SYSTEM CONCEPTS

System:

A number of elements or components are connected in a sequence to perform a specific function, the group thus form is called system.

Control system:

In a system when the o/p quantity is controlled by varying the i/p quantity the system is called control system.

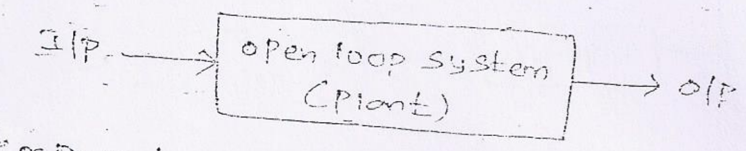
o/p quantity → Control variable (or) response

i/p quantity → Command signal (or) excitation

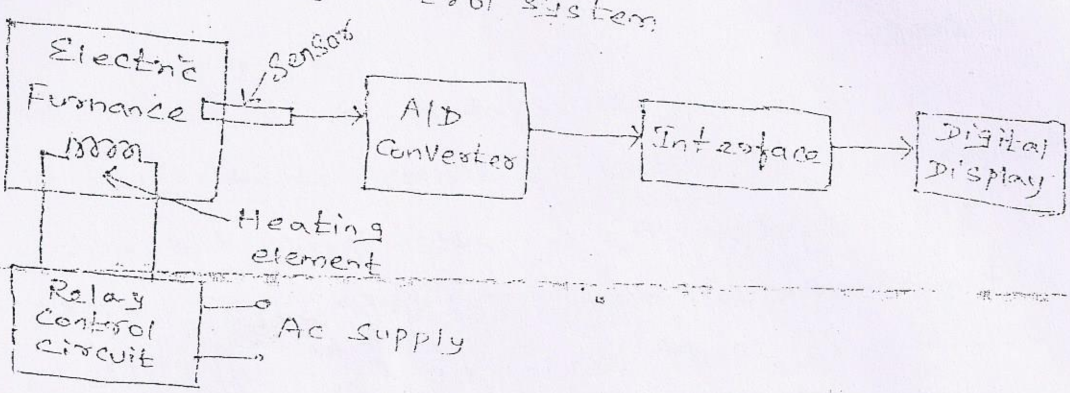
Open loop system

Any physical system which does not automatically correct the variation in its o/p is called open loop system.

In open loop system the o/p can be varied by varying the i/p.



eg) Temperature control system



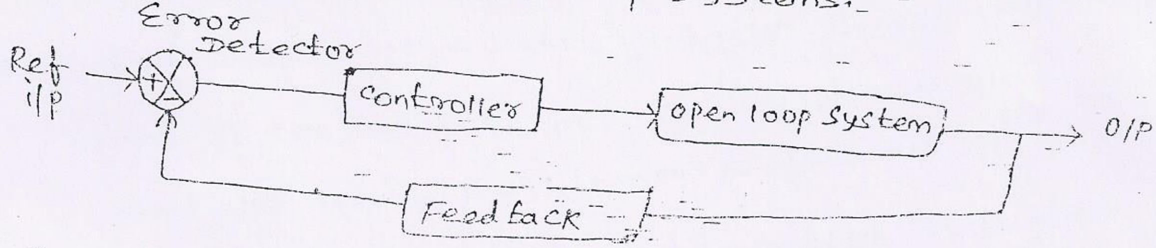
The o/p in the system is the desired temperature. The temperature of the system is raised by heats generated by the heating elements. The o/p depends on the time when heater remains ON.

The ON & OFF of the supply is governed by the setting of a relay. The temperature is measured by sensor which are signal is converted to digital by A/D Converter. Then the signal is given to display.

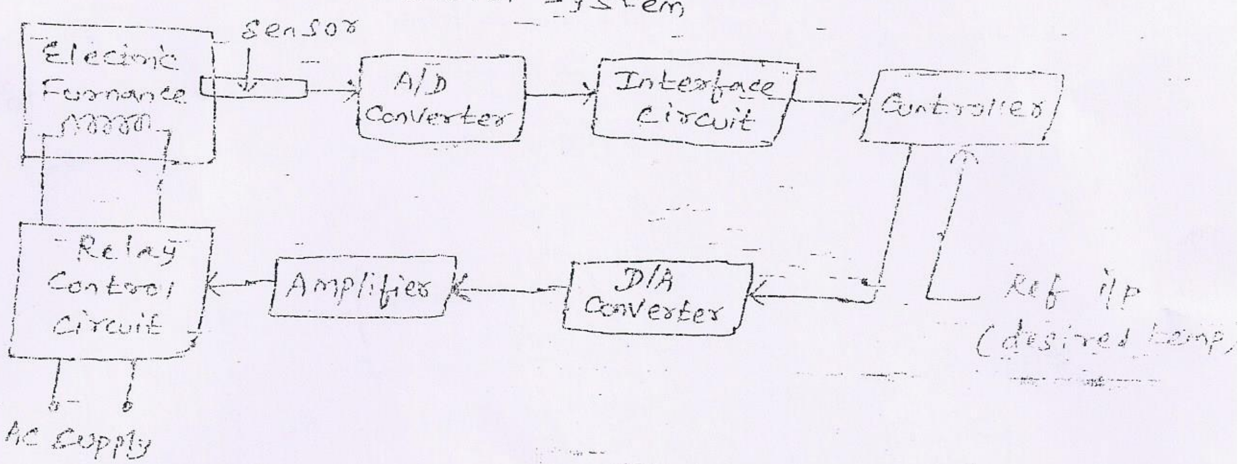
In this system if there is any change in o/p temperat then the time setting of the relay is not altered autom

Closed loop system

Control systems in which the o/p has an effect upon i/p quantity in such a manner as to maintain the desired value are called closed loop systems.



eg: Temperature control system



The desired temperature is i/p to the system. The ac temperature is sensed by sensor and converted to digital sig by A/D converter. The computer reads the actual temperatur and compares with desired temperature. If it [illegible] ence then it sends signal to switch on or OFF the relay throug D/A converter and amplifier. Thus the system is automatical corrects any changes in o/p. Hence it is a closed loop syste

Open loop System	Closed loop system
i) Inaccurate & unreliable	Accurate & reliable
ii) Simple & economical	Complex & Costlier

(iii) [illegible]

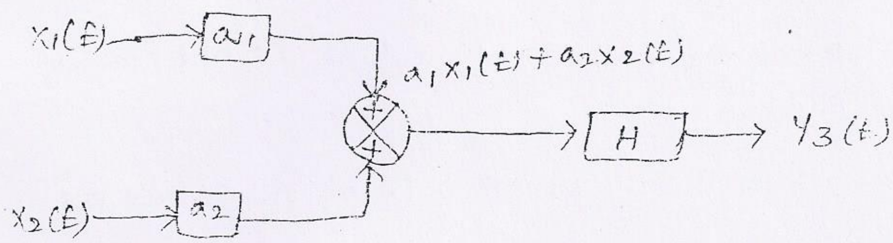
to external disturbances are not corrected automatically.	automatically.
iii) They are generally stable	Great efforts are needed to design a stable system.

Mathematical models of control systems

The mathematical model of a system is linear if it obeys the principle of superposition and homogenity.

Let H be a linear system

$X_1(t) \rightarrow \boxed{H} \rightarrow Y_1(t)$ $\quad$ $X_2(t) \rightarrow \boxed{H} \rightarrow Y_2(t)$



For linearity $Y_3(t) = a_1 y_1(t) + a_2 y_2(t)$

Transfer function

Transfer function of a system is defined as the ratio of laplace transform of o/p to the laplace transform of i/p with zero initial conditions.

$$\text{Transfer function} = \left.\frac{\text{Laplace transform of o/p}}{\text{Laplace transform of i/p}}\right|_{\text{with zero initial conditio}}$$

Signal flow graph

SFG is used to represent the control system graphically
SFG & block diagram reduction technique gets the same information
Using SFG overall system gain can be computed easily
Simple than Block dia red tech and gives relation among the signals.
Signal flows in only one direction and is indicated by arro
SFG will be time consuming.

Mason's Gain Formula

The mason's gain formula which can be directly used to find the T.F of the system.

$$\text{overall gain, } T = \frac{1}{\Delta} \Sigma_K P_K \Delta K$$

where $T = T(s)$ = T.F of the system

P_K = Forward Path gain of K^{th} forward path

K = Number of forward path in the SFG

Δ = 1 − (sum of individual loop gains) + (sum of gain products of all possible combinations of two non-touching loops) − (sum of gain products of all possible combinations of three non-touching loops) + ...

ΔK = Δ for that part of the graph which is not touching K^{th} forward path.

Nodes: Two or more branches meeting at a Point is called as nodes

Branch: Line segments connected in between two nodes. Arrow indicates the direction of SFG.

Path: Should not cross the nodes more than once

Loop: Loop is a closed path where its starts & ends at the same points.

Procedure to solve SFG

(i) Find the number of Forward Paths (K) and calculate the gain

(ii) Find the individual loops and calculate the gain

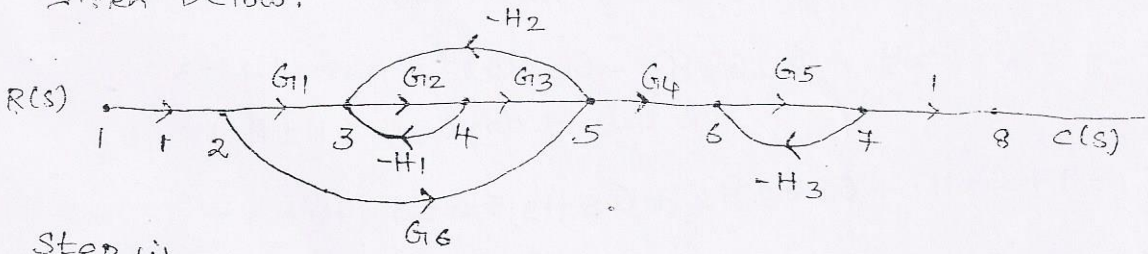
(iii) Find the two non touching loops & calculate gain (compare two individual loop and they should not have same nodes)

(iv) Find T.F $= \frac{C(s)}{R(s)} = \frac{1}{\Delta} \Sigma_K P_K \Delta K$

Calculate Δ

Calculate ΔK

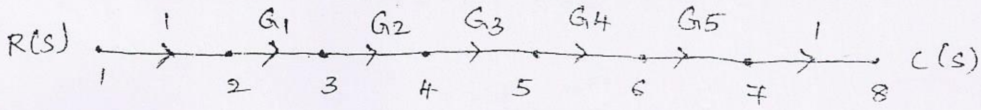
Find the overall T.f of the system whose signal flow graph given below.



Step (i) Find the no of forward Path

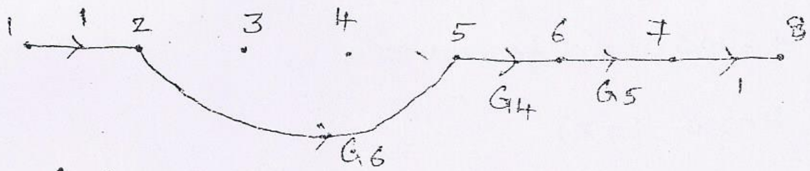
$K = 2$

Forward path 1



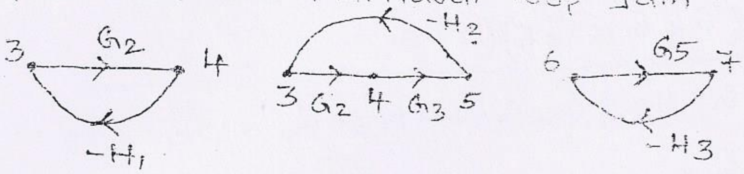
Gain for forward path 1, $P_1 = G_1 G_2 G_3 G_4 G_5$

Forward Path 2



Gain for forward Path 2, $P_2 = G_4 G_5 G_6$

Step (ii) Find the individual loop gain

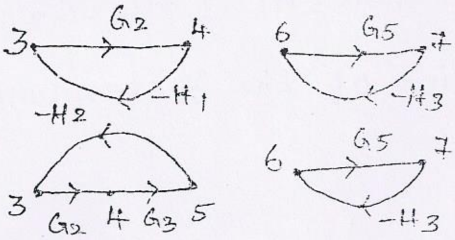


Individual loop gain of loop 1 $P_{11} = -G_2 H_1$

" loop 2 $P_{21} = -G_2 G_3 H_2$

" loop 3 $P_{31} = -G_5 H_3$

Step (iii) Find the gain Products of two non touching loops



Gain product of first combination of two non touching loops } $P_{12} = (-G_2 H_1)(-G_5 H_3)$

$= G_2 G_5 H_1 H_3$

Gain Product of Second combination of two non touching loops } $P_{22} = (-G_2 G_3 H_2)(-G_5 H_3)$

$= G_2 G_3 G_5 H_2 H_3$

step (iv)

Calculate $\Delta = 1-(P_{11}+P_{21}+P_{31})+(P_{12}+P_{22})$

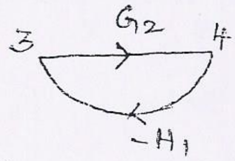
$$= 1-(-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

Calculate Δ_K when $K=2$

$\Delta_1 = 1$ (Since there is no part of the graph which is not touching with first forward path)

$\Delta_2 = 1 - P_{11}$



$= 1-(-G_2H_1)$

$= 1+G_2H_1$.

Find the T.F by using Mason's gain formula

$$T = \frac{1}{\Delta}\sum_K P_K \Delta_K \quad (\text{Here } K=2)$$

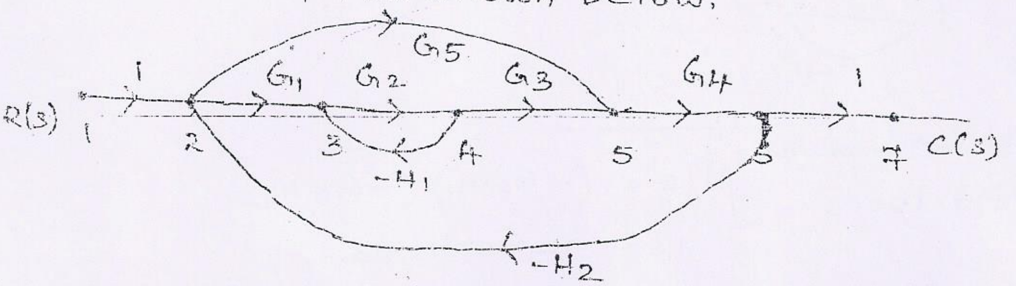
$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

$$= \frac{(G_1G_2G_3G_4G_5) + (G_4G_5G_6)(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_4G_5(G_1G_2G_3 + G_6 + G_2G_6H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

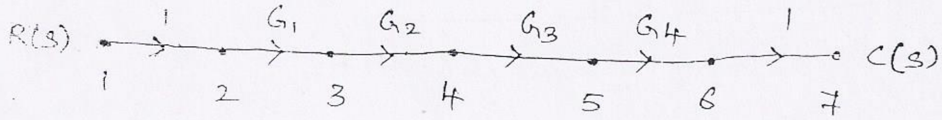
2. Find the overall transfer function of the system whose signa flow graph is shown below.



step (i) Find the no of forward path

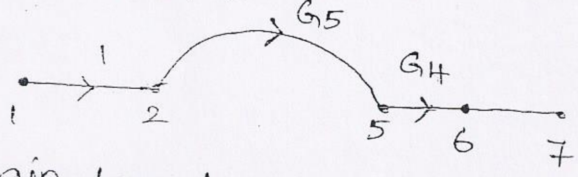
No of forward path $K=2$

Forward Path 1



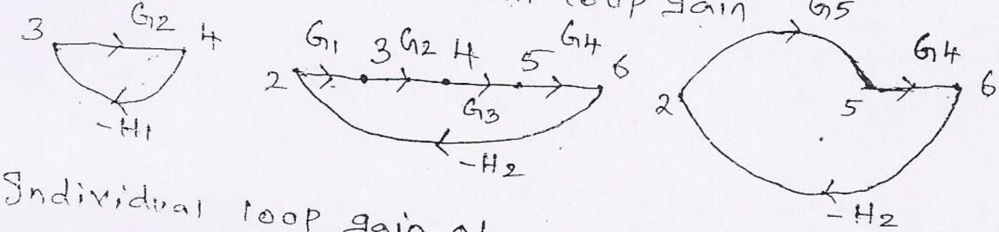
gain for forward path 1 $P_1 = G_1 G_2 G_3 G_4$

