SECX1063 PRINCIPLES OF DIGITAL SIGNAL PROCESSING

UNIT I INTRODUCTION TO CONTINUOUS TIME SIGNALS & SYSTEMS

Representation, Characterization and classifications of CT signals - Representation of CT signals - Sampling theorem - Aliasing effect - Reconstruction of signals from sampled sequence - Concept of signal processing - Advantage of DSP over ASP - Classification of CT systems - Linear time Invariant - Causal - BIBO stable - Impulse response - Transfer function - frequency response of CT LTI systems.

UNIT II INTRODUCTION TO DISCRETE TIME SIGNALS & SYSTEMS

Representation, Characterization and Classifications of DT signals - Classification of DT systems - Linear time invariant - Causal - BIBO stable - Impulse response - Transfer function - System response - Frequency response - Transfer function - Frequency response of DT LTI systems - Realization of discrete recursive and non recursive systems - Direct Form I and Form II - Cascade and parallel realization.

UNIT III DFT AND FFT

Introduction to DFT - Properties - Discrete Fourier transforms - Linear and circular convolution - Need for FFT - Radix 2 FFT - Properties - Decimation in time FFT and Decimation in frequency FFT algorithms - Inverse DFT.

UNIT IV ANALYSIS & DESIGN OF DIGITAL FILTERS

Review of Butterworth and Chebyshev approximations - Properties of IIR and FIR filters - Design of IIR filter using Impulse invariant and Bilinear transformation method - Design of FIR filter using window method - Rectangular, Hanning and Hamming Windows.

UNIT V EFFECT OF FINITE REGISTER LENGTH

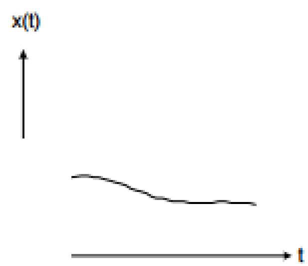
Effect of number representation in registers - ADC quantization noise - Coefficient Quantization Error - Product Quantization Error - Truncation - Limit cycles due to product round off error - Addition of over flow errors - Scaling - Dynamic range.

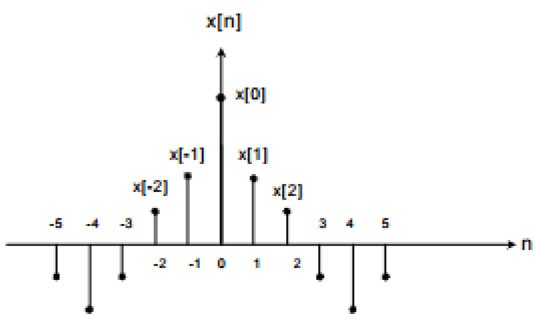
UNIT I INTRODUCTION TO CONTINUOUS TIME SIGNALS & SYSTEMS

Signal:
Any time varying physical phenomenon that can convey information is called signal. Some examples of signals are human voice, electrocardiogram, sign language, videos etc. There are several classifications of signals such as Continuous time signal, discrete time signal and digital
Ex: voice, television picture, telegraph.

Continuous Time signal – If the signal is defined over continuous-time, then the signal is a continuous-time signal.

Ex: Sinusoidal signal, Voice signal, Rectangular pulse function





ɔn of continuous time signal.

y take discrete values, such as t=kTs is called

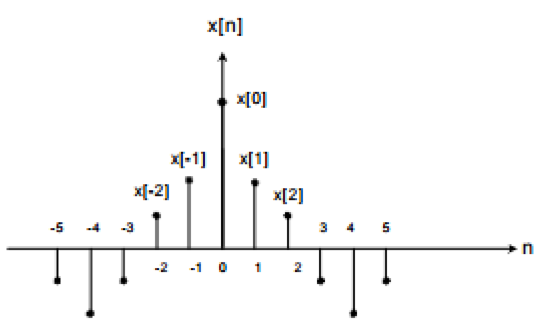


Fig:2 Graphical representation of Discrete time signal.

Digital Signal:

The signals that are discrete in time and quantized in amplitude are called digital signal. The term "digital signal" applies to the transmission of a sequence of values of a discrete-time signal in the form of some digits in the encoded form.

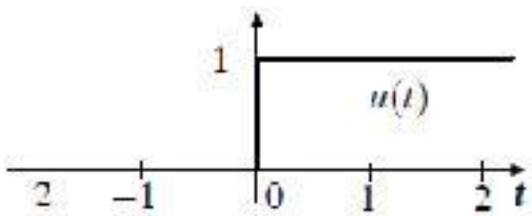
Elementary CT signals

Unit Step Signal:

The Unit Step Signal u(t) is defined as

$$u(t) = \begin{cases} 1 \ \text{ if } \ t \geq 0 \\ 0 \ \text{ if } \ t<0 \end{cases}$$

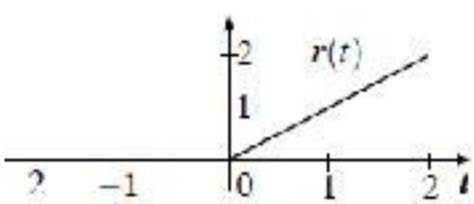
Graphically it is given by



Ramp Signal:

$$r(t)=\begin{cases} t \ \text{ if } \ t \geq 0 \\ 0 \ \text{ if } t <0 \end{cases}$$

Graphically it is given by



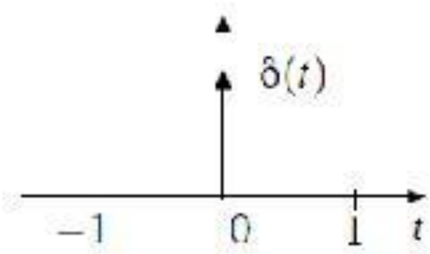
Pulse Signal:

A signal is having constant amplitude over a particular interval and for the remaining interval the amplitude is zero.

