UNIT II MICROWAVE COMPONENTS

Waveguide Attenuators- Resistive card, Rotary Vane types. Waveguide Phase Shifters : Dielectric, Rotary Vane types. Waveguide Multi port Junctions- E plane and H plane Tees, Magic Tee, Hybrid Ring. Directional Couplers- 2 hole, Bethe hole types. Ferrites-Composition and characteristics, Faraday Rotation. Ferrite components: Gyrator, Isolator, Circulator. S-matrix calculations for 2 port junction, E & H plane Tees, Magic Tee, Directional Coupler, Circulator and Isolator

WAVEGUIDE ATTENUATORS :

In order to control power levels in a microwave system by partially absorbing the transmitted microwave signal, attenuators are employed. Resistive films (dielectric glass slab coated with aquadag) are used in the design of both fixed and variable attenuators.

A co-axial fixed attenuator uses the dielectric lossy material inside the centre conductor of the co-axial line to absorb some of the centre conductor microwave power propagating through it dielectric rod decides the amount of attenuation introduced. The microwave power absorbed by the lossy material is dissipated as heat.

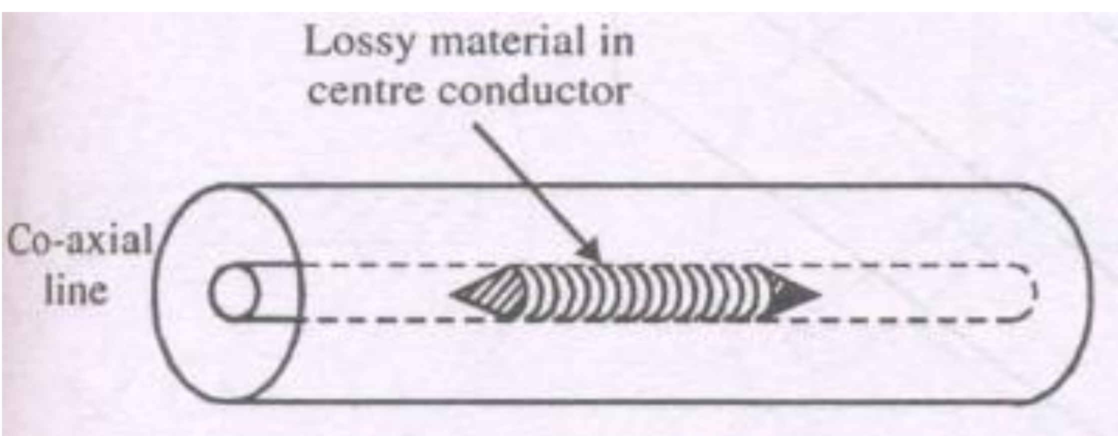


Figure 5.6 Co-axial line fixed attenuator

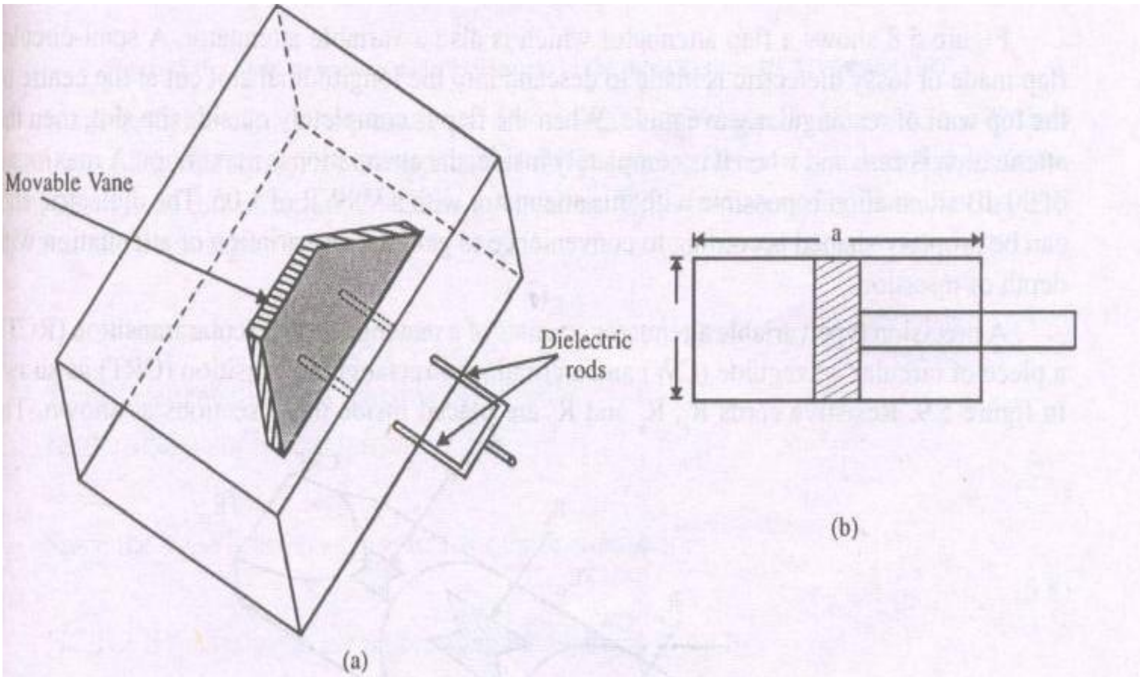


Figure 5.7 : Movable vane variable attenuator

In waveguides, the dielectric slab coated with aquadag is placed at the centre of the waveguide parallel to the maximum E-field for dominant TEIO mode. Induced current on the lossy material due to incoming microwave signal, results in power dissipation, leading to attenuation of the signal. The dielectric slab is tapered at both ends upto a length of more than half wavelength to reduce reflections as shown in figure 5.7. The dielectric slab may be made movable along the breadth of the waveguide by supporting it with two dielectric rods separated by an odd multiple of quarter guide wavelength and perpendicular to electric field. When the slab is at the centre, then the attenuation is maximum (since the electric field is concentrated at the centre for TEIO mode) and when it is moved towards one sidewall, the attenuation goes on decreasing thereby controlling the microwave power corning out of the other port.

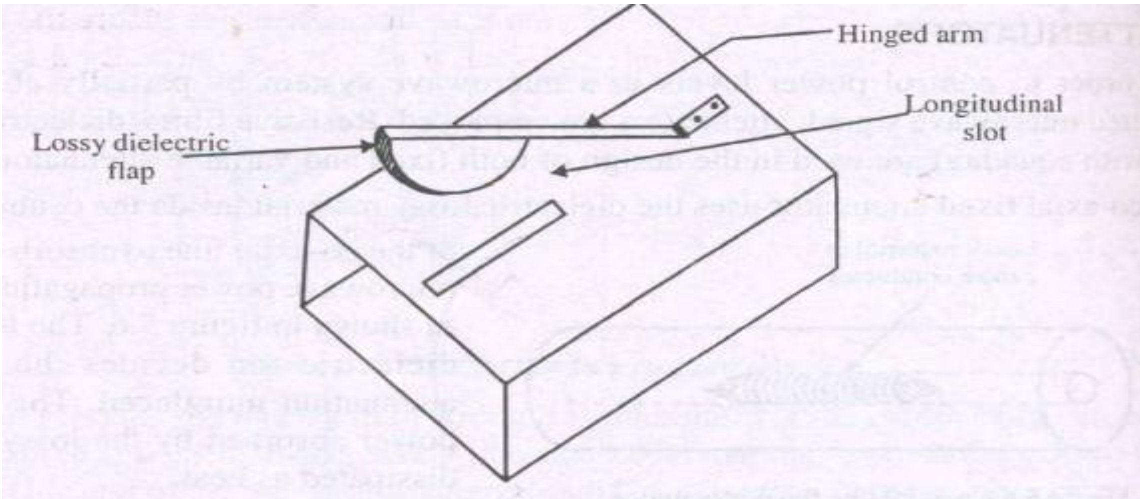


Figure 5.8 : Flap attenuator

Figure 5.8 shows a flap attenuator which is also a variable attenuator. A semi-circular flap made of lossy dielectric is made to descend into the longitudinal slot cut at the centre of the top wall of rectangular waveguide. When the flap is completely outside the slot, then the attenuation is zero and when it is completely inside, the attenuation is maximum. A maximum direction of 90 dB attenuation is possible with this attenuator with a VSWR of 1.05. The dielectric slab can be properly shaped according to convenience to get a linear variation of attenuation within the depth of insertion.

A precision type variable attenuator consists of a rectangular to circular transition (ReT), a piece of circular waveguide (CW) and a circular-to-rectangular transition (CRT) as shown in figure 5.9. Resistive cards R , Rand R are placed inside these sections as shown. The centre circular section containing the resistive card Rb can be precisely rotated by 36 00 with respect to the two fixed resistive cards. The induced current on the resistive card R due to the incident signal is dissipated as heat producing attenuation of the transmitted signal. TE mode in RCT is converted into TE in circular waveguide. The resistive cards R and R a kept perpendicular to the electric field of TEIO mode so that it does not absorb the energy. But any component parallel to its plane will be readily absorbed. Hence, pure TE mode is excited in circular waveguide section. II

If the resistive card in the centre section is kept at an angle 8 relative to the E-field direction of the TE11 mode, the component E cos8 parallel to the card get absorbed while the component E sin 8 is transmitted without attenuation. This component finally comes out as E sin2θ as shown in figure 5.10.

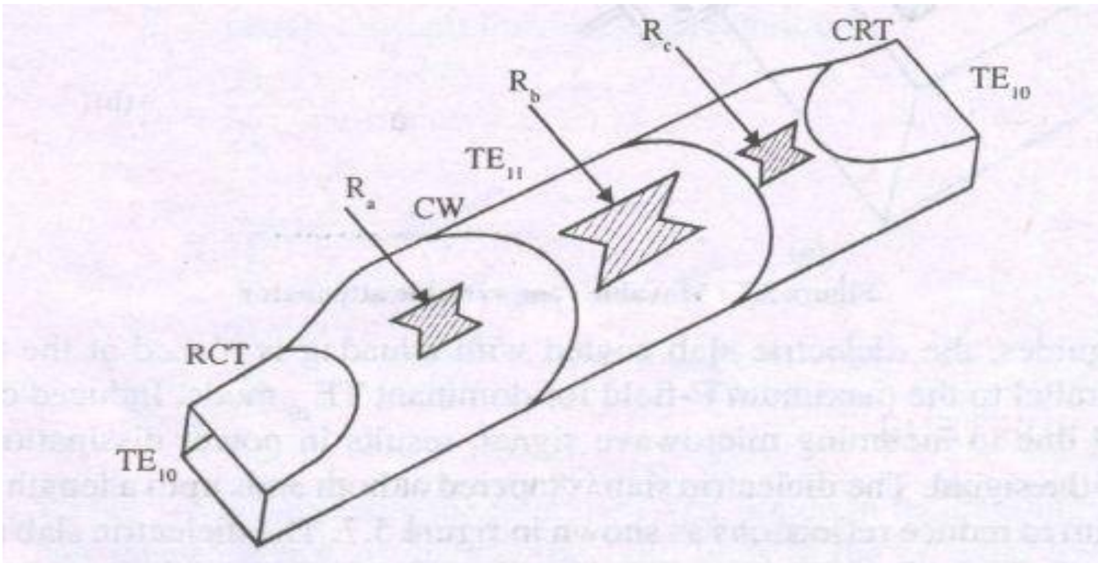


Figure 5.9 : Precision type variable attenuator

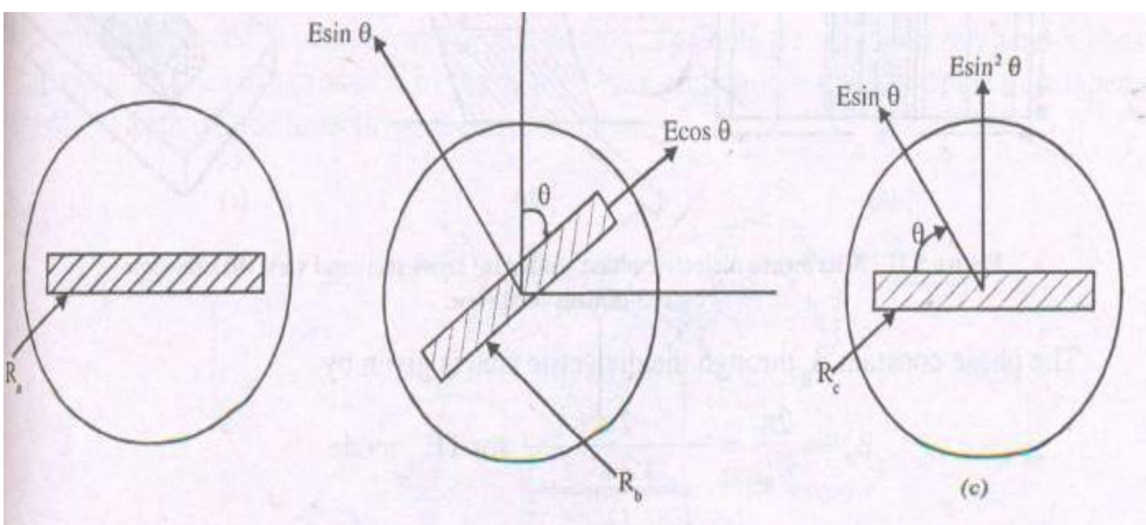


Figure 5.10 : Field components of electric field in the three sections RCT, CW and CRT

WAVEGUIDE PHASE SHIFTERS :

A microwave phase shifter is a two port device which produces a variable shift in phase of the incoming microwave signal. A lossless dielectric slab when placed inside the rectangular waveguide produces a phase shift.

PRECISION PHASE SHIFTER

The rotary type of precision phase shifter is shown in figure 5.12 which consists of a circular waveguide containing a lossless dielectric plate of length 2l called "half-wave section", a section of rectangular-to-circular transition containing a lossless dielectric plate of length l, called "quarter-wave section", oriented at an angle of 45° to the broader wall of the rectangular waveguide and a circular-to-rectangular transition again containing a lossless dielectric plate of same length 1 (quarter wave section) oriented at an angle 45°.

