Unit III

MICROWAVE TUBES

Microwave tubes: O-type – Two cavity Klystrons: structure, resonant cavities, velocity modulation and Applegate diagram, bunching process. Reflex Klystrons-structure, modes and o/p characteristics, electronic and mechanical tuning.M-type – cross-field effects, Magnetrons- types, 8-cavity Cylindrical Travelling Wave Magnetron- Hull cut-off and Hartree conditions, modes of resonance and PI-mode operation, o/p characteristics. HELIX TWT- types and characteristics of slow wave structures, structure of TWT and amplification process (qualitative treatment), Backward wave Oscillators

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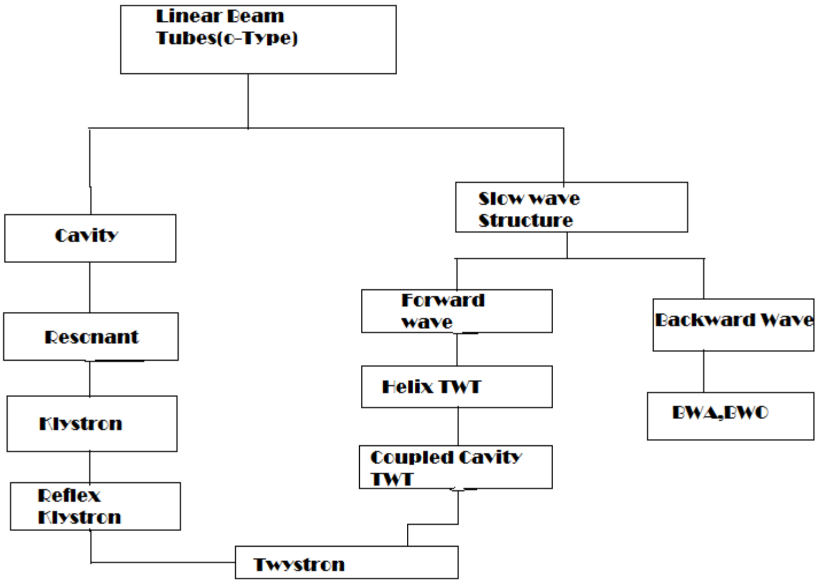
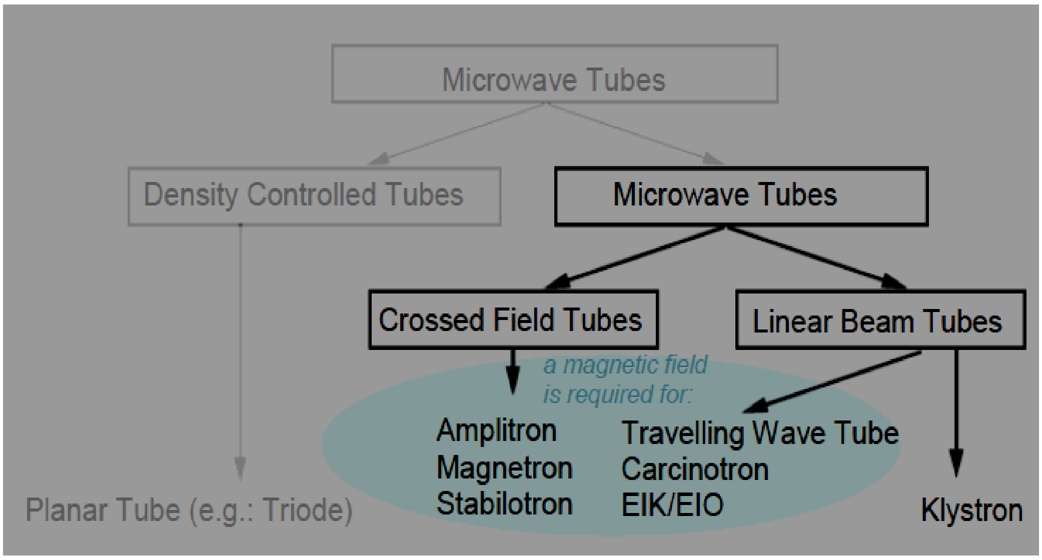
O-TYPE Linear Tubes (Travelling tube amplifiers,Klystrons) .In O-Type tube , a magnetic field whose axis coincides with the electron beam is used to hold the beam together as it travels the length of the tube



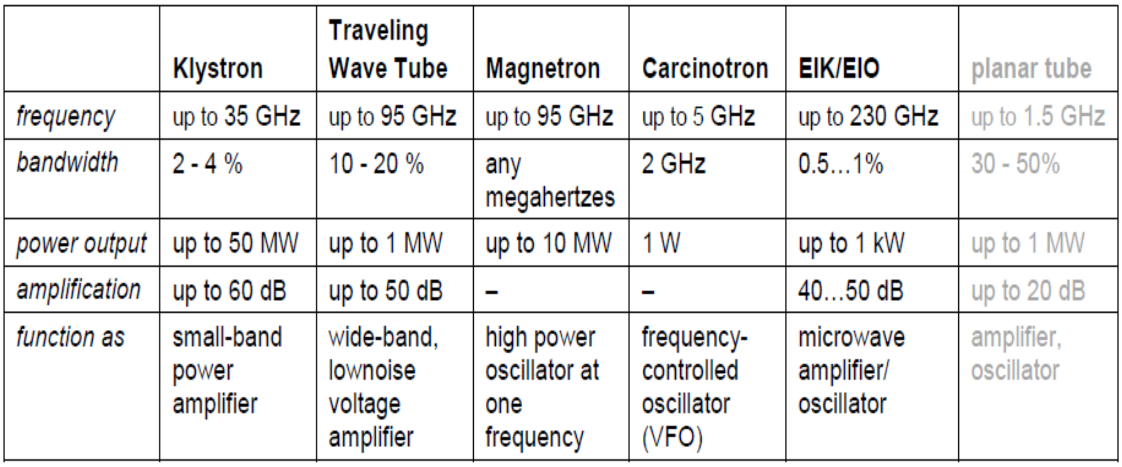
Velocity-modulated Tubes

Velocity-modulated tubes are microwave tubes using transit time in the conversion of dc power to radio-frequency power. The interchange of power is accomplished by using the principle of electron **velocity modulation** and low-loss resonant cavities in (or near the electron beam of) the microwave tube.

Velocity modulation is then defined as that variation in the velocity of a beam of electrons caused by the alternate speeding up and slowing down of the electrons in the beam. This variation is usually caused by a voltage signal applied between the grids through which the beam must pass. The direction of the electron beam and the static electrical field goes to each other parallelly (linearly) into linear beam tubes. Against this the fields influencing the electron beam stand vertically by the electron beam at the cross field tubes.



The following table compares with characteristic quantities of the velocity-modulated tubes used in radar technology. Although the planar tube isn't a velocity-modulated tube, it was included into this table for comparison purposes. The grid of the density controlled tube (like the planar triode) regulates the number of electrons on the path to the anode. The different speeds of the electrons by additional accelerating due the microwave voltage are annoying in this case. The cut-off frequency of density controlled tubes is relatively low. Higher frequencies need the use of velocity-modulated tubes, as shown in the table:



	Klystron	Traveling Wave Tube	Magnetron	Carcinotron	EIK/EIO	planar tube
frequency	up to 35 GHz	up to 95 GHz	up to 95 GHz	up to 5 GHz	up to 230 GHz	up to 1.5 GHz
bandwidth	2 - 4 %	10 - 20 %	any megahertzes	2 GHz	0.5…1%	30 - 50%
power output	up to 50 MW	up to 1 MW	up to 10 MW	1 W	up to 1 kW	up to 1 MW
amplification	up to 60 dB	up to 50 dB	–	–	40…50 dB	up to 20 dB
function as	small-band power amplifier	wide-band, lownoise voltage amplifier	high power oscillator at one frequency	frequency-controlled oscillator (VFO)	microwave amplifier/ oscillator	amplifier, oscillator

Klystron Amplifier

Klystron amplifiers are high power microwave vacuum tubes They are used in some coherent radar transmitters as power amplifiers. Klystrons make use of the transit-time effect by varying the velocity of an electron beam. A klystron uses special resonant cavities which modulate the electric field around the axis of the tube modulating the electric field around the axis the tube. In the middle of these cavities, there is a grid allowing the electrons to pass the cavity.

Due to the number of the resonant cavities klystrons are divided up into

Two- or Multicavity klystrons, and

Reflex or Repeller Klystrons.

Two-Cavity Klystron

As the name implies, this klystron uses two cavities. The first cavity together with the first coupling device is called a “buncher”, while the second cavity with its coupling device is called a “catcher”. The direction of the field changes with the frequency of the “buncher” cavity. These changes alternately accelerate and decelerate the electrons of the beam passing through the grids of the buncher cavity. The area beyond the cavities is called the “drift space”. The electrons form bunches in this area when the accelerated electrons overtake the decelerated electrons.

The function of the “catcher” cavity is to absorb energy from the electron beam. The “catcher” grids are placed along the beam at a point where the bunches are fully formed. The location is determined by the transit time of the bunches at the natural resonant frequency of the cavities (the resonant frequency of the catcher cavity is the same as the buncher cavity).The air-cooled collector collect the energy of the electron beam and change it into heat and X radiation.