Impulse Signal:

$$\delta(t)=\begin{cases} 1 & \text{if } t=0 \\ 0 & \text{if } t \neq 0 \end{cases}$$

Impulse Signal CT representation

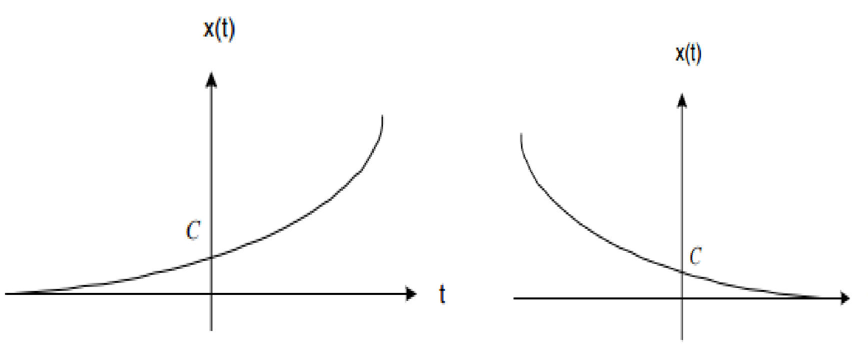


Exponential Signal:

Exponential signal is of two types. These two type of signals are real exponential signal and complex exponential signalwhich are given below.

Real Exponential Signal: A real exponential signal is defined as

$$\mathbf{x(t)=Ae^{at}}$$



When a > 0 ; When a < 0 .

Complex exponential Signal: The complex exponential signal is given by

$$\mathbf{x(t)=Ae^{st} \text{ where } s=\sigma+j\omega}$$

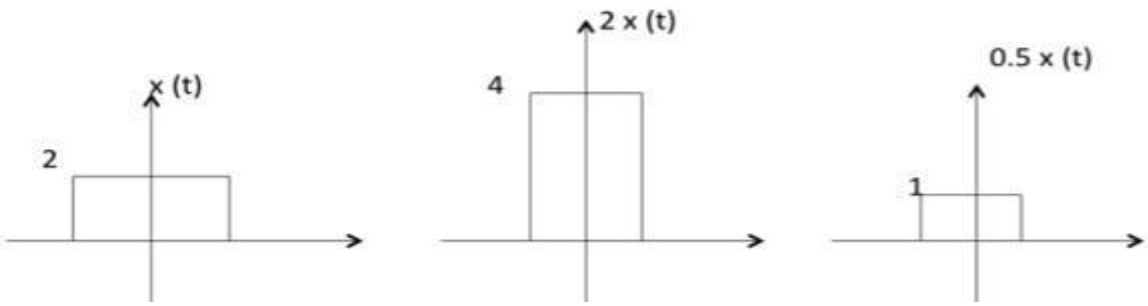
Basic Operations on signals:

Several basic operations by which new signals are formed from given signals are familiar from the algebra and calculus of functions.

1. **Amplitude Scaling :**

$$\mathbf{y(t)= a\ x(t)}$$

   where a is a real (or possibly complex) constant. ax(t) is a amplitude scaled version of x(t) whose amplitude is scaled by a factor a.



2. **Amplitude Shift**:

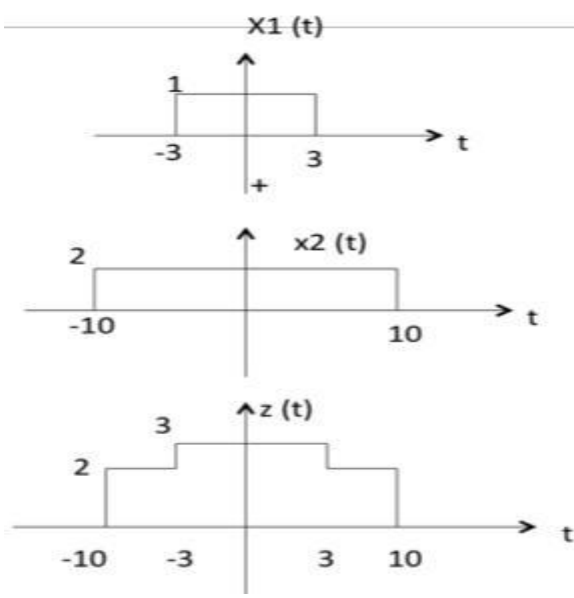
$$\mathbf{y(t)= x(t)+ b}$$

   where b is a real (or possibly complex) constant

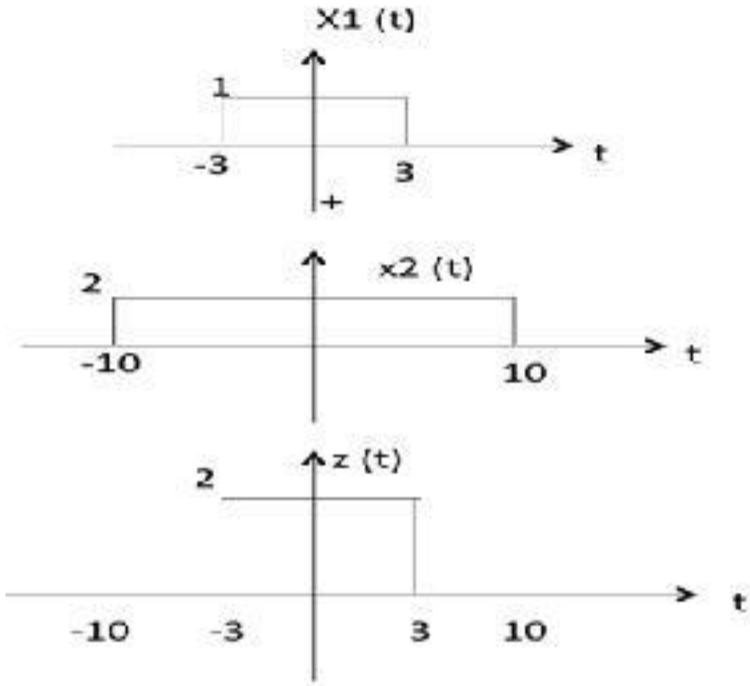
3. **Signal Addition: y(t)= X_1(t) + X_2(t)**

   As seen from the diagram above,

   $-10 < t < -3$ amplitude of z(t) = x1(t) + x2(t) = 0 + 2 = 2
   $-3 < t < 3$ amplitude of z(t) = x1(t) + x2(t) = 1 + 2 = 3
   $3 < t < 10$ amplitude of z(t) = x1(t) + x2(t) = 0 + 2 = 2



