SATHYABAMA UNIVERSITY **FACULTY OF MECHANICAL ENGINEERING**

SME1204	STRENGTH OF MATERIALS (For Mechanical)	L	T	P	Credits	Total Marks
		3	1	0	4	100

COURSE OBJECTIVES

- To gain knowledge of different types of stresses, strain and deformation induced in the mechanical components due to external loads.
- To study the distribution of various stresses in the mechanical elements such as beams, shafts etc.
- To study the effect of component dimensions and shapes on the stresses and deformations.

UNIT 1 STRESS STRAIN DEFORMATION OF SOLIDS **12 Hrs.**

Rigid and Deformable bodies - Strength, Stiffness and Stability - Stresses; Tensile, Compressive and Shear - Deformation of simple and compound bars under axial load - Thermal stress - Elastic constants , Strain energy and unit strain energy - Strain energy in uniaxial loads Principal stress and strain, Mohr's stress circle., Theory of failure.

UNIT 2 BEAMS - LOADS AND STRESSES **12 Hrs.**

Types of beams: Supports and Loads - Shear force and Bending Moment in beams - Cantilever, Simply supported and Overhanging beams - Stresses in beams - Theory of simple bending - Stress variation along the length and in the beam section - Effect of shape of beam section on stress induced - Shear stresses in beams - Shear flow

UNIT 3 SPRINGS **12 Hrs.**

Helical and Leaf Springs- deflection of springs by energy method- helical springs under axial load and under axial twist (respectively for circular and square cross sections) axial load and twisting moment acting simultaneously both for open and closed coiled springs- laminated springs. Application to close-coiled helical springs - Maximum shear stress in spring section including Wahl Factor - Deflection of helical coil springs under axial loads - Design of helical coil springs - stresses in helical coil springs under torsion loads.

UNIT 4 TORSION **12 Hrs.**

Analysis of torsion of circular bars - Shear stress distribution - Bars of Solid and hollow circular section - Stepped shaft - Twist and torsion stiffness - Compound shafts - Fixed and simply supported shafts - Columns and Struts: Combined bending and direct stress, middle third and middle quarter rules. Struts with different end conditions - Euler's theory and experimental results - Ranking Gardon Formulae

UNIT 5 CYLINDERS & CURVED BEAMS **12 Hrs.**

Thin cylinders & spheres - Hoop and axial stresses and strain.- Volumetric strain.- Thick cylinders- Radial, axial and circumferential stresses in thick cylinders subjected to internal or external pressures - Compound cylinders. Stresses due to interference fits Curved Beams - Bending of beams with large initial curvature, position of neutral axis for rectangular - trapezoidal and circular cross section- stress in crane hooks, stress in circular rings subjected to tension or compression

Max.60 Hours.

TEXT / REFERENCE BOOKS

1. Nash W.A, "Theory and problems in Strength of Materials", Schaum Outline Series,McGraw-Hill Book Co, New York, 1995
2. Kazimi S.M.A, "Solid Mechanics", Tata McGraw-Hill Publishing Co., New Delhi,1981.
3. Ryder G.H, "Strength of Materials, Macmillan India Ltd"., Third Edition, 2002
4. Ray Hulse, Keith Sherwin & Jack Cain, "Solid Mechanics", Palgrave ANE Books,2004.
5. Bansal.R.K., "Strength of Materials", 4th Edition, Laxmi Publications, 2007.

END SEMESTER EXAM QUESTION PAPER PATTERN

Max. Marks : 80 **Exam Duration : 3 Hrs.**

PART A : 2 Questions from each unit, each carrying 2 marks **20 Marks**

PART B : 2 Questions from each unit with internal choice, each carrying 12 marks **60 Marks**

COURSE MATERIAL

INTRODUCTION

The theory of strength of Materials was developed over several centuries by a judicious combination of mathematical analysis, scientific observations and experimental results. Ancient structures had been constructed based on thumb rules developed through experience and intuition of their builders.

A structure designed to carry loads comprises various members such as beams, columns and slabs. It is essential to know the load carrying capacity of various members of structure in order to determine their dimensions for the minimum rigidity and stability of isolated structural members such as beams and columns.

The theory of strength of materials is presented in this book in a systematic way to enable students understand the basic principles and prepare themselves to the tasks of designing large structures and systems subsequently. It should be appreciated that even awe inspiring structures such as bridges, high rise towers tunnels and space crafts, rely on these principle of their analysis and design

HISTORICAL REVIEW

Though ancient civilizations could boast of several magnificent structures, very little information is available on the analytical and design principles adopted by their builders. Most of the developments can be traced to the civilizations of Asia, Egypt, Greece and Rome.Greek philosophers Aristotle (384-322 BC) and Archimedes (287 – 212) who formulated significant fundamental principles of statics. Though Romans were generally excellent builders, they apparently had little knowledge about stress analysis. The strength of materials were formulated by Leonardo da Vinci (AD 1452 – 1549, Italy) arguably the greatest scientist and artist of all times. It was much later in the sixteenth century that Galileo Galilei (AD 1564 – 1642, Itlay) commenced his studies on the strength of materials and behavior of structures. Robe Hooke (1635 – 1703) made one of the most significant observations in 1678 that materials displayed a certain relation between the stress applied and the strain induced. Mariotte (1620 – 1684), Jacob Bernoulli (1667 – 1748), Daniel Bernoulli (1700 – 1782), Euler (1707 – 1783), Lagrange (1736 – 1813), Parent (1666 – 1748), Columb (1736 – 1806) and Navier (1785 – 1836), among several others made the most significant contributions.

The first complete elastic analysis for flexure of beams was presented by Columb in 1773 but his paper failed to receive the attention it deserved until 1825 when Navier published a book on strength of materials. Rapid industrial growth of the nineteenth century gave a further impetus to scientific investigations; several researchers and scientist advanced the frontiers of knowledge to new horizons.

The simple theories formulated in the earlier centuries have been extended to complex structural configuration and load conditions. Engineers are expected not only to design but also to check the performance of structures under various limit states such a s collapse, deflection and crack widths. The emphasis is always on safety, economy, durability, nevertheless.

SIMPLE STRESSES AND STRAINS

INTRODUCTION

Within elastic stage, the resisting force equals applied load. This resisting force per unit area is called stress or intensity of stress.

STRESS

The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load or force. The load is applied on the body while the stress is induced in the material of the body. A loaded member remains in equilibrium when the resistance offered by the member against the deformation and the applied load are equal.

Mathematically stress is written as, $\sigma = \dfrac{P}{A}$

where σ = Stress (also called intensity of stress),
P = Cross-Sectional or load, and
A = Cross-Sectional area.

In the S.I. Units, the force is expressed in newtons (Written as N) and area is expressed as m^2. Hence, unit stress becomes as N/m^2. The area is also expressed in millimetre square then unit of force becomes as N/mm^2.

$$1\ N/m^2 = 1\ N/(100cm)^2 = 1\ N/10^4\ cm^3$$

$$= 10^4\ N/cm^2 \text{ or } 10^{-6} N/mm^2 \qquad \left(\therefore = \frac{1}{cm^2} = \frac{1}{10^2\, mm^2}\right)$$

STRAIN

When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain. Strain is dimensionless.

$e = \dfrac{\delta\ell}{\ell}$ δl - Change in length in mm

l - original length in mm

Strain may be:-

1. Tensile strain,
2. Compressive strain
2. Volumetric strain, and
4. Shear strain

If there is some increase in length of a body due to external force, then the ratio of increase of length to the original length of body is known as tensile strain. But if there is some decrease in length of the body, then the ratio of decrease of the length of the body to the original length is known as compressive strain. The ratio of change of volume of the body to the original volume is known as volumetric strain. The strain produced by shear stress is known as shear strain.

TYPES OF STRESSES

The stress may be normal stress or a shear stress.

Normal stress is the stress which acts in a direction perpendicular to the areas. It is represented by σ (sigma). The normal stress is further divided into tensile stress and compressive stress.

Tensile Stress. The stress induced in a body, when subjected to two equal and opposite pulls as shown in Fig.1.1 (α) as a result of which there is an increase in length, is known as tensile stress. The ratio of increase in length to the original length is known as tensile strain. The tensile stress acts normal to the area and it pulls on the area.

Let P = Full (or force) acting on the body.
A = Cross-sectional area of the body.
L = Original length of the body
dL = Increase in length due to pull P acting on the body σ
= Stress induced in the body, and
e = Strain (i.e., tensile strain)

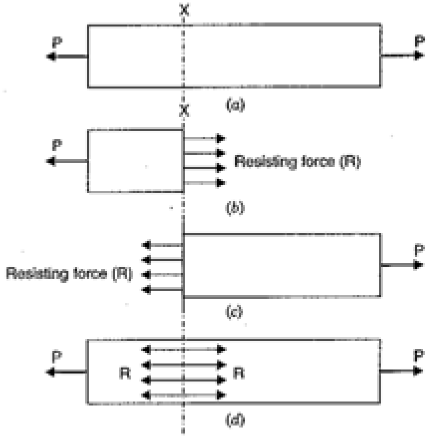


Fig. 1.1

Fig.1.1 (∝) shown a bar subjected to a tensile force P as its ends. Consider χ-χ, which divides the bar into two parts. The part left to the section χ-χ, will be in equilibrium if P = resisting force (R). This is shown in Fig.1.1 (b). Similarly the part right to the sections χ-χ, will be in equilibrium if P = Resisting force as shown in Fig.1.1 (c). This relating force per unit area is known as stress or intensity of stress.

$$\therefore \text{Tensile} = \sigma \frac{\text{Reisting force (R)}}{\text{Cross - sectional area}} = \frac{\text{Tensile Load (P)}}{\text{A}} \qquad (\therefore \text{P= R})$$

or $$\sigma = \frac{P}{A} \qquad \text{... (1.1)}$$

And tensile strain is given by,

$$\text{e} = \frac{\text{Increase in length}}{\text{Original Length}} = \frac{\text{dL}}{\text{L}} \qquad \text{... (1.2)}$$

Compressive Stress

The stress induced in a body, when subjected to two equal and opposite pushes as shown in Fig.1.2. (α) as a result of which there is a decrease in length of the body, is shown as compressive stress. And the ratio of decrease in length to the original length is known as compressive strain. The compressive stress acts normal to the area and it pushes on the area.

Let an axial push *P* is acting on a body is cross-sectional area A. Due to external push *P*, let the original length *L* of the body decrease by *dL*.

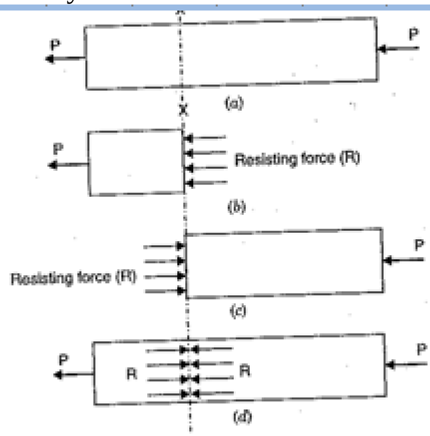


Fig. 1.2

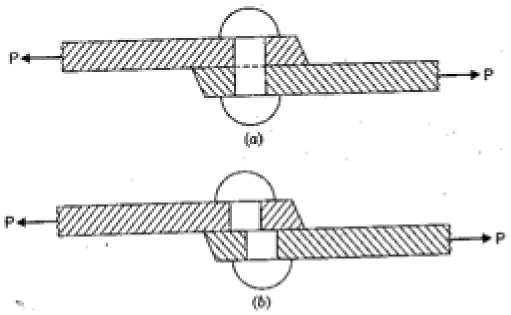
The compressive stress is given by,

$$\sigma = \frac{\text{Reisting force (R)}}{\text{Area (A)}} = \frac{\text{Push (P)}}{\text{Area (A)}} = \frac{P}{A}$$

And compressive strain is given by,

$$e = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{dL}{L}$$

Shear stress: The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as shown in Fig.1.3 as a result of which the body tends to shear off across the section, is known as shear stress. The corresponding strain is known as shear strain. The shear stress is the stress which acts tangential to the area. It is represented byτ.