Klystron amplification, power output, and efficiency can be greatly improved by the addition of intermediate cavities between the input and output cavities of the basic klystron. Additional cavities serve to velocity-modulate the electron beam and produce an increase in the energy available at the output.

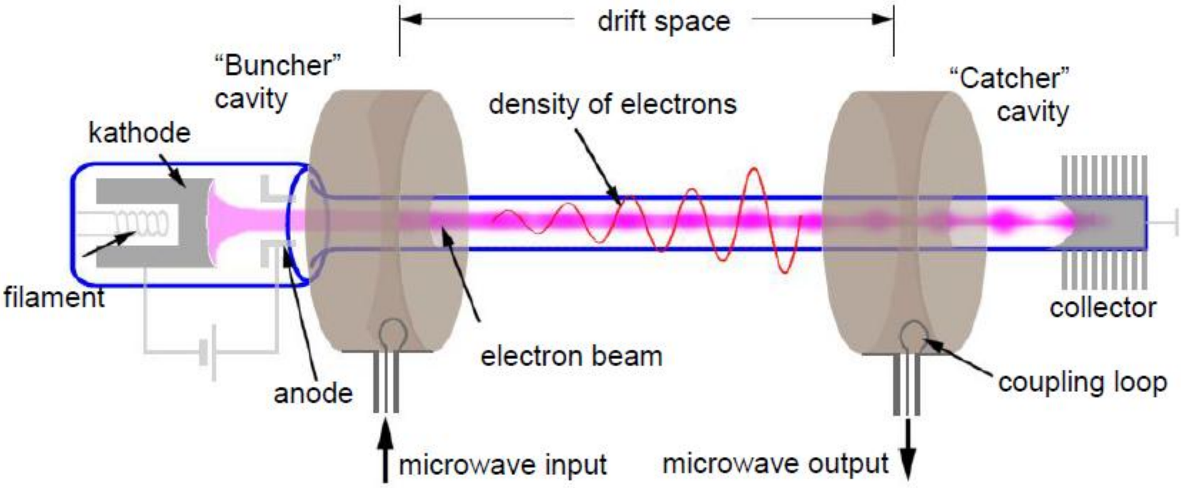


Figure 1: physical construction and mode of operation of a two-cavity klystron

As indicated in the introduction this voltage will produce velocity modulation on the beam.

Let the Z’ axis be taken in the direction of electron flow with grid position Z=0.

As the electron in between grids experiences a force due to the RF electric field

$$V_s = V_1 \sin \omega t \quad \text{----------------------- } 1$$

Where,V_1 is the amplitude of the signal and $V_1 << V_0$

By considering either time t_o or the exiting time t_1,te modulated velocity in the buncher cavity can be determined.The average microwave voltage in the buncher gap needs to be determined in below figure

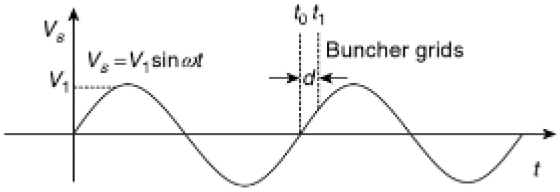


Figure:2 Signal Voltage in the Buncher gap

As $V_1 << V_0$,the average transit time all the way through the bunchergap of distance d is

$$\tau = \frac{d}{v_0} = t_1 - t_0$$ -----------------2

The phase delay caused during transit ti

me across the gap is referred to as gap transit angle θ_g and can be given as

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0}$$ -------------3

Eventually,The average microwave voltage in the buncher gap can be given as

$$V_s = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)dt = \frac{-V_1}{\omega\tau}[\cos(\omega t_1) - \cos(\omega t_0)]$$ -----------4

$$V_s = \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos\left(\omega t_0 + \frac{\omega d}{v_0}\right)\right]$$ ------------------------5

let

$$\omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A \text{ and } \frac{\omega d}{2v_0} = \frac{\theta_g}{2} = B$$

By using trigonometric relations ie cos(A-B)-cos(A+B)=2sinAsinB Eq 5 can be written as

$$V_s = V_1 \frac{\sin[\omega d / 2v_0]}{\omega d / 2v_0} \sin\left(\omega t_0 + \frac{\omega d}{2v_0} \right)$$

--------------------------6

$$V_s = V_1 \beta_1 \sin\left(\omega t_0 + \frac{\theta_g}{2} \right)$$

---7

Where β_1 the bamcoupling coefficient of the input cavity gap and is given as

$$\beta_1 = \frac{\sin[\omega d / 2v_0]}{\omega d / 2v_0} = \frac{\sin(\theta_g / 2)}{\theta_g / 2}$$

-------------------------------------8

We can observe that when the gap transit angle increases the coupling between the electron beam and buncher cavity reduces which means for a given microwave signal the velocity modulation decreases.The exit velocity modulatio,can be instantly calculated as

$$v(t_1) = \sqrt{\frac{2e}{m}(V_0 + V_s)}$$

--9

Substituting Eq 7 in Eq 9

$$v(t_1) = \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{\beta_1 V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2} \right) \right]}$$

----------------------------10

Where, the fector $\beta_1 V_1/V_0$ is called the depth of velocity modulation

$$v(t_1) = v_0 \sqrt{1 + \frac{\beta_1 V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2} \right)}$$

-----------------------------------11

Where β_1 the beam coupling coefficient of the input cavity gap and id given as

Assuming that $\beta_1 V_1 << V_0$ and by means of binomial expansion the Eq 11 is modified as

$$v(t_1) = v_0\left[1+\frac{\beta_1 V_1}{2V_0}\sin\left(\omega t_0+\frac{\theta_g}{2}\right)\right]$$ -------------------------------------12

This is called the velocity modulation equation,this equation can also be written as,

$$v(t_1) = v_0\left[1+\frac{\beta_1 V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right]$$ -------------------------------------13

Applegate diagram:

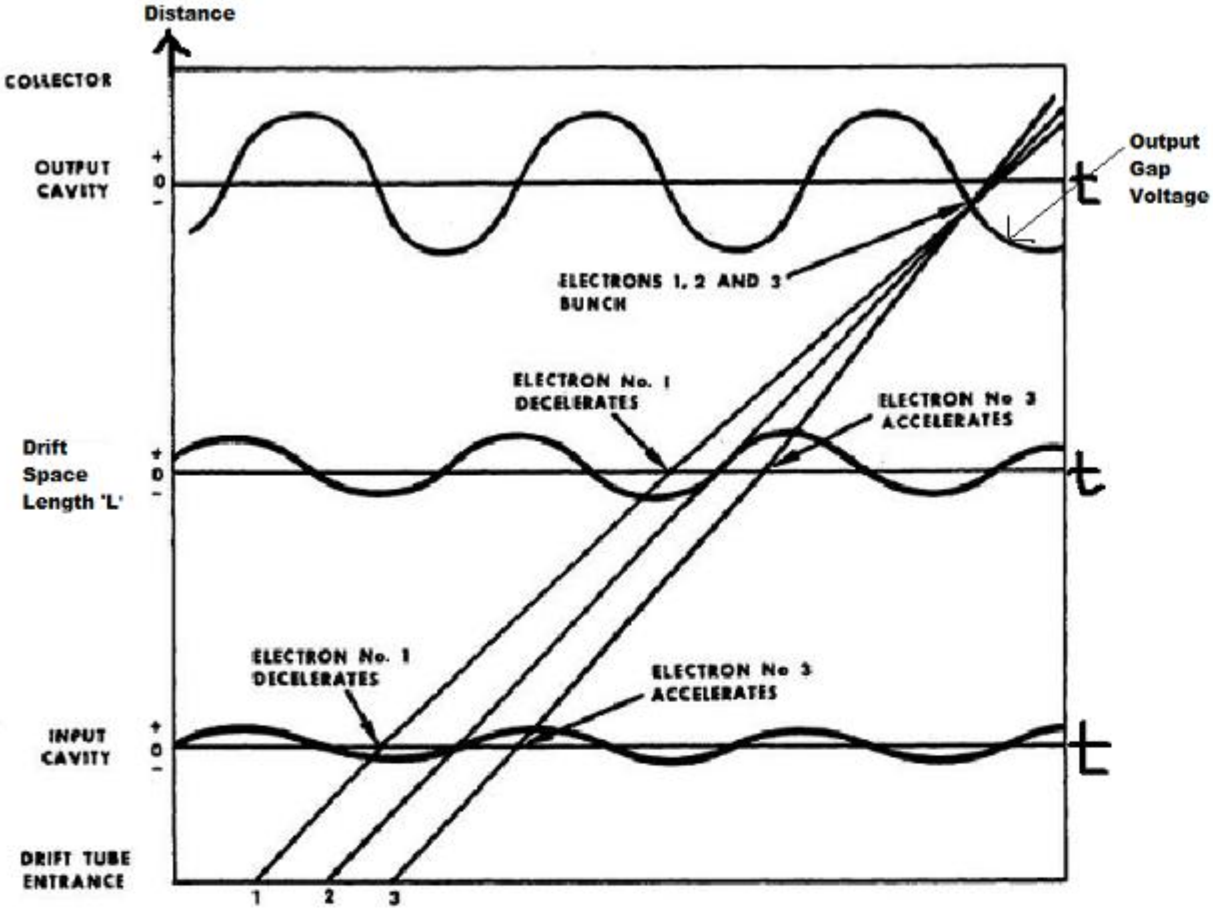


Figure 3: Applegate diagram

In a quarter of one period of the plasma frequency, the velocity modulation is converted to density modulation, i.e. bunches of electrons. Now let’s see this procedure with the help of Applegate diagram hus the electron beam is velocity modulated to form bunches or undergoes density(Current modulation) with input RF signal. This current modulation of beam produces amplification of RF signal input at the catcher cavity. Thus what we obtain finally is the amplification of RF input signal. One important observation is that the phase of output signal is opposite to that of input signal. Also many harmonics are generated during amplification. One way to remove this harmonics is to tune the catcher cavity to the fundamental frequency or any other harmonic desired.