4. **Signal Multiplication: $y(t)= x_1(t).\ x_2(t)$**



As seen from the diagram above,

$-10 < t < -3$ amplitude of z (t) = x1(t) ×x2(t) = 0 ×2 = 0

$-3 < t < 3$ amplitude of z (t) = x1(t) ×x2(t) = 1 ×2 = 2

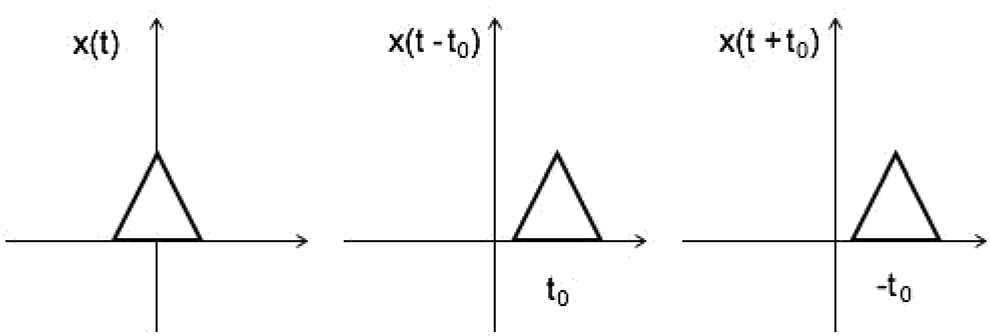
$3 < t < 10$ amplitude of z (t) = x1(t) × x2(t) = 0 × 2 = 0

5.Time Shift: If x(t) is a continuous function, the time-shifted signal is defined as $y(t)=x(t-t_0)$
If $t_0 > 0$, the signal is shifted to the right, and if $t_0 < 0$, the signal is shifted to the left.

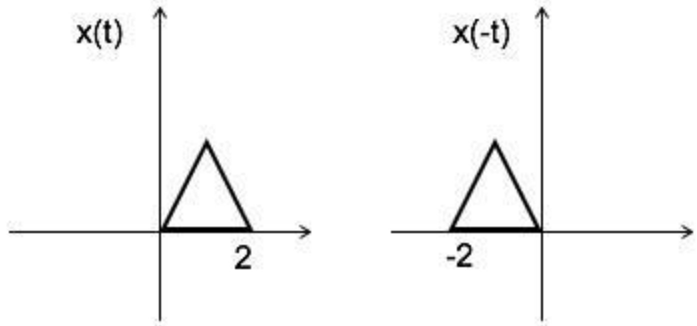
$X(t \pm t_0)$ is time shifted version of the signal x(t).

$x(t + t_0)$ →→ negative shift (Towards Left)

$x(t - t_0)$ →→ positive shift (Towards Right)

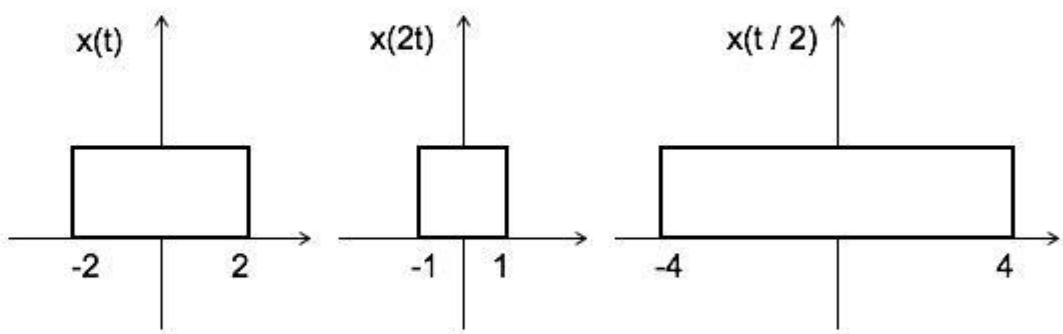


6. **Time Reversal:** If x(t) is a continuous function, the time-reversed signal is defined as **y(t) = x(-t).**
   x(-t) is the time reversal of the signal x(t).

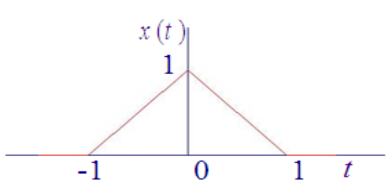


7. **Time Scaling:** If x(t) is a continuous function, a time-scale version of this signal is defined as y(t) = x(at). If a>1, the signal y(t) is a compressed version of x(t), i.e., the time interval is compressed. If 0<a< 1, the signal y(t) is a stretched version of x(t), i.e., the time interval is stretched by a . When operating on signals, the time-shifting operation must be performed first, and then the time-scaling operation is performed x(At) is time scaled version of the signal x(t). where A is always positive.
   |A| > 1 →→ Compression of the signal

   |A| < 1 →→ Expansion of the signal



1. A triangular pulse signal x(t) is depicted below.



Sketch each of the following signals:
(a) $x(3t)$
(b) $x(3t+2)$
(c) $x(-2t+1)$
(d) $x(0.5t+1)$

Classification CT Signals:

1. Even and Odd signal
2. Deterministic and Random Signal
3. Periodic and Aperiodic signal
4. Energy and Power signal
5. Causal and Non-Causal signals

Even and Odd Signal:

An even signal is any signal 'x' such that x (t) = x (-t).
Odd signal is a signal 'x' for which x (t) = -x (-t).

The even $x_e(t)$and odd $x_o(t)$ parts of a signal x(t) are given by

$$x_e(t)=\frac{1}{2}[x(t)+x(-t)]$$

$$x_o(t)=\frac{1}{2}[x(t)-x(-t)]$$

Proof:

$X(t)= x_e(t) + x_o(t)$ ………………………………..(1)

$X(-t)= x_e(-t) + x_o(-t)$

$X(-t)= x_e(t) - x_o(t)$ ……………………………(2) since $x_e(-t)=x_e(t)$, $x_o(-t)= - x_o(t)$

Eq 1 + Eq 2 $\longrightarrow$

$x(t) + x(-t) = 2\, x_e(t)$

$x_e(t)=\frac{1}{2}[x(t)+x(-t)]$ Thus proved

Eq 1 - Eq 2 $\longrightarrow$

$x(t) - x(-t) = 2\, x_o(t)$

$x_o(t)=\frac{1}{2}[x(t)-x(-t)]$ Thus proved

Deterministic Signal:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signals can be modeled exactly by a mathematical formula are known as deterministic signals.

