Introduction to Heat Transfer:

Practically all the operations that are carried out by the chemical engineers involve the production or absorption of energy in the form of heat. The study of temperature distribution and heat transfer is of great importance to engineers because of its almost universal occurrence in many branches of science and engineering. The first step in the optimal design of heat exchangers such as boilers, heaters, refrigerators and radiators is a detailed analysis of heat transfer. This is essential to determine the feasibility and cost of the undertaking, as well as the size of equipment required to transfer a specified amount of heat in a given time. Difference between thermodynamics and heat transfer Thermodynamic tells us (i) How much heat is transferred (ii) How much work is done (iii) Final state of the system.

Heat transfer tells us: (i) How much heat is transferred (with what modes) (ii) At what rate heat is transferred (iii) Temperature distribution inside the body.

The various modes of heat transfer are (i) conduction (ii) convection (iii) radiation. Conduction Heat transfer by the actual but invisible movement of molecules within the continuous substance due to temperature gradient is known as conduction. When a current or macroscopic particle of fluid crosses a specific surface, it carries with it a definite quantity of enthalpy. Such a flow of enthalpy is called convection. Convection is the mode of heat transfer in which the heat flow is associated with the movement of fluid. Transfer of energy through space by electromagnetic waves is known as radiation.

APPLICATIONS OF HEAT TRANSFER

Energy production and conversion -steam power plant, solar energy conversion etc. Refrigeration and air-conditioning Domestic applications -ovens, stoves, toaster Cooling of electronic equipment Manufacturing / materials processing -welding, casting, soldering, laser machining Automobiles / aircraft design

Conduction: It is the transfer of internal energy by microscopic diffusion and collisions of particles or quasi-particles within a body due to a temperature gradient. The microscopically diffusing and colliding objects include molecules, electrons, atoms, and phonons. They transfer disorganized microscopic kinetic and potential energy, which are jointly known as internal energy. Conduction can only take place within an object or material, or between two objects that are in direct or indirect contact with each other. On a microscopic scale, heat conduction occurs as hot, rapidly moving or vibrating atoms and molecules interact with neighboring atoms and molecules, transferring some of their energy (heat) to these neighboring particles. In other words, heat is transferred by conduction when adjacent atoms vibrate against one another, or as electrons move from one atom to another.

Fourier's law of heat Conduction

The rate of heat transfer due to conduction is governed by Fourier's Law, as shown

$$q = kA\left(\frac{\Delta T}{\Delta x}\right)$$

The terms in Eqn. 1 are:

q – rate of heat transfer (W)

k – thermal conductivity (W/m·K)

A – surface area across which heat is transferred (m^2)

ΔT – difference in temperature over which heat is transferred (K)

Δx – distance over which heat is transferred (m)

Thermal conductivity indicates the ease of heat transfer through a material and is a material dependent property. The ΔT term is the driving force for heat transfer.

Convection

The rate of heat transfer due to convection is described by

$$q = hA(\Delta T)$$

In Eqn. 2 the new term is:

h –heat transfer coefficient ($W/m^2 \cdot K$)

In Eqn. 2, the heat transfer coefficient replaces the k/Δx term in Eqn. 1. The reason this happens is because convection has a mobile phase, and thickness is no longer an effective way of describing how the heat is transferred. The heat transfer coefficient can be thought of as the inverse of the resistance to heat transfer. Also, because temperature is a function of distance from a surface, the ΔT term is calculated between the surface and the bulk temperature of the mobile phase.

Radiation

The concept for radiation is that all materials are constantly emitting infrared radiation that is absorbed by other materials. For this module, we will assume that radiation is emitted directly outward from the surface of objects. While conduction and convection are driven by a temperature gradient, radiation is only based on the temperature of the object emitting radiation.

The rate of heat transfer due to radiation can be described by

Radiation Emitted: $q_{out} = \varepsilon A\sigma({T_s}^4)$

Radiation Absorbed: $q_{in} = \alpha A\sigma({T_o}^4)$

Heat transfer is the exchange of thermal energy between physical systems. The rate of heat transfer is dependent on the temperatures of the systems and the properties of the intervening medium through which the heat is transferred. The three fundamental modes of heat transfer are *conduction*, *convection* and *radiation*.

