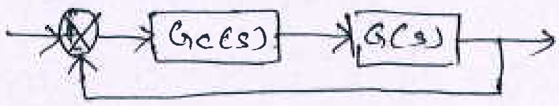
UNIT V
Compensation & Controllers

PI, PD, PID Controllers



A controller with t/f $G_c(s)$ can be introduced in cascade with open loop transfer function, $G(s)$ to modify the transient and steady state response of the system.

The different types of controllers employed in control system are the following

1. Proportional controller (P-controller)
2. Proportional-plus-integral controller (PI-controller)
3. Proportional-plus-derivative controller (PD-controller)
4. Proportional-plus-derivative-plus-integral controller (PID controller)

T.F of P-controller, $G_c(s) = \frac{U(s)}{E(s)} = K_p$

K_p – Proportional gain

T.F of PI controller, $G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s}$

K_i – Integral constant or gain

T.F of PD controller, $G_C(s) = \frac{U(s)}{E(s)} = K_p + K_d s$

K_d – Derivative constant or gain

T.F of PID controller, $G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s$

Procedure for Design of PD/PI/PID Controller in frequency domain

Step 1: Determine the magnitude and phase of uncompensated open loop sinusoidal transfer function (i.e $G(j\omega)$)

Let $A_1 = |G(j\omega)|$ at $\omega = \omega_1$

and $\phi_1 = \angle G(j\omega)$ at $\omega = \omega_1$

Step 2: Determine the phase margin of uncompensated system and the angle to be contributed by the controller to achieve the desired phase margin.

Let γ_u = Phase margin of uncompensated system

γ_d = Desired phase margin at ω_1

θ = Phase angle of the controller at $\omega = \omega_1$

Now $\gamma_u = 180 + \phi_1$

$\theta = \gamma_d - \gamma_u$

Step 3: Determine the transfer function of the controller

a) PD controller

Derivative constant, $K_d = \dfrac{\sin\theta}{\omega_1 A_1}$

Proportional constant, $K_p = \dfrac{\cos\theta}{A_1}$

T.F of PD controller } $G_c(s) = (K_p + K_d s) = K_p\left(1 + \dfrac{K_d}{K_p} s\right)$

b) PI controller

Integral constant, $K_i = \dfrac{-\omega_1 \sin\theta}{A_1}$

Proportional constant, $K_p = \dfrac{\cos\theta}{A_1}$

T.F of PI controller } $G_c(s) = \left(K_p + \dfrac{K_i}{s}\right) = \dfrac{K_i\left(1 + \dfrac{K_p}{K_i} s\right)}{s}$

c) PID controller

T.F of PID controller } $G_c(s) = \left(K_p + K_d s + \dfrac{K_i}{s}\right) = \dfrac{K_d\left(s^2 + \dfrac{K_p}{K_d} s + \dfrac{K_i}{K_d}\right)}{s}$

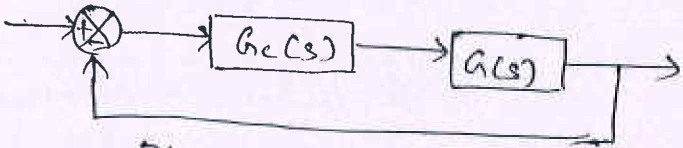
Evaluate K_i such that the compensated system satisfies the error requirement. For example if the compensated system is type 1 system then, $K_v = \lim_{s \to 0} s\, G_c(s)\, G(s)$ will give the value of K_i

Derivative constant $K_d = \dfrac{\sin\theta}{\omega_1 A_1} + \dfrac{K_i}{\omega_1^2}$

$\therefore$ Proportional constant, $K_p = \frac{\cos\theta}{A_1}$

Step 4: Determine the open loop transfer function of compensated system

The t.f of the controller is placed in series with G(s) as shown in fig



Block diagram of system with cascade controller

open loop t.f of compensated system, $G_o(s) = G_c(s) \times G(s)$

Step 5: Verify the design by calculating phase margin of compensated system

Let $A_o = |G_o(j\omega)|$ at $\omega = \omega_1$

$\phi_o = \angle G_o(j\omega)$ at $\omega = \omega_1$

γ_o = Phase margin of compensated system

Now, $\gamma_o = 180 + \phi_o$

It can be observed that $A_o = 1$ and γ_o satisfies the specifications.

1. Consider a unity feed back system with open loop transfer function, $G(s) = \frac{5}{s(s+0.5)(s+1)}$. Design a PD controller so that the phase margin of the system is 30° at a frequency of 1.2 rad/sec.

Step 1: $G(s) = \frac{5}{s(s+0.5)(s+1)} = \frac{5}{s \times 0.5\left(1+\frac{s}{0.5}\right)(1+s)}$

$$= \frac{10}{s(1+2s)(1+s)}$$

Put $s = j\omega$ in $G(s)$

$$\therefore G(j\omega) = \frac{10}{j\omega(1+j2\omega)(1+j\omega)} = \frac{10}{\omega\angle 90 \sqrt{1+4\omega^2}\angle \tan^{-1}2\omega \sqrt{1+\omega^2}\angle \tan^{-1}\omega}$$

$$|G(j\omega)| = \frac{10}{\omega\sqrt{1+4\omega^2}\sqrt{1+\omega^2}}$$

$$\angle G(j\omega) = -90 - \tan^{-1}2\omega - \tan^{-1}\omega$$

The gain cross over frequency of compensated system $\omega_1 = 1.2$ rad/sec

$$A_1 = \frac{10}{1.2 \times \sqrt{1+4\times 1.2^2} \times \sqrt{1+1.2^2}} = 2.052$$

$$\phi_1 = -90 - \tan^{-1}(2\times 1.2) - \tan^{-1}(1.2) = -207.5$$

To find γ_u & θ

$$\gamma_u = 180 + \phi_1 = 180 + (-207.5) = -27.5$$

$$\theta = \gamma_d - \gamma_u = 30 - (-27.5) = 57.5$$

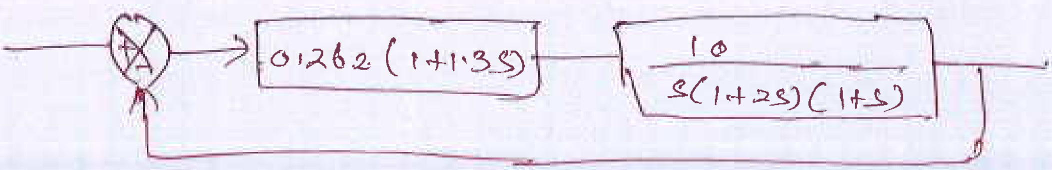
PD Controller

$$K_d = \frac{\sin\theta}{\omega_1 A_1} = \frac{\sin 57.5}{1.2 \times 2.052} = 0.343$$

$$K_p = \frac{\cos\theta}{A_1} = \frac{\cos 57.5}{2.052} = 0.262$$

$$G_c(s) = (K_p + K_d s) = K_p\left(1 + \frac{K_d}{K_p}s\right)$$

$$= 0.262\left(1 + \frac{0.343}{0.262}s\right) = 0.262(1+1.3s)$$



$$G_o(s) = G_c(s) \times G(s)$$

$$= 0.262(1+1.3s) \times \frac{10}{s(1+2s)(1+s)}$$

To Verify the design

Put $s = j\omega$ in $G_0(s)$

$$G_0(j\omega) = \frac{2.62(1+j1.3\omega)}{j\omega(1+j2\omega)(1+j\omega)} = \frac{2.62\sqrt{1+1.69\omega^2}\angle \tan^{-1}1.3\omega}{\omega\angle 90 \sqrt{1+4\omega^2}\angle \tan^{-1}2\omega \sqrt{1+\omega^2}\angle \tan^{-1}\omega}$$

$$A_0 = \frac{2.62\sqrt{1+1.69\omega^2}}{\omega\sqrt{1+4\omega^2}\sqrt{1+\omega^2}}$$

$$\phi_0 = \tan^{-1}1.3\omega - 90 - \tan^{-1}2\omega - \tan^{-1}\omega$$

$$\omega = \omega_1 \quad A = A_{01} = \frac{2.62\sqrt{1+1.69\times 1.2^2}}{1.2\times\sqrt{1+4\times 1.2^2}\sqrt{1+1.2^2}} = 1$$

$$\omega = \omega_1 \quad \phi_0 = \phi_{01} = \tan^{-1}(1.3\times 1.2) - 90 - \tan^{-1}(2\times 1.2) - \tan^{-1}1.2 = -150$$

$$\gamma_0 = 180 + \phi_{01} = 180 - 150 = 30°$$

Phase margin of the compensated system is satisfactory. Hence the design is acceptable.

T.F of PD controller $G_c(s) = 0.262(1+1.3s)$

$$G_0(s) = \frac{2.62(1+1.3s)}{s(1+2s)(1+s)}$$

2) Consider a unity feedback system with open loop transfer function $G(s) = \frac{100}{(s+1)(s+2)(s+10)}$. Design a PID controller, so that the phase margin of the system is 45° at a frequency of 4 rad/sec and the steady state error for unit ramp input is 0.1

$$G(s) = \frac{100}{(s+1)(s+2)(s+10)} = \frac{100}{(1+s)\times 2\times\left(1+\frac{s}{2}\right)\times 10\times\left(1+\frac{s}{10}\right)}$$

$$= \frac{5}{(1+s)(1+0.5s)(1+0.1s)}$$

Put $s = j\omega$ in $G(s)$

$$G(j\omega) = \frac{5}{(1+j\omega)(1+j0.5\omega)(1+j0.1\omega)}$$

$$= \frac{5}{\sqrt{1+\omega^2}\angle\tan^{-1}\omega\sqrt{1+0.25\omega^2}\angle\tan^{-1}0.5\omega\sqrt{1+0.01\omega^2}\angle\tan^{-1}0.1\omega}$$

$$|G(j\omega)| = \frac{5}{\sqrt{1+\omega^2}\sqrt{1+0.25\omega^2}\sqrt{1+0.01\omega^2}}$$

$$\angle G(j\omega) = -\tan^{-1}\omega - \tan^{-1}0.5\omega - \tan^{-1}0.1\omega$$