Bunching process:

The Electrons from the bunching centre they pass through at Vs=0 with an unchanged velocity Vo.During the Positive half cycles of the microwave input voltage Vs the electron passes the gap faster compared to the electrons that pass the gap at Vs=0.

The electrons that enter buncher cavity during neagative half cycle of Vs are slow compared to that pass the gap at Vs=0.

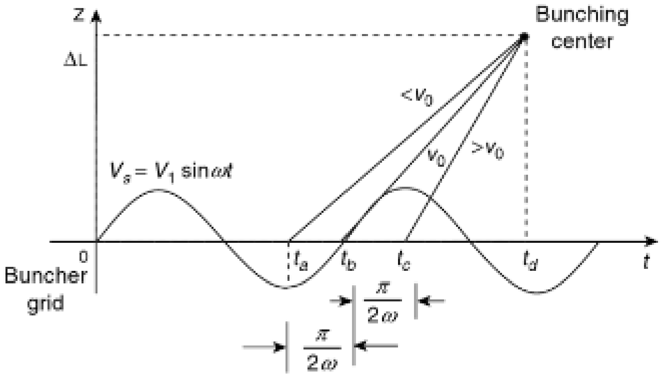


Figure 4: Bunching process

$$\Delta L = v_0(t_d - t_b)$$ **----------- 1**

Similarly, the distances for the electrons at ta and tc are

$$\Delta L = v_{\min}(t_d - t_a) = v_{\min}\left(t_d - t_b + \frac{\pi}{2w}\right)$$

-------------------------2

$$\Delta L = v_{\max}(t_d - t_c) = v_{\max}\left(t_d - t_b - \frac{\pi}{2w}\right)$$

-------------------------3

From the velocity modulation

Maximum velocity occurs at π/2,so that

$$v_{\max} = v_0\left(1 + \frac{\beta_1 V_1}{2V_0}\right)$$

---4

Minimum velocity occurs at -π/2,so that

$$v_{\min} = v_0\left(1 - \frac{\beta_1 V_1}{2V_0}\right)$$

---5

Substituting Eqs 5 and 4 in 3 and 2

$$\Delta L = v_0(t_d - t_b) + v_0\left[\frac{\pi}{2\omega} - \frac{\beta_1 V_1}{2V_0}(t_d - t_b) - \frac{\beta_1 V_1}{2V_0}\frac{\pi}{2\omega}\right]$$

-------------6

The necessary condition for those electrons at ta,tb,and tc to meet at the same distance ∆L is

$$\frac{\pi}{2\omega} - \frac{\beta_1 V_1}{2V_0}(t_d - t_b) - \frac{\beta_1 V_1}{2V_0}\frac{\pi}{2\omega} = 0$$

---------------------------7

$$-\frac{\pi}{2\omega} + \frac{\beta_1 V_1}{2V_0}(t_d - t_b) + \frac{\beta_1 V_1}{2V_0}\frac{\pi}{2\omega} = 0$$

--------------------------------8

Consequently

$$t_d - t_b \approx \frac{\pi V_0}{\omega \beta_1 V_1}$$

---9

$$\Delta L = v_0\left[\frac{\pi V_0}{\omega \beta_1 V_1}\right]$$

--10

The transit time for velocity-modulated electrons to travel at a distance L is given by above eqs

$$T = (t_2 - t_1) = \frac{L}{v(t_1)} = \frac{L}{v_0\left[1 + \frac{\beta_1 V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

-------------11

$$= \frac{L}{v_0}\left[1 + \frac{\beta_1 V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]^{-1}$$

------------------------------------12

Multiplying by ῳob both sides of the above equation,We get

$$\omega T = \omega t_2 - \omega t_1 = \frac{\omega L}{v_0}\left[1 - \frac{\beta_1 V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

---------------13

In the above equation,L/v_0=T_0 is the transit time

$$\omega T = \omega(t_2 - t_1) = \theta_0\left[1 - \frac{\beta_1 V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

-----------------14

$$\theta_0 = \frac{\omega L}{v_0} = 2\pi N$$

---15

Where θ_0=dc transit angle between cavities

N= number of electron transit cycle in the drift space

By expanding 14,we get the value of the bunching parameter

$$\omega T = \omega(t_2 - t_1) = \theta_0 - \theta_0\frac{\beta_1 V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

Where **-----------------16**

$$X = \frac{\beta_1 V_1}{2V_0}\theta_0$$

------------------------------------17

Is defined as the bunching parameter of the klystron.

Substituting eq 15 and 17,we get

$$X = \frac{\beta_1 V_1}{2V_0}\frac{\omega L}{v_0} \Rightarrow L = \frac{2v_0 V_0}{\omega\beta_1 V_1}X$$

Reflex Klystron or Repeller Klystron

Another tube based on velocity modulation, and used to generate microwave energy, is the reflex klystron (repeller klystron). The reflex klystron contains a reflector plate, referred to as the repeller, instead of the output cavity used in other types of klystrons. The electron beam is modulated as it was in the other types of klystrons by passing it through an oscillating resonant cavity, but here the similarity ends.

The feedback required to maintain oscillations within the cavity is obtained by reversing the beam and sending it back through the cavity. The electrons in the beam are velocity-modulated before the beam passes through the cavity the second time and will give up the energy required to maintain oscillations. The electron beam is turned around by a negatively charged electrode that repels the beam (“repeller”). This type of klystron oscillator is called a reflex klystron because of the reflex action of the electron beam.

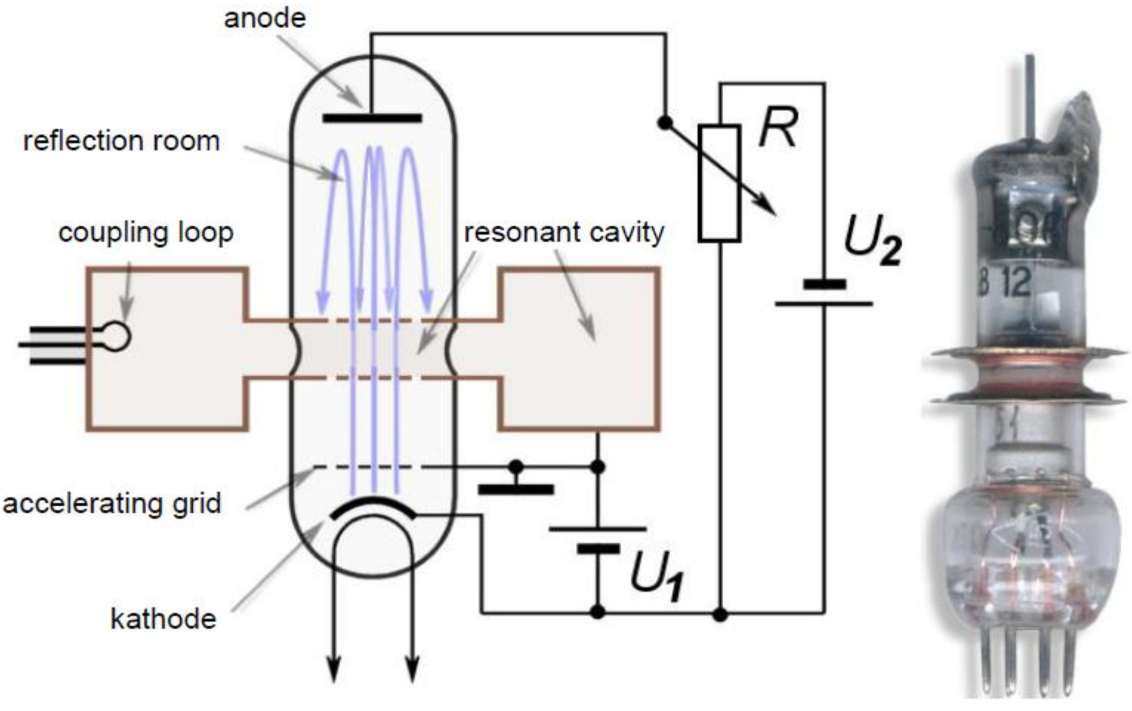


Figure 5: Wiring with a repeller klystron, and an example given low power repeller klystron tube

Repeller klystrons are often used in older radar sets as local oscillators or as oscillators in measurement sets. If the voltage feed is keyed, then the repeller klystron can be used for RF-pulse generation too, but as self-oscillating tube it provides a non-coherent oscillation only.

Modes and o/p characteristics

The output frequency and the output power vary with the change in repeller voltage for different modes are shown in below figure. These modes are called mode curves.

