UNIT I INTRODUCTION:

1. Classification of vibrations,
2. Mechanical vibrating systems,
3. Single degree of freedom,
4. Two degree of freedom,
5. Free, forced and damped vibrations, Modeling and simulation studies,
6. Model of an automobile,
7. Magnification factor,
8. Transmissibility,
9. Vibration absorber,

1.Vibration 2.Classification of vib 3.Basic terms 4.Components of mechanical vibration.

1. VIBRATION:

- it is defined as any motion that repeats itself after an interval of time.
- It involves transfer of potential energy to kinetic energy and vice versa.
- Vibration is the motion of a particle or a body or system of connected bodies displaced from a position of equilibrium
- Aim of vibration analysis:
- Why:
    - Vibrations can lead to excessive deflections and failure on the machines and structures.
    - To reduce vibration through proper design of machines and their mountings.
    - To utilize profitably in several consumer and industrial applications.
    - To improve the efficiency of certain machining, casting, forging & welding processes.
    - Cause rapid wear.
    - Create excessive noise.
    - Leads to poor surface finish (eg: in metal cutting process, vibration cause chatter).
    - Resonance – natural frequency of vibration of a machine/structure coincide with the frequency of the external excitation.
    - To stimulate earthquakes for geological research and conduct studies in design of nuclear reactors.

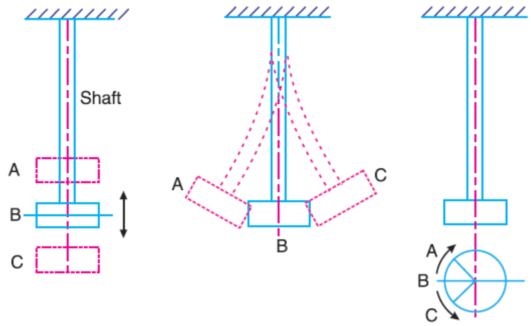
2. CLASSIFICATION OF VIBRATIONS:

Free vibration:

- When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibrations.
- The frequency of the free vibration is called free or natural frequency.

The following three types of free vibrations are important from the subject point of view :

- 1. Longitudinal vibrations, 2. Transverse vibrations, and 3. Torsional vibrations.



B = Mean position ; *A* and *C* = Extreme positions.
(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

Forced vibration:

- When the body vibrates under the influence of external force, then the body is said to be under forced vibrations.
- The external force applied to the body is a periodic disturbing force created by unbalance.
- The vibrations have the same frequency as the applied force.
- When the frequency of the external force is same as that of the natural vibrations, resonance takes place.
- *Resonance* occurs when the frequency of the external force coincides with one of the natural frequencies of the system

Damped vibration:

- When there is a reduction in amplitude over every cycle vibration, the motion is said to be damped vibration.
- This is due to the fact that a certain amount of energy possessed by the vibration system is always dissipated in overcome friction resistances to the motion.

Linear Vibration:

- When ***all*** basic components of a vibratory system, i.e. the spring, the mass and the damper behave linearly

Nonlinear Vibration:

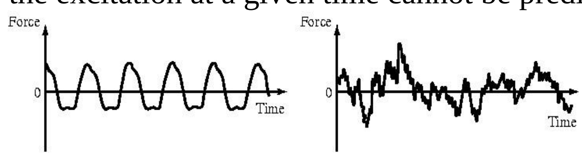
- If ***any*** of the components behave nonlinearly

Deterministic Vibration:

- If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time

Nondeterministic or random Vibration:

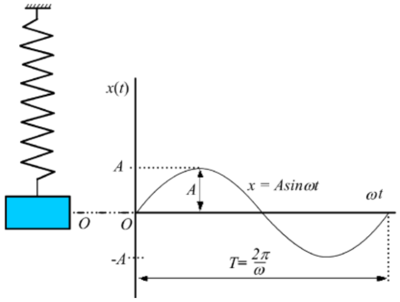
- When the value of the excitation at a given time cannot be predicted



(a) A deterministic (periodic) excitation (b) A random excitation

3. BASIC TERMS:

Oscillatory motion: repeats itself regularly.
Cycle: It is the motion completed during one time period.
Periodic motion: This motion repeats at equal interval of time T
Period : the time taken for one repetition. ***Period of vibration or time period.*** It is the time interval after which the motion is repeated itself. The period of vibration is usually expressed in seconds



$$\text{Period } \tau = \frac{2\pi}{\omega} \text{ s/cycle}$$

$$f = \frac{1}{\tau} = \frac{\omega}{2\pi} \text{ cycles/s, or Hz}$$

Time period: $t_p = \frac{2\pi}{\omega}$

Frequency: $\frac{1}{t_p}$ it is the reciprocal of time period: ***Frequency.*** It is the number of cycles described in one second. In S.I. units, the frequency is expressed in hertz (briefly written as Hz) which is equal to one cycle per second.

Natural frequency:

Damped vibration.

- When there is reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration.

Damping.

- Damping is the dissipation of energy with time or distance

Viscous damping

- The damping provided by fluid friction is known as viscous.

Critical damping.