As the bottom face of the block is fixed, the face ABCD will be distorted to ABC, D through an angle ϕ as a result of force P as shown in Fig.1.4 (d).

And shear strain (ϕ) is given by

$$\phi = \frac{\text{Transversal displacement}}{\text{Distance AD}}$$

$$\text{or } \phi = \frac{DD_1 = \frac{dl}{h}}{AD} \qquad ...(1.4)$$

ELASTICITY AND ELASTIC LIMIT

When an external force acts on a body tends to undergo some deformation. If the external force is removed and the body comes back to its origin shape and size (which means the deformation disappears completely), the body), the body is known as elastic body. The property by virtue of which certain materials return back to their original position after the removal of the external force, is called elasticity.

The body will regain its previous shape and size only when the deformation caused by the external force, is within a certain limit. Thus there is a limiting value of force upto and within which, the deformation completely disappears on the removal of the force. The value of stress corresponding to this limiting force is known as the elastic limit of the material.

If the external force is so large that the stress exceeds the elastic limit, the material loses to some extent its property of elasticity. If now the force is removed, the material will not return to the origin shape and size and there will be residual deformation in the material.

HOOKES LAW AND ELASTIC MODULII

Hooke's Law states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit. This constant is known as Module of Elasticity or Modulus of Rigidity or Elastic Moduli.

MODULUS OF ELASTICITY (OR YOUNG'S MODULUS)

The ratio of tensile or compressive stress to the corresponding strain is a constant. This ratio is known as Young's Modulus or Modulus of Elasticity and is denoted by E.

$$\therefore \qquad E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} \text{ or } \frac{\text{Compressive Stress}}{\text{Compressive Strain}}$$

$$\text{or} \qquad E = \frac{\sigma}{e} \qquad \ldots (1.5)$$

Modulus of Rigidity or Shear Modulus: The ratio of shear stress to the corresponding shear strain within the elastic limit, is known as Modulus or Rigidity or Shear Modulus. This is denoted by C or G or N.

$$\therefore \qquad C \text{ (or G or N)} = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{x}{\varphi} \qquad \ldots (1.6)$$

Let us define factor of safety also.

FACTOR OF SAFETY

It is defined as the ratio of ultimate tensile stress to the working (or permissible) stress. Mathematically it is written as

$$\text{Factor of Safety} = \frac{\text{Ultimate Stress}}{\text{Pemissible Stress}} \qquad \ldots (1.7)$$

CONSTITUTIVE RELATIONSHIP BETWEEN STRESS AND STRAIN

For One Dimensional Stress System. The relationship between stress and strain for unidirectional stress (i.e., for normal stress in one direction only) is given by Hooke's law, which states that when a material is loaded within its elastic limit, the normal stress developed is proportional to the strain produced. This means that the ratio of the normal stress to the corresponding strain is a constant within the elastic limit. This constant is represented by E and is known as modulus of elasticity or Young's modulus of elasticity.

$$\therefore \qquad = \frac{\text{Normal Stress}}{\text{Corresponding Strain}} = \text{Constant or } \frac{\sigma}{e} = E$$

where σ = Normal stress, e = strain and E = Young's Modulus

$$\text{or} \qquad e = \frac{\sigma}{E} \qquad \ldots [1.7 \text{ (A)}]$$

The above equation gives the stress and strain relation for the normal stress in one direction.

For Two Dimensional Stress System.: Before knowing the relationship between stress and strain for two-dimensional stress system, we shall have to define longitudinal strain, lateral strain, and Poisson's ratio.

Longitudinal Strain: When a body is subjected to an axial tensile load, there is an increase in the length of the body. But at the same time there is a decrease in other dimensions of the body at right angles to the line of action of the applied. Thus the body is having axial deformation and also deformation at right angles to the line of action of the applied load (i.e., lateral deformation).

The ratio of axial deformation to the original length of the body is known as longitudinal (or linear) strain. The longitudinal strain is also defined as the deformation of the body per unit length in the direction of the applied load.

Let L = Length of the body,
P = Tensile force acting on the body.
δL = Increase in the length of the body in the direction of P.

Then, longitudinal strain = $\dfrac{\delta L}{L}$

Lateral strain. The strain at right angles to the direction of applied load is known as lateral strain. Let a rectangular bar of length L, breadth b and depth δ is subjected to an axial tensile load *P* as shown in Fig.1.6. The length of the bar will increase while the breadth and depth will decrease.

Let L = Length of the body,

δb = Decrease in breadth, and

δd = Decrease in depth.

Then longitudinal strain = $\frac{\delta L}{L}$ ... [1.7 (B)]

and lateral strain = $\frac{\delta b}{b}$ or $\frac{\delta d}{d}$ ... [1.7 (C)]

Note:

(i) If longitudinal strain is tensile, the lateral strains will be compressive.
(ii) If longitudinal strain is compressive then lateral strains will be tensile.
(iii) Hence every longitudinal strain in the direction of load is accompanied by lateral strains of the opposite kind in all directions perpendicular to the load.

Poisson's Ratio. The ratio is lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by μ. Hence mathematically

Poisson's ratio, $\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$ ... [1.7 (D)]

or Lateral strain = μ x Longitudinal strain

As lateral strain is opposite in sign to longitudinal strain, t\hence algebraically, lateral strain is written as

Relationship between and strain: Consider a two dimensional figure ABCD, subjected to two mutually perpendicular stress σ_1 and σ_2

Longitudinal strain and will be equal to $\frac{\delta_1}{E}$ whereas the strain in the direction of y will be lateral strain and will be equal to - μ x $\frac{\delta_1}{E}$. (∴ Lateral strain = - μx longitudinal strain)

The above two equation, gives the stress and strain relationship for the two dimensional stress system. In the above equations, tensile stress is taken to be positive whereas the compressive stress negative.

For Three Dimensional Stress System: It shows a three-dimensional body subjected to three orthogonal normal stress σ_1, σ_2, σ_3 acting in the directions of x, y and z respectively.

Consider the strains produced by each stress separately

Similarly the stress σ_2 will produced strain $\frac{\delta_2}{E}$ in the direction of y and strain of - μ $\frac{\delta_2}{E}$ in the direction of x and y each.

Also the stress σ_2 will produce strain $\frac{\delta_3}{E}$ in the direction of z and strain of - μ x $\frac{\delta_3}{E}$ in the direction of x and y.

$$e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$ ... [1.7 (H)]

$$e_2 = \frac{\sigma_3}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$ ... [1.7 (J)]

$$e_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$ ... [1.7 (J)]

and The above three equations giver the stress and strain relationship for the three orthogonal normal stress system.

Problem 1: A rod 150cm long and of diameter 2.0cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is 2 x 10^5 N/mm^2, determine:

(i) the stress
(ii) the strain, and
(iii) the elongation of the rods.

Sol. Given: Length the rod, L = 150 cm,
Diameter of rod, D = 2.0 cm = 20mm

$\therefore$ Area, $A = \frac{\pi}{4}(20)^2 - 100\pi \ mm^2$

Axial pull, P = 20 kN = 20,000N

Modulus of elasticity, E = 2.0 x 10^5 N/mm^2

(i) The stress (σ) is given equation (1.1) as

$$\sigma = \frac{P}{A} = \frac{2000}{100\pi} - 63.662 \text{ N/mm}^2\text{, Ans.}$$

(ii) Using equation (1.5) the strain is obtained as

$$E = \frac{\sigma}{e}$$

$$\therefore \text{Strain, } e = E\frac{\sigma}{E} = \frac{63.662}{2 \times 10^6} = 0.000318. \text{ Ans}$$

(iii) Elongation is obtained by using equation (1.2) as

$$e = \frac{dL}{L}$$

$\therefore$ Elongation, dL = e x L
= 0.000318 x 150 = 0.0477cm. Ans

Problem 2: Find the minimum diameter of a steel wire, which is used to raise σ load of 4000 N if the stress in the rod is not to exceed 95MN/m^2.

Sol. Given : Load, P = 4000N

Stress, $\sigma = 95MN/m^2 = 95 \times 10^6 N/m^2$ ($\therefore$ M=Mega=10^6)
= 95N/mm^2 ($\therefore$ $10^6 N/m^2 = 1N/mm^2$)

Let D = Diameter of wire in mm

$\therefore$ Area, $A = \frac{\pi}{4}D^2$

Now Stress = $\frac{\text{Load}}{\text{Area}} = \frac{P}{A}$

$$95 = \frac{4000}{\frac{\pi}{4}D^2} = \frac{4000 \times 4}{\pi D^2} \quad \text{or} \quad D^2 = \frac{4000 \times 4}{\pi \times 95} = 53.61$$

D = 7.32mm Ans.

Problem 3: A tensile test was conducted on a mild steel bar. The following data was obtained from the test:

(i)	Diameter of the steel bar	=	3cm
(ii)	Gauge length of the bar	=	20cm
(iii)	Load at elastic limit	=	250 kN
(iv)	Extension at a load of 150 kN	=	0.21mm
(v)	Maximum load	=	380 kN
(vi)	Total extension	=	60mm
(vii)	Diameter of the rod at the failure	=	2.25cm

Determine: (a) the Young's Modulus, (b) the stress elastic limit
(c) the percentage elongation, and (d) the percentage decrease in area.

Sol. Area of rod, $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(3)^2 cm^2$

$= 7.06835\ cm^2 = 7.0685 \times 10^{-4}\ m^2 \left[\therefore cm^2 = \left(\frac{1}{100}m\right)^2\right]$

(a) To find Young's modulus, first calculate the value of stress and strain within elastic limit. The load at elastic limit it given but the extension corresponding to the load of elastic limit is not given. But a load 150 kN (which is within elastic limit) and corresponding extension of 0.21mm are given. Hence these values are used for stress and strain within elastic limit

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{150 \times 1000}{7.0685 \times 10^{-4}}\ N/m^2 \qquad (\therefore\ 1\ kN = 1000\ N)$$

$$= 21220.9 \times 10^4\ N/m^2$$

and $$\text{Strain} = \frac{\text{Increase in length (or Extension)}}{\text{Original Length (or Guage length)}}$$

$$= \frac{0.21mm}{20 \times 10mm} = 0.00105$$

$$E = \frac{\text{Stress}}{\text{Strain}} \times \frac{21220.9\times10^4}{0.00105} = 20209523 \times 10^4\ N/m^2$$

$$= 202.095 \times 10^9\ N/m^2 \qquad (\therefore\ 10^9 = \text{Giga} = G)$$

= 202.095 x GN/m² Ans.

(b) The stress at the elastic limit is given by

$$Stress = \frac{\text{Load at elastic limit}}{\text{Area}} = \frac{250\times1000}{7.0685\times10^{-4}}$$

$= 35368 \times 10^4\ N/m^2$

$= 353.68 \times 10^6\ N/m^2 \qquad (\therefore\ 10^6 = \text{Mega} = M)$

= 353.68 MN/m². Ans.