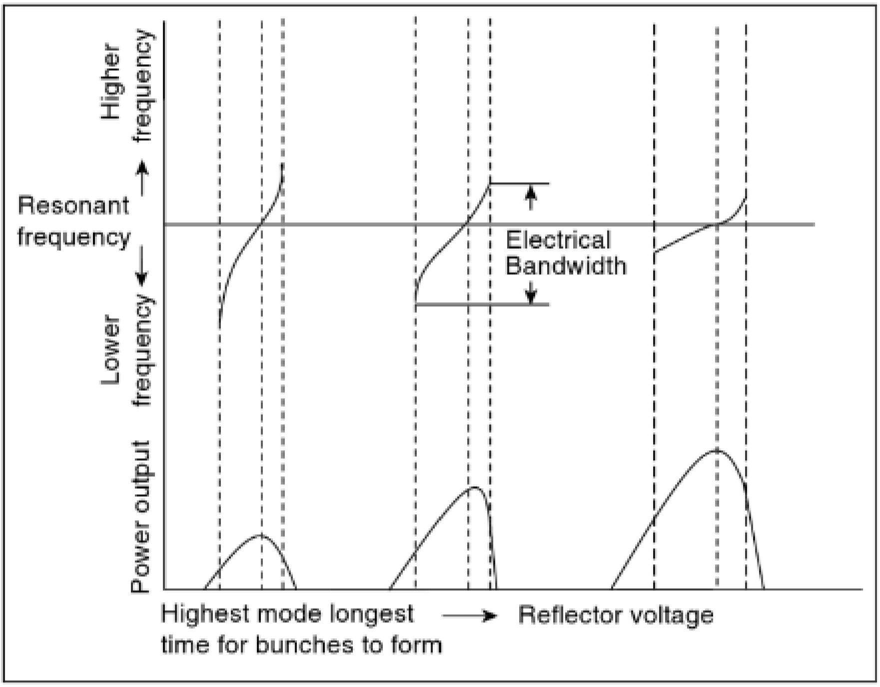


Figure 6: **Different Modes Curves**

The oscillation frequency is determined by the frequency of resonance of the output cavity,This is called as electronics tuning range of reflex klystron

o/p characteristics:

The adjustment of repeller and anode voltage is in such a way that the bunch appears exactly at any of the +ve maximum voltage of the RF signal,which is necessary for reflex klystron to undergo oscilation.The oscillations cab be achieved only for some combination of anode and repeller voltages.

The voltage or output characteristics os reflex klystron are shown in the below figure

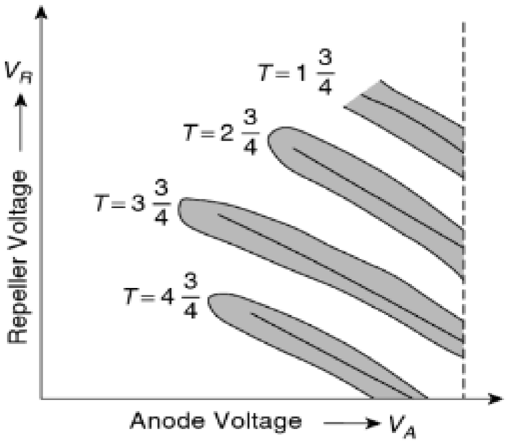


Figure :7 o/p characteristics

Electronic Tuning :

The nature of the variation of output power and frequency by adjustment of the repeller voltage is calld the electronic tuning.It can be measured by electronic tuning sensitivity(ETS).This can be determined by considering the slpe of the frequency of the modes.

We know the equations

$$(V_r + V_0)^2 = \frac{8mL_r^2V_0}{\left(2\pi n - \frac{\pi}{2}\right)^2 .e} .\omega^2$$

Traveling Wave Tube

Traveling wave tubes (TWT) are wideband amplifiers. They take therefore a special position under the velocity-modulated tubes. One reason of the special low-noise characteristic often they are in use as an active RF amplifier element in receivers additional. There are two different groups of TWT:

a) low-power TWT for receivers occurs as a highly sensitive, low-noise and wideband amplifier in radar equipments

b) high-power TWT for transmitters these are in use as a pre-amplifier or final stage for high-power transmitters.

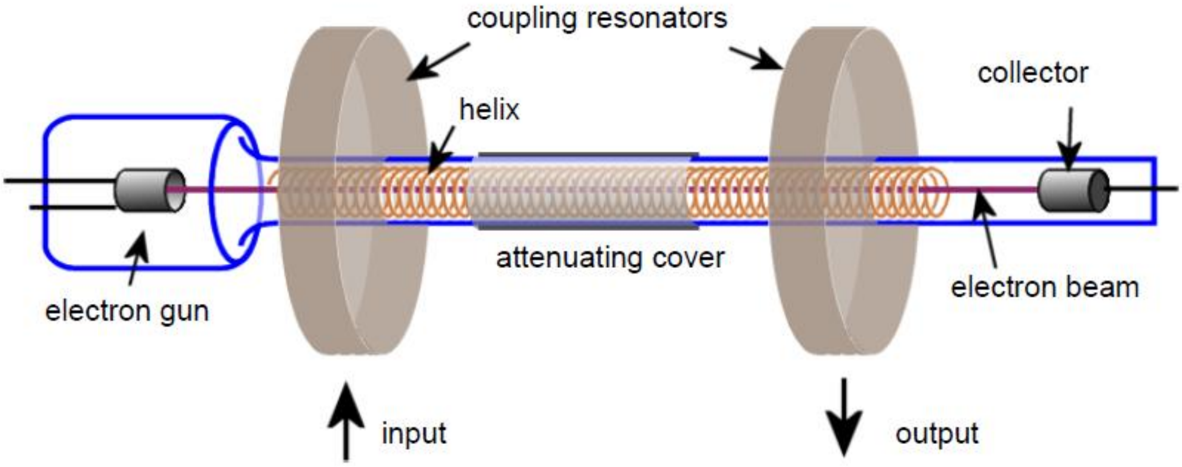


Figure 8. - Physical construction of a TWT

The physical construction of a typical TWT is shown in Figure 3. The TWT contains an electron gun which produces and then accelerates an electron beam along the axis of the tube. The surrounding magnet provides a magnetic field along the axis of the tube to focus the electrons into a tight beam. The helix, at the center of the tube, is a coiled wire that provides a low-impedance transmission line for the RF energy within the tube. The RF input and output are coupled onto and removed from the helix by waveguide directional couplers that have no physical connection to the helix. The attenuator prevents any reflected waves from traveling back down the helix.

The following figure shows the electric fields that are parallel to the electron beam inside the helical conductor.

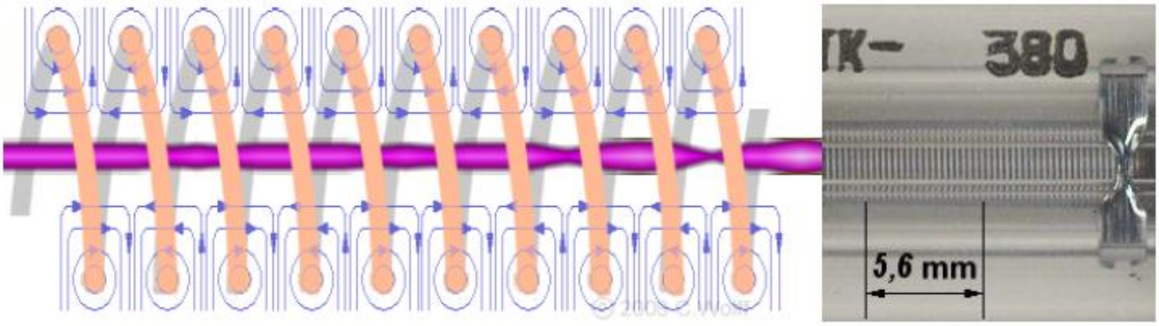


Figure 9. - Electron- beam bunching and a detail-foto of a helix (Measure detail for 20 windings)

The electron- beam bunching already starts at the beginning of the helix and reaches its highest expression on the end of the helix. If the electrons of the beam were accelerated to travel faster than the waves traveling on the wire, bunching would occur through the effect of velocity modulation. Velocity modulation would be caused by the interaction between the traveling-wave fields and the electron beam. Bunching would cause the electrons to give up energy to the traveling wave if the fields were of the correct polarity to slow down the bunches. The energy from the bunches would increase the amplitude of the traveling wave in a progressive action that would take place all along the length of the TWT.

Characteristics of a TWT

The attainable **power-amplification** is essentially dependent on the following factors:

constructive details (e.g. length of the helix)
electron beam diameter (adjustable by the density of
the focussing magnetic field)
power input (see figure 5)
voltage UA2 on the helix

As shown in the Figure 9, the gain of the TWT has got a linear characteristic of about 26 dB at small input power. If you increase the input power, the output power doesn't increase for the same gain. So you can prevent an oversteer of e.g the following mixer stage. The relatively low efficiency of the TWT partially offsets the advantages of high gain and wide bandwidth.

Given that the gain of an TWT effect by the electrons of the beam that interact with the electric fields on the delay structure, the frequency behaviour of the helix is responsible for the gain. The bandwidth of commonly used TWT can achieve values of many gigahertzes. The noise figure of recently used TWT is 3 ... 10 dB.

The helix may be replaced by some other slow wave structure such as a ring-bar, ring loop, or coupled cavity structure. The structure is chosen to give the characteristic appropriate to the desired gain/bandwidth and power Characteristic.