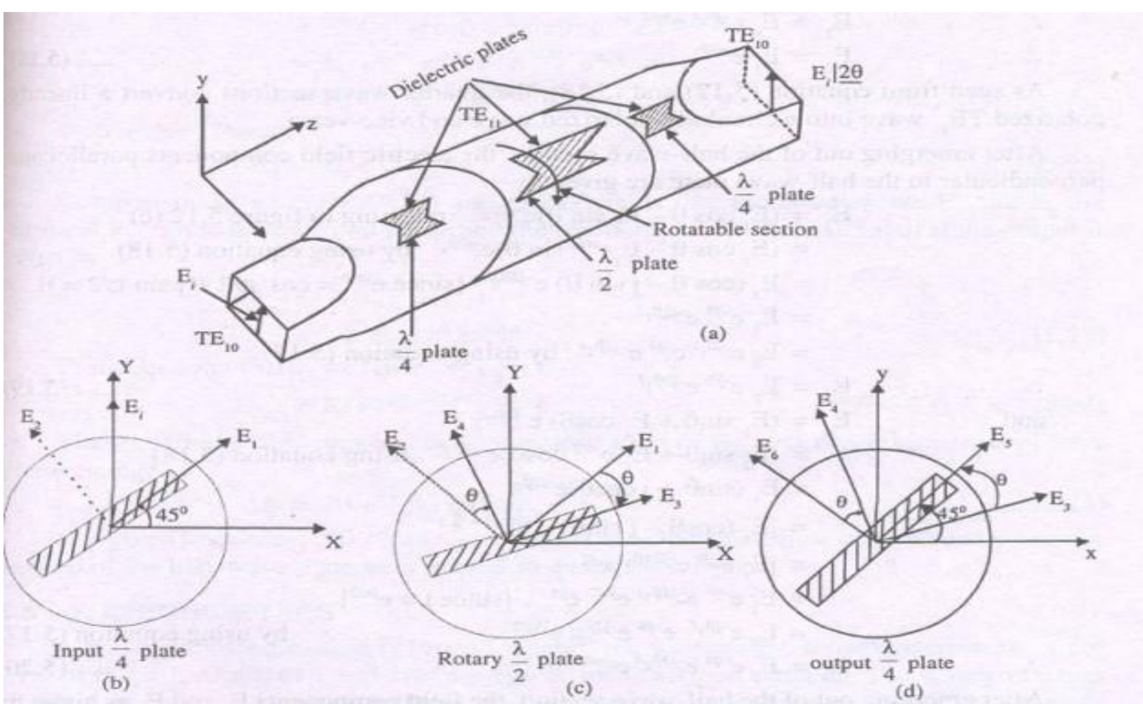


Figure 5.12 : Illustrating precision rotary phase shifter

The incident TEIO mode becomes TE11 mode in circular waveguide section. The half-wave section produces a phase shift equal to twice that produced by the quarter wave section. The dielectric plates are tapered at both ends to reduce reflections due to discontinuity.

When TEIO mode is propagated through the input rectangular waveguide of the rectangular to circular transition, then it is converted into TEll in the circular waveguide section. Let E; be the maximum electric field strength of this mode which is resolved into components, EI parallel to the plate and E2 perpendicular to EI as shown in figure 5.12 (b). After propagation through the plate these components are given by

$$E_1 = E_0 \, e^{-j\beta_1 l}$$

$$E_2 = E_0 \, e^{-j(\beta_1 l - 90^0)} = E_0 \, e^{-j(\beta_1 l - \frac{\pi}{2})}$$

$$\therefore \quad E_2 = E_0 \, e^{-j\beta_1 l} \, e^{j\pi/2}$$

$$\therefore \quad E_2 = E_1 \, e^{j\pi/2}$$

The length I is adjusted such that these two components E1 and Ez have equal amplitude but differing in phase by = 90°.

$$E_l = (E_i \cos 45^0) \, e^{-j\beta_1 l} = E_0 \, e^{-j\beta_1 l}$$

$$\text{and} \quad E_2 = (E_i \sin 45^0) \, e^{-j\beta_2 l} = E_0 \, e^{-j\beta_2 l}$$

$$\text{Where} \quad E_0 = \frac{E_i}{\sqrt{2}}$$

The quarter wave sections convert a linearly polarized TE11 wave into a circularly polarized wave and vice-versa. After emerging out of the half-wave section, the electric field components parallel and perpendicular to the half-wave plate are given by

$$
\begin{aligned}
E_3 &= (E_1 \cos\theta - E_2 \sin\theta)e^{-j2\beta_1 l} \quad \text{referring to figure 5.12 (c)}\\
&= (E_1 \cos\theta - E_1 e^{j\pi/2} \sin\theta)e^{-j2\beta_1 l} \quad \text{by using equation (5.18)}\\
&= E_1 (\cos\theta - j\sin\theta)\, e^{-j2\beta_1 l} \quad [\text{since } e^{j\pi/2} = \cos\pi/2 + j\sin\pi/2 = j]\\
&= E_1\, e^{-j\theta}\, e^{-j2\beta_1 l}\\
&= E_0\, e^{-j\beta_1 l}\, e^{-j\theta}\, e^{-j2\beta_1 l} \quad \text{by using equation (5.17)}
\end{aligned}
$$

$$\therefore \quad E_3 = E_0\, e^{-j\theta}\, e^{-j3\beta_1 l} \qquad \text{..... (5.19)}$$

and

$$
\begin{aligned}
E_4 &= (E_1 \sin\theta + E_2 \cos\theta)\, e^{-j2\beta_2 l}\\
&= (E_1 \sin\theta + E_1\, e^{j\pi/2} \cos\theta)e^{-j2\beta_2 l} \quad \text{using equation (5.18)}\\
&= E_1 (\sin\theta + j\cos\theta)e^{-j2\beta_2 l}\\
&= jE_1 (\cos\theta - j\sin\theta)\, e^{-j2(\beta_1 l - \frac{\pi}{2})}\\
&= jE_1\, e^{-j\theta}\, e^{-j2\pi\beta_1 l}\, e^{j\pi}\\
&= E_1\, e^{-j\theta}\, e^{-j2\beta_1 l}\, e^{j\pi/2}\, e^{j\pi} \quad [\text{since } j = e^{j\pi/2}]\\
&= E_0\, e^{-j\beta_1 l}\, e^{-j\theta}\, e^{-j2\beta_1 l}\, e^{j3\pi/2} \qquad \text{by using equation (5.17)}
\end{aligned}
$$

$$\therefore \quad E_4 = E_0\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{j3\pi/2} \qquad \text{..... (5.20)}$$

After emerging out of the half-wave section, the field components E3 and E4 as given by equations (5.19) and (5.20), may again be resolved into two TE11 modes, polarized parallel and perpendicular to the output quarterwave plate. At the output end of this quarterwave plate, the field components parallel and perpendicular to the quarter wave plate, by referring to figure 5.12 (d), can be expressed as

$$
\begin{aligned}
E_5 &= (E_3 \cos\theta + E_4 \sin\theta)\, e^{-j\beta_1 l}\\
&= (E_0\, e^{-j\theta}\, e^{-j3\beta_1 l} \cos\theta + E_0\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{j3\pi/2} \sin\theta)e^{-j\beta_1 l}
\end{aligned}
$$

$$
\begin{aligned}
&= E_0 (\cos\theta + e^{j3\pi/2} \sin\theta)\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{-j\beta_1 l} \\
&= E_0 (\cos\theta - j \sin\theta)\, e^{-j\theta} .\, e^{-4\beta_1 l}
\end{aligned}
$$

$$\therefore \quad E_5 = E_0\, e^{-j\theta}\, e^{-j\theta}\, e^{-j4\beta_1 l}$$

$$\therefore \quad E_5 = E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l} \qquad \text{..... (5.21)}$$

and

$$E_6 = (E_4 \cos\theta - E_3 \sin\theta)\, e^{-j\beta_2 l}$$

$$\therefore \quad E_6 = (E_0\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{j3\pi/2} \cos\theta - E_0\, e^{-j\theta}\, e^{-j3\beta_1 l} \sin\theta)\, e^{-j\beta_2 l} \text{ by using equations (5.19) and (5.20)}$$

$$
\begin{aligned}
\therefore \quad E_6 &= E_0 (e^{j3\pi/2} \cos\theta - \sin\theta)\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{-j(\beta_1 l - \frac{\pi}{2})} \\
&= E_0 (-j \cos\theta - \sin\theta)\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{-j\beta_1 l}\, e^{j\pi/2} \\
&= E_0 (-j) (\cos\theta - j \sin\theta)\, e^{-j\theta}\, e^{-j4\beta_1 l}\, e^{j\pi/2} \\
&= E_0\, e^{j3\pi/2}\, e^{-j\theta}\, e^{-j\theta}\, e^{-j4\beta_1 l}\, e^{j\pi/2} \\
&= E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l}\, e^{j2\pi}
\end{aligned}
$$

since $e^{j2\pi} = 1$, we get

$$E_6 = E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l} \qquad \text{..... (5.22)}$$

Comparison of equation (5.21) and (5.22) yields that the components Es and E6 are identical in both magnitude and phase and the resultant electric field strength at the output is given by

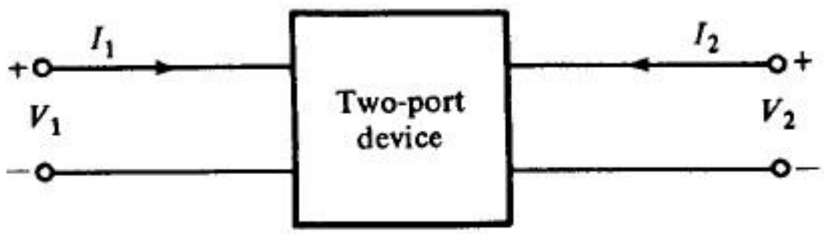
$$
\begin{aligned}
E_{out} &= \sqrt{(E_5)^2 + (E_6)^2} \\
&= \sqrt{2}\; E_0\, e^{-j2\theta}\, e^{-j4\beta_1 l}
\end{aligned}
$$

MICROWAVE HYBRID CIRCUITS :

A microwave circuit is formed when several microwave components and devices such as microwave generators , microwave amplifiers, variable attenuators, cavity resonators, microwave filters, directional couplers, isolators are coupled together without any mismatch for proper transmission of a microwave signal.

SCATTERING MATRIX OF A TWO PORT NETWORK :

Let us consider a two port network which represents a number of parameter



$$H \text{ parameters: } \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}$$

$$Y \text{ parameters: } \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_1 \end{aligned}$$

$$Z \text{ parameters: } \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned}$$

$$ABCD \text{ parameters: } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

All the above listed parameters can be represented as the ratio of either voltage to current or current or voltage under certain conditions of input or output ports.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad \text{(short circuit)}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad \text{(open circuit)}$$

At microwave frequencies it is impossible to measure:

1 total voltage and current as the required equipment is not available.

2 Over a broad band region, it is difficult to achieve perfect open and short circuit conditions.

3 The active devices used inside the two port network such as microwave power transistors will tend to become unstable under open and short circuit conditions.

WAVE GUIDE MULTI PORT JUNCTIONS:

A waveguide Tee is formed when three waveguides are interconnected in the form of English alphabet T and thus waveguide tee is 3-port junction. The waveguide tees are used to connects a branch or section of waveguide in series or parallel with the main waveguide transmission line either for splitting or combining power in a waveguide system

There are basically 2 types of tees namely

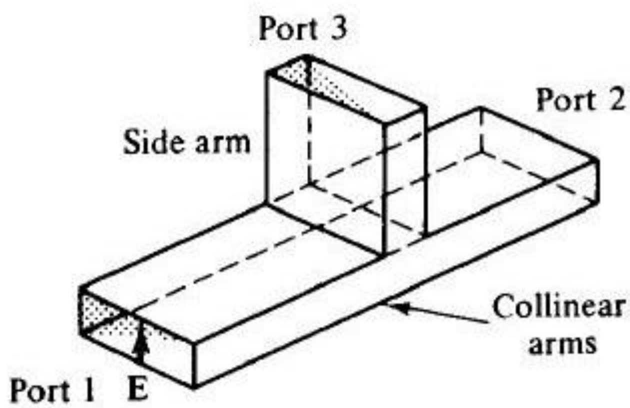
. H- Plane Tee junction

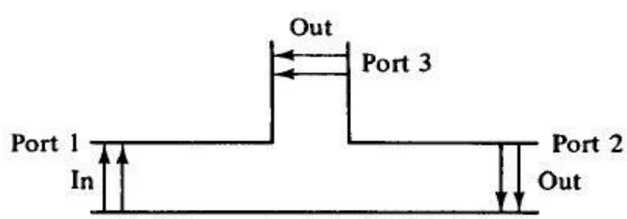
. E-plane Tee junction

A combination of these two tee junctions is called a hybrid tee or “Magic Tee”.

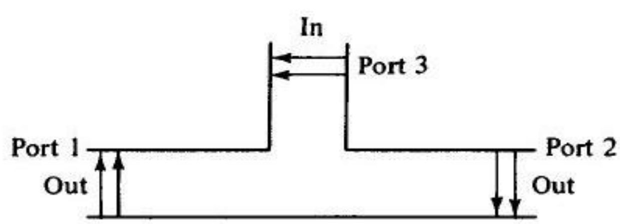
E-plane Tee(series tee):

An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the E field of the main guide . if the collinear arms are symmetric about the side arm.If the E-plane tee is perfectly matched with the aid of screw tuners at the junction, the diagonal components of the scattering matrix are zero because there will be no reflection . When the waves are fed into side arm, the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in same magnitude.





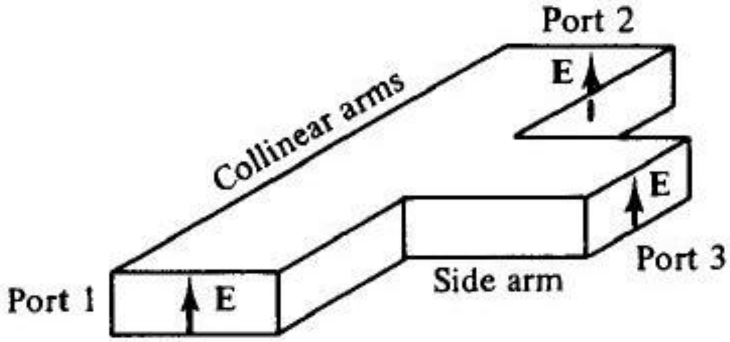
(a)



(b)

H-plane tee: (shunt tee)

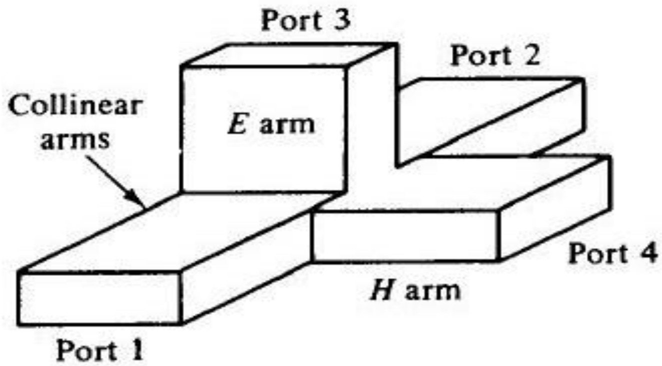
An H-plane tee is a waveguide tee in which the axis of its side arm is shunting the E field or parallel to the H-field of the main guide.



If two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive. If the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in same magnitude.

Magic Tee (Hybrid Tees)

A magic tee is a combination of E-plane and H-plane tee. The characteristics of magic tee are:



1. If two waves of equal magnitude and same phase are fed into port 1 and port the output will be zero at port 3 and additive at port 4.

2. If a wave is fed into port 4 it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3.

3. If a wave is fed into port 3 , it will produce an output of equal magnitude and opposite phase at port 1 and port 2. the output at port 4 is zero.

