Radiation Heat Transfer

It has been observed that the heat transfer studies were based on the fact that the temperature of a body, a portion of a body, which is hotter than its surroundings, tends to decrease with time. The decrease in temperature indicates a flow of energy from the body. In all the previous chapters, limitation was that a physical medium was necessary for the transport of the energy from the high temperature source to the low temperature sink. The heat transport was related to conduction and convection and the rate of heat transport was proportional to the temperature difference between the source and the sink.

Now, if we observe the heat transfer from the Sun to the earth atmosphere, we can understand that there is no medium exists between the source (the Sun) and the sink (earth atmosphere). However, still the heat transfer takes place, which is entirely a different energy transfer mechanism takes place and it is called thermal radiation.

Thermal radiation is referred when a body is heated or exhibits the loss of energy by radiation. However, more general form "radiation energy" is used to cover all the other forms. The emission of other form of radiant energy may be caused when a body is excited by oscillating electrical current, electronic bombardment, chemical reaction etc. Moreover, when radiation energy strikes a body and is absorbed, it may manifest itself in the form of thermal internal energy, a chemical reaction, an electromotive force, etc. depending on the nature of the incident radiation and the substance of which the body is composed.

In this chapter, we will concentrate on thermal radiation (emission or absorption) that on radiation produced by or while produces thermal excitation of a body.

There are many theories available in literature which explains the transport of energy by radiation. However, a dual theory is generally accepted which enables to explain the radiant energy in the characterisation of a wave motion (electromagnetic wave motion) and discontinuous emission (discrete packets or quanta of energy).

An electromagnetic wave propagates at the speed of light (3×10^8 m/s). It is characterised by its wavelength λ or its frequency ν related by

$$c = \lambda v \quad (1)$$

Emission of radiation is not continuous, but occurs only in the form of discrete quanta. Each quantum has energy

$$E = hv \quad (2)$$

where, = 6.6246×10^{-34} J.s, is known as Planck's constant.

Table 1 shows the electromagnetic radiation covering the entire spectrum of wavelength

Table 1: Electromagnetic radiation for entire spectrum of wavelength

Type	Band of wavelength (μm)
Cosmic rays	upto 4×10^{-7}
Gamma rays	4×10^{-7} to 1.4×10^{-4}
X-rays	1×10^{-5} to 2×10^{-2}
Ultraviolet rays	5×10^{-3} to 3.9×10^{-1}
Visible light	$3.9\times10^{-1}$to 7.8×10^{-1}
Infrared rays	7.8×10^{-1} to 1×10^{3}
Thermal radiation	1×10^{-1} to 1×10^{2}
Microwave, radar, radio waves	1×10^{3} to 5×10^{10}

It is to be noted that the above band is in approximate values and do not have any sharp boundary.

Basic definition pertaining to radiation

Before we further study about the radiation it would be better to get familiarised with the basic terminology and properties of the radiant energy and how to characterise it.

As observed in the table 1 that the thermal radiation is defined between wavelength of about 1×10^{-1} and 1×10^{2} μm of the electromagnetic radiation. If the thermal radiation is emitted by a surface, which is divided into its spectrum over the wavelength band, it would be found that the radiation is not equally distributed over all wavelength. Similarly, radiation incident on a system, reflected by a system, absorbed by a system, etc. may be wavelength dependent. The dependence on the wavelength is generally different from case to case, system to system, etc. The wavelength dependency of any radiative quantity or surface property will be referred to as a spectral dependency. The radiation quantity may be monochromatic (applicable at a single wavelength) or total (applicable at entire thermal radiation spectrum). It is to be noted that radiation quantity may be dependent on the direction and wavelength both but we will not consider any directional dependency. This chapter will not consider directional effect and the emissive power will always used to be (hemispherical) summed overall direction in the hemisphere above the surface.

Emissive power

It is the emitted thermal radiation leaving a system per unit time, per unit area of surface. The total emissive power of a surface is all the emitted energy, summed over all the direction and all wavelengths, and is usually denoted as E. The total emissive power is found to be dependent upon the temperature of the emitting surface, the subsystem which this system is composed, and the nature of the surface structure or texture.

The monochromatic emissive power E_λ, is defined as the rate, per unit area, at which the surface emits thermal radiation at a particular wavelength λ. Thus the total and monochromatic hemispherical emissive power are related by

$$E = \int_0^\infty E_\lambda d\lambda \qquad (3)$$

and the functional dependency of E_λ on λ must be known to evaluate E.

Radiosity

It is the term used to indicate all the radiation leaving a surface, per unit time and unit area.

$$J = \int_0^\infty J_\lambda \, d\lambda \qquad (4)$$

where, J and J_λ are the total and monochromatic radiosity.
The radiosity includes reflected energy as well as original emission whereas emissive power consists of only original emission leaving the system. The emissive power does not include any energy leaving a system that is the result of the reflection of any incident radiation.

Irradiation

It is the term used to denote the rate, per unit area, at which thermal radiation is incident upon a surface (from all the directions). The irradiative incident upon a surface is the result of emission and reflection from other surfaces and may thus be spectrally dependent.

$$G = \int_0^\infty G_\lambda d\lambda \qquad (5)$$

where, G and G_λ are the total and monochromatic irradiation.
Reflection from a surface may be of two types specular or diffusive as shown in fig.1.