The gain crossover frequency of compensated system,

$\omega_1 = 4$ rad/sec

Let, $A_1 = |G(j\omega)|$ at $\omega = \omega_1$

$\phi_1 = \angle G(j\omega)$ at $\omega = \omega_1$

$$A_1 = \frac{5}{\sqrt{1+4^2}\times\sqrt{1+0.25\times4^2}\times\sqrt{1+0.01\times4^2}} = 0.5$$

$$\phi_1 = -\tan^{-1}4 - \tan^{-1}(0.5\times4) - \tan^{-1}(0.1\times4) = -161^\circ$$

To find γ_u & θ

$$\gamma_u = 180 + \phi_1 = 180 - 161 = 19^\circ$$

$$\theta = \gamma_d - \gamma_u = 45 - 19 = 26^\circ$$

To find T/F of PID controller

$e_{ss} = 0.1$ for unit ramp input

$$K_v = \frac{1}{e_{ss}} = \frac{1}{0.1} = 10$$

$$K_v = \lim_{s\to0} s\, G_c(s)\, G(s)$$

$$G_c(s) = K_p + K_d s + \frac{K_i}{s} = \frac{K_d s^2 + K_p s + K_i}{s}$$

$$G(s) = \frac{5}{(1+s)(1+0.5s)(1+0.1s)}$$

$$K_v = \lim_{s\to0} s\frac{(K_d s^2 + K_p s + K_i)}{s} \times \frac{5}{(1+s)(1+0.5s)(1+0.1s)} = 10$$

$5K_i = 10$ (or) $K_i = \frac{10}{5} = 2$

$$K_d = \frac{\sin\theta}{\omega_1 A_1} + \frac{K_i}{\omega_1^2} = \frac{\sin(26)}{4 \times 0.5} + \frac{2}{4^2} = 0.344$$

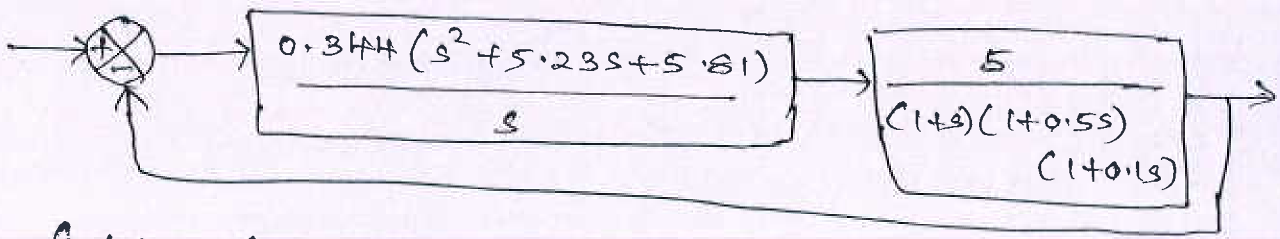
$$K_p = \frac{\cos\theta}{A_1} = \frac{\cos 26}{0.5} = 1.8$$

$$G_c(s) = \left(K_p + K_d s + \frac{K_i}{s}\right) = \left(1.8 + 0.344 s + \frac{2}{s}\right)$$

$$= \frac{0.344 s^2 + 1.8 s + 2}{s} = \frac{0.344\left(s^2 + \frac{1.8}{0.344} s + \frac{2}{0.344}\right)}{s}$$

$$= \frac{0.344\,(s^2 + 5.23 s + 5.81)}{s}$$

To find open loop t/f of compensated system



$$G_o(s) = G_c(s) \times G(s)$$

$$= \frac{0.344\,(s^2 + 5.23 s + 5.81)}{s} \times \frac{5}{(1+s)(1+0.5s)(1+0.1s)}$$

$$= \frac{1.72\,(s^2 + 5.23 s + 5.81)}{s(1+s)(1+0.5s)(1+0.1s)}$$

To verify the design

Put $s = j\omega$ in $G_o(s)$

$$G_o(j\omega) = \frac{1.72\,(-\omega^2 + j\,5.23\,\omega + 5.81)}{j\omega(1+j\omega)(1+j0.5\omega)(1+j0.1\omega)}$$

$$= \frac{1.72\sqrt{(5.81-\omega^2)^2 + (5.23\omega)^2}\ \angle \tan^{-1}\frac{5.23}{5.81-\omega^2}}{\omega\angle 90\ \sqrt{1+\omega^2}\angle\tan^{-1}\omega\ \sqrt{1+(0.5\omega)^2}\angle\tan^{-1}0.5\omega\ \sqrt{1+(0.1\omega)^2}\angle\tan^{-1}0.1\omega}$$

$A_0 = |G_0(j\omega)| \quad \phi_0 = \angle G_0(j\omega)$

$$A_0 = \frac{1.72\sqrt{(5.81-\omega^2)^2 + (5.23\omega)^2}}{\omega\sqrt{1+\omega^2}\sqrt{1+(0.5\omega)^2}\sqrt{1+(0.1\omega)^2}}$$

$$\phi_0 = \tan^{-1}\frac{5.23\omega}{5.81-\omega^2} - 90 - \tan^{-1}\omega - \tan^{-1}0.5\omega - \tan^{-1}0.1\omega \quad \text{for } \omega < \sqrt{5.81}$$

$$= 180 + \tan^{-1}\frac{5.23\omega}{5.81-\omega^2} - 90 - \tan^{-1}\omega - \tan^{-1}0.5\omega - \tan^{-1}0.1\omega \quad \text{for } \omega > \sqrt{5.81}$$

At $\omega = \omega_1$, $A_0 = A_{01} = \dfrac{1.72\sqrt{(5.81-4^2)^2 + (5.23\times 4)^2}}{4\times\sqrt{1+4^2}\times\sqrt{1+(0.5\times 4)^2}\times\sqrt{1+(0.1\times 4)^2}} = 1$

At $\omega = \omega_1$, $\phi_0 = \phi_{01} = 180 + \tan^{-1}\dfrac{5.23\times 4}{5.81-4^2} - 90 - \tan^{-1}4 - \tan^{-1}(0.5\times 4) - \tan^{-1}(0.1\times 4)$

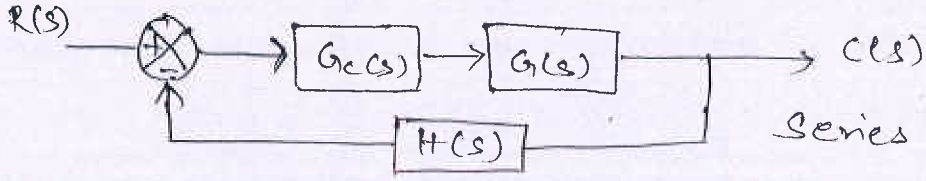
$= -135°$

$\gamma_0 = 180 + \phi_{01} = 180 - 135 = 45°$

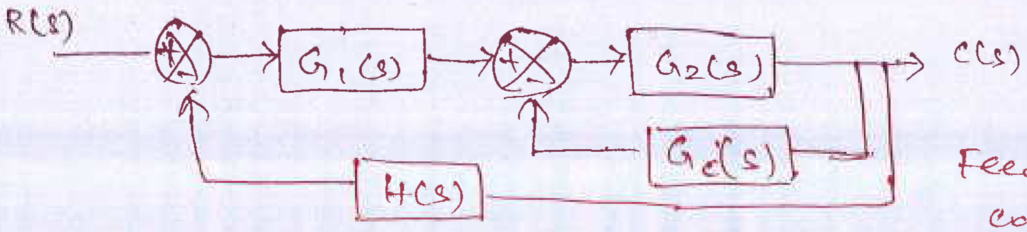
The phase margin of the compensated system, meets the given specification. Hence the design is acceptable.

Compensator

A device inserted into the system for the purpose of satisfying the specifications is called compensator. The different types of compensators are lag compensator, lead compensator & lag-lead compensator.