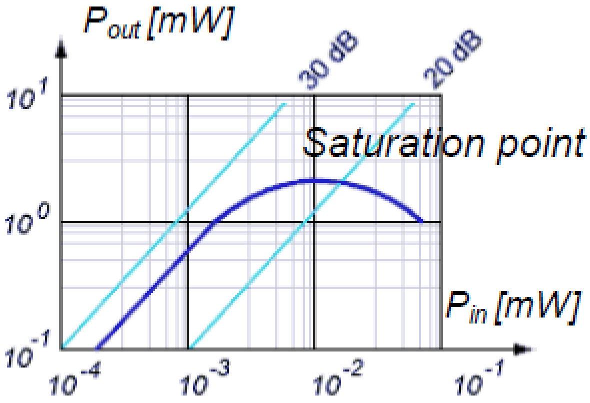


Figure 10: Characteristic of a traveling wave tube

Figure 11: Ring-Loop slow wave structure

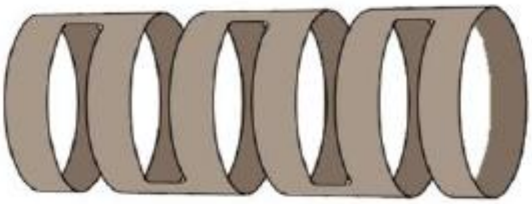


Figure 12: Ring-Bar slow wave structure

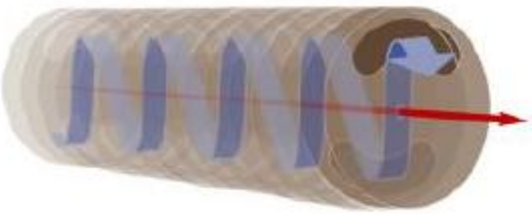


Figure 13: Coupled-cavity slow wave structure

Ring-Loop TWT

A Ring Loop TWT uses loops as slow wave structure to tie the rings together. These devices are capable of higher power levels than conventional helix TWTs, but have significantly less bandwidth of 5…15 percent and lower cut-off frequency of 18 GHz.

The feature of the ring-loop slow wave structure is high coupling impedance and low harmonic wave components. Therefore ring-loop traveling wave tube has advantages of high gain (40…60 Decibels), small dimension, higher operating voltage and less danger of the backward wave oscillation.

Ring-Bar TWT

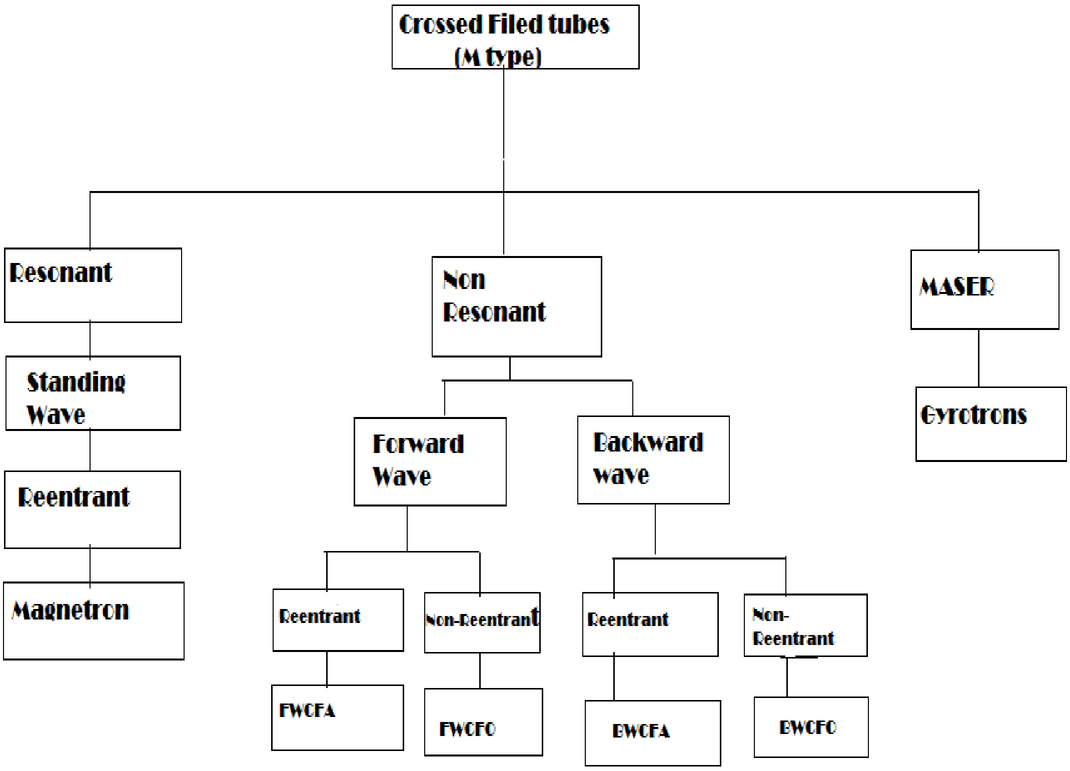
The Ring-Bar TWT has got characteristics likely the Ring- Loop TWT. The slow wave structure can be made easier by cut-out the structure of a copper tube.

Coupled-cavity TWT

The Coupled-cavity TWT uses a slow wave structure of a series of cavities coupled to one another. The resonant cavities are coupled together with a transmission line. The electron beam (shown in Figure 13 as red beam) is velocity modulated by an RF input signal at the first resonant cavity. This RF energy (displayed as blue arrow) travels along the cavities and induces RF voltages in each subsequent cavity.

If the spacing of the cavities is correctly adjusted, the voltages at each cavity induced by the modulated beam are in phase and travel along the transmission line to the output, with an additive effect, so that the output power is much greater than the power input.

M-type – cross-field effects:



Magnetron

In 1921 Albert Wallace Hull invented the magnetron as a microwave tube. During World War II it was developed by John Randall and Henry Boot to a powerful microwave generator for Radar applications.

Magnetrons function as self-excited microwave oscillators. Crossed electron and magnetic fields are used in the magnetron to produce the high-power output required in radar equipment. These multicavity devices may be used in radar transmitters as either pulsed or cw oscillators at frequencies ranging from approximately 600 to 96,000 megahertz. The relatively simple construction has the disadvantage, that the Magnetron usually can work only on a constructively fixed frequency.

Figure 14: Magnetron МИ 29Г of the old russian Radar “Bar Lock”

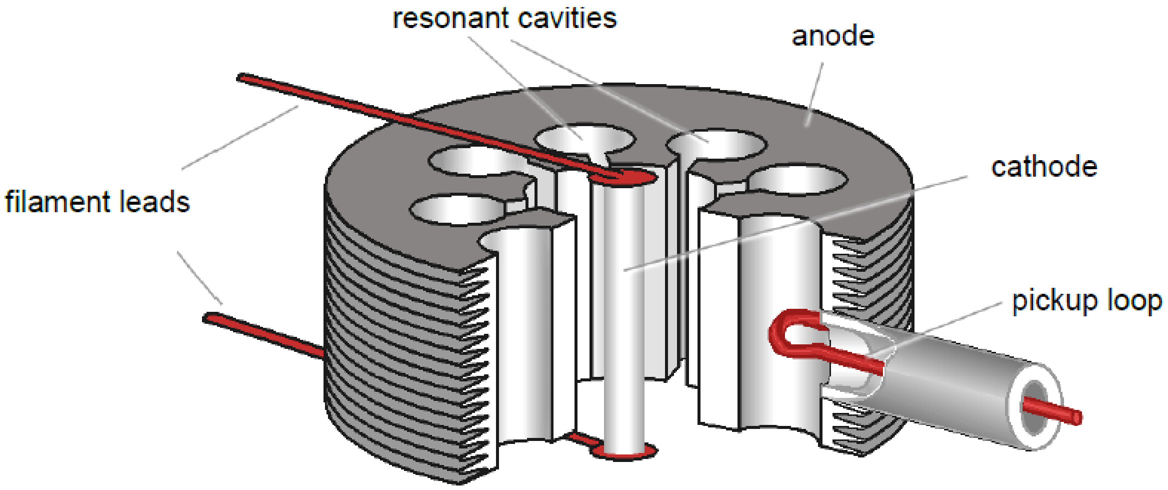
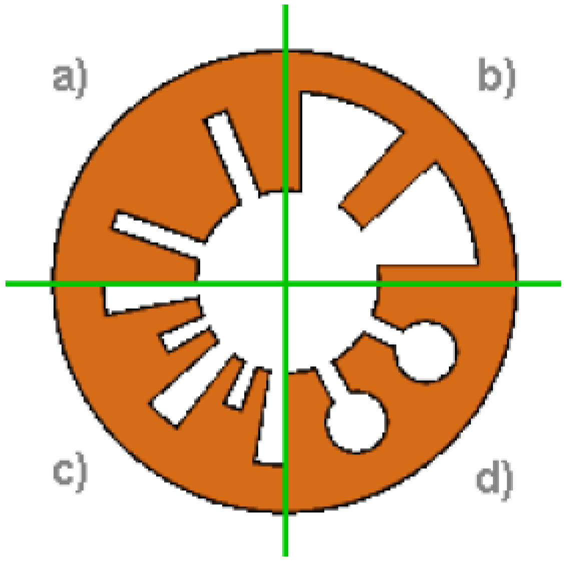


Figure 15: Cutaway view of a magnetron

Physical construction of a magnetron

The magnetron is classed as a diode because it has no grid. The anode of a magnetron is fabricated into a cylindrical solid copper block. The cathode and filament are at the center of the tube and are supported by the filament leads. The filament leads are large and rigid enough to keep the cathode and filament structure fixed in position. The cathode is indirectly heated and is constructed of a high-emission material. The 8 up to 20 cylindrical holes around its circumference are resonant cavities. The cavities control the output frequency. A narrow slot runs from each cavity into the central portion of the tube dividing the inner structure into as many segments as there are cavities.