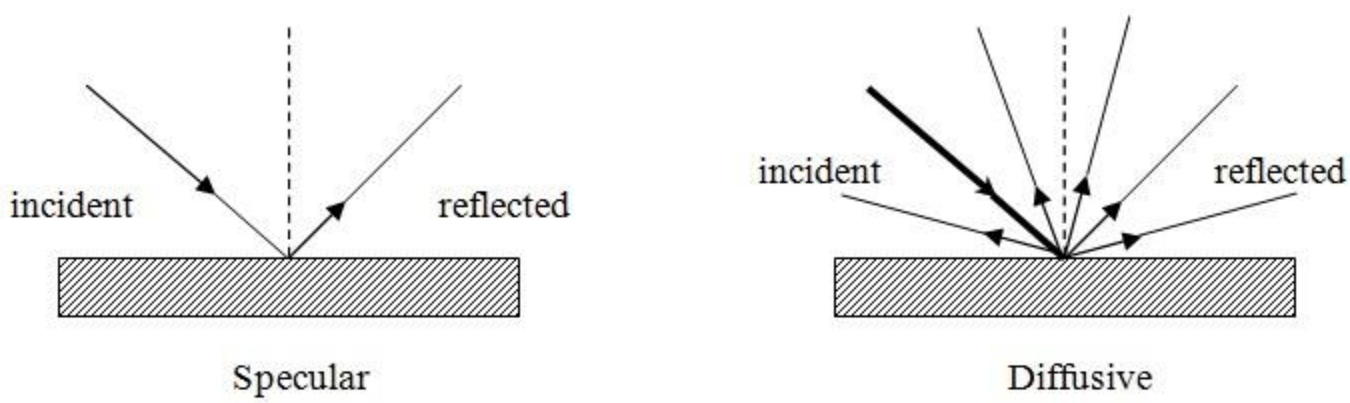

Fig. 1: (a) Specular, and (b) diffusive radiation

Thus,

$$J = E + \rho G \qquad (6)$$

Absorptivity, reflectivity, and transmittivity:

The emissive power, radiosity, and irradiation of a surface are inter-related by the reflective, absorptive, and transmissive properties of the system.

When thermal radiation is incident on a surface, a part of the radiation may be reflected by the surface, a part may be absorbed by the surface and a part may be transmitted through the surface as shown in fig.2. These fractions of reflected, absorbed, and transmitted energy are interpreted as system properties called reflectivity, absorptivity, and transmissivity, respectively.

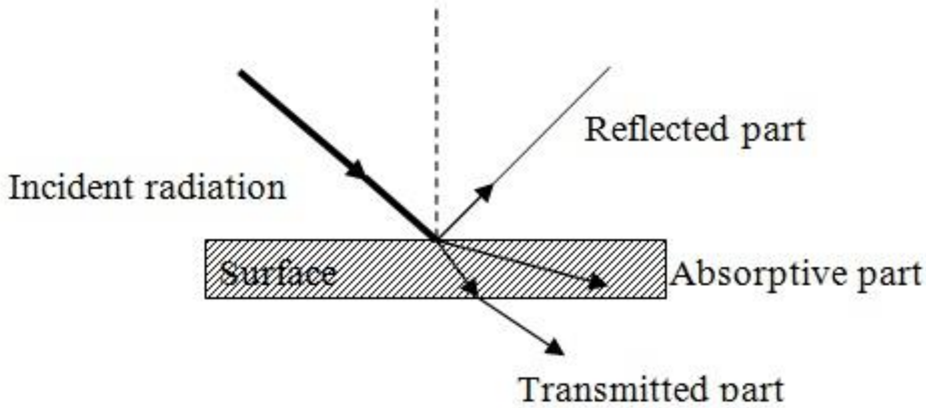

Fig. 2: Reflection, absorption and transmitted energy

Thus using energy conservation,

$$\rho + \alpha + \tau = 1 \quad (7)$$

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \quad (7)$$

where, ρ, α and τ are total reflectivity, total absorptivity, and total transmissivity. The subscript λ indicates the monochromatic property.

In general the monochromatic and total surface properties are dependent on the system composition, its roughness, and on its temperature.

Monochromatic properties are dependent on the wavelength of the incident radiation, and the total properties are dependent on the spectral distribution of the incident energy.

Most gases have high transmissivity, i.e. $\tau \approx 1$ and $\rho = \alpha = 0$ (like air at atmospheric pressure). However, some other gases (water vapour, CO_2 etc.) may be highly absorptive to thermal radiation, at least at certain wavelength.

Most solids encountered in engineering practice are opaque to thermal radiation $(\tau \approx 0)$. Thus for thermally opaque solid surfaces,

$$\rho + \alpha = 1 \tag{6}$$

Another important property of the surface of a substance is its ability to emit radiation. Emission and radiation have different concept. Reflection may occur only when the surface receives radiation whereas emission always occurs if the temperature of the surface is above the absolute zero. Emissivity of the surface is a measure of how good it is an emitter.