Series compensation

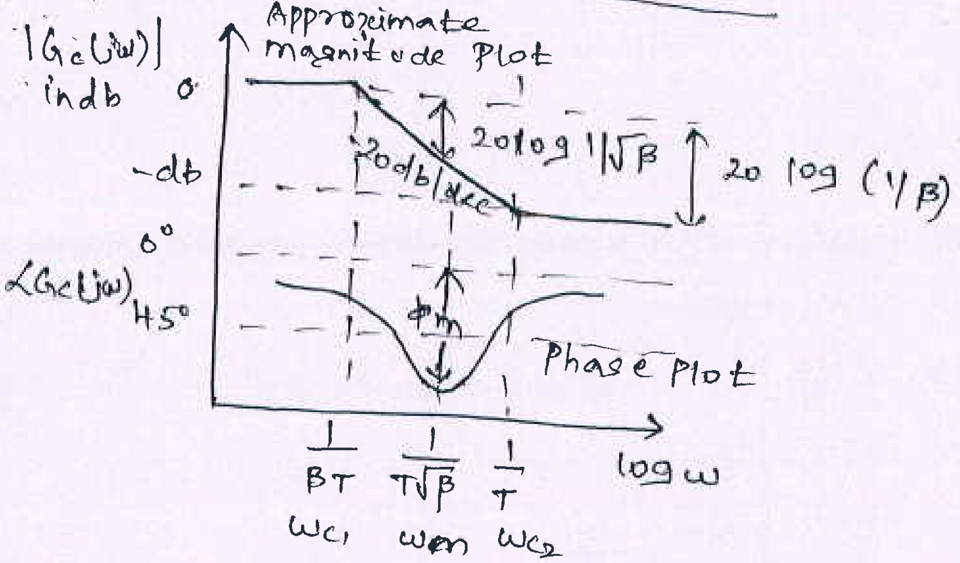


Feedback / parallel compensation

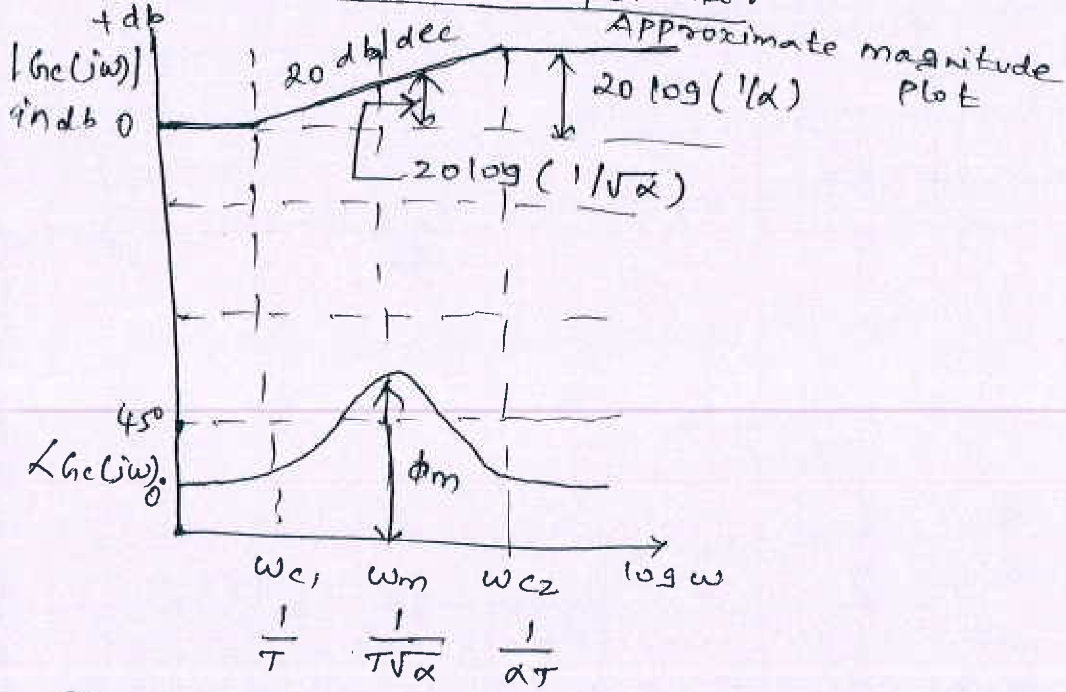
Bode plot of lag compensator

$T = R_2 C$

$\beta = \frac{(R_1 + R_2)}{R_2}$



Bode plot of lead compensator



Procedure for design of lead compensator using Bode plot

The following steps may be followed to design a lead compensator using bode plot and to be connected in series with T/F of uncompensated system, G(s).

step 1 : The open loop gain K of the given system is determined to satisfy the requirement of the error constant.

step 2 : The bode plot is drawn for the uncompensated system using the value of K, determined from the previous step.

step 3: The phase margin of the uncompensated system is determined from the bode plot

Step 4: Determine the amount of Phase angle to be contributed by the lead n/w by using the formula given below

$$\phi_m = \gamma_d - \gamma + \varepsilon$$

ϕ_m – max Phase lead angle of the lead compensator

γ_d - Desired Phase margin

γ – Phase margin of the uncompensated system

ε – Additional phase lead to compensate for shift in gain cross over frequency.

Step 5: Determine the t/f of lead compensator

calculate α using the equation, $\alpha = \dfrac{1-\sin\phi_m}{1+\sin\phi_m}$

From the bode plot, determine the frequency at which the magnitude of $G(j\omega)$ is $-20\log\dfrac{1}{\sqrt{\alpha}}$ db. This frequency is ω_m.

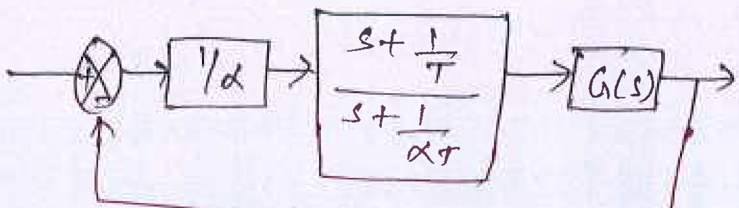
Calculate T from the relation, $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$

$$\therefore T = \frac{1}{\omega_m\sqrt{\alpha}}$$

T/f of lead compensator $\Big\}$ $G_c(s) = \dfrac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = \dfrac{\alpha(1+sT)}{(1+\alpha sT)}$

Step 6: Determine the open loop t/f of compensated system.

The lag compensator is connected in series with G(s) a shown. When the lead n/w is inserted in series with the plant, the open loop gain of the system is attenuated by the factor α ($\because \alpha < 1$). So an amplifier with the gain of $\frac{1}{\alpha}$ has to introduced in series with the compensator to nullify



Block diagram of lead compensated system the attenuation caused by the lead compensator.

open loop t/f of the over all system $\Big\}$ $G_o(s) = \dfrac{1}{\alpha} \dfrac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} G(s)$

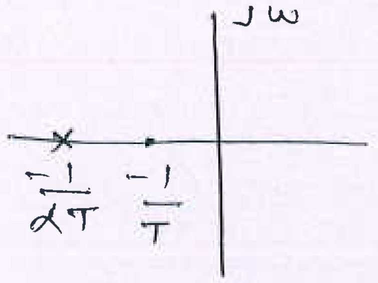
$$= \frac{1}{\alpha} \frac{\alpha(1+sT)}{(1+s\alpha T)} G(s)$$

$$= \frac{(1+sT)\,G(s)}{(1+s\alpha T)}$$

step 7: Verify the design

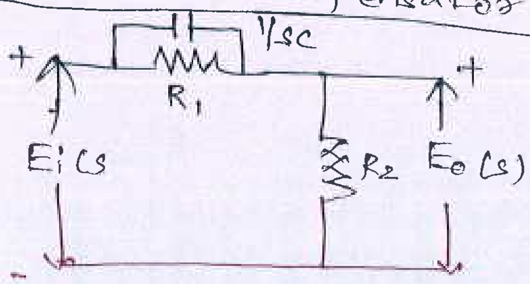
Finally the Bode plot of the compensated system is drawn and verify whether it satisfies the given specifications. If the phase margin of the compensated system is less than the required phase margin then repeat step 4 to by taking ε as 5° more than the previous design.

S-plane Representation of lead compensator



$$G_c(s) = \frac{s+z_c}{s+p_c} = \frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\alpha T}\right)}$$

Realisation of lead compensator using electrical n/w

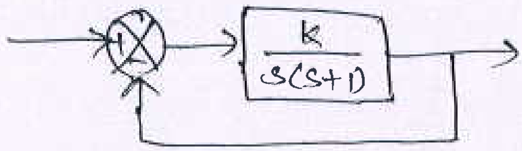


$$\frac{E_o(s)}{E_i(s)} = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

$T = R_1 C$ $\quad \alpha = \dfrac{R_2}{R_1+R_2}$

Design a phase lead compensator for the system to satisfy the following specifications. (i) The phase margin of the system $\geq 45^\circ$ (ii) steady state error for a unit ramp i/p $\leq \frac{1}{15}$ (iii) The gain crossover frequency of the ...

be less than 7.5 rad/sec



Step 1:
Determine K

$$e_{ss} = \frac{1}{K_V} = \frac{1}{15} \quad \therefore \quad K_V = 15$$

$$K_V = \lim_{S \to 0} S\, G(s)\, H(s)$$

$$= \lim_{S \to 0} S \frac{K}{S(S+1)} = K$$

$$K_V = K = 15$$

Step 2:
Draw bode plot

$$G(s) = \frac{K}{S(S+1)} = \frac{15}{S(S+1)} = \frac{15}{j\omega(1+j\omega)}$$

$\omega_{c1} = 1$ rad/sec

Term	Corner frequency rad/sec	slope db/dec	change in slope db/dec
$\frac{15}{j\omega}$	-	-20	-
$\frac{1}{(1+j\omega)}$	$\omega_{c1} = 1$	-20	-40

$\omega_l = 0.1 \qquad \omega_h = 10$

$$A = 20 \log \left| \frac{15}{j\omega} \right| = 20 \log \frac{15}{0.1} = 44 \text{ db}$$

$$A = 20 \log \left| \frac{15}{j\omega} \right| = 20 \log \frac{15}{1} = 24 \text{ db}$$

$$A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c1}} \right] + A\,(\text{at } \omega = \omega_{c1})$$

$$= -40 \times \log \frac{10}{1} + 24 = -16 \text{ db}$$

$\phi = \angle G(j\omega) = -90 - \tan^{-1}\omega$

ω rad/sec	0.1	0.5	1	2	5	10
φ deg	−96	−117	−135	−153	−169	−174

Step 3: Determine the phase margin of uncompensated system

$$\phi_{gc} = -167^\circ$$

$$\gamma = 180 + \phi_{gc} = 180 - 167 = 13^\circ$$

The system requires a phase margin of 45° but the available phase margin is 13° and so lead compensation should be employed to improve the phase margin.

Step 4 : Find ϕ_m

The desired phase margin $\gamma_d \geq 45^\circ$

Let additional phase lead required $\varepsilon = 5^\circ$

$$\phi_m = \gamma_d - \gamma + \varepsilon = 45^\circ - 13^\circ + 5^\circ = 37^\circ$$

Step 5

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} = \frac{1-\sin 37}{1+\sin 37} = 0.2486 \simeq 0.25$$

$$\omega_m = -20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.25}} = -6\,db$$

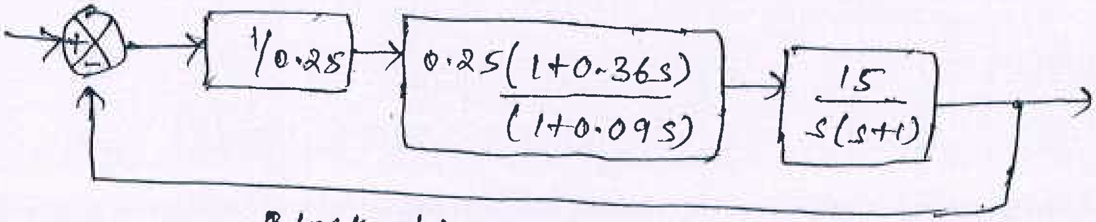
From the bode plot of uncompensated system the frequency ω_m corresponding to a db gain of −6db is found to be 5.6 rad/sec. $\omega_m = 5.6$ rad/sec

$$T = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{5.6\sqrt{0.25}} = 0.357 \simeq 0.36$$

T/f of lead compensator

$$G_c(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} = \frac{\alpha(1+sT)}{(1+s\alpha T)} = \frac{0.25(1+0.36s)}{(1+0.09s)}$$

