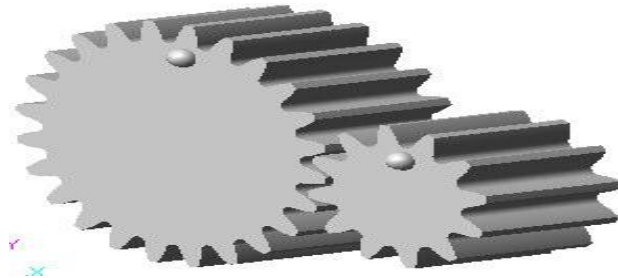# SME 1023 KINEMATICS OF MACHINE

UNIT-IV GEAR AND GEAR TRAIN

A toothed wheel that engages another toothed mechanism in order to change the speed or direction of transmitted motion.

$N = speed\ in\ rpm$

A gear is a component within a transmission device that transmits rotational force to another gear or device. A gear is different from a pulley in that a gear is a round wheel which has linkages that mesh with other gear teeth, allowing force to be fully transferred without slippage. Depending on their construction and arrangement, geared devices can transmit forces at different speeds, torques, or in a different direction, from the power source. The most common situation is for a gear to mesh with another gear

Gear's most important feature is that gears of unequal sizes (diameters) can be combined to produce a mechanical advantage, so that the rotational speed and torque of the second gear are different from that of the first.

To overcome the problem of slippage as in belt drives, gears are used which produce positive drive with uniform angular velocity.

GEAR CLASSIFICATION

Gears or toothed wheels may be classified as follows:

1. According to the position of axes of the shafts.
   The axes of the two shafts between which the motion is to be transmitted, may be
   a. Parallel

b. Intersecting

c. Non-intersecting and Non-parallel

Gears for connecting parallel shafts

1. **Spur Gear**

    Teeth is parallel to axis of rotation can transmit power from one shaft to another parallel shaft. Spur gears are the simplest and most common type of gear. Their general form is a cylinder or disk. The teeth project radially, and with these "straight-cut gears".
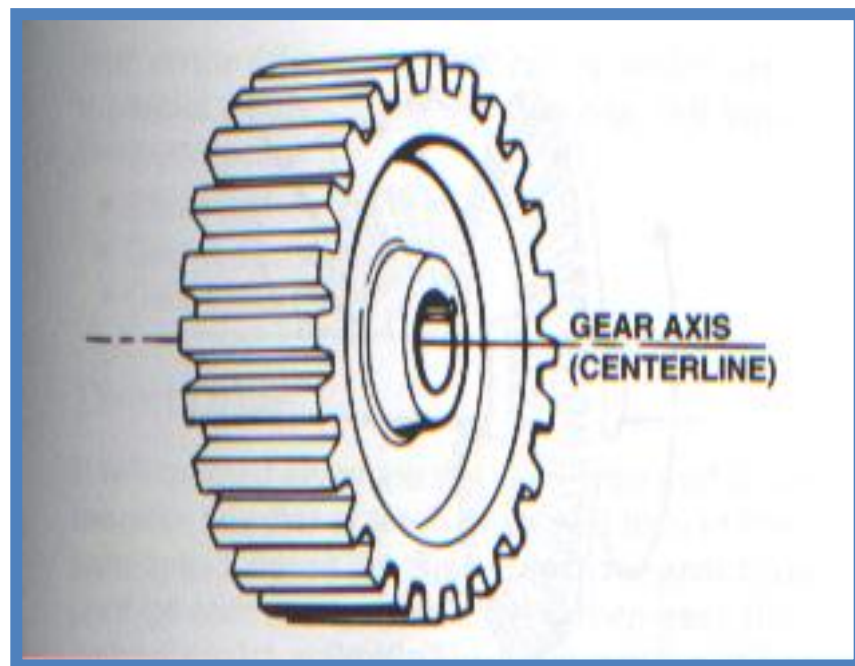
    Spur gears are gears in the same plane that move opposite of each other because they are meshed together. Gear 'A' is called the 'driver' because this is turned by a motor. As gear 'A' turns it meshes with gear 'B' and it begins to turn as well. Gear 'B' is called the 'driven' gear.
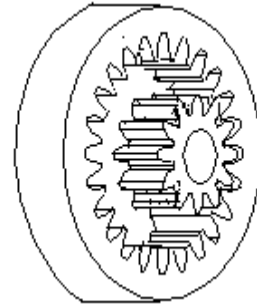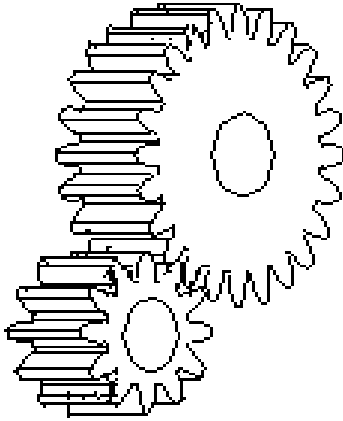
External gear makes external contact, and the internal gear (right side pair) makes internal contact.

APPLICATIONS OF SPUR GEAR

Electric screwdriver, dancing monster, oscillating sprinkler, windup alarm clock, washing machine and clothes dryer

2. Parallel Helical Gear

The teeth on helical gears are cut at an angle to the face of the gear. When two teeth on a helical gear system engage, the contact starts at one end of the tooth and gradually spreads as the gears rotate, until the two teeth are in full engagement.