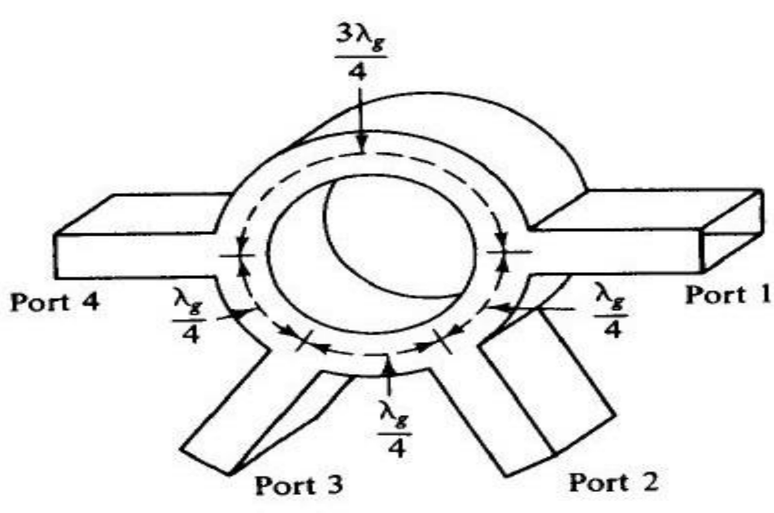
4. If a wave is fed into one of the collinear arms at port 1 and port 2, it will not appear in the other collinear arm at port 2 or 1 because the E-arm causes a phase delay while H arm causes a phase advance.

Therefore the **S** matrix of a magic tee can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

Hybrid Rings(Rat Race circuits):

A hybrid ring consists of an annular line of proper electrical length to sustain standing waves, to which four arms are connected at proper intervals by means of series or parallel junctions.



The hybrid ring has characteristics similar to those of the hybrid tee. When a I wave is fed into port 1, it will not appear at port 3 because the difference of phase shifts for the waves traveling in the clockwise and counterclockwise direction is 180°. Thus the waves are canceled at port 3. For the same reason, the w aves fed into port 2 will not emerge at port 4 and so on .

The S matrix for an ideal hybrid ring can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

It should be noted that the phase cancellation occurs only at a designated frequency for an ideal hybrid ring. In actual hybrid rings there are small leakage coupling s and therefore the zero elements in the matrix are not equal to zero.

WAVE GUIDE CORNERS , BENDS AND TWISTS:

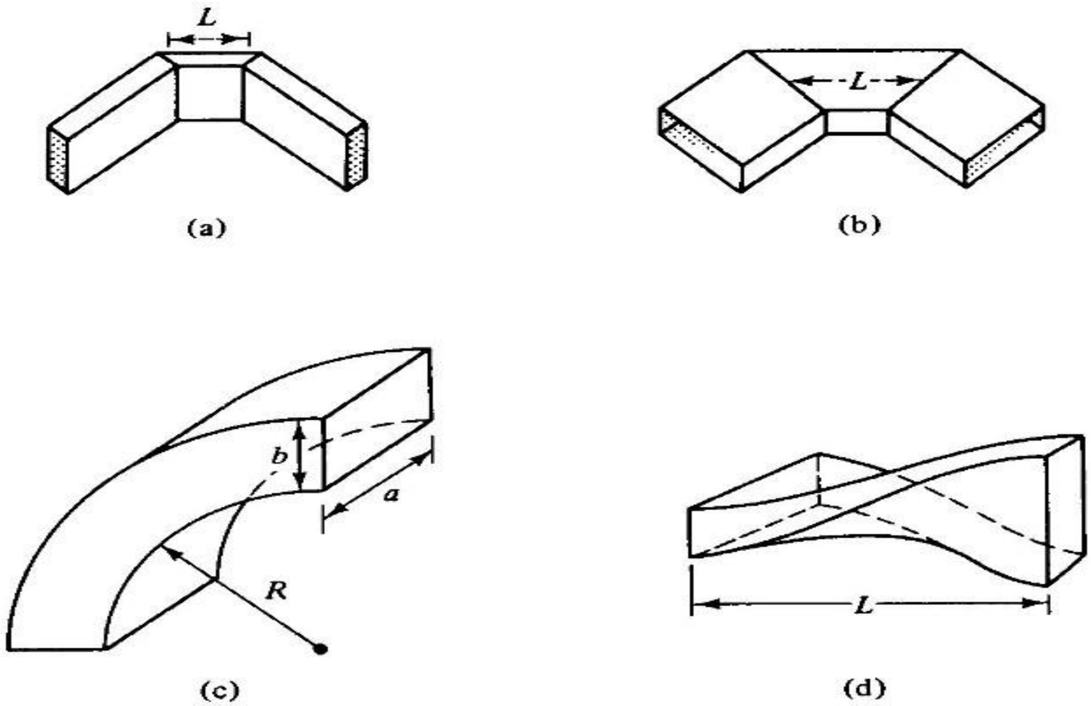
The waveguide corner, bend, and twist are shown in figure below, these waveguide components are normally used to change the direction of the guide through an arbitrary angle.

In order to minimize reflections from the discontinuities, it is desirable to have the mean length L between continuities equal to an odd number of quarter wave lengths. That is,

$$L = (2n + 1)\frac{\lambda_g}{4}$$

where n = 0, 1, 2, 3, ... , and Ag is the wavelength in the waveguide. If the mean length L is an odd number of quarter wavelengths, the reflected waves from both ends of the waveguide section are completely canceled. For the waveguide bend, the minimum radius of curvature for a small reflection is given by Southworth as

$R = 1.5b$ for an E bend $\qquad$ $R = 1.5a$ for an H bend

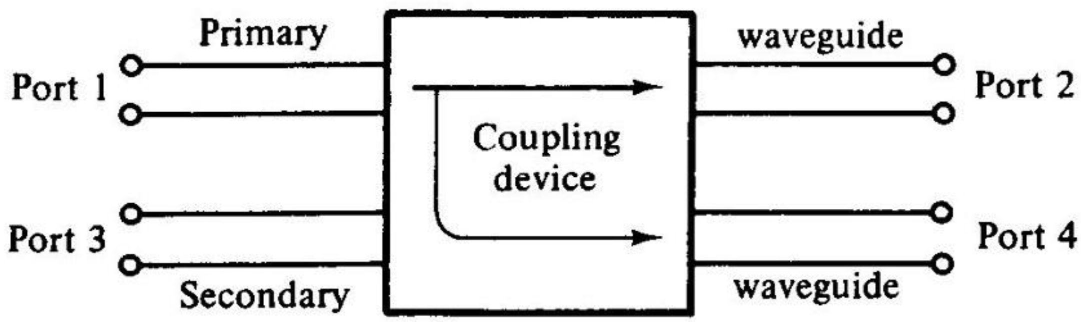


Waveguide corner, bend, and twist. (a) E-plane corner. (b) H-plane corner. (c) Bend. (d) Continuous twist.

DIRECTIONAL COUPLERS :

A directional coupler is a four-port waveguide junction as shown below. It Consists of a primary waveguide 1-2 and a secondary waveguide 3-4. When all Ports are terminated in their characteristic impedances, there is free transmission of the waves without reflection, between port 1 and port 2, and there is no transmission of power between port I and port 3 or between port 2 and port 4 because no couplin g exists between these two pairs of ports.The degree of coupling between port 1 an d port4 and between port 2 and port3 depends on the structure of the coupler.

The characteristics of a directional coupler can be expressed in terms of its Coupling factor and its directivity. Assuming that the wave is propagating from port to port2 in the primary line, the coupling factor and the directivity are defined ,



where P1 = power input to port 1

P3 = power output from port 3

P4 = power output from port 4

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4}$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances. The coupling factor is a measure of the ratio of power levels in the primary and secondary lines. Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1 .

This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line. The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and port A are perfectly matched.

Actually well-designed directional couplers have a directivity of only 30 to 35 .

Several types of directional couplers exist, such as a two-hole direct coupler, four-hole directional coupler, reverse-coupling directional coupler , and Bethe-hole directional coupler the very commonly used two-hole directional coupler is described here.

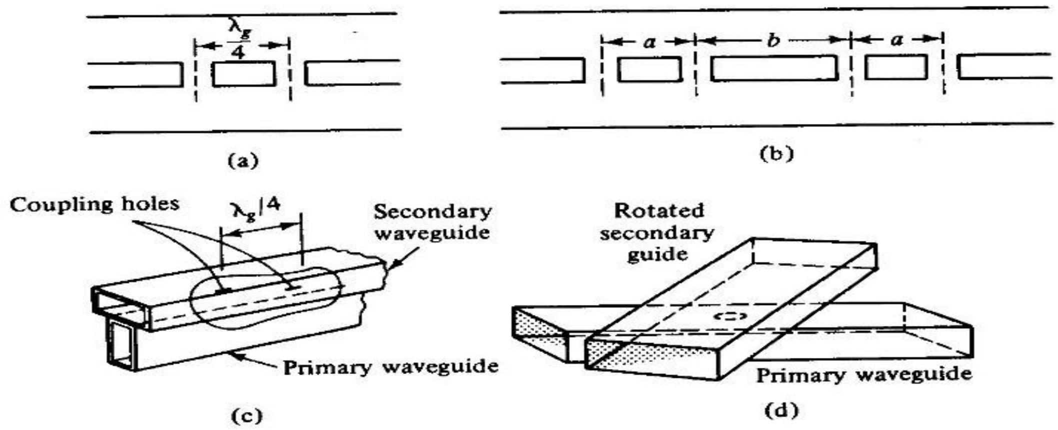


Figure 4-5-2 Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

TWO HOLE DIRECTIONAL COUPLERS:

A two hole directional coupler with traveling wave propagating in it is illustrated . the spacing between the centers of two holes is

$$L = (2n + 1)\frac{\lambda_g}{4}$$

A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as he holes act as slot antennas. The forward waves in the secondary guide are in same phase , regardless of the h ole space and are added at port 4. the backward waves in the secondary guide are out of phase and are cancelled in port 3.

6.11 FERRITE DEVICES

Ferrites are non-metallic materials with resistivities (ρ) nearly 10^{14} times greater than metals and with dielectric constants (ε_r) around 10-15 and relative permeabilities of the order of 1000. They have magnetic properties similar to those of ferrous metals. They are oxide based compounds having general composition of the form $MeO.Fe_2O_3$ *i.e.,*, a mixture of a metallic oxide and ferric oxide where MeO represents any divalent metallic oxide such as MnO, ZnO, CdO, NiO or a mixture of these. They are obtained by firing powdered oxides of materials at 1100°C or more and pressing them into different shapes. This processing gives them the added characteristics of ceramic insulators so that they can be used at microwave frequencies.

Ferrites have atoms with large number of spinning electrons resulting in strong magnetic properties. These magnetic properties are due to the magnetic dipole moment associated with the electron spin. Because of the above properties, ferrites find application in a number of microwave devices to reduce reflected power, for modulation purposes and in switching circuits. Because of high resistivity they can be used upto 100 GHz.

Ferrites have one more peculiar property which is useful at microwave frequencies *i.e.,*, the non-reciprocal property. When two circularly polarised waves one rotating clockwise and other anticlockwise are made to propagate through ferrite, the material reacts differently to the two rotating fields, thereby presenting different effective permeabilities to both the waves. *i.e.,*, ε_{r1}, μ_{r1}, ρ_1 for left circularly polarised wave and ε_{r2}, μ_{r2}, ρ_2 for the right circularly polarised wave.

6.11.1 Faraday Rotation in Ferrites

Consider an infinite lossless medium. A static field B_o is applied along the z-direction. A plane TEM wave that is linearly polarised along the x-axis at $t = 0$ is made to propagate through the ferrite in the z-direction. The plane of polarisation of this wave will rotate with distance, a phenomenon known as *Faraday Rotation*.

Any linearly polarised wave can be regarded as the vector sum of two counter rotating circularly polarised waves ($\mathbf{E}_O/2$ vectors shown in Fig. 6.30). The ferrite material offers different characteristics to these waves, with the result that the phase change for one wave is larger than the other wave resulting in rotation 'θ' of the linearly polarized wave, at $z = l$.

It is observed that a rotation of 100 degrees or more per cm of ferrite length is typical for ferrites at a frequency of 10 GHz. If the direction of propagation is reversed, the plane of polarisation continues to rotate in the same direction *i.e.,*, from $z = l$ to $z = 0$, the wave will arrive back

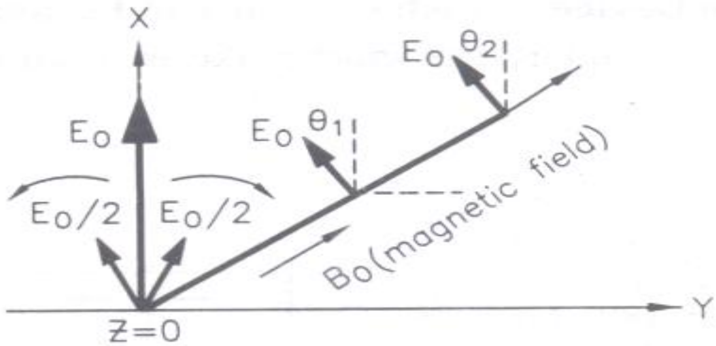


Fig. 6.30 *Faraday rotation*.

at $z = 0$ polarised at an angle $2\,\theta$ relative to x-axis.

In fact, the angle of rotation 'θ' is given by

$$\theta = \frac{l}{2}(\beta_+ \sim \beta_-) \qquad ...(6.75)$$

where, l = length of the ferrite rod

β_+ = Phase shift for the right circularly polarised (component in clockwise direction) wave with respect to some reference.

β_- = phase shift for the left circularly polarised (component in anticlockwise direction) wave with respect to the same reference.

In a practical ferrite medium, there will be finite losses. The propagation constant for circularly polarised wave will have unequal attenuation constants and unequal phase constant. Due to this, the direction of Faraday rotation will be different in the two regions above and below the resonant frequency (ω_o). A two port ferrite device is shown in Fig. 6.31 when a wave is transmitted from port ① port ②, it undergoes rotation in the anticlockwise direction as shown. Even if the same wave is allowed to propagate from port ② port ①, it will undergo rotation in the same direction (anticlockwise). Hence the direction of rotation of linearly polarised wave is independent of the direction of propagation of the wave.

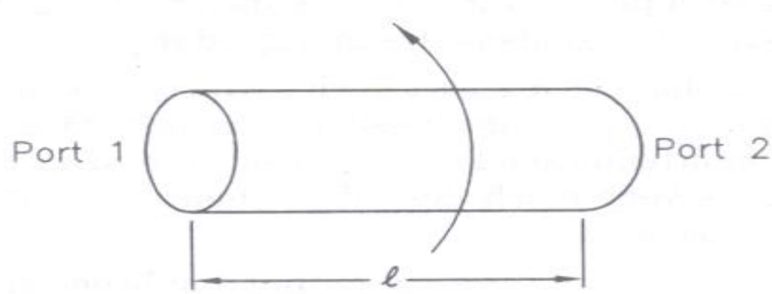


Fig. 6.31

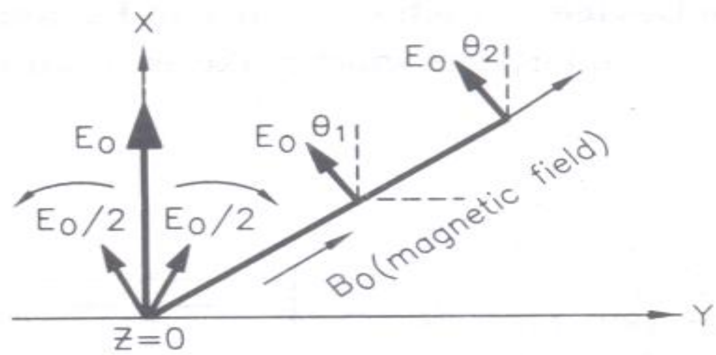


Fig. 6.30 *Faraday rotation*.

