Convective Heat transfer: When a current or macroscopic particle of fluid crosses a specific surface, it carries with it a definite quantity of enthalpy. Such a flow of enthalpy is called convection. Convection can refer to the flow of heat associated with the movement of fluid, such as when hot air from a furnace enters a room, or to the transfer of heat from a hot surface to a flowing fluid. The two types of convection are Natural convection and forced convection.

Natural convection: If the convection currents are the result of buoyancy forces generated by the differences in density and the differences in density are in turn caused by temperature gradients in the fluid mass, the action is called natural convection. **Forced convection**: If the convection currents are set in motion by the action of a mechanical device such as pump or agitator, the flow is independent of density gradients, it is called forced convection. Heat flux, average temperature of fluid stream Heat flux: Heat transfer calculations are based on the area of the heating surface area. The rate of heat transfer per unit area is called the heat flux. Bulk mean temperature: When a fluid is heated or cooled the temperature will vary throughout the cross section of the stream. Because of these temperature gradients through out the stream it is necessary to state what is meant by the temperature of the stream. It is the temperature that will be at a fluid stream flowing across the section are withdrawn and mixed adiabatically to a uniform temperature. This is also called as average or mixing cup stream temperature.

Film temperature: It is the average between the temperature of the surface and the fluid. indeed if the entire Overall Heat Transfer Coefficient Let us consider a plane wall of thickness x_w and thermal conductivity kw. The warm fluid at a mean temperature of T_h is flowing through the inside surface of the wall. The cold fluid at a mean temperature of T_c is flowing through the outside surface of the wall. The inside surface temperature is T_{wh} and outside surface temperature is T_{wc}. The overall heat transfer coefficient is constructed from the individual coefficients and the resistances of the wall in the following manner. (i) Overall Heat transfer Coefficient based on outside surface area. The rate of heat transfer from the warm fluid to the inner surface of the wall in differential form:

$$\frac{dq}{dA_i} = h_i\left(T_h - T_{wh}\right)$$

By rearranging the above,

$$dq = \frac{(T_h - T_{wh})}{\dfrac{1}{h_i dA_i}}$$

. The rate of heat transfer through the wall in differential form is given by

$$\frac{dq}{d\bar{A}_L} = \frac{k_w (T_{wh} - T_{wc})}{x_w}$$

on rearranging,

$$dq = \frac{(T_{wh} - T_{wc})}{\dfrac{x_w}{k_w d\bar{A}_L}}$$

the rate of heat transfer from the outer surface of the wall to the cold fluid in differential form is given by

$$\frac{dq}{dA_o} = h_o (T_{wc} - T_c)$$

on rearranging,

$$dq = \frac{(T_{wc} - T_c)}{\dfrac{1}{h_o dA_o}}$$

If the eqns are solved for the temperature differences and the temperature differences added, the result is

$$(T_h - T_{wh}) + (T_{wc} - T_c) + (T_{wc} - T_c) = T_h - T_c = \Delta T = dQ\left(\frac{1}{h_i dA_i} + \frac{x_w}{k_w d\bar{A}_L} + \frac{1}{h_o dA_o}\right)$$

Assume that the heat transfer rate is arbitrarily based on the outside area. If the eqn is solved for dQ, and if both sides of the resulting equations are divided by dAo, the result is

$$\frac{dQ}{dA_o} = \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{dA_o}{dA_i}\right) + \frac{x_w}{k_w}\left(\frac{dA_o}{d\bar{A}_L}\right) + \frac{1}{h_o}\left(\frac{dA_o}{dA_o}\right)}$$

$$\frac{dA_o}{dA_i} = \frac{r_o}{r_i} \quad ; \qquad \frac{dA_o}{d\bar{A}_L} = \frac{r_o}{\bar{r}_L}$$

Then the above Eqn becomes

$$\frac{dQ}{dA_o} = \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{r_o}{r_i}\right) + \frac{r_o}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}}$$

Overall heat transfer coefficient based on inside surface area The rate of heat transfer from the warm fluid to the inner surface of the wall in differential form:

$$\frac{dq}{dA_i} = h_i\left(T_h - T_{wh}\right)$$

On rearranging,

$$dq = \frac{\left(T_h - T_{wh}\right)}{\frac{1}{h_i dA_i}}$$

The rate of heat transfer through the wall in differential form

$$\frac{dq}{d\bar{A}_L} = \frac{k_w\left(T_{wh} - T_{wc}\right)}{x_w}$$

By rearranging the above,

$$dq = \frac{(T_{wh} - T_{wc})}{\frac{x_w}{k_w d\bar{A}_L}}$$

The rate of heat transfer from the outer surface of the wall to the cold fluid in differential form:

$$\frac{dq}{dA_o} = h_o(T_{wc} - T_c)$$

On rearranging

$$dq = \frac{(T_{wc} - T_c)}{\frac{1}{h_o dA_o}}$$

If the eqns to are solved for the temperature differences and the temperature differences added, the result is

$$(T_h - T_{wh}) + (T_{wc} - T_c) + (T_{wc} - T_c) = T_h - T_c = \Delta T = dQ\left(\frac{1}{h_i dA_i} + \frac{x_w}{k_w d\bar{A}_L} + \frac{1}{h_o dA_o}\right)$$

