Unit V- FRICTION

INTRODUCTION

When a body moves or tends to move on another body, a force appears between the surfaces. This force is called force of friction and it acts opposite to the direction of motion. Its line of action is tangential to the contacting surfaces. The magnitude of this force depends on the roughness of surfaces. In engineering applications friction is desirable and undesirable. Friction is useful in power transmission by belts. It is useful in appliances like brakes, bolts, screw jack, etc. It is undesirable in bearing and moving machine parts where it results in loss of energy and, thereby, reduces efficiency of the machine.

TYPES OF FRICTION

There are two types of friction :

(a) Friction in un-lubricated surfaces or dry surfaces, and

(b) Friction in lubricate surfaces.

The friction that exists when one dry surface slides over another dry surface is known as dry friction and the friction. If between the two surfaces a thick layer of an oil or lubricant is introduced, a film of such lubrication is formed on both the surfaces. When a surface moves on the other, in effect, it is one layer of oil moving on the other and there is no direct contact between the surfaces. The friction is greatly reduced and is known as film friction.

LAWS OF DRY FRICTION

The laws of dry friction, are based on experimental evidences, and as such they are empirical in nature :

(a) The friction force is directly proportional to the normal reaction between the surfaces.

(b) The frictional force opposes the motion or its tendency to the motion.

(c) The frictional force depends upon the nature of the surfaces in contact.

(d) The frictional force is independent of the area and the shape of the contacting surfaces.

(e) For moderate speeds, frictional force is independent of the relative velocities of the bodies in contact.

STATIC AND KINETIC FRICTION

Suppose a block of weight W rests on a plane surface as shown in Figure 4.1. The surface offers a normal reaction R_N equal to the weight W. Suppose, now, a pull P_1 is applied to the block such that it actually does not move but instead tends to move. This will be opposed by frictional force F_1, equal to P_1. The resultant reaction will be R_1 inclined at an angle ϕ_1 with the normal reaction. If the pull is increased to P_2, the frictional force will increase to F_2 and the resultant reaction will increase to R_2 inclined at an angle of ϕ_2. Thus, with increase of the pull or attractive force, the frictional force; the resultant reaction and its inclination will increase.

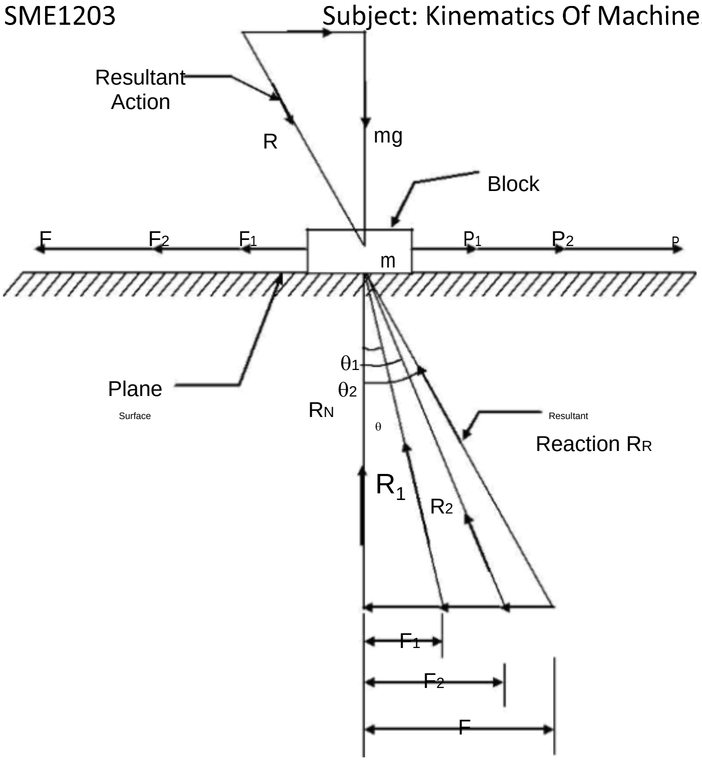


Figure 4.1 : Static and Kinetic Friction

Figure 4.1 shows a block of mass m resting on a plane surface with application of force P_1, P_2 and P_3 at which the body impends sliding, the self adjusting frictional forces will increase from F_1, to F_2 and finally F_3 when the body tends to move. Thus, in the limiting condition, the resultant active force will be R and reactive force R_R. The frictional resistance offered so long as the body does not move, is known as static friction. F_1 and F_2 are the static frictional forces. It may be noted that the direction of the resultant reaction R_R is such that it opposes the motion. The ultimate value of static friction (F) when the body just tends to move is called limiting friction or maximum static friction or friction of impending slide. The condition, when all the forces are just in equilibrium and the body has a tendency to move, is called limiting equilibrium position. When a body moves relative to another body, the resisting force between them is called kinetic or sliding friction. It has been experimentally found that the kinetic friction is less than the maximum static friction.

COEFFICIENT OF FRICTION

The ratio between the maximum static frictional force and the normal reaction R_N remains constant which is known as *coefficient of static friction denoted by Greek letter* μ.

$$\therefore \text{ Coefficient of friction } \frac{\text{Maximum static frictional force}}{\text{Normal reaction}}$$

or $\mu = \frac{F}{R_N}$ . . . (4.1)

The maximum angle ϕ which the resultant reaction R_N makes with the normal reaction R_N is known as *angle of friction*. It is denoted by ϕ.

From Figure 4.1 $\tan\phi = \frac{F}{R_N} = \mu$

⇒ $\phi = \tan^{-1}\mu$ . . . (4.2)

The coefficient of friction is different for different substances and even varies for different conditions of the same two surfaces.

Approximate values of static coefficient of friction for dry (un-lubricated) and greasy lubricated surfaces are given as below :

Materials	Dry	Greasy
Hard steel on hard steel	0.42	0.029-0.108
Mild steel on mild steel	0.57	0.09-0.19
Wooden wood	0.2-0.5	0.133
Mild steel on cast iron	0.24	0.09-0.116
Wood on metal	0.2-0.6	–
Glass on glass	0.4	–
Metal on stone	0.3-0.7	–
Metal on leather	0.3-0.6	–
Wood on leather	0.2-0.5	–
Earth on earth	0.1-1	–
Cast iron on cast iron	0.3-3.4	0.065-0.070
Rubber on concrete	0.65-0.85	–
Rubber on ice	0.05-0.2	–

ANGLE OF REPOSE

Consider a mass *m* resting on an inclined plane. If the angle of inclination is slowly increased, a stage will come when the block of mass *m* will tend to slide down. (Figure 4.2). This angle of the plane with horizontal plane is known as *angle of repose*. For satisfying the conditions of equilibrium all the forces are resolved parallel to the plane and perpendicular to it.

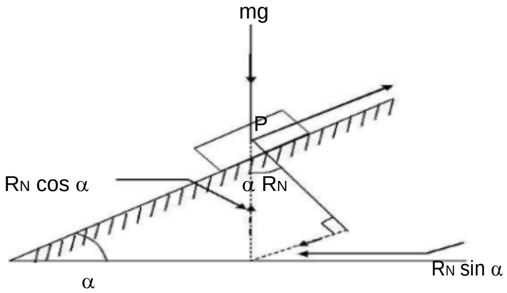


Figure 4.2 : Angle of Repose

$$P \quad R_N \sin\alpha \; mg \quad R_N \cos\alpha$$

$$\frac{P}{mg} \quad \tan\alpha$$

But $\frac{P}{mg} = \frac{F}{R_N} = \mu = \tan\phi$

$\Rightarrow$ $\quad \mathbf{tan}\ \alpha = \mathbf{tan}\ \phi$

Angle of repose α = angle of friction ϕ.

LEAST FORCE REQUIRED TO DRAG A BODY ON A ROUGH HORIZONTAL PLANE

Suppose a block, of mass *m*, is placed on a horizontal rough surface as shown in Figure 4.3 and a tractive force *P* is applied at an angle θ with the horizontal such that the block just tends to move.

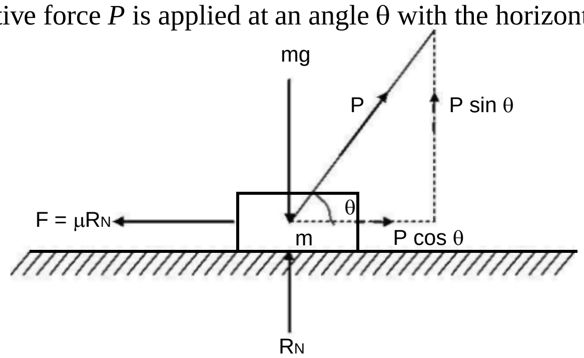


Figure 4.3

For satisfying the equilibrium conditions the forces are resolved vertically and horizontally.

For $\Sigma V = 0$

$$mg - P \sin\theta = R_N$$

or $\quad R_N \quad (mg - P \sin\theta)$ $\qquad$ . . . (4.3)

For $\Sigma H = 0$

$$P \cos\theta = F = \mu R_N = \mu\,(mg - P \sin\theta)$$

[substituting the value of R_N from Eq. (2.3)]

$$P \cos\theta = \frac{\sin\phi}{\cos\phi}\,(mg - P \sin\theta)$$

$$P \cos\theta \cos\phi = mg \sin\phi - P \sin\theta \sin\phi$$

$$P \cos\theta \cos\phi + P \sin\theta\ \phi = mg \sin\phi$$

$$P (\cos\theta \cos\phi + \sin\theta \sin\phi) \quad mg \sin\phi$$

$$P \cos(\theta - \phi) \quad mg \sin\phi$$

$$P \quad \frac{mg \sin\phi}{\cos(\theta - \phi)} \qquad \text{. . . (4.4)}$$

For P to be least, the denominator $\cos(\theta - \phi)$ must be maximum and it will be so if $\cos(\theta - \phi) \quad 1$ or $\theta - \phi =$ 0

$\Rightarrow$ $\theta = \phi$ for least value of P

$\Rightarrow$ $\boldsymbol{P_{least} = mg \sin \phi}$ . . . (4.5)

Hence, the force P will be the least if angle of its inclination with the horizontal : θ is equal to the angle of friction ϕ.