Forward Path 2



gain for forward path 2 $P_2 = G_4 G_5$

Step (ii) Find the individual loop gain

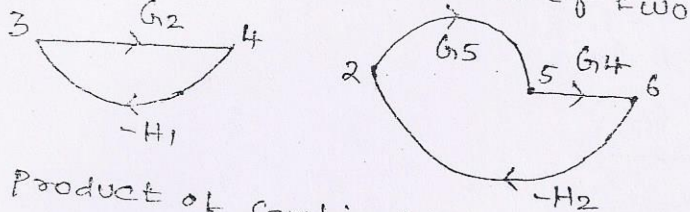


Individual loop gain of loop 1 $P_{11} = -G_2 H_1$

" loop 2 $P_{21} = -G_1 G_2 G_3 G_4 H_2$

" loop 3 $P_{31} = -G_4 G_5 H_2$

Step (iii) Find the gain product of two non touching loops



gain product of combination of two non touching loops $\Big\} = (-G_2 H_1)(-G_4 G_5 H_2)$

$P_{12} = G_2 G_4 G_5 H_1 H_2$

Step (iv)

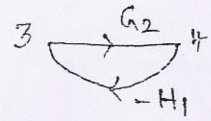
Calculate $\Delta = 1 - (-G_2 H_1 - G_1 G_2 G_3 G_4 H_2 - G_4 G_5 H_2) + (G_2 G_4 G_5 H_1 H_2)$

$= 1 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_4 G_5 H_2 + G_2 G_4 G_5 H_1 H_2$

Calculate ΔK ($K=2$)

$\Delta_1 = 1$ ($\because$ All the loops are touching the first forward path)

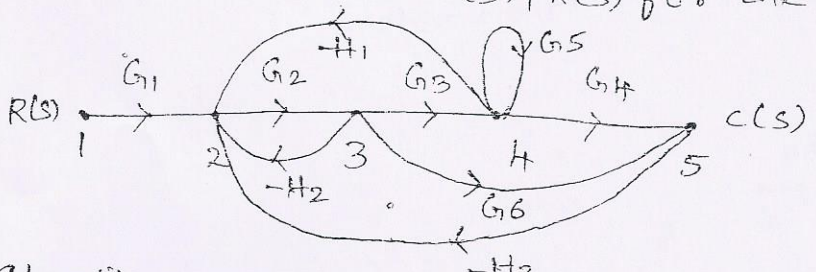
$\Delta_2 = 1 - (-G_2 H_1) = 1 + G_2 H_1$



$$T.F = \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 \cdot 1 + G_4 G_5 (1 + G_2 H_1)}{1 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_4 G_5 H_2 + G_2 G_4 G_5 H_1 H_2}$$

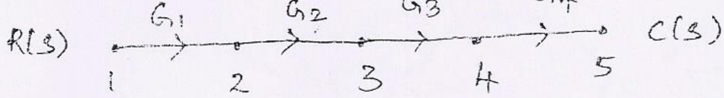
$$= \frac{G_1G_2G_3G_4 + G_4G_5 + G_2G_4G_5H_1}{1+G_2H_1 + G_1G_2G_3G_4H_2 + G_4G_5H_2 + G_2G_4G_5H_1H_2}$$

3. Find the overall gain $C(s)/R(s)$ for the signal flow graph

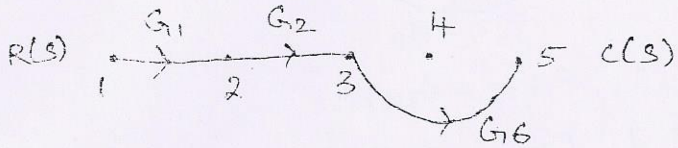


Step (i) Find Forward path

Forward path 1



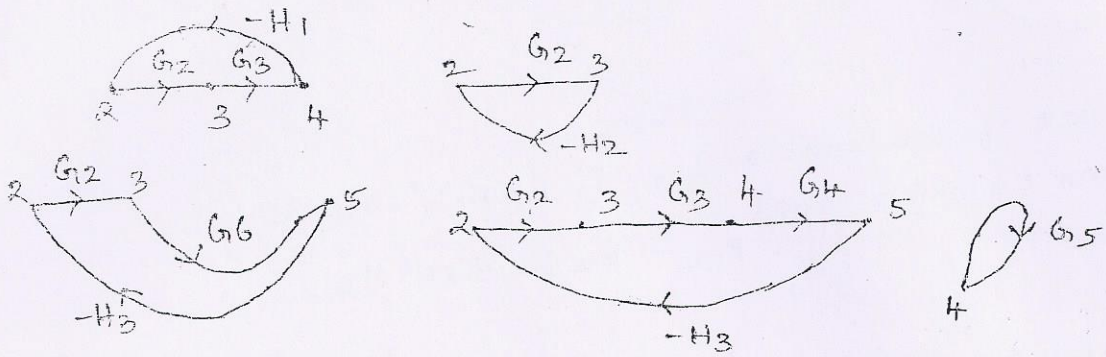
Forward Path 2



gain of forward path 1 $P_1 = G_1G_2G_3G_4$

" Path 2 $P_2 = G_1G_2G_6$

Step (ii) Find individual loop gain



loop gain of individual loop 1 $P_{11} = -G_2G_3H_1$

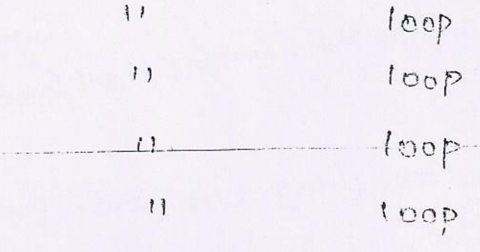
" loop 2 $P_{21} = -G_2H_2$

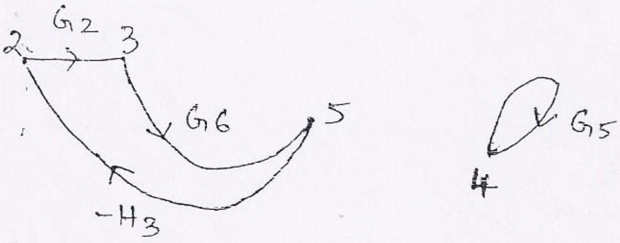
" loop 3 $P_{31} = -G_2G_6H_3$

" loop 4 $P_{41} = -G_2G_3G_4H_3$

" loop 5 $P_{51} = G_5$

Step (iii) gain Products of two non touching loops





Gain Product of first combination of two non touching loops } $P_{12} = (-G_2H_2)G_5$
$= -G_2G_5H_2$

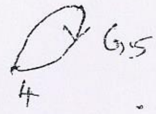
Gain Product of second combination of two non touching loops } $P_{22} = (-G_2G_6H_3)G_5$
$= -G_2G_5G_6H_3$

Step (iv) Calculation of Δ & ΔK

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41}+P_{51})+(P_{12}+P_{22})$$
$$= 1-(-G_2G_3H_1 - H_2G_2 - G_2G_3G_4H_3 + G_5 - G_2G_6H_3) + (-G_2H_2G_5 - G_2G_5G_6H_3)$$

$\Delta_1 = 1$ (since there is no part of graph which is not touching forward path 1)

$\Delta_2 = 1 - G_5$



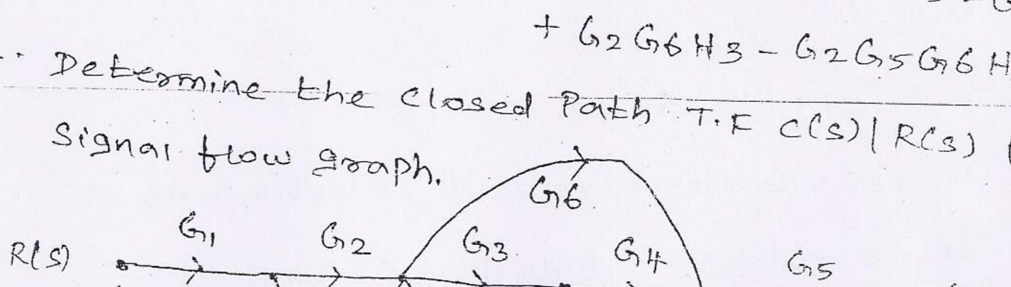
Step (v) Transfer fn, T

$$T = \frac{1}{\Delta} \Sigma_K P_K \Delta_K$$

$\because K=2$

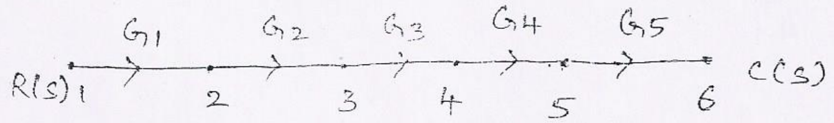
$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$
$$= \frac{1}{\Delta}(G_1G_2G_3G_4(1) + G_1G_2G_6(1-G_5))$$
$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1 + G_2G_3H_1 + H_2G_2 + G_2G_3G_4H_3 - G_5 - G_2H_2G_5 + G_2G_6H_3 - G_2G_5G_6H_3}$$

4. Determine the closed path T.F $C(s)/R(s)$ for the given signal flow graph.



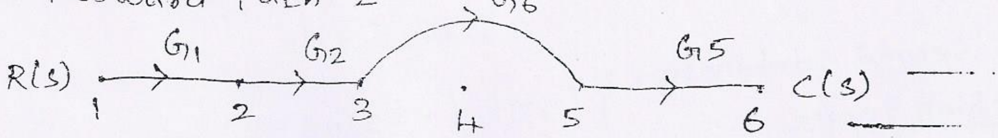
(i) Forward Path gain $K = 2$

Forward path 1



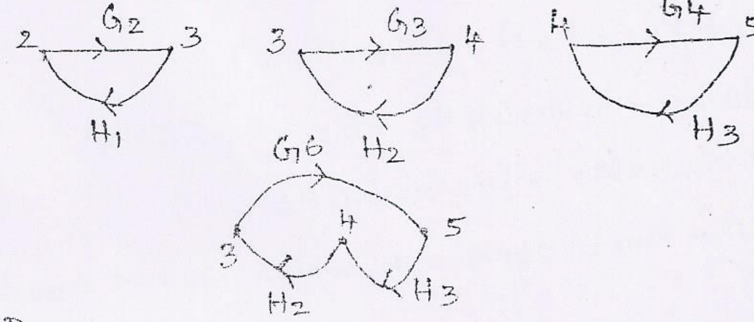
Forward Path 1 gain $P_1 = G_1 G_2 G_3 G_4 G_5$

Forward Path 2



Forward Path 2 gain $P_2 = G_1 G_2 G_6 G_5$

(ii) Individual loop gain



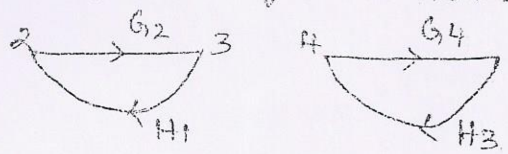
$P_{11} = G_2 H_1$

$P_{21} = G_3 H_2$

$P_{31} = G_4 H_3$

$P_{41} = G_6 H_2 H_3$

(iii) Gain Products of two non touching loops



$P_{12} = (G_2 H_1)(G_4 H_3)$

$= G_2 G_4 H_1 H_3$

(iv) calculation of Δ & ΔK

$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41}) + P_{12}$

$= 1 - (G_2 H_1 + G_3 H_2 + G_4 H_3 + G_6 H_2 H_3) + G_2 G_4 H_1 H_3$

$= 1 - G_2 H_1 - G_3 H_2 - G_4 H_3 - G_6 H_2 H_3 + G_2 G_4 H_1 H_3$

Since there is no part of graph which is non touching with Forward Path 1 & 2

$\therefore \Delta_1 = \Delta_2 = 1$

(v) Transfer function, T

By Mason's gain Formula

$$T = \frac{1}{\Delta} \Sigma_K P_K \Delta_K \quad (\text{Here } K = 2)$$

$$= \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2)$$

$$= \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_2 G_5 G_6}{1 - G_2 H_1 - G_3 H_2 - G_4 H_3 - G_6 H_2 H_3 + G_2 G_4 H_1 H_3}$$

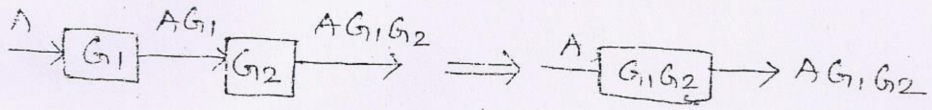
Block Diagram Reduction

Block diagram can be used to find the overall t/f function of the system.

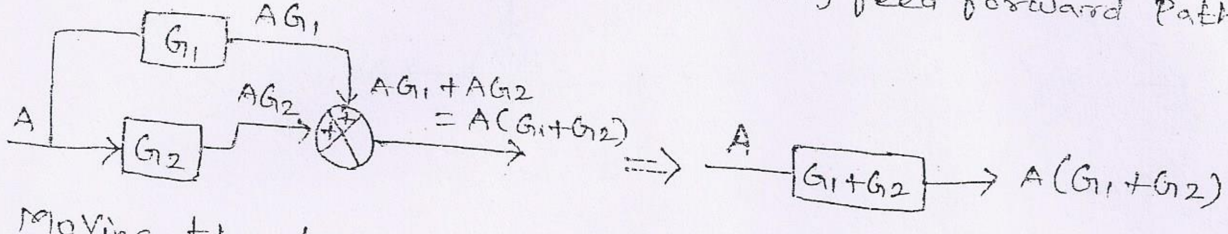
The rules are framed such that any modification made on the diagram does not alter the input output relation.

Rules of block diagram Algebra

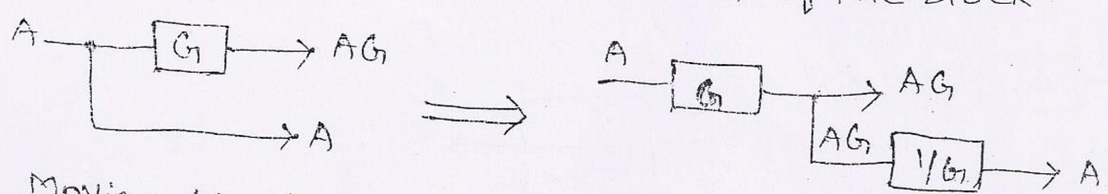
1. Combining the blocks in cascade



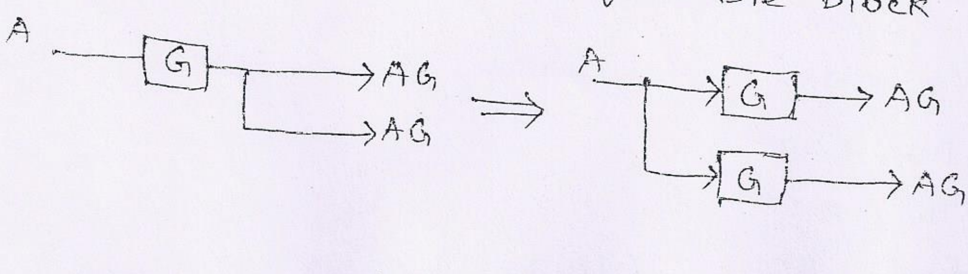
2. Combining Parallel blocks (or) combining feed forward Paths



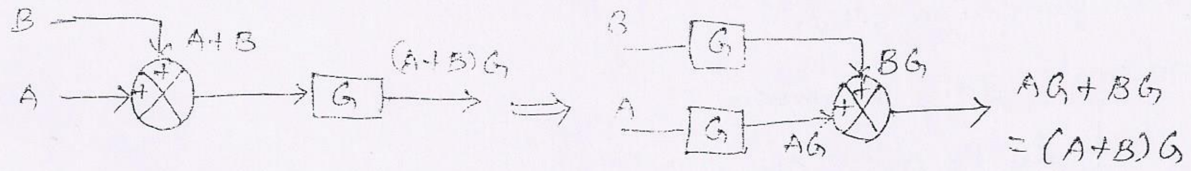
3. Moving the branch Point ahead of the block