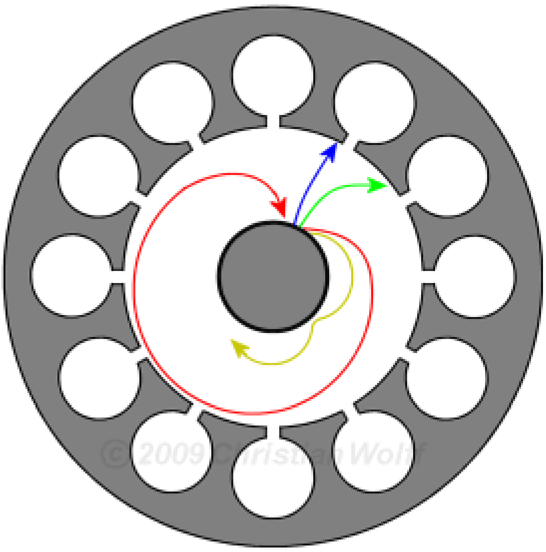


Figure 16: forms of the plate of magnetrons

Figure 17: the electron path under the influence of different strength of the magnetic field

The open space between the plate and the cathode is called the interaction space. In this space the electric and magnetic fields interact to exert force upon the electrons. The magnetic field is usually provided by a strong, permanent magnet mounted around the magnetron so that the magnetic field is parallel with the axis of the cathode.

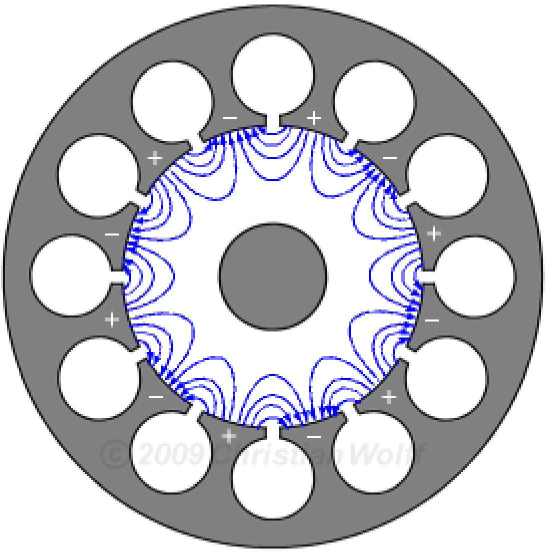
The form of the cavities varies, as shown in Figure 16. The output lead is usually a probe or loop extending into one of the tuned cavities and coupled into a waveguide or coaxial line.

a) slot- type

b) vane- type

c) rising sun- type

d) hole-and-slot- type



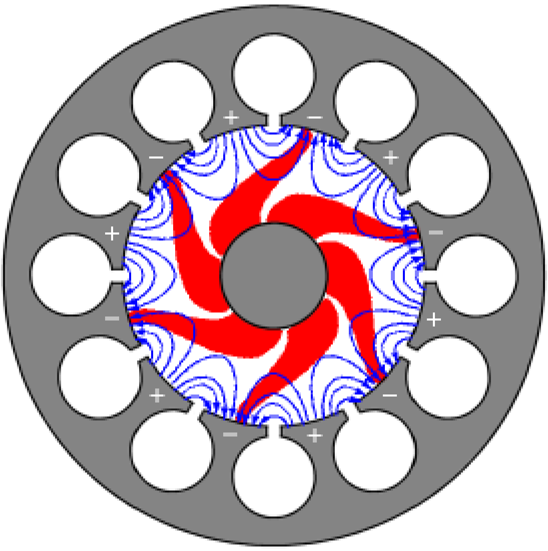


Figure 18: The high-frequency electrical field

Figure 19: Rotating space-charge wheel in an twelve-cavity magnetron

Basic Magnetron Operation

As when all velocity-modulated tubes the electronic events at the production microwave frequencies at a Magnetron can be subdivided into four phases too:

1. phase: production and acceleration of an electron beam
2. phase: velocity-modulation of the electron beam
3. phase: bunching the electrons, forming of a „Space-Charge Wheel”
4. phase: dispense energy to the ac field

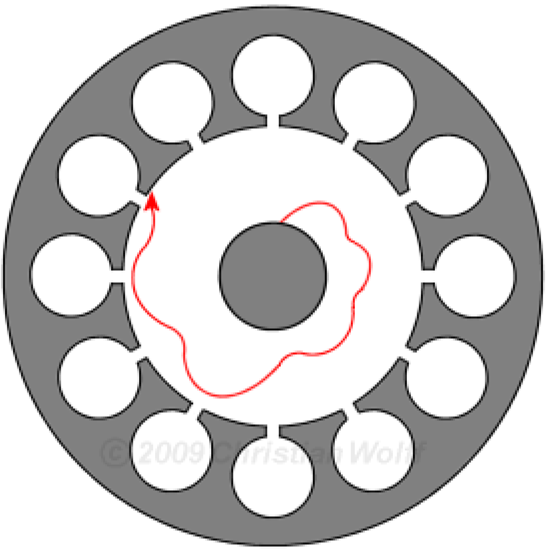


Figure 20: Path of a single electron under influence of the electric RF-field

1. Phase: Production and acceleration of an electron beam

When no magnetic field exists, heating the cathode results in a uniform and direct movement of the field from the cathode to the plate (the blue path in Figure 17). The permanent magnetic field bends the electron path. If the electron flow reaches the plate, so a large amount of plate current is flowing. If the strength of the magnetic field is increased, the path of the electron will have a sharper bend. Likewise, if the velocity of the electron increases, the field around it increases and the path will bend more sharply. However, when the critical field value is reached, as shown in the figure 20 as a red path, the electrons are deflected away from the plate and the plate current then drops quickly to a very small value. When the field strength is made still greater, the plate current drops to zero.

When the magnetron is adjusted to the cutoff, or critical value of the plate current, and the electrons just fail to reach the plate in their circular motion, it can produce oscillations at microwave frequencies.

2. Phase: Velocity-modulation of the electron beam

The electric field in the magnetron oscillator is a product of ac and dc fields. The dc field extends radially from adjacent anode segments to the cathode. The ac fields, extending between adjacent segments, are shown at an instant of maximum magnitude of one alternation of the rf oscillations occurring in the cavities.

In the Figure 18 is shown only the assumed high-frequency electrical ac field. This ac field work in addition to the to the permanently available dc field. The ac field of each individual cavity increases or decreases the dc field like shown in the figure.

Well, the electrons which fly toward the anode segments loaded at the moment more positively are accelerated in addition. These get a higher tangential speed. On the other hand the electrons which fly toward the segments loaded at the moment more negatively are slow down. These get consequently a smaller tangential speed.

3. Phase: Forming of a „Space-Charge Wheel”

On reason the different speeds of the electron groups a velocity modulation appears therefore.

The cumulative action of many electrons returning to the cathode while others are moving toward the anode forms a pattern resembling the moving spokes of a wheel known as a “Space-Charge Wheel”, as indicated in Figure 19. The space-charge wheel rotates about the cathode at an angular velocity of 2 poles (anode segments) per cycle of the ac field. This phase relationship enables the concentration of electrons to continuously deliver energy to sustain the rf oscillations.

One of the spokes just is near an anode segment which is loaded a little more negatively. The electrons are slowed down and pass her energy on to the ac field. This state isn't static, because both the ac- field and the wire wheel permanently circulate. The tangential speed of the electron spokes and the cycle speed of the wave must be brought in agreement so.

4. Phase: Dispense energy to the ac field

Recall that an electron moving against an E field is accelerated by the field and takes energy from the field. Also, an electron dispense energy to a field and slows down if it is moving in the same direction as the field (positive to negative). The electron spends energy to each cavity as it passes and eventually reaches the anode when its energy is expended. Thus, the electron has helped sustain oscillations because it has taken energy from the dc field and given it to the ac field. This electron describes the path shown in Figure 15 over a longer time period looked. By the multiple breaking of the electron the energy of the electron is used optimally. The effectiveness reaches values up to 80%.