HORIZONTAL FORCE REQUIRED TO MOVE THE BODY

Up the Inclined Plane

The force P has been applied to move the body up the plane. Resolving all the forces parallel and perpendicular to the plane and writing equations, we get

$$P \cos \alpha = \mu R_N + mg \sin \alpha \quad P \sin \alpha = R_N - mg \cos \alpha$$

$$\therefore \quad R_N \quad P \sin \alpha + mg \cos \alpha$$

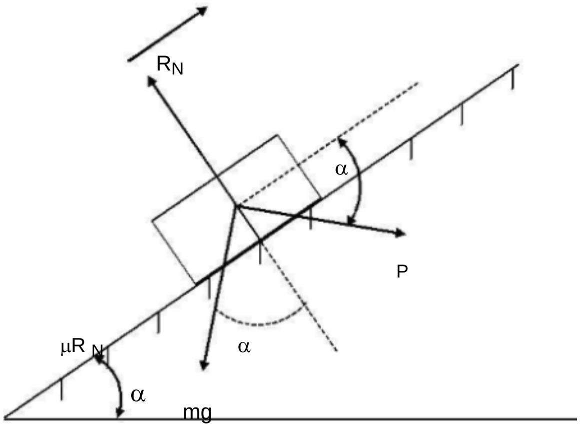


Figure 4.4

Substituting the value of R_N, we get

$$P \cos \alpha = \mu (P \sin \alpha + mg \cos \alpha) + mg \sin \alpha$$

Assuming $\mu = \tan \phi$

where ϕ is angle of friction.

$$P \cos \alpha = \tan \phi (P \sin \alpha + mg \cos \alpha) + mg \sin \alpha$$

or $P (\cos \alpha - \tan \phi \sin \alpha) \quad mg (\tan \phi \cos \alpha + \sin \alpha)$

$$\text{or} \quad P \quad mg \frac{\left(\frac{\sin \phi}{\cos \phi} \cos \alpha + \sin \alpha\right)}{\left(\cos \alpha - \frac{\sin \phi}{\cos \phi} \sin \alpha\right)}$$

$$mg \frac{(\sin \phi \cos \alpha + \sin \alpha \cos \phi)}{(\cos \alpha \cos \phi - \sin \phi \sin \alpha)}$$

$$mg \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} \quad mg \tan (\alpha + \phi) \qquad . . .(4.6)$$

Down the Plane

In this case body is moving down the plane due to the application of force P. Resolving the forces parallel and perpendicular to the plane and writing the equations, we get

$$P \cos \alpha = \mu R_N - mg \sin \alpha$$

$$P \sin \alpha = mg \cos \alpha - R_N$$

$$\therefore \quad R_N = mg \cos \alpha - P \sin \alpha$$

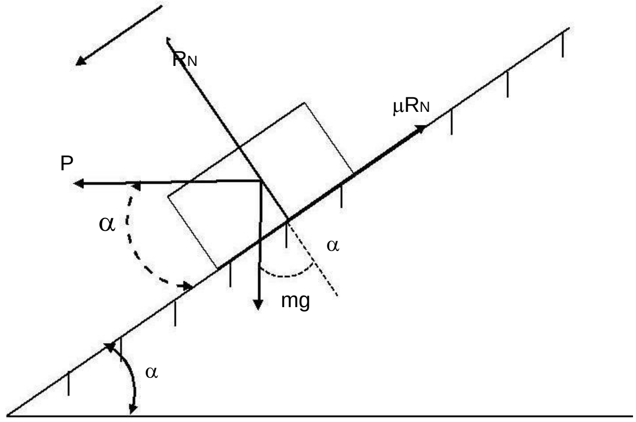


Figure 4.5

Substituting for R_N, we get

$$P \cos \alpha = \mu (mg \cos \alpha - P \sin \alpha) - mg \sin \alpha$$

Since $\mu = \tan \phi$

$$\therefore \quad P \cos \alpha = \tan \phi (mg \cos \alpha - P \sin \alpha) - mg \sin \alpha$$

or $\quad P (\cos \alpha + \tan \phi \sin \alpha) = mg (\tan \phi \cos \alpha - \sin \alpha)$

or $$P = mg \frac{\left(\frac{\sin \phi}{\cos \phi} \cos \alpha - \sin \alpha \right)}{\left(\cos \alpha + \frac{\sin \phi}{\cos \phi} \sin \alpha \right)}$$

$$= mg \frac{(\sin \phi \cos \alpha - \cos \phi \sin \alpha)}{(\cos \alpha \cos \phi + \sin \phi \sin \alpha)}$$

or $$P = mg \frac{\sin (\phi - \alpha)}{\cos (\phi - \alpha)} = mg \tan (\phi - \alpha) \qquad \ldots (4.7)$$

This means for force P to be applied $\alpha < \phi$.

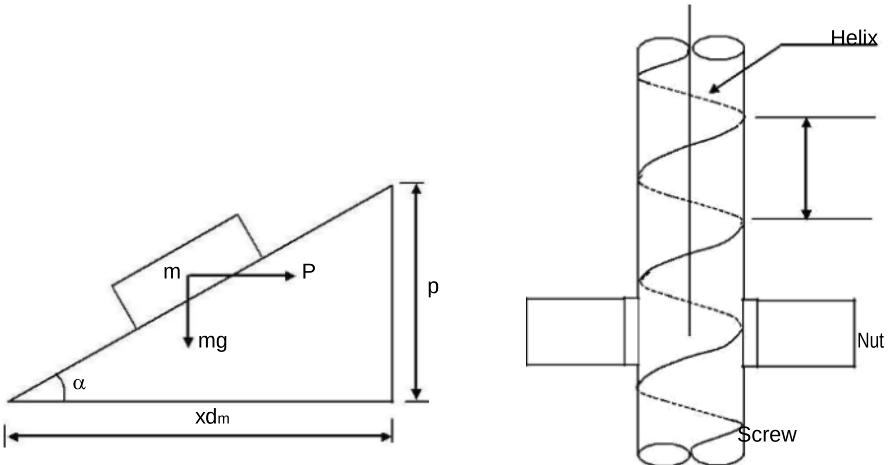
For $\phi < \alpha$ body will move without applying force P.

SCREW AND NUT FRICTION

Now consider screw and nut assembly. Both of them have threads in the form of helix. Screw has external thread and nut has internal thread. The pitch of threads (p) for both is same. When nut is rotated by one turn the screw traverses linear distance equal to the pitch (p). The nut at the same time traverses by one turn. These distances so traveled have been represented in Figure 4.6(a). Let mean diameter of the screw be '*dm*'.

Therefore, $\tan \alpha = \dfrac{p}{\pi \, dm}$

The theory of the inclined plane discussed in the preceding article can be applied for determining the effort to be applied.



(a) (b)

Figure 4.6 : Screw and Nut Friction

For Upward Motion

Effort $P_0 \quad mg \tan \alpha$ without friction [$\phi = 0$].

Effort $P \quad mg \tan (\alpha + \phi)$ with friction.

$$\therefore \text{Efficiency } e_{up} \quad \frac{\tan \alpha}{\tan (\alpha + \phi)}$$

For Downward Motion

$$P \quad mg \tan (\phi - \alpha)$$

SELF-LOCKING SCREWS

If $\alpha > \phi$, the mass placed on screw will start moving downward by its own weight and force P shall have to be applied to hold it. To guard against this undesirable effect, the screws angle α is always kept less than angle ϕ. Such a screw is known as *self-locking screw*.

CONDITION FOR MAXIMUM EFFICIENCY

Efficiency of screw : $e_{up} \quad \dfrac{\tan \alpha}{\tan (\alpha + \phi)}$

For determining condition of maximum efficiency,

$$\frac{d e}{d \alpha} \quad 0 \quad \frac{\sec^2 \alpha \tan (\alpha + \phi) - \sec^2 (\alpha + \phi) \tan \alpha}{\tan^2 (\alpha + \phi)} \quad 0$$

$$\sec^2 \alpha \tan (\alpha + \phi) \quad \sec^2 (\alpha + \phi) \tan$$

$$\alpha \sin (\alpha + \phi) \cos (\alpha + \phi) \quad \sin \alpha \cos \alpha$$

$$\sin 2 (\alpha + \phi) \quad \sin 2\alpha$$

$$2 (\alpha + \phi) \quad (\pi - 2\alpha)$$

or $\alpha = \dfrac{\pi}{4} - \dfrac{\phi}{2}$

Substituting the value of α in equation of efficiency.

$$e_{max} = \frac{\tan\left(45^o - \frac{\phi}{2}\right)}{\tan\left(45^o + \frac{\phi}{2}\right)} = \frac{\dfrac{1 - \tan\frac{\phi}{2}}{1 + \tan\frac{\phi}{2}}}{\dfrac{1 + \tan\frac{\phi}{2}}{1 - \tan\frac{\phi}{2}}} = \frac{\left(1 - \tan\frac{\phi}{2}\right)^2}{\left(1 + \tan\frac{\phi}{2}\right)^2} = \frac{\left(\cos\frac{\phi}{2} - \sin\frac{\phi}{2}\right)^2}{\left(\cos\frac{\phi}{2} + \sin\frac{\phi}{2}\right)^2}$$

$$= \frac{1 - 2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{1 + 2\sin\frac{\phi}{2}\cos\frac{\phi}{2}} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$\boldsymbol{e_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}}$$

Screw Jack With Square Threads

A screw jack with its spindle, having square threads, is shown in Figure 4.7. The theory discussed till now is directly applicable to this case.