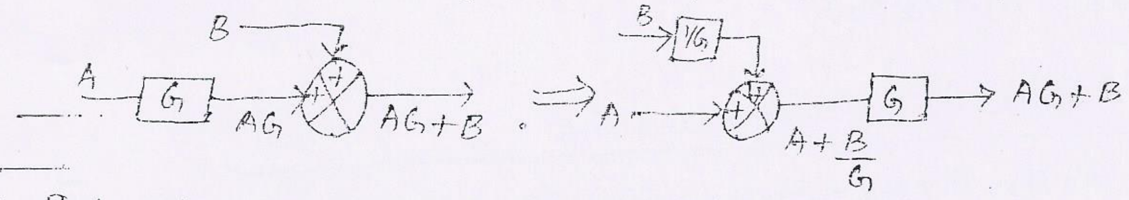
4. Moving the branch Point before the block



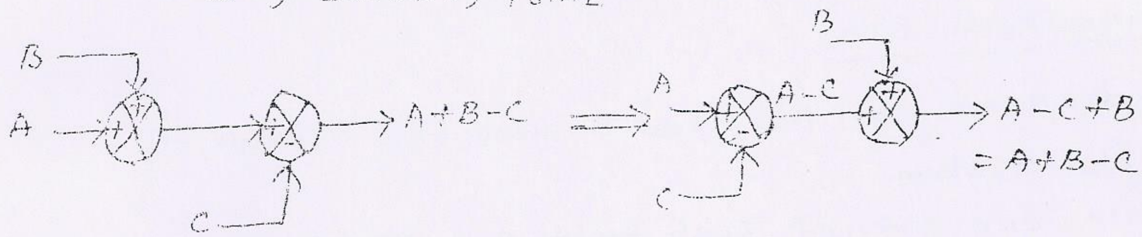
5. Moving the summing point ahead of the block



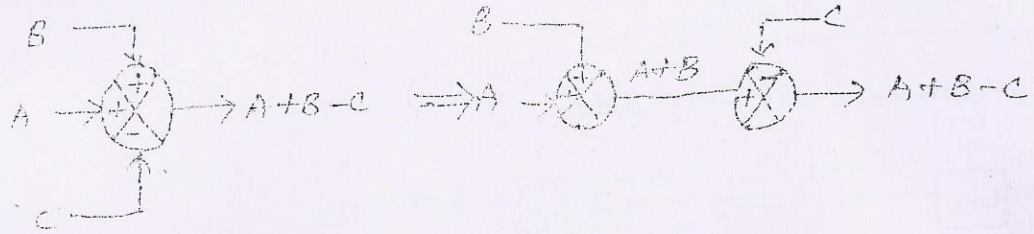
6. Moving the summing point before the block



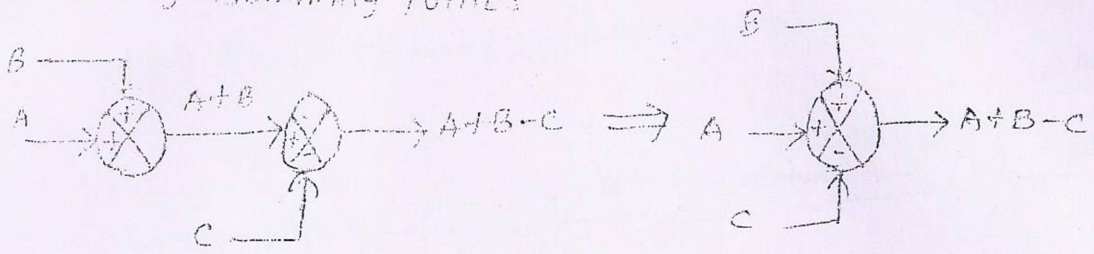
7. Interchanging summing point



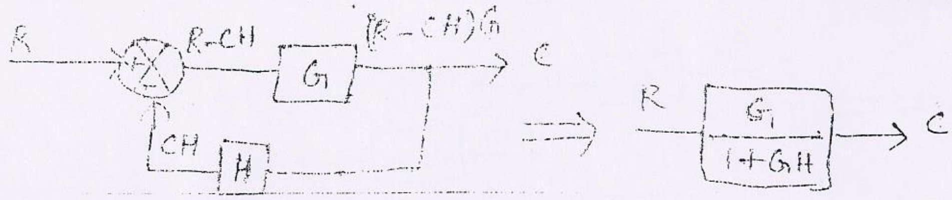
8. Splitting summing points



9. Combining summing points



10. Elimination of feedback loop



Proof

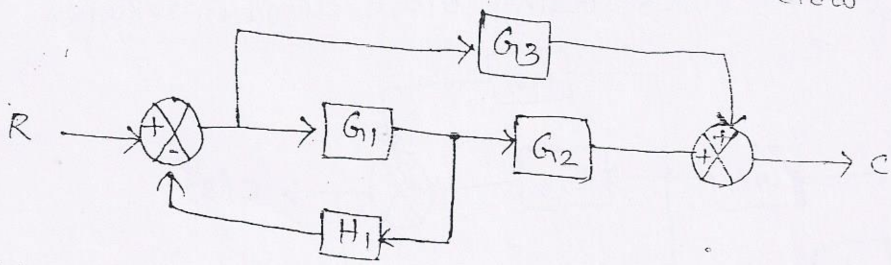
$$C = (R - CH)G$$

$$C = RG - CHG$$

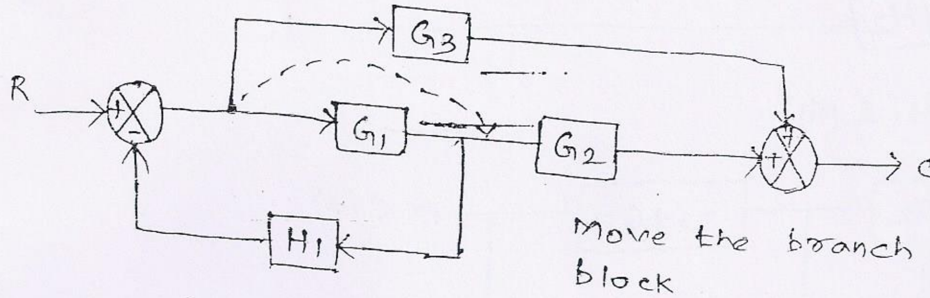
$$C + CHG = RG$$

$$C(1 + HG) = RG \qquad C/R = \frac{G}{1+HG}$$

1. Reduce the block diagram shown below & find C/R

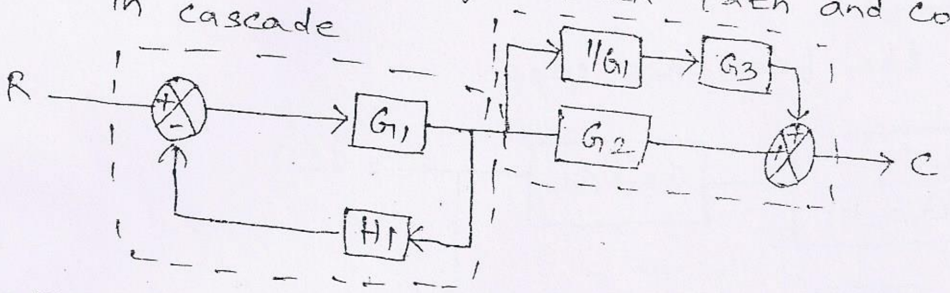


Step (i)

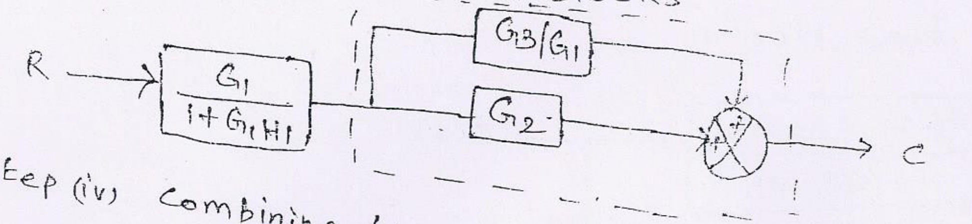


Move the branch point after the block

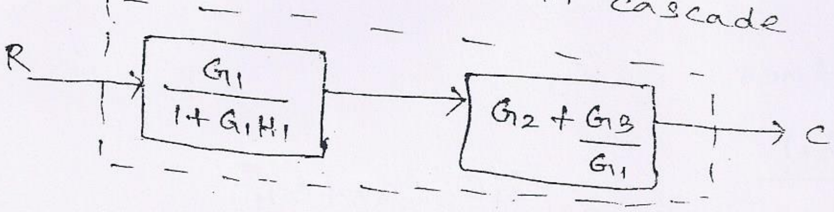
Step (ii) Eliminate the feedback path and combining blocks in cascade



Step (iii) Combining Parallel blocks



Step (iv) Combining blocks in cascade



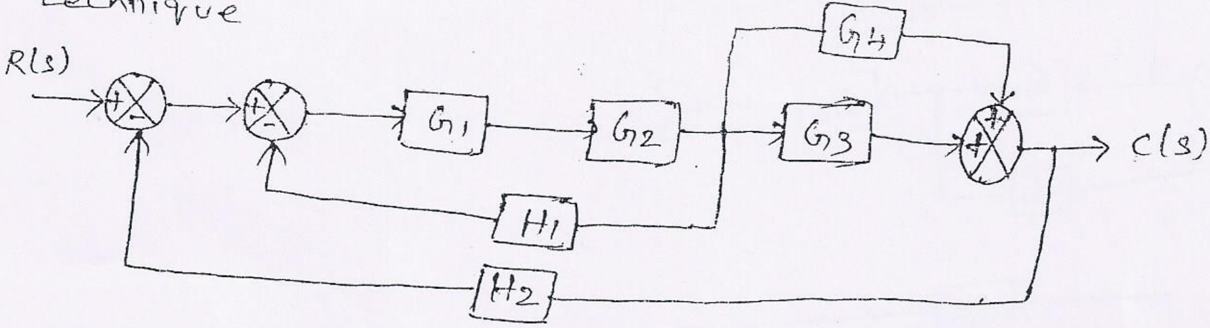
$$C/R = \left(\frac{G_1}{1+G_1H_1}\right)\left(G_2 + \frac{G_3}{G_1}\right)$$

$$= \left(\frac{\cancel{G_1}}{1+G_1H_1}\right)\left(\frac{G_2G_1 + G_3}{\cancel{G_1}}\right)$$

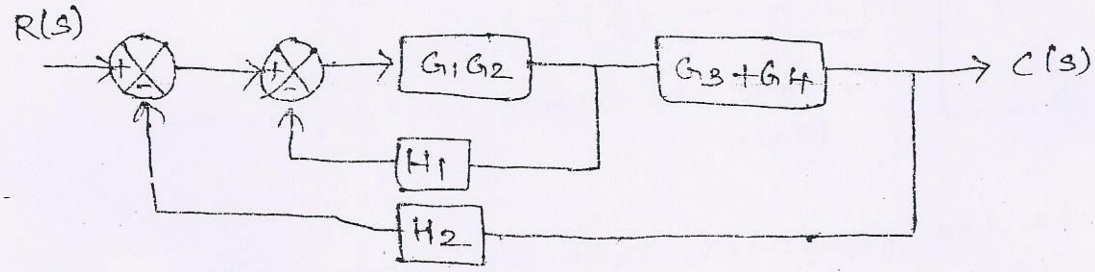
$$= \frac{G_1G_2 + G_3}{1+G_1H_1}$$

The overall transfer function of the $\frac{C}{} = \frac{G_1G_2 + G_3}{}$

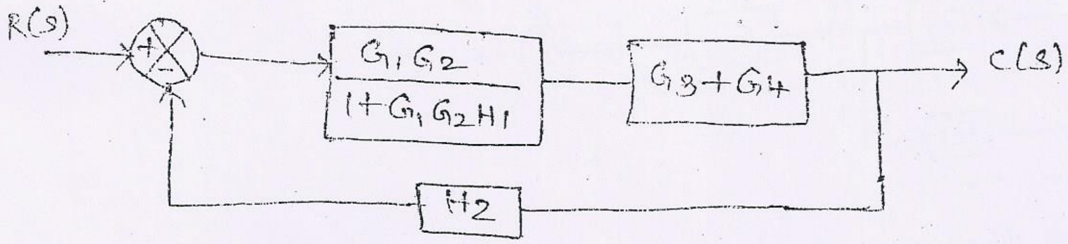
2. Find the T.f of the given block using block diagram reduction technique



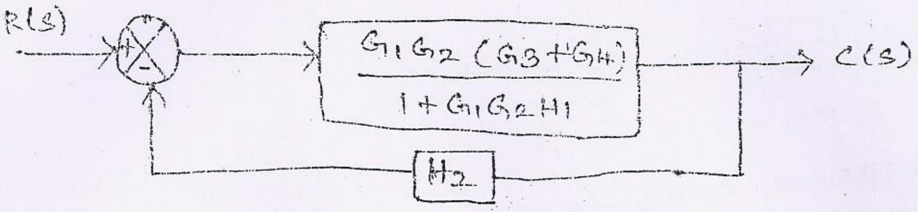
Step (i) Apply rule (i) & (ii)



Step (ii) Eliminate the feed back loop



Step (iii) Cascading two blocks

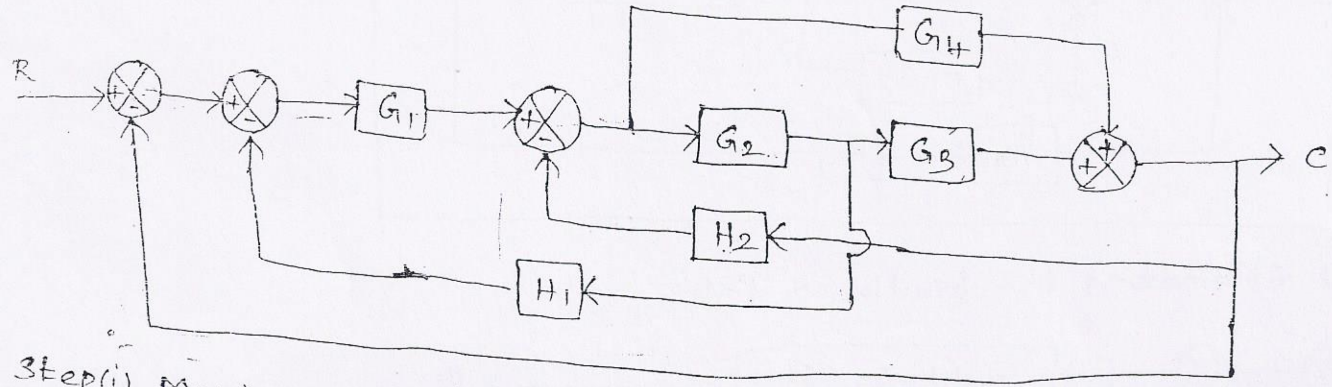


Step (iv) Eliminating Feed back

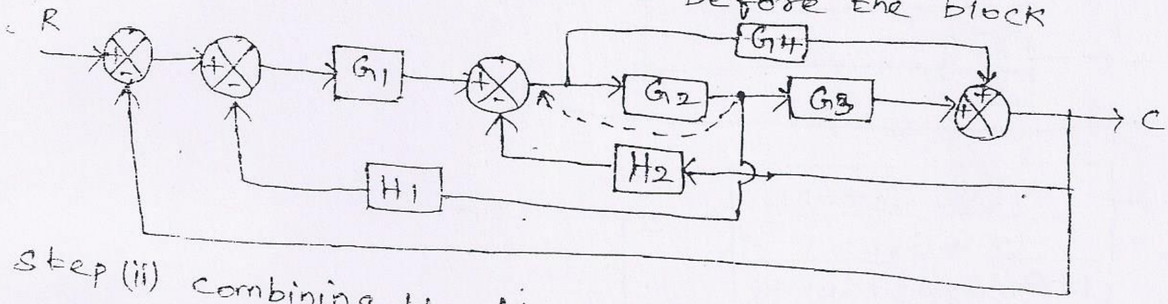
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2(G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{G_1G_2(G_3+G_4)}{1+G_1G_2H_1}H_2} = \frac{\dfrac{G_1G_2(G_3+G_4)}{1+\cancel{G_1G_2H_1}}}{\dfrac{1+G_1G_2H_1+G_1G_2(G_3+G_4)H_2}{1+G_1\cancel{G_2H_1}}}$$

$$T.F = \frac{C(s)}{R(s)} = \frac{G_1G_2(G_3+G_4)}{1+G_1G_2H_1+G_1G_2(G_3+G_4)H_2}$$

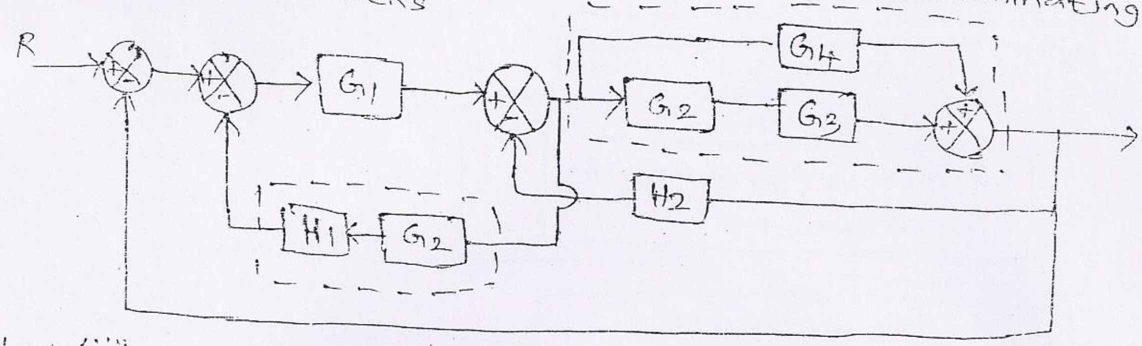
3. Using block diagram reduction technique find closed loop T.F of the system.