Blackbody radiation

In order to evaluate the radiation characteristics and properties of a real surface it is useful to define an ideal surface such as the perfect blackbody. The perfect blackbody is defined as one which absorbs all incident radiation regardless of the spectral distribution or directional characteristic of the incident radiation.

$$\alpha = \alpha_\lambda = 1$$

$$\rho = \rho_\lambda = 0$$

A blackbody is black because it does not reflect any radiation. The only radiation leaving a blackbody surface is original emission since a blackbody absorbs all incident radiation. The emissive power of a blackbody is represented by , and depends on the surface temperature only.

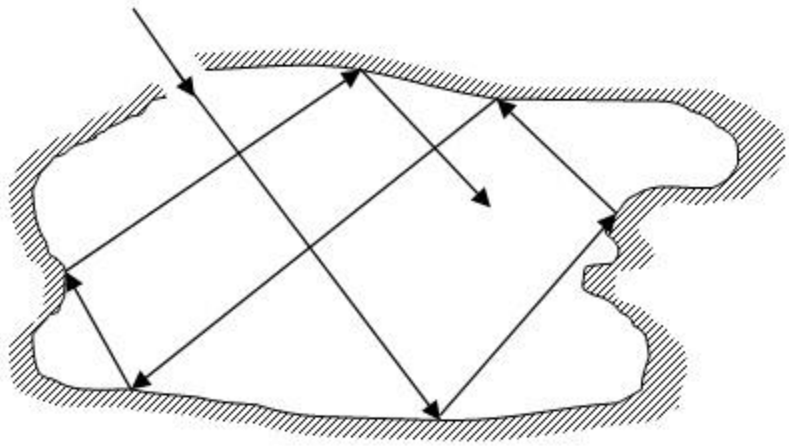

Fig. 3: Example of a near perfect blackbody

It is possible to produce a near perfect blackbody as shown in fig.3.

Figure.2 shows a cavity with a small opening. The body is at isothermal state, where a ray of incident radiation enters through the opening will undergo a number of internal reflections. A portion of the radiation absorbed at each internal reflection and a very little of the incident beam ever find the way out through the small hole. Thus, the radiation found to be evacuating from the hole will appear to that coming from a nearly perfect blackbody.

Planck's law

A surface emits radiation of different wavelengths at a given temperature (theoretically zero to infinite wavelengths). At a fixed wavelength, the surface radiates more energy as the temperature increases. Monochromatic emissive power of a blackbody is given by eq.10.

$$E_{b,\lambda} = \frac{2\pi hc^2\lambda^{-5}}{\exp\left(\frac{hc}{\lambda kT}\right)-1} \tag{7}$$

where, h = 6.6256 X 10^{-34} JS; Planck's constant

c = 3 X 10^{8} m/s; speed of light

T = absolute temperature of the blackbody

λ = wavelenght of the monochromatic radiation emitted k = Boltzmann constant

Equation.10 is known as Planck's law. Figure.4 shows the representative plot for Planck's distribution.

