 1 = t$

Then, $2x\ dx = dt$

$$\int \frac{x^3\ dx}{(x^2+1)^3} = \int \frac{(t-1)\frac{dt}{2}}{t^3} = \frac{1}{2}\int \left(\frac{1}{t^2} - \frac{1}{t^3}\right) dt$$

$$= \frac{1}{2}\left[\frac{-1}{t} + \frac{1}{2t^2}\right] + c$$

$$= \frac{1}{2}\left[\frac{-1}{x^2+1} + \frac{1}{2(x^2+1)^2}\right] + c$$

**3. Evaluate** $\int \frac{\sec^2(\log x)}{x} dx$

**Solution:**

$$Put\ t = \log x, \quad \therefore dt = \frac{1}{x} dx$$

$$\therefore \int \frac{\sec^2(\log x)}{x} dx = \int \sec^2 t\ dt$$

$$= \tan t + c = \tan(\log x) + c$$

**Integration of rational function of the type** $\frac{lx+m}{ax^2+bx+c}$

**1. Evaluate** $\int \frac{2x+3}{x^2+5x+7}dx$

**Solution:**

$$Let\ 2x+3 = A(2x+5)+B$$

*Equating the Coefficients of* $x$,

$$2 = 2A \quad \therefore A=1$$

Equating the constant term,

3 = 5A + B

$$\therefore B = 3-5 = -2$$

$$I = \int \frac{2x+3}{x^2+5x+7}dx$$

$$= \int \frac{1(2x+5)-2}{x^2+5x+7}dx$$

$$= \int \frac{2x+5}{x^2+5x+7}dx - 2\int \frac{dx}{x^2+5x+7}$$

$$Put\ x^2+5x+7 = t$$

$$(2x+5)dx = dt$$

$$\therefore I = \int \frac{dt}{t} - 2\int \frac{dx}{\left(x+\frac{5}{2}\right)^2+\frac{3}{4}}$$

$$= \log t - 2\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{5}{2}}{\frac{\sqrt{3}}{2}}\right)+c$$

$$= \log\left(x^2+5x+7\right) - \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{2x+5}{\sqrt{3}}\right)+c$$

**2. Evaluate** $\int \frac{5x+1}{x^2-2x-35}dx$

**Solution:**

Let 5x+1 = A (2x-2) + B

Equating the coefficient of x,

$$5 = 2A \quad \therefore A = \frac{5}{2}$$

Equating the Constant Terms, 1 = - 2A + B

Therefore, B = 6

$$\therefore I = \int \frac{5x+1}{x^2-2x-35}$$

$$= \int \frac{\frac{5}{2}(2x-2)+6}{x^2-2x-35}$$

$$= \frac{5}{2}\int \frac{(2x-2)}{x^2-2x-35}dx + 6\int \frac{dx}{(x-1)^2-36}$$

$$= \frac{5}{2}\log(x^2-2x-35)+6\frac{1}{2\ .\ 6}\log\left(\frac{x-1-6}{x-1+6}\right)+c$$

$$= \frac{5}{2}\log(x^2-2x-35)+\frac{1}{2}\log\left(\frac{x-7}{x+5}\right)+c$$

**Integration of irrational function of the type** $\frac{lx+m}{\sqrt{ax^2+bx+c}}$

**1. Evaluate** $\int \frac{2x+2}{\sqrt{x^2+4x+7}}dx$

**Solution:**

Let 2x + 2 = A (2x + 4) + B

Equating the coefficient of x, 2A = 2 $\therefore A = 1$

Equating the constant terms, 2 = 4 A + B $\therefore B = -2$

$$I = \int \frac{(2x+4-2)}{\sqrt{x^2+4x+7}}dx = \int \frac{2x+4}{\sqrt{x^2+4x+7}}dx - \int \frac{2}{\sqrt{x^2+4x+7}}dx$$

$$= 2\sqrt{x^2+4x+7} - 2\log\left( (x+2) + \sqrt{(x+2)^2+3}\right) + c$$

**2. Evaluate** $\int \sqrt{\frac{1-x}{1+x}}\ dx$

**Solution:**

$$\int \sqrt{\frac{1-x}{1+x}}\ dx = \int \frac{1-x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx - \int \frac{x}{\sqrt{1-x^2}}dx$$

$$= \sin^{-1} x + \sqrt{1-x^2} + c$$

**Integration of the function of the type** $\frac{1}{(ax+b)\sqrt{lx^2+mx+n}}$

**1. Find** $\int \frac{1}{(1+x)\sqrt{1+x^2}}dx$

**Solution:**

Put $\frac{1}{1+x} = t$ , x + 1 = $\frac{1}{t}$ , log ( 1+ x) = - log t , $\frac{dx}{1+x} = -\frac{dt}{t}$