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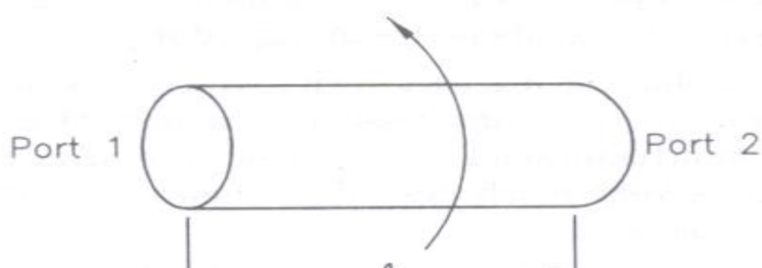


Fig. 6.31

6.11.2 Microwave Devices which make use of Faraday rotation

We discuss three important devices which make use of faraday rotation

(*a*) Gyrator

(*b*) Isolator

(*c*) Circulator

(***a***) ***Gyrator*** : It is a two port device that has a relative phase difference of 180° for transmission from port ① port ② and 'no' phase shift (0° phase shift) for transmission from port ② to port ① shown in Fig. 6.32.

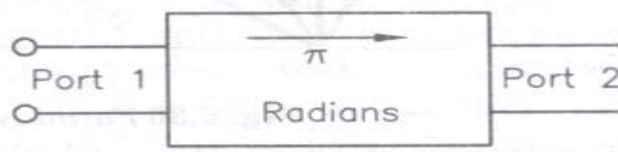


Fig. 6.32

The construction of a gyrator is as shown in Fig. 6.32. It consists of a piece of circular waveguide carrying the dominant TE_{11} mode with transitions to a standard rectangular waveguide with dominant mode (TE10) at both ends. A thin circular ferrite rod tapered at both ends is located inside the circular waveguide supported by polyfoam and the waveguide is surrounded by a permanent magnet which generates dc magnetic field for proper operation of ferrite. To the input end a 90° twisted rectangular waveguide is connected as shown. The ferrite rod is tapered at both ends to reduce the attenuation and also for smooth rotation of the polarized wave.

Operation : When a wave enters port ① its plane of polarization rotates by 90° because of the twist in the waveguide. It again undergoes Faraday rotation through 90° because of ferrite rod and the wave which comes out of port ② will have a phase shift of 180° compared to the wave entering port ①.

But when the same wave (TE_{10} mode signal) enters port ②, it undergoes faraday rotation through 90° in the same anticlock wise direction. Because of the twist, this wave gets rotated back by 90° comes out of port ① with 0° phase shift as shown in Fig. 6.30. Hence a wave at port ① undergoes a phaseshift of π radians (or 180°) but a wave fed from port ② does not change its phase in a gyrator.

(***b***) ***Isolator*** : An isolator is a 2 port device which provides very *small* amount of attenuation for transmission from port ① to port ② but provides *maximum* attenuation for transmission from port ② to port ①. This requirement is very much desirable when we want to match a source with a variable load.

In most microwave generators, the output amplitude and frequency tend to fluctuate very significantly with changes in load impedance. This is due to mismatch of generator output to the load resulting in reflected wave from load. But these reflected waves should not be

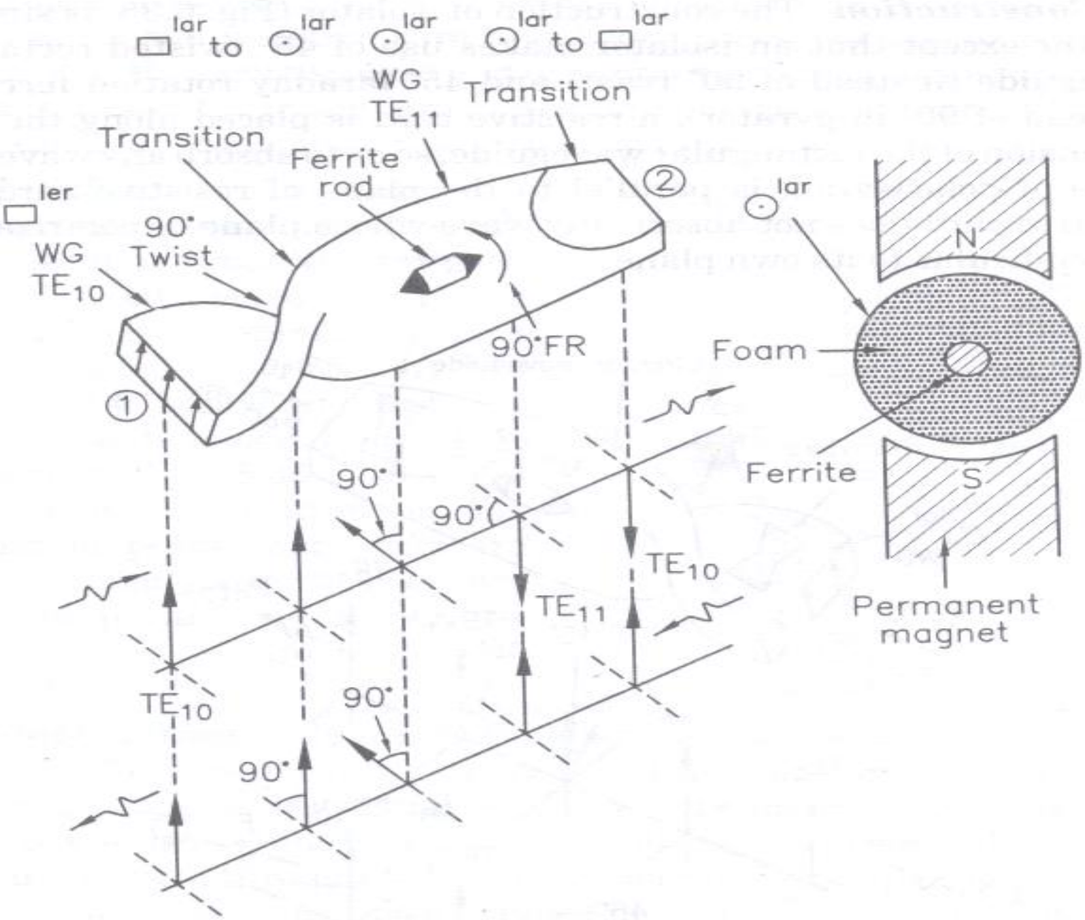


Fig. 6.33

allowed to reach the microwave generator, which will cause amplitude and frequency instability of the microwave generator.

When isolator is inserted between generator and load, the generator is coupled to the load with zero attenuation and reflections if any from the load side are completely absorbed by the isolator without affecting the generator output. Hence the generator appears to be matched for all loads in the presence of isolator so that there is no change in frequency and output power due to variation in load. Thisis shown in Fig. 6.34.

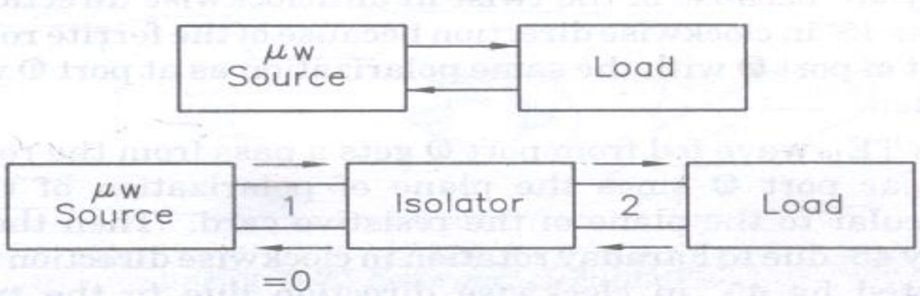


Fig. 6.34

Construction : The construction of isolator (Fig. 6.35) is similar to gyrator except that an isolator makes use of 45° twisted rectangular waveguide (instead of 90° twist) and 45° faraday rotation ferrite rod (instead of 90° in gyrator), a resistive card is placed along the larger dimension of the rectangular waveguide, so as to absorb any wave whose plane of polarisation is parallel to the plane of resistive card. The resistive card does not absorb any wave whose plane of polarization is perpendicular to its own plane.

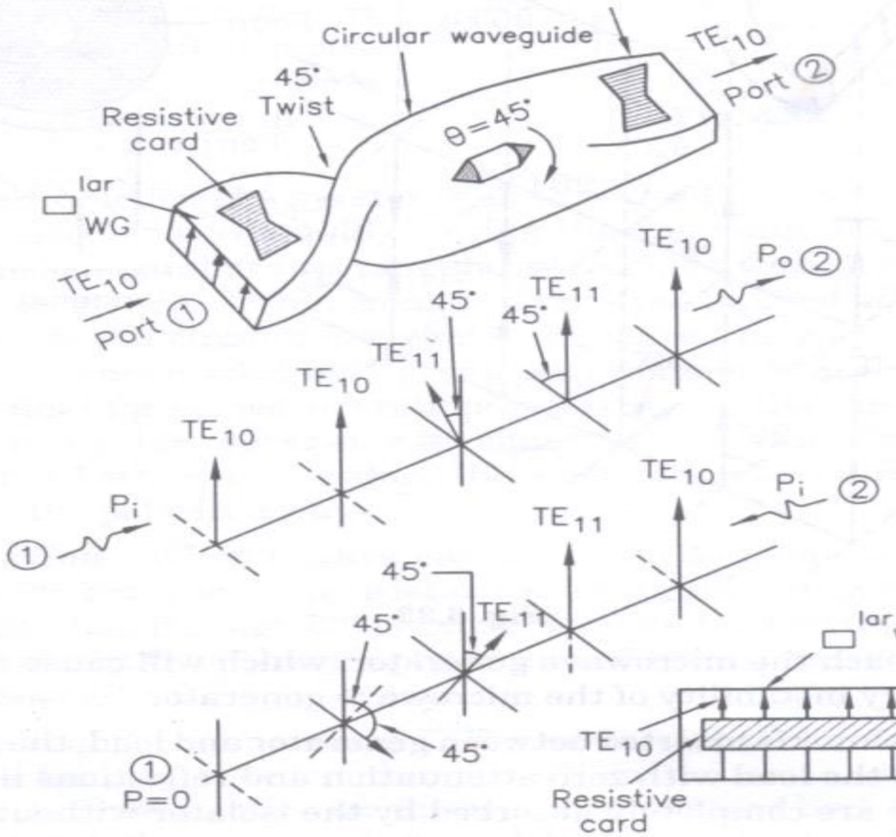


Fig. 6.35 *Constructional details of isolator.*

Operation : A TE_{10} wave passing from port ① through the resistive card and is not attenuated. After coming out of the card, the wave gets shifted by 45° because of the twist in anticlockwise direction and then by another 45° in clockwise direction because of the ferrite rod and hence comes out of port ② with the same polarization as at port ① without any attenuation.

But a TE_{10} wave fed from port ② gets a pass from the resistive card placed near port ② since the plane of polarization of the wave is perpendicular to the plane of the resistive card. Then the wave gets rotated by 45° due to Faraday rotation in clockwise direction and further gets rotated by 45° in clockwise direction due to the twist in the waveguide. Now the plane of polarization of the wave will be parallel with that of the resistive card and hence the wave will be completly absorbed by the resistive card and the output at port ① will be zero. This power is dissipated in the card as heat. In practice 20 to 30 dB isolation is obtained for transmission from port ② to port ①.

(*c*) ***Circulator*** : A circulator is a four port microwave device which has a peculiar property that each terminal is connected only to the next clockwise terminal. *i.e.*, port ① is connected to port ① only and not to port ③ and ④ and port ② is connected only to port ③ etc. This is shown in Fig. 6.36. Although there is no restriction on the number of ports, four ports are most commonly used. They are useful in parametric amplifiers, tunnel diode, amplifiers and duplexer in radars.

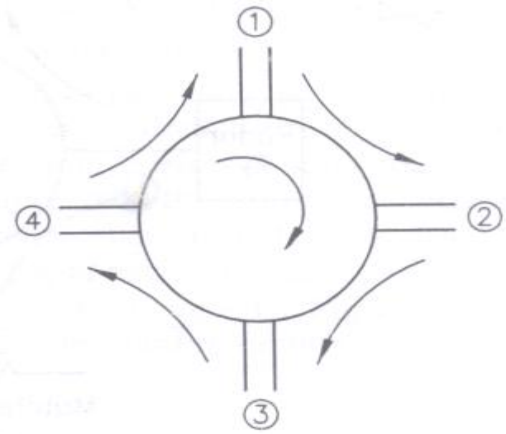


Fig. 6.36

Construction : A four port Faraday rotation circulator is shown in Fig. 6.37. The power entering port ① is TE_{10} mode and is converted to TE_{11} mode because of gradual rectangular to circular transition. This power passes port ③ unaffected since the electric field is not significantly cut and is rotated through 45° due to the ferrite, passes port ④ unaffected (for the same reason as it passes port ③) and finally emerges out of port ②. Power from port ② will have plane of polarization already tilted by 45° with respect to port ①. This power passes port ④ unaffected because again the electric field is not significantly cut. This wave gets rotated by another 45° due to ferrite rod in the clockwise direction. This power whose plane of polarization is tilted through 90° finds port ③ suitably aligned and emerges out of it. Similarly port ③ in coupled only to port ④ and port ④ to port ①.