Heat transfer, the flow of energy in the form of heat, is a process by which a system changes its internal energy, hence is of vital use in applications of the First Law of Thermodynamics. Conduction is also known as diffusion, not to be confused with diffusion related to the mixing of constituents of a fluid. The direction of heat transfer is from a region of high temperature to another region of lower temperature, and is governed by the Second Law of Thermodynamics. Heat transfer changes the internal energy of the systems from which and to which the energy is transferred. Heat transfer will occur in a direction that increases the entropy of the collection of systems. Thermal equilibrium is reached when all involved bodies and the surroundings reach the same temperature. Thermal expansion is the tendency of matter to change in volume in response to a change in temperature.

Newton's law of cooling states that *the rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings.* As such, it is equivalent to a statement that the heat transfer coefficient, which mediates between heat losses and temperature differences, is a constant. This condition is generally true in thermal conduction (where it is guaranteed by Fourier's law), but it is often only approximately true in conditions of convective heat transfer, where a number of physical processes make effective heat transfer coefficients somewhat dependent on temperature differences. Finally, in the case of heat transfer by thermal radiation, Newton's law of cooling is not true.

Thermal conductivity (often denoted k, λ, or κ) is the property of a material to conduct heat. It is evaluated primarily in terms of Fourier's Law for heat conduction. Heat transfer occurs at a lower rate across materials of low thermal conductivity than across materials of high thermal conductivity. Correspondingly, materials of high thermal conductivity are widely used in heat sink applications and materials of low thermal conductivity are used as thermal insulation. The thermal conductivity of a material may depend on temperature. The reciprocal of thermal conductivity is called thermal resistivity. Thermal conductivity is actually a tensor, which means it is possible to have different values in different directions.

Table 2.1: Thermal conductivity at room temperature for some metals and non-metals

Metals	Ag	Cu	Al	Fe	Steel
k [W/m-K]	420	390	200	70	50

Non-metals	H_20	Air	Engine oil	H_2	Brick	Wood	Cork
k [W/m-K]	0.6	0.026	0.15	0.18	0.4 -0 .5	0.2	0.04

EFFECT OF TEMPERATURE ON THERMAL CONDUCTIVITY

Thermal conductivity is the physical property of the substance. It depends upon temperature gradient. For pure metals thermal conductivity decreases with increase in temperature. For gases and insulators thermal conductivity increases with increase in temperature. For small ranges of temperature, k may be considered constant. For larger temperature ranges, thermal conductivity can be approximated by an equation of the form k = a + bT, where a and b are empirical constants.

Steady-State Conduction It is the form of conduction which happens when the temperature difference driving the conduction is constant so that after an equilibrium time, the spatial distribution of temperatures (temperature field) in the conducting object does not change any further. In steady state conduction, the amount of heat entering a section is equal to amount of heat coming out. Unsteady state conduction It is the form of conduction which happens when the temperature difference driving the conduction is not constant so that after an equilibrium time, the spatial distribution of temperatures (temperature field) in the conducting object changes as a function of time. Heat flux is denoted as q/A and it is defined as the rate of heat flow passing through a material per cross-sectional area and its unit is w/m^2.

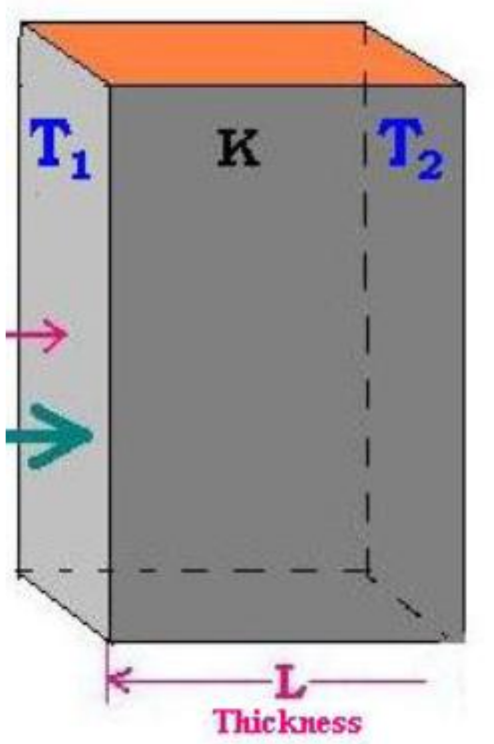
Silver is the material which possess highest thermal conductivity and being a solid it is composed of closed packing arrangement and due to this more molecular interactions within the molecules and hence the thermal conductivity is high.

HEAT TRANSFER THROUGH A PLANE WALL

Let us consider a plane wall of thickness L, thermal conductivity k, inside surface temperature Ti, outside surface temperature To. Let Q be the rate of heat transferred through the plane wall.