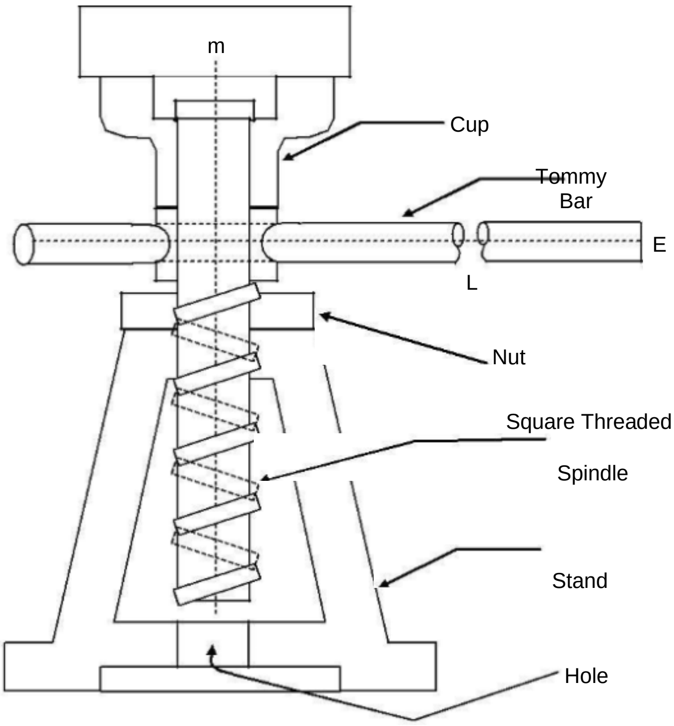


Figure 4.7 : Screw Jack

Let m = Mass on the jack,

P = Force applied at the screw tangentially in a horizontal plane,

P_e = Horizontal force applied tangentially at the end E of a tommy bar in a horizontal plane, and

L = Horizontal distance between central axis of the screw and the end E of the bar as shown.

In this screw jack nut is stationary and the screw is rotated with the help of tommy bar.

$$P_e \times L = P \times r$$

$$P_e = \frac{P \times r}{L}$$

$$P = mg\tan(\alpha + \phi) = \frac{mg(\tan\alpha + \tan\phi)}{1 - \tan\alpha\tan\phi}$$

But $\tan\alpha = \dfrac{p}{\pi d_m}$ and $\tan\phi = \mu$

Substituting for tan α and tan φ

$$\Rightarrow \quad P = \frac{mg\left(\dfrac{p}{\pi d_m} + \mu\right)}{1 - \dfrac{p}{\pi d_m} \times \mu} = \frac{mg\,(p + \mu\pi d_m)}{(\pi d_m - p\mu)}$$

Hence, $\boldsymbol{P = \dfrac{mg\,(p + \mu\pi d_m)}{(\pi d_m - p\mu)}}$

$$P_e = \frac{mg \times r}{L} \cdot \frac{(p + \mu\pi d_m)}{(\pi d_m - p\mu)}$$

Velocity ratio with tommy bar :

$$= \frac{\text{Distance covered by } P_e}{\text{Distance covered by load in one revolution}}$$

$$= \frac{2\pi L}{2\pi r_m \times \tan\alpha} = \frac{L}{r_m \tan\alpha}$$

$$\boldsymbol{V.R. = \frac{2\pi L}{p}}$$

With *V*-threads

The square threads, by their nature, take the axial load mg perpendicular to them where as in *V*-threads, the axial load does not act perpendicular to the surface of the threads as shown in Figure 4.8. The normal reaction R_N between the threads and the screw should be such that its axial component is equal and opposite to the axial load *mg*.

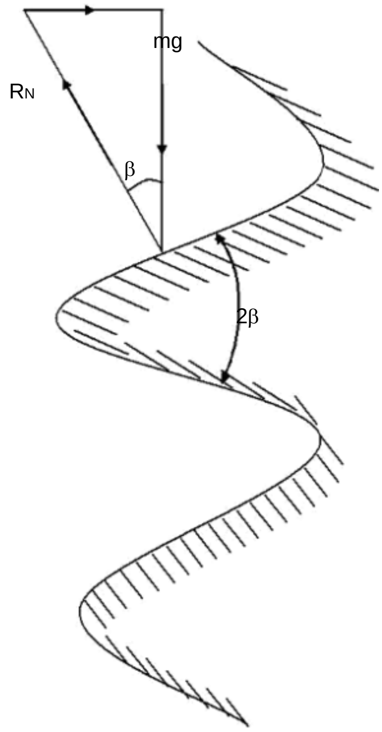


Figure 4.8 : V-threads

Let 2β be the angle included between the two sides of the thread.

$$R_N \cos \beta = mg$$

$$R_N = \frac{mg}{\cos \beta}$$

Frictional force which acts tangential to the surface of the threads = μR_N

$$\underline{\mu \times mg} = \mu_1 \times mg \cos \beta$$

where μ_1 may be regarded as virtual coefficient of friction :

$$\Rightarrow \qquad \mu_1 = \frac{\mu}{\cos\beta}$$

While treating *V*-threads for finding out effort *F* or '*e*', etc. μ may be substituting by μ_1 in all the relevant equations meant for the square threads.

It may be observed that force required to lift a given load with *V*-threads will be more than that with square threads.

Screw threads are also used for transmission of power such as in lathes (lead screw), milling machines, etc. The square threads will transmit power without any side thrust but is difficult to cut. The Acme threads, though not as efficient as the square threads are easier to cut.

Problem 1.

Outside diameter of a square threaded spindle of a screw jack is 40 mm. The screw pitch is 10 mm. If the coefficient of friction between the screw and the nut is 0.15, neglecting friction between the nut and the collar, determine :

(a) Force required to be applied at the end of tommy bar 1 m in length to raise a load of 20 kN.

(b) Efficiency of the screw.

Solution

Outside diameter of the screw : D = 40 mm

Inside diameter of the screw : d = 40 mm – 10 = 30 m Mean

diameter of the screw : d_m =(40+30)/2 = 35 mm 2

The force required for raising the load.

$$\frac{mg\,(\tan\alpha + \tan\phi)}{1 - \tan\alpha\tan\phi} \times \frac{d_m}{2L}$$

But $\tan\phi = \mu = 0.15$

$$\tan\alpha = \frac{p}{\pi d_m} \quad \frac{10 \times 7}{22 \times 35} \quad 0.091$$

Substituting the values,

$$P \quad 20 \times \frac{(0.091 + 0.15)}{1 - 0.091 \times 0.15} \; \frac{30}{2 \times 1000} \quad 73\text{ N}$$

We know that,

$$P_0 \quad mg\tan\alpha \; \frac{30}{2 \times 1000} \quad 20 \times 0.091 \times 15 \quad 27.3$$

(b) Efficiency e_{up} $\frac{P_0}{P}$ $\frac{27.3}{73}$ $\times$ 100 37.4%

PIVOT AND COLLAR FRICTION

The shafts of ships, steam and water turbines are subjected to *axial thrust*. In order to take up the axial thrust, they are provided with one or more bearing surfaces at right angle to the axis of the shaft. A bearing surface provided at the end of a shaft is known as a *pivot* and that provided at any place along with the length of the shaft with bearing surface of revolution is known as a *callar*. Pivots are of two forms : flat and conical. The bearing surface provided at the foot of a vertical shaft is called *foot step bearing*.

Due to the axial thrust which is conveyed to the bearings by the rotating shaft, rubbing takes place between the contacting surfaces. This produces friction as well as wearing of the bearing. Thus, power is lost in over-coming the friction, which is ultimately to be determined in this unit.

Obviously, the rate of wearing depends upon the intensity of thrust (pressure) and relative velocity of rotation. Since velocity is proportional to the radius, therefore,

$\therefore$ Rate of wear $\propto pr$.

Assumptions Taken

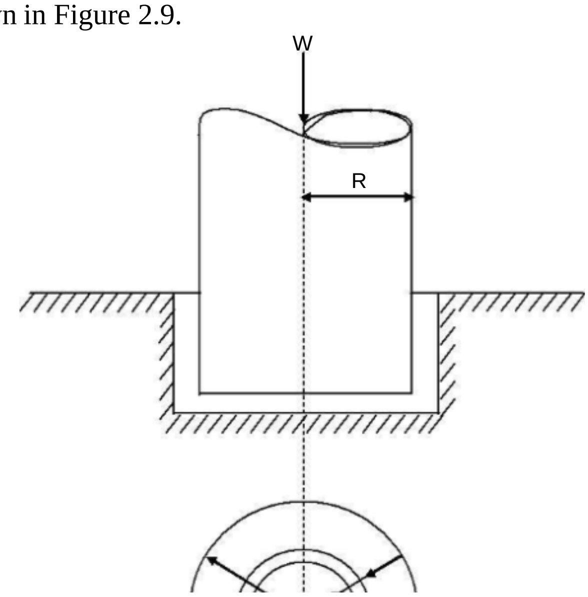
(a) Firstly, the intensity of pressure is uniform over the bearing surface. This assumption only holds good with newly fitted bearings where fit between the two contacting surfaces is assumed to be perfect. After the shaft has run for quite sometime the pressure distribution will not remain uniform due to varying wear at different radius.