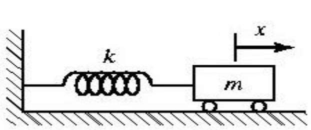
- It is the minimum viscous damping that will allow a displaced system to return to its initial position without oscillation.

Degree of freedom:

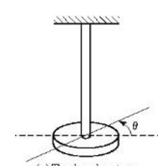
- The minimum number of independent co-ordinates required to define completely the position of all parts of the system at any instance of time.
- How many mass or masseswill be there in a system.

Single degree-of-freedom systems:

- The number of degree of freedom of a mechanical system is equal to the minimum number of independent co-ordinates required to define completely the positions of all parts of the system at any instance of time.

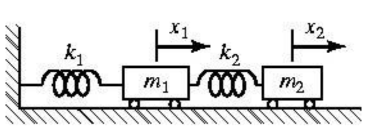


(b) Spring-mass system

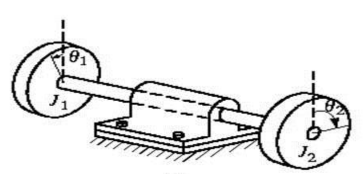


(c) Torsional system

Two degree-of-freedom systems:

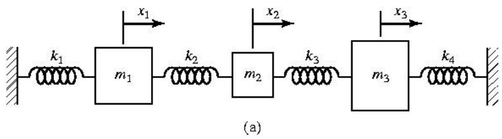


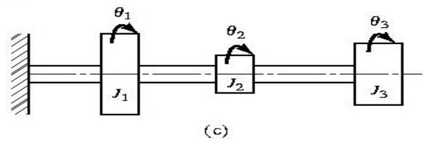
(a)



(b)

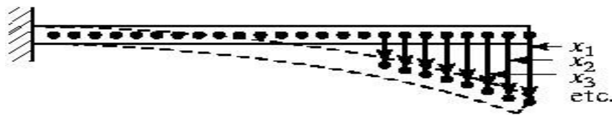
Three degree of freedom systems:





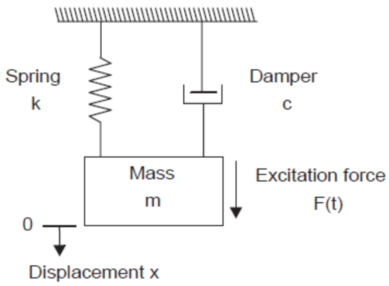
Multi-degree of freedom:

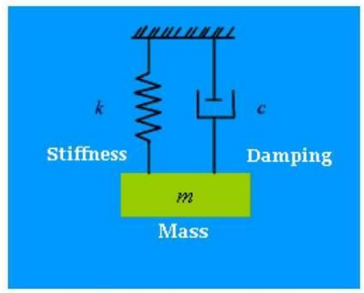
- Infinite number of degrees of freedom system
- For which 2 or 3 co-ordinates are required to define completely the position of the system at any instance of time.



4. COMPONENTS OF MECHANICAL VIBRATING SYSTEMS:

- It consist of mass, spring and damper.





Mass Element:

- The mass provides inertia force to the system, spring provides the restoring force and the damper provides the resistance.

Spring Elements:

Linear spring is a type of mechanical link that is generally assumed to have negligible mass and damping.

Spring force is given by: $F = kx$

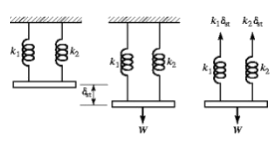
F = spring force,

k = spring stiffness or spring constant, and

x = deformation (displacement of one end with respect to the other)

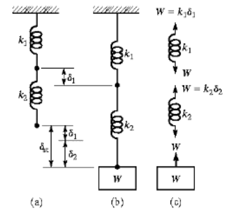
Combination of Springs:

Springs in parallel – if we have n spring constants k_1, k_2, …, k_n in *parallel*, then the equivalent spring constant k_{eq} is: $k_{eq} = k_1 + k_2 + ... + k_n$



Springs in series – if we have n spring constants k_1, k_2, …, k_n in *series*, then the equivalent spring constant k_{eq} is:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + ... + \frac{1}{k_n}$$



Damping elements:

- The process of energy dissipation is referred to in the study of vibration as *damping*. A damper is considered to have neither mass nor elasticity.
- The three main forms of damping are *viscous damping*, *Coulomb* or *dry-friction damping*, and *hysteresis damping*. The most common type of energy-dissipating element used in vibrations study is the *viscous damper*, which is also referred to as a *dashpot*.
- In viscous damping, the damping force is proportional to the velocity of the body. Coulomb or dry-friction damping occurs when sliding contact that exists between surfaces in contact are dry or have insufficient lubrication. In this case, the damping force is constant in magnitude but opposite in direction to that of the motion. In dry-friction damping energy is dissipated as heat.