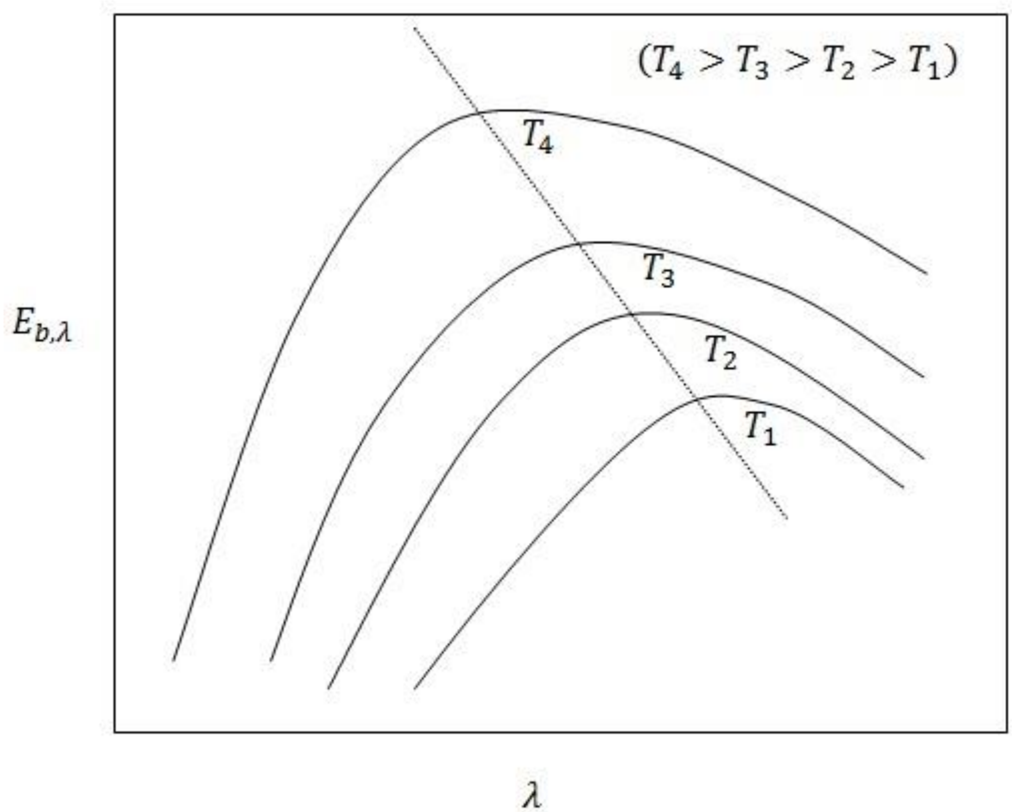

Fig. 4: Representative plot for Planck's distribution

Wien's law

Figure.4 shows that as the temperature increases the peaks of the curve also increases and it shift towards the shorter wavelength. It can be easily found out that the wavelength corresponding to the peak of the plot (λ_{max}) is inversely proportional to the temperature of the blackbody (Wein's law) as shown in eq.11.

$$\lambda_{max}\ T = 2898 \qquad (11)$$

Now with the Wien's law or Wien's displacement law, it can be understood if we heat a body, initially the emitted radiation does not have any colour. As the temperature rises the λ of the radiation reach the visible spectrum and we can able to see the red colour being height λ (for red colour). Further increase in temperature shows the white colour indicating all the colours in the light.

The Stefan-Boltzmann law for blackbody

Josef Stefan based on experimental facts suggested that the total emissive power of a blackbody is proportional to the fourth power of the absolute temperature. Later, Ludwig Boltzmann derived the same using classical thermodynamics. Thus the eq.12 is known as Stefan-Boltzmann law,

$$E_b = \int_0^{\infty} E_{b\lambda}(\lambda, T)\, d\lambda$$

$$E_b = \sigma T^4 \quad (12)$$

where, E_b is the emissive power of a blackbody, T is absolute temperature, and σ (= 5.67 X 10^{-8} W/m^2/K^4) is the Stefan-Boltzmann constant.

The Stefan-Boltzmann law for the emissive power gives the total energy emitted by a blackbody defined by eq.3.

Special characteristic of blackbody radiation

It has been shown that the irradiation field in an isothermal cavity is equal to E_b. Moreover, the irradiation was same for all planes of any orientation within the cavity. It may then be shown that the intensity of the blackbody radiation, I_b, is uniform. Thus, blackbody radiation is defined as,

$$E_b = \pi I_b \quad (13)$$

where, $I_b = \int_0^{\infty} I_{b\lambda}\, d\lambda$ is the total intensity of the radiation and is called the spectral radiation intensity of the blackbody.

Kirchhoff's law

Consider an enclosure as shown in fig.2 and a body is placed inside the enclosure. The radiant heat flux (q) is incident onto the body and allowed to come into temperature equilibrium. The rate of energy absorbed at equilibrium by the body must be equal to the energy emitted.

$$EA = \alpha q A$$

$$E = \alpha q \tag{7.14}$$

where, E is the emissive power of the body, α is absorptivity of the of the body at equilibrium temperature, and A is the area of the body.

Now consider the body is replaced by a blackbody i.e. $E \rightarrow E_b$ and $\alpha = 1$, the equation.14 becomes

$$E_b = q \tag{15}$$

Dividing eq.14 by eq.15,

$$\frac{E}{E_b} = \alpha \tag{16}$$

At this point we may define emissivity, which is a measure of how good the body is an emitter as compared to blackbody. Thus the emissivity can be written as the ratio of the emissive power to that of a blackbody,

$$\frac{E}{E_b} = \epsilon \tag{17}$$

On comparing eq.16 and eq.17, we get

$$\epsilon = \alpha \tag{18}$$

Equation 18 is the Kirchhoff's law, which states that the emissivity of a body which is in thermal equilibrium with its surrounding is equal to its absorptivity of the body. It should be noted that the source temperature is equal to the temperature of the irradiated surface. However, in practical purposes it is assumed that emissivity and absorptivity of a system are equal even if it is not in thermal equilibrium with the surrounding. The reason being the absorptivity of most real surfaces is relatively insensitive to temperature and wavelength. This particular assumption leads to the concept of grey body. The emissivity is considered to be independent of the wavelength of radiation for grey body.

Grey body

If grey body is defined as a substance whose monochromatic emissivity and absorptivity are independent of wavelength. A comparative study of grey body and blackbody is shown in the table.2.

Table-2: Comparison of grey and blackbody

Blackbody	**Grey body**
Ideal body	Ideal body
Emissivity independent of wavelength	Emissivity is independent of wavelength
Absorptivity s independent of wavelength	Absorptivity (α) is independent of wavelength
$\varepsilon = 1$	$\varepsilon < 1$
$\alpha = 1$	$\alpha < 1$

Radiative heat exchanger between surfaces

Till now we have discussed fundamental aspects of various definitions and laws. Now we will study the heat exchange between two or more surfaces which is of practical importance. The two surfaces which are not in direct contact, exchanges the heat due to radiation phenomena. The factors those determine the rate of heat exchange between two bodies are the temperature of the individual surfaces, their emissivities, as well as how well one surface can see the other surface. The last factor is known as view factor, shape factor, angle factor or configuration factor.