(b) Secondly, the rate of wear is uniform. As given previously, the rate of wear is proportional to pr which means that the pressure will go on increasing radially inward and at the centre where r = 0, the pressure will be infinite which is not true in practical sense. However, this assumption of uniform wear gives better practical results when bearing has become older.

The various types of bearings mentioned above will be dealt with separately for each assumption.

2.13.1 Flat Pivot

A flat pivot is shown in Figure 2.9.



R

dr

Figure 4.9 : Flat Pivot

Let W = Axial thrust or load on the bearing, R =
External radius of the pivot,

p = Intensity of pressure, and

μ = Coefficient of friction between the contacting surfaces.

Consider an elementary ring of the bearing surfaces, at a radius r and of thickness dr as shown in Figure 4.9.

Axial load on the ring

$$dW \quad p \times 2\pi r \times dr$$

$$\text{Total load } W \quad \int_0^R p \times 2\pi r \times dr \qquad \ldots (4.8)$$

Frictional force on the ring

$$dF = \mu \times dW = \mu \times p \times 2\pi r \times dr$$

Frictional moment about the axis of

$$\text{rotation } dM \quad dF \times r$$

$$= \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M \quad \int_0^R \quad dM \quad \int_0^R \mu \times p \times 2\pi r^2 \times dr \qquad \ldots (4.9)$$

Uniform Pressure

If the intensity of pressure p is assumed to be uniform and hence constant. From Eq. (4.8)

$$W \quad p \times 2\pi \int_0^R r \times dr \quad p \times 2\pi \left[\frac{r^2}{2}\right]_0^R \quad p \times 2\pi \times \frac{R^2}{2}$$

$$\Rightarrow \qquad \boldsymbol{W \quad p \times \pi R^2}$$

And from Eq. (2.9)

$$M = \mu \times p \times 2\pi \int_0^R r^2 \, dr$$

$$\left[\frac{r^3}{}\right]^R \quad \frac{2}{} \mu \times p \times \pi \times R^3$$

$$3 \qquad 3$$
$$\lfloor \quad \rfloor_0$$

But $\quad p\,\pi R^2 \quad W$

$$\Rightarrow \qquad M = \frac{2}{3}\,\mu WR = \mu W \times \frac{2}{3}R$$

The friction force μW can be considered to be acting at a radius of $\frac{2}{3}R$.

Uniform Rate of Wear

By Eq. (4.8),

$$W \quad \int_0^R p \times 2\pi r \times dr$$

As the rate of wear is taken as constant and proportional to *pr* = *a* constant say *c*. Substituting for *pr* = *c* in the above equation.

$$W \quad \int_0^R 2\pi \times c \times d\,r \quad 2\pi R \times c$$

$$\Rightarrow \qquad c \quad \frac{W}{2\pi R}$$

By Eq. (2.9), total frictional moment

$$M \quad \int_0^R \mu \times p \times 2\pi r^2 \times dr \qquad \int_0^R \mu \times 2\pi \times c \times r \times dr$$

$$= \mu \times 2\pi \times c \times \frac{R^2}{2} = \mu \times 2\pi \times \frac{W}{2\pi R} \times \frac{R^2}{2}$$

$$\therefore \qquad M = \mu W \times \frac{R}{2}$$

Thus, the frictional force : μW acts at a distance $\frac{R}{2}$ from the axis.

2.13.2 Conical Pivot

A truncated conical pivot is shown in Figure 2.10(a).

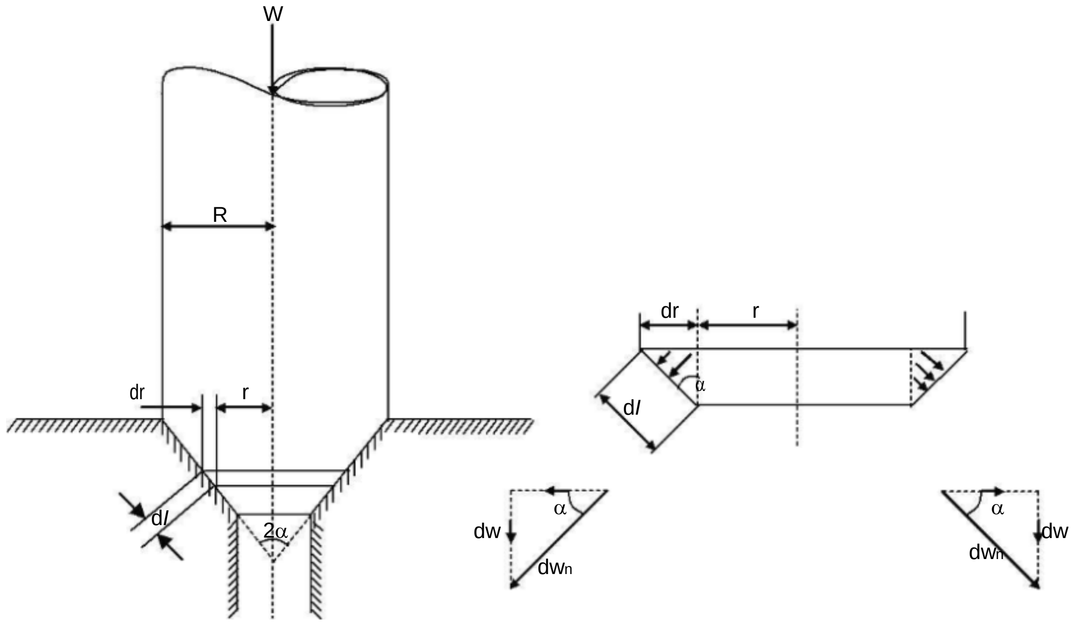
Let 2α = The cone angle,

W = The axial load/thrust,

R_1 = The outer radius of the cone,

R_2 = The inner radius of the cone, and

p = Intensity of pressure which will act normal to the inclined surface of the cone as shown in Figure 2.10(b).



(a) Conical Pivot **(b) Enlarged View of the Ring**

Figure 2.10 : Truncated Conical Pivot

Consider an elementary ring of the cone, of radius *r* thickness *dr* and of sloping length *dl* as shown. Enlarged view of the ring is shown in Figure 4.10(b).

Normal load on the ring

$$dW_n \quad p \times 2\,\pi r \times dl \quad p \times 2\pi r \times \frac{dr}{\sin\alpha}$$

Axial load on the ring

$$dW \quad dW_n \times \sin\alpha$$

$$p \times 2\pi r \times \frac{dr}{\sin\alpha} \times \sin\alpha = p \times 2\pi r \times dr$$

Total axial load on the bearing

$$W = \int_{R_2}^{R_1} p \times 2\pi r \times dr \qquad \ldots (4.12)$$

Frictional force on the elementary ring

$$dF = \mu\, dW_n = \mu \times p \times 2\pi r \times dl = \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha}$$

Moment of the frictional force about the axis of rotation

$$dM = \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin\alpha}$$

Total frictional moment

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin\alpha} \qquad \ldots (2.13)$$

Uniform Pressure

Uniform pressure, p is constant

From Eq. (2.12),

$$W = p \times 2\pi \int_{R_2}^{R_1} r\,dr = p \times 2\pi \left[\frac{r^2}{2}\right]_{R_2}^{R_1}$$

or $$W = p \times \frac{2\pi}{2}(R_1^2 - R_2^2) = p \times \pi (R_1^2 - R_2^2) \qquad \ldots (4.14)$$

And from Eq. (2.13),

$$\therefore \quad M = \frac{\mu \times p \times 2\pi}{\sin\alpha} \int_{R_2}^{R_1} r^2\,dr = \frac{\mu \times p \times 2\pi}{\sin\alpha} \times \frac{1}{3}(R_1^3 - R_2^3)$$

But from Eq. (2.14)

$$\boldsymbol{W = p \times \pi (R_1^2 - R_2^2)}$$

$$\therefore \quad p = \frac{W}{\pi (R_1^2 - R_2^2)}$$

Substituting for p above,

$$\Rightarrow \quad \boldsymbol{M = \frac{\mu W}{\sin\alpha} \times \frac{2(R_1^3 - R_2^3)}{3(R_1^2 - R_2^2)}}$$

Thus, frictional force $\frac{\mu W}{\sin\alpha}$ acts at a radius of $\frac{2}{3} \times \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$

For a full conical pivot shown in Figure 2.11, $R_2 = 0$

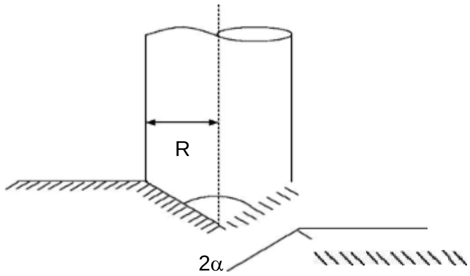


Figure 4.11 : Full Conical Pivot

$r = 0$

$$\Rightarrow \quad W = p \pi R^2$$

and

$$M = \frac{\mu W}{\sin \alpha} \times \frac{2}{3} R$$

In this case the frictional force $\left(\frac{\mu W}{\sin \alpha} \right)$ acts at a radius $\frac{2}{3}$ from the axis.