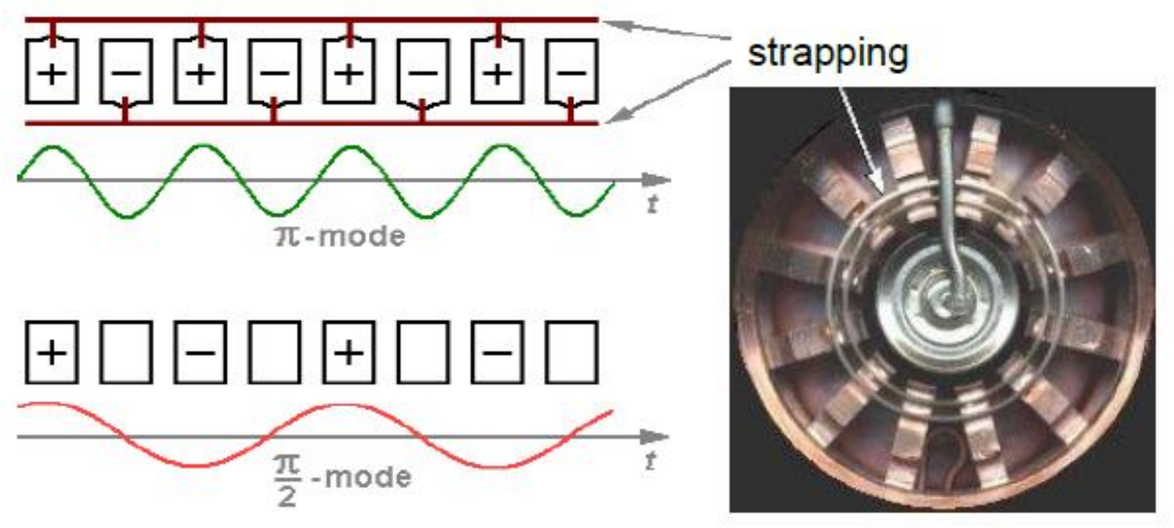


Figure 21: Waveforms of the magnetron (Anode segments are represented „unwound”) and a cuttaway view of a magnetron (vane-type), showing the strapping rings and the slots.

Modes of Operation

The operation frequency depends on the sizes of the cavities and the interaction space between anode and cathode. But the single cavities are coupled over the interaction space with each other. Therefore several resonant frequencies exist for the complete system. Two of the four possible waveforms of a magnetron with 8 cavities are in the figure 8 represented. Several other modes of oscillation are possible (3/4*π*, 1/2 *π*, 1/4 *π*), but a magnetron operating in the *π* mode has greater power and output and is the most commonly used.

So that a stable operational condition adapts in the optimal pi mode, two constructive measures are possible:

Strapping rings: The frequency of the *π* mode is separated from the frequency of the other modes by strapping to ensure that the alternate segments have identical polarities. For the pi mode, all parts of each strapping ring are at the same potential; but the two rings have alternately opposing potentials. For other modes, however, a phase difference exists between the successive segments connected to a given strapping ring which causes current to flow in the straps.

Use of cavities of **different resonance frequency** E.g. such a variant is the anode form “Rising Sun”

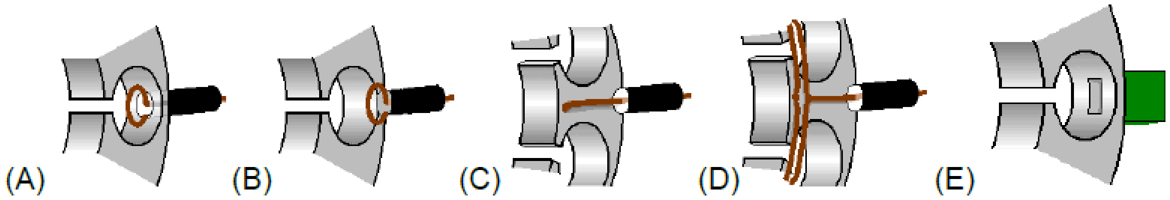


Figure 22: Magnetron coupling

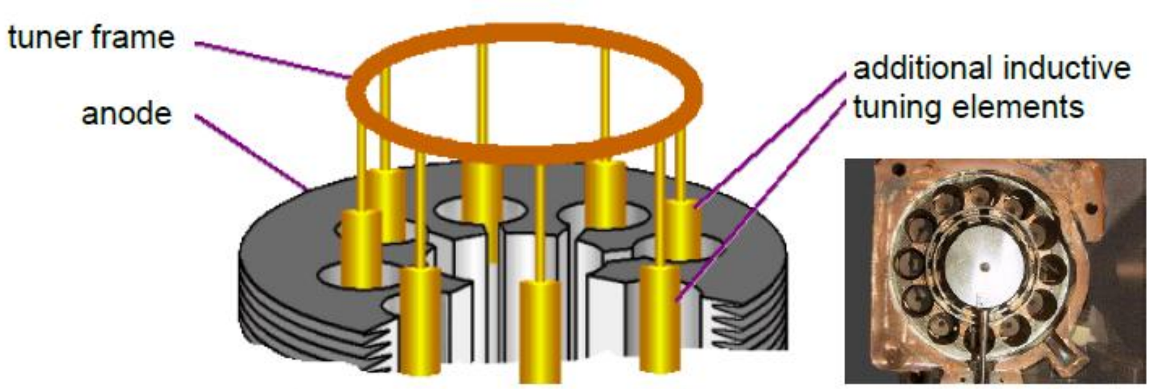
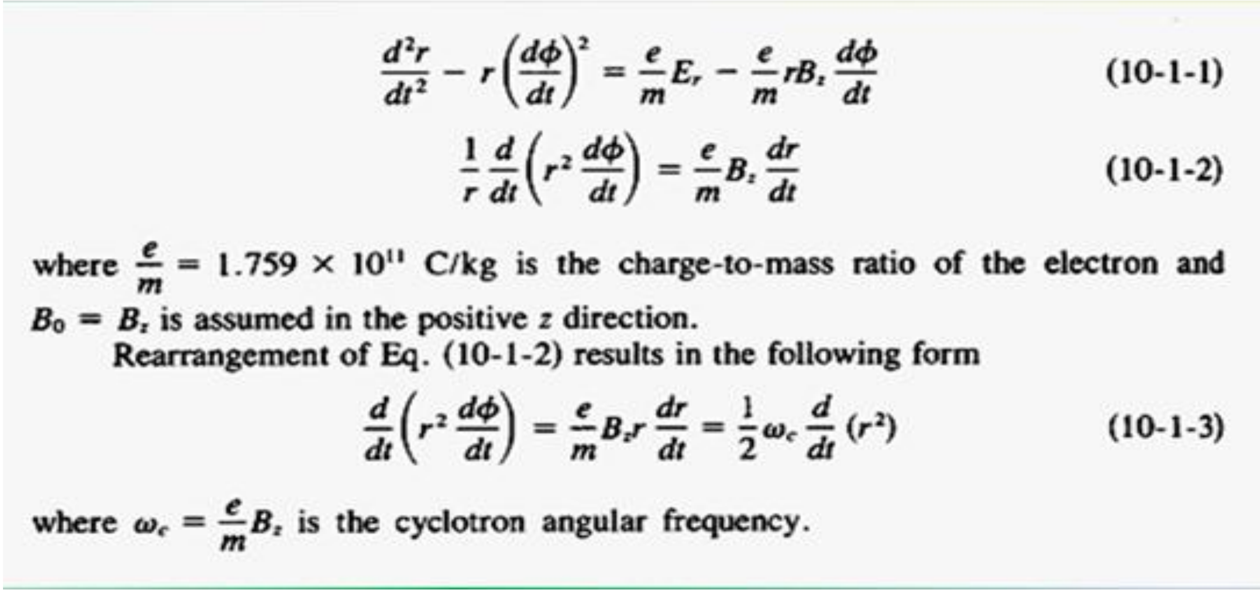


Figure 23: Resonant cavities of an hole-and-slot- type magnetron with inductive tuning elements

Hull cut-off

The Equation for the cutoff magnetic field can be obtained by considering the equations for the motion of electrons in the cylindrical magnetron which can be written as,



$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}E_r - \frac{e}{m}rB_z\frac{d\phi}{dt} \tag{10-1-1}$$