By Fourier's law of heat conduction

Q = -kA dT/dx



$$Q \int_0^L dx = -k\,A \int_{Ti}^{To} dT$$

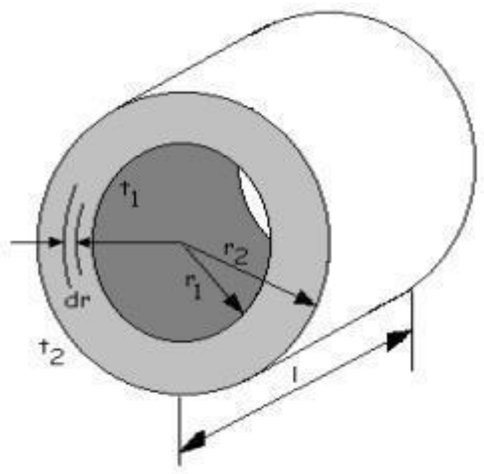
On integrating the above eqn, $Q = KA(T_i-T_o) / L$

$Q = KA\ \Delta T / L$ i.e $Q = \Delta T / (KA/L)$ where $KA/L = R_{th}$ where R_{th} – thermal resistance measured in ^{o}C / Watts or K / Watts. Hence $\mathbf{Q = \Delta T / R_{th}}$.

HEAT TRANSFER THROUGH A HOLLOW CYLINDER

Let us consider a hollow cylinder. The inside radius of the cylinder is r_1, the outside radius is r_2, and the length of the cylinder is L. The thermal conductivity of the material of which the cylinder is made is k. The temperature of the outside surface is T_2, and that of the inside surface is T_1.

By Fourier's law of heat conduction,



Q = -kA dT/dr

$$Q \int_{r1}^{r2} \frac{dr}{r} = -k\, 2\pi L \int_{T1}^{T2} dT$$

integrating the above eqn,

$$Q = \frac{2\pi k L\,(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

LOGARITHMIC MEAN RADIUS AND ARITHMETIC MEAN RADIUS

Logarithmic mean radius is the radius that when applied to the integrated equation for a flat wall, will give the correct rate of heat flow through a thick walled cylinder. It is given by the expression

$$\overline{r_L} = \frac{r_o - r_i}{\ln\left(\frac{r_o}{r_i}\right)}$$

where $\overline{r_L}$ is the logarithmic mean radius of the cylinder

r_o is the outer radius of the pipe and

r_i is the inner radius of the pipe.

Hence using the above expression Q = 2∏ k L (Ti-To) * (ro-ri) / ln (ro/ri) *(ro-ri)

Using the Logarithmic mean radius expression in above, we get

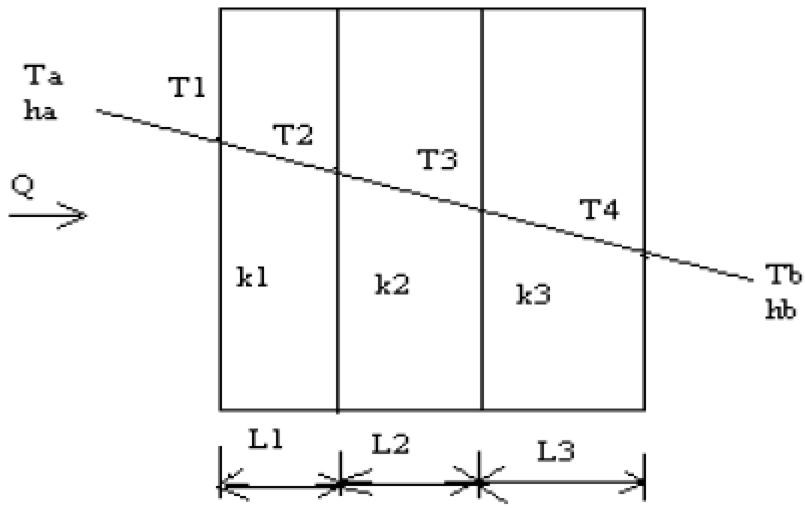
Q = 2∏ k L ΔT * r_{lm} / (ro-ri) hence, Q = ΔT / (ro-ri) / A_{lm} * k

Where A_{lm} = 2∏ L * r_{lm} , A_{lm} is the logarithmic mean area which is used for thin cylinders.