View factor

In this section we would like to find the energy exchange between two black surfaces having area $A_1$and A_2, respectively, and they are at different temperature and have arbitrary shape and orientation with respect to each other. In order to find the radiative heat exchange between the bodies we have to first define the view factor as

F_{12} = fraction of the energy leavings surface 1 which reaches surface 2

F_{21} = fraction of the energy leaving surface 2 which reaches surface 1 or in general,

F_{mn} = fraction of the energy leaving surface m which raches surface n

Thus the energy leaving surface 1 and arriving at surface 2 is $E_{b1}A_1F_{12}$ and the energy leaving surface 2 and arriving at surface 1 is $E_{b2}A_2F_{21}$. All the incident radiation will be absorbed by the blackbody and the net energy exchange will be,

$$Q = E_{b1}A_1F_{12} - E_{b2}A_2F_{21}$$

At thermal equilibrium between the surfaces $Q_{12} = 0$ and $E_{b1} = E_{b2}$, thus

$$0 = E_{b1} (A_1F_{12} - A_2F_{21})$$

$$A_1F_{12} - A_2F_{21} \qquad (19)$$

Equation.19 is known as reciprocating relation, and it can be applied in general way for any blackbody surfaces.

$$A_iF_{ij} - A_jF_{ji} \qquad (20)$$

Though the relation is valid for blackbody it may be applied to any surface as long as diffuse radiation is involved.

Relation between view factors

In this section we will develop some useful relation of view factor considering fig..5

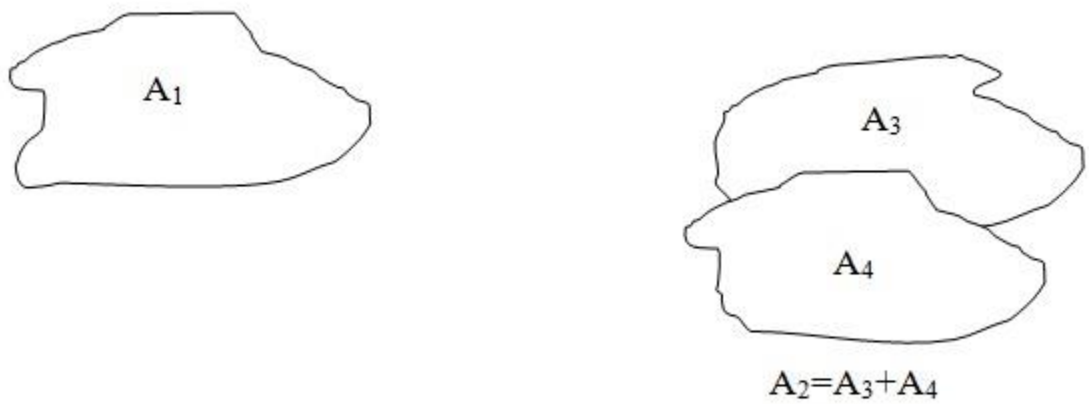

Fig. 5: Exchange of energy between area A1 and A2 (A is area of blackbody)

View factor for radiation from A_1 to the combined area A_2,

$$F_{12} = F_{13} + F_{14} \quad (21)$$

and using the reciprocating relations for surface 1 and 4,

$$A_1F_{14} = A_4F_{41} \quad (22)$$

Using eq..21 and22,

$$F_{41} = \frac{A_1}{A_4}F_{14}$$

$$F_{41} = \frac{A_1}{A_4}(F_{12} - F_{13})$$

Thus the unknown view factor F_{14} can be estimated if the view factors F_{12} and F_{13}, as well as their areas are (A_1, A_2) known.

Now, consider a flat plate (for eg.) which is emitting the radiation, it can be understood that the radiation of the flat plat cannot fall on its own surface (partly or fully). Such kind or surfaces are termed as "not able to see itself". In such situations,

$$F_{11} = F_{22} = F_{33} = F_{44} = 0$$

However, if the surface can see itself like concave curved surfaces, which may thus see themselves and then the shape factor will not be zero in those cases.

Another property of the shape factor is that when the surface is enclosed, then the following relation holds,

$$\sum_{j=1}^{n} F_{ij} = 1 \tag{23}$$

where, F_{ij} is the fraction of the total energy leaving surface i which arrives at surface j.

In case of N-walled enclosure, some of the view factors may be evaluated from the knowledge of the rest and the total N^2 view factors may be represented in square matrix form shown below,

$$\begin{bmatrix} F_{11} & F_{12} & \cdots F_{1N} \\ F_{21} & F_{22} & \ldots F_{2N} \\ \cdots & \cdots & \cdots\cdots \\ F_{N1} & F_{N2} & \cdots F_{NN} \end{bmatrix}$$

Heat exchange between non blackbodies

Evaluation of radiative heat transfer between black surfaces is relatively easy because in case of blackbody all the radiant energy which strikes the surface is absorbed. However, finding view factor is slightly complex, but once it can be done, finding heat exchange between the black bodies is quite easy.