(c) The percentage decrease is obtained as,
percentage elongation

$$= \frac{\text{Total Increase in length}}{\text{Original length (or guage length)}} \times 100$$

$$= \frac{60\text{mm}}{20 \text{ x } 10\text{mm}} \text{ x } 100 = 30\% \quad \text{Ans.}$$

(d) The percentage decrease in area is obtained as

percentage decrease in area.

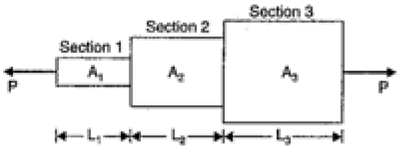
$$= \frac{(\text{Original area - Area at the failure})}{\text{Original area}} \text{ x } 100$$

$$= \frac{\left(\frac{\pi}{4} \text{ x } 3^2 - \frac{\pi}{4} \text{ x } 2.25^2\right)}{\frac{\pi}{4} \text{ x } 3^2} \text{ x } 100$$

$$= \left(\frac{3^2 - 2.25^2}{3^2}\right) \text{ x } 100 = \frac{(9 - 5.0625)}{9} \text{ x } 100 = 43.75\% \quad \text{Ans.}$$

ANALYSISZS OF BARS OF VARYING SECTIONS

A bar of different lengths and of different diameters (and hence of different cross-sectional areas) is shown in Fig.1.4 (α). Let this bar is subjected to an axial load *P*.

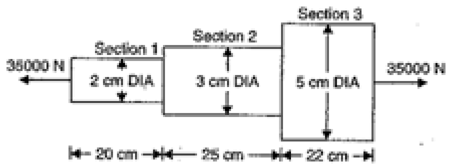


Though each section is subjected to the same axial load *P*, yet the stresses, strains and change in length will be different. The total change in length will be obtained by adding the changes in length of individual section

Let P = Axial load acting on the bar,
L_1 = Length of section 1,
A_1 = Cross-Sectional area of section 1,
L_2, A_2 = Length and cross-sectional areas of section 2,
L_3, A_3 = Length and cross-sectional areas of section 3, and
E = Young's modulus for the bar.

Problem 4: An axial pull of 35000 N is acting on a bar consisting of three lengths as shown in Fig.1.6 (b). If the Young's modulus = 2.1 x 10^5 N/mm^2, determine.

(i) Stresses in each section and
(ii) total extension of the bar



Sol. Given:
Axial pull, P = 35000 N
Length of section 1, L_1 = 20cm = 220mm
Dia. of Section 1, D_1 = 2cm = 20mm

$\therefore$ Area of Section 1, $A_1 = \frac{\pi}{4}(20^2) = 100\pi \text{ mm}^2$

Length of section 2, L_2 = 25cm = 250mm
Dia. of Section 2, D_2 = 3cm = 30mm

$\therefore$ Area of Section 2, $A_2 = \frac{\pi}{4}(30^2) = 225\,\pi \text{ mm}^2$

Length of section 3, L_3 = 22cm = 220mm
Dia. of Section 3, D_3 = 5cm = 50mm

$\therefore$ Area of Section 2, $A_3 = \frac{\pi}{4}(50^2) = 625\,\pi \text{ mm}^2$

Young's Modulus, E = 2.1 x 10^5 N/mm^2

(i) Stress in each section

Stress in section 1, $\sigma_1 = \frac{\text{Axial load}}{\text{Area of Section 1}}$

$$= \frac{P}{A_1} = \frac{35000}{100\pi} = 111.408 \text{N/mm}^2. \text{ Ans.}$$

Stress in section 2, $= \frac{P}{A_2} = \frac{35000}{225 \times \pi} = 49.516 \text{N/mm}^2.$ Ans.

Stress in section 3, $= \frac{P}{A_3} = \frac{35000}{625 \times \pi} = 17.825 \text{ N/mm}^2.$ Ans.

(ii) Total extension of the bar

Using equation (1.8), we get

$$\text{Total Extension} = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

$$= \frac{35000}{2.1 \times 10^5}\left(\frac{200}{100\pi} + \frac{250}{225 \times \pi} + \frac{230}{625 \times \pi}\right)$$

$$= \frac{35000}{2.1 \times 10^5}(6.366 + 3.536 + 1.120) = 0.183\text{mm Ans.}$$

Problem 5: A member formed by connecting a steel bar to aluminium for bar is shown in Fig.1.7. Assuming that the bars are presented from buckling, sideways, calculate the magnitude of force P that will causes the total length of the member to decrease 0.25mm. The values of elastic modulus for steel and aluminium are 2.1 x 10^6 N/mm^2 and 7 x 10^4 N/mm^2 respectively.

Sol. Given

Length of Steel bar, L_1 = 30c m = 300mm
Area of Steel bar, A_1 = 5 x 5 = 25m^2 = 250mm^2
Elastic modulus for steel bar, E_1 = 2.1 x 10^5 N/mm^2
Length of Aluminium bar, L_2 = 38cm = 380mm
Area of Aluminium bar A_2 = 10 x 10 = 100cm^2 = 1000mm^2
Elastic modulus for aluminium bar E_2 = 7 x 10^4 N/mm^2
Total Decrease in length, dL = 0.25mm
Let P = Required force

As both the bars are made of different materials, hence total change in the lengths of the bar is given by equation (1.9)

$$dL = P \; \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2}$$

or

$$0.25 = P \left(\frac{300}{2.1 \text{ x } 10^5 \text{ x } 2500} + \frac{380}{7 \text{ x } 10^4 \text{ x } 1000} \right)$$

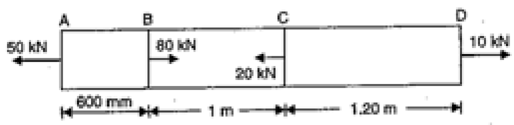
$$= \quad P\,(5.714 \text{ x } 10^{-7} + 5.428 \text{ x } 10^{-7}) = P \text{ x } 11.142 \text{ x } 10^{-7}$$

$$P \quad = \quad \frac{0.25}{11.142 \text{ x } 10^{-7}} + \frac{0.25 \text{ x } 10^7}{11.142} = 2.2437 \text{ x } 10^5 = 224.37 \text{ kN. Ans.}$$

Principle of Superposition:

When a number of Loads are acting on a body, the resulting strain, according to principle of superposition, will be the algebraic sum of strains caused by individual loads. While, using this principle for an elastic body which is subjected to a number of direct forces (tensile or compressive) at different sections along the length of the body, first the free body diagram of individual section is drawn. Then the deformation of the each section is obtained. The total deformation of the body will be then equal to the algebraic sum of deformation of the individual sections.

Problem 6: A brass bar, having cross-sectional area of 1000 mm^2 , is subjected to axial forces as shown in Fig.



Find the total elongation of the bar, Take $E = 1.05 \text{ x } 10^5 \text{ N/mm}^2$

Sol. Given:

Area $\quad A = 1000 mm^2$

Value of $\quad E = 1.05 \text{ x } 10^5 \text{ N/mm}^2$

Let $\quad d$ = Total elongation of the bar

The force of 80 kN acting at B is split up into three forces of 50 kN, 20 kN and 10 kN. Then the part AB of the bar will be subjected to a tensile load of 50 kN, part BC is subjected to a compressive load of 20 kN and part BD is subjected to a compressive load of 10 kN as shown in Fig.

Part AB. This part is subjected to a tensile load of 50kN. Hence there will be increase in length of this part.,

$\therefore$ Increase in the length of AB

$$= \frac{P_1}{AE} \text{ x } L_1 \quad = \frac{500 \text{ x } 1000}{1000 \text{ x } 1.05 \text{ x } 10^5} \text{ x } 600$$

($\because P_1 = 50,000$ N, $L_1 = 600$mm)

$= 0.2857$

Part BC. This part is subjected to a compressive load of 20kN or 20,000 N. Hence there will be decrease in length of this part.

∴ Decrease in the length of BC

$$= \frac{P_2}{AE} \times L_2 = \frac{20,000}{1000 \times 1.05 \times 10^5} \times 1000 \qquad (\because L_2 = 1m = 1000mm)$$

= 0.1904

Part BD. The part is subjected to a compressive load of 10kN or 10,000 N. Hence there will be decrease in length of this part.

∴ Decrease in the length of BC

$$= \frac{P_3}{AE} \times L_3 = \frac{10,000}{1000 \times 1.05 \times 10^5} \times 2200 \qquad (\because L_2 = 1.2 + 1.22m \text{ or } 2200m)$$

= 0.2095

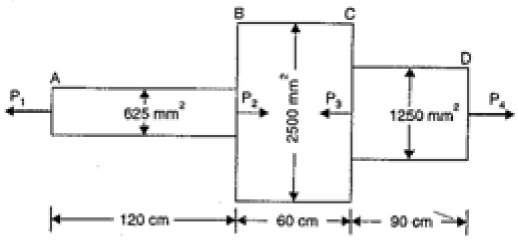
∴ Total elongation of bar = 0.2857 – 0.1904 – 0.2095)

(Taking +ve sign for increase in length and –ve sign for decrease in length

=- 0.1142mm. Ans.

Negative sign shows, that there will be decrease in length of the bar.

Problem 7: A Member ABCD is subjected to point loads P_1, P_2, P and P_4 as shown in Fig.



Calculate the force P_2 necessary for equilibrium, if P_1 = 45 kN, P_3 = 450 kN and P_4 = 130 kN. Determine the total elongation of the member, assuming the modulus of elasticity to be 2.1 x 10^5 N/mm^2.

Given:

Part AB: Area, A_1 = 625 mm^2 and
Length L_1 = 120cm = 1200mm
Part BC: Area, A_2 = 2500 mm^2 and
Length L_2 = 60cm = 600mm
Part CD: Area A_3 = 12.0mm^2 and
Length L_3 = 90cm = 900mm
Value of E = 2.1 x 10^5 N/mm^2
Value of P_2 necessary for equilibrium

Resolving the force on the rod along its (i.e., equating the forces acting towards right to those acting towards left) we get

$P_1 + P_3 = P_2 + P_4$

But, P_1 = 45kN, P_3 = 450 kN and P_4 = 130kN

∴ 45 + 450 = P_2 = 130 or P_2 = 495 – 130 = 365 kN

The force of 365 kN acting at B is split into two forces of 45 kN and 320 kN (i.e., 365 – 45 = 320 kN) The force of 450 kN acting at C is split into two forces of 320 kN and 130 kN (i.e., 450 – 320 = 130 kN) as shown Fig.
It is clear that part AB is subjected to a tensile load of 45kN, part BC is subjected to a compressive load of 320 kN and par CD is subjected to a tensile load 130 kN.

Hence for part AB, there will be increase in length; for part BC there will be decrease in length and for past CD there will be increase in length.

∴ Increase in length of AB

$$= \frac{P}{A_1 E} \text{ x } L_1 = \frac{45000}{625 \text{ x } 2.1 \text{ x } 10^5} \text{ x } 1200 \; (\because P = 45 \text{ kN} = 45000 \text{ N})$$

= 0.4114 mm

∴ Decrease in length of BC

$$= \frac{P}{A_2 E} \text{ x } L_2 = \frac{320{,}000}{2500 \text{ x } 2.1 \text{ x } 10^5} \text{ x } 600 \quad (\because P = 320 \text{ kN} = 320000 \text{ N})$$

= 0.3657 mm

Increase in length of CD

$$= \frac{P}{A_3 E} \text{ x } L_3 = \frac{130{,}000}{1250 \text{ x } 2.1 \text{ x } 10^5} \text{ x } 900 \quad (\because P = 130 \text{ kN} = 130000 \text{ N})$$

Total change in the length of member

= 0.4114 – 0.3657 + 0.4457

(Taking +ve for increase in length and –ve sign for decrease in length)

= 0.4914mm (extension) Ans.