COMPOUND RESISTANCES IN SERIES

(I) Heat Transfer Through A Composite Plane Wall

Let us consider a flat wall constructed of a series of 3 layers as shown. Let the thickness of the layers be L_1, L_2, L_3 and the average thermal conductivities of the materials of which the layers are made be k_1, k_2, k_3 respectively. Let us consider a hot fluid at a temperature T_a and heat transfer coefficient h_a inside the wall and cold fluid at a temperature T_b and heat transfer coefficient h_b outside the wall. Let T_1, T_2, T_3 and T_4 be the interface temperatures. It is desired to derive an equation for calculating the rate of heat flow through the series of resistances.



Rate of heat flow from the hot fluid to the inner surface of the wall
By Newton's law of cooling

$Q = h_a A (T_a – T_b)$

By rearranging the above eqn, we get

$$Q = \frac{(T_a - T_1)}{\frac{1}{h_a A}}$$

Rate of heat flow through the I layer, by Fourier's law of heat conduction,

$Q = KA(T_1-T_2) / L$ On rearranging this,

$Q = (T_1-T_2) / (L_1/K_1.A)$

Rate of heat flow through the II layer,

$Q = (T_2-T_3) / (L_2/K_2.A)$

Rate of heat flow through the III layer,

$Q = (T_3-T_4) / (L_3/K_3.A)$

Rate of heat flow from outer surface of the wall to the cold fluid By Newton's law of cooling
$Q = h_b A (T_4 – T_b)$ and $Q = (T_4 – T_b) / 1/ h_b A$

Overall rate of heat flow =*overall thermal resistance* / *overall temperature drop*

Overall rate of heat flow

$$Q = \frac{(T_a - T_b)}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}}$$

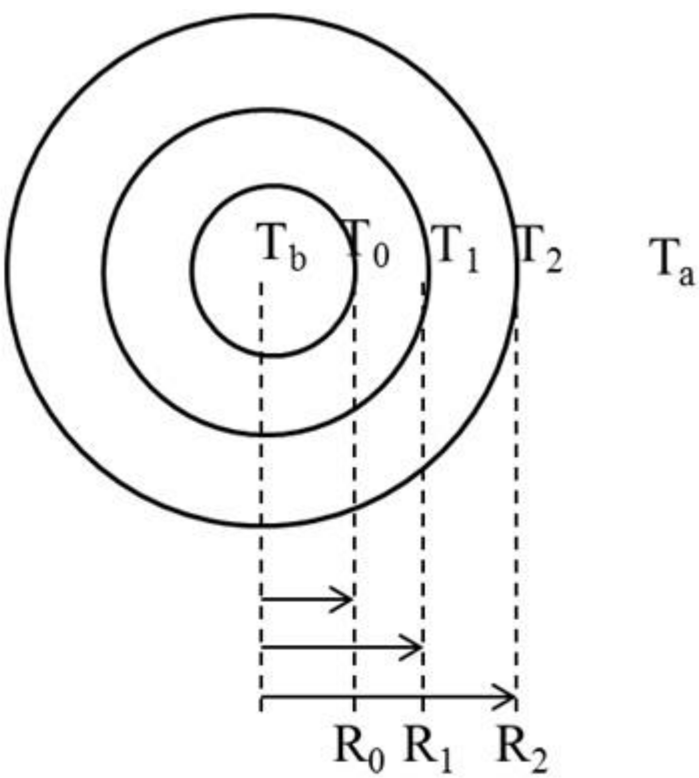
In steady state heat conduction through a composite wall, this can be written as

$Q = (T_1 – T_4) / (L_1/K_1.A + L_2/K_2.A + L_3/K_3.A)$

i.e $Q = \Delta T / R_{th1} + R_{th2} + R_{th3}$

Hence, $Q = \Delta T / \sum R_{th}$

II) Heat transfer through co-axial cylinder Provided with one layer of insulation:



Let us consider coaxial cylinders constructed of a series of 3 layers as shown in fig. Let R_o R_1 and R_2 be the radii of the cylinders and the average thermal conductivities of the materials of which the layers are made be k_1 and k_2 respectively. Let us consider a hot fluid at a temperature T_b and heat transfer coefficient h_b inside the cylinder and cold fluid at a temperature T_a and heat transfer coefficient h_a outside the cylinder. Let T_o, T_1 and T_2 be the interface temperatures. It is desired to derive an equation for calculating the rate of heat flow through the composite cylinder provided with series of resistances.