When non blackbodies are involved the heat transfer process becomes very complex because all the energy striking on to the surface does not get absorbed. A part of this striking energy reflected back to another heat transfer surface, and part may be reflected out from the system entirely. Now, one can imagine that this radiant energy can be reflected back and forth between the heat transfer surfaces many times.

In this section, we will assume that all surfaces are in the analysis are diffuse and uniform in temperature and that the reflective and emissive properties are constant over all surfaces.

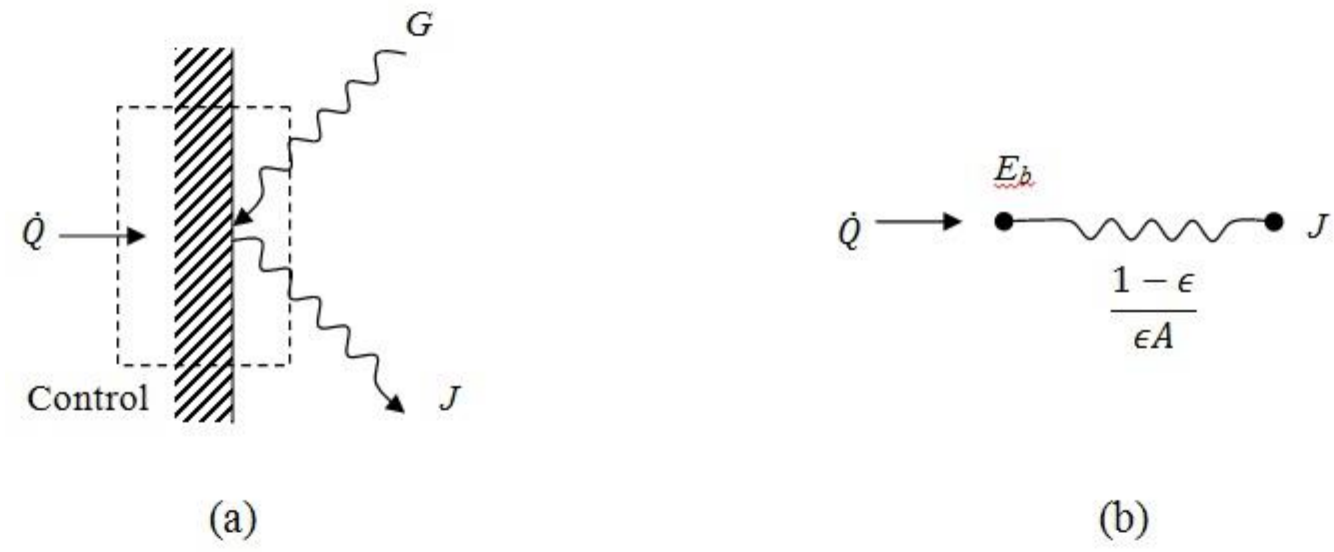

Fig. 6: (a) Surface energy balance for opaque surface (b) equivalent electrical circuit

It is also assumed that the radiosity and irradiation are uniform over each surface. As we have already discussed that the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted (as opaque body), or

$$J = \epsilon E_b + \rho G \tag{24}$$

where, ϵ is the emissivity and E_b is the blackbody emissive power. Because the transmissivity is zero due to opaque surface and absorptivity of the body (grey) will be equal to its emissivity by Kirchhoff's law.

$$\rho = 1 - \alpha = 1 - \epsilon$$

Thus, eq.24 becomes

$$J = \epsilon E_b + (1 - \epsilon)G \tag{25}$$

The net energy leaving the surface is the difference between the radiosity and the irradiance (fig.6a),

$$\frac{\dot{Q}}{A} = J - G = \epsilon E_b + (1-\epsilon)G - G$$

$$\frac{\dot{Q}}{A} = \frac{\epsilon A}{1-\epsilon}(E_b - J)$$

$$\dot{Q} = \frac{E_b - J}{(1-\epsilon)/\epsilon A} \qquad (26)$$

The eq.26 can be analogous to the electrical circuit as shown in fig.6(b). The numerator of the eq.26 is equivalent to the potential difference, denominator is equivalent to the surface resistance to radiative heat, and left part is equivalent to the current in the circuit.

In the above discussion we have considered only one surface. Now we will analyse the exchange of radiant energy by two surfaces, A_1 and A_2, as shown in the fig.7a.