This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears. For this reason, helical gears are used in almost all car transmissions. Because of the angle of the teeth on helical gears, they create a thrust load on the gear when they mesh. Devices that use helical gears have bearings that can support this thrust load.

One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees.

CROSSED HELICAL GEAR

Herringbone gears:

To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces. These are called double helical or herringbone gears
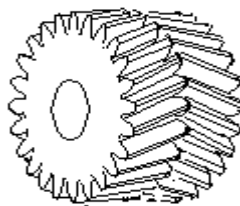
Herringbone gears (or double-helical gears)

Applications of Herringbone Gears

The most common application is in power transmission. They utilize curved teeth for efficient, high capacity power transmission. This offers reduced pulsation due to which they are highly used for extrusion and polymerization. Herringbone gears are mostly used on heavy machinery.

3. Rack and pinion

Rack and pinion gears are used to convert rotation (From the pinion) into linear motion (of the rack). A perfect example of this is the steering system on many cars. The steering wheel rotates a gear which engages the rack. As the gear turns, it slides the rack either to the right or left, depending on which way you turn the wheel. Rack and pinion gears are also used in some scales to turn the dial that displays your weight.

RACK AND PINION

GEARS FOR CONNECTING INTERSECTING SHAFTS

1. Straight Bevel Gear

Bevel gears are useful when the direction of a shaft's rotation needs to be changed. They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well. The teeth on bevel gears can be **straight**, **spiral** or **hypoid**. Straight bevel gear teeth actually have the same problem as straight spur gear teeth as each tooth engages, it impacts the corresponding tooth all at once.

BEVEL GEAR

Just like with spur gears, the solution to this problem is to curve the gear teeth. These spiral teeth engage just like helical teeth: the contact starts at one end of the gear and progressively spreads across the whole tooth.

SPIRAL BEVEL GEAR

On straight and spiral bevel gears, the shafts must be perpendicular to each other, but they must also be in the same plane. If you were to extend the two shafts past the gears, they would intersect

The bevel gear has many diverse applications such as locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.

NON-INTERSECTING AND NON-PARALLEL

1. WORM AND WORM GEAR

Worm gears are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater.

Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm. This is because the angle on the worm is so shallow that when the gear tries to spin it, the friction between the gear and the worm holds the worm in place.

WORM AND WORM GEAR

This feature is useful for machines such as conveyor systems, in which the locking feature can act as a brake for the conveyor when the motor is not turning. One other very interesting usage of worm gears is in the Torsen differential, which is used on some high-performance cars and trucks. They are used in right-angle or skew shaft drives. The presence of sliding action in the system even though results in quieter operation, it gives rise to considerable frictional heat, hence they need good lubrication for heat dissipation and for improving the efficiency. High reductions are possible which results in compact drive.

APPLICATION OF WORM GEARS

Worm gears are used widely in material handling and transportation machinery, machine tools, automobiles etc.

NOMENCLATURE OF SPUR GEARS
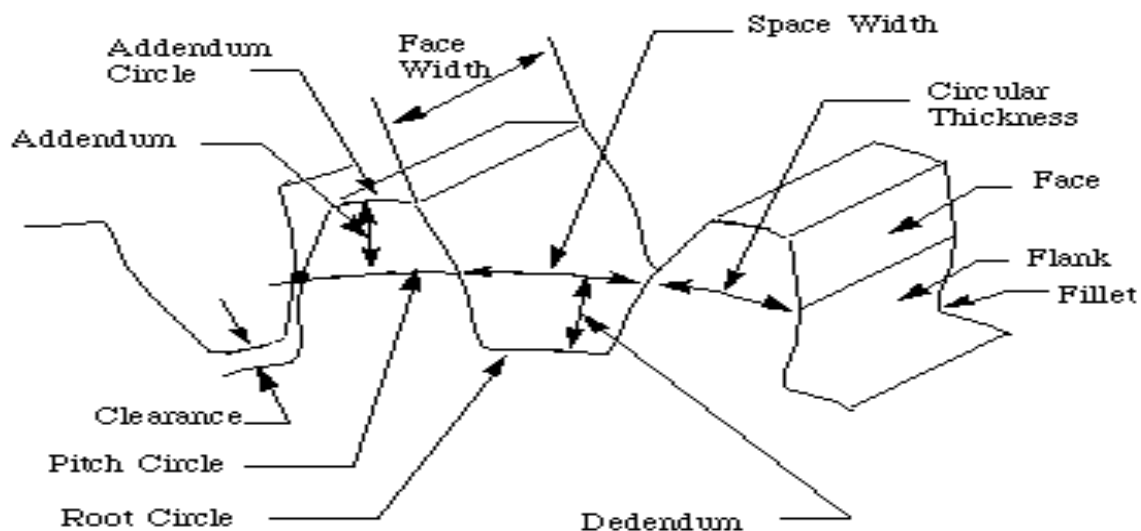
NOMENCLATURE OF SPUR GEAR

In the following section, we define many of the terms used in the analysis of spur gears.

Pitch surface: The surface of the imaginary rolling cylinder (cone, etc.) that the toothed gear may be considered to replace.