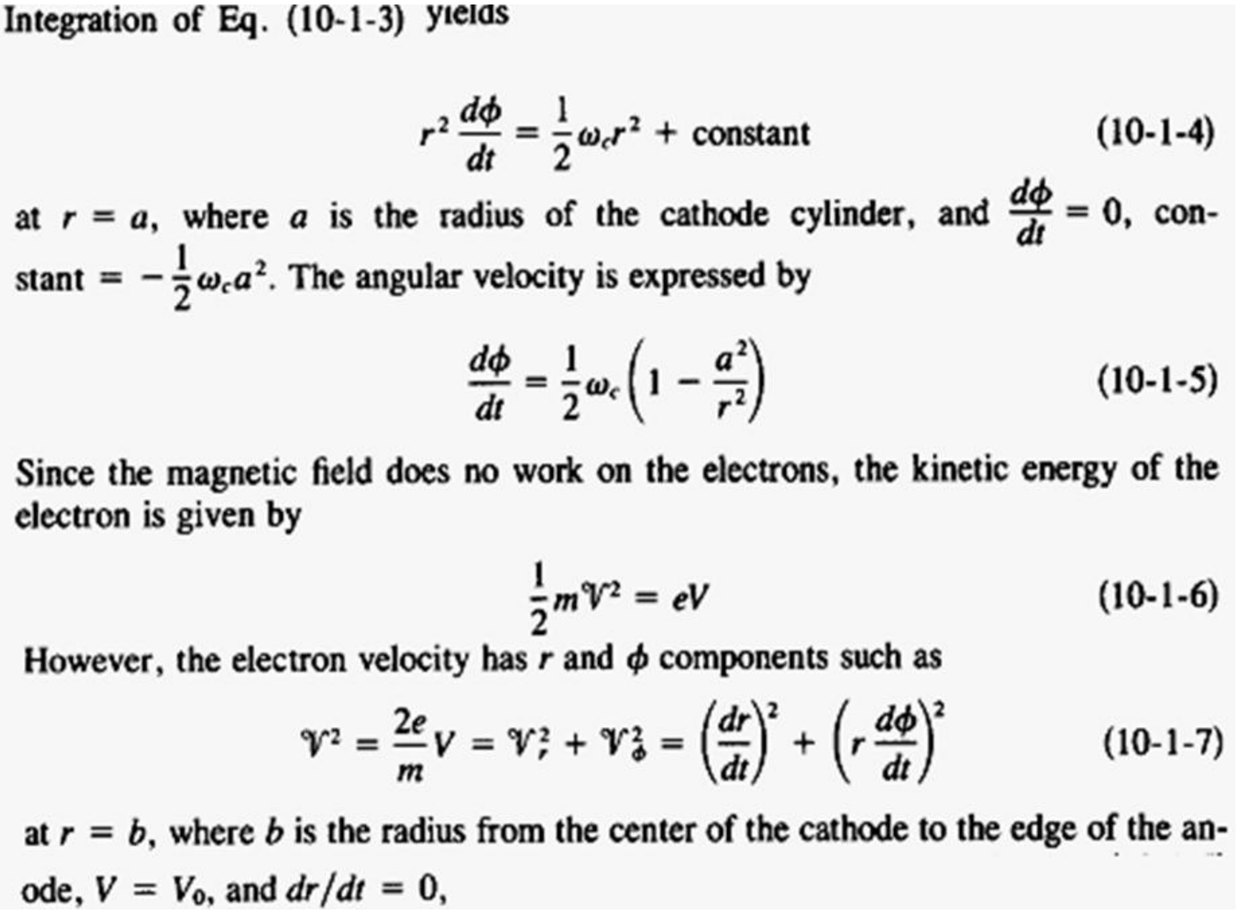
$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{dr}{dt} \tag{10-1-2}$$

where $\frac{e}{m} = 1.759 \times 10^{11}$ C/kg is the charge-to-mass ratio of the electron and $B_0 = B_z$ is assumed in the positive z direction.

Rearrangement of Eq. (10-1-2) results in the following form

$$\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z r\frac{dr}{dt} = \frac{1}{2}\omega_c\frac{d}{dt}(r^2) \tag{10-1-3}$$

where $\omega_c = \frac{e}{m}B_z$ is the cyclotron angular frequency.



Integration of Eq. (10-1-3) yields

$$r^2\frac{d\phi}{dt} = \frac{1}{2}\omega_c r^2 + \text{constant} \tag{10-1-4}$$

at $r = a$, where a is the radius of the cathode cylinder, and $\frac{d\phi}{dt} = 0$, constant $= -\frac{1}{2}\omega_c a^2$. The angular velocity is expressed by

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{a^2}{r^2}\right) \tag{10-1-5}$$

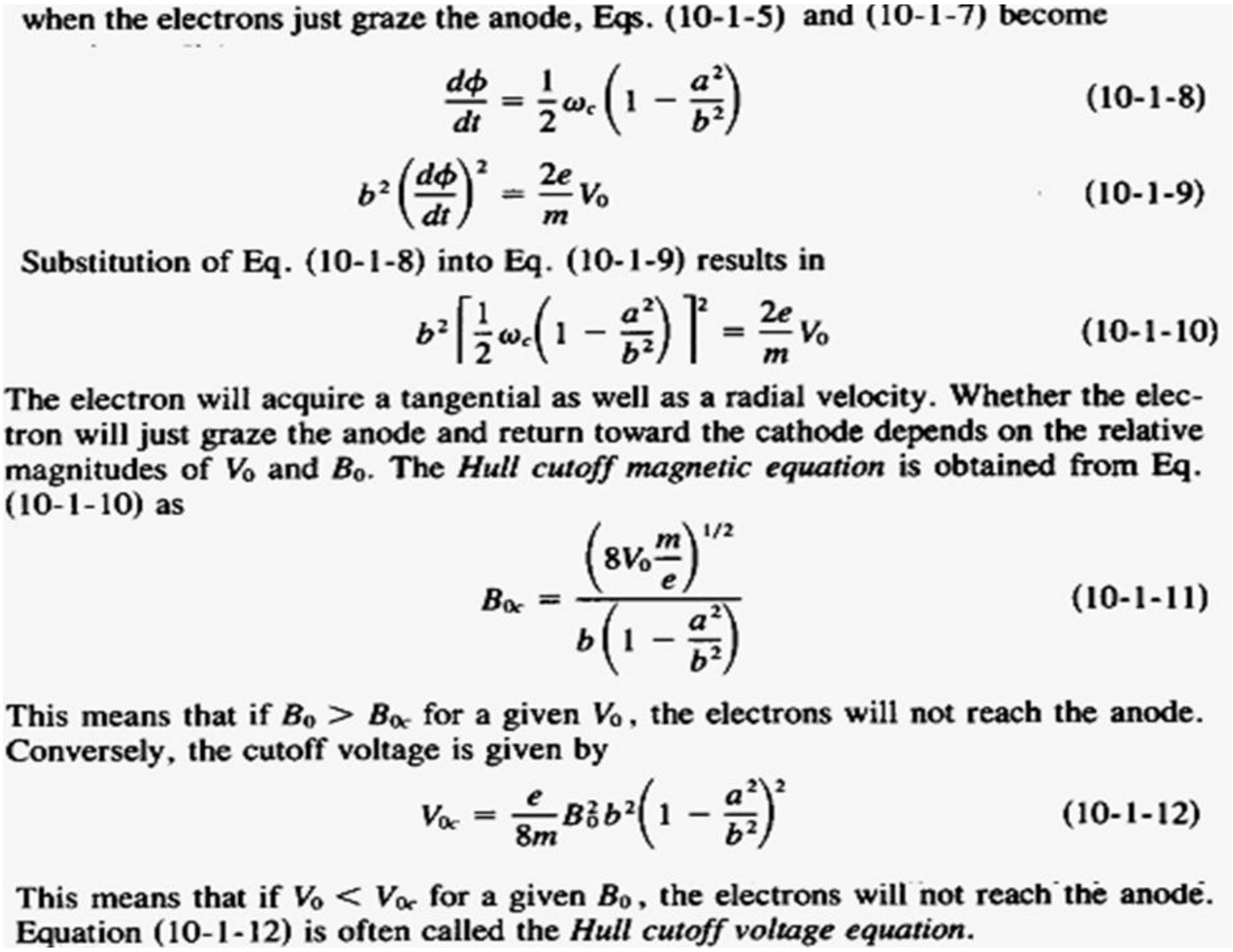
Since the magnetic field does no work on the electrons, the kinetic energy of the electron is given by

$$\frac{1}{2}m\mathcal{V}^2 = eV \tag{10-1-6}$$

However, the electron velocity has r and ϕ components such as

$$\mathcal{V}^2 = \frac{2e}{m}V = \mathcal{V}_r^2 + \mathcal{V}_\phi^2 = \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2 \tag{10-1-7}$$

at $r = b$, where b is the radius from the center of the cathode to the edge of the anode, $V = V_0$, and $dr/dt = 0$,


when the electrons just graze the anode, Eqs. (10-1-5) and (10-1-7) become

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{a^2}{b^2}\right) \qquad (10\text{-}1\text{-}8)$$

$$b^2\left(\frac{d\phi}{dt}\right)^2 = \frac{2e}{m}V_0 \qquad (10\text{-}1\text{-}9)$$

Substitution of Eq. (10-1-8) into Eq. (10-1-9) results in

$$b^2\left[\frac{1}{2}\omega_c\left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2e}{m}V_0 \qquad (10\text{-}1\text{-}10)$$

The electron will acquire a tangential as well as a radial velocity. Whether the electron will just graze the anode and return toward the cathode depends on the relative magnitudes of V_0 and B_0. The *Hull cutoff magnetic equation* is obtained from Eq. (10-1-10) as

$$B_{0c} = \frac{\left(8V_0\dfrac{m}{e}\right)^{1/2}}{b\left(1 - \dfrac{a^2}{b^2}\right)} \qquad (10\text{-}1\text{-}11)$$

This means that if $B_0 > B_{0c}$ for a given V_0, the electrons will not reach the anode. Conversely, the cutoff voltage is given by

$$V_{0c} = \frac{e}{8m}B_0^2 b^2\left(1 - \frac{a^2}{b^2}\right)^2 \qquad (10\text{-}1\text{-}12)$$

This means that if $V_0 < V_{0c}$ for a given B_0, the electrons will not reach the anode. Equation (10-1-12) is often called the *Hull cutoff voltage equation*.

Hartree conditions:

Hartee voltage is an important specification of magnetron.Magnetrons are designed to operate in π mode where the phase difference between adjacent resonators is 180°. For strong interaction between the wave on anode structure and the electron beam, the phase velocity of wave should be nearly equal to drift velocity vΦ and the oscillations for π mode start at beam voltage

Voh = $(2\pi f/N)*Bo*(b^2- a^2)$ ....*Hartee volatge*

where

f =operating frequency

N =number of resonators

b= anode radius

a= cathode radius

Bo = applied magnetic field.