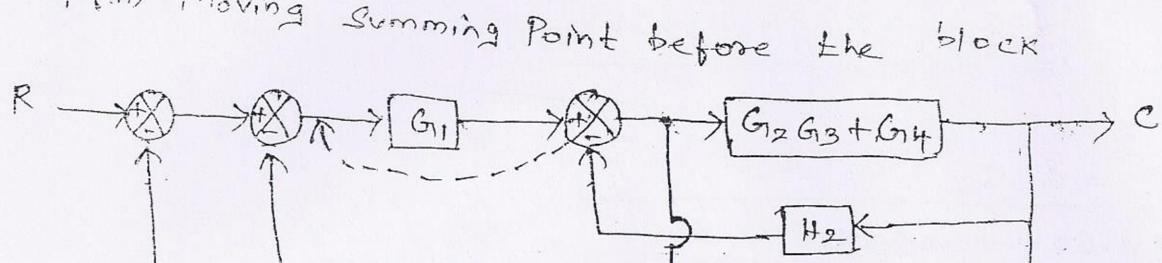
Step(i) Moving the branch Point before the block



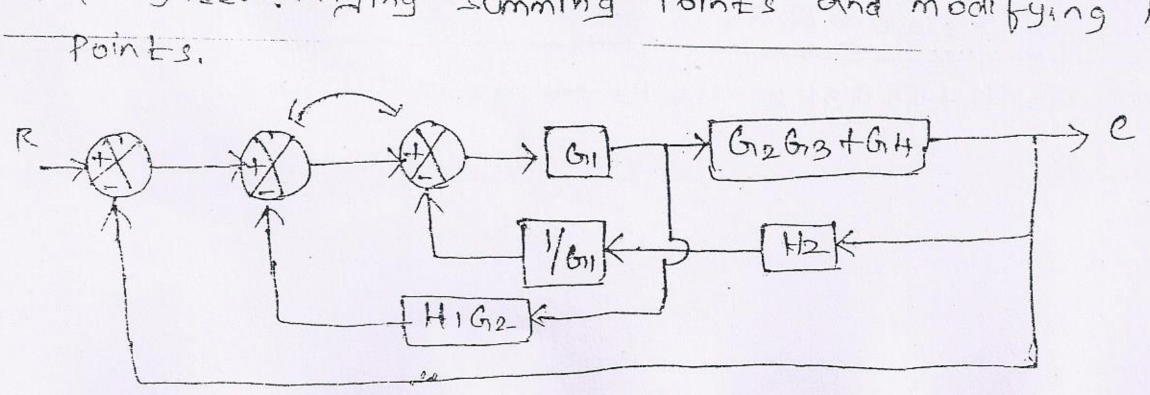
Step (ii) Combining the blocks in cascade and eliminating Parallel blocks



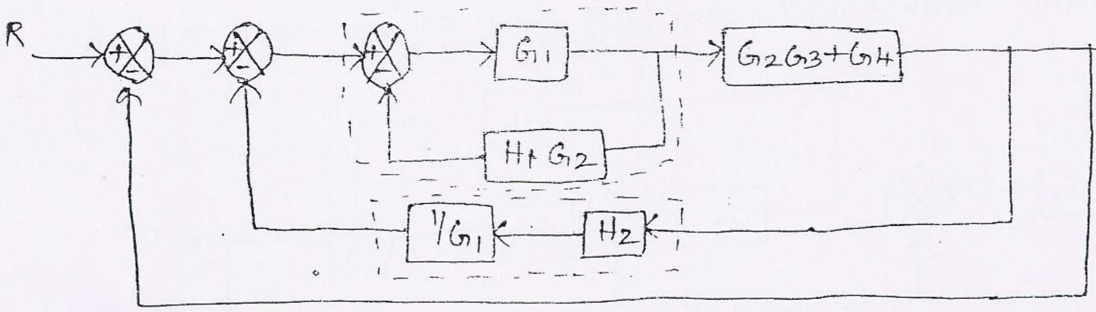
Step (iii) Moving Summing Point before the block



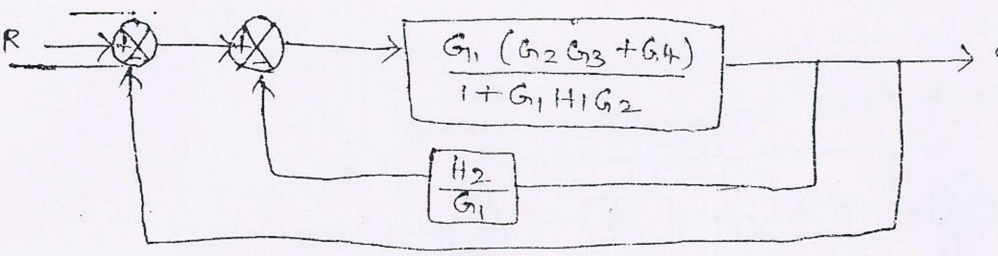
Step (iv) Interchanging summing Points and modifying branch Points.

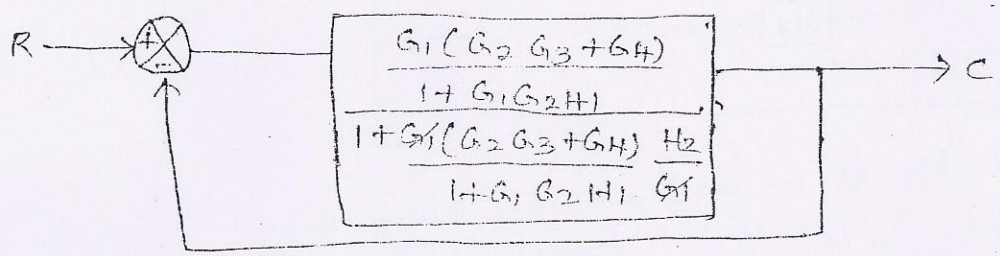


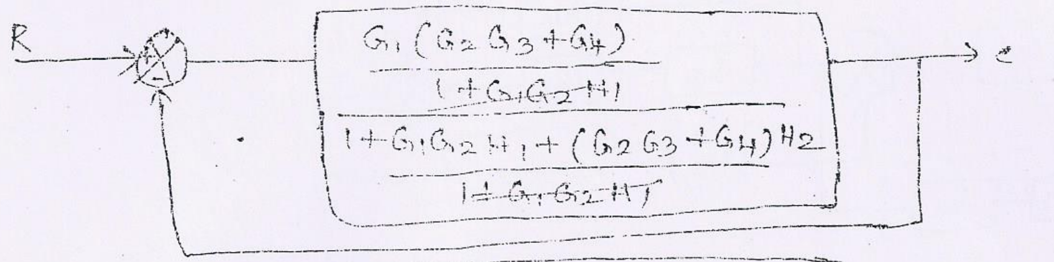
Step (v) Eliminating the feedback Path and Combining blocks in Cascade.

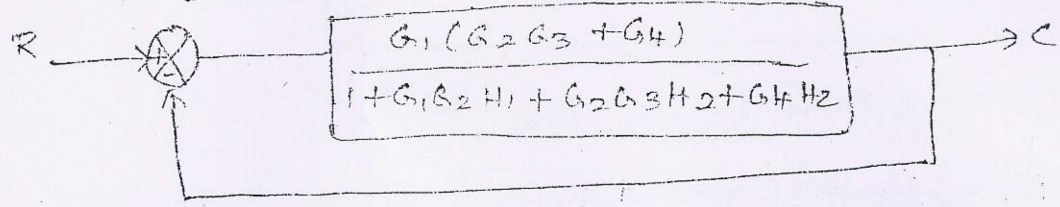


Step (vi) Eliminating the feedback Path





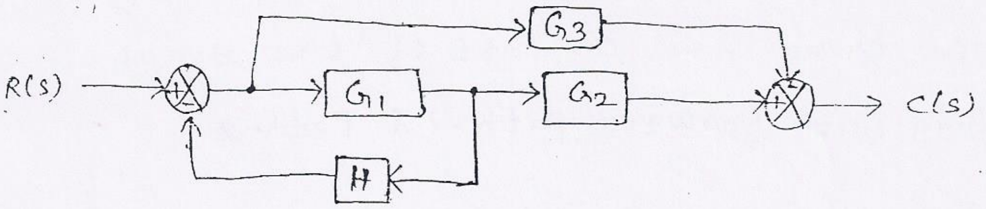




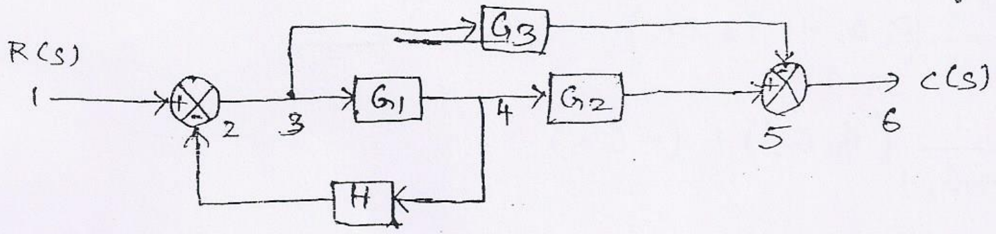
$$\frac{C}{R} = \frac{\dfrac{G_1G_2G_3 + G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}{1+\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}$$

$$\frac{C}{R} = \frac{G_1G_2G_3 + G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

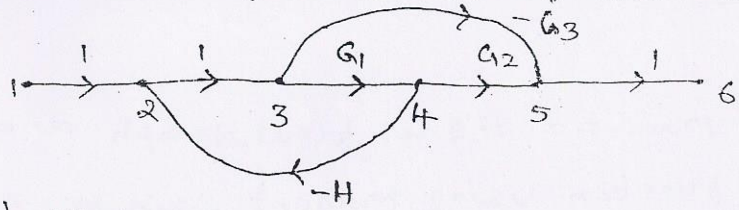
4. Convert the given block diagram to signal flow graph and determine $C(s) / R(s)$



Soln The nodes are assigned at input, output, at every summing point and branch points.

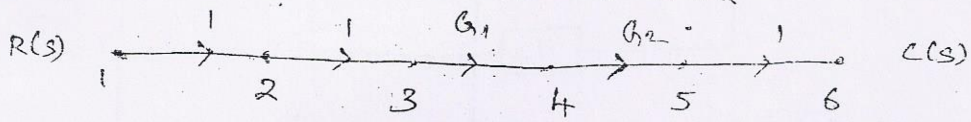


The signal flow graph of the above system is

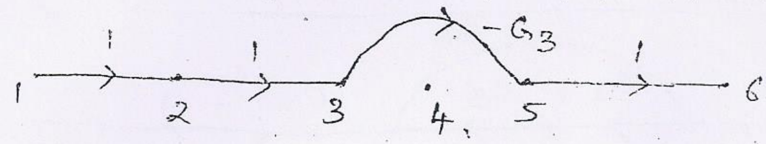


step (i) Find the forward path gains

No of Forward path $K = 2$

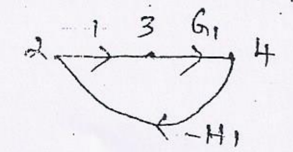


gain for forward path-1 $P_1 = G_1 G_2$



gain for forward path-2 $P_2 = -G_3$

step (ii) Find the individual loop gain



loop gain of individual loop 1 $P_{11} = -G_1 H$

Step (iii) There are no combination of non touching loops.

Step (iv) Calculation Δ & Δ_K

$\Delta = 1 -$ individual loop gain

$= 1 - (-G_1 H)$

$= 1 + G_1 H$

$\Delta_1 = 1$ & $\Delta_2 = 1$ since there are no Part of the graph which is non touching with forward path-1 & path-2

step (iv) Transfer Fn

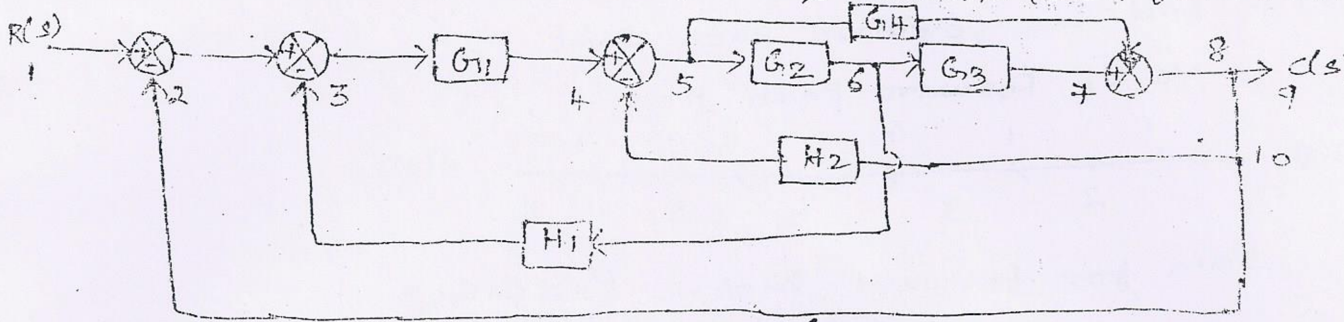
$$T(s) = \frac{1}{\Delta} \Sigma_K P_K \Delta_K$$

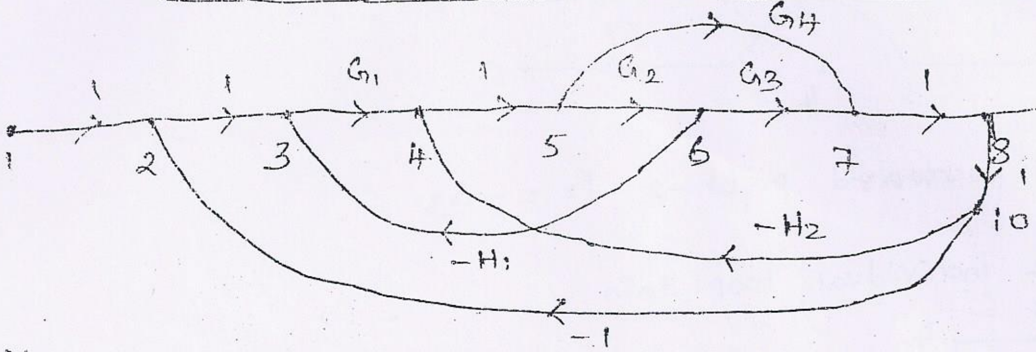
$$= \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2)$$

$$= \frac{1}{1+G_1 H} (G_1 G_2) + (-G_3)$$

$$T(s) = \frac{G_1 G_2 - G_3}{1 + G_1 H}$$

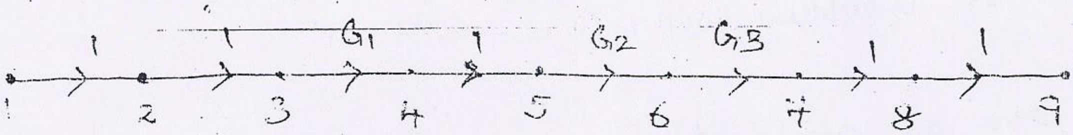
5. Convert the block diagram to signal flow graph and determine the transfer function using mason's gain formula.