- **Pitch circle**: A right section of the pitch surface.
- **Addendum circle**: A circle bounding the ends of the teeth, in a right section of the gear.
- **Root (or dedendum) circle**: The circle bounding the spaces between the teeth, in a right section of the gear.
- **Addendum**: The radial distance between the pitch circle and the addendum circle.
- **Dedendum**: The radial distance between the pitch circle and the root circle.
- **Clearance**: The difference between the dedendum of one gear and the addendum of the mating gear.
- **Face of a tooth**: That part of the tooth surface lying outside the pitch surface.
- **Flank of a tooth**: The part of the tooth surface lying inside the pitch surface.
- **Circular thickness** (also called the **tooth thickness**): The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line.
- **Tooth space**: pitch diameter The distance between adjacent teeth measured on the pitch circle.
- **Backlash**: The difference between the circle thickness of one gear and the tooth space of the mating gear.
- **Circular pitch** (Pc) : The width of a tooth and a space, measured on the pitch circle.

$$P_c = \frac{\pi D}{N}$$

- **Diametral pitch (P_d)**: The number of teeth of a gear unit pitch diameter. A toothed gear must have an integral number of teeth. The circular pitch, therefore, equals the pitch circumference divided by the number of teeth. The diametral pitch is, by definition, the number of teeth divided by the pitch diameter. That is,

$$P_d = \frac{N}{D}$$

Where

Pc = circular pitch

P_d = diametral pitch

N = number of teeth

D = pitch diameter

- **Module** (m): Pitch diameter divided by number of teeth. The pitch diameter is usually specified in inches or millimeters; in the former case the module is the inverse of diametral pitch.

$$m = D/N$$

- **Fillet:** The small radius that connects the profile of a tooth to the root circle.
- **Pinion**: The smaller of any pair of mating gears. The larger of the pair is called simply the gear.
- **Velocity ratio**: The ratio of the number of revolutions of the driving (or input) gear to the number of revolutions of the driven (or output) gear, in a unit of time.
- **Pitch point**: The point of tangency of the pitch circles of a pair of mating gears.
- **Common tangent**: The line tangent to the pitch circle at the pitch point.
- **Line of action**: A line normal to a pair of mating tooth profiles at their point of contact.
- **Path of contact**: The path traced by the contact point of a pair of tooth profiles.
- **Pressure angle** (α): The angle between the common normal at the point of tooth contact and the common tangent to the pitch circles. It is also the angle between the line of action and the common tangent.
- **Base circle**: An imaginary circle used in involute gearing to generate the involutes that form the tooth profiles.

VELOCITY RATIO OF GEAR DRIVE

Velocity ratio is defined as the ratio of the speed of the driven shaft to the speed of the driver shaft.
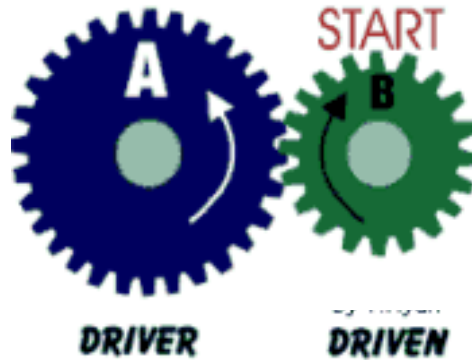
DRIVER   DRIVEN

One gear is a driver, which has d_1, N_1, ω_1 as diameter, speed and angular speed respectively. Another gear is driven connected to the driven shaft has d_2, N_2, ω_2 as diameter, speed angular speed respectively.

Angular speeds of the two gears will be

$$\omega_1 = 2\pi N_1 \qquad\qquad \omega_2 = 2\pi N_2$$

The peripheral velocity of the driver and driven shafts for the meshing pair of gear is

equal and is given by $V_P = \omega_1 \dfrac{d_1}{2} = \pi d_1 N_1 = \omega_2 \dfrac{d_2}{2} = \pi d_2 N_2$

Hence velocity ratio (n) = $\dfrac{\omega_2}{\omega_1} = \dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

T_1 and T_2 are the number of teeth on driver gear and driven gear, since the pair of gear

as the same module (m),then

$$d_1 = mT_1 \; ; \; d_2 = mT_2$$

and $n = \dfrac{N_2}{N_1} = \dfrac{d_1}{d_2} = \dfrac{T_1}{T_2}$

GEAR TRAINS

A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed. It reduces speed and increases torque. To create large gear ratio, gears are connected together to form gear trains. They often consist of multiple gears in the train. The smaller gears are one-fifth of the size of the larger gear. Electric motors are used with the gear systems to reduce the speed and increase the torque. Electric motor is connected to the driving end of each train and is mounted on the test platform. The output end of the gear train is connected to a large magnetic particle brake that is used to measure the output torque.