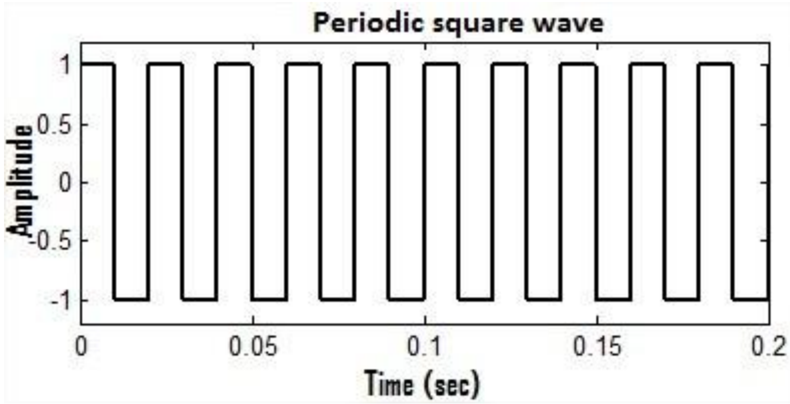


Fig. Deterministic signal

Random (or) Nondeterministic Signals:

Nondeterministic signals and events are either random or irregular. Random signals are also called non deterministic signals are those signals that take random values at any given time and must be characterized statistically.Random signals, on the other hand, cannot be described by a mathematical equation they are modeled in probabilistic terms.

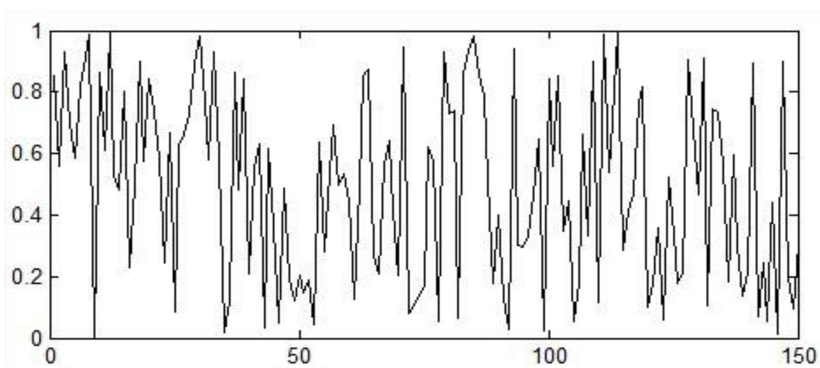


Fig.Random signal

Periodic signal:

A signal is said to be periodic if it repeats itself after some amount of time $x(t+T)=x(t)$, for some value of T. The period of the signal is the minimum value of time for which it exactly repeats itself.

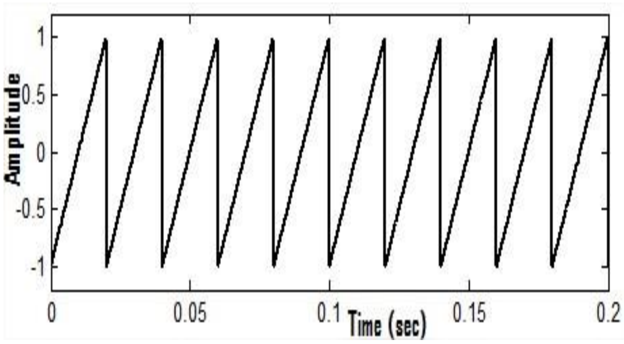


Fig. Periodic signal

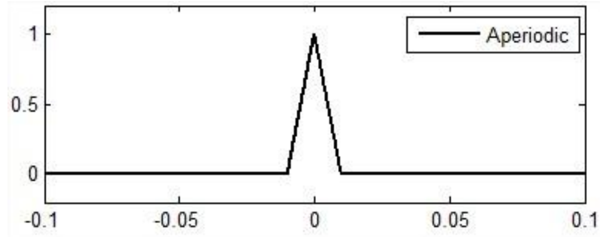


Fig.2Aperiodic signal

Signal which does not repeat itself after a certain period of time is called aperiodic signal. The periodic and aperiodic signals are shown in Figure

A CT signal x(t) is said to be periodic if it satisfies the following property: **x(t)=x(t+T)** at all time t, where T=Fundamental Time Interval (T=2π/ω)

Ex:

1. x(t)=sin(4πt). It is periodic with period of 1/2
2. x(t)=cos(3πt). It is periodic with period of 2/3

Energy Signal:

The Energy in the signal is defined as :

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \mathrm{dt}$$

Power Signal:

The Power in the signal is defined as

$$P = \lim_{T\to\infty} 1 \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \mathrm{dt}$$

If **0<E<**∞ then the signal x(t) is called as **Energy signal**. However there are signals where this condition is not satisfied. For such signals we consider the power. If **0<P<**∞ then the signal is called a **power signal.**Note that the **power for an energy** signal is **zero** (P=0) and that the **energy for a power signal is infinite**

Problem 1:

Determine if the following signal is Energy signal, Power signal, or neither, and evaluate E and P for the signal a(t)=3 sin(2πt) ,-∞<t<∞

$$E_a = \int_{-\infty}^{\infty} |a(t)|^2\, dt = \int_{-\infty}^{\infty} |3\sin(2\pi t)|^2\, dt$$

$$= 9\int_{-\infty}^{\infty} \frac{1}{2}[1-\cos(4\pi t)]dt$$

$$= 9\int_{-\infty}^{\infty} \frac{1}{2}dt - 9\int_{-\infty}^{\infty} \cos(4\pi t)dt$$

$$= \infty \quad \text{J}$$

$$P_a = \frac{1}{1}\int_{0}^{1} |a(t)|^2\, dt = \int_{0}^{1} |3\sin(2\pi t)|^2\, dt$$

$$= 9\int_{0}^{1} \frac{1}{2}[1-\cos(4\pi t)]dt$$

$$= 9\int_{0}^{0} \frac{1}{2}dt - 9\int_{0}^{1} \cos(4\pi t)dt$$

$$= \frac{9}{2} - \left[\frac{9}{4\pi}\sin(4\pi t)\right]_0^1$$

$$= \frac{9}{2} \quad \text{W}$$

So, the energy of that signal is infinite and its average power is finite (9/2). This means that it is a power signal as expected. It is a power signal.\

Causal, Non-causal and Anti-causal Signal:

Signal that are zero for all negative time, that type of signals are called causal signals, while the signals that are zero for all positive value of time are called anti-causal signal.