Rate of heat flow from the hot fluid to the inner surface of the wall
By Newton's law of cooling,

$Q = h_b A (T_b - T_0)$

$Q = h_b 2\pi R_o L (T_b - T_o)$

By rearranging the above,

$Q = (T_b - T_o) / 1/ h_b 2\pi R_o L$

Rate of the heat flow through the cylinder - By Fourier's law of heat conduction

$$Q = \frac{2\pi k L \ (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

using the above conditions from the fig,

$Q = 2\prod k_1 L (T_o - T_1) / \ln (R_1/R_o)$

By rearranging the above,

$Q = (T_o - T_1) / \ln (R_1/R_o) / 2\prod k_1 L$

Rate of the heat flow through the II layer By Fourier's law of heat conduction

$Q = (T_1 - T_2) / \ln (R_2/R_1) / 2\prod k_2 L$

Rate of heat flow from the outer surface of the wall to the cold fluid
By Newton's law of cooling

$Q = h_a A (T_2 - T_a)$

$Q = h_a 2\pi R_2 L (T_2 - T_a)$ on rearranging, $Q = (T_2 - T_a) / 1 / h_a 2\prod R_2 L$

Overall rate of heat flow =*overall thermal resistance* / *overall temperature drop*

Hence overall rate of heat flow is given by

$Q = (T_b - T_a) / (1/ h_b 2\pi R_o) + (\ln (R_1/R_o) / 2\prod k_1 L) + (\ln (R_2/R_1) / 2\prod k_2 L) + (1/ h_a 2\prod R_2 L)$

For steady state conduction , heat transfer coefficients can be neglected and the heat flow is given by

$Q = (T_b - T_a) / (\ln (R_1/R_o) / 2\prod k_1 L) + (\ln (R_2/R_1) / 2\prod k_2 L)$

Hence $Q = \Delta T / R_{th1} + R_{th2}$

Where $R_{th1} = (\ln (R_1/R_o) / 2\prod k_1 L)$ and $R_{th2} = (\ln (R_2/R_1) / 2\prod k_2 L)$

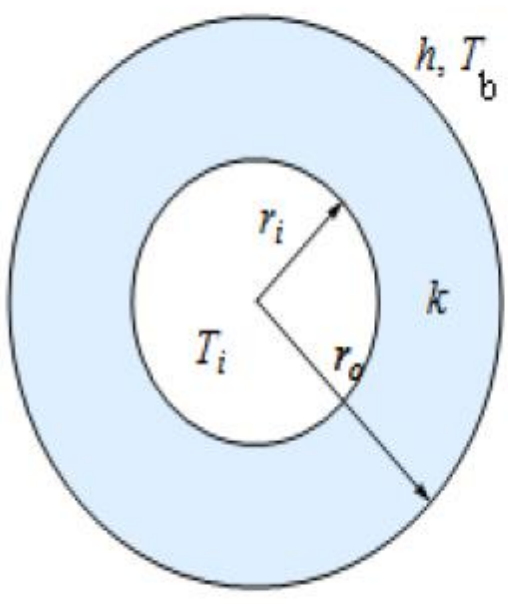
$\mathbf{Q = \Delta T / \sum R_{th}}$

Insulation

The addition of insulation material on a surface reduces the amount of heat flow to the ambient. There are certain instances in which the addition of insulation to the outside surface of cylindrical of spherical walls does not reduce the heat loss. Under certain circumstances it actually increases the heat loss up to a certain thickness of insulation. It is well known fact that the rate of heat transfer will approach zero if an infinite amount of insulation are added. This means that there must be a value of radius for which rate of heat transfer is maximum. This value is known as the critical radius of insulation, rc.

CRITICAL RADIUS OF INSULATION IN PIPES

Let us consider an insulating layer in the form of a hollow cylinder of length L. Let ri and ro be the inner and outer radii of insulation. The thermal conductivity of the material of which the layer is made be k. Let the inside surface of insulation be at a temperature Ti, and the outside surface at a temperature To be dissipating heat by convection to the surroundings at a temperature Tb with a heat transfer coefficient h. Then the rate of heat transfer Q through this insulation layer is



Insulation layer

$$Q = \frac{2\pi L\left(T_i - T_b\right)}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{1}{hr_o}} \qquad (1)$$

The value of critical radius r_c, that is r_o for which Q is a maximum may be obtained by equating dQ/dr_o to zero.

$$\frac{dQ}{dr_o} = \frac{0 - \left(T_i - T_b\right)\left[\frac{1}{2\pi k L r_o} - \frac{1}{2\pi h L r_o^2}\right]}{\left[\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{2\pi h L r_o}\right]^2} \qquad (2)$$

$\left(T_i - T_b\right) \neq 0$(Since it is the driving force)

$$\therefore \quad \frac{1}{2\pi k L r_o} - \frac{1}{2\pi h L r_o^2} = 0 \qquad (3)$$

$$r_0 = \frac{k}{h} = r_c$$

The radius at which the rate of heat transfer is maximum is known as the critical radius of insulation.