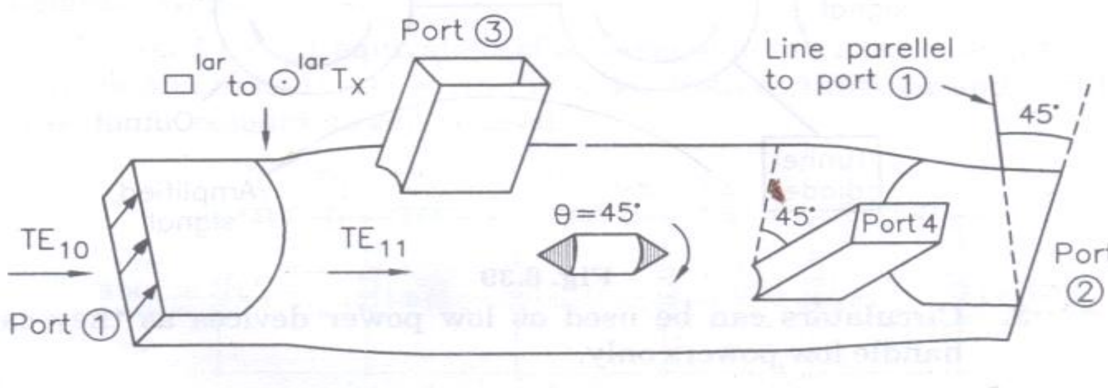


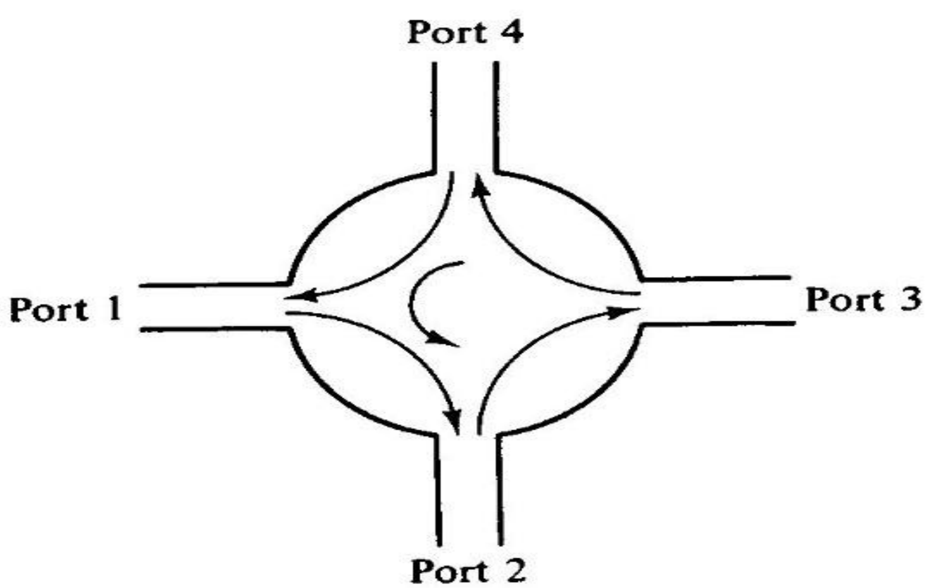
Fig. 6.37 *Four port circulator.*

CIRCUALTORS AND ISOLATORS

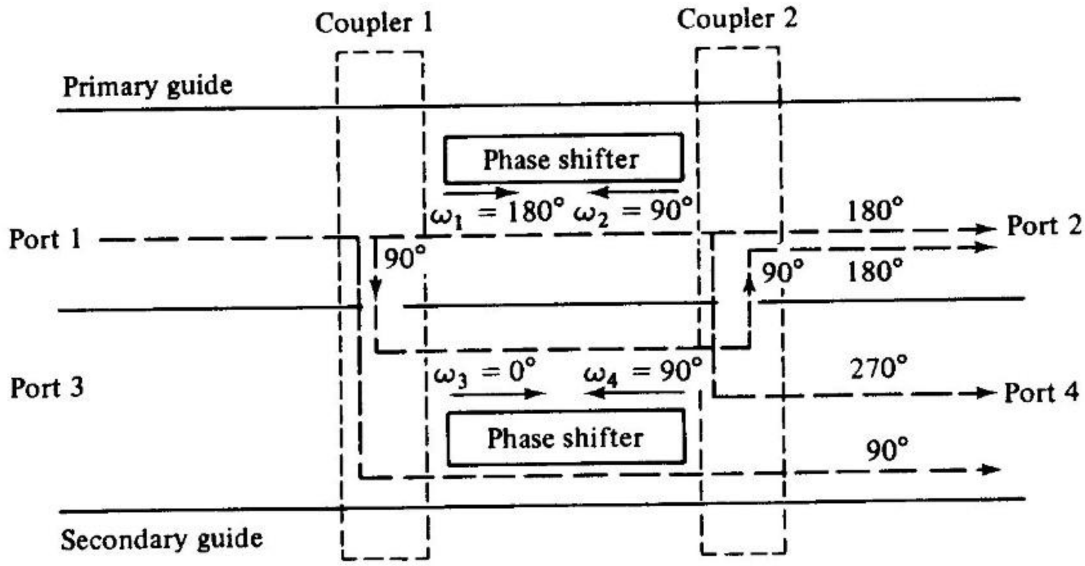
Both microwave circulators and isolators are non reciprocal transmission devices that use the property of Faraday rotation in the ferrite material. A non reciprocal phase shifter consists of thin slab of ferrite placed in a rectangular waveguide at a point where the dc magnetic field of the incident wave mode is circularly polarized. When a piece of ferrite is affected by a dc magnetic field the ferrite exhibits Faraday rotation. It does so because the ferrite is nonlinear material and its permeability is an asymmetric tensor.

MICROWAVE CIRCULATORS:

A *microwave circulator* is a multiport waveguide junction in which the wave can flow only from the *nth* port to the *(n* + I)th port in one direction Although there is no restriction on the number of ports, the four-port microwave circulator is the most common. One type of four-port microwave circulator is a combination of two 3-dB side hole directional couplers and a rectangular waveguide with two non reciprocal phase shifters.



The symbol of a circulator.



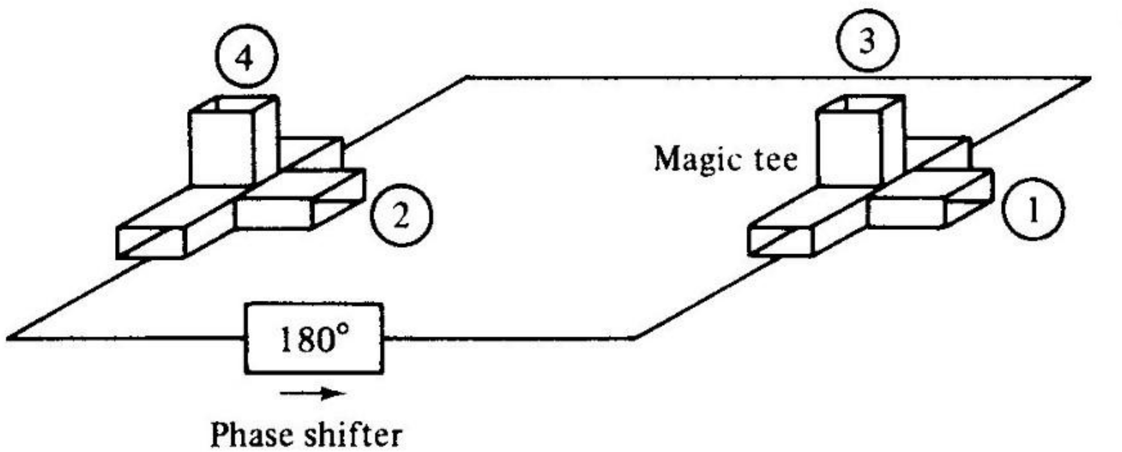
Schematic diagram of four port circulator

The operating principle of a typical microwave circulator can be analyzed with the aid of Fig shown above .Each of the two 3-dB couplers in the circulator introduces a phase shift of 90°, and each of the two phase shifters produces a certain amount o f phase change in a certain direction as indicated. When a wave is incident to port 1,the wave is split into two components by coupler I. The wave in the primary guide arrives at port 2 with a relative phase' change of 180°. The second wave propagates through the two couplers and the secondary guide and arrives at port 2 with a relative phase shift of 180°. Since the two waves reaching port 2 are in phase, the power transmission is obtained from port 1 to port 2. However, the wave propagates through the primary guide, phase shifter, and coupler 2 and arrives at port 4 with a phase change of 270°. The wave travels through coupler 1 and the secondary guide, and it arrives at port 4 with a phase shift of 90°. Since the two waves reaching port 4 are out of phase by 180°, the power transmission from port 1 to port 4 is zero. In general, the differential propagation constants in the two directions o f propagation in a waveguide containing ferrite phase shifters should be

$$\omega_1 - \omega_3 = (2m + 1)\pi \qquad \text{rad/s}$$
$$\omega_2 - \omega_4 = 2n\pi \qquad \text{rad/s}$$

where *m* and *n* are any integers, including zeros. A similar analysis shows that a wave incident to port 2 emerges at port 3 and so on. As a result, the sequence of power flow is designated as 1 ~ 2 ~ 3 ~ 4 ~ 1.

Many types of microwave circulators are in use today. However, their principles of operation remain the same. .A four-port circulator is constructed by the use of two magic tees and a phase shifter. The phase shifter produces a phase shift of 180°.



A four port circulator

A perfectly matched, lossless, and nonreciprocal four-port circulator has an S matrix of the form

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

Using the properties of S parameters the S-matrix is

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

MICROWAVE ISOLATORS :

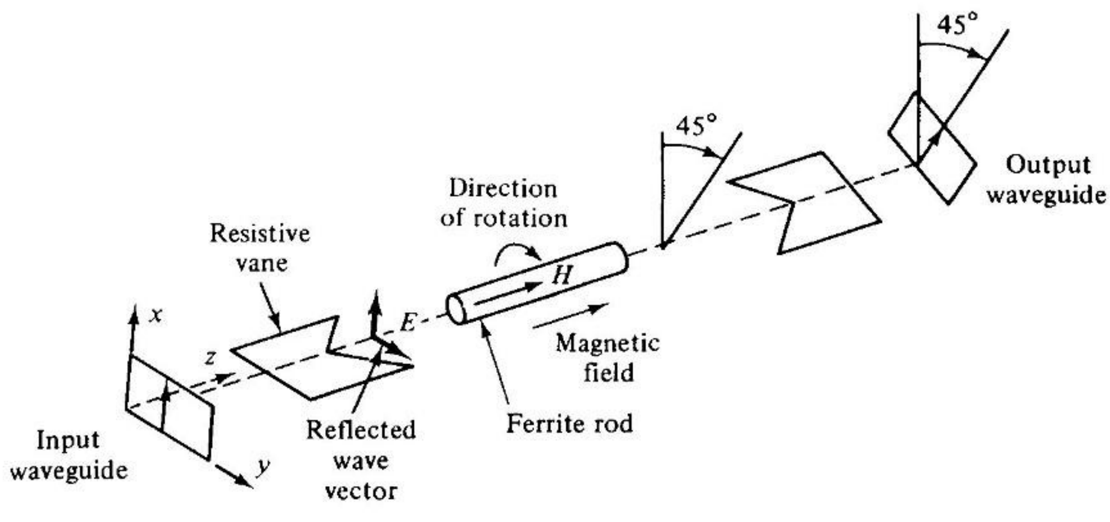
An *isolator* is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Thus the isolator is usually called *uniline.*

Isolators are generally used to improve the frequency stability of microwave generators, such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency. In such cases, the isolator placed between the generator and load prevents the reflected power from the unmatched load from returning to the generator. As a result, the isolator maintains the frequency stability of the generator.

Isolators can be constructed in many ways. They can be made by terminating ports 3 and 4 of a four-port circulator with matched loads. On the other hand, isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguide as shown below.

The isolator here is a Faraday-rotation isolator. Its operating principle can be explained as follows . The input resistive card is in the *y-z* plane, and the output resistive card is displaced 45° with respect to the input card. The dc magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by 45°.

The degrees of rotation depend on the length and diameter of the rod and on the applied de magnetic field. An input TE10 dominant mode is incident to the left end of the isolator. Since the TE10 mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation The wave in the ferrite rod section is rotated clockwise by 45° and is normal to the output resistive card. As a result of rotation, the wave arrives at the output.



end without attenuation at all. On the contrary, a reflected wave from the output end is similarly rotated clockwise 45° by the ferrite rod. However, since the reflected wave is parallel to the input resistive card, the wave is thereby absorbed by the input card. The typical performance of these isolators is about 1-dB insertion loss in forward transmission and about 20- to 30-dB isolation in reverse attenuation.

MICROWAVE PASSIVE DEVICES

INTRODUCTIO N

A microwave network consists of coupling of various microwave components and devices such as attenuators, phase shifters, amplifiers, resonators etc., to sources through transmission lines or waveguides. Connection of two or more microwave devices and components to a single point results in a *microwave junction.*

In a low frequency network, the input and output variables are voltage and current which can be related in terms of impedance Z-parameters, or admittance Y-parameters or hybrid h-parameters or ABCD parameters. These relationships for a two-port network of figure 4.1 can be represented by

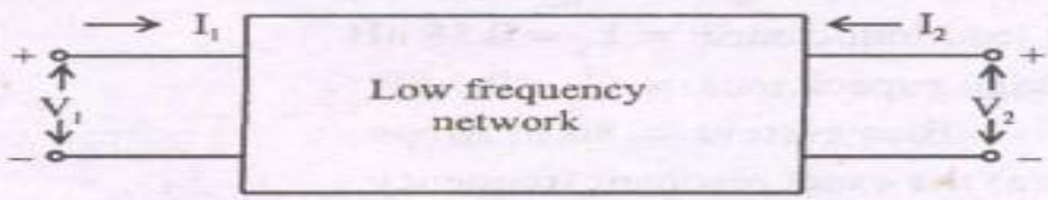


Figure 4.1: Low frequency two port network

$$\left.\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}\right\} \quad \text{..... (4.1)}$$

$$\text{or} \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{..... (4.2)}$$

$$\left.\begin{aligned} I_1 &= Y_{11} V_1 + Y_{12} V_2 \\ I_2 &= Y_{21} V_1 + Y_{22} V_2 \end{aligned}\right\} \quad \text{..... (4.3)}$$

$$\text{or} \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{..... (4.4)}$$

$$\left.\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right\} \quad \text{..... (4.5)}$$

$$\text{or} \quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{..... (4.6)}$$

$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}\right\} \quad \text{..... (4.7)}$$

$$\text{or} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \text{..... (4.8)}$$

These parameters, Z, Y,h and ABCD parameters can be easily measured at low frequencies under short or open circuit conditions and can be used for analyzing the circuit.

The physical length of the device or the line at microwave frequencies, is comparable to or much larger than the wavelength. Due to this, the voltage and current are difficult to measure as also the above mentioned parameters. The reasons for this are listed as below

(a) Equipment is not available to measure the total voltage and total current at any point.

(b) Over a wide range of frequencies, short and open circuits are difficult to realize.

(c) Active devices such as power transistors, tunnel diodes etc, will become unstable under short or open circuit conditions.

Therefore, a new representation is needed to overcome these problems at microwave frequencies. The logical variables are traveling waves rather than voltages and currents and these variables are labeled as "Scattering or S-parameters". These parameters for a two port network are represented as shown in figure 4.2 .

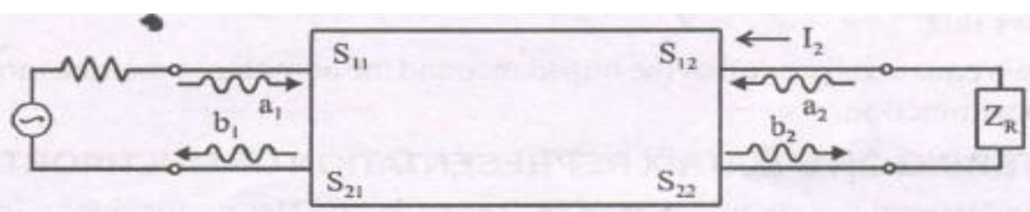


Figure 4.2 : S-parameters of a two-port network

These S-parameters can be represented in an equation form related to the traveling waves a1,a2 and b1 b2 through

$$\left.\begin{aligned} b_1 &= S_{11}\ a_1 + S_{12}\ a_2 \\ b_2 &= S_{21}\ a_1 + S_{22}\ a_2 \end{aligned}\right\} \qquad \text{..... (4.9)}$$

SYMMETRICAL Z AND Y MATRICES FOR RECIPROCAL NETWORK

In a reciprocal network, the junction media are characterized by scalar electrical parameters namely absolute permeability I..l and absolute permittivity E . In such a network, the impedance and the admittance matrices became symmetrical. This property can be proved by considering an N-port network.