A non-causal signal is one that has non zero values in both positive and negative time. Causal, non-causal and anti-causal signals are shown below in the Figure 4(a), 4(b) and 4(c) respectively.

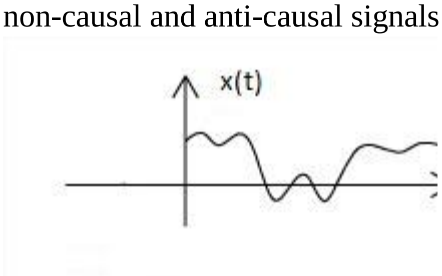


Fig. Causal signal

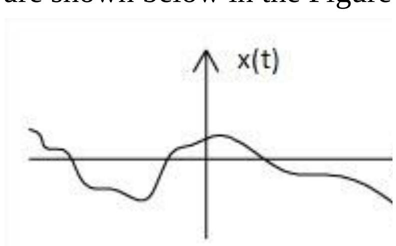


Fig.Non-caual signal

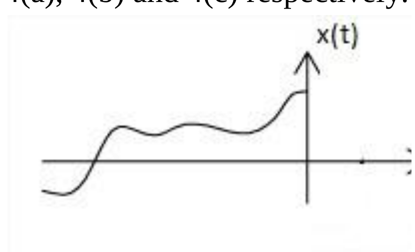


Fig.Anti-causal signal

Sampling Theorem and its Importance

Sampling Theorem: "A bandlimited signal can be reconstructed exactly if it is sampled at a rate atleast twice the maximum frequency component in it."

Figure 1 shows a signal g(t) that is bandlimited.

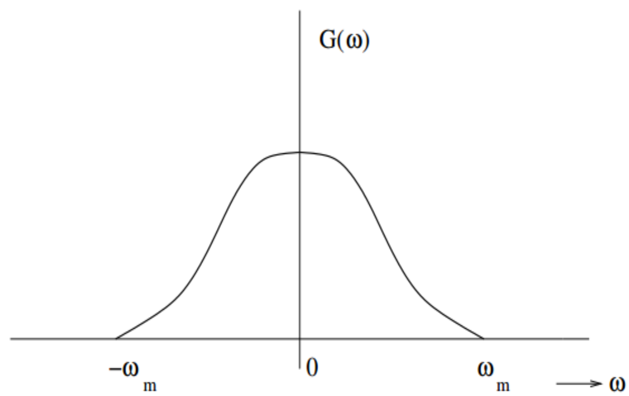
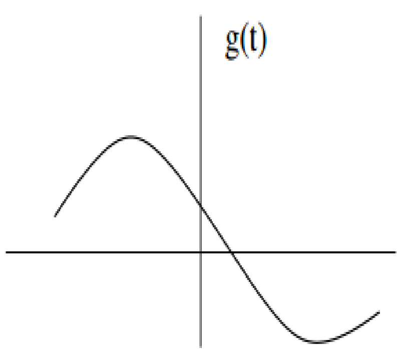


Figure 1: Spectrum of bandlimited signal g(t)

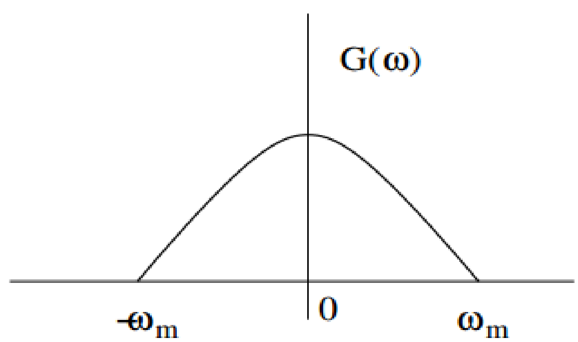
The maximum frequency component of g(t) is fm. To recover the signal g(t) exactly from its samples it has to be sampled at a rate fs $\geq$ 2fm.

The minimum required sampling rate fs = 2fm is called Nyquist rate.

Proof: Let g(t) be a band limited signal whose bandwidth is fm ($\omega_m = 2\pi$fm).

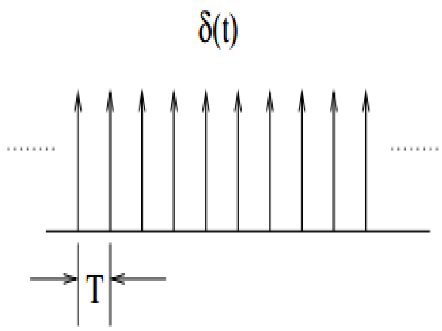


Original signal g(t)

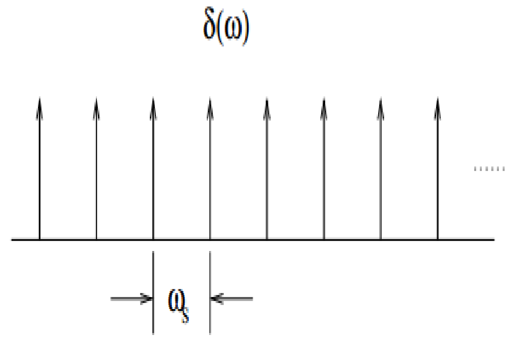


Spectrum G(ω)

$\delta_T(t)$ is the sampling signal with fs = 1/T > 2fm.



sampling signal δ_T (t)

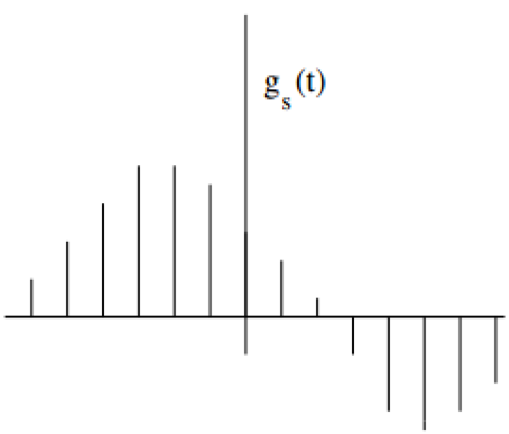


Spectrum δ_T (ω)