Types of gear trains

1. Simple gear train
2. Compound gear train
3. Planetary gear train

Simple Gear Train

The most common of the gear train is the gear pair connecting parallel shafts. The teeth of this type can be spur, helical or herringbone. only one gear for each axis. The angular velocity is simply the reverse of the tooth ratio. The main limitation of a simple gear train is that the maximum speed change ratio is 10:1. For larger ratio, large sizes of gear trains are required. The sprockets and chain in the bicycle is an example of simple gear train. When the paddle is pushed, the front gear is turned and that meshes with the links in the chain. The chain moves and meshes with the links in the rear gear that is attached to the rear wheel. This enables the bicycle to move.
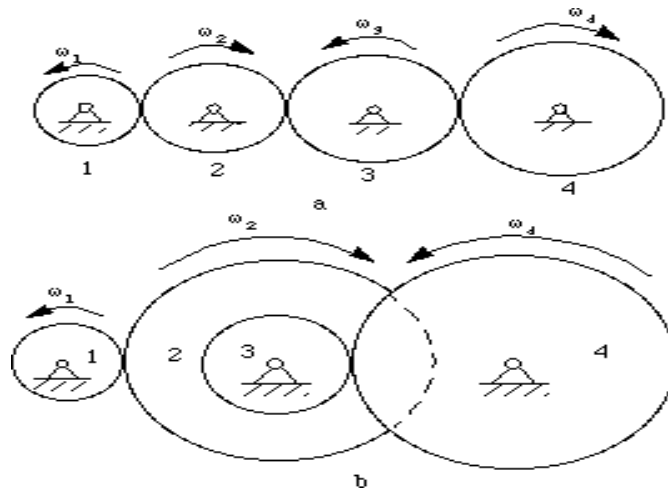
Simple and compound gear trains

Compound Gear Train

For large velocities, compound arrangement is preferred. Two keys are keyed to a single shaft. A double reduction train can be arranged to have its input and output shafts in a line, by choosing equal center distance for gears and pinions. Two or more gears may rotate about a single axis
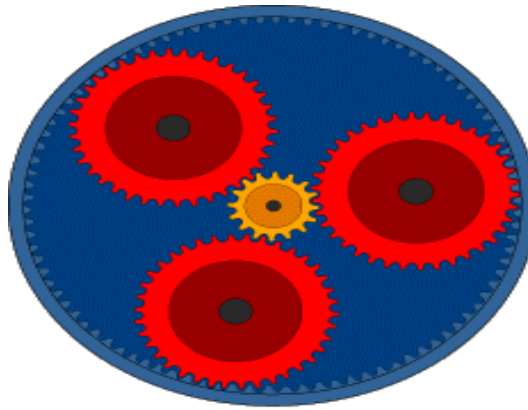
Planetary Gear Train (Epicyclic Gear Train)

Planetary gears solve the following problem. Let's say you want a gear ratio of 6:1 with the input turning in the same direction as the output. One way to create that ratio is with the following three-gear train:

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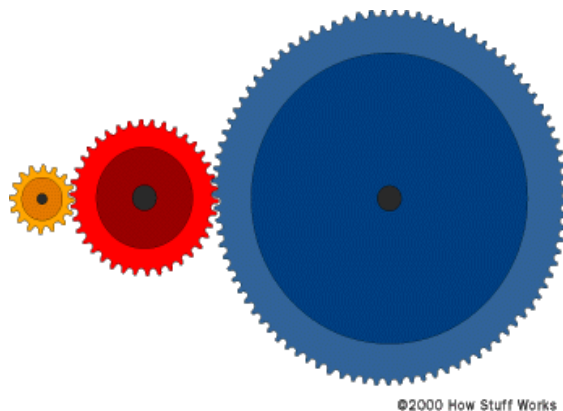
Planetary Gear Train

In this train, the blue gear has six times the diameter of the yellow gear (giving a 6:1 ratio). The size of the red gear is not important because it is just there to reverse the direction of rotation so that the blue and yellow gears turn the same way. However, imagine that you want the axis of the output gear to be the same as that of the input gear. A common place where this same-axis capability is needed is in an electric screwdriver. In that case, you can use a planetary gear system, as shown here:

Planetary Gear Train

In this gear system, the yellow gear (the **sun**) engages all three red gears (the **planets**) simultaneously. All three are attached to a plate (the **planet carrier**), and they engage the inside of the blue gear (the **ring**) instead of the outside. Because there are three red gears instead of one, this gear train is extremely rugged. The output shaft is attached to the blue ring gear, and the planet carrier is held stationary -- this gives the same 6:1 gear ratio. Another interesting thing about planetary gear sets is that they can produce different gear ratios depending on which gear you use as the input, which gear you use as the output, and which one you hold still. For instance, if the input is the sun gear, and we hold the ring gear stationary and attach the output shaft to the planet carrier, we get a different gear ratio. In this case, the planet carrier and planets orbit the sun gear, so instead of the sun gear having to spin six times for the planet carrier to make it around once, it has to spin seven times. This is because the planet carrier circled the sun gear once in the same direction as it was spinning, subtracting one revolution from the sun gear. So in this case, we get a 7:1 reduction.

You could rearrange things again, and this time hold the sun gear stationary, take the output from the planet carrier and hook the input up to the ring gear. This would give you a 1.17:1 gear reduction. An automatic transmission uses planetary gear sets to create the different gear ratios, using clutches and brake bands to hold different parts of the gear set stationary and change the inputs and outputs.

Planetary gear trains have several advantages. They have higher gear ratios. They are popular for automatic transmissions in automobiles. They are also used in bicycles for controlling power of pedaling automatically or manually. They are also used for power train between internal combustion engine and an electric motor.

Applications

Gear trains are used in representing the phases of moon on a watch or clock dial. It is also used for driving a conventional two-disk lunar phase display off the day-of-the-week shaft of the calendar.