Assume that the heat transfer rate is arbitrarily based on the inside area. If the Eqn is solved for dQ, and if both sides of the resulting equations are divided by dAi, the result is

$$\frac{dQ}{dA_i} = \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{dA_i}{dA_i}\right) + \frac{x_w}{k_w}\left(\frac{dA_i}{d\bar{A}_L}\right) + \frac{1}{h_o}\left(\frac{dA_i}{dA_o}\right)}$$

$$\frac{dA_i}{dA_o} = \frac{r_i}{r_o} \quad ; \qquad \frac{dA_i}{d\bar{A}_L} = \frac{r_i}{\bar{r}_L}$$

The above eqn becomes

$$\frac{dQ}{dA_i} = \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{r_i}{r_i}\right) + \frac{r_i}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}\left(\frac{r_i}{r_o}\right)}$$

the overall heat transfer coefficient based on outside surface area is,

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{r_o}{r_i}\right) + \frac{r_o}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}}$$

Overall heat transfer coefficient based on inside surface area

The rate of heat transfer from the warm fluid to the inner surface of the wall in differential form

$$\frac{dq}{dA_i} = h_i(T_h - T_{wh})$$

On rearranging,

$$dq = \frac{(T_h - T_{wh})}{\frac{1}{h_i dA_i}}$$

The rate of heat transfer from the outer surface of the wall to the cold fluid in differential form:

$$\frac{dq}{dA_o} = h_o(T_{wc} - T_c)$$

on rearranging,

$$dq = \frac{(T_{wc} - T_c)}{\frac{1}{h_o dA_o}}$$

substituting the above equations,

$$(T_h - T_{wh}) + (T_{wc} - T_c) + (T_{wc} - T_c) = T_h - T_c = \Delta T = dQ\left(\frac{1}{h_i dA_i} + \frac{x_w}{k_w d\overline{A}_L} + \frac{1}{h_o dA_o}\right)$$

Assume that the heat transfer rate is arbitrarily based on the inside area. If the eqn is solved for dQ, and if both sides of the resulting equations are divided by dAi, the result is

$$\frac{dQ}{dA_i} = \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{dA_i}{dA_i}\right) + \frac{x_w}{k_w}\left(\frac{dA_i}{d\overline{A}_L}\right) + \frac{1}{h_o}\left(\frac{dA_i}{dA_o}\right)}$$

$$\frac{dA_i}{dA_o} = \frac{r_i}{r_o} \quad ; \qquad \frac{dA_i}{d\overline{A}_L} = \frac{r_i}{\overline{r}_L}$$

$$\frac{dQ}{dA_i} = \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{r_i}{r_i}\right) + \frac{r_i}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}\left(\frac{r_i}{r_o}\right)}$$

Hence, the overall heat transfer coefficient based on inside area is

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}\left(\frac{r_i}{r_o}\right)}$$

Fouling factors

In actual service, heat transfer surfaces do not remain clean. Scale, dirt and other solid deposits form on one or both the sides of the tubes, provide additional resistances to heat flow and reduce the overall coefficient. The effect of such deposits is taken into account as fouling factors in design calculation of heat exchangers. hdi, hdo are the fouling factors for the scale deposits on the inside and outside tube surfaces.

Overall heat transfer coefficient based on outside surface area

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{r_o}{r_i}\right) + \frac{1}{h_{di}}\left(\frac{r_o}{r_i}\right) + \frac{r_o}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_{do}} + \frac{1}{h_o}}$$

Overall heat transfer coefficient based on inside surface area

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{1}{h_{di}} + \frac{r_i}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_{do}}\left(\frac{r_i}{r_o}\right) + \frac{1}{h_o}\left(\frac{r_i}{r_o}\right)}$$

Concept of heat transfer by convection, natural and forced convection

When a current or macroscopic particle of fluid crosses a specific surface, it carries with it a definite quantity of enthalpy. Such a flow of enthalpy is called convection. Convection can refer to the flow of heat associated with the movement of fluid, such as when hot air from a furnace enters a room, or to the transfer of heat from a hot surface to a flowing fluid.

The two types of convection are Natural convection and forced convection.

Natural convection: If the convection currents are the result of buoyancy forces generated by the differences in density and the differences in density are in turn caused by temperature gradients in the fluid mass, the action is called natural convection.

Forced convection: If the convection currents are set in motion by the action of a mechanical device such as pump or agitator, the flow is independent of density gradients, it is called forced convection.

Newton's law of cooling

According to **Newton's law of cooling**, convective heat flux is proportional to the difference between the surface temperature and the temperature of the fluid.