$$\therefore \int \frac{1}{(1+x)\sqrt{1+x^2}}dx = \int \frac{-dt/t}{\sqrt{1+\left(\frac{1}{t}-1\right)^2}}$$

$$= \int \frac{-dt}{t\sqrt{1+\frac{1}{t^2}+1-\frac{2}{t}}} = -\int \frac{dt}{\sqrt{2t^2-2t+1}}$$

$$= -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{t^2-t+\frac{1}{2}}} = -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{\left(t-\frac{1}{2}\right)^2+\frac{1}{4}}}$$

$$= -\frac{1}{\sqrt{2}}\log\left[\left(t-\frac{1}{2}\right)+\sqrt{\left(t-\frac{1}{2}\right)^2+\frac{1}{4}}\right]+c = -\frac{1}{\sqrt{2}}\log\left[(2t-1)+\sqrt{(2t-1)^2+1}\right]+c$$

where $t = \frac{1}{1+x}$

**2. Evaluate** $\int \frac{1}{x\sqrt{x^2+6x+109}}dx$

**Solution:**

$$Put\ t = \frac{1}{x}\ \ or\ \ x = \frac{1}{t}$$

$$\therefore dx\ = \frac{-1}{t^2}dt$$

$$I = \int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1}{t^2}+\frac{6}{t}+109}}$$

$$= \int \frac{-dt}{\sqrt{109\ t^2\ +6t+1}}$$

$$= -\frac{1}{\sqrt{109}}\int \frac{dt}{\sqrt{t^2+\frac{6t}{109}+\frac{1}{109}}}$$

$$= -\frac{1}{\sqrt{109}}\int \frac{dt}{\sqrt{\left(t+\frac{3}{109}\right)^2+\frac{1}{109}-\frac{9}{109^2}}}$$

$$= -\frac{1}{\sqrt{109}}\int \frac{dt}{\sqrt{\left(t+\frac{3}{109}\right)^2+(\frac{10}{109})^2}}$$

$$= -\frac{1}{\sqrt{109}}\log\left[\left(t+\frac{3}{109}\right)+\sqrt{t^2+\frac{6t}{109}+\frac{1}{109}}\right]+c$$

$$where, t\ = \frac{1}{x}$$

**Integrals of the type** $\int \frac{a\ \sin x+\ b\cos x}{c\ \sin x+\ d\ \cos x}dx$

**1. Evaluate** $\int \frac{2\sin x+3\cos x}{4\sin x+5\cos x}dx$

**Solution:**

2 sin x + 3 cosx = A ( 4 sin x + 5 cos x) + B( 4 cos x – 5 sin x)

Equating the coefficients of sin x and cos x, we get A= $\frac{23}{41}$ , B = $\frac{2}{41}$

$$I = \int \frac{\frac{23}{41}(4\sin x + 5\cos x) + \frac{2}{41}(4\cos x - 5\sin x)}{4\sin x + 5\cos x}dx$$

$$= \frac{23}{41}\int dx + \frac{2}{41}\int \frac{4\cos x - 5\sin x}{4\sin x + 5\cos x}dx = \frac{23}{41}x + \frac{2}{41}\log(4\sin x + 5\cos x) + c.$$

**2. Evaluate** $\int \frac{dx}{1+\tan x}$

**Solution:**

$$I = \int \frac{dx}{1+\tan x} = \int \frac{\cos x}{\cos x + \sin x}dx$$

Cos x = A( cos x + sin x) + B(-sin x + cos x)

A = $\frac{1}{2}$ , B = $\frac{1}{2}$

$$I = \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\cos x - \sin x)}{\sin x + \cos x}dx$$

$$= \frac{1}{2}\int dx + \frac{1}{2}\log(\sin x + \cos x) + c = \frac{1}{2}x + \frac{1}{2}\log(\sin x + \cos x) + c$$

**INTEGRATION USING PARTIAL FRACTIONS**

| S.No. | Form of the Rational Function | Form of the partial fraction |
|---|---|---|
| **1.** | $\frac{px+q}{(x-a)(x-b)}, a \neq b$ | $\frac{A}{x-a} + \frac{B}{x-b}$ |
| **2.** | $\frac{px+q}{(x-a)^2}$ | $\frac{A}{x-a} + \frac{B}{(x-a)^2}$ |
| **3.** | $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$ | $\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ |
| **4.** | $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$,where $x^2+bx+c$ cannot be factorized further. | $\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$ |