NATURAL FREQUENCY

Vehicle vibration with SINGLE DEGREE of freedom:

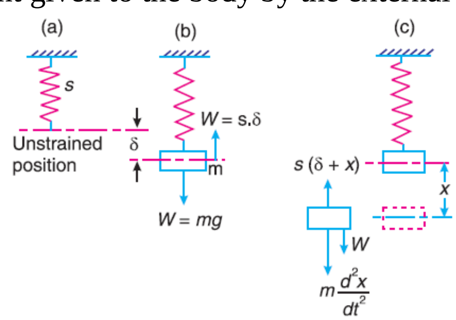
The natural frequency of the free longitudinal vibrations may be determined by the following three methods

1. ***Equilibrium Method***
2. ***Energy method*** (***summation of kinetic energy and potential energy must be a constant quantity which is same at all the times.)*** $\frac{d}{dt}(K.E.+P.E.)=0$
3. ***Rayleigh's method*** (***the maximum kinetic energy at the mean position is equal to the maximum potential energy (or strain energy) at the extreme position)***

a. Frequency of **Free-undamped vibrations:**

Consider a constraint (i.e. spring) of negligible mass in an unstrained position

s = Stiffness of the constraint. It is the force required to produce unit displacement in the direction of vibration. It is usually expressed in N/m.
m = Mass of the body suspended from the constraint in kg,
W = Weight of the body in newtons = $m.g$,
δ = Static deflection of the spring in metres due to weight W newtons, and
x = Displacement given to the body by the external force, in metres



In the equilibrium position, the gravitational pull $W = m.g$, is balanced by a force of spring, such that $W = s.\ \delta$ .Since the mass is now displaced from its equilibrium position by a distance x, and is then released, therefore after time t, Restoring force $= W - s\ (\delta + x) = W - s.\delta - s.x = s.\delta - s.\delta - s.\ x = -s.x$ *(Taking upward force as negative)*

Accelerating force = Mass × Acceleration

$$= m \times \frac{d^2x}{dt^2}$$

$$m \times \frac{d^2x}{dt^2} = -s.x$$

$$m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0$$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2} + \omega^2 .x = 0$$

$$\omega = \sqrt{\frac{s}{m}}$$

Time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$

natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$ *(m.g = s.δ)*

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}}\ \text{Hz}$$

b. Frequency ofFreeDampedVibrations(ViscousDamping)

- The motion of a body is resisted by frictional forces, the effect of friction is referred to as damping.
- The damping provided by fluid resistance is known as ***viscous damping.***
- In damped vibrations, the amplitude of the resulting vibration gradually diminishes.
- Certain amount of energy is always dissipated to overcome the frictional resistance. The resistance to the motion of the body is provided partly by the medium in which the vibration takes place and partly by the internal friction

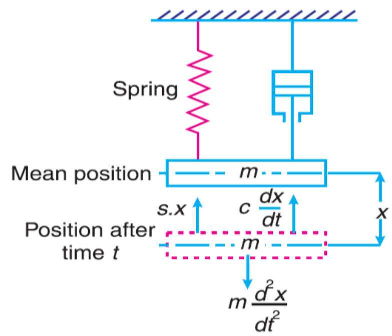
Let m = Mass suspended from the spring,

s = Stiffness of the spring,

x = Displacement of the mass from the mean position at time t,

™ = Static deflection of the spring = $m.g/s$, and

c = Damping coefficient or the damping force per unit velocity



Damping force or frictional force on the mass acting in ***opposite*** direction to the motion of the mass

$$= c \times \frac{dx}{dt}$$

Accelerating force on the mass, acting ***along*** the motion of the mass

$$= m \times \frac{d^2x}{dt^2}$$

Spring force on the mass, acting in ***opposite*** direction to the motion of the mass,

$$= s.x$$

Therefore the equation of motion becomes

$$m \times \frac{d^2x}{dt^2} = -\left(c \times \frac{dx}{dt} + s.x\right)$$

$$m \times \frac{d^2x}{dt^2} + c \times \frac{dx}{dt} + s.x = 0$$

$$\frac{d^2x}{dt^2} + \frac{c}{m} \times \frac{dx}{dt} + \frac{s}{m} \times x = 0$$

This is a differential equation of the second order. Assuming a solution of the form $x = e^{k\,t}$ where k is a constant to be determined

$$k^2.e^{kt} + \frac{c}{m} \times k.e^{kt} + \frac{s}{m} \times e^{kt} = 0 \qquad \left[\because \; \frac{dx}{dt} = ke^{kt}, \text{ and } \frac{d^2x}{dt^2} = k^2.e^{kt}\right]$$

$$k^2 + \frac{c}{m} \times k + \frac{s}{m} = 0$$

$$k = \frac{-\dfrac{c}{m} \pm \sqrt{\left(\dfrac{c}{m}\right)^2 - 4 \times \dfrac{s}{m}}}{2}$$

$$= -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{s}{m}}$$

The two roots of the equation are:

$$k_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{s}{m}}$$

$$k_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{s}{m}}$$

The most general solution of the differential equation, with its right hand side equal to zero has only complementary function and it is given by

$$x = C_1 e^{k_1 t} + C_2 e^{k_2 t}$$

The roots k_1 and k_2 may be real, complex conjugate (imaginary) or equal.