$Q = h A (T_s – T_f)$

Where

Q = rate of heat transfer

h= heat transfer coefficient

A = heat transfer area

$T = (T_s - T_f)$ = temperature difference

T_s = surface temperature

T_f = bulk temperature of fluid

Application of dimensional analysis for convection

Many engineering problems can't be solved completely by theoretical or mathematical methods. Problems of this type are common in fluid flow, heat flow. One method of attacking a problem for which no mathematical equation can be derived is that of empirical experimentation. The empirical method of obtaining an equation, is laborious, and it is difficult to organize or correlate the results so obtained into a useful relationship for calculation. There exists a method intermediate between mathematical equation and empirical equation. It is based on the fact that if a theoretical equation does exist among the variables involved in the process, that equation must be dimensionally homogeneous. It is possible to group many factors into a smaller number of dimensionless groups of variables. The groups themselves appear in the final equation. This method is called dimensional analysis.

(i) Natural convection

Let us consider the case of natural convection from a vertical plane wall to an adjacent fluid.

Variables	Symbol	Dimension
Length	L	L
Fluid density	ρ	ML^{-3}
Fluid viscosity	μ	$ML^{-1}t^{-1}$
Fluid heat capacity	Cp	$L^{2}t^{-2}\theta^{-1}$
Fluid coefficient of thermal expansion	β	θ^{-1}
Acceleration due to gravity	g	Lt^{-2}
Temperature difference	ΔT	θ
Heat transfer coefficient	h	$Mt^{-3}\theta^{-1}$
Fluid thermal conductivity	k	$MLt^{-3}\theta^{-1}$

Number of groups = number of variables(N) – number of fundamental dimensions(m)

Number of ∏ groups = 9-4 = 5

$$\pi_1 = L^{a1}\ \mu^{b1}\ k^{c1}\ g^{d1}\ \ \rho$$

$$\pi_2 = L^{a2}\ \mu^{b2}\ k^{c2}\ g^{d2}\ \ Cp$$

$$\pi_3 = L^{a3}\ \mu^{b3}\ k^{c3}\ g^{d3}\ \ \beta$$

$$\pi_4 = L^{a4}\ \mu^{b4}\ k^{c4}\ g^{d4}\ \ \Delta T$$

$$\pi_5 = L^{a5}\mu^{b5}\ k^{c5}\ g^{d5}\ \ h$$

$$\pi_1 = (L)^{a1}\ (ML^{-1}t^{-1})^{b1}\ (MLt^{-3}\theta^{-1})^{c1}\ (Lt^{-2})^{d1}\ \ ML^{-3}$$

By equating the net dimensions of mass, length, time and temperature to zero, we get

$b_1 + c_1 + 1 = 0$

$a_1 - b_1 + c_1 + d_1 - 3 = 0$

$-b_1 - 3c_1 - 2d_1 = 0$

$-c_1 = 0$

By solving these equations, we get

$c_1 = 0$

$b_1 = -1$

$d_1 = ½$

$a_1 = 3/2$

$\pi_1 = L^{3/2} g^{1/2} \rho / \mu$

Squaring of both sides

$$\pi_1 = \frac{L^3 g \rho^2}{\mu^2}$$

Hence,

$$\pi_2 = (L)^{a2} (ML^{-1}t^{-1})^{b2} (MLt^{-3}\theta^{-1})^{c2} (Lt^{-2})^{d2} \; L^2t^{-2}\theta^{-1}$$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_2 = \frac{\mu C_p}{k} = N_{Pr}$$

$\pi_3 = (L)^{a3} (ML^{-1}t^{-1})^{b3} (MLt^{-3}\theta^{-1})^{c3} (Lt^{-2})^{d3} \theta^{-1}$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_3 = \frac{L\mu g\beta}{k}$$

$\pi_4 = (L)^{a4} (ML^{-1}t^{-1})^{b4} (MLt^{-3}\theta^{-1})^{c4}(Lt^{-2})^{d4} \theta$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_4 = \frac{k\Delta T}{L\mu g}$$

$\pi_5 = (L)^{a5} (ML^{-1}t^{-1})^{b5} (MLt^{-3}\theta^{-1})^{c5}(Lt^{-2})^{d5} Mt^{-3}\theta^{-1}$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_5 = \frac{hL}{K} = N_{Nu}$$

Combining π_1, π_3, π_4 ;

$$\frac{L^3 \rho^2 g\beta\Delta T}{\mu^2} = N_{Gr}$$

$\therefore$ **$N_{Nu} = f(N_{Gr}, N_{pr})$**

Where N_{Nu} = Nusselt number

N_{Gr} = Grashof number

N_{pr} = Prandtl number

(ii) Forced Convection

Let us consider the flow of fluid through a hot tube.