Problem 8: A rod, which tapers uniformly from 40mm diameter to 20mm diameter in a length of 400 mm is subjected to an axial load of 5000 N. If E = 2.1 x 10^6 N/mm^2, find the extension of rod.

Sol. Given

Larger diameter D_1 = 40mm
Smaller diameter D_2 = 20mm
Length of rod, L = 400mm
Axial load P = 5000 N
Young's modulus E – 2.1 x 10^5 N/mm^2
Let dL = Total extension of the rod
Using equation (1.10),

$$dL = \frac{4PL}{\pi E D_1 D_2} = \frac{4 \text{ x } 5000 \text{ x } 400}{\pi \text{ x } 2.1 \text{ x } 10^5 \text{ x } 40 \text{ x } 20}$$

= 0.01515mm Ans.

Problem 9: Find the modulus of elasticity for a rod, which tapers uniformly from 20mm, to 15mm diameter in a length of 350mm. The rod is subjected to an axial load of 5.5 kN and extension of the rod is 0.025mm.

Given:

Larger diameter D_1 = 30mm
Smaller diameter D_2 = 15mm
Length of rod, L = 350mm
Axial load P = 5.5 kN = 5500 N
Extension dL = 0.025mm
Using equation (1.10), We get

$$dL = \frac{4PL}{\pi E D_1 D_2}$$

or $$E = \frac{4PL}{\pi D_1 D_2 dL} = \frac{4 \text{ x } 5000 \text{ x } 350}{\pi \text{ x } 30 \text{ x } 15 \text{ x } 0.025}$$

= 217865 N/mm^2 or 2.17865 x 10^5 N/mm^2. Ans.

Problem 10: A rectangular bar made of steel is 2.8m long and 15mm thick. The rod is subjected to an axial tensile load of 40kN. The width of the rod varies from 75mm at one end to 30mm at the other. Find the extension of the rod if E = 2 x 10^5 N/mm^2.

Given

Larger L_1 = 2.8 m = 2800mm
Thickness t = 15mm
Axial load P = 40 kN = 40,000 N
Width at bigger end a = 75mm
Width at smaller end b = 30mm
Value of E = 2 x 10^5 N/mm^2
Let dL = Extension of the rod
Using equation (1.), We get

$$dL = \frac{PL}{Et(a-b)} \log_e \frac{a}{b}$$

$$= \frac{4000 \text{ x } 2800}{2 \text{ x } 10^5 \text{ x } 15(75-30)} \log_e \frac{75}{30}$$

= 0.8296 x 0.9163 = 0.76mm Ans.

Problem 11: The extension is a rectangular steel bar of length 400mm and thickness 10mm, is found to be 0.21 mm. The bar tapers uniformly in width from 100mm to 50mm. If E for the bar is 2 x 10^5 N/mm^2, determine the axial load on the bar.

Given

Extension dL = 0.21mm
Length L = 400mm
Thickness t = 10mm
Width at bigger end a = 100mm
Width at smaller end b = 50mm
Value of E = 2 x 10^5 N/mm^2
Let P = axial load

Using equation (1), We get

$$dL = \frac{PL}{Et(a-b)} \log_e \left(\frac{a}{b}\right)$$

or $$0.21 = \frac{P \times 400}{2 \times 10^5 \times 10(100-50)} \log_e \frac{100}{50}$$

$= 0.000004 \text{ P} \times 0.6931$

$$\therefore P = \frac{0.21}{0.000004 \times 0.6931} = 75746 \text{ N}$$

$= 75.746$ kN Ans.

ANALYSIS OF BARS OF COMPOSITE SECTIONS

A bar, made up two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compressive when subjected to an axial tensile or compressive loads, is called a composite bar. For the composite bar the following two points are important:

1. The extension or compression in each bar is equal. Hence determination per unit length i.e. strain in each bar is equal.
2. The total external load on the composite bar is equal to the sum of the loads carried by each different material.

Problem 12: A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter of 4cm. The composite bar is ten subjected to an axial pull of 45000 N. If the length of each bar is equal to 15cm, determine.

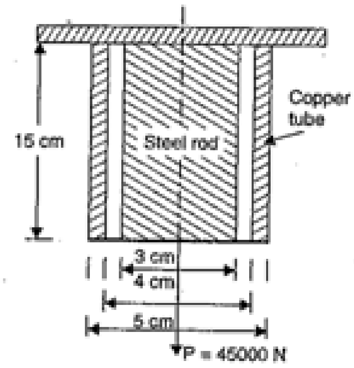
(i) The stresses in the rod and tube, and
(ii) Load carried by each bar

Take E for steel = 2.1×10^5 N/mm^2 and for copper = 1.1×10^5 N/mm^2

Given:

Dia of steel rod = 3cm = 30mm

$\therefore$ Area of steel rod, $A_e = \frac{\pi}{4}(30)^2 = 706.86\text{mm}^2$



External dia. of copper tube = 5cm = 50mm

Internal dia. of copper tube = 4cm = 40mm

$\therefore$ Area of copper tube, $A_e = \frac{\pi}{4}(50^2 - 40^2)\text{mm}^2 = 706.86\text{mm}^2$

Axial pull on composite bar, P = 45000 N

Length of each bar L = 15cm

Young's modulus for steel, $E_S = 2.1 \times 10^5$ N/mm^2

Young's modulus for copper $E_c = 1.1 \times 10^5$ N/mm^2

(i) The stress in the rod and tube

Let σ_S = Stress in steel

P_S = Load carried by steel rod

σ_c = Stress in copper, and

P_c = Load carried by copper tube.

Now strain in steel = Strain in copper $\left(\because \frac{\sigma}{E} = \text{Strain}\right)$

or $\frac{\sigma_S}{E_S} = \frac{\sigma_c}{E_c}$

$\therefore \sigma_S = \frac{E_S}{E_c} \text{ x } \sigma_c = \frac{2.1 \text{ x } 10^6}{11 \text{ x } 106} \text{ x } \sigma_c$ =- 1.900 σ_c

Now Stress = $\frac{\text{Load}}{\text{Area}}$, $\therefore$ Load = Stress x Area

Load on steel + load on copper = Total load

$\sigma_S \text{ x } A_S + \sigma_c \text{ x } A_c = P$ $(\because \text{Total Load} = P)$

or 1.909 σ_c x 706.86 + 706.86 = 45000

or σ_c (1.909 x 706.86 + 706.86) = 45000

or 2056.25 σ_c = 45000

$\therefore \sigma_c \frac{45000}{2056.25} = 21.88 \text{N/mm}^2 \text{Ans}$

Substituting the value of σ_c in equation (i), we get

σ_c = 1.909 x 21.88 N/mm^2

= 41.77 N/mm^2. Ans

(ii) Load carried by each bar

As Load = Stress x Area

$\therefore$ Load carried by steel rod,

$P_s = \sigma_S \text{ x } A_S$

= 41.77 x 706.86 = 29525.5 N. Ans

Load Carried by copper tube,

P_c = 45000 – 29525.5

= 15474.5 N. Ans

Problem 13: A compound tube consists of a steel tube 140mm internal diameter and 160mm external diameter and an out brass tube 160mm internal diameter and 180mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 900 kN. Find the stresses and the load carried by each tube and the amount if shortens. Length of each tube is 140mm. Take E for Steel as 2 x 10^5 N/mm^2 and for brass as 1 x 10^5 N/mm^2.

Given:

Internal dia. of steel tube = 140mm

External dia. of steel tube = 160mm

$\therefore$ Area of steel tube, $A_a = \frac{\pi}{4}(160^2 - 140^2)$ = 4712.4mm^2

Internal dia. of brass tube = 160mm

External dia. of brass tube = 180mm

$\therefore$ Area of steel tube, $A_b = \frac{\pi}{4}(180^2 - 160^2)$ = 5340.7.4mm^2

Axial load carried by compound tube,

P = 900 kN = 900 x 1000 = 900000N

Length of each tube L = 140mm

E for steel $E_a = 2 \times 10^5$ N/mm^2

E for brass $E_b = 1 \times 10^5$ N/mm^2

Let σ_a = Stress in steel in N/mm^2 and

σ_b = Stress in brass in N/mm^2

Now strain in steel = Strain in brass $\left(\because \text{Strain} \dfrac{\text{Stress}}{E}\right)$

or $$\frac{\sigma_S}{E_S} = \frac{\sigma_b}{E_b}$$

$$\therefore \sigma_S = \frac{E_a}{E_b} \times \sigma_b = \frac{2 \times 10^6}{1 \times 10^5} \times \sigma_b = 2\sigma_b$$

Now load on steel + Load on brass = Total load

or $\sigma_S \times A_a + \sigma_b \times A_b$ =900000 ($\because$ Load = Stress x Area)

or $2\sigma_b \times 4712.4 + \sigma_b \times 5340.7$ = 900000 ($\because \sigma_S = 2\sigma_b$)

or 147655 σ_b = 900000

$$\therefore \sigma_b = \frac{900000}{14765.5} = 60.95 \text{N/mm}^2.\text{Ans}$$

Substituting the value of P_b in equation (i), we get

$\sigma_s = 2 \times 60.95 = 121.9$ N/mm^2. Ans.

Load carried by brass tube

= Stress x Area

= $\sigma_b \times A_b = 60.95 \times 5340.7$N

= 325515 N = 325.515 kN Ans.

Load carried by steel tube

= 900 – 325.515 = 574.485 kN. Ans.

Decrease in the length of the compound tube

= Decrease in length of either of the tubes

= Decrease in length of brass tube

= Strain in brass tube x original length

$$= \frac{\sigma_b}{E_b} \times L = \frac{60.95}{1 \times 10^5} \times 140 = 0.0853 \text{ mm.} \quad \text{Ans}$$

Thermal Stresses

A solid structure is changes in original shape due to change in temperature its might expand or contract.

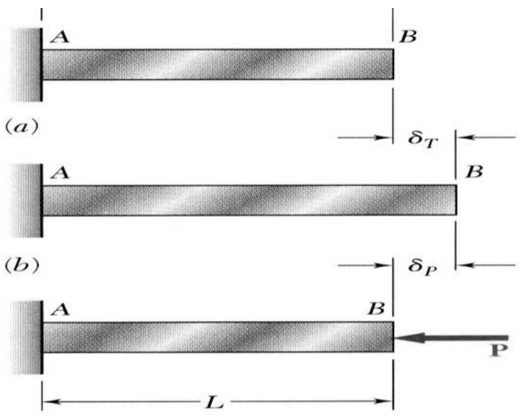


Fig: Thermal expansion and contraction

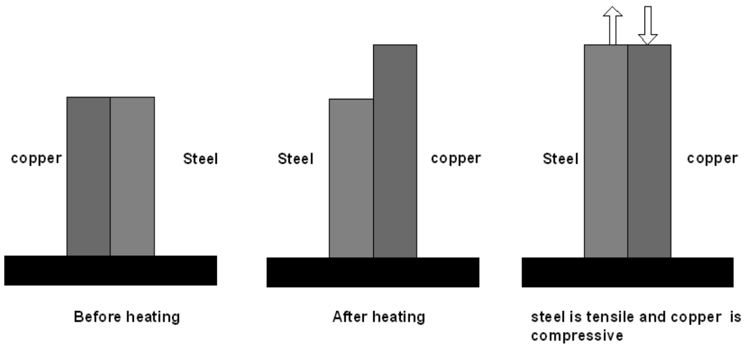
Definition: A temperature change results in a change in length or thermal strain. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.
Raise at temperature ∝ materials is expands (elongate)
Decreases at temperature ∝ materials is contract (shorten)

$\delta_T = \alpha(\Delta T)L$ $\delta_P = \frac{PL}{AE}$

Thermal strain e =α.T and thermal stress p=α.T.E
Where, α = thermal expansion coefficient
T=Rise or fall of temperature
E= young's modulus

Thermal stress in composite bar

In certain application it is necessary to use a combination of elements or bars made from different materials, each material performing a different function. Temperature remains the same for all the materials but strain rate is different due to thermal expansion of materials. The blow figure shows the thermal expansion on composite bar.