Velocity ratio of Gear trains

We know that the **velocity ratio** of a pair of gears is the inverse proportion of the diameters of their pitch circle, and the diameter of the pitch circle equals to the number of teeth divided by the diametral pitch. Also, we know that it is necessary for the mating gears to have the same diametral pitch so that to satisfy the condition of correct meshing. Thus, we infer that the **velocity ratio** of a pair of gears is the inverse ratio of their number of teeth.

For the ordinary gear trains we have (Fig a)

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} \quad \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} \quad \frac{\omega_3}{\omega_4} = \frac{N_4}{N_3}$$

These equations can be combined to give the velocity ratio of the first gear in the train to the last gear:

$$\frac{\omega_1}{\omega_4} = \frac{N_2 N_3 N_4}{N_1 N_2 N_3} = \frac{N_4}{N_1}$$

$$\frac{(N_2 N_3 N_{4)}}{(N_1 N_2 N_3)} = \frac{(T_1 T_2 T_3)}{(T_2 T_3 T_4)} = \frac{N_4}{N_1} = \frac{T_1}{T_4} = n$$

Epicyclic gear train:

Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun. Here is a picture of a typical gear box.

This design can produce large gear ratios in a small space and are used on a wide range of applications from marine gearboxes to electric screwdrivers.

Basic Theory

The diagram shows a gear B on the end of an arm. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.
First consider what happens when the planet gear orbits the sun gear

Planet wheel
B

Arm

B

Arm 'A'

C

C

Sun wheel

P

P

P

P

B

A

P
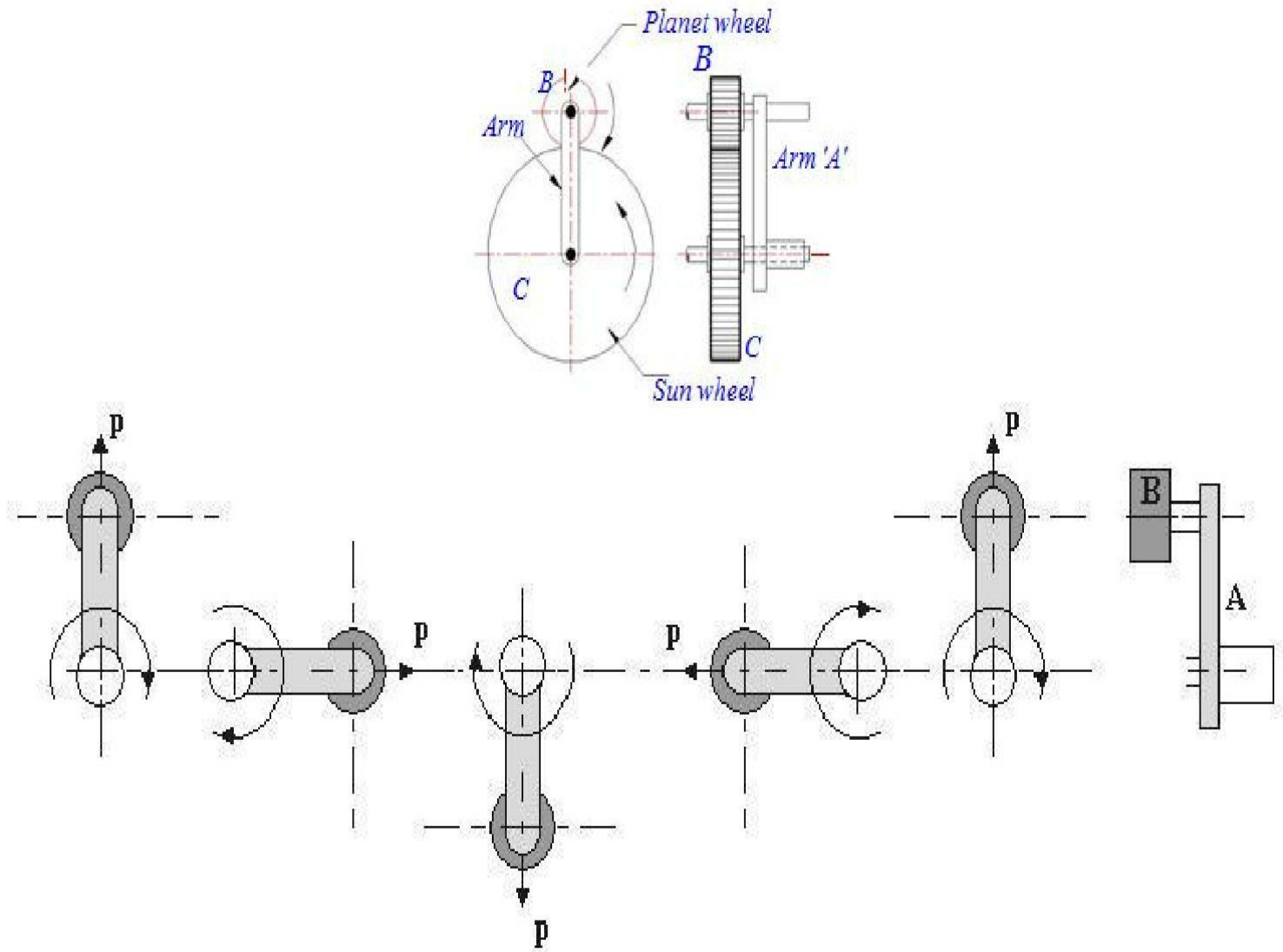
Observe point p and you will see that gear *B* also revolves once on its own axis. Any object orbiting around a center must rotate once. Now consider that *B* is free to rotate on its shaft and meshes with C. Suppose the arm is held stationary and gear *C* is rotated once. *B* spins about its own center and the number of revolutions it makes is the ratio t_c/B will rotate by this number for every complete t_s revolution of *C*.