S-MATRIX REPRESENTATION OF MULTIPORT NETWORK:

Let us now consider a junction of "n" number of rectangular waveguides as shown in figure 4.4. In this case, all "a' s" represent the incident waves at respective ports and all "b 's" the reflected waves from the microwave junction coming out of the respective ports. In this case also, equations (4.18) and (4.19) are still valid where S..and S. have the following meanings:

Sii= Scattering coefficient corresponding to the input power applied at IJ the i1hport and output power coming out of i^{th} port

Sjj = Scattering coefficient corresponding to the power applied at the ^{t} port " . and output taken out of i1hport itself. This coefficient is a measure of amount of mismatch between the i^{h} port and the junction

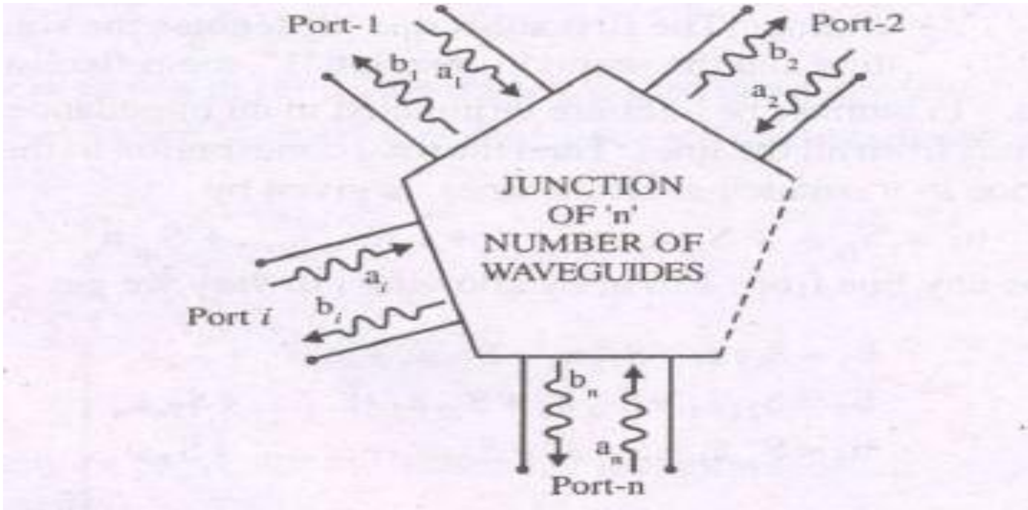


Figure 4.4 : Junction of "n" number of waveguides.

As an example let us consider a two-port network as shown in figure 4:5.

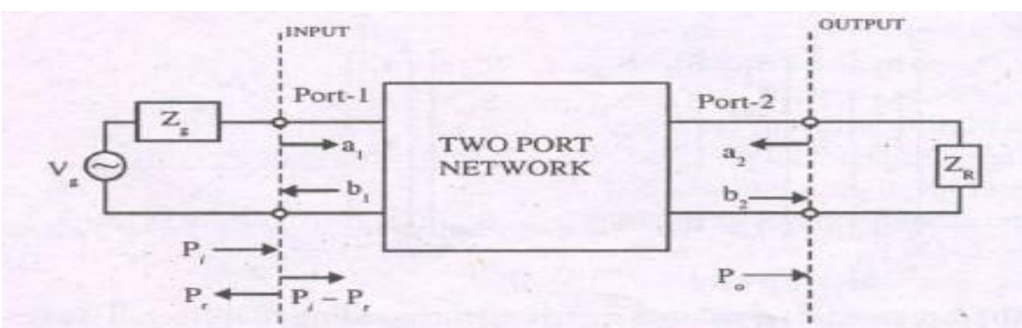


Figure 4.5 : Illustrating two-port network

The relationship. between the incident and reflected waves in terms of scattering coefficients can be written as

$$\text{Insertion loss indB} = 10\log_{10}\frac{1}{|S_{21}|^2}$$

$$= 20\log_{10}\frac{1}{|S_{21}|} = 20\log_{10}\frac{1}{|S_{12}|} \qquad \text{..... (4.24)}$$

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad \text{..... (4.20)}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad \text{..... (4.21)}$$

From these equations, the scattering coefficients are found as

$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$ = reflection coefficient at port-1 when port-2 is terminated with a matched load ($a_2 = 0$)

$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$ = reflection coefficient at port-2 when port-1 is terminated with a matched load ($a_1 = 0$)

$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$ = attenuation of the wave travelling from port-2 to port-1.

$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$ = attenuation of the wave travelling from port-1 to port-2.

In figure 4.5, we have

P_i = incident power at port-1

P_r = power reflected by the network coming out of port-1 itself.

P_o = output power at port-2.

The various losses can be expressed in terms of S-parameters as given below:

$$\text{Insertion loss in dB} = 10\log_{10}\frac{P_i}{P_o} \qquad \text{..... (4.22)}$$

But $P_i \ \alpha \ |a_1|^2$

$P_o \ \alpha \ |b_2|^2$

$$\therefore \quad \frac{P_i}{P_o} = \frac{|a_1|^2}{|b_2|^2} = \frac{1}{\left|b_2/a_1\right|^2} = \frac{1}{|S_{21}|^2} = \frac{1}{|S_{12}|^2} \qquad \text{..... (4.23)}$$

2. Transmission loss (or attenuation loss) in dB

$$= 10 \log_{10} \frac{P_i - P_r}{P_o}$$

$$= 10 \log_{10} \frac{|a_1|^2 - |b_1|^2}{|b_2|^2}$$

$$= 10 \log_{10} \frac{\dfrac{|a_1|^2 - |b_1|^2}{|a_1|^2}}{\dfrac{|b_2|^2}{|a_1|^2}} = 10 \log_{10} \frac{1 - \dfrac{|b_1|^2}{|a_1|^2}}{\dfrac{|b_2|^2}{|a_1|^2}}$$

$$= 10 \log_{10} \frac{1 - |S_{11}|^2}{|S_{21}|^2} \quad \text{..... (4.25)}$$

3. Reflection loss in dB $= 10 \log_{10} \dfrac{P_i}{P_i - P_r}$

$$= 10 \log_{10} \frac{|a_1|^2}{|a_1|^2 - |b_1|^2}$$

$$= 10 \log_{10} \frac{1}{1 - \dfrac{|b_1|^2}{|a_1|^2}}$$

$$= 10 \log_{10} \frac{1}{1 - |S_{11}|^2} \quad \text{..... (4.26)}$$

4. Return loss in dB $= 10 \log_{10} \dfrac{P_i}{P_r}$

$$= 10 \log_{10} \frac{|a_1|^2}{|b_1|^2}$$

$$= 10 \log_{10} \frac{1}{|S_{11}|^2}$$

$$= 20 \log_{10} \frac{1}{|S_{11}|} \quad \text{..... (4.27)}$$

S-MATRIX FOR E-PLANE TEE

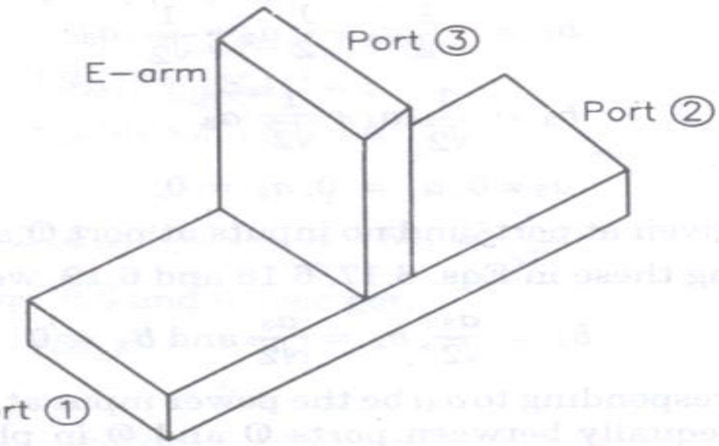


Fig.6.6 *E-plane Tee.*

When TE_{10} mode is made to propagate into port ③, the two outputs at port ① and ② will have a phase shift of 180° as shown in Fig. 6.7. Since the electric field lines change their direction when they come out of port ① and ②, it is called a E-plane Tee. E-plane Tee is a voltage or series junction symmetrical about the central arm. Hence any signals that is to be split or any two signal that are to be combined will be fed from the E arm.

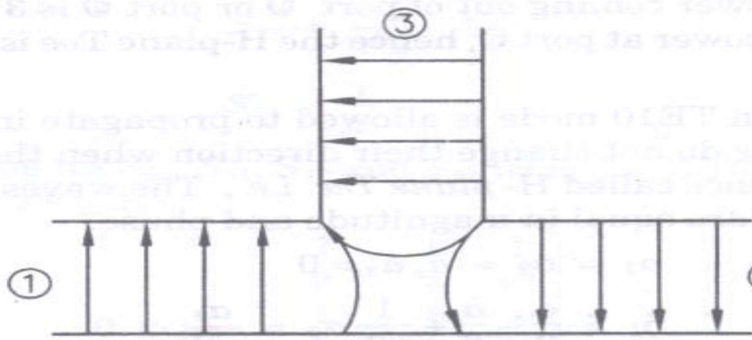


Fig. 6.7

The scattering matrix of an E-plane Tee can be used to describe its properties. In general, the power out of port ③ (side or E arm) is proportional to the difference between instantaneous powers entering from ports ① and ②.

Also, the effective value of the power leaving the E arm is proportional to the phasor difference between the powers entering ports ① and ②. When powers entering the main arms (ports ① and ② are in phase opposition, maximum energy comes out of port ③ or E-arm.

Since it is a three port junction the scattering matrix can be derived as follows.

Substituting the values from Eq. 6.29 to 6.31, the $[S]$ matrix of Eq. 6.23 becomes,

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \quad ...(6.32)$$

We know, (from Eq. 6.3)

$$[b] = [S]\,[a]$$

$$\therefore \quad \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad ...(6.33)$$

$$\therefore \quad b_1 = \frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \quad ...(6.34)$$

$$b_2 = \frac{1}{2}a_1 + \frac{1}{2}a_2 - \frac{1}{\sqrt{2}}a_3 \quad ...(6.35)$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2 \quad ...(6.36)$$

Case 1 : $a_1 = a_2 = 0,\ a_3 \neq 0$

$$b_1 = \frac{1}{\sqrt{2}}a_3\,;\ b_2 = -\frac{1}{\sqrt{2}}a_3\,;\ b_3 = 0$$

i.e., An input at port ③ equally divides between ① and ② but introduces a *phase shift* of 180° between the two outputs. Hence E-plane Tee also acts as a 3 dB splitter.

Case 2 : $a_1 = a_2 = a,\ a_3 = 0$

Substituting again in Eqs. 6.34 to 6.36, we get

$$b_1 = \frac{a}{2} + \frac{a}{2}\,;\ b_2 = \frac{a}{2} + \frac{a}{2}\,;\ b_3 = \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}a = 0$$

i.e., equal inputs at port ① and port ② result in no output at port ③

Case 3 : $a_1 \neq 0,\ a_2 = 0,\ a_3 = 0$

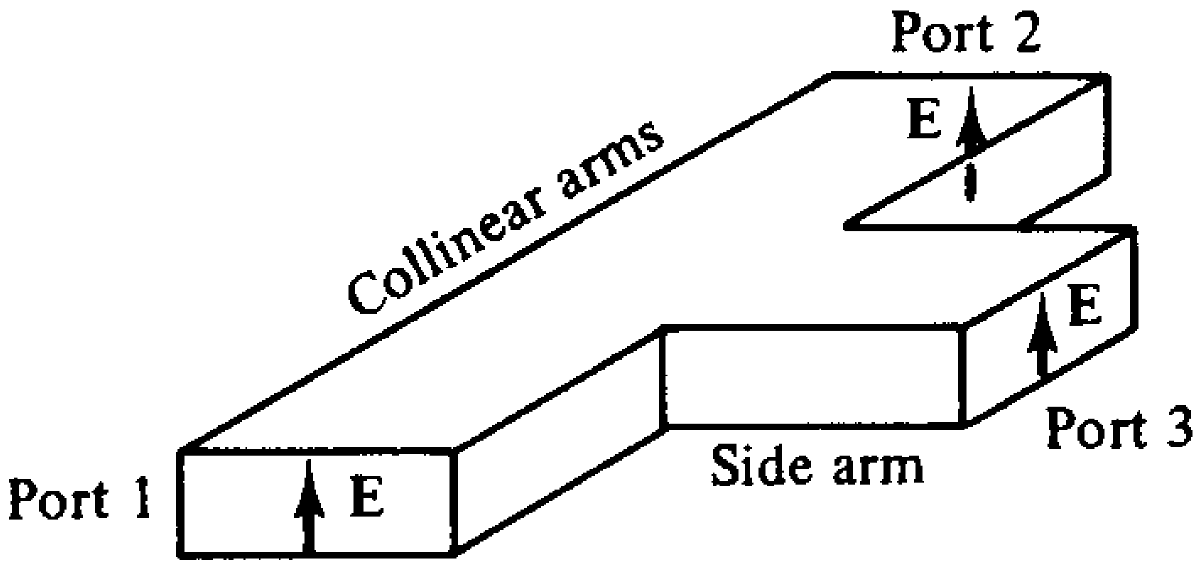
Hence, $b_1 = \frac{a_1}{2}\,;\ b_2 = \frac{a_1}{2}\,;\ b_3 = \frac{-a_1}{\sqrt{2}},$

Similarly we can have all combinations of inputs and outputs.

H-Plane Tee

Shunt tee

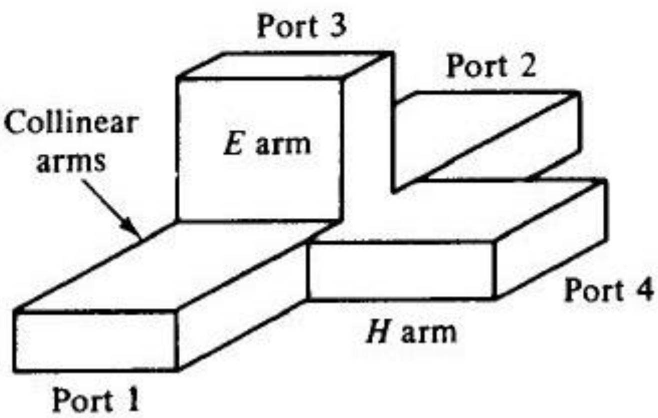
A waveguide tee in which the axis of its side arm is “shunting” the E-field or parallel to the H-field of the main guide.



If two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive. If the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in the same magnitude. Therefore the S matrix of H-plane tee is similar to Eplane tee except $S_{13} = S_{23}$

Magic Tee (Hybrid Tees)

A magic tee is a combination of E-plane and H-plane tee. The characteristics of magic tee are:



1. If two waves of equal magnitude and same phase are fed into port 1 and port 2
the output will be zero at port 3 and additive at port 4.
2. If a wave is fed into port 4 it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3.
3. If a wave is fed into port 3 , it will produce an output of equal magnitude and opposite phase at port 1 and port 2. the output at port 4 is zero.
4. If a wave is fed into one of the collinear arms at port 1 and port 2, it will not appear in the other collinear arm at port 2 or 1 because the E-arm causes a phase delay while H arm causes a phase advance.