Thermal expansion on Composite bar

The Expression for thermal stress is Load on the brass = load on the steel

From the stress equation

$\sigma_c \, x \, A_c = \sigma_s \, x \, A_s$

Thermal stress for copper $\sigma_c = \frac{\sigma_s \, x \, A_s}{A_c}$

Thermal stress for steel $\sigma_s = \frac{\sigma_c \, x \, A_c}{A_s}$

Actual expansion of copper = Actual expansion of steel

Free expansion of copper – contraction due to compressive stress = Free expansion of steel – expansion due to tensile stress

$\alpha_s x \text{ T x L } + \frac{\sigma_s}{E_s} x \text{ L } = \alpha_c x \text{ T x L } + \frac{\sigma_c}{E_c} x \text{ L}$

"L" is the common for both the sides therefore rewriting the above equation

$\alpha_s x \text{ T } + \frac{\sigma_s}{E_s} = \alpha_c x \text{ T } + \frac{\sigma_c}{E_c}$

Problem: A copper rod of 15 mm diameter passes centrally through a steel tube of 30 mm outer diameter and 20 mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. Calculate the stress developed in copper and steel when the temperature of the assembly is raised from 10oC to200oC. Take E for steel = 2 .1 x 105 N/mm2, E for copper = 1 x 105N/mm2, αs = 11 x 10-6/ oC , αc = 18 x 10-6/ oC

<u>Given</u>

Diameter of copper rod dc =15 mm
Steel tube OD do = 30 mm

Steel tube ID di = 20 mm
T_1 and T_2 respectively 10 °C and 200 °C {$T = T_2 - T_1$}
Young's modules for steel $E_s = 2.1 \times 10^5$ N/mm^2
Young's modules for copper $E_c = 1 \times 10^5$ N/mm^2
$\alpha_s = 11 \times 10^{-6}$/ °C and $\alpha_c = 18 \times 10^{-6}$/ °C

To find

Thermal stress in copper [α_c]andsteel [α_s]

Solution

For temperature is the same for both the materials
Compressive load on copper = tensile load on steel

α_sx T + $\frac{\sigma_s}{E_s}$ =α_cx T + $\frac{\sigma_c}{E_c}$

Area of Steel (hollow tube) = $\frac{\pi}{4}\{30^2 - 20^2\} = 125\,\pi$ mm^2

Area of copper = $\frac{\pi}{4}15^2 = 56.25\,\pi$ mm^2

$$\sigma_c = \frac{\sigma_s\, X\, 56.25\,\pi}{125\,\pi} = 2.22\,\sigma_s$$

$$\sigma_c = 2.22\,\sigma_s$$

α_sx T + $\frac{\sigma_s}{E_s}$ =α_cx T - $\frac{\sigma_c}{E_c}$

$11 \times 10^{-6} \times 190 + \frac{\sigma_s}{2.1 \times 10^5} = 18 \times 10^{-6} \times 190 - \frac{\sigma_c}{1 \times 10^5}$

$\frac{\sigma_s}{2.1 \times 10^5} + \frac{\sigma_c}{1 \times 10^5} = 18 \times 10^{-6} \times 190 - 11 \times 10^{-6} \times 190$

Substitute $\sigma_c = 2.22\,\sigma_s$

$\frac{\sigma_s}{2.1 \times 10^5} + \frac{2.22\,\sigma s}{1 \times 10^5} = 5 \times 10^{-6} \times 190$

$5.662\,\sigma_s = 1.995$

$\sigma_s = 35.235$ N/mm^2 and $\sigma_s = 78.22$ N/mm^2

Elastic constants

When the structural stressed by axial load it's under goes the deformation and it's comes back to original shape or structural stressed by within the elastic limit then there is the changes in length along x-direction, y-direction and z - direction.

Types of elastic constant related to isotropic materials

1.Elasticity Modulus (E)0r Young's Modulus
2. Poisson's Ratio ($\boldsymbol{\mu}$)
3. Shear Modulus (G)
4. Bulk Modulus (K)

Elasticity Modulus or Young's Modulus(E)

$$E = \frac{Tensile\ Stress}{Tensile\ Strain}$$

$$E = \frac{\sigma}{\varepsilon}$$

$$E = \frac{F/A}{e/L}$$

$\frac{FL}{eA}$= E

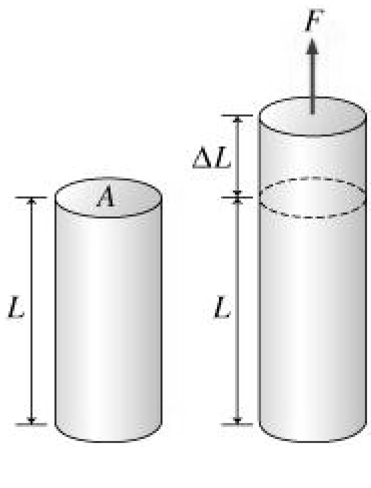


Fig. Before applied load and after applied load

2. Poisson's Ratio (μ)

(μ) (or) $\frac{1}{m}$ $=\frac{lateral\ \ strain}{longitudinal\ \ \ strain}$

Lateral strain (e_t) $= \partial d/d$ (or)$\partial t/t$

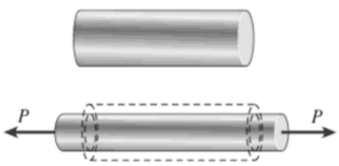


Fig. linear change and lateral change

Longitudinal strain (e_l) $= \partial l/L$

Shear Modulus (G)

Shear modulus G $=\frac{Shear\ \ Stress}{Shear\ \ Strain}$

G $=\frac{\gamma}{\tau}$

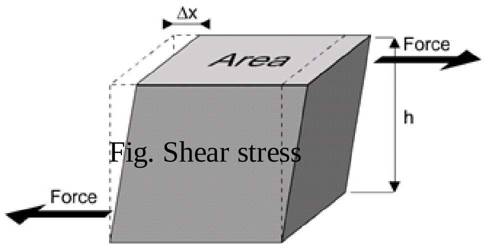


Fig. Shear stress

Volumetric Strain e_v

$$e_v = \frac{Change\ \ in\ volume}{Original\ \ volume} = \frac{\partial V}{v}$$

The volumetric strain is defined as materials tends to change in volume at three direction by external load within the elastic limit

$$e_v = \{\frac{\delta L}{L} - \frac{\delta b}{b} - \frac{\delta d}{d}\}$$

Volume of uniform rectangular section = L X b X d

Here b=d

$$e_v = \{\frac{\delta L}{L} - 2\frac{\delta d}{d}\}$$

$$\mu = \frac{lateral\ strain}{Longitudinal\ strain}$$

μ X $Longitudinal\ strain$ = lateral strain

$$e_v = \{\frac{\delta L}{L} - 2\mu\frac{\delta L}{L}\}$$

Rewriting the above equation

$$e_v = \frac{\delta L}{L}\{1 - 2\mu\}$$

Volumetric strain of rectangular structural subjected to three forces which are mutually perpendicular

$$e_x = \{\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}\}$$

$$e_x = \frac{\sigma_x}{E} - \frac{\mu}{E}\{\sigma_y + \sigma_z\}$$

$$e_x = \frac{1}{E}\{\sigma_x - \mu\{\sigma_y + \sigma_z\}$$

Similarly for e_y and e_y

$$e_y = \frac{1}{E}\{\sigma_y - \mu\{\sigma_x + \sigma_z\}$$

$$e_z = \frac{1}{E}\{\sigma_z - \mu\{\sigma_x + \sigma_y\}$$

$$\frac{\partial V}{v} = \{e_x + e_y + e_z\}$$

$$\{e_x + e_y + e_z\} = \frac{1}{E}\{\sigma_x + \sigma_y + \sigma_z\} - \frac{2\mu}{E}\{\sigma_x + \sigma_y + \sigma_z\}$$

$$\{e_x + e_y + e_z\} = \frac{1}{E}\{\sigma_x + \sigma_y + \sigma_z\}\{1 - 2\mu\}$$

Volumetric strain of cylindrical rod

$$e_v = \{\frac{\delta L}{L} - 2\frac{\delta d}{d}\}$$

Bulk modulus [K]

$$[K] = \frac{Direct\ stress}{volimetric\ strain}$$

$$[K] = \frac{\sigma}{\frac{dV}{v}}$$

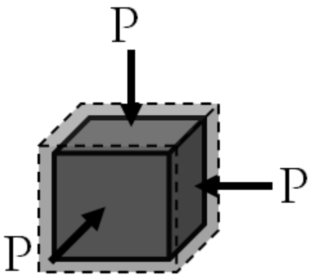


Fig. Change in volume

<u>Relationship between young's modulus and bulk modulus</u>

Volume = L x L x L

$V = L^3$

$dV = 3L^2 \times dL$

$$\frac{dL}{L} = \{\frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}\}$$

$$= \frac{\sigma}{E}\{1 - 2\mu\}$$

$$dL = \frac{\sigma}{E}\{1 - 2\mu\} \times L$$

$dV = 3L^2 \times dL$

$dV = 3L^2 \times \frac{\sigma}{E}\{1 - 2\mu\} \times L$

$\frac{dV}{v} = \frac{3\,dL}{L}$

$dV = \frac{3}{L}\frac{\sigma}{E} \times \{1 - 2\mu\} \times L$

$$[K] = \frac{\sigma}{\frac{dV}{v}}$$

$$[K] = \frac{\sigma}{\left[\frac{3\sigma}{E}\right][1 - 2\mu]}$$

$$[K] = \frac{E}{3[1 - 2\mu]}$$

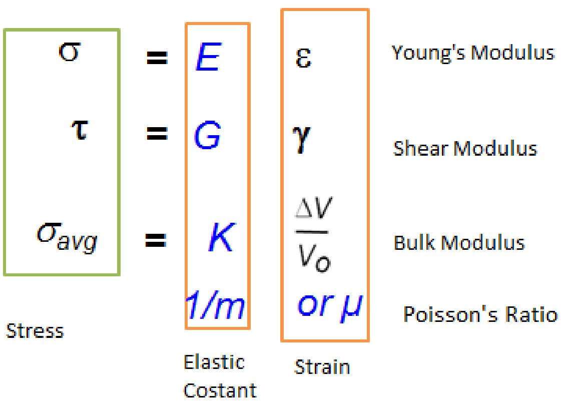
$3K[1 - 2\mu] = E$ From this equation

$$\mu = \frac{3K - E}{6K}$$

Relationship between modulus of elasticity and modulus of rigidity {E and G}

$$G = \frac{E}{2\{1 + \mu\}}$$

Easy to identify with the four elastic constant are calculated by single module as shown in fig.