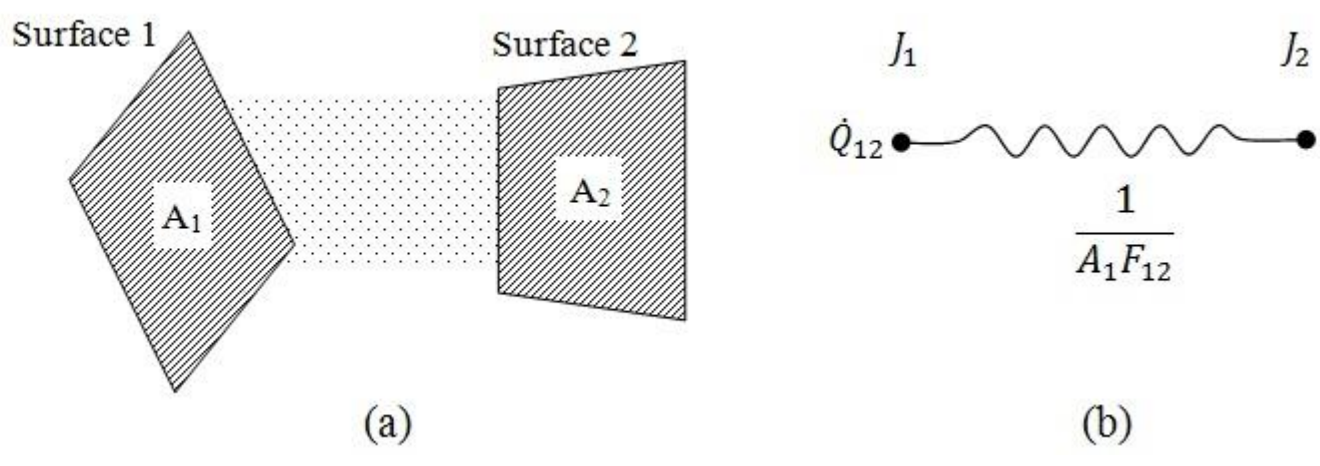

Fig. 7: (a) Energy exchange between two surfaces, (b) equivalent circuit diagram

The radiation which leaves surface 1, the amount that reaches surface 2 is

$$J_1 A_1 F_{12}$$

Similarly, the radiation which leaves system 2, the amount that reaches surface 1 is

$$J_2A_2F_{21}$$

The net energy transfer between the surfaces,

$$\dot{Q}_{12} = J_1A_1F_{12} - J_2A_2F_{21}$$

Reciprocity theorem states that

$$A_1F_{12} = A_2F_{21}$$

$$\Rightarrow \dot{Q}_{12} = (J_1 - J_2)A_1F_{12} = (J_1 - J_2)A_2F_{21}$$

$$\Rightarrow \dot{Q}_{12} = \frac{(J_1 - J_2)}{1/\,A_1F_{12}} \qquad (27)$$

It also resembles an electrical circuit shown in fig.7b. The difference between eq.26 and 27 is that in eq.27 the denominator term is space resistance instead of surface resistance.

Now, to know, the net energy exchange between the two surfaces we need to add both the surface resistances along with the overall potential as shown in the fig.8. Here the surfaces see each other and nothing else.

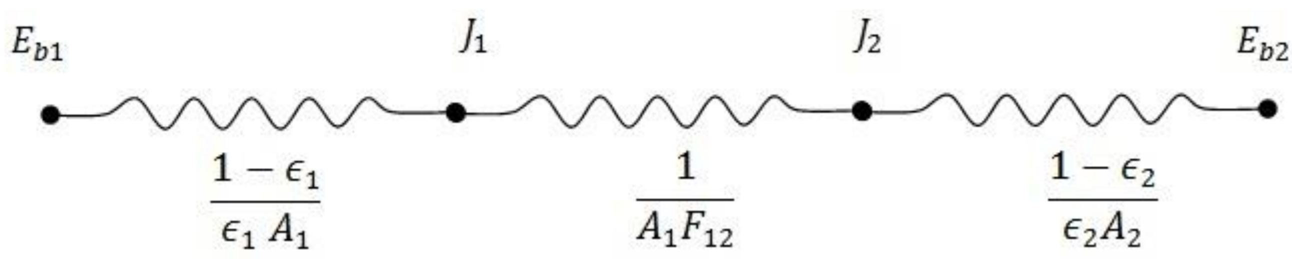

Fig.8: Radiative nature for two surfaces which can see each other nothing else

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}} \tag{28}$$

Radiation shield

Till now we have discussed about the radiative heat transfer from one surface to another without any interfering surface in between. Here we will discuss about an interfering shield in between, which is termed as radiation shield. A radiation shield is a barrier wall of low emissivity placed between two surfaces which reduce the radiation between the bodies. In fact, the radiation shield will put additional resistance to the radiative heat transfer between the surfaces as shown in fig.9.

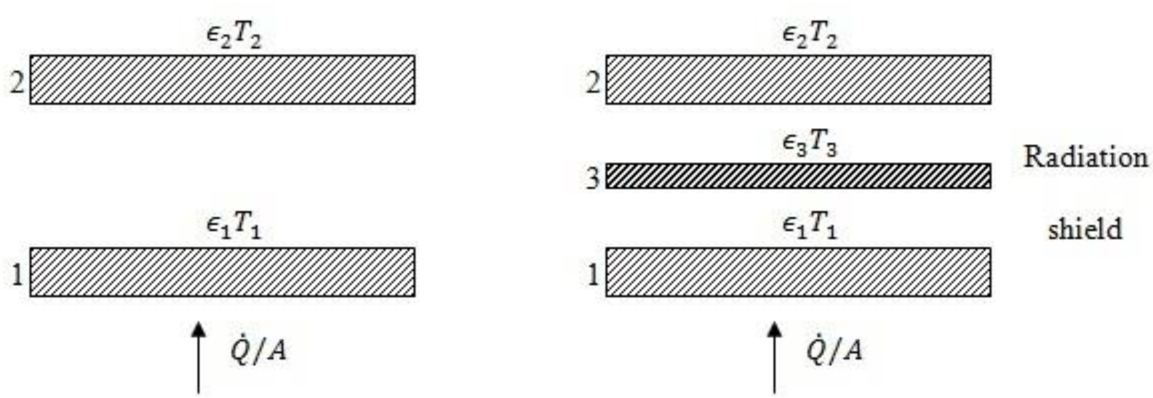

Fig. 9: Radiation between two large infinite plates (a) without and (b) with radiation shield