Variables	Symbol	Dimension
Tube Diameter	D	L
Fluid density	ρ	ML^{-3}
Fluid velocity	u	Lt^{-1}
Fluid viscosity	μ	$ML^{-1}t^{-1}$
Fluid heat capacity	Cp	$L^2t^{-2}\theta^{-1}$
Fluid thermal conductivity	k	$MLt^{-3}\theta^{-1}$
Heat transfer coefficient	h	$Mt^{-3}\theta^{-1}$

Number of groups = number of variables(N) – number of fundamental dimensions(m)

Number of ∏ groups = 7-4 = 3

$\pi_1 = D^{a1}\ \mu^{b1}\ u^{c1}\ k^{d1}\ \ \rho$

$\pi_2 = D^{a1}\ \mu^{b1}\ u^{c1}\ k^{d1}\ \ Cp$

$\pi_3 = D^{a1}\ \mu^{b1}\ u^{c1}\ k^{d1}\ \ h$

$\pi_1 = (L)^{a1}\ (ML^{-1}t^{-}1)^{b1}\ (Lt^{-1})^{c1}\ (MLt^{-3}\theta^{-1})^{d1}\ \ ML^{-3}$

By equating the net dimensions of mass,length,time and temperature to zero, we get

$b_1 + d_1 + 1 = 0$

$a_1 – b_1 + c_1 + d_1 -3 =0$

$-b_1 –c_1 -3d_1 = 0$

$-d_1 =0$

By solving these equations, we get

$d_1 = 0$

$b_1 = -1$

$c_1 = 1$

$a_1 = 1$

$$\pi_1 = \frac{D\rho u}{\mu} = N_{\mathrm{Re}}$$

$\pi_2 = (L)^{a2} (ML^{-1}t^{-1})^{b2} (Lt^{-1})^{c2} (MLt^{-3}\theta^{-1})^{d2}\ L^2t^{-2}\theta^{-1}$

By equating the net dimensions of mass,length,time and temperature to zero and solving those equations, we get

$$\pi_2 = \frac{\mu C_P}{k} = N_{\mathrm{Pr}}$$

$\pi_3 = (L)^{a3} (ML^{-1}t^{-1})^{b3} (Lt^{-1})^{c3} (MLt^{-3}\theta^{-1})^{d3}\ Mt^{-3}\theta^{-1}$

By equating the net dimensions of mass,length,time and temperature to zero and solving those equations, we get

$$\pi_3 = \frac{hD}{k} = N_{Nu}$$

$\therefore$ $N_{Nu} = f(N_{Re}, N_{pr})$

Where N_{Nu} = Nusselt number

N_{Re} = Reynolds number

N_{pr} = Prandtl number

HEAT TRANSFER TO FLUIDS WITHOUT PHASE CHANGE

Heat transfer coefficient calculation for forced convection:

Empirical equations for laminar flow:

An empirical equation for moderate Graetz numbers is

$$Nu = 2\ Gz^{1/3}$$

For viscous liquids with large temperature drops, a modification of the above equation is required to account for differences between heating and cooling. Therefore a correction factor is added with that equation to give the final equation for laminar flow heat transfer.

$$Nu = 2\ Gz^{1/3}\ \phi_v = 2\left(\frac{mCp}{kL}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Empirical equations for turbulent flow:

One empirical correlation for long tubes with sharp-edged entrances is the **Dittus-Boelter** equation.

$$Nu = \frac{h_i D}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{n}$$

n= 0.4 when the fluid is heated, n= 0.3 when the fluid is cooled

A better relationship for turbulent flow is known as the **Sieder- Tate** equation.

$$Nu = \frac{h_i D}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

The correction factor $\left(\frac{\mu}{\mu_w}\right)^{0.14}$ accounts for the heating and cooling for fluid. μ is the absolute of viscosity of fluid at bulk mean temperature and μ_w is the absolute of viscosity of fluid at surface temperature.

Heat transfer coefficient calculation for natural convection

Equations for heat transfer in natural convection between fluids and solids of definite geometric shape are of the form

$$Nu = b\ (Gr\ Pr)^n$$

b,n are constants.

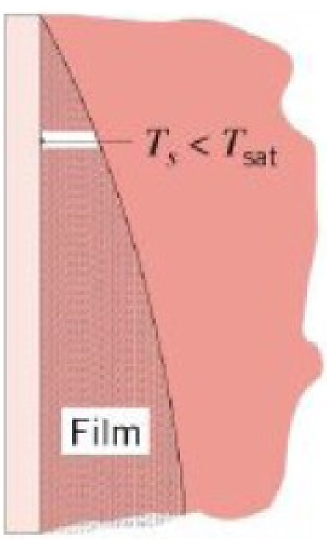
HEAT TRANSFER TO FLUIDS WITH PHASE CHANGE:

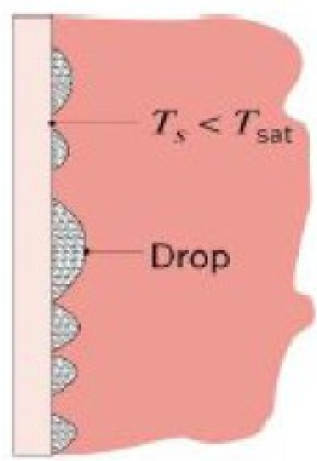
Heat transfer from condensing vapours, dropwise and film-type condensation,

A vapour may condense on a cold surface in one of two ways, which are drop-wise and film-wise condensation. In **film wise condensation**, the liquid condensate forms a film, or continuous layer of liquid that flows over the surface of the tube under the action of gravity.