Therefore the **S** matrix of a magic tee can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

1. $[S]$ is a 4×4 matrix since there are 4 ports

$$i.e.,\quad [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad ...(6.37)$$

2. Because of H-plane Tee section

$$S_{23} = S_{13} \quad ...(6.38)$$

3. Because of E-plane Tee section

$$S_{24} = -S_{14} \quad ...(6.39)$$

4. Because of geometry of the junction an input at port ③ cannot come out of port ④ since they are isolated ports and vice versa

$$\therefore \quad S_{34} = S_{43} = 0 \quad ...(6.40)$$

5. From symmetric property, $S_{ij} = S_{ji}$

$$S_{12} = S_{21}\,;\; S_{13} = S_{31}\,;\; S_{23} = S_{32}\,;$$

$$S_{34} = S_{43}\,;\; S_{24} = S_{42}\,;\; S_{41} = S_{14} \quad ...(6.41)$$

6. If ports ③ and ④ are perfectly matched to the junction.

$$S_{33} = S_{44} = 0 \quad ...(6.42)$$

Substituting the above properties from Eqs. 6.38 to 6.42 in Eq. 6.37, we get

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad ...(6.43)$$

7. From unitary property, $[S][S]^* = [I]$

$$i.e., \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1: \quad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad ...(6.44)$

$R_2C_2: \quad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad ...(6.45)$

$R_3C_3: \quad |S_{13}|^2 + |S_{13}|^2 = 1 \quad ...(6.46)$

$R_4C_4: \quad |S_{14}|^2 + |S_{14}|^2 = 1 \quad ...(6.47)$

From Eq. 6.46 and Eq. 6.47,

$$S_{13} = \frac{1}{\sqrt{2}} \quad ...(6.48)$$

$$S_{14} = \frac{1}{\sqrt{2}} \quad ...(6.49)$$

Comparing Eqs. 6.44 and 6.45, we get

$$S_{11} = S_{22} \qquad ...(6.50)$$

Using these values from Eqs. 6.48 and 6.49 in Eq. 6.44 we get,

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$\therefore \qquad |S_{11}|^2 + |S_{12}|^2 = 0$$

i.e.,

$$S_{11} = S_{12} = 0 \qquad ...(6.51)$$

$\therefore$ From Eq. 6.45,

$$S_{22} = 0 \qquad ...(6.52)$$

This means ports ① and ② are also perfectly matched to the junction. Hence in any four port junction, if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction. Such a junction where in all the four ports are perfectly matched to the junction is called a *Magic Tee*.

The [S] of Magic Tee is obtained by substituting the scattering parameters from Eqs. 6.48 to 6.52 in Eq. 6.43.

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \qquad ...(6.53)$$

We know that, [b] = [S] [a] (from Eq. 6.3)

i.e.,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\therefore \left.\begin{aligned} b_1 &= \frac{1}{\sqrt{2}}(a_3 + a_4)\,; \quad b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2) \\ b_2 &= \frac{1}{\sqrt{2}}(a_3 - a_4)\,; \quad b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2) \end{aligned}\right\} \qquad ...(6.54)$$

Using Eq. 6.54, we look at the properties of Magic Tee for some important cases.

***Case* 1 :** $a_3 \neq 0, a_1 = a_2 = a_4 = 0$

Substituting these in Eq. 6.54, we get,

$$b_1 = \frac{a_3}{\sqrt{2}} ; b_2 = \frac{a_3}{\sqrt{2}} ; b_3 = b_4 = 0$$

This is the property of H-plane Tee.

***Case* 2** : $a_4 \neq 0, a_1 = a_2 = a_3 = 0$

$$\therefore \quad b_1 = \frac{a_4}{\sqrt{2}} ; b_2 = -\frac{a_4}{\sqrt{2}} ; b_3 = b_4 = 0$$

This is the property of E-plane Tee.

***Case* 3** : $a_1 \neq 0 , a_2 = a_3 = a_4 = 0$

$$\therefore \quad b_1 = 0 ; b_2 = 0 ; b_3 = \frac{a_1}{\sqrt{2}} ; b_4 = \frac{a_1}{\sqrt{2}}$$

i.e., when power is fed into port ①, nothing comes out of port ② even though they are collinear ports (Magic !!). Hence ports ① and ② are called *isolated ports*. Similarly an input at port ② cannot come out at port ①. Similarly E and H ports are isolated ports.

***Case* 4** : $a_3 = a_4 , a_1 = a_2 = 0$

Then $$b_1 = \frac{1}{\sqrt{2}} (2 a_3) ; b_2 = 0 ; b_3 = b_4 = 0$$

This is nothing but the additive property. Equal inputs at ports ③ and ④ result in an output at port ① (in phase and equal in amplitude).

***Case* 5** : $a_1 = a_2 , a_3 = a_4 = 0 ;$

$$\therefore \quad b_1 = 0 = b_2 = b_4 ; b_3 = \frac{1}{\sqrt{2}} (2 a_1)$$

that is equal inputs at ports ① and ② results in an output at port ③ (additive property) and no outputs at ports ①, ② and ④. This is similar to case 4.

6.3.4 Applications of Magic Tee

A magic Tee has several applications. A few of them have been discussed here.

(*a*) ***Measurement of Impedance*** : A Magic Tee has been used in the form of a bridge, as shown in Fig. 6.9 for measuring impedance.

Microwave source is connected in arm ③ and a null detector in arm ④. The unknown impedance is connected in arm ② and a standard variable known impedance in arm ①. Using the properties of Magic Tee, the power from microwave source (a_3) gets divided equally between arms ① and ② $\left(\frac{a_3}{\sqrt{2}}\right)$ (to the unknown impedance and standard variable impedances). These impedances are not equal to characteristic impedance Z_o and hence there will be reflections from arms ① and ②. If ρ_1 and ρ_2 are the reflection coefficients, powers $\frac{\rho_1 a_3}{\sqrt{2}}$ and $\frac{\rho_2 a_3}{\sqrt{2}}$ enter the

$$b_1 = \frac{a_3}{\sqrt{2}}\ ;\ b_2 = \frac{a_3}{\sqrt{2}}\ ;\ b_3 = b_4 = 0$$

This is the property of H-plane Tee.

***Case* 2** : $a_4 \neq 0, a_1 = a_2 = a_3 = 0$

$$\therefore \quad b_1 = \frac{a_4}{\sqrt{2}}\ ;\ b_2 = -\frac{a_4}{\sqrt{2}}\ ;\ b_3 = b_4 = 0$$

This is the property of E-plane Tee.

***Case* 3** : $a_1 \neq 0\ ,\ a_2 = a_3 = a_4 = 0$

$$\therefore \quad b_1 = 0\ ;\ b_2 = 0\ ;\ b_3 = \frac{a_1}{\sqrt{2}}\ ;\ b_4 = \frac{a_1}{\sqrt{2}}$$

i.e., when power is fed into port ①, nothing comes out of port ② even though they are collinear ports (Magic !!). Hence ports ① and ② are called *isolated ports*. Similarly an input at port ② cannot come out at port ①. Similarly E and H ports are isolated ports.

***Case* 4** : $a_3 = a_4\ ,\ a_1 = a_2 = 0$

Then $b_1 = \frac{1}{\sqrt{2}}(2\,a_3)\ ;\ b_2 = 0\ ;\ b_3 = b_4 = 0$

This is nothing but the additive property. Equal inputs at ports ③ and ④ result in an output at port ① (in phase and equal in amplitude).

***Case* 5** : $a_1 = a_2\ ,\ a_3 = a_4 = 0\ ;$

$$\therefore \quad b_1 = 0 = b_2 = b_4\ ;\ b_3 = \frac{1}{\sqrt{2}}(2\,a_1)$$

that is equal inputs at ports ① and ② results in an output at port ③ (additive property) and no outputs at ports ①, ② and ④. This is similar to case 4.

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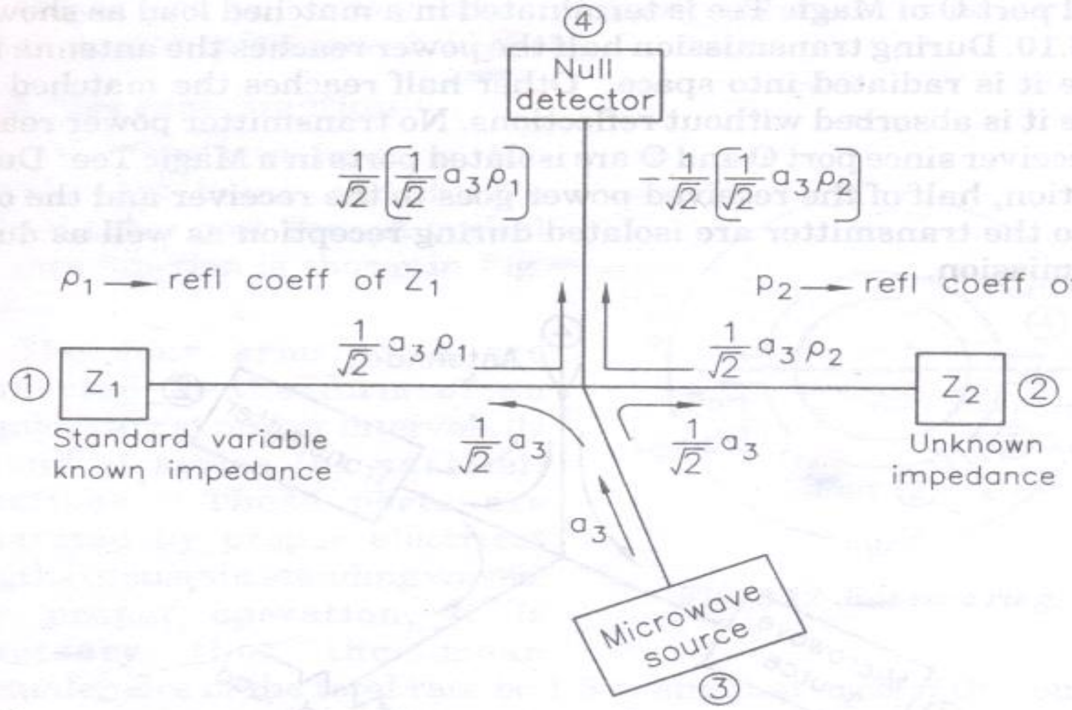

Fig. 6.9 *Magic Tee for measurement of impedances.*

Magic Tee junction from arms ① and ② as shown in Fig. 6.9. The resultant wave into arm ④ *i.e.*, the null detector can be calculated as follows :

The net wave reaching the null detector (Refer Fig. 6.9)

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}a_3\rho_1\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}a_3\rho_2\right) = \frac{1}{2}a_3(\rho_1 - \rho_2) \quad ...(6.55)$$

For perfect balancing of the bridge (null detection) Eq. 6.55 is equated to zero.

i.e., $$\frac{1}{2}a_3(\rho_1 - \rho_2) = 0$$

or $$\rho_1 - \rho_2 = 0 \text{ or } \rho_1 = \rho_2$$

or $$\frac{Z_1 - Z_z}{Z_1 + Z_z} = \frac{Z_2 - Z_z}{Z_2 + Z_z}$$

$\therefore$ $$Z_1 = Z_2$$

i.e., $$R_1 + jX_1 = R_2 + jX_2$$

or $$R_1 = R_2 \text{ and } X_1 = X_2.$$

Thus the unknown impedance can be measured by adjusting the standard variable impedance till the bridge is balanced and both impedances become equal.

(*b*) *Magic Tee as a Duplexer* : The transmitter and receiver are connected in ports ② and ① respectively, antenna in the E-arm or port

④ and port ③ of Magic Tee is terminated in a matched load as shown in Fig. 6.10. During transmission half the power reaches the antenna from where it is radiated into space. Other half reaches the matched load where it is absorbed without reflections. No transmitter power reaches the receiver since port ① and ② are isolated ports in a Magic Tee. During reception, half of the received power goes to the receiver and the other half to the transmitter are isolated during reception as well as during transmission.

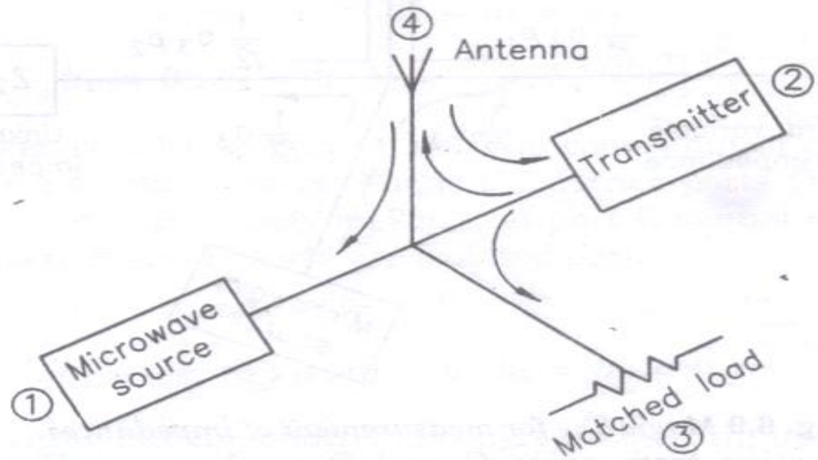

Fig. 6.10 *Magic Tee as a Duplexer.*

(c) *Magic Tee as a Mixer* : A Magic Tee can also be used in microwave receivers as a mixer where the signal and local oscillator are fed into the E and H arms as shown in Fig. 6.11

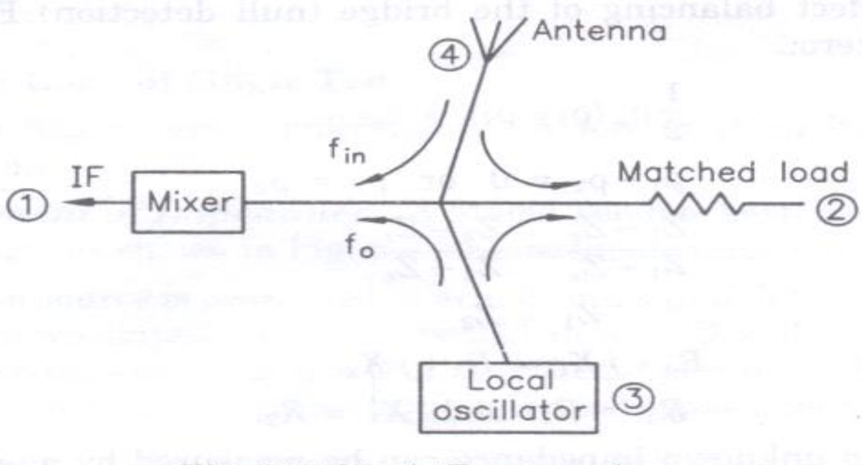

Fig. 6.11 *Magic Tee as a mixer.*

Half of the local oscillator power and half of the received power from antenna goes to the mixer where they are mixed to generate the *IF* frequency.

$$IF = f_{in} \sim f_0$$

Magic Tee has many other applications such as a microwave discriminator, microwave bridge etc.

6.3.5 Rat race junction

This is a four port junction, the fourth port being added to a normal three port Tee. A typical rat race junction is shown in Fig. 6.12.

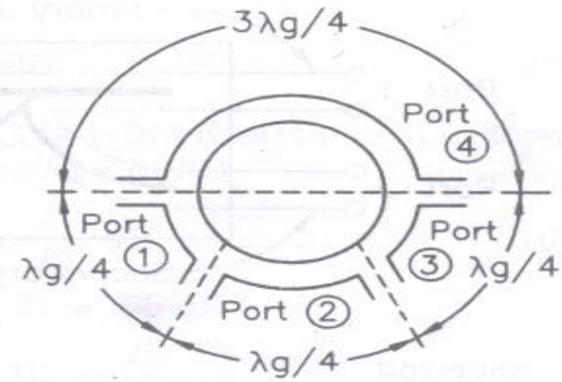

Fig. 6.12 *Rat-race ring.*

The four arms/ports are connected in the form of an angular ring at proper intervals by means of series (or parallel) junctions. These ports are separated by proper electrical lengths to sustain standing waves. For proper operation, it is necessary that the mean circumference of the total race be 1.5 λ_g and that each of the four ports be separated from its neighbour by a distance of $\lambda_g/4$.