Now consider that C is unable to rotate and the arm *A* is revolved once. Gear *B* will revolve $1+t_C/t_B$ because of the orbit. It is this extra rotation that causes confusion. One way to get round this is to imagine that the whole system is revolved once. Then identify the gear that is fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up. The following tabular method makes it easy.

Suppose gear *C* is fixed and the arm *A* makes one revolution. Determine how many revolutions the planet gear *B* makes.

Step 1 is to revolve everything once about the center.

Step 2 identify that *C* should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions of *B*.

Step 3 is simply add them up and we find the total revs of *C* is zero and for the arm is 1.

Step	*Action*	*A*	*B*	*C*
1	Revolve all once	1	1	1
2	Revolve *C* by −1 revolution, keeping the arm fixed	0	$+\dfrac{t_C}{t_B}$	-1
3	Add	1	$1+\dfrac{t_C}{t_B}$	0

Problem 1: In an ecicyclic gear train shown in figure, the arm A is fixed to the shaft S. The wheel B having 100 teeth rotates freely on the shaft S. The wheel F having 150 teeth driven separately. If the arm rotates at 200 rpm and wheel F at 100 rpm in the same direction; find (a) number of teeth on the gear C and (b) speed of wheel B.
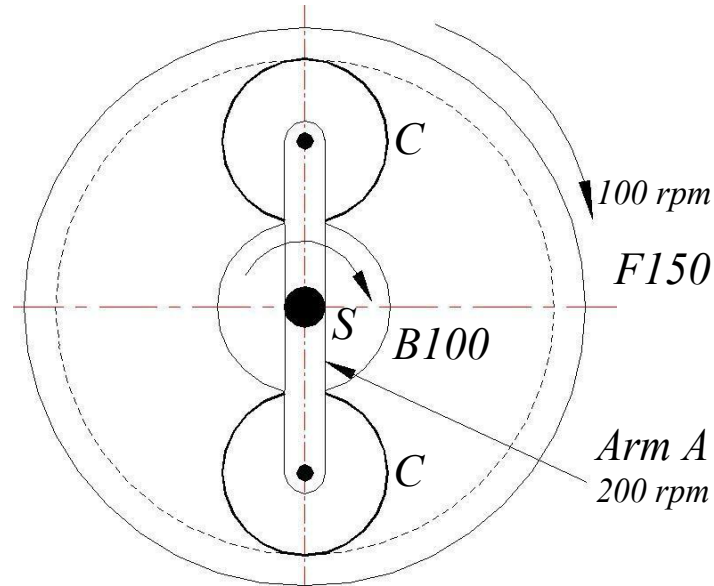
Solution:

$$T_B=100; \qquad T_F=150; \qquad N_A=200rpm; \; N_F=100rpm:$$

Since the mod *ule is same for all gears* :

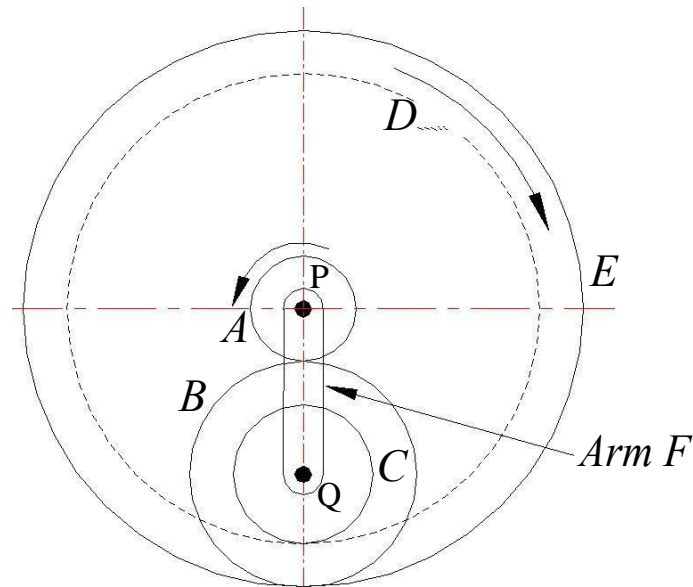the number of teeth on the gears is proportional to the pitch cirlce :

$$\therefore \qquad r_F = r_B + 2r_C$$
$$\Rightarrow \qquad T_F = T_B + 2T_C$$
$$150 = 100 + 2 \times T_C$$
$$T_C = 25 \rightarrow Number\ of\ teeth\ on\ gears\ C$$

The gear B and gear F rotates in the opposite directions:

$$\therefore Train\ value = -\frac{T_B}{T_F}$$

$$also \quad TV = \frac{N_L - N_{Arm}}{N_F - N_{Arm}} = \frac{N_F - N_A}{N_B - N_A} \quad (general \exp ression\ for\ epicyclic\ gear\ train)$$

$$\therefore \qquad -\frac{T_B}{T_F} = \frac{N_F - N_A}{}$$

$$-\frac{100}{150} = \frac{100 - 200}{N_B - 200} \qquad \Rightarrow \qquad N_E = 350$$

The Gear B rotates at 350 rpm in the same direction of gears F and Arm A.