Relationship between modulus of elasticity (E) and bulk modulus (K):

$$E = 3K(1 - 2\mu)$$

Relationship between modulus of elasticity (E) and modulus of rigidity (G):

$$E = 2G(1 + \mu)$$

Relation among three elastic constants:

$$E = \frac{9KG}{G + 3K}$$

Problem:

Determine the changes in length, breadth and thickness of a steel bar which 5cm long, 40mm wide and 30mm thick and is subjected to an axial pull of 35KN in the direction in length take the young 'modulus and position's ratio 200Gpa and 0.32 respectively .

Given:

L = 5cm=50mm
b=40mm
d=30mm
E=200Gpa= 2 x 10^5N/mm^2
μ = 0.32

To find: δL , δb and δd

Solultion:

$\mu = \frac{\delta L}{L} / \frac{\delta b}{b}$

$\mu = \frac{\delta L}{L}$

$E = \frac{\sigma}{e}$

$\sigma = \frac{P}{A} \sigma = \frac{35 X 10^3}{40 X 30} = \frac{350}{12} = 29.16\ N/mm^2$

(i) Change in length (δL)

$= \frac{PL}{AE} = \frac{35 X 10^3 X 50}{40 X 30 X 2 X 10^5} = \frac{35 X 5}{40 X 30 X 2 X 10} = \frac{175}{24000}$ =7.29x10^3 mm

ii) Change in breadth (δb)

$$\mu = \frac{lateral \quad strain}{Longitudinal \quad strain}$$

μ X $Longitudinal \quad strain$ = lateral strain

μ X $\frac{\delta L}{L} = \frac{\delta b}{b}$

$\delta b = \mu$ X $\frac{\delta L}{L}$ X b = 0.32 x 40 x $\frac{72.29 X 10^{-3}}{50}$ = 1.866 x 10^{-3} mm

ii) Change in diameter (δd)

μ X $\frac{\delta L}{L} = \frac{\delta d}{d}$

$\delta d = \mu$ X $\frac{\delta L}{L}$ X d=0.32 x $\frac{72.29 X 10^{-3}}{50}$ x 40 =1.39 X10^{-3} mm

Problem:

Calculate the modulus of rigidity and bulk modulus of cylindrical bar of diameter of 25mm and of length 1.6m. if the longitudinal strain in a bar during a tensile test is four times the lateral strain find the change in volume when the bar subjected to hydrostatic pressure of 100 N/mm^2 the young's modulus of cylindrical bar E is 100 GPa

Given:

D=25 mm

L=1.6m=1600 mm

Longitudinal strain = 4 X lateral strain

E=100Gpa=1 x 10^5N/mm^2

To find:

(i) Modulus of Rigidity (ii) Bulk modulus (iii) Change in volume

(i) Modulus of Rigidity[G]

E= 2G (1+μ) ---------- Relationship between E, G & μ

Longitudinal strain = 4 X lateral strain

$$\frac{1}{4} = \frac{later al \quad strain}{Longitudinal \quad strain}$$

E= 2G (1+μ) =2G (1+$\frac{1}{4}$) =2G (1+0.25)

E=2G (1+0.25)

G = = $\frac{E}{2(1.25)} = \frac{1 X 10^5}{2(1.25)}$ = 4 x 10^4 N/mm^2

(ii) Bulk modulus [K]

E = 3K [1-2 μ]

= K x 3[1-0.5]

1 x 10^5 = 1.5 x K

$\frac{1 X 10^5}{1.5}$ = K

K =0.666 X 10^5N/mm^2

(iii) Change in volume [dV]

$$[K] = \frac{Direct \quad stress}{volimetric \quad strain}$$

$$[K] = \frac{\sigma}{\frac{dV}{v}}$$

$\frac{dV}{v} = \frac{\sigma}{K} = \frac{100}{0.666 X 10^5}$=1.5 X 10^{-3}

$V = \frac{\pi}{4} X\ d^2\ X\ L$

$V = \frac{\pi}{4} X\ 25^2\ X\ 1600 = 785000$

$$\frac{dV}{v} = 1177.5\ mm^3$$

Strain energy

When material is deformed by external loading, energy is stored *internally* throughout its volume the stored energy is called strain energy.

Strain energy = work done

Resilience: the total strain energy stored in a volume or capacity of work after removing straining force is called Resilience

Proof Resilience:
The maximum strain energy stored in the volume or quantity of strain energy stored in volume in a body when strained up to elastic limit its called Proof Resilience.

$$\text{Proof Resilience} = \frac{\sigma^2}{2E}\ X\ Volume$$

Modulus of Resilience

$$= \frac{\text{Proof Resilience}}{Volume\ of\ the\ body}$$

Principal stresses and Strain & Mohr's Circle:

Principal planes and stresses

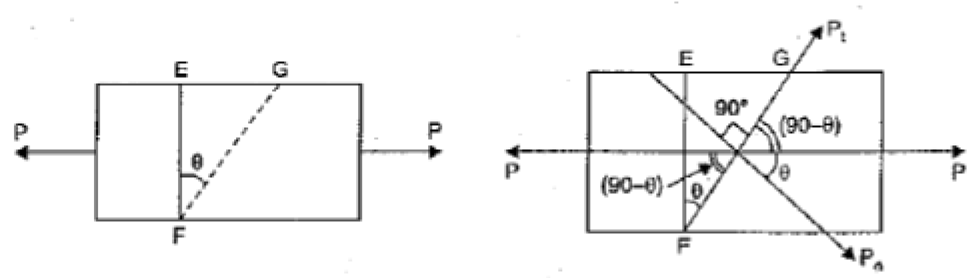
The planes, which have no shear stress, are known as principal planes. Hence principal planes are the planes of zero shear stress. These planes carry only normal stresses. The normal stresses, acting on a principal plane, are known as principal stresses.

Methods for determining principal planes and stresses

- Analytical method
- Graphical method

Analytical method on oblique section
The following are the two cases considered
1. A member subjected to a direct stress in one plane
2. A member subjected to like direct stresses in two mutually perpendicular directions.

Direct stress in one plane



Normal stress, $\sigma_n = \sigma \cos^2 \theta$

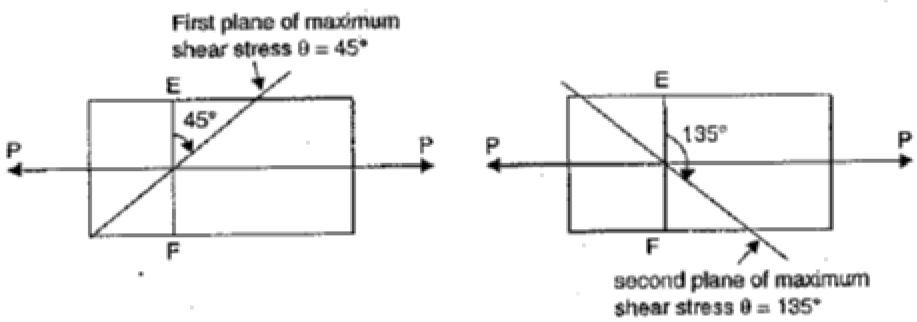
Tangential stress, $\sigma_t = \frac{\sigma}{2} \sin 2\theta$

σ_n will be maximum, when $\cos^2 \theta$ (or) $\cos\theta$ is maximum.
Cos θ will be maximum when θ = 0° as cos 0° = 1
Therefore, max. normal stress = $\sigma \cos^2\theta = \sigma$

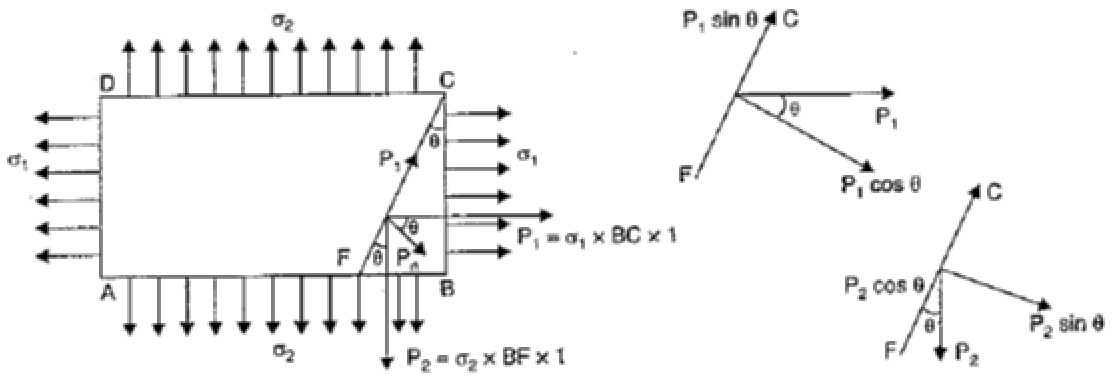
σ_t will be max, when sin 2θ is maximum.
Sin2θ be max. when sin2θ = 1 or 2θ = 90° (or) 270°
θ = 45° (or) 135°

Max. value of shear stress = $\frac{\sigma}{2} \sin 2\theta$

= $\frac{\sigma}{2}$



Member subjected to direct stresses in two mutually perpendicular directions



Normal Stress, $\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$

$$\text{Tangential Stress}, \sigma_t = \frac{(\sigma_1 - \sigma_2)}{2} \sin 2\theta$$

$$\text{Resultant Stress}, \sigma_R = \sqrt{\sigma_{n^2} + \sigma_{t^2}}$$

$$\text{Obliquity}, \quad \tan\phi = \frac{\sigma_t}{\sigma_n}$$

Problem

A small block is 4 cm long, 3 cm high and 0.5 cm thick. It is subjected to uniformly distributed tensile forces of resultants 1200 N and 500 N as shown in Fig. below. Compute the normal and shear stresses developed along the diagonal AB.