1. ***When the roots are real (overdamping)***

If $\left(\frac{c}{2m}\right)^2 > \frac{s}{m}$ then the roots k_1 and k_2 are real but negative. This is a case of ***overdamping*** or ***large damping*** and the mass moves slowly to the equilibrium position. This motion is known as ***aperiodic*.** When the roots are real, the most general solution of the differential equation is

$$x = C_1 e^{k_1 t} + C_2 e^{k_2 t}$$

$$= C_1 e^{\left[-\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{s}{m}}\right]t} + C_2 e^{\left[-\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{s}{m}}\right]t}$$

In actual practice, the overdamped vibrations are avoided

2. ***When the roots are complex conjugate (underdamping)***

If $\frac{s}{m} > \left(\frac{c}{2m}\right)^2$ then the radical (*i.e.* the term under the square root) becomes negative. The two roots k_1 and k_2 are then known as complex conjugate. This is a most practical case of damping and it is known as ***underdamping*** or ***small damping*.** The two roots are

$$k_1 = -\frac{c}{2m} + i\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2}$$

$$k_2 = -\frac{c}{2m} - i\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2}$$

For the sake of mathematical calculations,let

$$\frac{c}{2m} = a; \ \frac{s}{m} = (\omega_n)^2; \ \text{and} \ \sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2} = \omega_d = \sqrt{(\omega_n)^2 - a^2}$$

Therefore the two roots may be written as

$$k_1 = -a + i\,\omega_d \ ; \quad \text{and} \quad k_2 = -a - i\,\omega_d$$

We know that the general solution of a differential equation is

$$x = C_1 e^{k_1 t} + C_2 e^{k_2 t} = C_1 e^{(-a + i\omega_d)t} + C_2 e^{(-a - i\omega_d)t}$$

$$= e^{-at}(C_1 e^{i\omega_d . t} + C_2 e^{-i\omega_d t}) \quad \ldots(\text{Using } e^{m+n} = e^m \times e^n) \ldots($$

Now according to Euler's theorem

$$e^{+i\theta} = \cos\theta + i\sin\theta \ ; \text{and} \ e^{-i\theta} = \cos\theta - i\sin\theta$$

Therefore the equation ***(iii)*** may be written as

$$x = e^{-at}\left[C_1(\cos\omega_d .t + i\sin\omega_d .t) + C_2(\cos\omega_d .t - i\sin\omega_d .t)\right]$$

$$= e^{-at}\left[(C_1 + C_2)\cos\omega_d .t + i(C_1 - C_2)\sin\omega_d .t)\right]$$

$$C_1 + C_2 = A, \ \text{and} \ i(C_1 - C_2) = B$$

$$\therefore \quad x = e^{-at}(A\cos\omega_d.t + B\sin\omega_d.t) \quad \ldots (iv)$$

Again, let $A = C\cos\theta$, and $B = C\sin\theta$, therefore

$$C = \sqrt{A^2 + B^2}, \text{ and } \tan\theta = \frac{B}{A}$$

Now the equation (*iv*) becomes

$$x = e^{-at}(C\cos\theta\cos\omega_d.t + C\sin\theta\sin\omega_d.t)$$

$$= Ce^{-at}\cos(\omega_d.t - \theta) \quad \ldots (v)$$

If t is measured from the instant at which the mass m is released after an initial displacement A, then

$$A = C\cos\theta \quad \ldots \text{[Substituting } x = A \text{ and } t = 0 \text{ in equation } (v)]$$

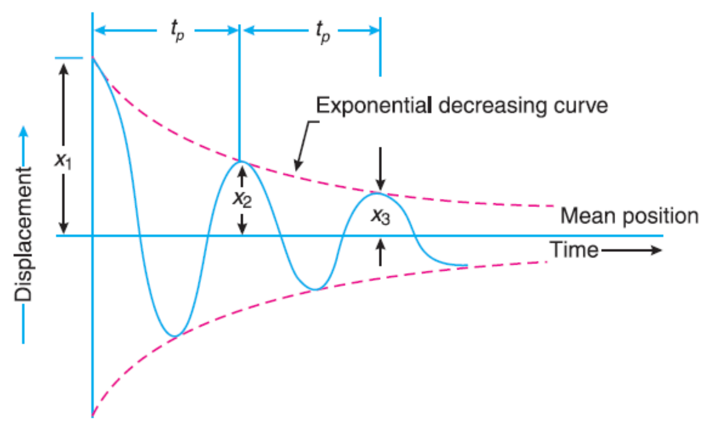
and when $\theta = 0$, then $A = C$

$\therefore$ The equation (*v*) may be written as

$$x = Ae^{-at}\cos\omega_d.t \quad \ldots (vi)$$

where $$\omega_d = \sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2} = \sqrt{(\omega_n)^2 - a^2}\ ; \text{ and } a = \frac{c}{2m}$$

The motion of the mass is simple harmonic whose circular damped frequency is ω_d and the amplitude is Ae^{-at} which diminishes exponentially with time as shown in Fig. 23.18. Though the mass eventually returns to its equilibrium position because of its inertia, yet it overshoots and the oscillations may take some considerable time to die away.