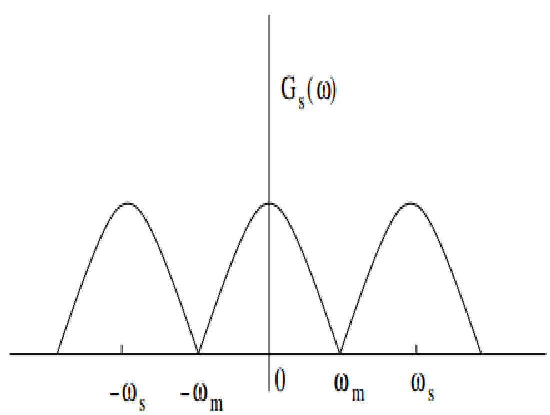
- Let $g_s(t)$ be the sampled signal. Its Fourier Transform $G_s(\omega)$ is given by

$$
\begin{aligned}
\mathcal{F}(g_s(t)) &= \mathcal{F}\left[g(t)\delta_T(t)\right] \\
&= \mathcal{F}\left[g(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT)\right] \\
&= \frac{1}{2\pi}\left[G(\omega)*\omega_0\sum_{n=-\infty}^{+\infty}\delta(\omega-n\omega_0)\right]
\end{aligned}
$$

$$
\begin{aligned}
G_s(\omega) &= \frac{1}{T}\sum_{n=-\infty}^{+\infty}G(\omega)*\delta(\omega-n\omega_0) \\
G_s(\omega) &= \mathcal{F}\left[g(t)+2g(t)\cos(\omega_0 t)+2g(t)\cos(2\omega_0 t)+\cdots\right] \\
G_s(\omega) &= \frac{1}{T}\sum_{n=-\infty}^{+\infty}G(\omega-n\omega_0)
\end{aligned}
$$



sampled signal $g_s(t)$



Spectrum $G_s(\omega)$

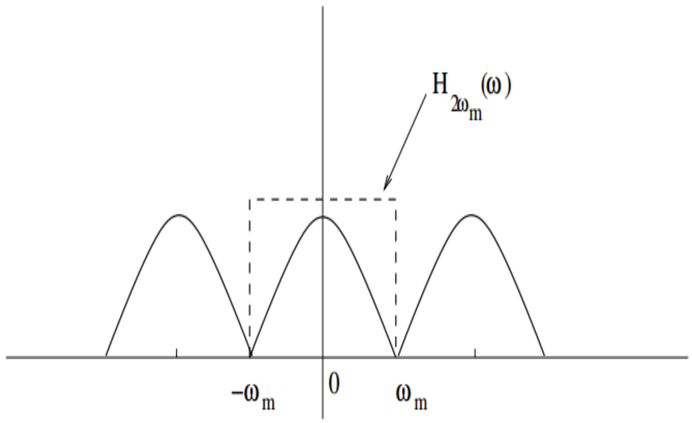
If $\omega_s = 2\omega m$, i.e., T = 1/2fm. Therefore, $G_s(\omega)$ is given by

$$G_s(\omega) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} G(\omega - n\omega_m)$$

To recover the original signal $G(\omega)$:

1. Filter with a Gate function, $H_2\omega_m(\omega)$ of width $2\omega m$.
2. Scale it by T.

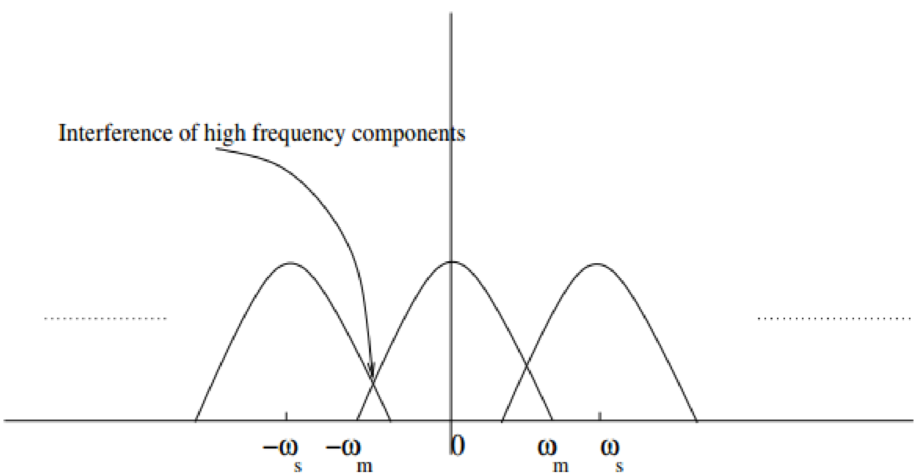
   $G(\omega) = TG_s(\omega)H_2\omega_m(\omega)$.



Recovery of signal by filtering with a filter of width 2ωm

Aliasing

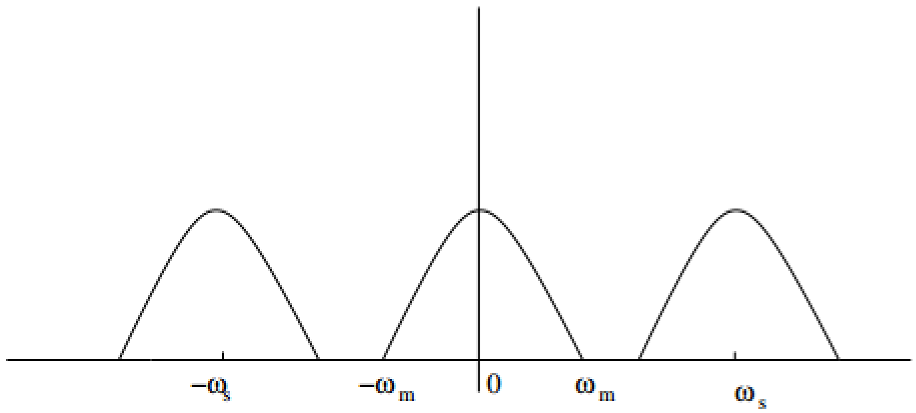
Aliasing is a phenomenon where the high frequency components of the sampled signal interfere with each other because of inadequate sampling $\omega_s < 2\omega_m$.



Aliasing due to inadequate sampling

Aliasing leads to distortion in recovered signal. This is the reason why sampling frequency should be atleast twice the bandwidth of the signal.