Step 6 : Open loop t/f of compensated system



Block diagram of lead compensated system

$$G_o(s) = \frac{1}{0.25} \times \frac{0.25(1+0.36s)}{(1+0.09s)} \times \frac{15}{s(s+1)}$$

$$= \frac{15(1+0.36s)}{s(1+0.09s)(1+s)}$$

Step 7 : Draw the bode plot of compensated system to verify the design

$s = j\omega \quad \therefore G_o(j\omega) = \dfrac{15(1+j0.36\omega)}{j\omega(1+j0.09\omega)(1+j\omega)}$

$\omega_{c1} = \frac{1}{1}$ rad/sec $\omega_{c2} = \frac{1}{0.36} = 2.8$ rad/sec $\omega_{c3} = \frac{1}{0.09} = 11.1$ rad/sec

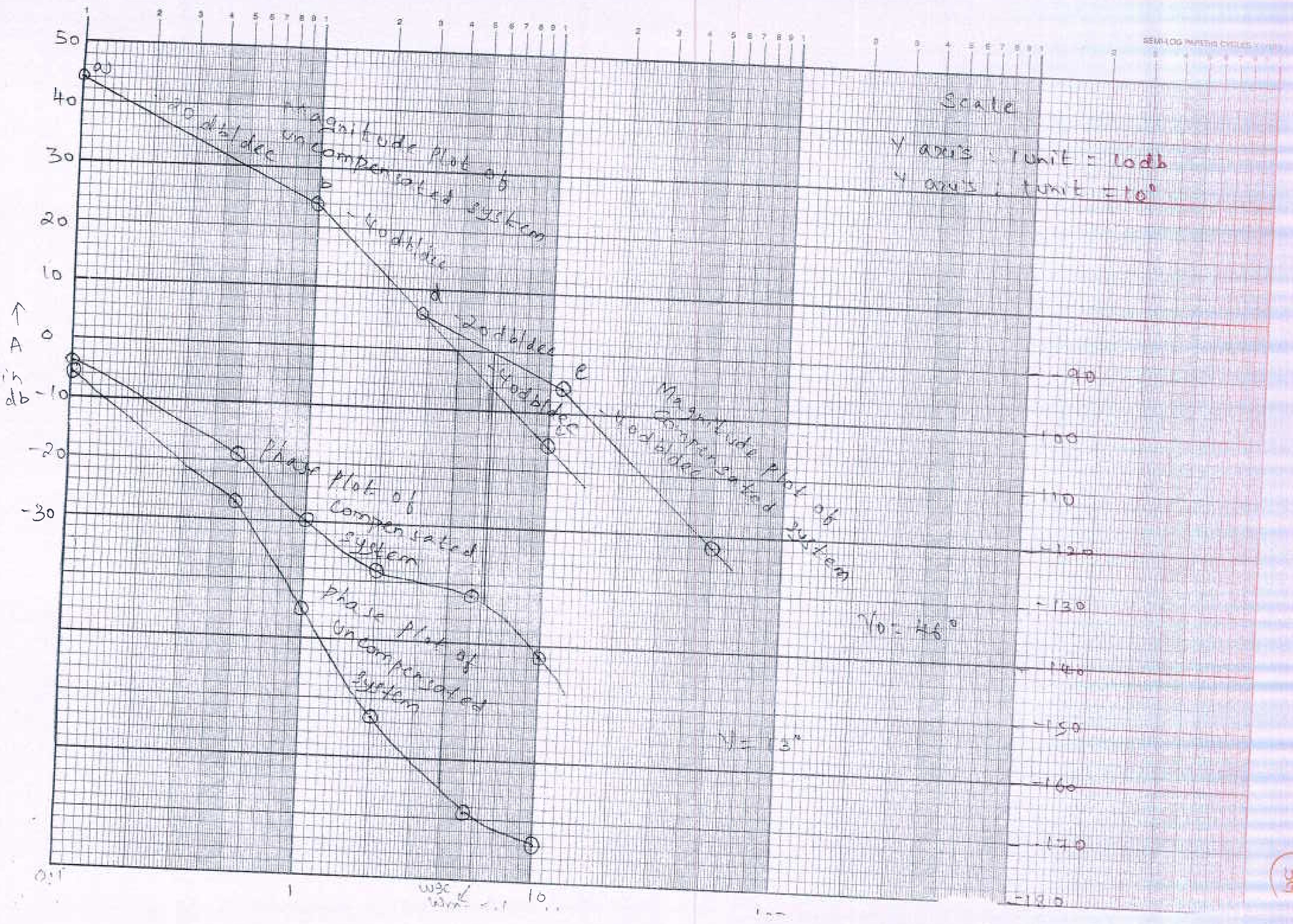
Term	Corner frequency rad/sec	Slope db/dec	change in slope db/dec
$\frac{15}{j\omega}$	-	-20	
$\frac{1}{1+j\omega}$	$\omega_{c1} = 1$	-20	-40
$1+j0.36\omega$	$\omega_{c2} = 2.8$	+20	-20
$\frac{1}{1+j0.09\omega}$	$\omega_{c3} = 11.1$	-20	-40

At $\omega = \omega_l = 0.1$ $\quad A_o = 20\log\left|\frac{15}{j\omega}\right| = 20\log\left|\frac{15}{0.1}\right| = 44\,db$

At $\omega = \omega_{c1} = 1$ $\quad A_o = 20\log\left|\frac{15}{j\omega}\right| = 20\log\frac{15}{1} = 24\,db$

At $\omega = \omega_{c2} = 2.8$ $\quad A_o = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right]$

$+$ (gain at



SEMI-LOG PAPER 5 CYCLES
Scale
Y axis : 1 unit = 10 db
Y axis : 1 unit = 10°
A in db
50
40
30
20
10
0
-10
-20
-30
-90
-100
-110
-120
-130
-140
-150
-160
-170
-180
0.1
1
10
100
magnitude plot of uncompensated system
Magnitude plot of compensated system
Phase plot of compensated system
Phase plot of uncompensated system
-20 db/dec
-40 db/dec
-20 db/dec
-40 db/dec
-40 db/dec
a
b
d
e
γ₀ = 46°
γ = 13°

$$= -40 \times \log \frac{2.8}{1} + 24 = 6\,db$$

At $\omega = \omega_{c3} = 11.1$ rad/sec

$$A_0 = \left[\text{slope from } \omega_{c2} \text{ to } \omega_{c3} \times \log \frac{\omega_{c3}}{\omega_{c2}}\right] + \left(\begin{array}{c}\text{gain at}\\ \omega = \omega_{c2}\end{array}\right)$$

$$= -20 \times \log \frac{11.1}{2.8} + 6 = -6\,db$$

A $\omega = \omega_h = 50$ rad/sec

$$A_0 = \left[\text{slope from } \omega_{c3} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c3}}\right] + \left[\begin{array}{c}\text{gain at}\\ \omega = \omega_{c3}\end{array}\right]$$

$$= -40 \times \log \frac{50}{11.1} + (-6) = -32\,db$$

$$\phi_0 = \angle G_0(j\omega) = \tan^{-1} 0.36\omega - 90 - \tan^{-1} 0.09\omega - \tan^{-1}\omega$$

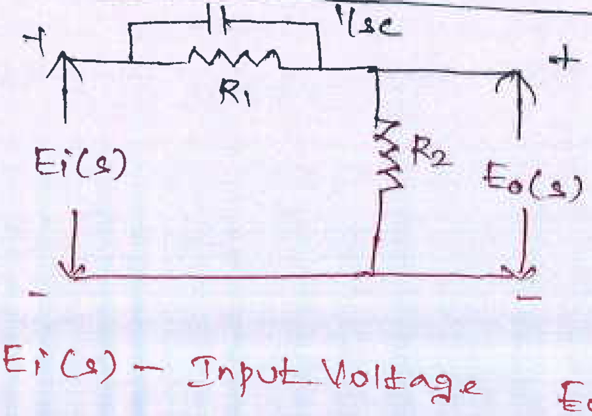
ω	0.1	0.5	1	2	5	10
ϕ_0	-94	-109	-120	-128	-132	-142

From the bode plot of compensensated system we get $\phi_{gc0} = -134°$

$$\gamma_0 = 180 + \phi_{gc0} = 180 - 134 = 46°$$

The phase margin of the compensated system is satisfactory. Hence the design is acceptable.

Realisation of lead compensator using Electrical Network



Ei(s) – Input Voltage Eo(s) – o/p Voltage

The i/p Voltage is applied to the series combination of $R_1 \| C$ & R_2. The o/p Voltage is obtained across R_2.

By Voltage division rule

o/p Voltage $E_o(s) = E_i(s) \times \dfrac{R_2}{R_2 + \dfrac{\left(R_1 \times \frac{1}{sC}\right)}{\left(R_1 + \frac{1}{sC}\right)}}$

$$E_o(s) = E_i(s) \times \frac{R_2}{R_2 + \dfrac{R_1}{R_1 sC + 1}} = E_i(s) \frac{R_2}{\dfrac{R_2(R_1 sC+1) + R_1}{R_1 sC + 1}}$$

T.F of Electrical n/w

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2(R_1 sC + 1)}{R_1 R_2 C s + R_2 + R_1} = \frac{R_1 C R_2 \left[s + \dfrac{1}{R_1 C}\right]}{R_1 C R_2 \left[s + \dfrac{(R_1 + R_2)}{R_1 C R_2}\right]}$$

$$= \frac{\left[s + \dfrac{1}{R_1 C}\right]}{s + \left[\dfrac{1}{\dfrac{R_2}{R_1 + R_2}}\right] \dfrac{1}{R_1 C}} \quad -①$$

General form of Lead compensator t/f is

$$G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \quad -②$$

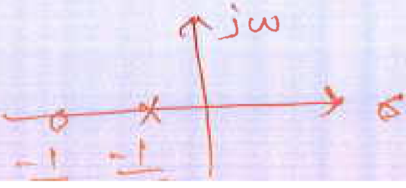
On comparing ① & ②

$$\frac{E_o(s)}{E_i(s)} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \qquad T = R_1 C \qquad \alpha = \frac{R_2}{R_1 + R_2}$$

A compensator having the characteristics of a lead n/w is called lead compensator.

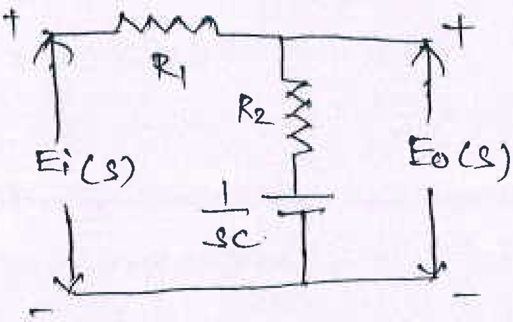
A compensator having the characteristics of a lag n/w is called lag compensator.