Considering fig.9(b) and the system is at steady state, and the surfaces are flat (F_{ij} because each plate is in full view of the other). Moreover, the surface are large enough and $\frac{A_1}{A_2} \approx 1$ may be considered and the equivalent blackbody radiation energy may be written as $E_b = \sigma T^4$.

Thus, eq.28 becomes

$$\left.\frac{\dot{Q}}{A}\right|_{net} = \left.\frac{\dot{Q}_{13}}{A_1}\right|_{net} = \left.\frac{\dot{Q}_{32}}{A_3}\right|_{net} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{\sigma(T_3^4 - T_2^4)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1} \tag{29}$$

In order to have a feel of the role of the radiation shield, consider that the emissivities of all the three surfaces are equal.

$\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon$

Then it can be seen that the heat flux is just one half of that which would be experienced if there were no shield present.
In similar line we can deduce that when n-shields are arranged between the two surfaces then,

$$\left(\frac{\dot{Q}}{A}\right)_{net_{with\ shield}} = \frac{1}{n+1}\left(\frac{\dot{Q}}{A}\right)_{without\ shield} \quad (30)$$

Electrical network for radiation through absorbing and transmitting medium
The previous discussions were based on the consideration that the heat transfer surfaces were separated by a completely transparent medium. However, in real situations the heat transfer medium absorbs as well as transmits. The examples of such medium are glass, plastic film, and various gases.

Consider two non-transmitting surfaces (same as in fig.8) are separated by a transmitting and absorbing medium. The medium may be considered as a radiation shield which see themselves and others. If we distinguish the transparent medium by *m* and if the medium is non-reflective (say gas) then using Kirchhoff's law,

$$\alpha_m + \tau_m = 1 = \epsilon_m + \tau_m \quad (31)$$

The energy leaving surface 1 which is transmitted through the medium and reaches the surface 2 is,

$J_1 A_1 F_{12} \tau_m$

and that which leaves surface 2 and arrives at surface 1 is,

$J_2 A_2 F_{21} \tau_m$

Therefore, the net exchange in the transmission process is therefore,

$$\dot{Q}_{12} = A_1 F_{12} \tau_m (J_1 - J_2)$$

Using eq.31,

$$\dot{Q}_{12} = \frac{(J_1 - J_2)}{\left(\frac{1}{A_1 F_{12}(1-\epsilon_m)}\right)}$$

Thus the equivalent circuit diagram is shown in fig.9

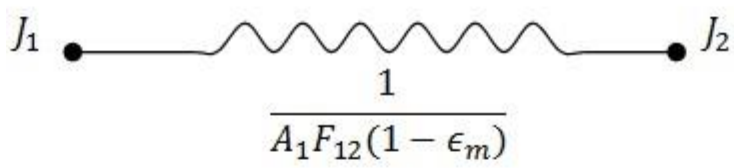

Fig.9. Equivalent electrical circuit for radiation through gas

Radiation combined with conduction and convection

In industrial processes, in general, the heat transfer at higher temperature has significant portion of radiation along with conduction and convection. For example, a heated surface is shown in the fig.10 with all the three mechanism of heat transfer.

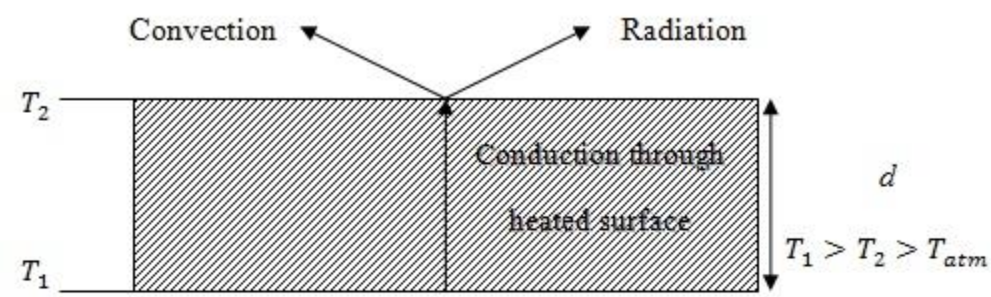

Fig.11: Radiation combined with conduction and convection

At steady state

Heat flux by conduction = heat flux by convention + heat flux by radiation

$$\frac{K}{d}(T_1 - T_2) = h(T_2 - T_{atm}) + \epsilon\sigma(T_2^4 - T_{atm}^4)$$

where, h is the heat transfer coefficient at the surface in contact (outer surface) with atmosphere due to natural and forced convection combined together, ϵ is the emissivity of the outer surface, and T_{atm} is the atmospheric temperature.