Problem 2: In a compound epicyclic gear train as shown in the figure, has gears A and an annular gears D & E free to rotate on the axis P. B and C is a compound gear rotate about axis Q. Gear A rotates at 90 rpm CCW and gear D rotates at 450 rpm CW. Find the speed and direction of rotation of arm F and gear E. Gears A,B and C are having 18, 45 and 21 teeth respectively. All gears having same module and pitch.

Solution:

$$T_A=18; \qquad T_B=45; \qquad T_C=21; \qquad N_A = -90rpm; \qquad N_D=450rpm:$$

Since the module *and* pitch are same for all gears :

the number of teeth on the gears is proportional to the pitch cirlce :

$$\therefore \qquad r_D = r_A + r_B + r_C$$
$$\Rightarrow \qquad T_D = T_A + T_B + T_C$$
$$T_D = 18 + 45 + 21 = 84 \ \textit{teeth on gear D}$$

Gears A and D rotates in the opposite directions:

$$\therefore Train\ value = - \frac{T_A}{T_B} \times \frac{T_C}{T_D}$$

$$also \quad TV = \frac{N_L - N_{Arm}}{N_F - N_{Arm}} = \frac{N_D - N_F}{N_A - N_F}$$

$$\therefore \qquad - \frac{T_A}{T_B} \times \frac{T_C}{T} = \frac{N_D - N_E}{N_A - N_F}$$

$$- \frac{18 \times 21}{45 \times 84} = \frac{450 - N_F}{-90 - N_F}$$

$$\Rightarrow \quad N_F = \textit{Speed of Arm} = 400.9\ rpm\ -CW$$

Now consider gears A, B and E:

$$r_E = r_A + 2r_B$$
$$\Rightarrow \quad T_E = T_A + 2T_B$$
$$T_E = 18 + 2 \times 45$$
$$T_E = 108 \rightarrow \textit{Number of teeth on gear E}$$

Gears A and E rotates in the opposite directions:

$$\therefore \textit{Train value} = -\frac{T_A}{T_E}$$

$$\textit{also} \quad TV = \frac{N_E - N_F}{N_A - N_F}$$

$$\therefore \quad -\frac{T_A}{T_E} = \frac{N_E - N_F}{N_A - N_F}$$

$$-\frac{18}{108} = \frac{N_E - 400.9}{-90 - 400.9}$$

$$\Rightarrow \quad N_E = \textit{Speed of gear E} = 482.72\ rpm\ -CW$$

Problem 3: In an epicyclic gear of sun and planet type shown in figure 3, the pitch circle diameter of the annular wheel *A* is to be nearly 216mm and module 4mm. When the annular ring is stationary, the spider that carries three planet wheels *P* of equal size to make *one revolution* for every *five revolution* of the driving spindle carry the sun wheel.

Determine the number of teeth for all the wheels and the exact pitch circle diameter of the annular wheel. If an input torque of 20 N-m is applied to the spindle carrying the sun wheel, determine the fixed

torque on the annular wheel.

Annular 'A'

Spider 'L'

Sun Wheel 'S'

Planet Wheel 'P'
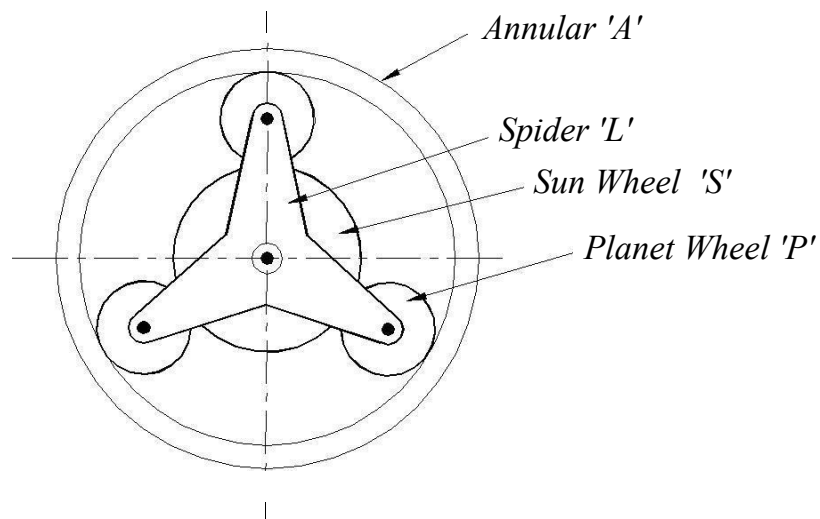
Solution: Module being the same for all the meshing gears:

$$T_A = T_S + 2T_P$$

$$T_A = PCD \text{ of } A = \frac{216}{4} = 54 \text{ teeth}$$

    If L rotates $+1$ revolution: $\therefore$ $n = 1$ (1)
  The sun wheel S to rotate $+5$ revolutions correspondingly:
           $\therefore$ $n + m = 5$ (2)
    From (1) and (2) $m = 4$
  When A is fixed:

$$n - \frac{T_S}{T_A} m = 0 \quad\Rightarrow\quad T_A = 4T_S$$

$$\therefore \quad T_S = \frac{54}{4} = 13.5 \text{ teeth}$$

    But fractional teeth are not possible; therefore T_S should be either 13 or 14 and T_A correspondingly 52 and 56.