**Problems:**

**1. Find** $\int \frac{dx}{(x+1)(x+2)}$

**Solution:**

The Integrand is a proper rational function. So we write,

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

1 = A(x+2) + B(x+1)

Equating the coefficient of x-term and the constant term, we get

A + B = 0 and 2A+B = 1

Solving we get A = 1 and B = -1

$$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$$

$$\int \frac{dx}{(x+1)(x+2)} = \int \frac{dx}{(x+1)} - \int \frac{dx}{(x+2)}$$

= log( x+1) - log ( x+2) + c

$$= \log\left(\frac{x+1}{x+2}\right) + c$$

**2: Find** $\int \frac{x^2 + 1}{x^2 - 5x + 6} dx$

**Solution:**

Here the integrand is not a proper rational function. So we divide $x^2+1$ by $x^2 - 5x + 6$

$$\frac{x^2 + 1}{x^2 - 5x + 6} = 1 + \frac{5x - 5}{x^2 - 5x + 6}$$

Now $\frac{5x - 5}{x^2 - 5x + 6} = \frac{A}{x-2} + \frac{B}{x-3}$

5x – 5 = A (x-3) + B ( x-2 )

Equating the coefficient of x-term and the constant term, we get

A + B = 5 and 3A + 2B = 5 solving we get A = -5 and B = 10

$$\frac{x^2 + 1}{x^2 - 5x + 6} = 1 - \frac{5}{x-2} + \frac{10}{x-3}$$

$$\int \frac{x^2 + 1}{x^2 + 5x + 6} dx = \int dx - \int \frac{5}{x-2} dx + \int \frac{10}{x-3} dx$$

= x - 5 log (x-2) + 10 log (x-3) + c.

**3. Find** $\int \frac{3x - 2}{(x+1)^2 (x+3)} dx$

**Solution:**

$$\frac{3x-2}{(x+1)^2 \ (x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+3)}$$

3x – 2 = A ( x+1)(x+3) + B( x+3) + C(x+1)$^2$

Equating the coefficient of $x^2$ , x-term and the constant term, we get

A +C = 0 , 4A + B +2C =3, 3A + 3B + C = -2. Solving these equations we get

A = $\frac{11}{4}$ , B = $\frac{-5}{2}$ and C = $\frac{-11}{4}$

$$\frac{3x-2}{(x+1)^2 \ (x+3)} = \frac{11}{4(x+1)} - \frac{5}{2(x+1)^2} - \frac{11}{4(x+3)}$$

$$= \frac{11}{4}\log(x+1) + \frac{5}{2(x+1)} - \frac{11}{4}\log(x+3) + c$$

$$= \frac{11}{4}\log\left(\frac{x+1}{x+3}\right) + \frac{5}{2(x+1)} + c \ .$$

**INTEGRATION BY PARTS:**

$$\int u \ dv = u \ v - \int v \ du$$

**1. Find** $\int x \cos x \, dx$

**Solution:**

Let u =x, dv =cosx dx

Then integration by parts gives,

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + c$$

**2. Find** $\int \log x\ dx$

**Solution:**

Let u = logx , dv = dx

Then, $\int \log x\ dx = (\log x)x - \int \frac{1}{x}x\ dx$

$= x(\log x) - x + c$

**3. Find** $\int x e^{x}\ dx$

**Solution:**

Let u = x , dv = $e^x$dx

$$\int x e^{x}\ dx = x e^{x} - \int 1 e^{x}\ dx = x e^{x} - e^{x} + c$$

**4. Find** $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

**Solution:**

Let u = $\sin^{-1}$x , dv = x/√(1-$x^2$)dx

For finding v,

Put t = 1 – $x^2$ then dt = -2x dx

Then v = $\int \frac{-dt}{2\sqrt{t}} = -\sqrt{t} = -\sqrt{1-x^2}$

$$\therefore \int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \sin^{-1}x\left(-\sqrt{1-x^2}\right) - \int \frac{1}{\sqrt{1-x^2}}\left(-\sqrt{1-x^2}\right)dx$$

$$= -\sqrt{1-x^2}\ \sin^{-1}x + x + c$$

**BERNOULLI’S FORMULA**

$$\int u\ dv = uv - u' v_1 + u'' v_2 + \ldots$$

**Problems**

**1. Solve** $\int x^2 e^x \, dx$

**Solution:**

$$\int x^2 e^x \, dx = x^2 e^x - 2x ( e^x ) + 2 e^x + C$$

**2. Solve** $\int x \sin ax \, dx$

**Solution:**

$$\int x \sin ax \, dx = x\left(\frac{-\cos ax}{a}\right) - \left(\frac{-\sin ax}{a^2}\right) + C$$