Uniform Wear

From Eq. (2.12),

$$W = \int_r^R p \times 2\pi r \times dr$$

Since the rate of wear is uniform.

Therefore, pr = constant c

Substituting for pr

$$W = \int_{R_2}^{R_1} 2\pi c \times dr = 2\pi c \int_{R_2}^{R_1} dr = 2\pi c \times (R_1 - R_2)$$

$$\Rightarrow \quad c = \frac{W}{2\pi (R_1 - R_2)} \qquad \text{. . . (4.15)}$$

From Eq. (4.13), frictional moment

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin\alpha}$$

$$= \int_{R_2}^{R_1} \mu \times 2\pi r \times c \times \frac{dr}{\sin\alpha}$$

$$= \frac{\mu \times 2\pi c}{\sin\alpha} \int_{R_2}^{R_1} r\,dr$$

$$= \frac{\mu \times 2\pi c}{\sin\alpha} \frac{(R_1^2 - R_2^2)}{2} = \frac{\mu \times \pi c}{\sin\alpha}$$

But $c = \dfrac{W}{2\pi(R_1 - R_2)}$

[since $pr = c$]

$$\Rightarrow \quad \boldsymbol{M = \frac{\mu W}{\sin\alpha} \times \frac{(R_1 + R_2)}{2}}$$

[Thus, the frictional force $\dfrac{\mu W}{\sin\alpha}$ acts at a radius $\dfrac{(R_1 + R_2)}{2}$]

For the full conical pivot (Figure 2.10), $R_2 = 0$

$$\boldsymbol{M = \frac{\mu W}{\sin\alpha} \times \frac{R_1}{2}}$$

Thus, the friction force $\dfrac{\mu W}{\sin\alpha}$ acts at a radius $\dfrac{R_1}{2}$.

Collar Bearing

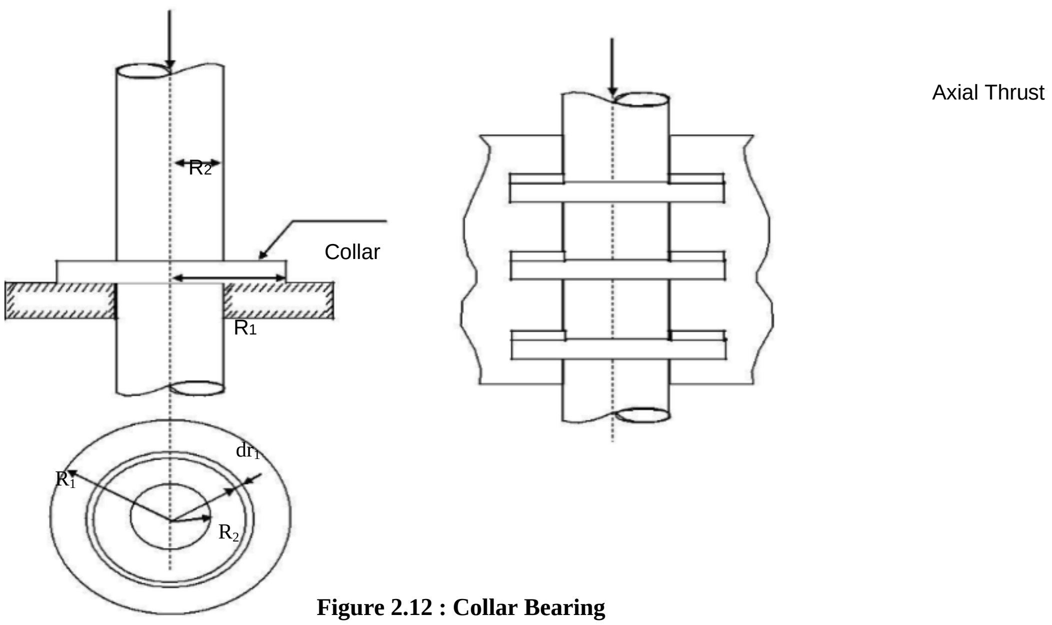


Figure 2.12 : Collar Bearing

A collar bearing which is provided on to a shaft, is shown in Figure 2.12. Let W = The axial load/thrust,

R_1 = External radius of the collar, and

R_2 = Internal radius of the collar.

Consider an elementary ring of the collar surface, of radius r and of thickness dr as shown in Figure4.12.

Axial load on the ring

$$dW = p \times 2\pi r \times dr$$

Total axial load

$$W = \int_{R_2}^{R_1} p \times 2\pi r \times dr \qquad \text{. . . (4.17)}$$

Frictional force on the ring

$$dF = \mu \times p \times 2\pi r \times dr$$

Frictional moment of the ring

$$dM = \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times dr \qquad \text{. . . (4.18)}$$

Uniform Pressure

Uniform pressure, p is constant From Eq. (4.17),

$$W = p \times 2\pi \int_{R_2}^{R_1} r \times dr = p \times 2\pi$$

$$\Rightarrow \quad \boldsymbol{W = p \times \pi (R_1^2 - R_2^2)}$$

$$\frac{(R_1^2 - R_2^2)}{3} \qquad \text{. . . (4.19)}$$

From Eq. (2.18),

$$M = \mu \times p \times 2\pi \int_{R_2}^{R_1} r^2 \times dr = \mu \times p \times 2\pi \frac{(R_1^3 - R_2^3)}{3}$$

Substituting for p from above

$$\Rightarrow \quad \boldsymbol{M = \mu W \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1 - R_2)}} \qquad \text{. . . (4.20)}$$

Uniform Wear

From Eq. (4.17),

Since the rate of wear is uniform

$\therefore$ pr = constant c Substituting for pr above,

$$W = \int_{r}^{R} c \times 2\pi dr = c \times 2\pi \int_{R_2}^{R_1} dr = c \times 2\pi \times (R_1 - R_2)$$

$$c = \frac{W}{2\pi(R_1 - R_2)} \qquad \text{. . . (4.21)}$$

From Eq. (2.18),

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times dr = \mu \times 2\pi \times c \int_{R_2}^{R_1} r \times dr$$

Substituting for c from Eq. (4.21),

$$\Rightarrow \quad \boldsymbol{M = \mu W \times \frac{(R_1 + R_2)}{2}} \qquad \text{. . . (4.22)}$$

There is a limit to the bearing pressure on a single collar and it is about 40 N/cm^2. Where the axial load is more and pressure on each collar is not to be allowed to exceed beyond the designed limit, then more collars are provided as shown in Figure 4.12.

$$\text{Number of collars : } n = \frac{\text{Total axial load}}{\text{Permissible axial load on each collar}}$$

It may be pointed out there is no change in the magnitude of frictional moments with more number of collars. The number of collars, as given above, only limit the maximum intensity of pressure in each collar.

Table4.1 : Pivot and Collars Summary of Formulae

Sl. No.	Particular	Frictional Moments : *M*	
		Uniform Pressure	**Uniform Wear**
1.	Flat pivot	$\mu W \times \frac{2}{3} R$	$\mu W \times \frac{R}{2}$
2.	Conical pivot (a) Truncated	$\frac{\mu W}{\sin \alpha} \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$	$\frac{\mu W}{\sin \alpha} \times \frac{(R_1 + R_2)}{2}$
	(b) Full conical	$\mu W \times \frac{2}{3} R$	$\mu W \times \frac{R}{2}$
3.	Collar	$\mu W \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$	$\mu W \times \frac{(R_1 + R_2)}{2}$

CLUTCH

It is a mechanical device which is widely used in automobiles for the purpose of engaging driving and the driven shaft, at the will of the driver or the operator. The driving shaft is the engine crankshaft and the driven shaft is the gear-box driving shaft. This means that the clutch is situated between the engine flywheel mounted on the crankshaft and the gear box.

In automobile, gears are required to be changed for obtaining different speeds. It is possible only if the driving shaft of the gear box is stopped for a while without stopping the engine. These two objects are achieved with the help of a clutch.

Broadly speaking, a clutch consists of two members; one fixed to the crankshaft or the flywheel of the engine and the other mounted on a splined shaft, of the gear box so that this could be engaged or disengaged as the case may be with the member fixed to the engine crankshaft.

TYPES OF CLUTCHES

Clutches can be classified into two types as follows :

(a) conical clutch, and

(b) the place or disc clutches can be of single plate or of multiple plates.

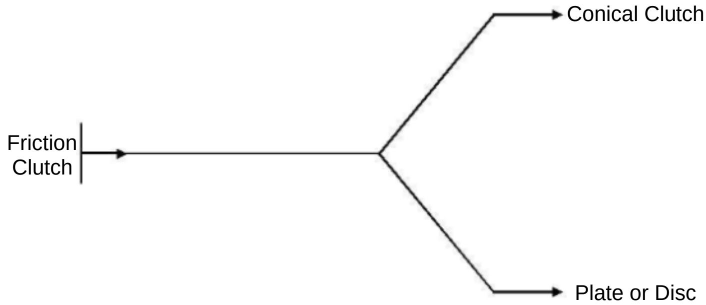


Figure 4.13 : The Single Cone Clutch

Conical Clutch

A conical clutch is shown in Figure 4.14. It consists of a cone *A* mounted on engine crankshaft. Cone *B* has internal splines in its boss which fit into the corresponding splines provided in the gear box shaft. Cone *B* could rotate the gear box shaft as well as may slide along with it. The outer surface of cone *B* is lined with friction material. In the normal or released position of the clutch pedal *P*, cone *B* fits into the inner conical surface of cone *A* and by means of the friction between the contacting surfaces, power is transmitted from crankshaft to the gear box shaft. When the clutch pedal is pressed, pivot *D* being the fulcrum provided in it, the collar *E* is pressed towards the right side, thus disengaging cone *B* from cone *A* and keeping the compression spring *S* compressed. On releasing the pedal, by the force of the spring, the cone *B* is thrusted back to engage cone *A* for power transmission.