In **drop-wise condensation**, the condensate begins to form at microscopic nucleation sites. The drops grow and coalesce with their neighbors to form visible fine drops. The fine drops in turn, coalesce into rivulets, which flow down the tube under the action of gravity.

In drop-wise condensation, large areas of the tube surface are covered with an extremely thin film of liquid of negligible thermal resistance. Because of this the heat transfer coefficient at these areas is very high the average coefficient for Drop-wise condensation may be 5 – 8 times that for film-type condensation. For normal design, film-type condensation is assumed.





(a) Filmwise condensation (b) Dropwise condensation

Heat transfer coefficient in drop-wise condensation is 10 times higher than in Film-wise condensation.

Much of the experimental work on the drop-wise condensation of steam is summarized as follows:

(1) Film-type condensation of water occurs on tubes of the common metals if both the steam and the tube are clean, in the presence or absence of air, on rough or on the polished surfaces.

(2) Drop-wise condensation is obtainable only when the cooling surface is not wetted by the liquid. In the condensation of steam it is often induced by contamination of the vapour with droplets of oil. It is more easily maintained on a smooth surface than on a rough surface.

(3) The quantity of contaminant or promoter required to cause drop-wise condensation is minute, and apparently only a monomolecular film is necessary.

(4) Effective drop promoters are strongly adsorbed by the surface, and substances that merely prevent wetting are ineffective. Some properties are especially effective on certain metals.

(5) The average coefficient obtainable in pure drop-wise condensation may be as high as 115 kw/m^2K.

The effect of non condensable gases on condensation.

The presence of even small amounts of non-condensing gas in a condensing vapour seriously reduces the rate of condensation. When a vapour containing non-condensable gas condenses, the non-condensable gas is left at the surface.

Any further condensation will occur only after the incoming vapour has diffused through this non-condensable gas which does not move toward the condensate. As condensation proceeds, the relative amount of this inert gas in the vapour phase increases significantly. The non-condensable gas acts as a thermal resistance to the condensation process.

Heat transfer coefficients calculation for film-type condensation.
The assumptions used in Nusselt's equation for condensation to determine film thickness.

(i) The vapour and liquid at the outside boundary of the liquid layer are in thermodynamic equilibrium.

(ii) The only resistance to heat flow is offered by the layer of condensate flowing in laminar flow.

(iii) The velocity of the liquid at the wall is zero.

(iv) The temperatures of the wall and the vapour are constant.

(v) Condensate is assumed to leave the tube at the condensing temperature.

(vi) The fluid properties are taken at the mean film temperature.

Film wise condensation on horizontal pipe

$$h = 0.729\left[\frac{k_f^3 \rho_f^2 g \lambda}{\Delta T D \mu_f}\right]^{1/4}$$

Film wise condensation on vertical pipe

$$h = 0.943\left[\frac{k_f^3 \rho_f^2 g \lambda}{\Delta T L \mu_f}\right]^{1/4}$$