$$t_p = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2}} = \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$$f_d = \frac{1}{t_p} = \frac{\omega_d}{2\pi} = \frac{1}{2\pi}\sqrt{(\omega_n)^2 - a^2} = \frac{1}{2\pi}\sqrt{\frac{s}{m} - \left(\frac{c}{2m}\right)^2}$$

3. ***When the roots are equal (critical damping)***

If $\left(\frac{c}{2m}\right)^2 = \frac{s}{m}$ then the radical becomes zero and the two roots k_1 and k_2 are equal. This is acase of **critical damping.** In other words, the critical damping is said to occur when frequency of damped vibration (f_d) is zero (*i.e.*

motion is aperiodic). This type of damping is also avoided because the mass moves back rapidly to its equilibrium position, in the shortest possible time

$$x = (C_1 + C_2)\, e^{-\frac{c}{2m}t} = (C_1 + C_2)\, e^{-\omega_n t}$$

Thus the motion is again aperiodic. The critical damping coefficient (c_c) may be obtained by substituting c_c for c in the condition for critical damping,

$$\left(\frac{c_c}{2m}\right)^2 = \frac{s}{m} \qquad \text{or} \qquad c_c = 2m\sqrt{\frac{s}{m}} = 2m \times \omega_n$$

The critical damping coefficient is the amount of damping required for a system to be critically damped

Damping Factor or Damping Ratio

The ratio of the actual damping coefficient (c) to the critical damping coefficient (c_c) is known as ***damping factor* or *damping ratio*.** Mathematically

$$\text{Damping factor} \qquad = \frac{c}{c_c} = \frac{c}{2m.\omega_n}$$

Logarithmic Decrement

It is defined as the natural logarithm of the amplitude reduction factor. The amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position. If x_1 and x_2 are successive values of the amplitude on the same side of the mean position, as shown in Fig. 23.18, then amplitude reduction factor,

$$\frac{x_1}{x_2} = \frac{Ae^{-at}}{Ae^{-a(t+t_p)}} = e^{at_p} \quad = \text{constant}$$

$$\delta = \log\left(\frac{x_1}{x_2}\right) = \log e^{at_p}$$

where t_p is the period of forced oscillation or the time difference between two consecutive amplitudes.
As per definition, logarithmic decrement

$$\delta = \log_e\left(\frac{x_1}{x_2}\right) = a.t_p = a \times \frac{2\pi}{\omega_d} = \frac{a \times 2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$$= \frac{\frac{c}{2m} \times 2\pi}{\sqrt{(\omega_n)^2 - \left(\frac{c}{2m}\right)^2}}$$

$$= \frac{\frac{c}{2m} \times 2\pi}{\omega_n\sqrt{1 - \left(\frac{c}{2m.\omega_n}\right)^2}} = \frac{c \times 2\pi}{c_c\sqrt{1 - \left(\frac{c}{c_c}\right)^2}}$$

$$= \frac{2\pi \times c}{\sqrt{(c_c)^2 - c^2}}$$

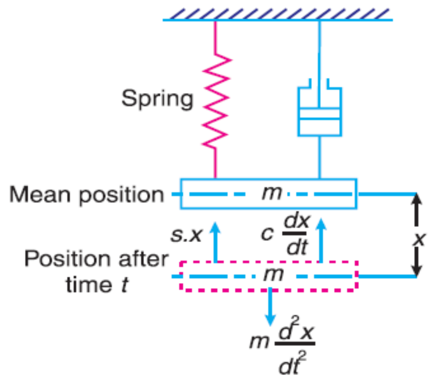
In general, amplitude reduction factor

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \ldots = \frac{x_n}{x_{n+1}} = e^{at_p} \quad = \text{constant}$$

Logarithmic decrement

$$\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right) = a.t_p = \frac{2\pi \times c}{\sqrt{(c_c)^2 - c^2}}$$

C. Forced damped vibration system: (Frequency of Under Damped Forced Vibrations)



Consider a system consisting of spring, mass anddamper as shown in Fig. 23.19. Let the system is acted upon by an external periodic (*i.e.* simple harmonic) disturbing force,

$$F_x = F\cos\omega.t$$

F = Static force, and

ω = Angular velocity of the periodic disturbing force.

When the system is constrained to move in vertical guides, it has only one degree of freedom. Let at sometime t, the mass is displaced downwards through a distance $x$from its mean position.

the equation of motion may be written as

$$m\times\frac{d^2x}{dt^2} = -c\times\frac{dx}{dt} - s.x + F\cos\omega.t$$

$$m\times\frac{d^2x}{dt^2} + c\times\frac{dx}{dt} + s.x = F\cos\omega.t$$

This equation of motion may be solved either by differential equation method or by graphical method as discussed below :

1. *Differential equation method*

The equation ***(i)*** is a differential equation of the second degree whose right hand side is some function in t. The solution of such type of differential equation consists of two parts ; one part is the complementary function and the second is particular integral. Therefore the solution may be written as

$$x = x_1 + x_2$$

x_1 = Complementary function, and

x_2 = Particular integral.