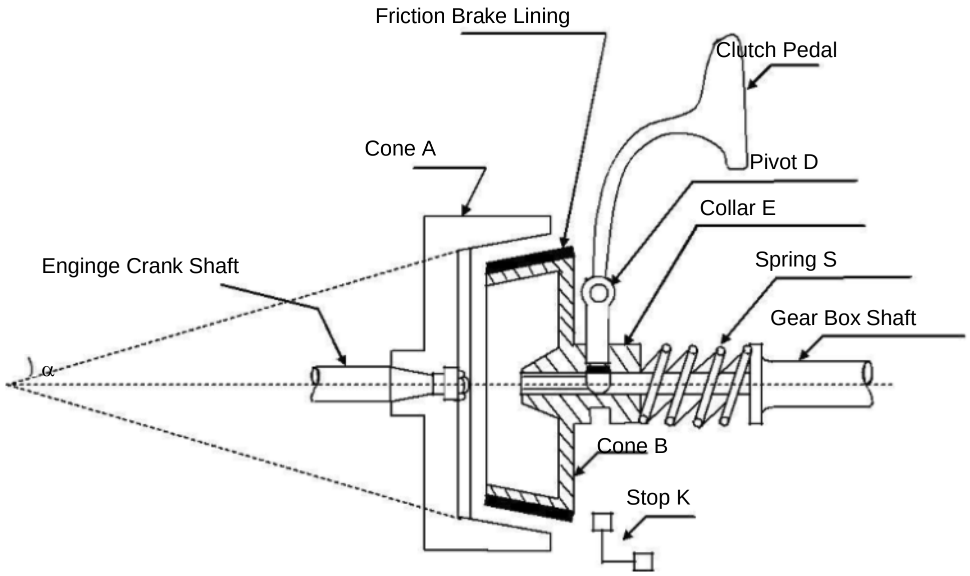


Figure 4.14 : Conical Clutch

For calculating frictional moments or torque transmitted on account of friction in clutches, unless otherwise specifically stated uniform rate of wear is assumed. For torque transmitted formulae of conical pivot can be used.

Single Plate Clutch

A single plate clutch is known as single disc clutch. It is shown in Figure 4.15. It has two sides which are driving and the driven side. The driving side comprises of the driving shaft or engine crankshaft A. A boss B is keyed to it to which flywheel C is bolted as shown. On the driven side, there is a driven shaft D. It carries a boss E which can freely slide axially along with the driven shaft through splines F. The clutch plate is mounted on the boss E. It is provided with rings of friction material – known as friction linings, on the both sides indicated H. One friction lining is pressed on the flywheel face and the other on the pressure plate I. A small spigot, bearing J, is provided in the end of the driving shaft for proper alignment.

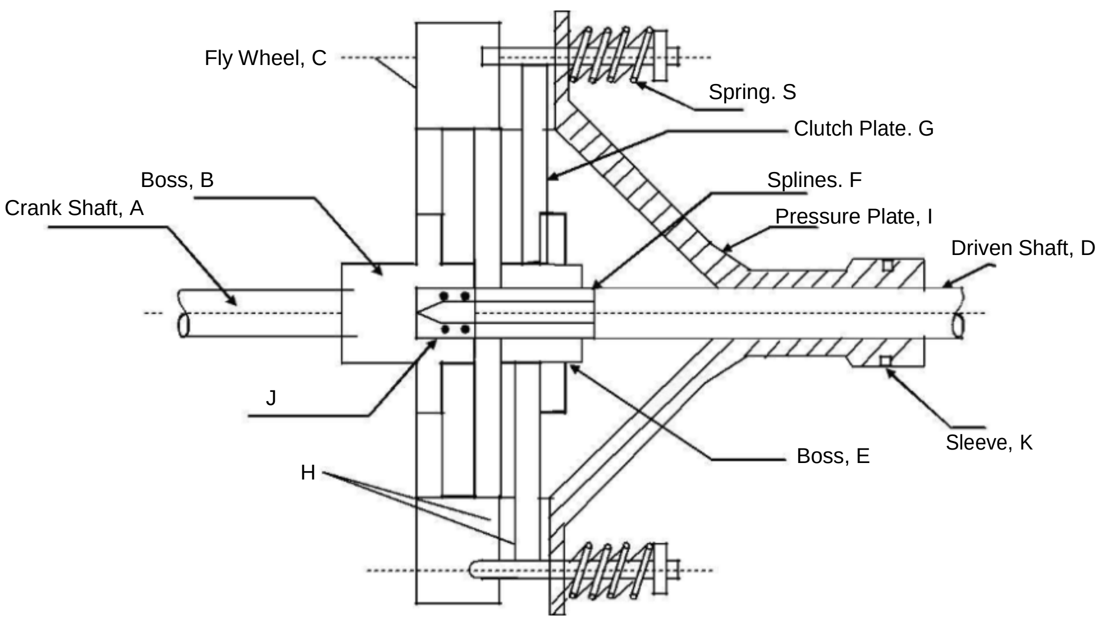


Figure 4.15 : Single Plate Clutch

The pressure plate provides axial thrust or pressure between clutch plate *G* and the flywheel *C* and the pressure plate *I* through the linings on its either side, by means of the springs, *S*.The pressure plate remains engaged and as such clutch remains in operational position Power from the driving shaft is transferred to the driven shaft from flywheel to the clutch plate through the friction lining between them. From pressure plate the power is transmitted to clutch plate through friction linings. Both sides of the clutch plate are effective. When the clutch is to be disengaged the sleeve *K* is moved towards right hand side by means of clutch pedal mechanism (it is not shown in the figure). By doing this, there is no pressure between the pressure plate, flywheel and the clutch plate and no power is transmitted. In medium size and heavy vehicles, like truck, single plate clutch is used.

Multi Plate Clutch

As already explained in a plate clutch, the torque is transmitted by friction between one or more pairs of co-axial annular faces kept in contact by an axial thrust provided by springs. In a single plate clutch, both sides of the plate are effective so that it has two pairs of surfaces in contact or $n = 2$.

Obviously, in a single plate clutch limited amount of torque can be transmitted. When large amount of torque is to be transmitted, more pair of contact surfaces are needed and it is precisely what is obtained by a multi-plate clutch.

JOURNAL BEARING

The portion of a shaft, which revolves in the bearing and subjected to load at right angle to the axis of the shaft, is known as journal as clearly indicated in Figure 4.16. The whole unit consisting of the journal and its supporting part (or bearing) is known as journal bearing.

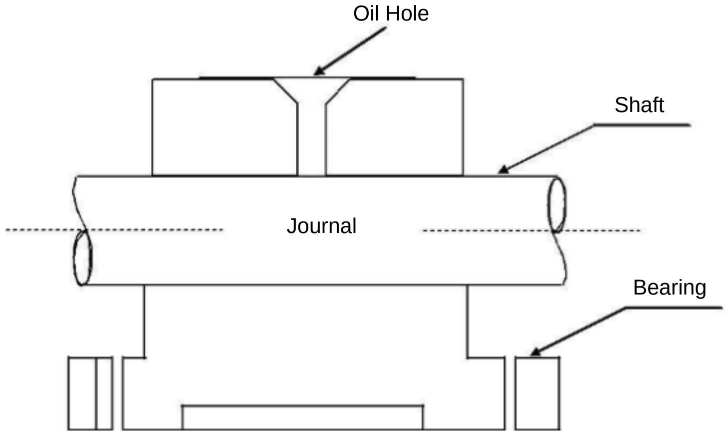


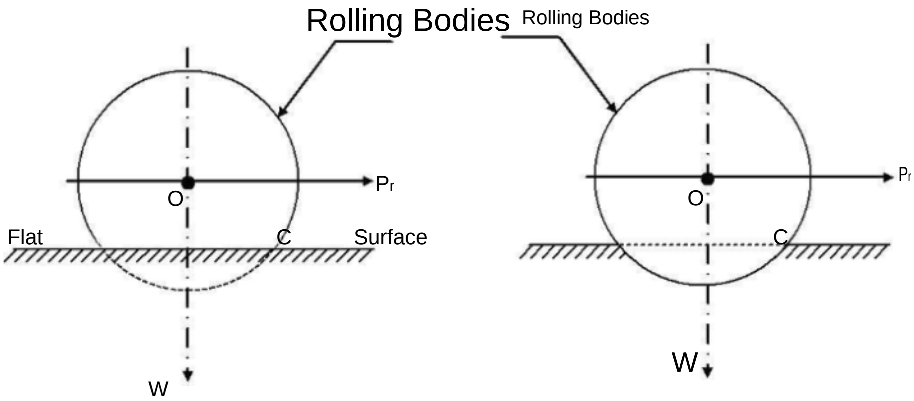
Figure 4.16 : Journal Bearing

ROLLING FRICTION

The frictional resistance arises only when there is relative motion between the two connecting surfaces. When there is no relative motion between the connecting surfaces or stated plainly when one surface does not slide over the other question of occurrence of frictional resistance or frictional force does not arise.