When power is fed into port ① it splits equally (in clockwise and anti clockwise directions) into ports ② and ④ and nothing enters port ③. At ports ② and ④ the powers combine in phase but at port ③ cancellation occurs due to $\lambda_g/2$ path difference. For similar reasons any input applied at port ③ is equally divided between ports ② and ④ but the output at port ① will be zero. The rat race can also be used for combining two signals or dividing a single signal into two equal halves. If two unequal signals are applied at port ①, an output proportional to their sum will emerge from ports ② and ④ while a differential output will appear at port ③.

The scattering matrix of a rat race junction (also called hybrid junction) can be written as shown below in ideal conditions (*i.e.*, neglecting leakage coupling values).

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix} \qquad \text{...(6.56)}$$

6.4 DIRECTIONAL COUPLERS

Directional couplers are flanged, built in waveguide assemblies which can sample a small amount of microwave power for measurement purposes. They can be designed to measure incident and/or reflected powers, SWR (Standing Wave Ratio) values, provide a signal path to a receiver or perform other desirable operations. They can be undirectional (measuring only incident power) or bi-directional (measuring both incident and reflected) powers. In its most common form, the directional coupler is a four port waveguide junction consisting of a primary main waveguide and a secondary auxiliary waveguide as shown in Fig. 6.13a.

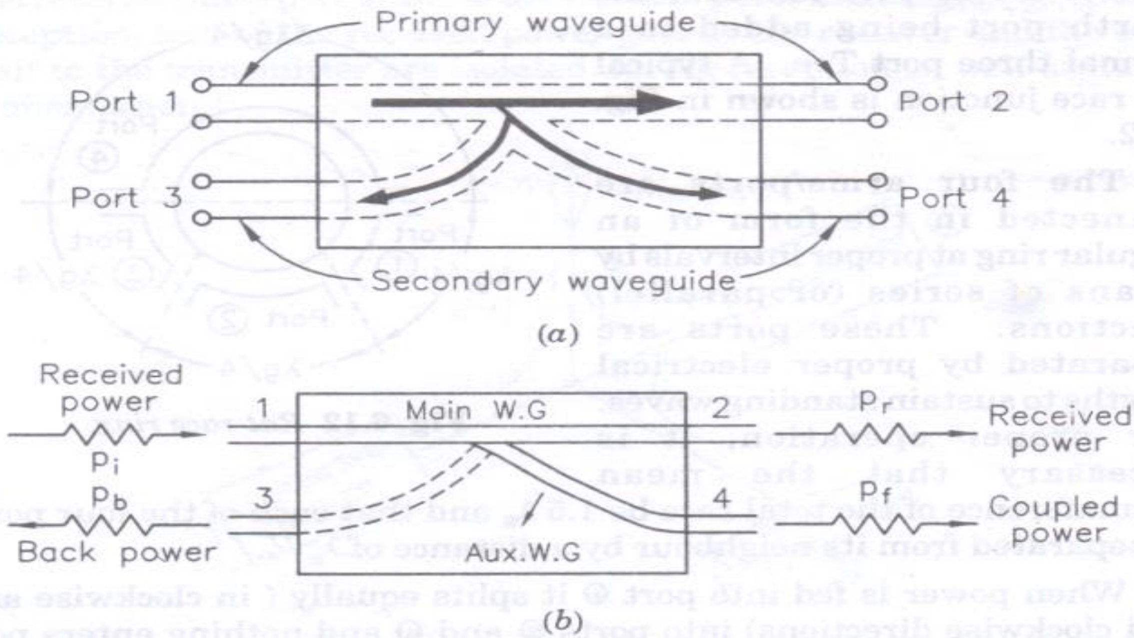

Fig. 6.13 *(a) A schematic of a directional coupler (b) Directional coupler indicating powers.*

With matched terminations at all its ports, the properties of an ideal directional coupler can be summarized as follows.

1. A portion of power travelling from port ① to port ② is coupled to port ④ but **not** to port ③.
2. A portion of power travelling from port ② to port ① is coupled to port ③ but **not**to port ④ (bidirectional case).
3. A portion of power incident on port ③ is coupled to port ② but **not** to port ① and a portion of the power incident on port ④ is coupled to port ① but **not** to port ②. Also ports ① and ③ are decoupted as are ports ② and ④.

A small portion of input power at port ① is coupled to port ④ so that measurement of this small power is possible. Ideally no power should come out of port ③. Fig. 6.13b indicates the various input/output powers.

P_i = incident power at port ①.

P_r = received power at port ②.

P_f = forward coupled power at port ④.

P_b = back power at port ③.

The performance of a directional coupler is usually defined in terms of two parameters which are defined as follows.

Coupling Factor C : The coupling factor of a directional coupler (D.C.) is defined as the ratio of the incident power 'P_i' to the forward power 'P_f' measured in dB.

i.e., $$C = 10 \log_{10} \frac{P_i}{P_f} \text{ dB} \quad \text{...(6.57)}$$

Directivity D : The directivity of a D.C. is defined as the ratio of forward power 'P_f' to the back power 'P_b' expressed in dB.

i.e., $$D = 10 \log_{10} \frac{P_f}{P_b} \text{ dB} \quad \text{...(6.58)}$$

For a typical D.C., $C = 20$ dB, $D = 60$ dB

i.e., $$C = 20 = 10 \log \frac{P_i}{P_f}$$

$$\therefore \quad \frac{P_i}{P_f} = 10^2 = 100$$

or $$P_f = \frac{P_i}{100}$$

Also, $$D = 60 = 10 \log \frac{P_f}{P_b}$$

$$\therefore \quad \frac{P_f}{P_b} = 10^6$$

or $$P_b = \frac{P_f}{10^6} = \frac{P_i}{10^8} \left(\text{since } P_f = \frac{P_i}{100}\right)$$

Since P_b is very small, $\left(\frac{1}{10^8}\right) P_i$, the power coming out of port ③ can be neglected.

The *Coupling factor* is a measure of how much of the incident power is being sampled while *directivity* is a measure of how well the directional coupler distinguishes between the forward and reverse traveling powers.

Isolation : Another parameter called *Isolation* is sometimes defined to describe the directive properties of a directional coupler. It is defined as the ratio of the incident power P_i to the back power P_b expressed in dB.

$$I = 10 \log_{10} \frac{P_i}{P_b} \text{ dB} \quad \text{...(6.59)}$$

It may be noted that isolation in dB equals coupling factor plus directivity.

In addition to the above parameters the *SWR*, frequency range and transmission loss are also specified for a directional coupler. Low *SWR* magnitude of the power coming out of 2 holes depends upon the dimension of the two holes. Since the distance between holes is $\lambda_g/4$, P_b is made '0' (since the incident power will have to travel a distance of $(\lambda_g/4 + \lambda_g/4)$ when it comes back from hole ② resulting in 180° phase shift. compared to incident power leakage through hole ① entering port ③).

The number of holes can be one (as in Bethe crossguide coupler) or more than two (as in a Multihole coupler). The degree of coupling is determined by size and location of the holes in the waveguide walls.

Although a high degree of directivity can be achieved at a fixed frequency, it is quite difficult over a band of frequencies. In this connection, it should be realized that the frequency determines the separation of the two holes as a fraction of the wavelength.

6.4.2 Bethe or Single-hole Coupler

A single-hole directional coupler is shown in Fig. 6.20. Here the directivity is improved as the Bethe coupler relies on a single hole for coupling process rather than the separation between two holes. The power entering port ① is coupled to the co-axial probe output and the power entering port ② in absorbed by the matched load. The auxiliary guide is placed at such an angle that the magnitude of the magnetically excited wave is made equal to that of the electrically excited wave for improved directivity. In this coupler, the waves in the auxiliary guide are generated through a single hole which includes both electric and magnetic fields. Because of the phase relationships involved in the coupling process, the signals generated by the two types of coupling cancel in the forward direction and reinforce in the reverse direction.

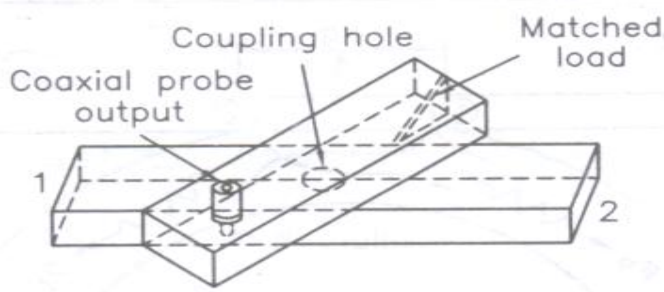

Fig. 6.20 *Bethe or single-hole coupler.*

6.4.3 Scattering Matrix of a Directional Coupler

We use the properties of the directional coupler to arrive at the [S] matrix.

1. Directional coupler is a four port network. Hence [S] is a 4×4 matrix

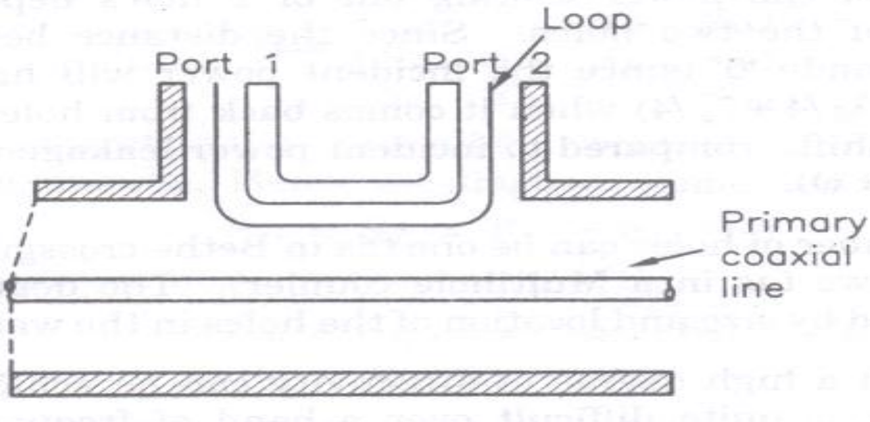

Fig. 6.18 *Loop directional coupler.*

It may be noted that in most of the directional couplers only three of the four ports are used, the unwanted port is normally terminated in a matched load built into it. The two waveguides (primary and secondary) share a common wall. This common wall has got hole or holes for coupling the energy flowing into the main waveguide to the side waveguide and hence called a side hole coupler. A two hole directional coupler is most commonly used.

6.4.1 Two-hole Directional Coupler

The principle of operation of a two-hole directional coupler is shown in Fig. 6.19. It consists of two guides the main and the auxiliary with two tiny holes common between them as shown. The two holes are at a distance of $\lambda_g/4$ where λ_g is the guide wavelength.

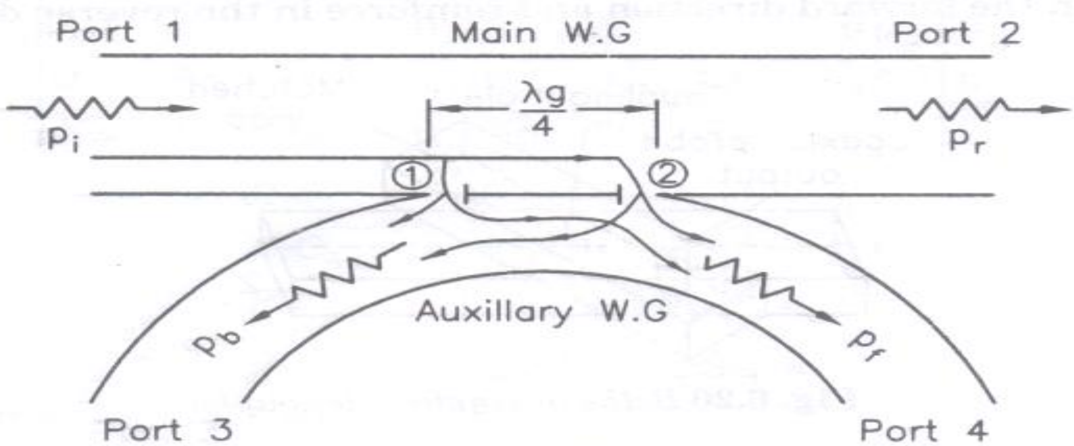

Fig. 6.19 *Two hole directional coupler.*

The two leakages out of holes ① and ② both in phase at the position of 2nd hole and hence they add up contributing to P_f. But the two leakages are out of phase by 180° at the position of the 1st hole and therefore they cancel each other making $P_b = 0$ (ideally). The i.e., $$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad ...(6.60)$$

2. In a directional coupler all four ports are perfectly matched to the junction. Hence the diagonal elements are zero

i.e., $$S_{11} = S_{22} = S_{33} = S_{44} = 0 \quad ...(6.61)$$

3. From symmetric property, $S_{ij} = S_{ji}$

$$\therefore \quad S_{23} = S_{32}\,;\ S_{13} = S_{31}\,;\ S_{24} = S_{42}\,;$$

$$S_{34} = S_{43}\,;\ S_{41} = S_{14}\,; \quad ...(6.62)$$

Ideally back power is zero ($P_b = 0$) i.e., There is no coupling between port ① and port ③.

$$\therefore \quad S_{13} = S_{31} = 0 \quad ...(6.63)$$

4. Also there is no coupling between port ② and port ④

$$\therefore \quad S_{24} = S_{42} = 0 \quad ...(6.64)$$

Substituting in Eq. 6.55, the values of scattering parameters as per Eqs. 6.56 to 6.59, we get,

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \quad ...(6.65)$$

5. Since $[S][S^*] = I$, we get,

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1: \quad |S_{12}|^2 + |S_{14}|^2 = 1 \quad ...(6.66)$$

$$R_2C_2: \quad |S_{12}|^2 + |S_{23}|^2 = 1 \quad ...(6.67)$$

$$R_3C_3: \quad |S_{23}|^2 + |S_{34}|^2 = 1 \quad ...(6.68)$$

$$R_1C_3: \quad S_{12}\,S_{23}^* + S_{14}\,S_{34}^* = 0 \quad ...(6.69)$$

Comparing Eq. 6.61 and 6.62,

$$S_{14} = S_{23} \quad ...(6.70)$$

Comparing Eq. 6.62 and 6.63,

$$S_{12} = S_{34} \quad ...(6.71)$$

Let us assume that S_{12} is real and positive = 'P'

$$\therefore \quad S_{12} = S_{34} = P = S_{34}^* \quad ...(6.72)$$

From Eqs. 6.69 and 6.72,

$$P \quad S_{23}^* + S_{23}\, P \; = \; 0$$

$$\therefore \qquad P \quad [S_{23} + S_{23}^*] \; = \; 0$$

Since, $\quad P \neq 0,\; S_{23} + S_{23}^* \; = \; 0$

$$S_{23} \; = \; jy$$

$$S_{23}^* \; = \; -jy$$

i.e., S_{23} must be imaginary.

Let $\quad S_{23} \; = \; jq \; = \; S_{14}$ ...(6.73)

Therefore,

$$S_{12} \; = \; S_{34} \; = \; P \qquad \text{(transmission parameter)}$$

and $\quad S_{23} \; = \; S_{14} \; = \; jq. \quad$ Also, $P^2 + q^2 = 1$

Substituting these values in Eq. 6.65, [S] matrix of a directional coupler is reduced to

$$[S] \; = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix} \qquad \text{...(6.74)}$$