S Plane representation of lag compensator



$G_c(s) = \dfrac{s + Z_c}{s + P_c} = \dfrac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$

Realisation of lag compensator using Electrical network



$E_i(s)$ = input Voltage

$E_o(s)$ = output Voltage

The i/p voltage is applied to the series combination of R_1, R_2 & c, The o/p Voltage is obtained across series combination of R_2 & c.

By Voltage division rule

$$E_o(s) = \frac{E_i(s) \times \left(R_2 + \frac{1}{sc}\right)}{\left(R_1 + R_2 + \frac{1}{sc}\right)} = E_i(s) \frac{(scR_2+1)/sc}{[sc(R_1+R_2)+1]/sc}$$

$$= E_i(s) \frac{[scR_2+1]}{[sc[R_1+R_2]+1]}$$

T.F of electrical n/w } $$\frac{E_o(s)}{E_i(s)} = \frac{\cancel{cR_2}\, cR_2\left(s + \frac{1}{cR_2}\right)}{\cancel{cR_2}\, c(R_1+R_2)\left[s + \frac{1}{c(R_1+R_2)}\right]}$$

$$= \frac{\left(s + \frac{1}{R_2c}\right)}{\left(\frac{R_1+R_2}{R_2}\right)\left[s + \frac{1}{\left(\frac{R_1+R_2}{R_2}\right) \times \cancel{R_2}\, c}\right]}$$

But the t/f of lag compensator is given by

$$G_c(s) = \frac{\left(s + \frac{1}{T}\right)}{\left(s + \frac{1}{\beta T}\right)}$$

Comparing we get

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{\beta}\frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\beta T}\right)}$$

where $T = R_2 C$

$\beta = \frac{(R_1+R_2)}{R_2}$

Procedure for the design of lag compensator using Bode Plot

Step 1: Choose the value of K in uncompensated system to meet the steady state requirement.

Step 2: Sketch the bode plot of uncompensated system.

Step 3: Determine the phase margin of the uncompensated system from the bode plot. If the phase margin does not satisfy the requirement then lag compensation is required.

Step 4: Choose a suitable value for the phase margin of the compensated system

$$\gamma_n = \gamma_d + \epsilon$$

Step 5: Determine the new gain cross over frequency ω_{gcn}. The new ω_{gcn} is the frequency corresponding to a Phase margin of γ_n on the bode plot of uncompensated system.

$$\gamma_n = 180 + \phi_{gcn} \text{ (or) } \phi_{gcn} = \gamma_n - 180°$$

The new gain cross over frequency, ω_{gcn} is given by the frequency at which the Phase of $G(j\omega)$ is ϕ_{gcn}

Step 6: Determine the Parameter, β of the compensator. The value of β is given by the magnitude of $G(j\omega)$ at new gain cross over frequency, ω_{gcn}. Find the db gain (A_{gcn}) at new gain cross over frequency, ω_{gcn}.

Now, $A_{gcn} = 20 \log \beta$ (or) $\frac{A_{gcn}}{20} = \log \beta$

$\therefore \beta = 10^{A_{gcn}/20}$

etermine the transfer function of lag compensator
e the zero of the compensator arbitrarily at $1/10^{th}$ of the new gain crossover frequency, ω_{gcn}.

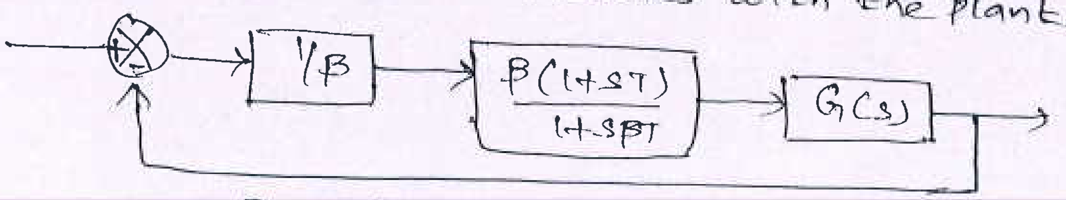
Zero of the lag compensator, $Z_c = \frac{1}{T}$

Now, $T = \frac{10}{\omega_{gcn}}$

Pole of the lag compensator, $P_c = \frac{1}{\beta T}$

Transfer function of lag compensator $\Big\}$ $G_c(s) = \dfrac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = \beta\left(\dfrac{1+sT}{1+s\beta T}\right)$

Step 8: Determine the open loop t/f of compensated system. The lag compensator is connected in series with the plant.



Block diagram of lag compensated system

When the lag compensator is inserted in series with the plant, the open loop gain of the system is amplified by the factor β. If the gain produced is not required then attenuator with gain 1/β can be introduced in series with the lag compensator to nullify the gain produced by lag compensator.

The open loop t/f of the compensated system,

$$G_o(s) = \frac{1}{\beta} G_c(s)\, G(s) = \frac{1}{\beta}\, \frac{\beta(1+sT)}{(1+s\beta T)}\, G(s)$$

$$= \frac{(1+sT)}{(1+s\beta T)}\, G(s)$$

Step 9: Determine the actual phase margin of compensated system. Calculate the actual phase angle of the compensated t/f at new gain crossover frequency

Let, ϕ_{gco} = Phase of $G_o(j\omega)$ at $\omega = \omega_{gcn}$

Actual Phase margin of the compensated system $\gamma_o = 180 + \phi_{gco}$

If the actual phase margin satisfies the given specification then the design is accepted. Otherwise repat the procedure from step 4 . by taking ϵ as 5° more than previous design.

A unity feedback system has an open loop t/f, $G(s) = \frac{K}{s(1+2s)}$.

Design a suitable lag compensator so that phase margin is 40° and the steady state error for ramp input is less than or equal to 0.2

Step 1 : Calculation of gain, K

$e_{ss} \le 0.2$ Let $e_{ss} = 0.2$

$e_{ss} = \frac{1}{K_v}$ $K_v = \frac{1}{e_{ss}} = \frac{1}{0.2} = 5$

$K_v = \lim_{s \to 0} s\,G(s)\,H(s)$ $H(s) = 1$

$= \lim_{s \to 0} s \frac{K}{s(1+2s)} = K$ $\therefore K = 5$

Step 2 : Bode plot of uncompensated system

$$G(s) = \frac{5}{s(1+2s)} = \frac{5}{j\omega(1+j2\omega)}$$

$\omega_{c1} = 1/2 = 0.5$ rad/sec

Term	Corner frequency rad/sec	slope db/dec	change in slope db/dec
$\frac{5}{j\omega}$	–	−20	
$\frac{1}{1+j2\omega}$	$\omega_{c1} = \frac{1}{2} = 0.5$	−20	−40

$\omega_l = 0.1$ $\omega_h = 10$

$\omega = \omega_l$, $A = 20 \log \left|\frac{5}{j\omega}\right| = 20 \log \frac{5}{0.1} = 34$ db

At $\omega = \omega_{c_1}$, $A = 20 \log \left| \frac{5}{j\omega} \right| = 20 \log \frac{5}{0.5} = 20\,db$

At $\omega = \omega_h$, $A = \left[\text{slope from } \omega_{c_1} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_c} \right] + \left[A \text{ (at } \omega = \omega_c \right.$

$= -40 \times \log \frac{10}{0.5} + 20 = -32\,db$

$\phi = \angle G(j\omega) = -90 - \tan^{-1} 2\omega$

ω rad/sec	0.1	0.5	1.0	5	10
ϕ deg	-101	-135	-153	-174	-177

Step 3: Determine of Phase margin of uncompensated system

$\phi_{gc} = -162$

$\gamma = 180 + \phi_{gc} = 180 - 162 = 18°$

The system requires a phase margin of 40°, but the available phase margin is 18° and so lag compensation should be employed to improve the phase margin.

Step 4:

$\gamma_d = 40$

$\gamma_n = \gamma_d + \epsilon = 40 + 5 = 45$

Step 5: Determine new gain cross over frequency

$\gamma_n = 180 + \phi_{gcn}$

$\therefore \phi_{gcn} = \gamma_n - 180 = 45 - 180 = -135°$

From the bode plot we found that, the frequency corresponding to a phase of -135° is 0.5 rad/sec

$\therefore$ New gain cross over frequency, $\omega_{gcn} = 0.5$ rad/sec

Step 6: Determine the parameter, β

From the bode plot we found that, the db magnitude at ω_{gcn} is 20 db

$\therefore |G(j\omega)|$ in db at $(\omega = \omega_{gcn}) = A_{gcn} = 20\,db$

Also, $A_{gcn} = 20 \log \beta \quad \therefore \beta = 10^{A_{gcn}/20} = 10^{20/20} = 10$

Step 7: Determine the transfer function of lag compensator.