When a wheel rolls over a flat surface, there is a line contact between the two surfaces, parallel to the central axis of the cylinder. On the other hand when a spherical body rolls over a flat surface, there is a point contact between the two. In both the above mentioned cases there is no relative motion of slip between the line or point of contact on the flat surface because of the rolling motion. If while rolling of a wheel or that of a spherical body on the flat surface there is no deformation of depression of either of the two under the load, it is said to be a case pure rolling.

In practice it is not possible to have pure rolling and it can only be approached. How-so-ever hard the material be, either the rolling body will be deformed as happens in case of a car or cycle tyre, indicated in Figure 4.17(a) or the flat surface gets depressed or deformed. When the road roller passes over unsettled road or kacha road as shown in Figure 4.17(b). The surface is depressed.



(a) Rolling Body Deformed **(b) Flat Surface Deformed**

Figure 4.17 : Rolling Friction

At times when a load or a heavy machine or its part is to be shifted from one place to another place, for a short distance, and no suitable mechanical lifting device is available, the same is placed on a few rollers in the form of short pieces of circular bars or pipes as shown in Figure 4.18 and comparatively with a less force the load is moved. The rollers, roll and the one which becomes free at the rear side is again placed in the front of the load and so on. Because of the reduced friction, it requires less force.

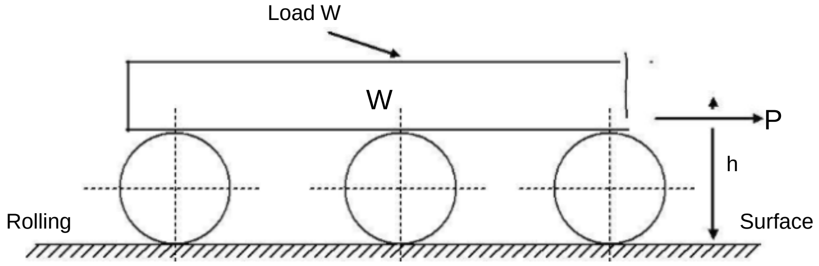


Figure 4.18 : Ball and Roller Bearing

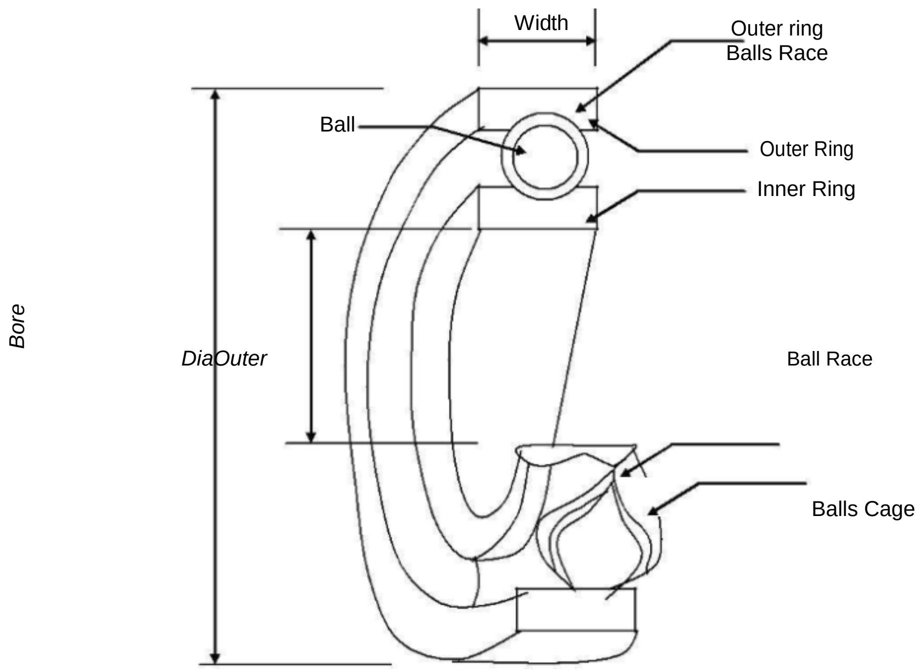
When a shaft revolves in a bush bearing, there is sliding motion between the journal and the bearing surface, resulting in loss of power due to friction. If between the journal and the bearing surface, balls of rollers are provided, instead of sliding motion, rolling motion will take place. To reduce the coefficient of rolling friction the balls or rollers are made of chromium steel or chrome-nickel steel and they are further heat treated with a view to make them more hard. They are finally ground and polished with high precision.

BALL AND ROLLER BEARINGS

Ball Bearing

A ball bearing is shown in Figure 4.19. As may be seen, it consists of mainly the following parts :

(a) Inner ring,
(b) Outer ring,
(c) Anti-friction element in the form of balls placed between the two races, and
(d) A cage which separates the balls from one another.



Roller Bearing

A roller bearing is shown in Figure 4.20. There is no difference between ball bearing and the roller bearing except that in roller bearing rollers, instead of balls are used. In roller bearings, there is a line contact instead of point contact as in ball bearings. The roller bearings are widely used for more load carrying capacity than that of ball bearings.

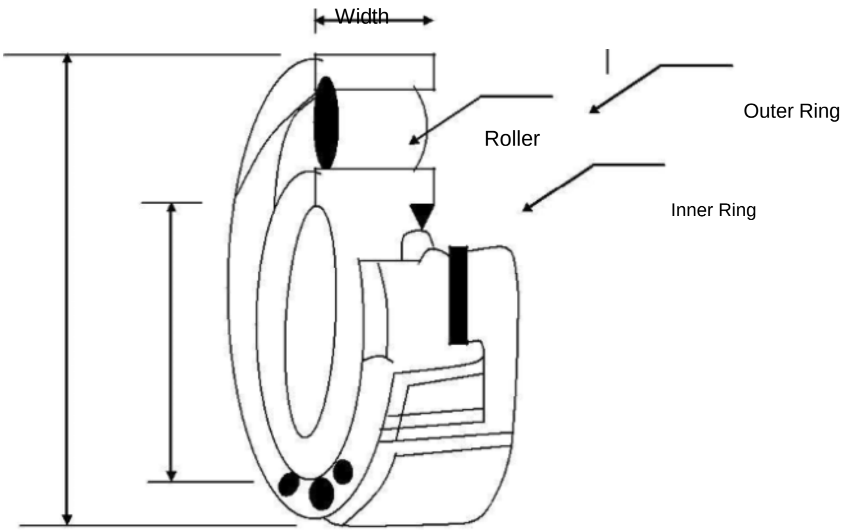


Figure 4.20 : Roller Bearing

The rollers may be cylindrical, straight or tapered. When tapered rollers are used, the bearings are called tapered roller bearings and those in which needles are used are called needle bearing.

Tapered Roller Bearings

Parts of a tapered roller bearing as well as its assembly are shown in Figure 2.21. The rollers are in the form of frustrum of a cone. The contact angle is between 12^{o} to 16^{o} for thrusts of moderate magnitude and 28^{o} to 30^{o} for heavy thrust.

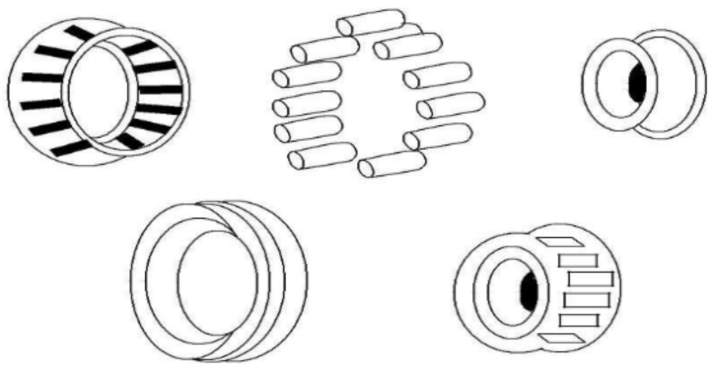


Figure 4.21 : Tapered Roller Bearing Parts

Needle Bearings

The rollers of small diameter are known as *needless*. It is : 2 to 4 mm. Ratio between length and diameter of needle is 3-10 : 1. Needle bearings are not provided with cage

A double square-thread power screw (Fig. 6.11) has the major diameter $d = 40$ mm and the pitch $p = 6$ mm. The coefficient of friction of the thread is $\mu = 0.08$ and the coefficient of collar friction is $\mu_c = 0.1$. The mean collar diameter is $d_c = 45$ mm. The external load on the screw is $F = 8$ kN. Find: a) the lead, the pitch (mean) diameter and the minor diameter; b) the moment required to raise the load; c) the moment required to lower the load; d) the efficiency of the device.

Solution

a) From Fig. 6.7:

the minor diameter is $d_r = d - p = 40 - 6 = 34$ mm,

the pitch (mean) diameter is $d_m = d - p/2 = 40 - 3 = 37$ mm,

the lead is $l = 2p = 2(6) = 12$ mm.

b) The moment required to raise the load is [Eqs. (6.11) and (6.18]

$$M_r = \frac{F d_m}{2}\left(\frac{l + \pi \mu d_m}{\pi d_m - \mu l}\right) + \frac{F \mu_c d_c}{2}$$

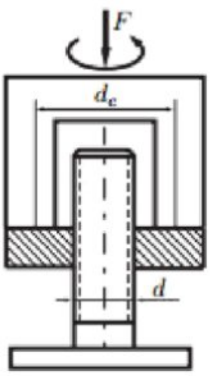


Fig. 6.11 Example 6.1.1

$$= \frac{8(10^3)(37)(10^{-3})}{2}\left[\frac{12 + 0.08(37)\pi}{37\pi - 0.08(12)}\right] + \frac{8(10^3)(0.1)(45)(10^{-3})}{2}$$

$= 45.344$ N m.

c) The moment required to lower the load is [Eqs. (6.12) and (6.18)]:

$$M_l = \frac{F d_m}{2}\left(\frac{\pi \mu d_m - l}{\pi d_m + \mu l}\right) + \frac{F \mu_c d_c}{2}$$

$$= \frac{8(10^3)(37)(10^{-3})}{2}\left[\frac{0.08(37)\pi - 12}{37\pi + 0.08(12)}\right] + \frac{8(10^3)(0.1)(45)(10^{-3})}{2}$$

$= 14.589$ N m.