Where

k_f = thermal conductivity of condensate

ρ_f = density of condensate

g = acceleration due to gravity

λ = latent heat of condensation

μ_f = absolute viscosity of condensate

D = Pipe diameter

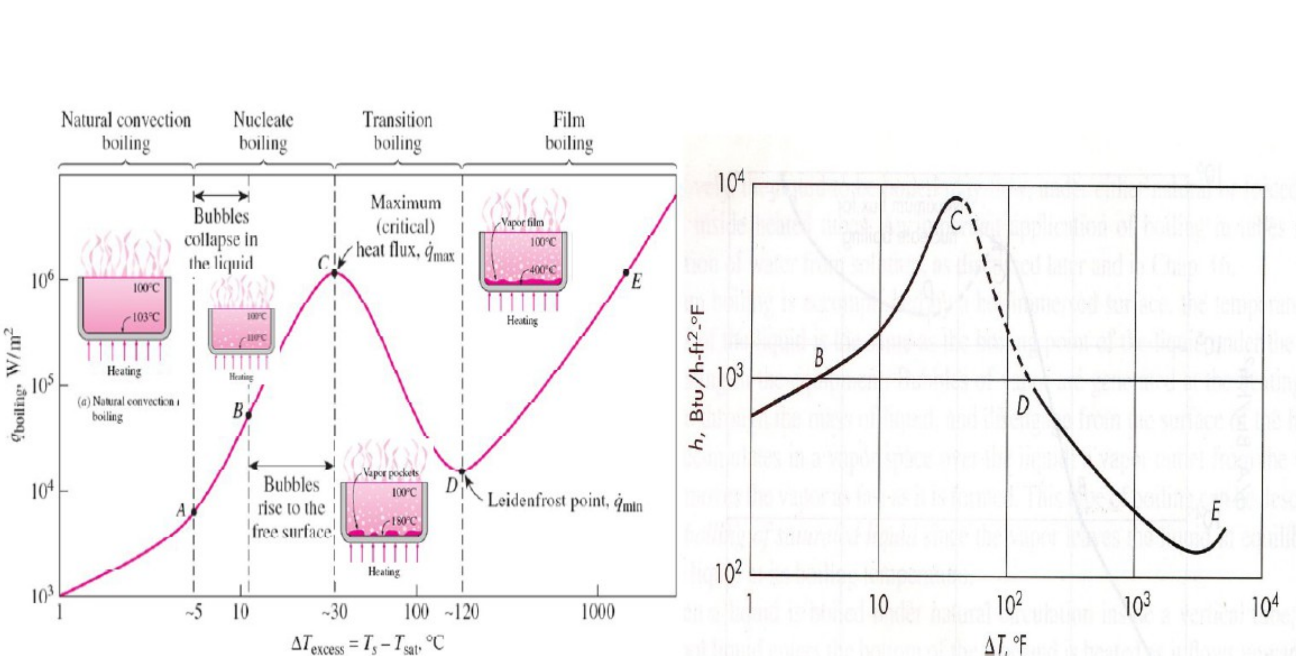
L = pipe length

ΔT = temperature difference

Heat Transfer To Boiling Liquids

Heat transfer to a boiling liquid is a necessary step in evaporation, distillation, and steam generation, and it may be used to control the temperature of a chemical reactor. **Boiling** occurs at the solid–liquid interface when a liquid is brought into contact with a surface maintained at a temperature sufficiently above the saturation temperature of the liquid.

Saturated Boiling Boiling of liquid When the temperature of the liquid is equal to the saturation temperature.



(a) q/A Vs ΔT diagram (b) h Vs diagram

When boiling is accomplished by a hot immersed surface, the temperature of the mass of the liquid is same as the boiling point of the liquid under the pressure existing in the equipment. Bubbles of vapour are generated at the heating surface, rise through the mass of liquid, and disengage from the surface of the liquid. Vapour accumulates in a vapour space over the liquid, a vapour outlet from the vapour space removes the vapour as fast as it is formed. This type of boiling can be described as **pool boiling of saturated liquid** since the vapour leaves the liquid in equilibrium with the liquid at its boiling temperature.

Consider a horizontal, electrically heated wire immersed in a vessel containing a boiling liquid. Assume that q, heat flux, and ΔT, the difference between the temperature of the wire surface, Tw and that of the boiling liquid T are measured. Start with a very low temperature drop. Now raise Tw and increase the temperature drop by steps, measuring q and ΔT at each step, until very large values of ΔT are reached. A plot of q vs ΔT on logarithmic coordinates will give a curve of the type shown in figure. This curve can be divided into four segments.

Each of the four segments of the graph corresponds to a definite mechanism of boiling. In the first section, at low temperature drops, the mechanism is that of heat transfer to a liquid in natural convection. Bubbles form on the surface of the heater, are released from it, rise to the surface of the liquid, and are disengaged into the vapour space, but they are too few to disturb the normal currents of free convection. This segment is called **natural convection zone**. At larger temperature drops the rate of bubble production is large enough for the stream of bubbles moving up through the liquid to increase the velocity of circulation currents in the mass of liquid, and the coefficient f heat transfer becomes greater than in undisturbed natural convection. As ΔT is increased, the rate of bubble formation increases and the coefficient increases rapidly.

The action occurring at temperature drops below the critical temperature drop is called **nucleate boiling**, in reference to the formation of tiny bubbles on the heating surface. During nucleate boiling, the bubbles occupy a small portion of the heating surface at a time, and most of the surface is in direct contact with the liquid. The bubbles are generated at localized active sites, usually small pits or scratches on the heating surface. As the temperature drop is raised, more sites become active, improving the agitation of the liquid and increasing the heat flux and the heat transfer coefficient.

Eventually, however, so many bubbles are present that they tend to coalesce and cover portions of the heating surface with a layer of insulating vapour. This layer has highly unstable surface. This type of action is called **transition boiling**. The heat flux and the heat transfer coefficient both fall as the temperature drop is raised. Near the Leidenfrost point another distinct change in mechanism occurs. The hot surface becomes covered with the film vapour, through which heat is transferred by conduction and by radiation. The random explosion characteristic of transition boiling disappear and are replaced by the slow and orderly formation of bubbles at the interface between the liquid and the film of hot vapour. As temperature drop increases, the heat flux rises, slowly at first and then more rapidly as radiation heat transfer becomes important. The boiling action in this region is known as **film boiling.**

Sub-cooled Boiling

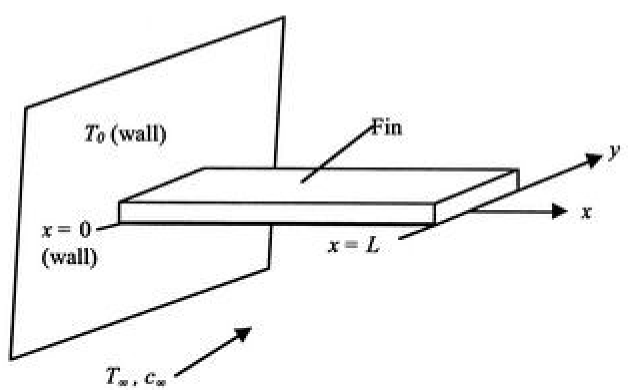
In some types of forced-circulation equipment, the temperature of the mass of the liquid is below that of its boiling point, but the temperature of the heating surface is considerably above the boiling point of the liquid. Bubbles form on the heating surface, but on release from the surface are absorbed by the mass of the liquid. This type of heat transfer is called subcooled boiling, even though the fluid leaving the heat exchanger is entirely liquid.