Area of cross-section normal to x-axis = 3 x 0.5 = 1.5 cm^2

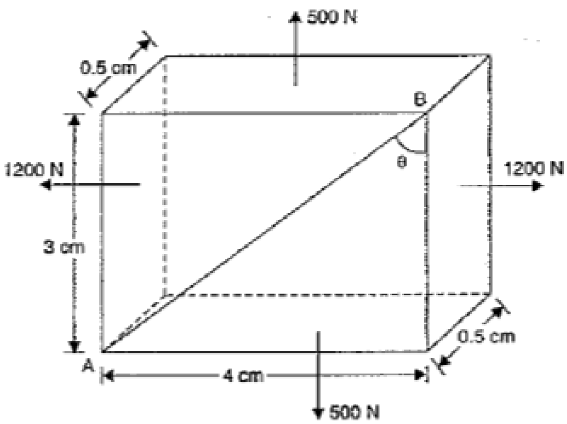
Area of cross-section normal to y-axis = 4 x 0.5 = 2 cm^2

$$\text{Stress along x-axis}, \quad \sigma_1 = \frac{F_x}{A_x} = 800 \text{N/cm}^2$$

$$\text{Stress along y-axis}, \quad \sigma_2 = \frac{F_y}{A_y} = 250 \text{N/cm}^2$$

$$\tan\theta = \frac{4}{3} = 1.33$$

$$\theta = \tan^{-1}(1.33) = 53.06°$$



$$\text{Normal Stress}, \sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$= \frac{800 + 250}{2} + \frac{800 - 250}{2} \cos(2 \times 53.06)$$

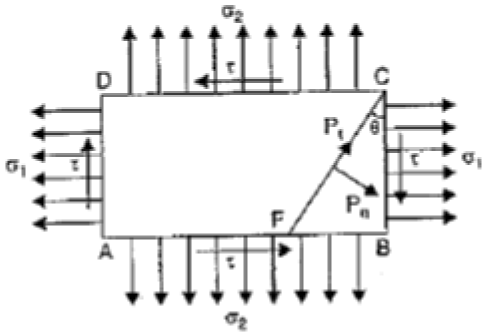
$$= 448.65 \text{N/cm}^2$$

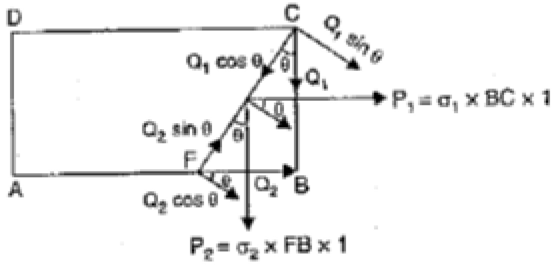
$$\text{Tangential Stress}, \sigma_t = \frac{(\sigma_1 - \sigma_2)}{2} \sin 2\theta$$

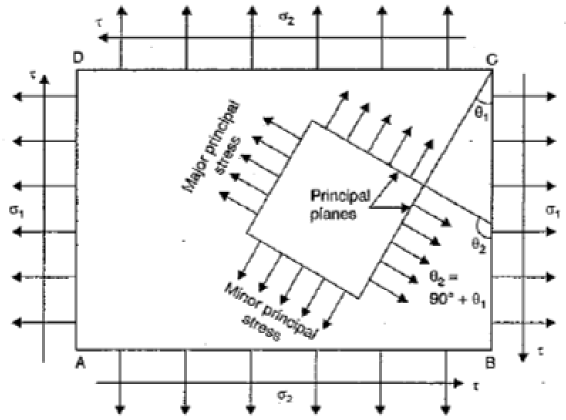
$$= \frac{800 - 250}{2} \sin(2 \times 53.06)$$

$$= 264.18 \text{N/cm}^2$$

Members subjected to direct stresses in two mutually perpendicular directions accompanied by simple shear stress







Normal Stress, $\sigma_n = \dfrac{\sigma_1+\sigma_2}{2} + \dfrac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$

Tangential Stress, $\sigma_t = \dfrac{(\sigma_1-\sigma_2)}{2}\sin 2\theta - \tau \cos 2\theta$

$\tan 2\theta = \dfrac{2\tau}{(\sigma_1-\sigma_2)}$

Major principal stress $= \dfrac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\dfrac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

Minor principal stress $= \dfrac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\dfrac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

Max. Shear stress $= \dfrac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2 + 4\tau^2}$

Problem

A rectangular block of material is subjected to a tensile stress of 110 N/mm2 on one plane and a tensile stress of 47 N/mm2 on the plane at right angles to the former. Each of the above stresses is accompanied by a shear stress of 63 N/mm2 and that associated with the former tensile stress tends to rotate the block anticlockwise. Find:

(i) The direction and magnitude of each of the principal stress and

(ii) Magnitude of the greatest shear stress

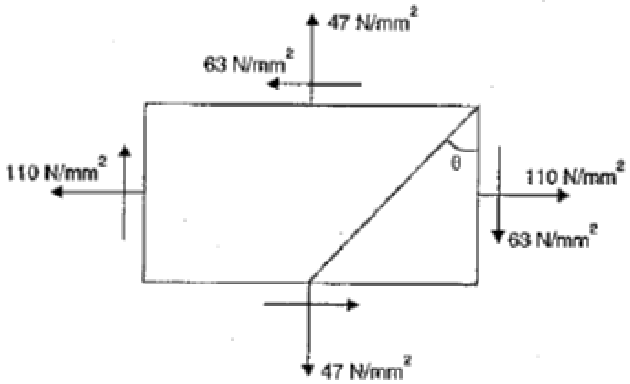
Given

$\sigma_1 = 110$ N/mm^2

$\sigma_2 = 47$ N/mm^2

$\tau = 63$ N/mm^2

$\theta = 45°$



$$\text{Major principal stress} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{110 + 47}{2} + \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$

$$= \frac{157}{2} + \sqrt{\left(\frac{63}{2}\right)^2 + 63^2}$$

$$= 78.5 + \sqrt{31.5^2 + 63^2}$$

$$= 78.5 + 70.436$$

$$= 148.936 \text{ N/mm}^2$$

$$\text{Minor principal stress} = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{110 + 47}{2} - \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$

$$= 78.5 - 70.436$$

$$= 8.064 \text{ N/mm}^2$$

$$\tan 2\theta = \frac{2\tau}{(\sigma_1 - \sigma_2)}$$

$$2\theta = \tan^{-1}(2)$$

$$\theta = 31°43'$$

Magnitude of the greatest shear stress

$$(\sigma_t)_{max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(100 - 47)^2 + 4 \times 63^2}$$

$$(\sigma_t)_{max} = 70.436 \text{N/mm}^2$$

Mohr's circle method

It is a graphical method of finding normal, tangential and resultant stresses or an oblique plane. It is drawn for following cases

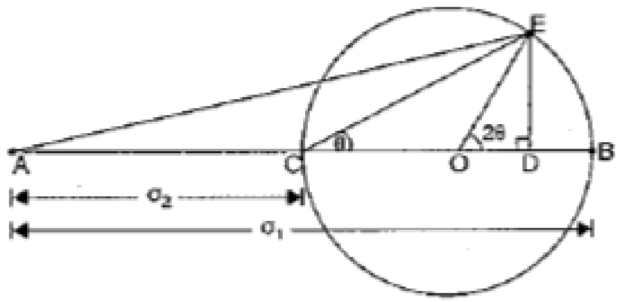
1. A body subjected to two mutually perpendicular principal stresses of unequal intensities
2. A body subjected to two mutually perpendicular stresses which are unequal and unlike (one is tension and other is compression)
3. A body subjected to two mutually perpendicular tensile stresses accompanied by a simple shear stress.

Case 1: A body subjected to two mutually perpendicular principal stresses of unequal intensities

Let σ_1= Major tensile stress
σ_2 = Minor tensile stress
θ = Angle made by the oblique plane with the axis of minor tensile stress

Mohr's Circle procedure

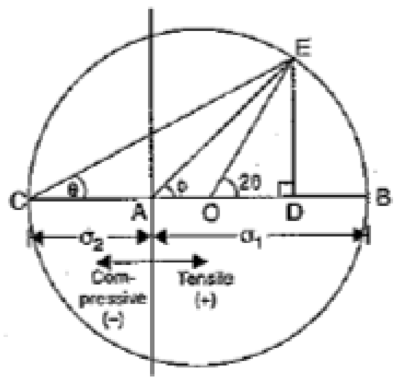
Take any point A and draw a horizontal line through A. Take AB = σ_1 and AC = σ_2 towards right from A to some suitable scale. With BC as diameter draw a circle. Let O is the centre of circle. Now through O, draw a line OE marking an angle 2θ with OB. From E, draw ED perpendicular on AB. Join AE. Then the normal and tangential stresses on the oblique plane are given by AD and ED respectively. The resultant stress on the oblique plane is given by AE.



From Figure, we have
Length AD = Normal stress on oblique plane
Length ED = Tangential stress on oblique plane
Length AE = Resultant stress on oblique plane
Angle φ = obliquity

Case 2: Mohr's circle when a body is subjected to two mutually perpendicular principal stresses which are unequal and unlike (one is tensile and other is compressive)

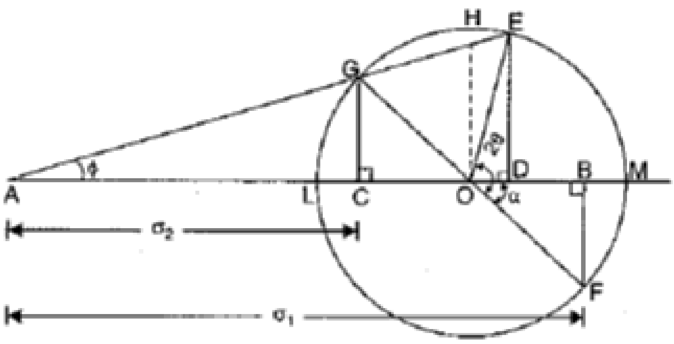
Take any point A and draw a horizontal line through A on both sides of A as shown in Fig. Take AB = $\sigma_{1(+)}$ towards right of A and



AC = σ_2(-) towards left of A to some suitable scale. Bisect BC at O. With O as centre and radius equal to CO or OB, draw a circle. Through O draw a line OE making an angle 2θ with OB. From E, draw ED perpendicular to AB. Join AE and CE. Then normal and shear stress on the oblique plane are given by AD and ED. Length AE represents the resultant stress on the oblique plane.

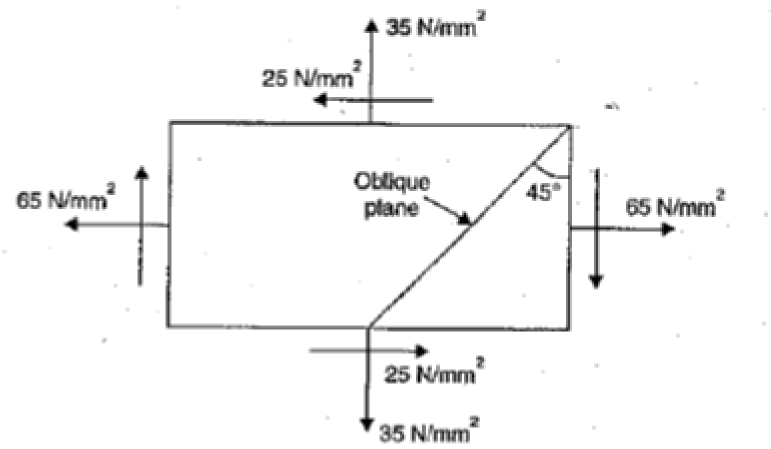
Case 3: Mohr's circle when a body subjected to two mutually perpendicular tensile stresses accompanied by a simple shear stress.

Take any point A and draw a horizontal line through A. Take AB = $\sigma_1$and AC = σ_2 towards right of A to some suitable scale. Draw perpendiculars at B and C and cut off BF and CG equal to shear stress to the same scale. Bisect BC at O. Now with O as centre and radius equal to OG or OF draw a circle. Through O, draw a line OE making an angle of 2θ with OF as shown in Fig. From E, draw ED perpendicular to CB. Join AE. Then length AE represents the resultant stress on the oblique plane. And lengths AD and ED represents the normal stress and tangential stress respectively.



Problems

1. A point in a strained material is subjected to stresses shown in Fig. Using Mohr's circle method, determine the normal and tangential stress across the oblique plane.