VARIABLE THERMAL CONDUCTIVITY

Let us a hollow cylinder. The inside radius of the cylinder is ri, the outside radius is ro, and the length of the cylinder is L. The thermal conductivity of the material of which the cylinder is varies with temperature as k= ko(1 + αT). The temperature of the outside surface is To and that of the inside surface is Ti. This can be used with many equations such as

$k = ko\ (\alpha + \beta T)$

$k = ko\ (\alpha + \beta T + \gamma T^2)$

$k = ko\ (a + bT)$

$k = ko\ (a + bT + cT^2)$

By Fourier’s law of heat conduction

$$Q = -kA \frac{dT}{dr} \qquad (1)$$

$$Q = -k_o (1 + \beta T) A \frac{dT}{dr}$$

$$Q \int_{ri}^{ro} \frac{dr}{r} = -k_o 2\pi L \int_{Ti}^{To} (1+\beta T)\, dT \qquad (2)$$

$$Q \ln \frac{r_o}{r_i} = k_0 2\pi L \left(1 + \beta \frac{[T_i + T_o]}{2}\right)(T_i - T_o)$$

$$Q = \frac{k_0 2\pi L \left(1 + \beta \frac{[T_i + T_o]}{2}\right)(T_i - T_o)}{\ln \frac{r_o}{\;}} \qquad (3)$$

Introduction to Unsteady state heat transfer

A solid body is said to be in a steady state if its temperature does not vary with time. If however there is an abrupt change in its surface temperature or environment it takes some time before the body to attain an equilibrium temperature or steady state. During this interim period the temperature varies with time and the body is said to be in an unsteady or transient state. The analysis of unsteady state heat transfer is of great interest to engineers because of its widespread occurrence such as in boiler tubes, rocket nozzles, automobile engines, cooling of IC engines, cooling and freezing of food, heat treatment of metals by quenching, etc. For practical purposes it is necessary to know the time taken to attain a certain temperature when the environment suddenly changes. The solution of an unsteady sate problem will be more complex than that of steady state one because of the presence of another variable time, t.

Transient heat conduction problems can be divided into periodic heat flow and non periodic heat flow problems. Periodic heat flow problems are those in which the temperature varies on a regular basis, eg., the variation of temperature of the surface of the earth during a twenty four hour period.. In the non periodic type, the temperature at any point within the system varies non linearly with time.

Introduction To this point, we have considered conductive heat transfer problems in which the temperatures are independent of time. In many applications, however, the temperatures are varying with time, and we require the understanding of the complete time history of the temperature variation. For example, in metallurgy, the heat treating process can be controlled to directly affect the characteristics of the processed materials. Annealing (slow cool) can soften metals and improve ductility. On the other hand, quenching (rapid cool) can harden the strain boundary and increase strength. In order to characterize this transient behavior, the full unsteady equation is needed.

$$\frac{1}{a} \cdot \frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\hat{q}}{k}$$

Where $\alpha = \frac{k}{\rho c}$ is the thermal diffusivity. Without any heat generation and considering spatial variation of temperature only in x-direction, the above equation reduces to:

Systems with negligible internal resistance – Lumped Heat Analysis

Heat transfer in heating and cooling of a body is dependent upon both the internal and surface resistances. The simplest unsteady state problem is one in which the internal resistance is negligible, that is, the convective resistance at the surface boundary is very large when compared to the internal resistance due to conduction. In other words, the solid has an infinite thermal conductivity so that there is no variation of temperature inside the solid and temperature is a function of time only. This situation cannot exist in reality because all the solids have a finite thermal conductivity and there will always be a temperature gradient inside whenever heat is added or removed. Problems such as heat treatment of metals by quenching, time response of thermocouples and thermometers, etc can be analyzed by this idealization of negligible internal resistance. The process in which the internal resistance is ignored being negligible in comparison with its surface resistance is called the Newtonian heating and cooling process. In Newtonian heating and cooling process the temperature throughout the solid is considered to be uniform at a given time. Such an analysis is called the lumped heat capacity analysis.