Extended surfaces : **Heat transfer through fins**

Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law: $q = hA(Ts-T\alpha)$, where Ts is the surface temperature and $T\alpha$ is the fluid temperature. Therefore, to increase the convective heat transfer, one can Increase the temperature difference $(Ts-T\alpha)$ between the surface and the fluid. Increase the convection coefficient h. This can be accomplished by increasing the fluid flow over the surface since h is a function of the flow velocity and the higher the velocity, the higher the *h*. Example: a cooling fan. Increase the contact surface area A. Example: a heat sink with fins. Many times, when the first option is not in our control and the second option (i.e. increasing *h*) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces. Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid. Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers) and elsewhere.

Heat Transfer From a Fin

Fins are used in a large number of applications to increase the heat transfer from surfaces. Typically, the fin material has a high thermal conductivity. The fin is exposed to a flowing fluid, which cools or heats it, with the high thermal conductivity allowing increased heat being conducted from the wall through the fin. The design of cooling fins is encountered in many situations and we thus examine heat transfer in a fin as a way of defining some criteria for design.



Geometry of heat transfer through longitudinal fin

A model configuration is shown in Figure. The fin is of length L. The other parameters of the problem are indicated. The fluid has velocity c_∞ and temperature T_∞. We assume (using the

Reynolds analogy or other approach) that the heat transfer coefficient for the fin is known and has the value h. The end of the fin can have a different heat transfer coefficient, which we can call h_L. The approach taken will be quasi-one-dimensional, in that the temperature in the fin will be assumed to be a function of x only. This may seem a drastic simplification, and it needs some explanation. With a fin cross-section equal to A and a perimeter P, the characteristic dimension in the transverse direction is A/P (For a circular fin, for example, $A/P = r/2$). The regime of interest will be taken to be that for which the Biot number is much less than unity, $\mathrm{Bi} = h(A/P)/k \ll 1$, which is a realistic approximation in practice.

The physical content of this approximation can be seen from the following. Heat transfer per unit area out of the fin to the fluid is roughly of magnitude $\sim h(T_w - T_\infty)$ per unit area. The heat transfer per unit area **within** the fin in the transverse direction is (again in the same approximate terms)

$$k\frac{(T_1 - T_w)}{A/P},$$

where T_1 is an internal temperature. These two quantities must be of the same magnitude.

If $h(A/P)/k \ll 1$, then $(T_1 - T_w)/(T_w - T_\infty) \ll 1$. In other words, if $\mathrm{Bi} \ll 1$, there is a much larger capability for heat transfer per unit area across the fin than there is between the fin and the fluid, and thus little variation in temperature inside the fin **in the transverse direction**. To emphasize the point, consider the limiting case of zero heat transfer to the fluid, i.e., an insulated fin. Under these conditions, the temperature within the fin would be uniform and equal to the wall temperature.

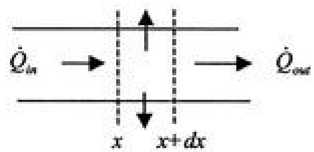


Figure 18.4: Element of fin showing heat transfer

If there **is** little variation in temperature **across** the fin, an appropriate model is to say that the temperature within the fin is a function of x only, $T = T(x)$, and use a quasi-one-dimensional approach. To do this, consider an element, dx, of the fin as shown. There is heat flow of magnitude $\dot{Q}_{\rm in}$ at the left-hand side and heat flow out of magnitude $\dot{Q}_{\rm out} = \dot{Q}_{\rm in} + \frac{d\dot{Q}}{dx}dx$ at the right hand side. There is also heat transfer around the perimeter on the top, bottom, and sides of the fin. From a quasi-one-dimensional point of view, this is a situation similar to that with internal heat sources, but here, for a cooling fin, in each elemental slice of thickness dx there is essentially a heat sink of magnitude $Pdxh(T - T_\infty)$, where Pdx is the area for heat transfer to the fluid.