The screw is not self-locking: $\pi \mu d_m - l = 0.08(37)\pi - 12 = -2.700 < 0$.

d) The overall efficiency is [Eq. (6.15)]:

$$e = \frac{F l}{2\pi M_r} = \frac{8(10^3)(12)(10^{-3})}{2(45.344)\pi} = 0.336.$$

A number of trucks are to be hauled by a rope passed round a hydraulic capstan; assuming a coefficient of friction of 0.25 between the rope and the capstan and a constant manual pull of 30 lb., derive an expression for the maximum pull on the trucks in terms of complete turns of rope round the capstan. What will be the horse power **exerted by the capstan under these conditions when there are** $2\frac{1}{2}$ **turns of rope and it is being wound off at 100 ft/min.?**

From equation (3)

$$\frac{T_1}{T_2} = e^{\mu\,\theta}$$

From the question:

$T_2 = 30\ lb.$, $\mu = 0.25$ and $\theta = 2\,\pi\,n\ radians$

From which:

$$T_1 = 30\ e^{0.25 \times 2\,\pi\,n} = 30(4.82)^n$$

When $n = 2.5$, $T_1 = 1520\ lb.$

Using equation (7) the horse-power of the capstan is given by:

$$\frac{(T_1 - T_2) \times v}{550} = \frac{(1520 - 30) \times 100}{550 \times 60} = 4.52\ h.p.$$

To which must be added the power supplied manually which is:

$$\frac{30 \times 100}{550 \times 60} = 0.091\ h.p.$$

So the horse-power taken by the trucks $= 4.52 + 0.091 = 4.611\ h.p.$

An elastic belt makes contact with a pair of flat pulleys over arcs of 180^0**, the coefficient of friction being 0.45. The initial tension is 90 lb. and it may be assumed that the sum of the tensions on the two sides remains constant.Find the** difference in the tensions **at which the belt slips on the pulleys.**

If the belt is 1 inch wide and $\frac{1}{4}$ **thick and the elastic modulus for the material is 35,000 lb./sq.in.estimate the** effect **on the maximum tension difference which would be caused if the pulley centre moved together by** $\frac{1}{8}$ **in., if they were initially 3 ft. apart.**

From equation (3) the ratio of the tensions at slip are given by: $\frac{T_1}{T_2} = e^{0.45\,\pi} = 4.12$

But $T_1 + T_2 =$ twice the original tension = 180 lb.

$T_2 = \frac{180}{5.12} = 35.2\ lb.$ and $T_1 = 180 - 35.2 = 144.8\ lb.$

The difference in the tensions $= 180 - 35.2 = 144.8\ lb.$

If the pulleys are now move together by $\frac{1}{8}$in.the decrease in stress = the strain X the modulus.

The decrease in Stress $= \frac{0.125}{3 \times 12} \times 35,000$

The decrease in initial tension = stress X area

$$= \frac{0.125}{3 \times 12} \times 35,000 \times 1 \times 0.25 = 30.4\ lb.$$

Thus the new initial tension is 90 - 30.4 = 59.6 lb.

But the running tensions (and hence the maximum tension differences) are proportional to the initial tension. (See paragraph on "Power Transmitted") and hence the maximum tension difference is reduced in the same ratio as the initial tension,

i.e. The new tension difference $= 109.6 \times \frac{59.6}{90} = 72.6\ lb.$

An open belt connects two flat pulleys. The smaller pulley is 1 ft. in diameter and runs at 200 r.p.m. The angle of lap on this pulley is 160^0 and the coefficient of friction between belt and pulley face is 0.25. The belt is on the point of slipping when $3\frac{1}{2}$ h.p. is being transmitted.
Which of the following alternatives would be more effective in increasing the power which could be transmitted?

- **Increase the initial tension in the belt by 10%**
- **Increase the coefficient of friction by 10% by the application of a suitable dressing onto the belt.**

From equations (7) The horse-power transmitted $= \dfrac{(T_1 - T_2)v}{550}$

Combining this with equation (3) The horse-power transmitted

$$= \frac{T_1(1 - e^{-\mu\theta})v}{550} \qquad (1)$$

Assuming that: $T_1 + T_2 = 2T_0$

And using equation (3) again, $T_1 = \left(\dfrac{2}{1 + e^{-\mu\theta}}\right) T_0$

Substituting the above into equation (1)

The horse-power transmitted $= 2T_0 \left(\dfrac{1 - e^{-\mu\theta}}{1 + e^{-\mu\theta}}\right) \times \dfrac{v}{550}$

from the above it can be seen that the power transmitted will increase by the same proportion as T_0 and as a result an increase in the initial tension of 10% will increase the power transmitted by 10%.

The effect of increasing the coefficient of friction is to alter the value of $e^{-\mu\theta}$

The original value of $e^{-\mu\theta} = \dfrac{1}{e^{0.25 \times 160 \times \frac{\pi}{180}}} = 0.497$

The modified value of $e^{-\mu\theta} = \dfrac{1}{e^{0.275 \times 160 \times \frac{\pi}{180}}} = 0.464$

The ratio of the increase in h.p.transmitted is thus:

$$\frac{1 - 0.464}{1 + 0.464} \times \frac{1 + 0.497}{1 - 0.497} = 1.09$$

A leather belt transmits 25 h.p. from a pulley 3 ft. in diameter which runs at 300 r.p.m.; $\theta = 165^0$, $\mu = 0.27$, **the density of the belt is 0.035 lb/cu.in. and the maximum stress is not to exceed 350 lb/sq.in. In the belt is** $\frac{1}{4}$ **in. thick, find the least possible width required.**

From Equation (9), Horse-power

$$= (T_T - T_C)\left(1 - e^{-\mu\theta}\right) \times \frac{v}{550} \quad (1)$$

The velocity of the belt,

$$= \pi \times 3 \times \frac{300}{60} = 47.1\ ft.sec.^{-1} \quad (2)$$

From equation (4), $T_C = \frac{w\,v^2}{g}$

Where w is the weight per ft. length

$$\therefore \quad T_C = 0.035 \times 0.25b \times 12 \times \frac{47.1^2}{32.2} = 7.2lb. \quad (3)$$

Note. The density thickness and width b of the belt are given in inches. In the above equation we require the weight of 1 foot i.e. 12 inches.

As the maximum stress must not exceed 350 lb./sq.in

$$T_T = 350 \times 0.25b = 87.5b\ lb. \quad (4)$$

Putting equations (2) (3) and (4) into (1)

Horse-power $= b(87.5 - 7.2)91 - 0.460 \times \frac{47.1}{550} = 25$ (given)

$$\therefore \quad b = \frac{25 \times 550}{80.3 \times 0.54 \times 47.1} = 6.75in.$$

An open belt drive connects two pulleys of 48 in. and 20 in.diameters, on parallel shafts 12 ft. apart. The belt weighs 0.6 lb/ft length and the maximum tension in it is not to exceed 4r500 lb. The coefficient of friction is 0.3.

The 48 in pulley, which is the driver, runs at 200 r.p.m.. Due to belt slip on one of the pulleys, the velocity of the driven shaft is only 450 r.p.m..

Calculate the torque on each of the two shafts, the horse- power transmitted and the horse-power lost in friction.

What is the efficiency of the drive ?

Slipping will occur on the driven pulley since this has the smaller angle of lap. Hence the belt speed, v, is calculated for the larger pulley.

$$v = 200 \times \pi \times \frac{48}{12 \times 60} = 41.8 ft.\ sec.^{-1}$$

Centrifugal Tension, $T_C = \frac{wv^2}{g} = .6 \times \frac{41.8^2}{32.2} = 32.7\ lb.$

Useful Tension :

$$T_1 = T_T - T_C = 400 - 32.7 = 367.3\ lb. \qquad (1)$$

Angle of lap, θ on the Driven pulley (radians)

$$\theta = \pi - 2\sin^{-1}\frac{24 - 10}{144} = 2.947\ rad.$$

Slack side tension, T_2. From equation (3)

$$T_2 = \frac{T_1}{e^{0.3 \times 2.947}} = 152\ lb. \qquad (2)$$

Output Torque (From equations (1) and (2)) $= (T_1 - T_2) \times \frac{20}{24} = 179\ lb.ft.$

Input Torque, $= (T_1 - T_2) \times \frac{48}{24} = 430\ lb.ft.$

Output Horse-power, $= 179 \times 2\pi \times \frac{450}{33,000} = 15.35$

Input Horse-power (From equation (2),Example 7) $= 430 \times 2\pi \times \frac{200}{33,000} = 16.35$

Frictional Horse-power=Input h.p. - Output h.p. = 1.0

The efficiency of the drive is output/input $= \frac{15.35}{16.35} \times \frac{100}{1} = 94\%$