**3. Solve** $\int (a x^2 + b x + c) \cos x \, dx$

**Solution:**

$$\int (a x^2 + b x + c) \cos x \, dx = (a x^2 + b x + c)( \sin x) + ( 2 a x + b) ( -\cos x) + 2a ( -\sin x ) +c$$

**DEFINITE INTEGRAL**

**PROPERTIES OF DEFINITE INTEGRALS:**

**1.** $\int_a^b f(x)dx = -\int_b^a f(x) \, dx$

**2.** $\int_0^a f(x)dx = \int_0^a f(a-x) \, dx$

**3.** $\int_{-a}^a f(x)dx = 2\int_0^a f(x) \, dx \;\; if \;\; f(x) \; is \; even$

$= 0 \; if \;\; f(x) \; is \; odd$

**4.** $\int_a^b f(x)dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$ , a < c < b

**5.** $\int_0^{2a} f(x)dx = 2\int_0^a f(x) \, dx \;\; if \;\; f(2a-x) = f(x)$

$= 0 \; if \;\; f(2a-x) = -f(x)$

**6.** $\int_0^{\pi} f(\sin x) dx = 2\int_0^{\frac{\pi}{2}} f(\sin x) \, dx$

**Problems:**

**1.Solve** $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\ x}}{\sqrt{\sin\ x}+\sqrt{\cos x}}\,dx$

**Solution:**

I $=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ (1)

$$=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$$

$=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}++\sqrt{\cos x}}\,dx$ (2)

(1)+(2) $\Rightarrow$ 2I $=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \frac{\pi}{2}$

Hence I= $\frac{\pi}{4}$ .

**2. Solve** $\int_0^{\frac{\pi}{2}} \log\sin x\ dx$

**Solution:**

I= $\int_0^{\frac{\pi}{2}} \log\sin x\ dx$

= $\int_0^{\frac{\pi}{2}} \log\cos x\ dx$ ( by property 2)

2I = $\int_0^{\frac{\pi}{2}} (\ \log\sin x+\log\ \cos x\ )\,dx$

= $\int_0^{\frac{\pi}{2}} \log\ (\sin x\cos x)\ dx = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2x}{2}\right)dx = \int_0^{\frac{\pi}{2}} \log\sin 2x\ dx - \int_0^{\frac{\pi}{2}} \log 2\ dx$

$$= \int_0^{\pi} \log \sin y \left(\frac{dy}{2}\right) - \frac{\pi}{2}\log 2 = \frac{1}{2}\int_0^{\pi} \log \sin y \; dy - \frac{\pi}{2}\log 2$$

$$= \frac{2}{2}\int_0^{\frac{\pi}{2}} \log \sin y \; dy - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2 \Rightarrow I = -\frac{\pi}{2}\log 2.$$

**3. Solve** $\int_0^{\frac{\pi}{2}} \log \; (\tan x + \cot x)\, dx$

**Solution:**

$$\int_0^{\frac{\pi}{2}} \log \; (\tan x + \cot x)\, dx = \int_0^{\frac{\pi}{2}} \log \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \log \left(\frac{\sin^2 x + \cos^2 x}{\sin x . \cos x}\right) dx = \int_0^{\frac{\pi}{2}} \log \left(\frac{1}{\sin x . \cos x}\right) dx = -\int_0^{\frac{\pi}{2}} \log \sin x \, dx - \int_0^{\frac{\pi}{2}} \log \cos x \, dx$$

$$= \frac{\pi}{2}\log 2 + \frac{\pi}{2}\log 2 = \pi \log 2.$$

**4. Solve** $\int_0^{\frac{\pi}{4}} \log \; (1 + \tan x)\, dx$

**Solution:**

$$I = \int_0^{\frac{\pi}{4}} \log \; (1 + \tan x)\, dx = \int_0^{\frac{\pi}{4}} \log \left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$$

$$= \int_0^{\frac{\pi}{4}} \log \left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx = \int_0^{\frac{\pi}{4}} \log \left(\frac{2}{1 + \tan x}\right) dx = \int_0^{\frac{\pi}{4}} \log 2 - \log (1 + \tan x) \; dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2 \;\; dx - I$$

$$2I = \frac{\pi}{4}\log 2 ,$$

$$I = \frac{\pi}{8}\log 2$$