Systems with negligible surface resistance

Another class of transient problems met with in practice is one in which the surface resistance is negligible compared to the overall resistance. This amounts to saying that the convective heat transfer coefficient at the surface is infinity. For such a process the surface temperature remains constant for all the time and its value is equal to that of ambient temperature.

Dimensionless parameters:

$$\frac{T_i - T_s}{T_s - T_\infty} = \frac{\bar{h}L}{k} = \text{Biot number}$$

$$\mathbf{Bi} = \frac{\textbf{resistance to internal heat flow}}{\textbf{resistance to external heat flow}}$$

The Biot number is dimensionless, and it can be thought of as the ratio to the internal and external heat flows. Whenever the Biot number is small, the internal temperature gradients are also small and a transient problem can be treated by the "lumped thermal capacity" approach. The lumped capacity assumption implies that the object for analysis is considered to have a single mass-averaged temperature.

In general, a characteristic length scale may be obtained by dividing the volume of the solid by its surface area: $L = V/A_s$

Using this method to determine the characteristic length scale, the corresponding Biot number may be evaluated for objects of any shape, for example a plate, a cylinder, or a sphere. As a thumb rule, if the Biot number turns out to be less than 0.1, lumped capacity assumption is applied. In this context, *a dimensionless time*, known as the **Fourier number**, can be obtained by multiplying the dimensional time by the thermal diffusivity and dividing by the square of the characteristic length:

$$\textbf{dimensionless time} = \frac{\alpha t}{L^2} = \textbf{Fo}$$

Lumped heat capacity analysis: temperature distribution inside or outside the solid is neglected.

The simplest situation in an unsteady heat transfer process is to use the lump assumption, wherein we neglect the temperature distribution inside the solid ar with the heat transfer between the solid and the ambient fluids. In other wc assuming that the temperature inside the solid is constant and is equal to temperature.

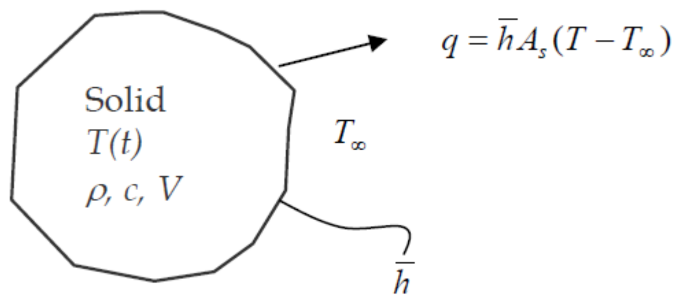


Fig. 5.2

The solid object shown in figure 5.2 is a metal piece which is being cooled in forming. Thermal energy is leaving the object from all elements of the surface shown for simplicity by a single arrow. The first law of thermodynamics apı problem is

$$\begin{pmatrix}\text{heat out of object} \\ \text{during time } dt\end{pmatrix} = \begin{pmatrix}\text{decrease of internal thermal} \\ \text{energy of object during time } dt\end{pmatrix}$$

Now, if Biot number is small and temperature of the object can be considered to this equation can be written as

$$\bar{h}A_s\left[T(t) - T_\infty\right]dt = -\rho c V dT$$

or,
$$\frac{dT}{(T - T_\infty)} = -\frac{\bar{h}A_s}{\rho c V}dt$$

Integrating and applying the initial condition $T(0) = T_i$,

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{\bar{h}A_s}{\rho c V}t$$

Taking the exponents of both sides and rearranging,

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

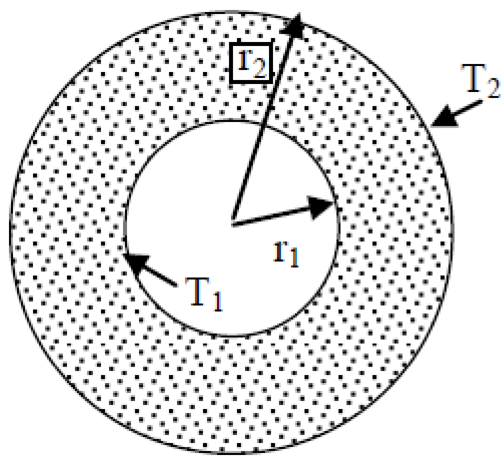
where

$$b = \frac{\bar{h}A_s}{\ldots} \qquad (1/s)$$

$$b = \frac{\overline{h} A_s}{\rho c V} \qquad (1/s)$$

b is a positive quantity having dimension (time)-1. The reciprocal of *b* is usually called *time constant*, which has the dimension of time.