Given:

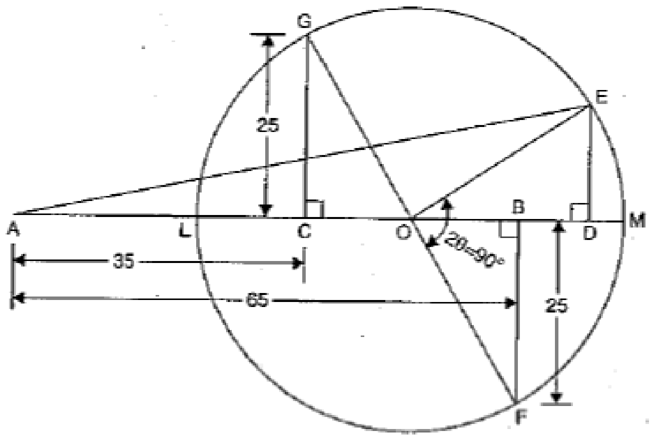
$\sigma_1 = 65$ N/mm^2

$\sigma_2 = 35$ N/mm^2

$\tau = 25$ N/mm^2

$\theta = 45°$

Let 1 cm = 10 N/mm^2



$$\sigma_1 = \frac{65}{10} = 6.5\text{cm}$$

$$\sigma_2 = \frac{35}{10} = 3.5\text{cm}$$

$$\tau = \frac{25}{10} = 2.5\text{cm}$$

By measurements, Length AD = 7.5 cm and
Length ED = 1.5 cm

Normal stress (σ_n) = Length AD x Scale = 7.5 x 10 = 75 N/mm^2

Tangential stress (σ_t) = Length ED x Scale = 1.5 x 10 = 15 N/mm^2

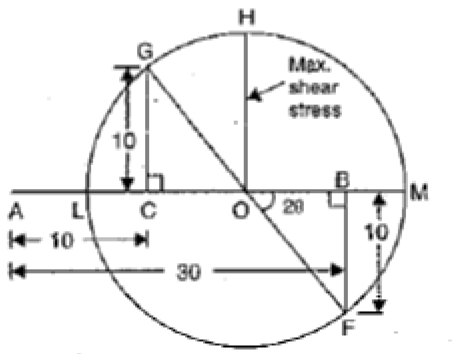
2. An elemental cube is subjected to tensile stresses of 30 N/mm2 and 10 N/mm2 acting on two mutually perpendicular planes and a shear stress of 10 N/mm2 on these planes. Draw the Mohr's circle of stresses and hence or otherwise determine the magnitudes and directions of principal stresses and also the greatest shear stress.

Given:

$\sigma_1 = 30$ N/mm^2

$\sigma_2 = 10$ N/mm^2

$\tau = 10$ N/mm^2



Let 1 cm = 2 N/mm^2

$$\sigma_1 = \frac{30}{2} = 15\text{cm}$$

$$\sigma_2 = \frac{10}{2} = 5\text{cm}$$

$$\tau = \frac{10}{2} = 5\text{cm}$$

By measurements,

Length AM = 17.1 cm

Length AL = 2.93 cm

Length OH = Radius of Mohr's circle = 7.05 cm

$\angle FOB (or) 2\theta = 45°$

Major principal stress = Length AM x Scale = 17.1 x 2 = 34.2 N/mm^2
Minor principal stress = Length AL x Scale = 2.93 x 2 = 5.86 N/mm^2

$$\theta = \frac{45}{2} = 22.5°$$

The second principal plane is given by θ+90°

= 22.5 + 90

= 112.5°

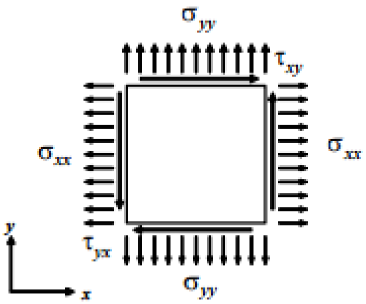
Greatest shear stress = Length OH x Scale

= 7.05 x 20

$= 14.1 \text{ N/mm}^2$

BIAXIAL STRESS SYSTEMS

A biaxial stress system has a stress state in two directions and a shear stress typically showing in Fig..



Element of a structure showing a biaxial stress system

When a Biaxial Stress state occurs in a thin metal, all the stresses are in the plane of the material. Such a stress system is called PLANE STRESS. We can see plane stress in pressure vessels, aircraft skins, car bodies, and many other structures.

THEORIES OF FAILURES

When some external load is applied on a body, the stresses and strains are produced in the body. The stresses are directly proportional to the strains within the elastic limit. This means when the load is removed, the body will return to its original state. There is no permanent deformation in the body.

According to the important theories, the failure takes place when a certain limiting value is reached by one of the following:

- Maximum principal stress
- Maximum principal strain
- Maximum shear stress theory
- Maximum strain energy theory
- Maximum shear strain energy theory

Maximum Principal Stress theory

According to this theory, the failure of a material will occur when the maximum principal tensile stress in the complex system reaches the value of the maximum stress at the elastic limit in simple tension or the minimum principal stress (the maximum principal compressive stress) reaches the value of the maximum stress at the elastic limit in simple compression.

Let in a complex three dimensional stress system,

σ_1, σ_2 and σ_3 = principal stresses at a point in three perpendicular directions. The stresses σ_1 and σ_2 are tensile and σ_3 is compressive. Also σ_1 is more than σ_2.

σ_t^* = tensile stress at elastic limit in simple tension.

σ_c^* = compressive stress at elastic limit in simple compression.

Then according to this theory, the failure will take place if

$$\sigma_1 \geq \sigma_t^* \text{ in simple tension}$$

or $$|\sigma_3| \geq \sigma_c^* \text{ in simple compression}$$

Maximum principal strain theory

According to this theory, the failure will occur in a material when the maximum principal strain reaches the strain due to yield stress in simple tension or when the minimum principal strain (maximum compressive strain) reaches the strain due to yield stress in simple compression. Yield stress is the maximum stress at elastic limit. Consider a three dimensional stress system.

Principal strain in the direction of principal stress σ_1 is,

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} - \frac{\mu \sigma_3}{E}$$

$$= \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$$

Principal strain in the direction of principal stress σ_3 is

$$e_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)]$$

Strain due to yield stress in simple tension $= \frac{1}{E} \times$ yield stress in tension

$$= \frac{1}{E} \times \sigma_t^*$$

and strain due to yield stress in simple compression $= \frac{1}{E} \times \sigma_c^*$

where yield stress is the maximum stress at elastic limit.

According to this theory, the failure of the material will take place when

$$e_1 \geq \frac{\sigma_t^*}{E}$$

or

$$|e_3| \geq \frac{\sigma_c^*}{E}$$

Maximum Shear Stress Theory

According to this theory, the failure of a material will occur when the maximum shear stress in a material reaches the value of maximum shear stress in simple tension at the elastic limit. The maximum shear stress in the material is equal to half the difference between maximum and minimum principal stress.

If σ_1, σ_2 and σ_3 are principal stresses at a point in a material for which σ_t^* is the principal stress in simple tension at elastic limit, then

Max. shear stress in the material = Half of difference of maximum and minimum principal stresses

$$= \frac{1}{2}[\sigma_1 - \sigma_3]$$

In case of simple tension, at the elastic limit the principal stresses are σ_t^*, 0, 0

[In simple tension, the stress is existing in one direction only]

$\therefore$ Max. shear stress in simple tension at elastic limit

= Half of difference of maximum and minimum principal stresses

$$= \frac{1}{2}[\sigma_t^* - 0] = \frac{1}{2}\sigma_t^*$$

$\therefore$ For the failure of material,

$\therefore$ For the failure of material,

$$\frac{1}{2}[\sigma_1 - \sigma_3] \geq \frac{1}{2}\sigma_t^* \quad \text{or} \quad (\sigma_1 - \sigma_3) \geq \sigma_t^*$$

Maximum Strain Energy Theory

According to this theory, the failure of a material occurs when the total strain energy per unit volume in the material reaches the strain energy per unit volume of the material at the elastic limit in simple tension. It stated that the strain energy in a body is equal to work done by the load (P) in straining the material.

$$\therefore \quad U = \text{Strain energy}$$

$$= \frac{1}{2} \times P \times \delta L$$

$$= \frac{1}{2} \times (\sigma \times A) \times (e) \times L \qquad \left[\because \ \sigma = \frac{P}{A} \ \therefore \ P = \sigma \times A \ \text{and} \ e = \frac{\delta L}{L} \ \therefore \ \delta L = e \times L\right]$$

$$= \frac{1}{2} \times \sigma \times e \times A \times L$$

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} \qquad (\because \ A \times L = \text{volume})$$

∴ Strain energy per unit volume

$$= \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \times \sigma \times e$$

For a three dimensional stress system, the principal stresses acting at a point are σ_1, σ_2 and σ_3. The corresponding strains are e_1, e_2 and e_3, where e_1 = principal strain in the direction of σ_1

Now, $$e_2 = \frac{\sigma_1}{E} - \frac{\mu}{E}(\sigma_2 + \sigma_3)$$

Similarly, $$e_2 = \frac{\sigma_2}{E} - \frac{\mu}{E}(\sigma_3 + \sigma_1)$$

and $$e_3 = \frac{\sigma_3}{E} - \frac{\mu}{E}(\sigma_1 + \sigma_2)$$

∴ Total strain energy per unit volume in three dimensional system,

$$U = \frac{1}{2} \times \sigma_1 \times e_1 + \frac{1}{2} \times \sigma_2 \times e_2 + \frac{1}{2}\sigma_3 \times e_3$$

$$= \frac{1}{2}\sigma_1 \times \left[\frac{\sigma_1}{E} - \frac{\mu}{E}(\sigma_2 + \sigma_3)\right] + \frac{1}{2}\sigma_2 \times \left[\frac{\sigma_2}{E} - \frac{\mu}{E}(\sigma_3 + \sigma_1)\right] + \frac{1}{2}\sigma_3 \times \left[\frac{\sigma_3}{E} - \frac{\mu}{E}(\sigma_1 + \sigma_2)\right]$$

$$= \frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

The strain energy per unit volume corresponding to stress at elastic limit in simple tension

$$= \frac{1}{2} \times \sigma_t^* \times e_t^* \qquad [\text{where } e_t^* = \text{strain due to } \sigma_t^*$$

$$= \frac{1}{2} \times \sigma_t^* \times \frac{\sigma_t^*}{E} \qquad \therefore \ E = \frac{\sigma_t^*}{e_t^*} \text{ or } e_t^* = \frac{\sigma_t^*}{E}\Big]$$

$$= \frac{1}{2E} \times (\sigma_t^*)^2$$

For the failure of the material

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \geq \frac{1}{2E} \times (\sigma_t^*)^2$$

or

$$\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \geq (\sigma_t^*)^2$$

For a two-dimensional stress system, $\sigma_3 = 0$. Hence above equation becomes as

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2) \geq (\sigma_t^*)^2$$

Maximum Shear Strain Theory

This theory is also called as energy of distortion theory. According to this theory, the failure of a material occurs when the total shear strain energy per unit volume in the stressed material reaches a value equal to the shear strain energy per unit volume at the elastic limit in the simple tension test.

The total shear strain energy per unit volume due to principal stresses σ_1, σ_2 and σ_3 in a stressed material is given as

$$= \frac{1}{12C}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

The simple tension test is a uniaxial stress system which means the principal stresses are σ_1, 0, 0.

At the elastic limit the tensile stress in simple test is σ_t^*.

Hence at the elastic limit in simple tension test, the principal stresses are σ_t^*, 0, 0.

The shear strain energy per unit volume at the elastic limit in simple tension will be

$$= \frac{1}{12C}\left[(\sigma_t^* - 0)^2 + (0 - 0)^2 + (0 - \sigma_t^*)^2\right]$$

[Here $\sigma_1 = \sigma_t^*$, $\sigma_2 = 0$, $\sigma_3 = 0$]

$$= \frac{1}{12C}\left[2 \times \sigma_t^{*2}\right]$$

For the failure of the material,

$$\frac{1}{12C}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \geq \frac{1}{12C}\left[2 \times (\sigma_t^*)^2\right]$$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2 \times (\sigma_t^*)^2$$

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