Operation	Spider arm L	Sun Wheel S T_S	Planet wheel P T_P	Annular wheel A $T_A = 54$
Arm L is fixed & Sun wheel S is given +1 revolution	0	+1	$-\dfrac{T_S}{T_P}$	$-\dfrac{T_S}{T_P} \times \dfrac{T_P}{T_A} = -\dfrac{T_S}{T_A}$
Multiply by m (S rotates through m revolution)	0	m	$-\dfrac{T_S}{T_P} m$	$-\dfrac{T_S}{T_A} m$
Add n revolutions to all elements	n	$m+n$	$n - \dfrac{T_S}{T_P} m$	$n - \dfrac{T_S}{m} \dfrac{}{T_A}$

Trial 1:   Let $T_A = 52$ and $T_S = 13$
        $\therefore$ $T_P = \dfrac{T_A - T_S}{2} = \dfrac{52 - 13}{4}$ $= 19.5 \text{ teeth}$ -   ***This is impracticable***

Trial 2:   Let $T_A = 56$ and $T_S = 14$
        $\therefore$ $T_P = \dfrac{T_A - T_S}{2} = \dfrac{56 - 14}{4} = 21 \text{ teeth}$ -   ***This is practicable***

     $\therefore$ **$T_A = 56$, $T_S = 14$ and $T_P = 21$**
     $\Rightarrow$ PCD of $A = 56 \times 4 = 224$ mm
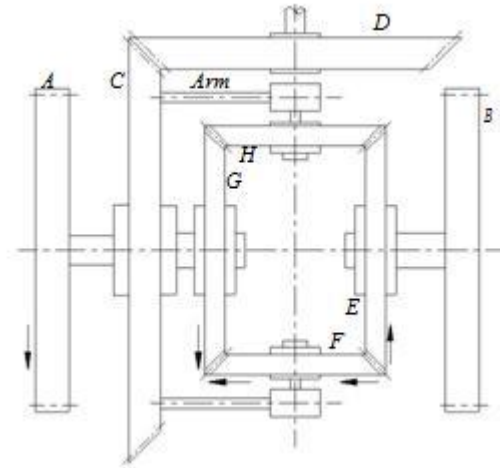 Also

      Torque on $L \times \omega_L$ = Torque on S $\times$ ω_S
      Torque on $L \times \omega_L$ = $20 \times \dfrac{5}{1} = 100 \, N - m$

$$\therefore \qquad \text{Fixing torque on A} = (T_L - T_S) = 100 - 20 = \textbf{80 N-m}$$

Problem 4: The gear train shown in figure 4 is used in an indexing mechanism of a milling machine. The drive is from gear wheels *A* and *B* to the bevel gear wheel *D* through the gear train. The following table gives the number of teeth on each gear.

Gear	A	B	C	D	E	F
Number of teeth	72	72	60	30	28	24
Diametral pitch in mm	08	08	12	12	08	08

How many revolutions does D makes for one revolution of A under the following situations

    a.   If *A* and *B* are having the same speed and same direction

    b.   If *A* and *B* are having the same speed and opposite direction

    c.   If *A* is making 72 rpm and *B* is at rest

    d.   If *A* is making 72 rpm and *B* 36 rpm in the same direction

Solution:

    Gear D is external to the epicyclic train and thus C and D constitute an ordinary train.

Operation	*Arm C (60)*	*E (28)*	*F (24)*	*A (72)*	*B (72)*	*G (28)*	*H (24)*
Arm or C is fixed & wheel A is given +1 revolution	0	-1	$-\dfrac{28}{24} = -\dfrac{7}{6}$	+1	-1	+1	$\dfrac{28}{24} = \dfrac{7}{6}$
Multiply by *m* (*A* rotates through *m* revolution)	*0*	-*m*	$-\dfrac{7}{6}m$	+*m*	-*m*	+*m*	$\dfrac{7}{6}m$
Add *n* revolutions to all elements	*n*	*n* - *m*	$n - \dfrac{7}{6}m$	*n* + *m*	*n* - *m*	*n* + *m*	$n + \dfrac{7}{6}m$

(i) For one revolution of A: $n + m = 1$ (1)

For A and B for same speed and direction: $n + m = n - m$ (2)

From (1) and (2): $n = 1$ and $m = 0$

$\therefore$ If C or arm makes one revolution, then revolution made by D is given by:

$$N_D = \frac{T_C}{T_D} = \frac{60}{30} = 2$$

$\therefore$ $N_D = 2\,N_C$

(ii) A and B same speed, opposite direction: $(n + m) = -(n - m) \quad (3)$

$n = 0; \quad m = 1$

When C is fixed and A makes one revolution, D does not make any revolution.

(iii) A is making 72 rpm: $(n + m) = 72$

B at rest $(n - m) = 0 \implies n = m = 36$ rpm

$\therefore$ C makes 36 rpm and D makes $36 \times \dfrac{60}{30} = 72 \; rpm$

(iv) A is making 72 rpm and B making 36 rpm

$(n + m) = 72$ rpm and $(n - m) = 36$ rpm

$(n + (n - m)) = 72; \quad n = 54$

$\therefore$ D makes $54 \times \dfrac{60}{30} = 108 \; rpm$