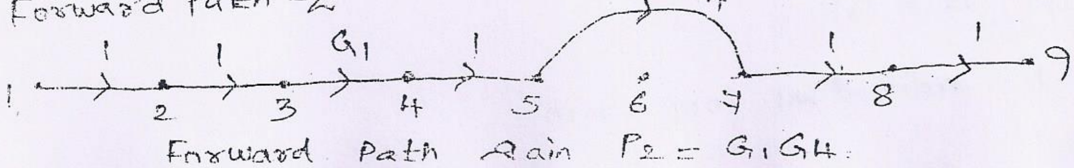
No of forward path K=2

Forward Path-1



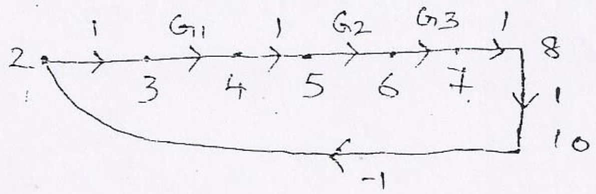
Forward Path gain $P_1 = G_1 G_2 G_3$

Forward Path-2

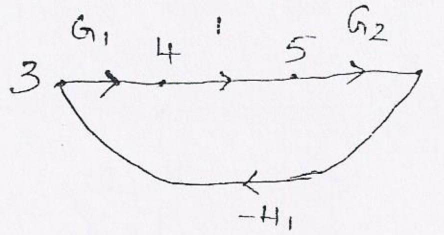


Forward Path gain $P_2 = G_1 G_4$.

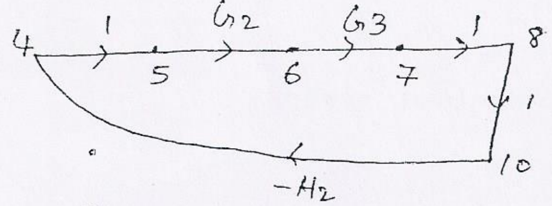
Individual loop gain



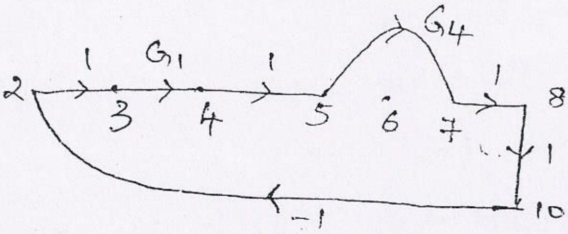
$P_{11} = -G_1 G_2 G_3$



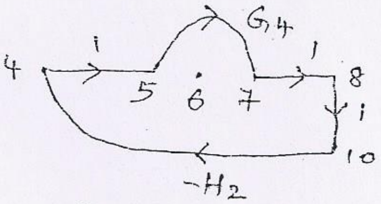
$P_{21} = -G_1 G_2 H_1$



$P_{31} = -G_2 G_3 H_2$



$P_{41} = -G_1 G_4$



$P_{51} = -G_4 H_2$

There is no combination of two non touching loops

Calculation of Δ & Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51})$$

$$= 1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_4 + G_4 H_2$$

$\Delta_1 = \Delta_2 = 1$ since there is no part of graph is non touching with forward path 1 & 2

Transfer function $T = \frac{1}{\Delta} \Sigma_K P_K \Delta_K$

$$= \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_4 + G_4 H_2}$$

Electrical systems

Voltage relationship for R, L & C

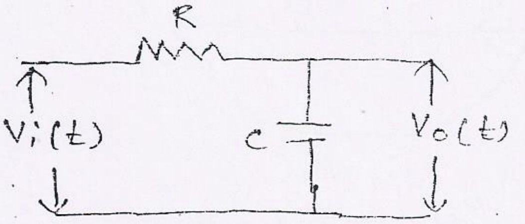
For R $\quad V = IR \quad v(t) = R\,i(t)$

$\quad V(s) = R\,i(s)$

L $\quad V = L\frac{di}{dt} \quad V(t) = L\frac{d\,i(t)}{dt} = Ls\,i(s)$

C $\quad V = \frac{1}{C}\int i\,dt \quad V(t) = \frac{1}{Cs}\,i(s)$

1. Derive the transfer function of electrical system :



According to KVL

Sum of Potential rise = Sum of Potential droop

Apply KVL for loop 1

$$IR + \frac{1}{C}\int i\,dt = Vi \quad \text{—} \ (1)$$

Apply KVL for loop 2

$$Vo(t) = \frac{1}{C}\int i\,dt \quad \text{—} \ (2)$$

Taking L.T for (1) & (2)

$$Vi(s) = RI(s) + \frac{1}{Cs} I(s) \quad \text{—} \ (3)$$

$$Vo(s) = \frac{1}{Cs} I(s) \quad \text{—} \ (4)$$

$$Vo(s)\, Cs = I(s)$$

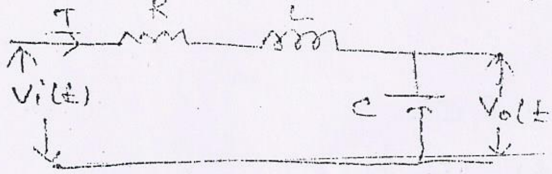
$$Vi(s) = \left[R + \frac{1}{Cs}\right] I(s)$$

$$Vi(s) = \left[R + \frac{1}{Cs}\right] Vo(s) \cdot Cs$$

$$Vi(s) = \left[\frac{R.Cs + 1}{\cancel{Cs}}\right] Vo(s) \cdot \cancel{Cs}$$

$$\frac{Vo(s)}{Vi(s)} = \frac{1}{(RCs + 1)}$$

2. Find the T.F of the given electrical system



$$Vi = RI + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{—} \ (1)$$

$$Vo(t) = \frac{1}{C}\int i\,dt \quad \text{—} \ (2)$$

Taking L.T for (1) & (2)

$$\therefore V_i(s) = RI(s) + LsI(s) + \frac{1}{Cs}I(s) \quad -③$$

$$V_o(s) = \frac{1}{Cs}I(s)$$

$$V_o(s)\,Cs = I(s)$$

$$V_i(s) = \left[R + Ls + \frac{1}{Cs}\right] V_o(s)\,Cs$$

$$V_i(s) = \frac{RCs + Ls^2C + 1}{\cancel{Cs}}\, V_o(s)\,\cancel{Cs}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{RCs + LCs^2 + 1}$$

Mechanical Translational Systems

The model of the mechanical translational systems can be obtained by the basic elements mass, spring and dash pot.

→ Weight of the system represented by mass and it is assumed to be concentrated at the centre of the body

→ elastic deformation of the body can be represented by spring

→ Friction exsisting can be represented by dash-pot

When a force is applied to a translational system, it is opposed by opposing forces due to mass, friction and elasticity of the system.

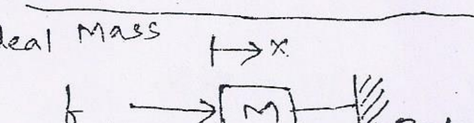
→ It is governed by Newton's second law of motion

It states that the sum of forces acting on a body is zero (or)

Sum of applied force = Sum of opposing Force

Force balance equations of idealized elements

Ideal Mass



f - applied force

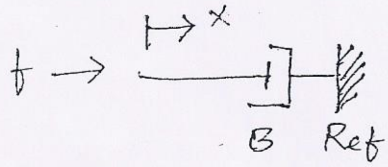
f_m - opposing force due to mass

$$f_m \propto \frac{d^2x}{dt^2} \qquad f_m = M\frac{d^2x}{dt^2}$$

By Newton's second law

$$f = f_m = M\frac{d^2x}{dt^2}$$

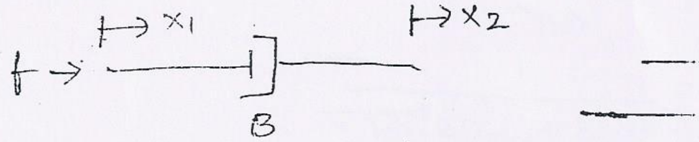
Ideal dash pot



$$f_b = B\frac{dx}{dt}$$

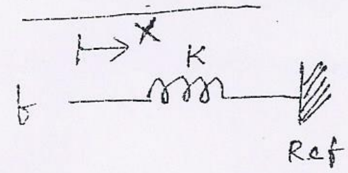
$$f = f_b = B\frac{dx}{dt}$$

When dashpot has displacement in both ends



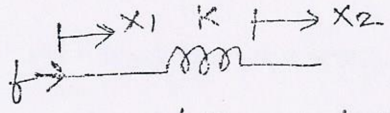
$$f = f_b = B\frac{d}{dt}(x_1 - x_2)$$

Ideal spring



$$f = f_K = Kx$$

Ideal spring with displacement at both ends



$$f = f_K = K(x_1 - x_2)$$

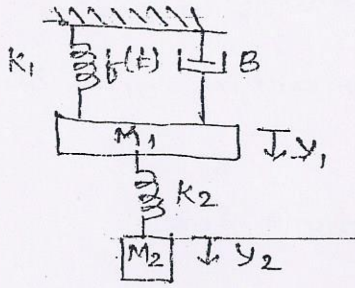
Note

L.T of $x(t) = L[x(t)] = X(s)$

L.T of $\frac{d}{dt}x(t) = L\left[\frac{d}{dt}x(t)\right] = sX(s)$

L.T of $\frac{d^2x(t)}{dt^2} = L\left[\frac{d^2}{dt^2}x(t)\right] = s^2X(s)$

1. Determine the T.F $\frac{Y_2(s)}{F(s)}$ of the given system

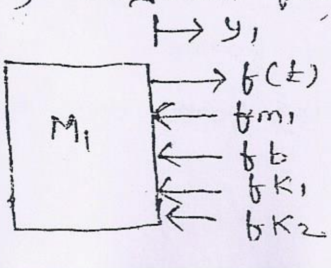


Let L.T of $f(t) = L[f(t)] = F(s)$

$y_1 = L[y_1] = Y_1(s)$

$y_2 = L[y_2] = Y_2(s)$

Free body diagram for mass M_1



$$f_{m_1} = M_1\frac{d^2y}{dt^2}$$

$$f_b = B\frac{dy_1}{dt}$$

$$f_{K_1} = K_1y_1$$

$$f_{K_2} = K_2(y_1 - y_2)$$

By Newton's second law

$$f(t) = f_{m_1} + f_B + f_{K_1} + f_{K_2}$$

$$\therefore M_1 \frac{d^2 y_1}{dt^2} + B \frac{dy_1}{dt} + K_1 y_1 + K_2 (y_1 - y_2) = f(t)$$

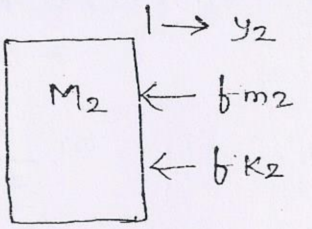
On taking L.T with zero initial condition

$$M_1 s^2 y_1(s) + B s y_1(s) + K_1 y_1(s) + K_2 (y_1(s) - y_2(s)) = F(s)$$

$$M_1 s^2 y_1(s) + B s y_1(s) + K_1 y_1(s) + K_2 y_1(s) - K_2 y_2(s) = F(s)$$

$$y_1(s) \left[M_1 s^2 + Bs + (K_1 + K_2) \right] - Y_2(s) K_2 = F(s) \quad \text{--- (1)}$$

The free body diagram of mass M_2



$$f_{m2} = M_2 \frac{d^2 y_2}{dt^2}$$

$$f_{K2} = K_2 (y_2 - y_1)$$

By newton's second law

$$M_2 \frac{d^2 y_2}{dt^2} + K_2 (y_2 - y_1) = 0$$

On taking L.T

$$M_2 s^2 y_2(s) + K_2 (y_2(s) - y_1(s)) = 0$$

$$M_2 s^2 y_2(s) + K_2 y_2(s) - K_2 y_1(s) = 0$$

$$y_2(s) \left[M_2 s^2 + K_2 \right] - y_1(s) K_2 = 0$$

$$\therefore y_1(s) = y_2(s) \left[\frac{M_2 s^2 + K_2}{K_2} \right]$$

Sub $y_1(s)$ in (1)

$$y_2(s) \left[\frac{M_2 s^2 + K_2}{K_2} \right] \left[M_1 s^2 + Bs + (K_1 + K_2) \right] - Y_2(s) K_2 = F(s)$$

$$y_2(s) \left[\frac{(M_2 s^2 + K_2)(M_1 s^2 + Bs + (K_1 + K_2)) - K_2^2}{K_2} \right] = F(s)$$

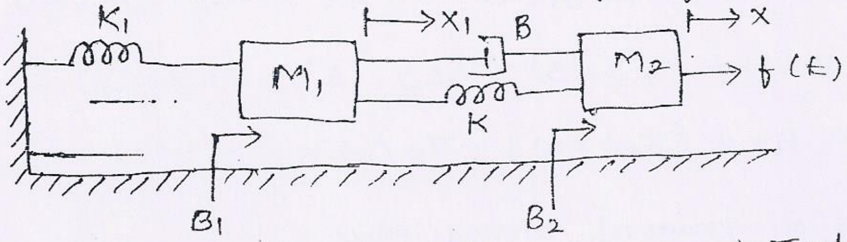
$$\frac{Y_2(s)}{F(s)} = \frac{K_2}{(M_1 s^2 + Bs + K_1 + K_2)(M_2 s^2 + K_2) - K_2^2}$$

The differential equations governing the system are

1. $M_1 \frac{d^2 y_1}{dt^2} + B \frac{dy_1}{dt} + K_1 y_1 + K_2 (y_1 - y_2) = f(t)$

2. $M_2 \frac{d^2 y_2}{dt^2} + K_2 (y_2 - y_1) = 0$

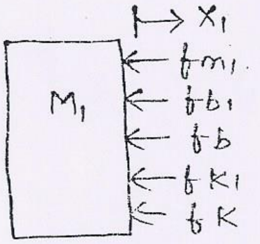
2. Write the differential equations governing the mechanical system and determine the transfer function.