Heat Conduction in a Spherical Shell



Consider a spherical shell with inside radius r_1 and outside radius r_2. Let T_1 be the inside temperature and T_2 be the outside temperature. K be the thermal conductivity of the material. Q be the heat flow through the spherical shell with a cross sectional area to be A. based on Fourier's law of heat conduction and following the assumptions,

(i) Heat flow is uniform

(ii) Heat flow is normal to the surface

(iii) The material is uniform and possess constant thermal conductivity

(iv) Heat flow is uni-directional

Let us consider a differential element of thickness dr which is lying between inside and outside radius. For such an element the heat flow is given by

Q = -kA dT/dr where area of the sphere is $A = 4\pi r^2$

Substituting the area of a sphere

Integrating, between r = r_1 and r_2, and T_1 and T_2,

$$\frac{q_r}{4\pi}\left|-\frac{1}{r}\right|_{r_1}^{r_2} = -k\left|T\right|_{T_1}^{T_2}$$

$$\frac{q_r(r_2 - r_1)}{4\pi r_1 r_2} = -k(T_2 - T_1)$$

$$q_r = \frac{4\pi k r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}$$

The thermal resistance is expressed as

Q = ΔT / ($r_2 - r_1$) / 4πk $r_1 r_2$ where the resistance is given by ($r_2 - r_1$) / 4πk $r_1 r_2$

Geometric mean radius is given by $r_m^2 = r_1 . r_2$ and hence

R_{th} = (r_2-r_1) / A_m . k where $A_m = 4\pi\ r_m^2$ and A_m is the geometric mean area.

Similarly, rate of heat flow for a composite spherical shell with one layer of insulation is given by Q = ΔT / [($r_2 - r_1$) / $4\pi k_1$ $r_1 r_2$ + ($r_3 - r_2$) / $4\pi k_2$ $r_2 r_3$]

P.No. 1. A furnace wall consists of two layers, 22.5cm of fire brick(k=1.2kcal/hr m °C)and 12.5cm of insulating brick (k=0.15kcal/hr m °C) . The temperature inside the furnace is 1650 °C and the inside heat transfer coefficient is 60kcal/hr m °C . The temperature of the surrounding atmosphere is 27°C and the outside heat transfer coefficient is 10kcal/hr m^2 °C . Determine the rate of heat of loss per square meter of the wall.

Solution:

L_1 - 22.5 x 10^{-2} m

L_2 - 12.5 x 10^{-2} m

k_1 - 1.2 kcal/hr m °C

k_2 - 0.15 kcal/hr m °C

h_a - 60 kcal/hr m^2 °C

h_b - 10kcal/hr m^2 °C

T_a – 1650 °C

T_b – 27 °C

$$Q = \frac{(T_a - T_b)}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_b A}}$$

Q / A = 1426.8 W / m^2

P.No.2. A pipe carrying steam at 220 °C has an I.D. of 15cm. The convection coefficient on the inside wall is 60W/m^2K . The pipe wall thickness is 15mm and the thermal conductivity is 35W/mK . The outside is exposed to a chemical at 130°C with a convection coefficient of 15W/m^2K . If the pipe wall is covered with two insulation layers, the first 3cm thickness with k=0.12W/mK and the second 4cm thickness with k= 0.35W/m K. Determine the rate of heat transfer.

Solution :

r_1 - 75 x 10^{-3} m

r_2 - 90 x 10^{-3} m

r_3 - 120 x 10^{-3} m

r_4 - 160 x 10^{-3} m

k_1 - 35 W / m K

k_2 - 0.12 W / m K

k_2 - 0.35 W / m K

h_a - 60 W / m^2 K

h_b – 15 W / m^2 K

$$Q = \frac{2\pi L\,(T_a - T_b)}{\frac{1}{h_a r_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k_3} + \frac{1}{h_b r_4}}$$

Q = 146.32 W

Analogy between heat flow and electricity:

Heat flow is represented by Fourier's law of heat conduction whereas electrical flow is represented by ohm's law.

Heat flow = Temperature gradient / thermal resistance

Ohm's law is given by

Electrical flow = voltage drop / electrical resistance

The various parameters analogous to each other in both the laws are heat flow and electrical flow, voltage drop with temperature gradient, thermal and electrical resistance.