The zero of the compensator is placed at a frequency one-tenth of ω_{gcn}

$\therefore$ Zero of the lag compensator, $Z_c = \frac{1}{T} = \frac{\omega_{gcn}}{10}$

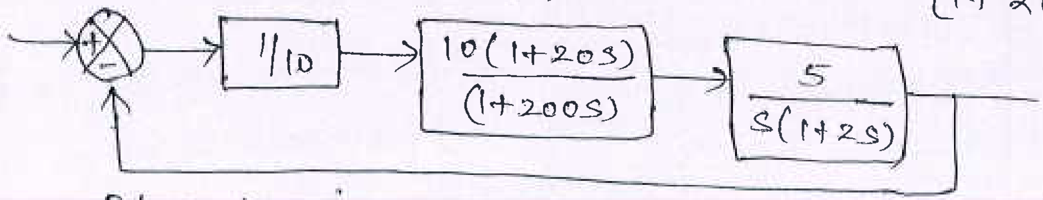
$$T = \frac{10}{\omega_{gcn}} = \frac{10}{0.5} = 20$$

Pole of the lag compensator, $P_c = \frac{1}{\beta T} = \frac{1}{10 \times 20} = \frac{1}{200} = 0.005$

T/f of lag compensator, $G_c(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = \beta \frac{1+sT}{1+s\beta T}$

$$= \frac{10(1+20s)}{(1+200s)}$$

Step 8: Determine the open loop t/f of compensated system



Block diagram of compensated system

$$G_o(s) = \frac{1}{10} \times \frac{10(1+20s)}{(1+200s)} \times \frac{5}{s(1+2s)}$$

$$= \frac{5(1+20s)}{s(1+200s)(1+2s)}$$

Step 9: Actual phase margin of compensated system

$s = j\omega$ $\quad G_o(j\omega) = \frac{5(1+j20\omega)}{j\omega(1+j200\omega)(1+j2\omega)}$

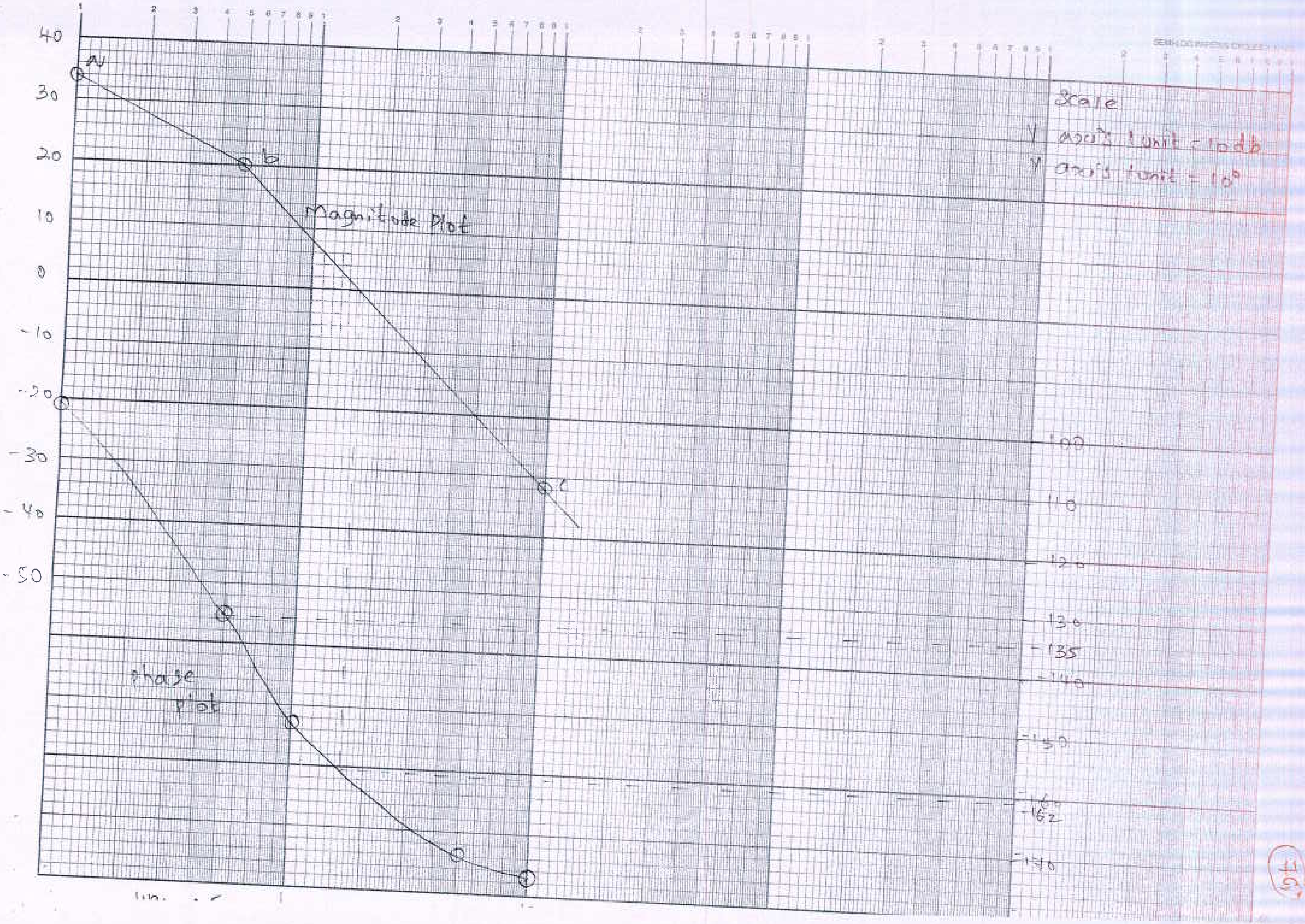
$$\phi_o = \tan^{-1} 20\omega - 90 - \tan^{-1} 200\omega - \tan^{-1} 2\omega$$

$$\phi_{gco} = \tan^{-1}(20 \times 0.5) - 90 - \tan^{-1}(200 \times 0.5) - \tan^{-1}(2 \times 0.5)$$

$$= -140$$

$$\gamma_o = 180 + \phi_{gco} = 180 - 140 = 40^\circ$$

The actual phase margin of the compensated system satisfies the requirement. Hence the design is acceptable.



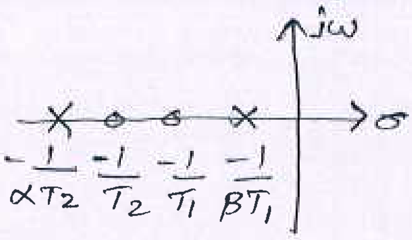
Scale
Y axis 1 unit = 10db
Y axis 1 unit = 10°
Magnitude plot
phase plot
40
30
20
10
0
-10
-20
-30
-40
-50
-100
-110
-120
-130
-135
-140
-150
-160
-162
-170

Lag-Lead compensator

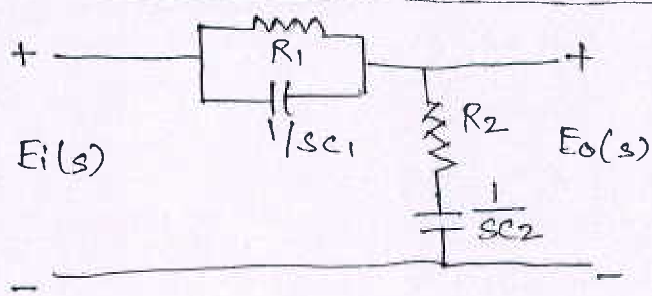
A compensator having the characteristics of lag-lead network is called lag-lead compensator.

S-Plane Representation of lag-lead compensator

$$G_c(s) = \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\underbrace{\left(s+\frac{1}{\beta T_1}\right)}_{\text{lag section}}\underbrace{\left(s+\frac{1}{\alpha T_2}\right)}_{\text{lead section}}}$$



Realisation of lag-lead compensator using electrical n/w



$$E_o(s) = E_i(s)\frac{R_2+\frac{1}{sC_2}}{\left(R_1 \parallel \frac{1}{sC_1}\right)+R_2+\frac{1}{sC_2}}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{sR_2C_2+1}{sC_2}}{\frac{R_1\frac{1}{sC_1}}{R_1+\frac{1}{sC_1}}+\frac{sR_2C_2+1}{sC_2}} = \frac{\frac{sR_2C_2+1}{sC_2}}{\frac{R_1}{sR_1C_1+1}+\frac{sR_2C_2+1}{sC_2}}$$

$$= \frac{\frac{sR_2C_2+1}{sC_2}}{\frac{sR_1C_2+(sR_1C_1+1)(sR_2C_2+1)}{(sR_1C_1+1)(sC_2)}}$$

$$= \frac{(sR_1C_1+1)(sR_2C_2+1)}{sR_1C_2+(sR_1C_1+1)(sR_2C_2+1)}$$

$$= \frac{R_1C_1R_2C_2\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{sR_1C_2+R_1C_1R_2C_2\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}$$

On dividing the numerator and denominator by $R_1C_1R_2C_2$ we get;

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{\frac{s}{R_2C_1}+\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}$$

$$= \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)+\frac{1}{R_1R_2C_1C_2}}$$

The transfer function of lag-lead compensator is given by

$$G_c(s) = \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{1}{\beta T_1}\right)\left(s+\frac{1}{\alpha T_2}\right)}$$

$$= \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{s^2+s\left((1/\beta T_1)+(1/\alpha T_2)\right)+1/\alpha\beta T_1T_2}$$

On comparing equations we get

$$T_1 = R_1C_1 \quad , T_2 = R_2C_2 \, , \quad R_1R_2C_1C_2 = \alpha\beta T_1T_2$$

$$\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1} = \frac{1}{\beta T_1}+\frac{1}{\alpha T_2}$$

$$\alpha\beta = \frac{T_1T_2}{R_1R_2C_1C_2}$$

Procedure for Design of Lag-lead Compensator Using Bode Plot

Step 1: Determine the open loop gain k of the uncompensated system to satisfy the specified error requirement.

Step 2: Draw the bode plot of uncompensate system.

Step 3: From the bode plot determine the gain margin of the uncompensated system.

Let ϕ_{gc} = Phase of $G(j\omega)$ at gain crossover frequency

γ = Phase margin of uncompensated system

If the gain margin is not satisfactory then compensation is required.

Step 4: Choose a new phase margin

Let γ_d = Desired phase margin

Now, new phase margin, $\gamma_n = \gamma_d + \varepsilon$

Choose an initial value of $\varepsilon = 5°$

Step 5: From the bode plot, determine the new gain crossover frequency which is the frequency corresponding to a phase margin of γ_n.

Let ω_{gcn} = New gain crossover frequency

ϕ_{gcn} = Phase of $G(j\omega)$ at ω_{gcn}

$\gamma_n = 180 + \phi_{gcn}$ (or) $\phi_{gcn} = \gamma_n - 180°$

In the phase plot of uncompensated system, the frequency corresponding to a phase of ϕ_{gcn} is the new gain crossover frequency ω_{gcn}.

Choose the gain crossover frequency of the lag compensator, ω_{gcl}, somewhat greater than ω_{gcn} (ie choose ω_{gcl} such that $\omega_{gcl} > \omega_{gcn}$)

Step 6: Calculate β of lag compensator.

Let, $A_{gcl} = |G(j\omega)|$ in db at $\omega = \omega_{gcl}$

From the bode plot find A_{gcl}

Now, $A_{gcl} = 20 \log \beta$ (or) $\beta = 10^{(A_{gcl}/20)}$

Step 7: Determine the transfer function of lag section

The zero of the lag compensator is placed at a frequency one-tenth of ω_{gcl}.