L.T of $f(t) = F(s)$

$x = X(s)$

Free body diagram of M_1



$f_{m1} = M_1 \frac{d^2 x_1}{dt^2}$ $\quad f_{b1} = B_1 \frac{dx_1}{dt}$

$f_b = B \frac{d}{dt}(x_1 - x)$ $\quad f_{K1} = K_1 x_1$

$f_K = K(x_1 - x)$

By newtons second law

$f_{m1} + f_{b1} + f_b + f_{K1} + f_K = 0$

$\therefore M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x) + K_1 x_1 + K(x_1 - x) = 0$

Taking L.T

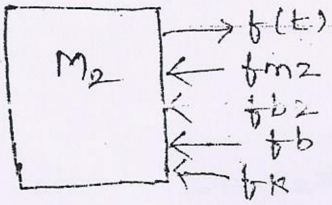
$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs[X_1(s) - X(s)] + K_1 X_1(s) + K[X_1(s) - X(s)] =$

$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs X_1(s) - Bs X(s) + K_1 X_1(s) + K X_1(s) - K X(s) = 0$

$X_1(s)[M_1 s^2 + s(B_1 + B) + (K_1 + K)] - X(s)[Bs + K] = 0$

$\therefore X_1(s) = X(s) \left[\frac{Bs + K}{M_1 s^2 + (B_1 + B)s + (K_1 + K)}\right]$

Free body diagram of M_2



$f_{m2} = M_2 \frac{d^2 x}{dt^2}$ $\quad f_{b2} = B_2 \frac{dx}{dt}$

$f_b = B \frac{d}{dt}(x - x_1)$ $\quad f_K = K(x - x_1)$

By Newton's Second law

$f_{m2} + f_{b2} + f_b + f_K = f(t)$

$$M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$$

Taking L.T

$$M_2 s^2 X(s) + B_2 s X(s) + Bs X(s) - Bs X_1(s) + K X(s) - K X_1(s) = F(s)$$

$$X(s)\left[M_2 s^2 + s(B_2 + B) + K\right] - X_1(s)\left[Bs + K\right] = F(s)$$

Sub $X_1(s)$ in the above eqn

$$X(s)\left[M_2 s^2 + s(B_2 + B) + K\right] - \frac{X(s)\ Bs + K\ (Bs + K)}{M_1 s^2 + (B_1 + B)s + (K_1 + K)} = F(s)$$

$$X(s)\left[M_2 s^2 + s(B_2 + B) + K\right] - \frac{X(s)(Bs + K)^2}{M_1 s^2 + (B_1 + B)s + (K_1 + K)} = F(s)$$

$$X(s)\left[\frac{\left[M_1 s^2 + (B_1 + B)s + (K_1 + K)\right]\left[M_2 s^2 + s(B_2 + B) + K\right] - (Bs + K)^2}{M_1 s^2 + (B_1 + B)s + (K_1 + K)}\right] = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1 + B)s + (K_1 + K)}{\left[M_1 s^2 + (B_1 + B)s + (K_1 + K)\right]\left[M_2 s^2 + s(B_2 + B) + K\right] - \left[Bs + K\right]^2}$$

Mechanical Rotational systems

θ - angular displacement (rad)

$\frac{d\theta}{dt}$ = angular velocity (rad/sec)

$\frac{d^2\theta}{dt^2}$ = angular acceleration (rad/sec^2)

T = applied Torque $\qquad$ T_J - opposing torque

$T_J \propto \frac{d^2\theta}{dt^2}$ $\qquad$ $T_b \propto \frac{d\theta}{dt}$ $\qquad$ $T_K \propto \theta$

$T_J = J\frac{d^2\theta}{dt^2}$ $\qquad$ $T_b = B\frac{d\theta}{dt}$ $\qquad$ $T_K = K\theta$

$T_b = B\frac{d}{dt}(\theta_1 - \theta_2)$ (displacement at both the ends) $\qquad$ $T_K = K(\theta_1 - \theta_2)$

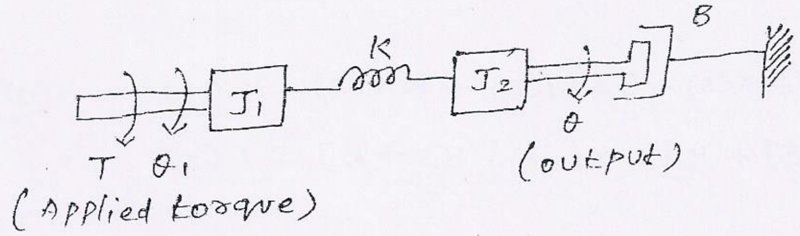
L.T

$$L[\theta] = \theta(s)$$

$$L\left[\frac{d\theta}{dt}\right] = s\theta(s)$$

$$L\left[\frac{d^2\theta}{dt^2}\right] = s^2\theta(s)$$

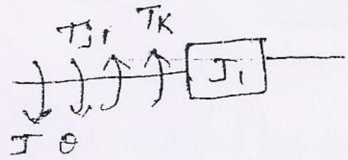
1. Write the differential equations governing the mechanical rotational system. Obtain the T.f of the system.



$L[T] = T(s)$

$L[\theta] = \theta(s)$

Free body diagram of mass with moment of inertia J_1



$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2}$$

$$T_K = K(\theta_1 - \theta)$$

By Newton's second law

$$T_{j1} + T_K = T$$

$$J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$

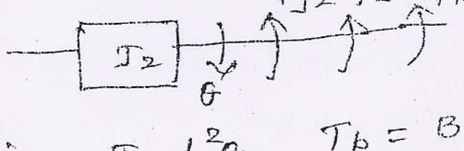
$$J_1 \frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = T$$

Taking L.T

$$J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$$

$$\theta_1(s)[J_1 s^2 + K] - K\theta(s) = T(s) \quad \text{--- (1)}$$

Free body diagram of mass with moment of inertia



$$T_{j2} = J_2 \frac{d^2\theta}{dt^2} \qquad T_b = B\frac{d\theta}{dt} \qquad T_K = K(\theta - \theta_1)$$

According to newtons second law

$$T_{j2} + T_b + T_K = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\,d\theta + K\theta = K\theta_1$$

Taking L.T

$$J_2 s^2 \theta(s) + Bs\theta(s) + K\theta(s) = K\theta_1(s)$$

$$\therefore \theta_1(s) = \frac{J_2 s^2 \theta(s) + Bs\theta(s) + K\theta(s)}{K}$$

$$= \theta(s)\left[\frac{J_2 s^2 + Bs + K}{K}\right]$$

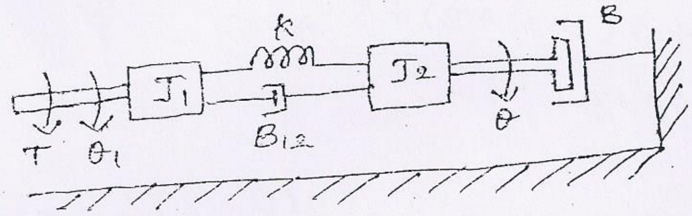
Sub $\theta_1(s)$ in (1)

$$(J_1 s^2 + K)\left(\frac{J_2 s^2 + Bs + K}{K}\right)\theta(s) - K\theta(s) = T(s)$$

$$\left[\frac{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}{K}\right]\theta(s) = T(s)$$

$$\frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}$$

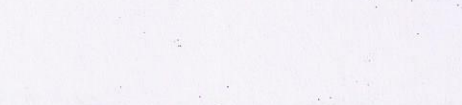
2. Write the differential equations governing the mechanical rotational system and determine the T.F $\theta(s)/T(s)$



$L[T] = T(s)$

$L[\theta] = \theta(s)$

Free body diagram of J_1



$$T_{J1} = J_1 \frac{d^2\theta_1}{dt^2}$$

$$T_{b12} = B_{12}\frac{d}{dt}(\theta_1 - \theta)$$

$$T_K = K(\theta_1 - \theta)$$

By Newton's Second law

$$T_{J1} + T_{b12} + T_K = T$$

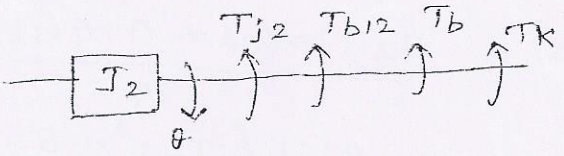
$$J_1 \frac{d^2\theta_1}{dt^2} + B_{12}\frac{d}{dt}(\theta_1 - \theta) + K(\theta_1 - \theta) = T$$

Taking L.T

$$J_1 s^2 \theta_1(s) + sB_{12}[\theta_1(s) - \theta(s)] + K\theta_1(s) - K\theta(s) = T(s)$$

$$\theta_1(s)\left[J_1 s^2 + sB_{12} + K\right] - \theta(s)\left[sB_{12} + K\right] = T(s) \quad -\ ①$$

Free body diagram J_2



$$T_{j2} = J_2 \frac{d^2\theta}{dt^2}$$

$$T_{b12} = B_{12}\frac{d}{dt}(\theta - \theta_1)$$

$$T_b = B\frac{d\theta}{dt} \qquad T_K = K(\theta - \theta_1)$$

By Newton's second law

$$T_{j2} + T_{b12} + T_b + T_K = 0$$

$$J_2\frac{d^2\theta}{dt^2} + B_{12}\frac{d}{dt}(\theta - \theta_1) + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

Taking L.T

$$J_2 s^2\theta(s) - B_{12} s\,\theta_1(s) + s\theta(s)\left[B_{12} + B\right] + K\theta(s) - K\theta_1(s) = 0$$

$$\theta(s)\left[s^2 J_2 + s(B_{12} + B) + K\right] - \theta_1(s)\left[sB_{12} + K\right] = 0$$

$$\theta_1(s) = \frac{\left(s^2 J_2 + s(B_{12} + B) + K\right)}{(sB_{12} + K)}\theta(s) \quad -\ ②$$

sub $\theta_1(s)$ in ①

$$\left[J_1 s^2 + sB_{12} + K\right]\left[\frac{J_2 s^2 + s(B_{12} + B) + K}{(sB_{12} + K)}\right]\theta(s) - (sB_{12} + K)\theta(s) = T(s)$$

$$\left[\frac{(J_1 s^2 + sB_{12} + K)(J_2 s^2 + s(B_{12} + B) + K) - (sB_{12} + K)^2}{(sB_{12} + K)}\right]\theta(s) = T(s)$$

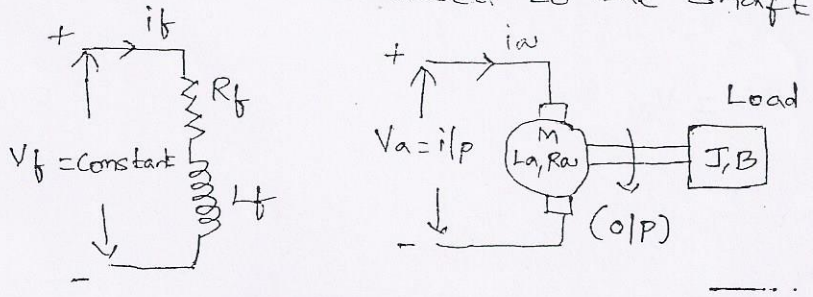
$$\therefore \frac{\theta(s)}{T(s)} = \frac{(sB_{12} + K)}{(J_1 s^2 + sB_{12} + K)(J_2 s^2 + s(B_{12} + B) + K) - (sB_{12} + K)^2}$$

Transfer function of armature Controlled DC motor

The Speed of DC motor is directly Proportional to armature Voltage and inversly proportional to flux in fiel winding.

In armature controlled DC motor the desired speed i obtained by Varying the armature Voltage.

In electrical system only the armature ckt is considered and the mechanical system consist of the rotating parts of the motor and load connected to the shaft of the motor.



Let

R_a - armature resistance (Ω)

L_a - armature inductance (H)

i_a - armature current (A)

V_a - armature Voltage (V)

e_b - Back emf (V)

K_t - Torque constant (N-m/A)

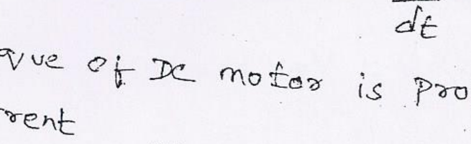
T - Torque developed by motor (N-m)

θ - angular displacement of the shaft (rad)

J - Moment of inertia of motor and load (Kg-m^2/rad)

B - frictional coefficient of motor & load. (rad/sec)

K_b - back emf constant V/(rad/sec)



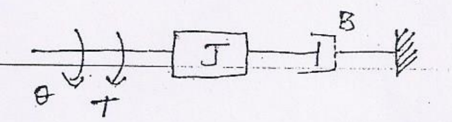
By KVL

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \quad - (1)$$

Torque of DC motor is proportional to the product of flux and current

$$T \propto i_a$$

$$\therefore T = K_t i_a \quad - (2)$$



The differential equation governing the mechanical system of motor is given by $J \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad - (3)$

The back emf of DC m/c is proportional to speed (angular Velocity) of shaft

$\therefore e_b \propto \frac{d\theta}{dt}$ $\quad e_b = K_b \frac{d\theta}{dt}$ — (4)

The differential equations governing the armature controlled DC motor speed control system are

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a$$

$$T = K_t i_a$$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

$$e_b = K_b \frac{d\theta}{dt}$$

Taking L.T

$$I_a(s) R_a + L_a s I_a(s) + E_b(s) = V_a(s) \quad — (5)$$

$$T(s) = K_t I_a(s) \quad — (6)$$

$$Js^2\theta(s) + Bs\theta(s) = T(s) \quad — (7)$$

$$E_b(s) = K_b s\theta(s) \quad — (8)$$

equating (6) & (7)

$$K_t I_a(s) = (Js^2 + Bs)\theta(s)$$

$$I_a(s) = \frac{(Js^2 + Bs)\theta(s)}{K_t} \quad — (9)$$

eq (5) can be written as

$$(R_a + L_a s) I_a(s) + E_b(s) = V_a(s) \quad — (10)$$

Sub $E_b(s)$ & $I_a(s)$ in (10)

$$(R_a + sL_a)\left(\frac{Js^2 + Bs}{K_t}\right)\theta(s) + K_b\theta(s)s = V_a(s)$$

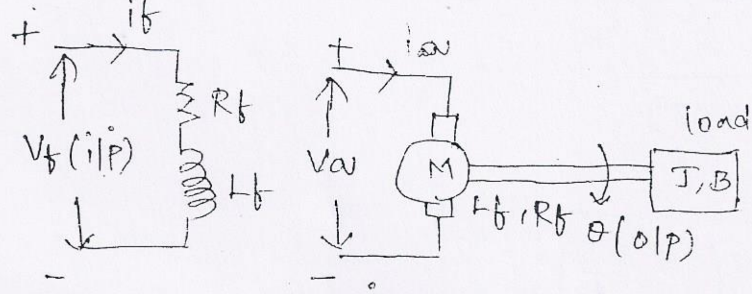
$$\left[\frac{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}{K_t}\right]\theta(s) = V_a(s)$$

$$\text{T.F} \quad \frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}$$

T.F of Field controlled DC motor

In field controlled DC motor the armature voltage is kept constant and the speed is varied by varying the flux of the machine.

For electrical system only field circuit is considered and the mechanical system consists of the rotating part of the motor and the load connected to the shaft of the motor.



Rf - field resistance (Ω)
Lf - field inductance (H)
if - field current (A)
Vf - field Voltage (Vf)
T - Torque
Ktf - Torque Constant
J - moment of inertia
B - frictional coefficient

By KVL

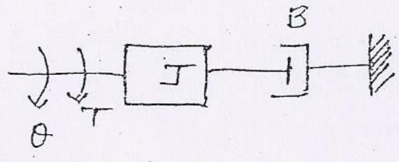
$$R_f i_f + L_f \frac{di_f}{dt} = V_f \quad \text{--- (1)}$$

Since the armature current is constant in this system, the torque is proportional to flux alone, but flux is proportional to field current.

$$T \propto i_f \qquad \therefore T = K_{tf} i_f \quad \text{--- (2)}$$

The diff eqn governing the mechanical system of the motor is governed by

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{--- (3)}$$



The diff eqns governing the field controlled DC motor are

$$R_f i_f + L_f \frac{di_f}{dt} = V_f, \quad T = K_{tf} i_f, \quad J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

On taking L.T

$$R_f I_f(s) + L_f s I_f(s) = V_f(s) \text{ - (4)}$$

$$T(s) = K_{tf} I_f(s) \quad \text{--- (5)}$$

$$Js^2\theta(s) + Bs\theta(s) = T(s) \text{ - (6)}$$

Equating (5) & (6)

$$K_{tf} I_f(s) = Js^2\theta(s) + Bs\theta(s)$$

$$I_f(s) = \frac{s(Js+B)}{K_{tf}}\theta(s)$$

eqn (4) $\Rightarrow (R_f + sL_f) I_f(s) = V_f(s)$ - (7)

sub $I_f(s)$ in the above eqn

$$(R_f + sL_f)\, s\, \frac{(Js+B)}{K_{tf}}\, \theta(s) = V_f(s)$$

$$\frac{\theta(s)}{V_f(s)} = \frac{K_{tf}}{s(R_f + sL_f)(B+sJ)}$$

$$= \frac{K_{tf}}{sR_f\left(1+\frac{sL_f}{R_f}\right)B\left(1+\frac{sJ}{B}\right)}.$$

$$= \frac{K_m}{s(1+sT_f)(1+sT_m)}$$

where,

motor gain constant $K_m = K_{tf} / R_f B$

Field time constant $T_f = L_f / R_f$

Mechanical time constant $T_m = J/B$

Electrical analogous of mechanical translational system

Force Voltage analogy

Mech syst
I/p – Force
o/p – Velocity

Elect sys
I/p – Voltage source
o/p – current through the element

Mechanical system	Electrical system (Mesh)
Force, (f)	Voltage, e
Velocity (v)	current, i
displacement (x)	Charge, q
Dashpot (B)	Resistance (R)
Mass (M)	inductance (L)
Spring (K)	Inverse of capacitance (1/C)
Newtons second law	Kirchoff's Voltage law

Force current analogy

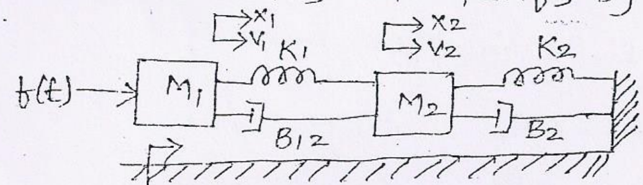
Mechanical system	Electrical system (Nod.
Force (f)	Current (i)
Velocity (v)	Voltage (v)
displacement (x)	flux ϕ

Dashpot (B)	Conductance $G = 1/R$
Mass (M)	Capacitance (C)
Spring (K)	Inverse of inductance ($1/L$)
Newtons Second law	KCL

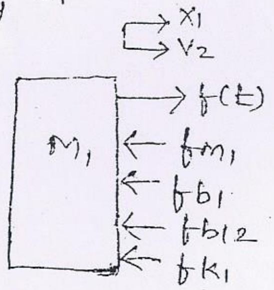
$\Sigma F = 0$

Voltage Relation		Current Relation	
$V = iR$	for R	$i = \frac{V}{R}$	for R
$V = L\frac{di}{dt}$	for L	$i = C\frac{dv}{dt}$	for C
$V = \frac{1}{C}\int i\,dt$	for C	$i = \frac{1}{L}\int V\,dt$	for L

1. Write the differential equations governing the mechanical system. Draw the force Voltage and force current electrical analogous circuits and verify by writing mesh and node eqns.