The complementary function is same as discussed in the previous article,

$$x_1 = Ce^{-at}\cos(\omega_d t - \theta)$$

$$c.\omega = X\sin\phi; \text{ and } s - m.\omega^2 = X\cos\phi$$

$$X = \sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}$$

$$\phi = \tan^{-1}\left(\frac{c.\omega}{s - m.\omega^2}\right)$$

$$x_2 = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}} \times \cos(\omega.t - \phi)$$

In actual practice, the value of the complementary function x_1 at any time t is much smaller as compared to particular integral x_2. Therefore, the displacement x, at any time t, is given by the particular integral x_2 only

$$x = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}} \times \cos(\omega.t - \phi)$$

The equations ***(vii)*** and ***(viii)*** hold good when steady vibrations of constant amplitude takes

Maximum displacement or the amplitude of forced vibration,

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

$$x_{max} = \frac{F / s}{\sqrt{\frac{c^2.\omega^2}{s^2} + \frac{(s - m.\omega^2)^2}{s^2}}} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{m.\omega^2}{s}\right)^2}}$$

where *x*o is the deflection of the system under the static force *F*.

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

At resonance $\omega = \omega_n$. Therefore the angular speed at which the resonance occurs is

$$\omega = \omega_n = \sqrt{\frac{s}{m}} \text{ rad/s}$$

$$x_{max} = x_o \times \frac{s}{c.\omega_n}$$

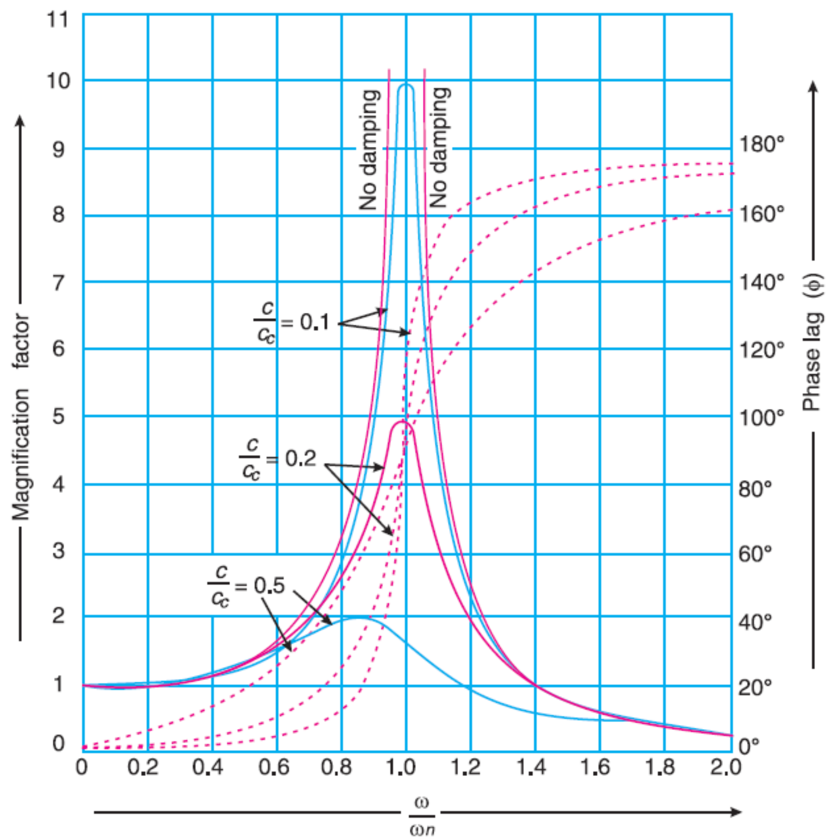
Magnification Factor or Dynamic Magnifier

It is the ratio of ***maximum displacement of the forced vibration (x_{max}) to the deflection due to the static force F(x_o).*** We have proved in the previous article that the maximum displacement or the amplitude of forced vibration,

Magnification factor:

- It is the ratio between the maximum actual amplitude of the body and the maximum actual amplitude of the road.

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$



Magnification factor or dynamic magnifier,

$$D = \frac{x_{max}}{x_o} = \frac{1}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$$= \frac{1}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

The magnification factor or dynamic magnifier gives the factor by which the static deflection produced by a force F (*i.e.* x_o) must be multiplied in order to obtain the maximum amplitude of the forced vibration (*i.e.* x_{max}) by the harmonic force F coswt

$$x_{max} = x_o \times D$$

If there is no damping (*i.e.* if the vibration is undamped), then c = 0. In that case, magnification factor,

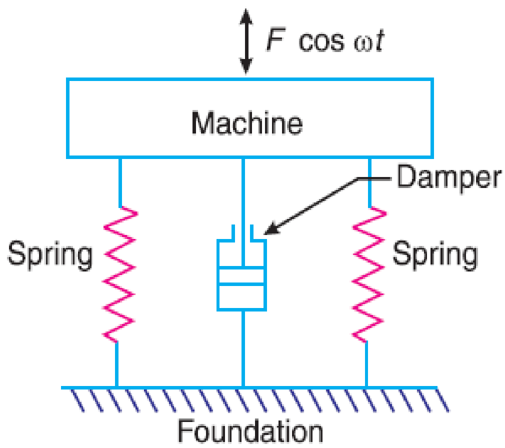
$$D = \frac{x_{max}}{x_o} = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}} = \frac{(\omega_n)^2}{(\omega_n)^2 - \omega^2}$$