$\therefore$ Zero of lag compensator, $z_{c1} = 1/T_1 = \omega_{gcl}/10$

Now, $T_1 = 10/\omega_{gcl}$

Pole of lag compensator, $p_{c1} = 1/\beta T_1$

Transfer function of lag section $\Big\}$ $G_1(s) = \dfrac{(s + 1/T_1)}{(s + 1/\beta T_1)} = \beta \dfrac{(1 + sT_1)}{(1 + s\beta T_1)}$

Step 8: Determine the transfer function of lead section

Take, $\alpha = 1/\beta$

From the bode plot find ω_m which is the frequency at which the db gain is $-20\log(1/\sqrt{\alpha})$

Now $T_2 = \frac{1}{\omega_m \sqrt{\alpha}}$

T/f of lead section } $G_2(s) = \frac{(s + 1/T_2)}{(s + 1/\alpha T_2)} = \alpha \frac{(1 + sT_2)}{(1 + s\alpha T_2)}$

Step 9: Determine the transfer function of lag-lead compensator

Transfer function of lag-lead compensator, $G_c(s) = G_1(s) \times G_2(s)$

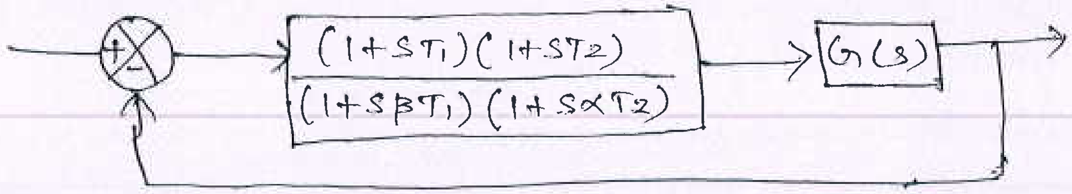
$$= \beta \frac{(1+sT_1)}{(1+s\beta T_1)} \times \alpha \frac{(1+sT_2)}{(1+s\alpha T_2)}$$

Since $\alpha = \frac{1}{\beta}$,

$$G_c(s) = \frac{(1+sT_1)(1+sT_2)}{(1+s\beta T_1)(1+s\alpha T_2)}$$

Step 10: Determine the open loop transfer function of compensated system.

The lag-lead compensator is connected in series with $G(s)$ as shown



Block diagram of lag-lead compensated system.

open loop transfer function of compensated system } $G_o(s) = \frac{(1+sT_1)(1+sT_2)}{(1+s\beta T_1)(1+s\alpha T_2)} \times G(s)$

Step 11: Draw the bode plot of compensated system & verify whether the specifications are satisfied or not. If the specification are not satisfied then choose another choice of α such that, $\alpha \nless 1/\beta$ and repeat the steps 8 to 11.

Consider the unity feedback system whose open loop t/f is $G(s) = K / s(s+3)(s+6)$. Design a lag-lead compensator to meet the following specifications. (i) Velocity error constant $K_V = 80$ (ii) Phase margin, $\gamma \geq 35^\circ$

Step1: Determine K

For unity feed back system,

Velocity error constant, $K_V = \underset{s \to 0}{Lt}\ sG(s)$

Given that, $K_V = 80$.

$$\therefore \underset{s \to 0}{Lt}\ sG(s) = \underset{s \to 0}{Lt}\ s \frac{K}{s(s+3)(s+6)} = 80$$

$$\frac{K}{3 \times 6} = 80 \quad \text{or} \quad K = 80 \times 3 \times 6 = 1440$$

$$\therefore G(s) = \frac{1440}{s(s+3)(s+6)} = \frac{1440}{s \times 3(1+s/3) \times 6(1+s/6)}$$

$$= \frac{80}{s(1+0.33s)(1+0.167s)}$$

Step 2: Bode plot of uncompensated system.

In $G(s)$, put $s = j\omega$

$$\therefore G(j\omega) = \frac{80}{j\omega(1+j0.33\omega)(1+j0.167\omega)}$$

Magnitude Plot

The corner frequencies are ω_{c1} and ω_{c2}.

Here $\omega_{c1} = 1/0.33 = 3$ rad/sec and $\omega_{c2} = 1/0.167 = 6$ rad/sec

Term	Corner frequency rad/sec	slope db/dec	Change in slope db/dec
$\frac{80}{j\omega}$	–	-20	–
$\frac{1}{1+j0.33\omega}$	$\omega_{c1} = 3$	-20	$-20-20=-40$
$\frac{1}{1+j0.167\omega}$	$\omega_{c2} = 6$	-20	$-40-20=-60$

Choose a low frequency ω_l and high frequency ω_h

$\omega_l = 0.5$ rad/sec $\qquad \omega_h = 20$ rad/sec

$\omega_l = 0.5 \quad A = 20 \log \left|\frac{80}{j\omega}\right| = 44$ db

$\omega_{c1} = 3 \quad A = 20\log\left|\frac{80}{j\omega}\right| = 28\,db$

$\omega_{c2} = 6 \quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A(\text{at } \omega = \omega_{c1})$

$= -40 \times \log\frac{6}{3} + 28 = 16\,db$

$\omega_h = 20 \quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A(\text{at } \omega = \omega_{c2})$

$= -60 \times \log\frac{20}{6} + 16 = -15\,db$

Phase plot

$\phi = \angle G(j\omega) = -90 - \tan^{-1} 0.33\omega - \tan^{-1} 0.167\omega$

ω rad/sec	0.5	1.0	3.0	6	10	20
$\angle G(j\omega)$ deg	-104	-118	-161 $\simeq -160$	-198	-222	-244

Step 3: Find phase margin of uncompensated system

$\phi_{gc} = -226$

$\gamma = 180 + \phi_{gc} = 180 - 226 = -46^\circ$

Step 4: Choose a new phase margin

$\gamma_d = 35^\circ \qquad \varepsilon = 5^\circ$

$\gamma_n = \gamma_d + \varepsilon = 35 + 5 = 40^\circ$

Step 5: Determine new gain crossover frequency

$\gamma_n = 180 + \phi_{gcn}$

$\phi_{gcn} = \gamma_n - 180 = 40 - 180 = -140^\circ$

From the bode plot frequency correspoding to a phase of -140° is 1.8 rad/sec. $\omega_{gcn} = 1.8$

Choose ω_{gcl} such that $\omega_{gcl} > \omega_{gcn}$

Let $\omega_{gcl} = 4$ rad/sec

Step 6: Calculate β of lag compensator

$\beta = 10^{A_{gcl}/20} = 10^{23/20} = 14$

From Bode plot ω_{gcl} is 23db

Step 7: Determine the t/f of lag section

$$z_{c1} = \frac{-1}{T_1} = \frac{\omega_{gcl}}{10}$$

$$T_1 = \frac{10}{\omega_{gcl}} = \frac{10}{4} = 2.5$$

$$P_{c1} = \frac{1}{\beta T_1} = \frac{1}{14 \times 2.5} = \frac{1}{35}$$

$$G_1(s) = \frac{\beta(1+sT_1)}{(1+s\beta T_1)} = 14\frac{(1+2.5s)}{(1+35s)}$$

Step 8. Determine the t/f of lead section

Let $\alpha = 1/\beta \quad \alpha = 1/14 = 0.07$

The db gain (magnitude) corresponding to ω_m $\Big\} = -20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.07}} = -11.5\,db = -12\,db$

From the bode plot of uncompensated system the frequency ω_m corresponding to a db pair of -12 db is 17 rad/sec

$\omega_m = 17$ rad/sec

$$T_2 = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{17\sqrt{0.07}} = 0.22$$

$$G_2(s) = \frac{\alpha(1+sT_2)}{(1+s\alpha T_2)} = \frac{0.07(1+0.22s)}{(1+0.0154s)}$$

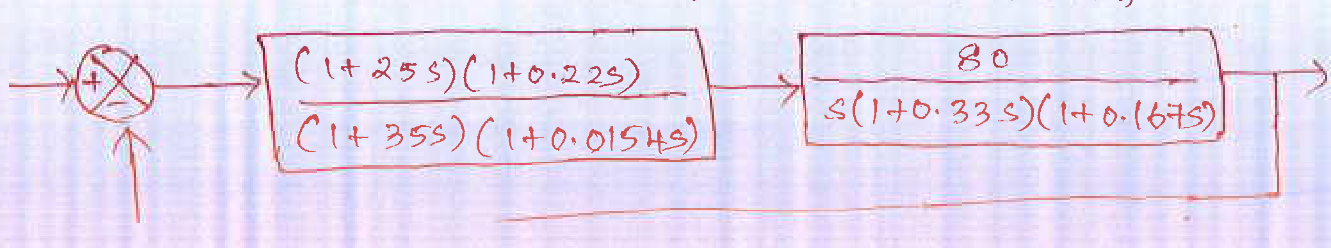
Step 9: Determine the t/f of lag-lead compensator

$$G_c(s) = G_1(s) \times G_2(s)$$

$$= \frac{14(1+2.5s)}{(1+35s)} \times \frac{0.07(1+0.22s)}{(1+0.0154s)}$$

$$= \frac{(1+2.5s)(1+0.22s)}{(1+35s)(1+0.0154s)}$$

Step 10: Determine open loop t/f of compensated system



$$G_0(s) = \frac{80(1+2.5s)(1+0.22s)}{s(1+35s)(1+0.0154s)(1+0.33s)(1+0.167s)}$$

Step 11: Bode Plot of Compensated system

Put $s = j\omega$ in $G_0(s)$

$$G_0(j\omega) = \frac{80(1+j2.5\omega)(1+j0.22\omega)}{j\omega(1+j35\omega)(1+j0.33\omega)(1+j0.167\omega)}$$

Magnitude plot

$\omega_{c1} = \frac{1}{35} = 0.03$ $\quad \omega_{c2} = \frac{1}{2.5} = 0.4$ $\quad \omega_{c3} = \frac{1}{0.33} = 3$

$\omega_{c4} = \frac{1}{0.22} = 4.5$ $\quad \omega_{c5} = \frac{1}{0.167} = 6$ $\quad \omega_{c6} = \frac{1}{0.0154} = 65$