Free body diagram of M_1



$$f_{m1} = M_1 \frac{d^2x_1}{dt^2}, \quad f_{b1} = B_1 \frac{dx_1}{dt}$$

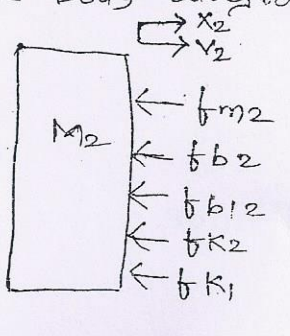
$$f_{b12} = B_{12}\frac{d}{dt}(x_1 - x_2), \quad f_{K1} = K_1(x_1 - x_2)$$

By Newton's second law

$$f_{m1} + f_{b1} + f_{b12} + f_{K1} = f(t)$$

$$M_1 \frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B_{12}\frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = f(t) \quad \text{—(1)}$$

Free body diagram of M_2



$$f_{m2} = M_2\frac{d^2x_2}{dt^2}, \quad f_{b2} = B_2\frac{dx_2}{dt}$$

$$f_{b12} = B_{12}\frac{d}{dt}(x_2 - x_1), \quad f_{K2} = K_2 x_2$$

$f_{K1} = K$

By Newtons Second law

$f_{m2} + f_{b2} + f_{K2} + f_{b12} + f_{K1} = 0$

$$M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + B_{12} \frac{d}{dt}(x_2 - x_1) + K_1 (x_2 - x_1) = 0 \quad -②$$

on replacing the displacements by Velocity in the diff equations of the mechanical system

$$X = \int v\, dt$$

$$V = \frac{dx}{dt}$$

$$\frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

$$M_1 \frac{dv_1}{dt} + B_1 V_1 + B_{12}(V_1 - V_2) + K_1 \int (V_1 - V_2)\, dt = f(t) \quad -③$$

$$M_2 \frac{dv_2}{dt} + B_2 V_2 + K_2 \int V_2\, dt + B_{12}(V_2 - V_1) + K_1 \int (V_2 - V_1)\, dt = 0 \quad -④$$

Force Voltage analogous circuit

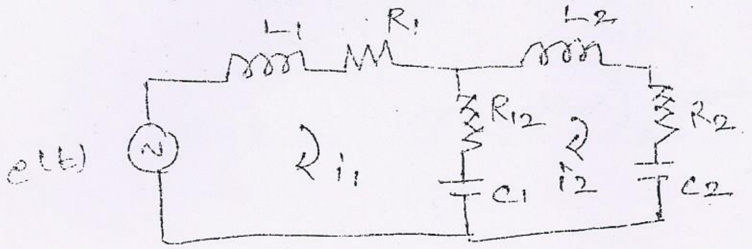
The elements M_1, B_1, K_1 and B_{12} in mesh one

The elements K_1, B_{12}, M_2, K_2 & B_2 in mesh two

$f(t) \rightarrow e(t)$, $M_1 \rightarrow L_1$, $B_1 \rightarrow R_1$, $K_1 \rightarrow 1/C_1$

$V_1 \rightarrow i_1$, $M_2 \rightarrow L_2$, $B_2 \rightarrow R_2$, $K_2 \rightarrow 1/C_2$

$V_2 \rightarrow i_2$, $B_{12} \rightarrow R_{12}$



$$L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1} \int (i_1 - i_2)\, dt = e(t) \quad -⑤$$

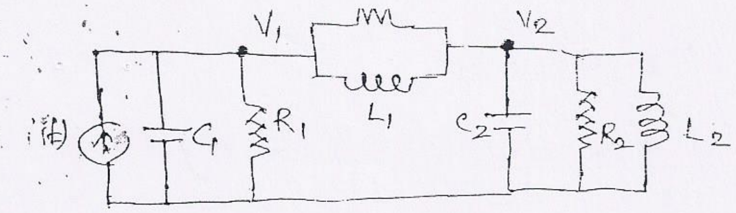
$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2\, dt + R_{12}(i_2 - i_1) + \frac{1}{C_1} \int (i_2 - i_1)\, dt = 0 \quad -⑥$$

Force current analogous circuit

M_1, B_1, K_1 & B_{12} are connected to first node

K_1, B_{12}, M_2, K_2 & B_2 are connected to second node

Two masses $\rightarrow$ two nodes



$f(t) \rightarrow i(t)$ $\quad M_1 \rightarrow C_1$ $\quad B_1 \rightarrow 1/R_1$ $\quad K_1 \rightarrow 1/L_1$

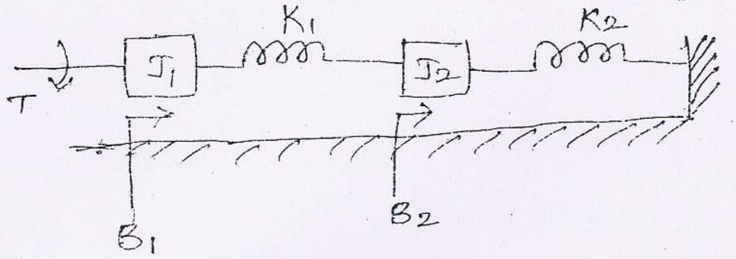
$V_1 \rightarrow V_1$ $\quad M_2 \rightarrow C_2$ $\quad B_2 \rightarrow 1/R_2$ $\quad K_2 \rightarrow 1/L_2$

$V_2 \rightarrow V_2$ $\quad B_{12} \rightarrow 1/R_{12}$

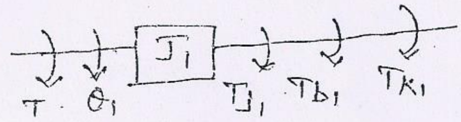
$$C_1 \frac{dV_1}{dt} + \frac{1}{R_1} V_1 + \frac{1}{R_{12}}(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_2)\,dt = i(t)$$

$$C_2 \frac{dV_2}{dt} + \frac{1}{R_2} V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{R_{12}}(V_2 - V_1) + \frac{1}{L_1}\int (V_2 - V_1)\,dt =$$

2. Write the differential eqns governing the mechanical rotational system. Draw torque-voltage & torque current electrical analogous circuits & verify by writing mesh & node equations.



For J_1



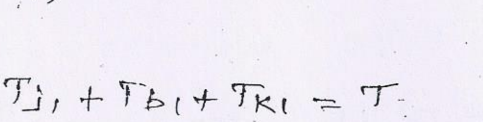
$$T_{J_1} = J_1 \frac{d^2\theta_1}{dt^2}$$

$$T_{b_1} = B_1 \frac{d\theta_1}{dt} \qquad T_{K_1} = K_1(\theta_1 - \theta_2)$$

By newton's second law, $T_{J_1} + T_{b_1} + T_{K_1} = T$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) = T$$

For J_2



$$T_{J_2} = J_2 \frac{d^2\theta_2}{dt^2} \qquad T_{b_2} = B_2 \frac{d\theta_2}{dt}$$

$$T_{K_2} = K_2 \theta_2 \qquad T_{K_1} = K_1(\theta_2 - \theta_1)$$

By newton's second law, $T_{J_2} + T_{b_2} + T_{K_2} + T_{K_1} = 0$

$$J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2 - \theta_1) = 0$$

On replacing angular displacement by angular velocity $\left(\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}, \frac{d\theta}{dt} = \omega\right)$

$$\theta = \int \omega\, dt$$

$$J_1 \frac{d\omega_1}{dt} + B_1\omega_1 + K_1\int(\omega_1 - \omega_2)\,dt = T$$

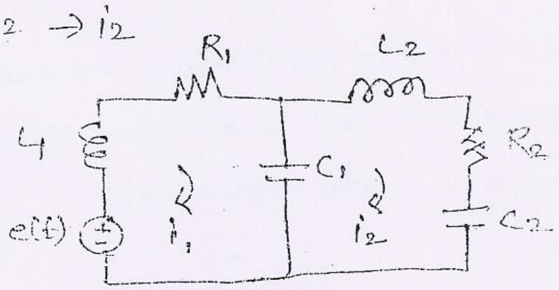
$$J_2 \frac{d\omega_2}{dt} + B_2\omega_2 + K_2\int \omega_2\, dt + K_1\int(\omega_2 - \omega_1)\,dt = 0$$

Torque voltage andlogous circuit

$T \rightarrow e(t)$ $J_1 \rightarrow L_1$ $B_1 \rightarrow R_1$ $K_1 \rightarrow 1/C_1$

$\omega_1 \rightarrow i_1$ $J_2 \rightarrow L_2$ $B_2 \rightarrow R_1$ $K_2 \rightarrow 1/C_2$

$\omega_2 \rightarrow i_2$



using KVL

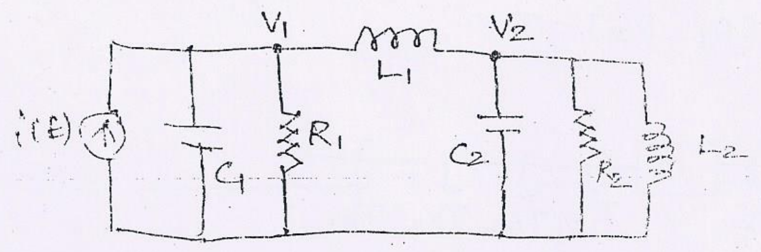
$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int(i_1 - i_2) = e(t)$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\, d(t) + \frac{1}{C_1}\int(i_2 - i_1)\,dt = 0$$

Torque current analogous circuit

$T \rightarrow i(t)$ $B_1 = 1/R_1$ $\omega_1 \rightarrow V_1$ $J_1 \rightarrow C_1$ $K_1 \rightarrow 1/L_1$

$B_2 \rightarrow 1/R_2$ $\omega_2 \rightarrow V_2$ $J_2 \rightarrow C_2$ $K_2 \rightarrow 1/L_2$

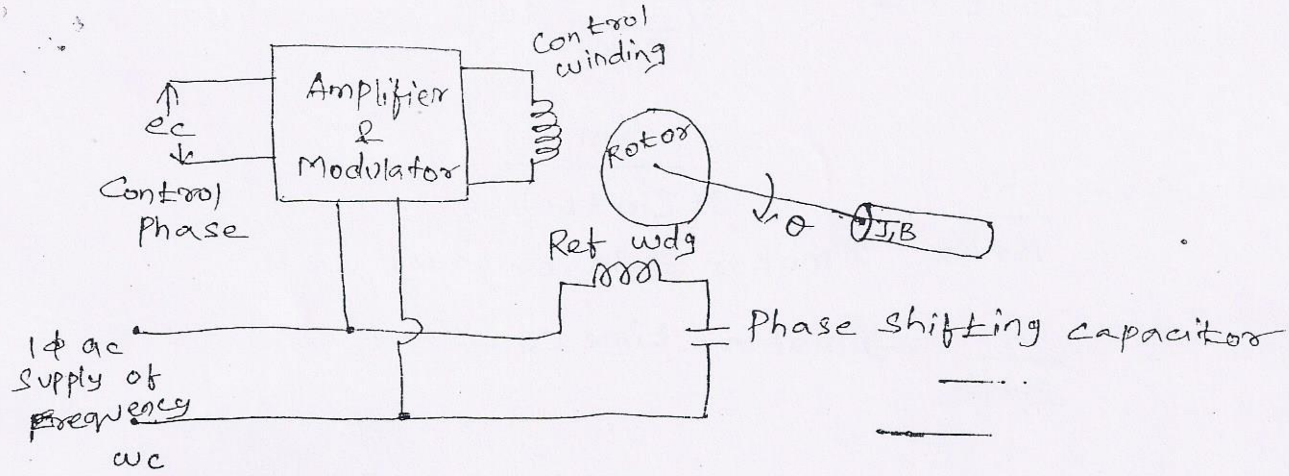


Using KCL

$$C_1 \frac{dV_1}{dt} + \frac{1}{R_1}V_1 + \frac{1}{L_1}\int(V_1 - V_2)\,dt = i(t)$$

$$C_2 \frac{dV_2}{dt} + \frac{1}{R_2}V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_1}\int(V_2 - V_1)\,dt = 0$$

Ac servomotor



Transfer function of Ac servomotor

Let T_m - Torque developed by servomotor

θ - angular displacement of rotor

$\omega - \frac{d\theta}{dt}$ - angular speed

T_l - load torque

J - moment of inertia of load & rotor

B - Viscous frictional coefficient of load and the rotor

K_1 - Slope of control phase Vs torque characteristics

K_2 - slope of speed torque characteristics

Torque developed by motor, $T_m = K_1 e_c - K_2 \frac{d\theta}{dt}$

Load Torque, $T_l = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$

At equilibrium the motor torque is equal to load torque

$$\therefore J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = K_1 e_c - K_2\frac{d\theta}{dt}$$

Taking L.T

$$Js^2\theta(s) + Bs\theta(s) = K_1 E_c(s) - K_2 s\theta(s)$$

$$(Js^2 + Bs + K_2 s)\theta(s) = K_1 E_c(s)$$

$$\therefore \frac{\theta(s)}{E_c(s)} = \frac{K_1}{s[Js+B+K_2)} = \frac{K_1/(B+K_2)}{s\left(\frac{J}{B+K_2}s+1\right)}$$

$$= \frac{K_m}{s(\tau_m s+1)}$$

where $K_m = \frac{K_1}{B+K_2}$ → motor gain constant

$\tau_m = \frac{J}{B+K_2}$ → motor time constant

Electrical analogous of Mechanical Rotational system

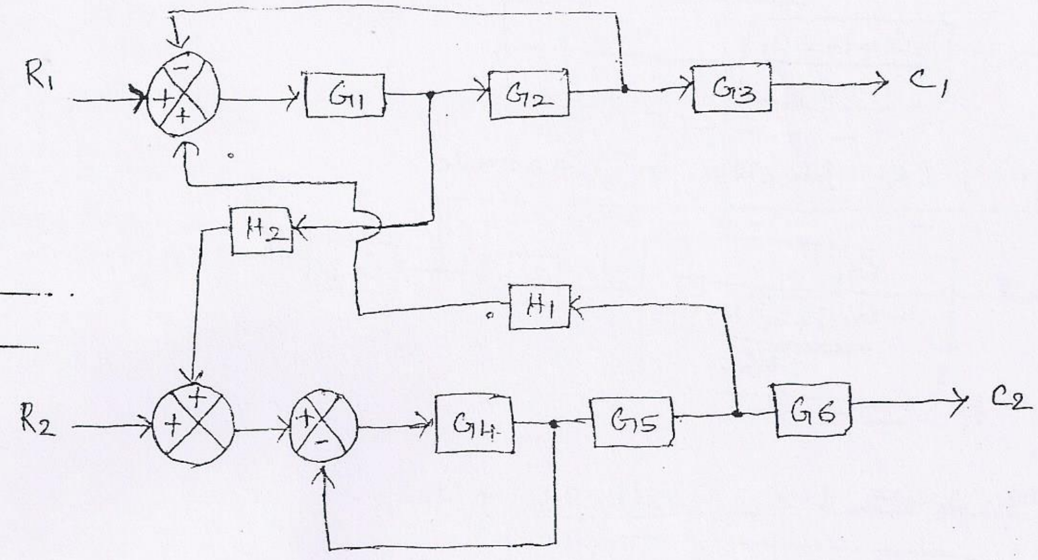
Torque - Voltage analogy

Mechanical rotational system	Electrical system
I/p - Torque	I/p - voltage force
o/p - Angular velocity	o/p - current through the element