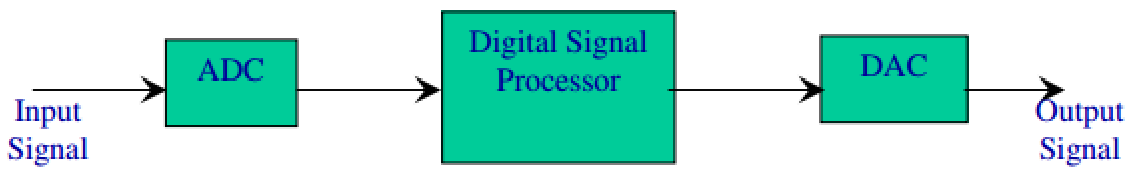
Oversampling – In practice signal are oversampled, where fs is significantly higher than Nyquist rate to avoid aliasing.



Oversampled signal-avoids aliasing

What is DSP?

DSP is a technique of performing the mathematical operations on the signals in digital domain. As real time signals are analog in nature we need first convert the analog signal to digital, then we have to process the signal in digital domain and again converting back to analog domain. Thus ADC is required at the input side whereas a DAC is required at the output end. A typical DSP system is as shown in figure



Need for DSP

Analog signal Processing has the following drawbacks:

- They are sensitive to environmental changes
- Aging
- Uncertain performance in production units
- Variation in performance of units
- Cost of the system will be high
- Scalability

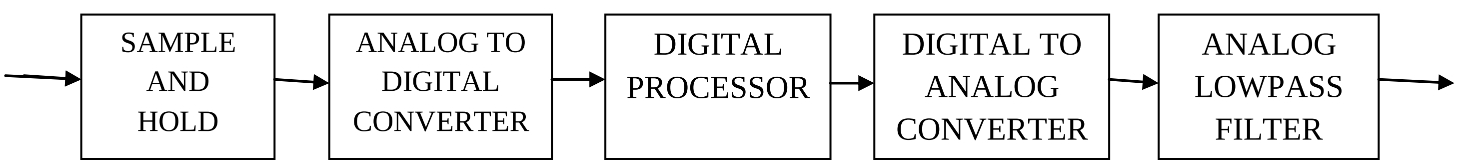
If Digital Signal Processing would have been used we can overcome the above shortcomings of ASP.

What is signal processing?

By processing we mean operating in some fashion on a signal to extract some useful information. For example when we hear same thing we use our ears and auditory path ways in the brain to extract the information. The signal is processed by a system. In the example mentioned above the system is biological in nature. We can use an electronic system to try to mimic this behavior. The signal processor may be an electronic system, a mechanical system or even it might be a computer program. The word digital in digital signal processing means that the processing is done either by a digital hardware or by a digital computer.

we can have two kinds of signals-continuous time and discrete time. And then we also emphasized the need for processing: why should one process a signal? This is to obtain a better quality signal than the original one. It could be for example, noise filtering or improving the quality of a picture in terms of brightness or contrast or whatever it is.

Even if the signal is analog, one prefers to use DSP. The reasons are many. DSP has many advantages over ASP or Analog Signal Processing. And many a times it happens to be a less costly proposition. Digital IC’s are available off the shelf at very low cost and therefore in many cases, analog signal processing gives way to Digital Signal Processing even if the original signal is analog.



How it is done is represented by this block diagram where the input is analog CT; then we have a sample and hold device which basically consists of a switch which samples the analog signal and holds the samples for some time so that it can be converted to a digital form, because conversion from analog to digital requires time. During that time the signal must be held. After the A to D, you get a coded signal, whose usual form is binary. This is then processed by a digital processor. What basically the DSP does is to combine signals by multiplication, addition and by recalling past signals, through delays. There are basically three operations: delay, multiplication, and addition. So this is what the digital processor does. It is a very simple device, it simply performs 3 operations. Multiplication is repeated addition, with only two signals at a time; subtraction is also a form of addition with one of these signals negated or the sign bit changed. So, there is no integration, there is no differentiation; there are no complicated operations which limits the usefulness of analog processing. The output of the digital processor is also digital, a sequence of numbers, and in order that output will be useful, you must convert it back to analog form. So you have a D to A or digital to analog converter. The output of the digital to analog converter is in the form of stair cases.

The signal changes from one level to the next and then stays. The abrupt change gives rise to high frequencies in the signal. These high frequencies present in the signal have to be gotten rid of by an analog low pass filter, and finally this analog output is the output that is to be used. It could be, for example, a piece of music which has to be filtered from surrounding noise and perhaps mixed with other signals, and finally it has to be played. It has to be heard on the loud speaker and therefore you do require an analog output. Pictorially this slide represents the basic operations.

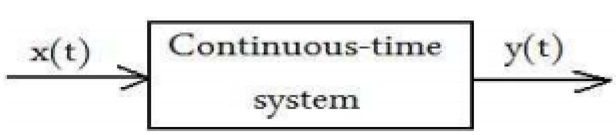
Advantages Of Digital Signal Processing Over Analog Signal Processing

Digital signal processing has following advantages:

1. Digital signal processing operations can be changed by changing the program in digital programmable system, *i.e.*, these is flexible systems.
2. Better control of accuracy in digital systems compared to analog systems.
3. Digital signals are easily stored on magnetic media such as magnetic tape without loss of quality of reproduction of signal.
4. Digital signals can be processed off line, *i.e.*, these are easily transported.
5. Sophisticated signal processing algorithms can be implemented by DSP method.
6. Digital circuits are less sensitive to tolerances of component values.
7. Digital systems are independent of temperature, ageing and other external parameters.
8. Digital circuits can be reproduced easily in large quantities at comparatively lower cost.
9. Cost of processing per signal in DSP is reduced by time-sharing of given processor among a number of signals.
10. Processor characteristics during processing, as in adaptive filters can be easily adjusted in digital implementation.
11. Digital system can be cascaded without any loading problems.

Continuous time systems

Physically, a system is an interconnection of components, devices, etc., such as a computer or an aircraft or a power plant. · Conceptually, a system can be viewed as a black box which takes in an input signal x(t) and as a result generates an output signal y(t).



$$y(t) = T\{x(t)\}$$

Classification of Continuous-Time Systems

Linear/Nonlinear

• A system is said to be linear if the following two properties hold:
1. If $x(t) \rightarrow y(t)$, then $\alpha x(t) \rightarrow \alpha y(t)$ for any real number α
2. If $x1(t) \rightarrow y1(t)$ and $x2(t) \rightarrow y2(t)$, then
$(x1(t)+x2(t)) \rightarrow (y1(t)+y2(t))$.