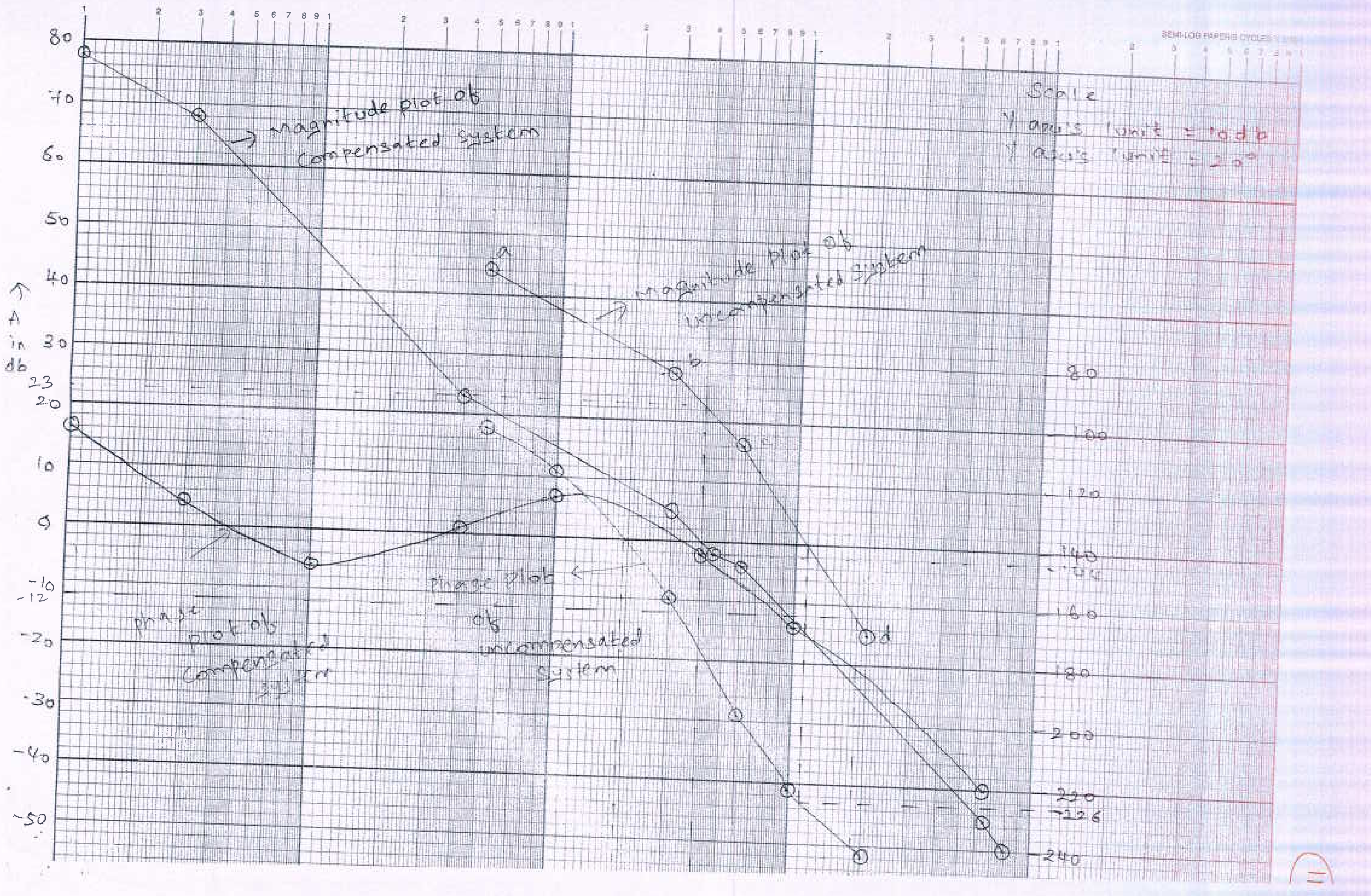
Term	Corner frequency	slope	Change in slope
$\frac{80}{j\omega}$		-20	—
$\frac{1}{1+j3.5\omega}$	$\omega_{c1} = 0.03$	-20	-40
$1+j2.5\omega$	$\omega_{c2} = 0.4$	20	-20
$\frac{1}{1+j0.33\omega}$	$\omega_{c3} = 3$	-20	-40
$1+j0.22\omega$	$\omega_{c4} = 4.5$	20	-20
$\frac{1}{1+j0.167\omega}$	$\omega_{c5} = 6$	-20	-40
$\frac{1}{1+j0.0154\omega}$	$\omega_{c6} = 65$	-20	-60

$\omega = 0.01$ $\quad A_0 = 20 \log \frac{80}{0.01} = 78\,db$

$\omega_{c1} = 0.03$ $\quad A_0 = 20 \log \frac{80}{0.03} = 68\,db$

$\omega_{c2} = 0.4$ $\quad A_0 = -40 \times \log \frac{0.4}{0.03} + 68 = 23\,db$

$\omega_{c3} = 3$ $\quad A_0 = -20 \times \log \frac{3}{0.4} + 23 = 5\,db$



SEMI-LOG PAPERS CYCLES
magnitude plot of Compensated system
magnitude plot of uncompensated system
phase plot of Compensated system
phase plot of uncompensated system
A in db
Scale
Y axis 1 unit = 10 db
Y axis 1 unit = 20°
80
70
60
50
40
30
23
20
10
0
-10
-12
-20
-30
-40
-50
80
100
120
140
144
160
180
200
220
226
240
a
b
c
d

$\omega_{c4} = 4.5 \quad A_0 = -40 \times \log \frac{4.5}{3} + 5 = -2\,db$

$\omega_{c5} = 6 \quad A_0 = -20 \times \log \frac{6}{4.5} + (-2) = -4\,db$

$\omega_{c6} = 65 \quad A_0 = -40 \times \log \frac{65}{6} + (-4) = -45\,db$

$\omega_h = 80 \quad A_0 = -60 \times \log \frac{80}{65} + (-45) = -50\,db$

Phase plot

$$\phi_0 = \angle G_0(j\omega) = \tan^{-1} 2.5\omega + \tan^{-1} 0.22\omega - 90 - \tan^{-1} 35\omega - \tan^{-1} 0.0154\omega - \tan^{-1} 0.33\omega - \tan^{-1} 0.167\omega$$

ω	0.01	0.03	0.1	0.4	1	4	10	65
$\angle G_0(j\omega)$ deg	-108	-132	-152	-138	-126	-144	-168	-221 ≈ -220

$\phi_{gc0} = -144$ (from bode plot)

$\gamma_0 = 180 + \phi_{gc0} = 180 - 144 = 36°$

The phase margin of the compensated system is satisfactory. Hence the design is acceptable.

Consider a unity f/b system with O.L.T.F $G(s) = \frac{100}{(s+1)(s+2)(s}$ ...

Design a PI controller, so that the phase margin of the system is 60° at a freq of 0.5 rad/sec

$$G(s) = \frac{100}{(s+1)(s+2)(s+5)} = \frac{100}{(1+s)\times 2(1+\frac{s}{2})\times 5(1+\frac{s}{5})}$$

$$= \frac{10}{(1+s)(1+0.5s)(1+0.2s)}$$

$$G(j\omega) = \frac{10}{(1+j\omega)(1+0.5j\omega)(1+0.2j\omega)}$$

$$= \frac{10}{\sqrt{1+\omega^2}\angle\tan^{-1}\omega \sqrt{1+(0.5\omega)^2}\angle\tan^{-1}0.5\omega \sqrt{1+(0.2\omega)^2}\angle\tan^{-1}0.2\omega}$$

step 1

$A_1 = |G(j\omega)|$ at $\omega = \omega_1$

$$|G(j\omega)| = \frac{10}{\sqrt{1+\omega^2}\sqrt{1+(0.5\omega)^2}\sqrt{1+(0.2\omega)^2}} = 8.63$$

$\omega = 0.5$

$\phi_1 = \angle G(j\omega)$ at $\omega = \omega_1$

$$\phi_1 = -\tan^{-1}\omega - \tan^{-1}0.5\omega - \tan^{-1}0.2\omega$$

$$= -\tan^{-1}0.5 - \tan^{-1}0.5\times0.5 - \tan^{-1}0.2\times0.5$$

$$= -46^\circ$$

step 2:

$\gamma_d = 60$

$$\gamma_u = 180 + \phi_1 = 180 - (-46) = 134^\circ$$

$$\theta = \gamma_d - \gamma_u = 60 - 134 = -74$$

step 3:

T.F of PI Controller $= G_c(s) = K_p + \frac{K_i}{s}$

$$K_p = \frac{\cos\theta}{A_1} \qquad K_i = \frac{-\omega_1 \sin\theta}{A_1}$$

$$K_p = \frac{\cos(-74)}{8.63} = 0.032 \qquad K_i = \frac{-0.5 \times \sin(-74)}{8.63}$$

$$= 0.056$$

$$\therefore G_c(s) = K_p + \frac{K_i}{s} = 0.032 + \frac{0.056}{s}$$

$$= \frac{0.032s + 0.056}{s}$$

$$= \frac{1}{s}\, 0.056\left(1 + \frac{0.032s}{0.056}\right)$$

$$= \frac{0.056(1 + 0.57s)}{s}$$

step 4:

$$G_o(s) = G_c(s) \times G(s)$$

$$= \frac{0.056(1 + 0.57s)}{s} \times \frac{10}{(1+s)(1+0.5s)(1+0.2s)}$$

$$= \frac{0.56(1 + 0.57s)}{s(1+s)(1+0.5s)(1+0.2s)}$$

$$G_o(j\omega) = \frac{0.56(1 + 0.57j\omega)}{j\omega(1+j\omega)(1+0.5j\omega)(1+0.2j\omega)}$$

$$A_o = |G_o(j\omega)| = \frac{0.56\sqrt{(1+0.57\omega)^2}}{\sqrt{\omega^2}\sqrt{1+\omega^2}\sqrt{1+(0.5\omega)^2}\sqrt{1+(0.2\omega)^2}} = 1$$

at $\omega = \omega_1$

$\omega = 0.5$

$$\phi_o = \tan^{-1} 0.57\omega - 90 - \tan^{-1}\omega - \tan^{-1} 0.5\omega - \tan^{-1} 0.2\omega$$

$\omega = 0.5$

$= -1\ldots$

$$\gamma_0 = 180 + \phi_0 = 180 + (-120) = 60^\circ$$