At resonance, $\omega = \omega_n$ Therefore magnification factor,

$$D = \frac{x_{max}}{x_o} = \frac{s}{c.\omega_n}$$

Vibration Isolation and Transmissibility

A little consideration will show that when an unbalanced machine is installed on the foundation, it produces vibration in the foundation. In order to prevent these vibrations or to minimise the transmission of forces to the foundation, the machines are mounted on springs and dampers or on some vibration isolating material, as shown in Fig. 23.22. The arrangement is assumed to have one degree of freedom, *i.e.* it can move up and down only.
It may be noted that when a periodic (*i.e.* simple harmonic) disturbing force $F \cos \omega t$ is applied to a machine



of mass m supported by a spring of stiffness s, then the force is transmitted by means of the spring and the damper or dashpot to the fixed support or foundation. The ratio of the force transmitted (F_T) to the force applied (F) is known as the ***isolation factor*** or ***transmissibility ratio*** of the spring support.
We have discussed above that the force transmitted to the foundation consists of the following two forces :

1. Spring force or elastic force which is equal to $s.x_{max}$, and

2. Damping force which is equal to $c.\omega.x_{max}$.

Since these two forces are perpendicular to one another, as shown in Fig.23.23, therefore the force transmitted,

$$F_T = \sqrt{(s.x_{max})^2 + (c.\omega.x_{max})^2}$$

$$= x_{max}\sqrt{s^2 + c^2.\omega^2}$$

Transmissibility ratio,

$$\varepsilon = \frac{F_T}{F} = \frac{x_{max}\sqrt{s^2 + c^2.\omega^2}}{F}$$

$$x_{max} = x_o \times D = \frac{F}{s} \times D$$

$$\varepsilon = \frac{D}{s}\sqrt{s^2 + c^2.\omega^2} = D\sqrt{1 + \frac{c^2.\omega^2}{s^2}}$$

$$= D\sqrt{1 + \left(\frac{2c}{c_c} \times \frac{\omega}{\omega_n}\right)^2}$$

the magnification factor,

$$D = \frac{1}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$$\varepsilon = \frac{\sqrt{1 + \left(\frac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

When the damper is not provided, then $c = 0$, and

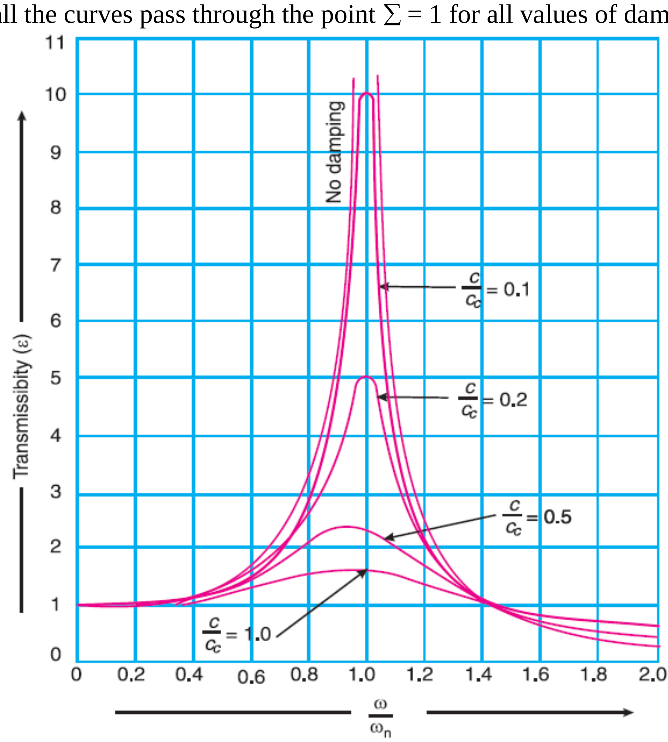
$$\varepsilon = \frac{1}{1 - (\omega / \omega_n)^2}$$

From above, we see that when $\omega/\omega_n > 1$, Σ is negative. This means that there is a phase difference of 180° between the transmitted force and the disturbing force ($F \cos\omega.t$) . The value of ω/ω_n must be greater than 2 if Σ is to be less than 1 and it is the numerical value of Σ , independent of any phase difference between the forces that may exist which is important. It is therefore more convenient to use equation **(*ii*)** in the following form, *i.e.*

$$\varepsilon = \frac{1}{(\omega / \omega_n)^2 - 1}$$

l Fig. 23.24 is the graph for different values of damping factor c/c_c to show the variation of transmissibility ratio (Σ) against the ratio ω/ω_n .

1. When $\omega/\ \omega_n = 2$, then all the curves pass through the point $\Sigma = 1$ for all values of damping factor c/c_c



2. When $\omega/\omega_n < 2$, then $\Sigma > 1$ for all values of damping factor c/c_c. This means that the force transmitted to the foundation through elastic support is greater than the force applied.