Mechanical rotational system	Electrical system (Mesh basis system)
Torque, T	Voltage, e
Angular Velocity, ω	Current, i
Angular displacement, θ	Charge, q
Rotational Coefficient of dashpot, B	Resistance, R
Moment of inertia, J	Inductance, L
Stiffness of spring, K	Inverse of Capacitance, 1/C
Newton's Second law	Kirchoff's Voltage law

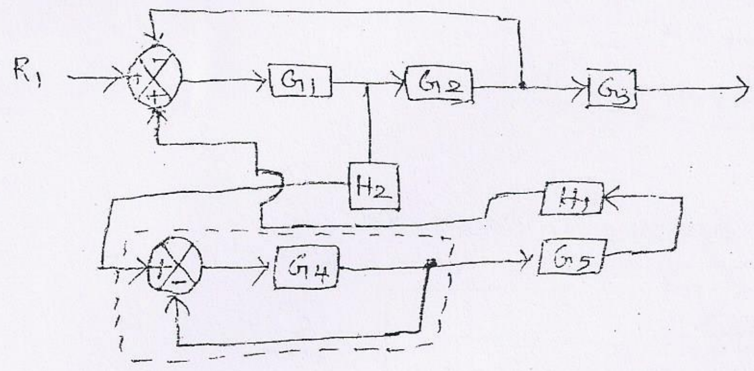
Mechanical rotational system	Electrical system (Node basis system)
Torque, T	Current, i
Angular Velocity, ω	Voltage, V
Angular displacement, θ	Flux, φ
Rotational frictional Coefficient of dashpot, B	Conductance, G = 1/R
Moment of inertia, J	Capacitance, C
Stiffness of spring, K	Inverse of inductance, 1/L
Newton's second law	Kirchoff's Current law

For the system represented by the block diagram determine C_1/R_1 & C_2/R_1

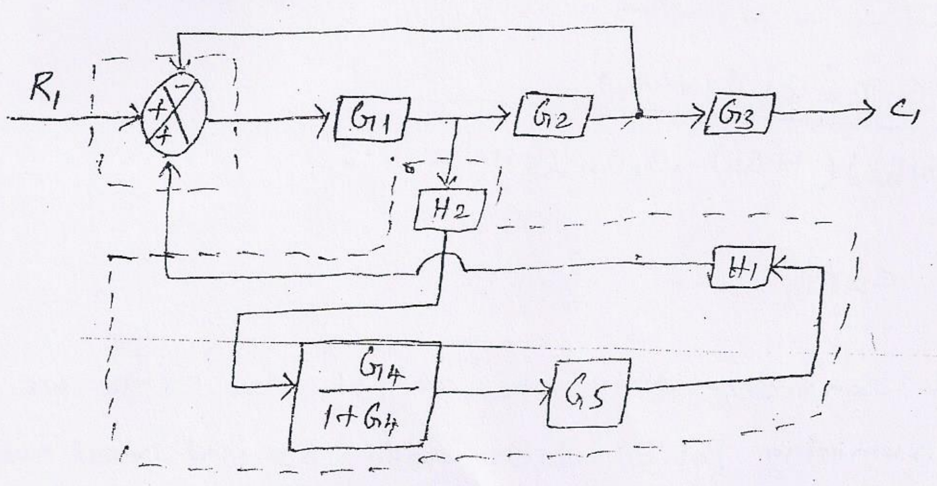


Case (i) To find C_1/R_1 $R_2 = 0$ Consider only one output C_1. Hence we can remove the summing point which add R_2 and need not consider G_6, since G_6 is on the open path.

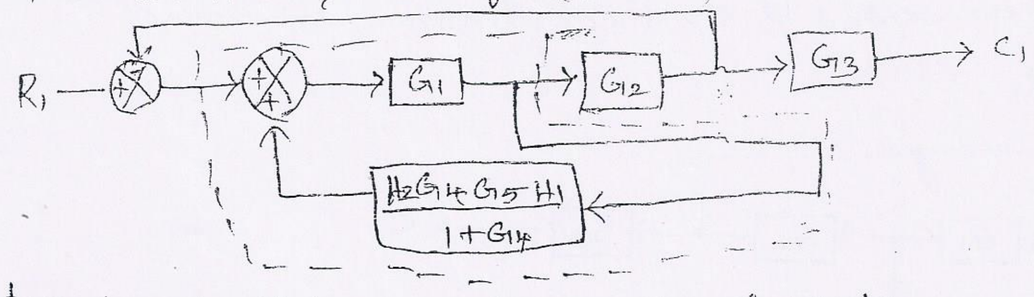
Step 1: Eliminating the feedback path



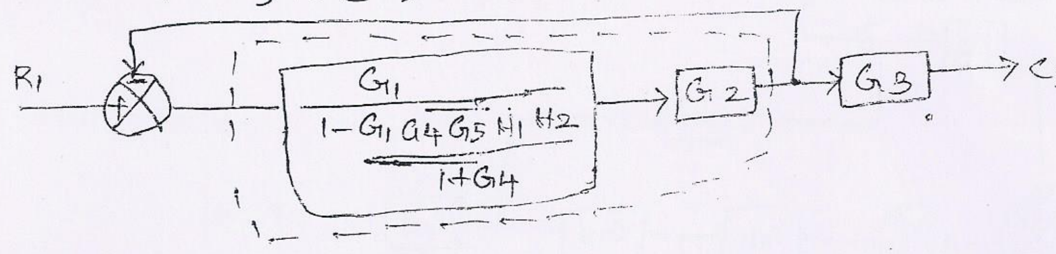
Step 2: Combining the blocks in cascade and splitting the summing point



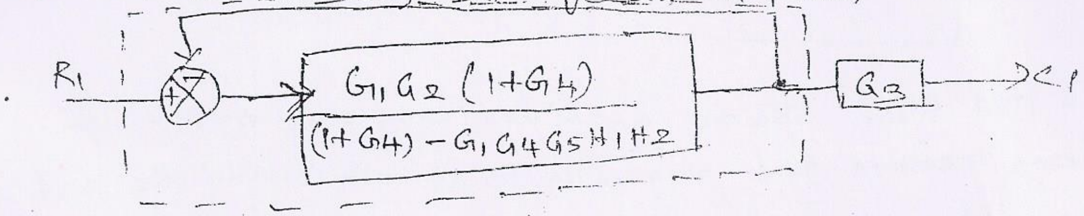
Step 3: Eliminating the feedback path

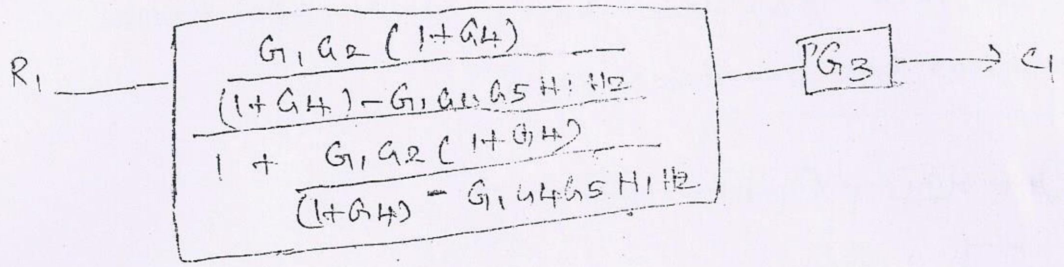


Step 4: Combining the blocks in cascade

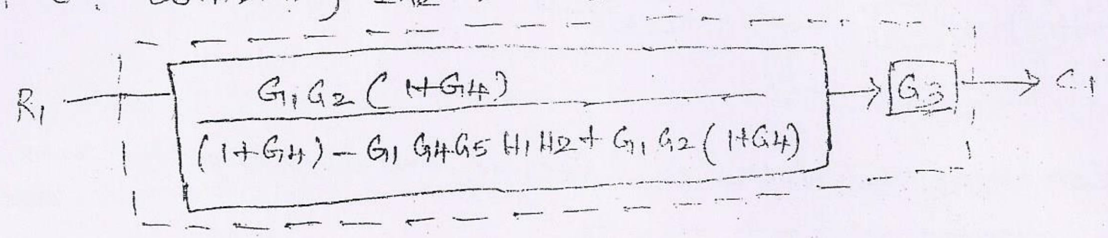


Step 5: Eliminating the feedback path





Step 6: Combining the blocks in cascade



$$\frac{C_1}{R_1} = \frac{G_1G_2G_3(1+G_4)}{(1+G_1G_2)(1+G_4) - G_1G_4G_5H_1H_2}$$

Case 2: To find C_2/R_1

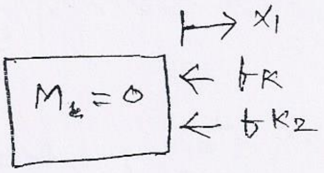
$R_2 = 0$ & consider only one output C_2. Hence we can remove the summing point which adds R_2 and need not consider G_3, since G_3 is on the open path.

$$\therefore M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B_2\frac{d}{dt}(x - x_1) = f(t) \quad \text{—①}$$

$$Ms^2X(s) + B_1 sX(s) + B_2 s[X(s) - X_1(s)] = F(s)$$

$$[Ms^2 + (B_1 + B_2)s]X(s) - B_2 s X_1(s) = F(s) \quad \text{—②}$$

$$f_{b2} = B_2\frac{d}{dt}(x_1 - x) \qquad f_K = Kx_1$$



$$f_{b2} + f_K = 0$$

$$B_2\frac{d}{dt}(x_1 - x) + Kx_1 = 0 \quad \text{—③}$$

$$B_2 s[X_1(s) - X(s)] + KX_1(s) = 0$$

$$(B_2 s + K)X_1(s) - B_2 s X(s) = 0 \quad \text{—④}$$

$$\therefore X_1(s) = \frac{B_2 s}{B_2 s + K} X(s) \quad \text{—⑤}$$

Sub $X_1(s)$ in the above equation no ②

$$[Ms^2 + (B_1 + B_2)s]X(s) - B_2 s\left[\frac{B_2 s}{B_2 s + K}\right]X(s) = F(s)$$

$$X(s)\left[\frac{[Ms^2 + (B_1 + B_2)s][B_2 s + K] - (B_2 s)^2}{B_2 s + K}\right] = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{B_2 s + K}{[Ms^2 + (B_1 + B_2)s][B_2 s + K] - (B_2 s)^2}$$

The differential equations governing the system are

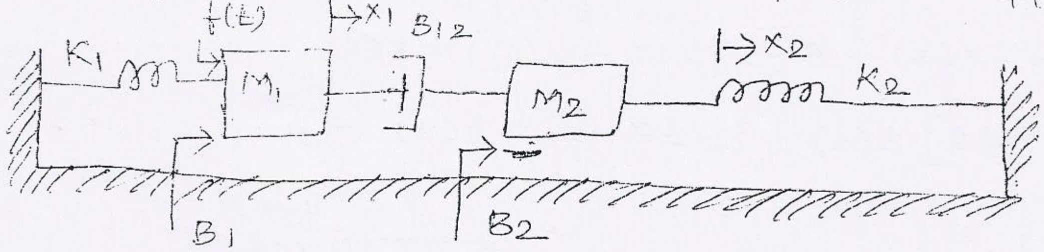
1. $M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B_2\frac{d}{dt}(x - x_1) = f(t)$
2. $B_2\frac{d}{dt}(x_1 - x) + Kx_1 = 0$

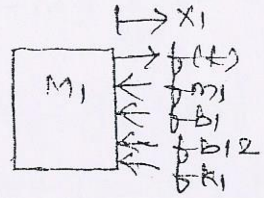
The equations of motion in s domain are

1. $[Ms^2 + (B_1 + B_2)s]X(s) - B_2 X_1(s) = F(s)$
2. $(B_2 s + K)X_1(s) - B_2 s X(s) = 0$

$$\frac{X(s)}{F(s)} = \frac{B_2 s + K}{[Ms^2 + (B_1 + B_2)s][B_2 s + K) - (B_2 s)^2}$$

Determine the transfer function $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$ for the system.





$$f_{m1} = M_1\frac{d^2x_1}{dt^2} \; ; \; f_{b1} = B_1\frac{dx_1}{dt}$$

$$f_{b12} = B_{12}\frac{d}{dt}(x_1-x_2) \quad f_{k1} = K_1x_1$$

$$f_{m1}+f_{b1}+f_{b12}+f_{k1} = f(t)$$

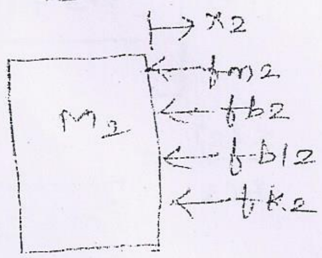
$$M_1\frac{d^2x_1}{dt^2}+B_1\frac{dx_1}{dt}+B_{12}\frac{d}{dt}(x_1-x_2)+K_1x_1 = f(t)$$

$$M_1s^2X_1(s)+B_1sX_1(s)+B_{12}s[X_1(s)-X_2(s)]+K_1X_1(s) = F(s)$$

$$X_1(s)[M_1s^2+(B_1+B_{12})s+K_1]-B_{12}sX_2(s) = F(s) \quad -(1)$$

$$f_{m2} = M_2\frac{d^2x_2}{dt^2} \quad f_{b2} = B_2\frac{dx_2}{dt} \quad f_{b12} = B_{12}\frac{d}{dt}(x_2-x_1)$$

$$f_{k2} = K_2x_2$$



$$f_{m2}+f_{b2}+f_{b12}+f_{k2} = 0$$

$$M_2\frac{d^2x_2}{dt^2}+B_2\frac{dx_2}{dt}+B_{12}\frac{d}{dt}(x_2-x_1)+K_2x_2 = 0$$

$$M_2s^2X_2(s)+B_2sX_2(s)+B_{12}s[X_2(s)-X_1(s)]+K_2X_2(s) = 0$$

$$X_2(s)[M_2s^2+(B_2+B_{12})s+K_2]-B_{12}sX_1(s) = 0$$

$$X_2(s)[M_2s^2+(B_2+B_{12})s+K_2] = B_{12}sX_1(s)$$

$$X_2(s) = \frac{B_{12}sX_1(s)}{[M_2s^2+(B_2+B_{12})s+K_2]} \quad -(2)$$

Sub $X_2(s)$ in eq (1)

$$X_1(s)[M_1s^2+(B_1+B_{12})s+K_1]-\frac{(B_{12}s)^2X_1(s)}{M_2s^2+(B_2+B_{12})s+K_2} = F(s)$$

$$X_1(s)\frac{\left[[M_1s^2+(B_1+B_{12})s+K_1][M_2s^2+(B_2+B_{12})s+K_2]-(B_{12}s)^2\right]}{M_2s^2+(B_2+B_{12})s+K_2} = F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + (B_2 + B_{12})s + K_2}{[M_1 s^2 + (B_1 + B_{12})s + K_1][M_2 s^2 + (B_2 + B_{12})s + K_2] - [B}$$

From (2) $X_1(s) = \frac{[M_2 s^2 + (B_2 + B_{12})s + K_2] X_2(s)}{B_{12} s}$

sub (3) in (1)

$$\frac{X_2(s)[M_2 s^2 + (B_2 + B_{12})s + K_2]}{B_{12}s}[M_1 s^2 + (B_1 + B_{12})s + K_1]$$

$$- B_{12}S\, X_2(s) =$$

$$\cancel{X_2(s)}\left[\frac{[M_2 s^2 + (B_2 + B_{12})s + K_2][M_1 s^2 + (B_1 + B_{12})s + K_1] - (B_{12}s}{B_{12}(s)}\right.$$

$$\frac{X_2(s)}{F(s)} = \frac{B_{12}\cancel{t}S}{[M_2 s^2 + (B_2 + B_{12})s + K_2][M_1 s^2 + (B_1 + B_{12})s + K_1] - \{B_{12}}$$