Memoryless/With Memory

• A system is said to be memoryless if the value of the output signal at any time depends only on the value of the input signal at that time, i.e., if x is the input and y the output, then y(t0) depends only on x(t0) for any t0.
• A system is said to have memory if it is not memoryless, i.e., a system is said to have memory if the value of the output signal at some time depends on the values of the input signal at other time instants also.

Causal/Noncausal

• A system is said to be causal if y(t0), i.e., the value of the output signal at time t0, does not depend on the values of the input signal at any time instants $t > t0$, i.e., the value of the output signal at any time does not depend on future values of the input signal.
• Equivalently, a system is said to be causal if the following property holds:
If $x1(t) \rightarrow y1(t)$ and $x2(t) \rightarrow y2(t)$, and if $x1(t) = x2(t)$ for all $t \leq t0$,
then $y1(t) = y2(t)$ for all $t \leq t0$.
• A system which is not causal is said to be noncausal

Time Invariant/Time Varying

• A system is said to be time invariant if the following property holds:
If $x(t) \rightarrow y(t)$, then $x(t - t0) \rightarrow y(t - t0)$ for any t0.
A system which is not time invariant is said to be time varying.
• Time invariance of a system essentially means that if the input signal is shifted, then the output signal is also shifted by the same amount.

If a system is both linear (L) and time-invariant (TI), then it is said to be LTI. We will mainly focus on LTI systems.

BIBO Stability

BIBO stands for bounded input, bounded output. BIBO stable is a condition such that any bounded input yields a bounded output. This is to say that as long as we input a stable signal, we are guaranteed to have a stable output.
Continuous-Time Condition for BIBO Stability is

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty$$

Definition of the Laplace Transform

The (bilateral) Laplace transform of the function *x*(*t*) is denoted as L{*x*(*t*)} or *X*(*s*) and is defined as

$$X(s) = \mathscr{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt.$$

The inverse Laplace transform is then given by

$$x(t) = \mathscr{L}^{-1}\{X(s)\} = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} \, ds.$$

where s = Re{*s*}. We refer to *x*(*t*) and *X*(*s*) as a Laplace transform pair and denote this relationship as

$$x(t) \overset{\mathscr{L}}{\longleftrightarrow} X(s).$$

Two different versions of the Laplace transform are commonly used. The first is the bilateral version, as introduced above. The second is the unilateral version. The unilateral Laplace transform is most frequently used to solve systems of linear differential equations with nonzero initial conditions. As it turns out, the only difference between the definitions of the bilateral and unilateral Laplace transforms is in the lower limit of integration. In the bilateral case, the lower limit is −¥, whereas in the unilateral case, the lower limit is 0.

Laplace Transform Examples

Example 1. Find the Laplace transform *X*(*s*) of the signal *x*(*t*) = *e*−*atu*(*t*).

Solution. Let *s* =s + *j*w where s and w are real. From the definition of the Laplace transform, we have

$$\begin{aligned} X(s) &= \mathscr{L}\{e^{-at} u(t)\} \\ &= \int_{-\infty}^{\infty} e^{-at} u(t) e^{-st} \, dt \\ &= \int_{0}^{\infty} e^{-(s+a)t} \, dt \\ &= \left[\left(-\tfrac{1}{s+a} \right) e^{-(s+a)t} \right] \Big|_0^{\infty}. \end{aligned}$$

At this point, we substitute *s* = s + *j*w in order to more easily determine when the above expression converges to a finite value. This yields

$$\begin{aligned} X(s) &= \left[\left(-\frac{1}{\sigma + a + j\omega} \right) e^{-(\sigma + a + j\omega)t} \right] \Big|_0^{\infty} \\ &= \left(\frac{-1}{\sigma + a + j\omega} \right) \left[e^{-(\sigma + a)t} e^{-j\omega t} \right] \Big|_0^{\infty} \\ &= \left(\frac{-1}{\sigma + a + j\omega} \right) \left[e^{-(\sigma + a)\infty} e^{-j\omega\infty} - 1 \right] \end{aligned}$$

$$
\begin{aligned}
X(s) &= \left(\tfrac{-1}{\sigma+a+j\omega}\right)[0-1] \\
&= \left(\tfrac{-1}{s+a}\right)(-1) \\
&= \frac{1}{s+a}.
\end{aligned}
$$

Example 2. Find the inverse Laplace transform of

$$X(s) = \frac{2}{s^2-s-2} \quad \text{for } -1 < \mathrm{Re}\{s\} < 2.$$

*Solution : R*ewriting $X(s)$ in the factored form

$$X(s) = \frac{2}{(s+1)(s-2)}.$$

We know that $X(s)$ has an expansion of the form

$$
\begin{aligned}
X(s) &= \frac{A_1}{s+1} + \frac{A_2}{s-2}. \\
A_1 &= (s+1)X(s)\big|_{s=-1} \\
&= \left.\frac{2}{s-2}\right|_{s=-1} \\
&= -\frac{2}{3} \quad \text{and} \\
A_2 &= (s-2)X(s)\big|_{s=2} \\
&= \left.\frac{2}{s+1}\right|_{s=2} \\
&= \frac{2}{3}.
\end{aligned}
$$

$$X(s) = \frac{2}{3}\left(\frac{1}{s-2}\right) - \frac{2}{3}\left(\frac{1}{s+1}\right).$$

Taking the inverse Laplace transform of both sides of this equation, we have

$$x(t) = \frac{2}{3}\mathscr{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{2}{3}\mathscr{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$-e^{2t}u(-t) \overset{\mathscr{L}}{\longleftrightarrow} \frac{1}{s-2}$$

$$e^{-t}u(t) \overset{\mathscr{L}}{\longleftrightarrow} \frac{1}{s+1}$$

$$
\begin{aligned}
x(t) &= \tfrac{2}{3}[-e^{2t}u(-t)] - \tfrac{2}{3}[e^{-t}u(t)] \\
&= -\tfrac{2}{3}e^{2t}u(-t) - \tfrac{2}{3}e^{-t}u(t).
\end{aligned}
$$