3. When $\omega/\omega_n > 2$, then $\sum < 1$ for all values of damping factor c/c_c. This shows that the force transmitted through elastic support is less than the applied force. Thus vibration isolation is possible only in the range of $\omega/\omega_n > 2$.We also see from the curves in Fig. 23.24 that the damping is detrimental beyond $\omega/\omega_n > 2$ and advantageous only in the region $\omega/\omega_n < 2$. It is thus concluded that for the vibration isolation, dampers need not to be provided but in order to limit resonance amplitude,
stops may be provided.

Transmissibility:

- It is the ratio between the force transmitted to the body and force acting on the road.
- It is the ratio between the force transmitted to the body and force acting on the road. Transmissibility is the nondimensional ratio of the response amplitude of a system in steady state forced vibration on the excitation amplitude. The ratio may be one for forces, displacement, velocities or accelerations.

Vibration absorber:

- It is an additional spring mass system used to make the amplitude values of vibration equal to zero.

Vehicle dynamics and its classification.

- Vehicle dynamics has been a pivotal domain in the field of automotive engineering. It is primarily divided into three subgroups: Performance, Ride and Handling. Performance mainly deals with the efficiency and effectiveness of the vehicle in its ability to accelerate, brake and overcome obstacles. Ride is related to the vibration of the vehicle due to road excitations and its effect on occupants and cargo. Handling is concerned with the overall behaviour or response of the vehicle to driver inputs.

Generalized co-ordinates

Rolling: Angular oscillation of vehicle about longitudinal axis.
Pitching: Angular oscillation of vehicle about transverse axis.
Yawing: Angular oscillation of vehicle about vertical axis.

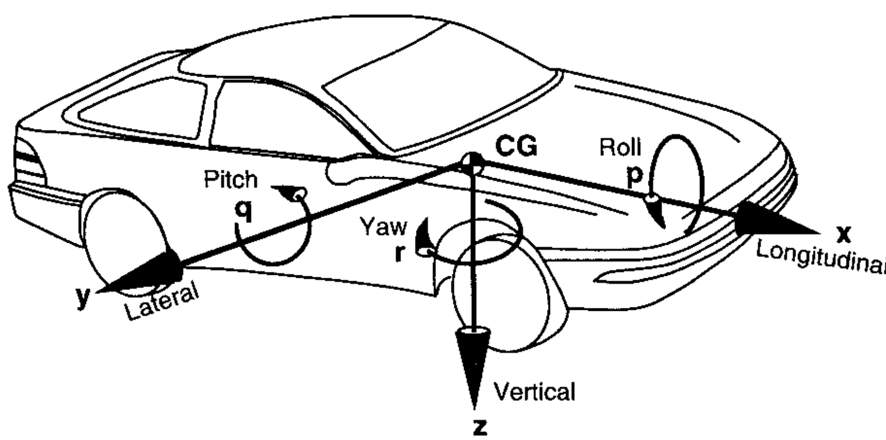


Fig. 1.4 SAE Vehicle Axis System.

The source of vibration of the vehicle may be due to

The road roughness
The unbalance of the engine
The whirling of shafts
The cam forces
Torsional fluctuations

What is Modeling and simulation?

- Writing the equation corresponding to a physical system.
- This equation may be algebraic or differential equation.
- Writing the program and then fed in to the computer.
- Solving the problem by use of computer is called simulation

Model of an automobile used in vehicle dynamics analysis.

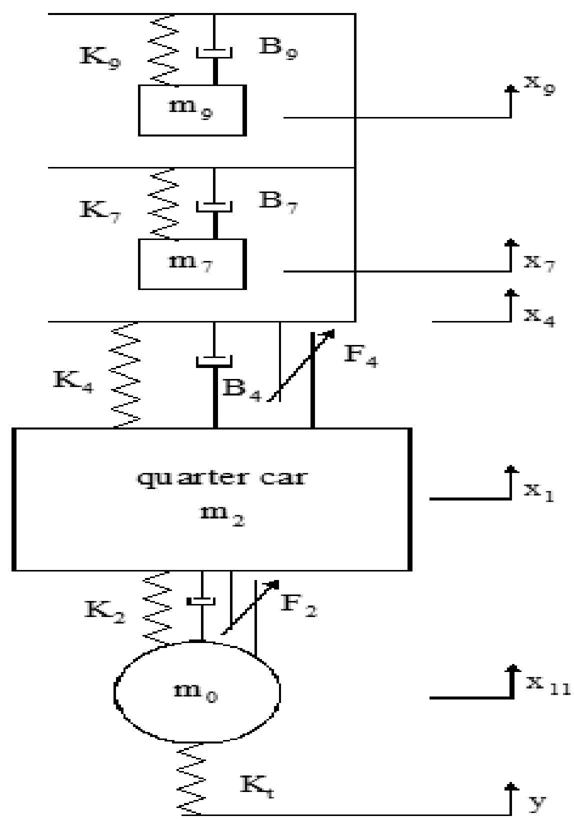


Figure 1 Model of vehicle and driver