The heat balance for the element can be written in terms of the heat flux using $\dot{Q} = \dot{q}A$, for a fin of constant area:

$$\dot{q}A = Ph(T - T_\infty)dx + \left(\dot{q}A + \frac{d\dot{q}}{dx}dxA\right).$$

$$\frac{d^2T}{dx^2} - \frac{Ph}{Ak}(T - T_\infty) = 0.$$

$$A\frac{d\dot{q}}{dx} + Ph(T - T_\infty) = 0.$$

The quantity of interest is the temperature difference $(T - T_\infty)$, and we can change variables

$$\frac{d}{dx}(T - T_\infty) = \frac{dT}{dx}.$$

$$\frac{d^2}{dx^2}(T - T_\infty) - \frac{Ph}{Ak}(T - T_\infty) = 0.$$

the temperature variation along the fin. It is a second order equation and needs two boundary conditions. The first of these is that the temperature at the end of the fin that joins the wall is equal to the wall temperature.

$$(T - T_\infty)_{x=0} = T_0 - T_\infty.$$

The second boundary condition is at the other end of the fin. We will assume that the heat transfer from this end is negligible. The boundary condition at $x = L$ is

$$\left.\frac{d}{dx}(T - T_\infty)\right|_{x=L} = 0.$$

The relation between derivatives that is needed to cast the equation in terms of ξ is

$$\frac{d}{dx} = \frac{d\xi}{dx}\frac{d}{d\xi} = \frac{1}{L}\frac{d}{d\xi}.$$

$$\frac{d^2\Delta\tilde{T}}{d\xi^2} - \left(\frac{hP}{kA}L^2\right)\Delta\tilde{T} = 0.$$

There is one non-dimensional parameter and it is m

$$m^2L^2 = \frac{hPL^2}{kA}.$$

The equation for the temperature distribution we have obtained is

$$\frac{d^2\Delta\tilde{T}}{d\xi^2} - m^2L^2\Delta\tilde{T} = 0.$$

This second order equation has the solution

$$\Delta\tilde{T} = ae^{mL\xi} + be^{-mL\xi}.$$

the boundary condition at $\xi = 0$ is given by

$$\Delta\tilde{T}(0) = a + b = 1.$$

The boundary condition at $\xi = 1$ is that the temperature gradient is zero or

$$\frac{d\Delta\tilde{T}}{d\xi}(L) = mLae^{mL} - mLbe^{-mL} = 0.$$

Solving the two equations given by the boundary conditions for a and b gives an expression for $\Delta\tilde{T}$ in terms of the hyperbolic cosine or $\cosh$:

$$\cosh x = \frac{(e^x + e^{-x})}{2},$$

$$\Delta\tilde{T} = \frac{\cosh mL(1-\xi)}{\cosh mL}.$$

This is the solution to above Equation for a fin with no heat transfer at the tip. In terms of the actual heat transfer parameters it is written as

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh\left(\left(1 - \frac{x}{L}\right)\sqrt{\frac{hP}{kA}}L\right)}{\cosh\left(\sqrt{\frac{hP}{kA}}L\right)}.$$

The amount of heat removed from the wall due to the fin, which is the quantity of interest, can be found by differentiating the temperature and evaluating the derivative at the wall, x = 0

$$\dot{Q} = -kA\frac{d}{dx}(T - T_\infty)\bigg|_{x=0}$$

$$\frac{\dot{Q}L}{kA(T_0 - T_\infty)} = -\frac{d\Delta\tilde{T}}{d\xi}\bigg|_{\xi=0} = \frac{mL\sinh(mL)}{\cosh(mL)} = mL\tanh(mL),$$

$$\frac{\dot{Q}}{\sqrt{kAhP}(T_0 - T_\infty)} = \tanh(mL).$$

this is the expression for heat flow through a longitudinal fin.

It can also be derived for fin with an insulation and fin provided with free convection off the end.

PROBLEMS

Water heated to 80°C flows through a 2.54cm I.D and 2.88cm O.D steel tube (k = 50 W/m K). The tube is exposed to an environment which is known to provide an average convection coefficient of ho = 30800 W/m2 K on the outside of the tube. The water velocity is 50 cm/s. Calculate the overall heat transfer coefficient based on the outer area of the pipe. Properties of water at bulk mean temperature of 80 °C: ρ =974 kg/m^3, γ = 0.364 x 10- 6 m2/sec, k= 668.7 x 10- 3 W/m K, Pr = 2.20.

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{r_o}{r_i}\right) + \frac{1}{h_{di}}\left(\frac{r_o}{r_i}\right) + \frac{r_o}{k_w}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_{do}} + \frac{1}{h_o}}$$

r_i - 0.0127 m

r_o - 0.0144 m

h_o - 30800 W/m^2 K

k_w - 50 W/m K

Dittus-Boelter equation

$$N_{Nu} = 0.023(N_{Re})^{0.8}(N_{Pr})^{0.3} = 125.48$$

$$N_{Re} = \frac{D_i \rho u}{\mu} = 34890.10$$

$$hi = \frac{N_{Nu} k}{D_i} = 3303.48 \text{ W/m}^2\text{ K}$$

U_o = 2428.23 W/m^2 K

2. Air at 2 atm and 200·C is heated as it flows at a velocity of 12m/s through a tube with a diameter of 3cm.The tube wall temperature is 20·C above the air temperature all along the length of the tube. Calculate the rate of heat transfer per unit length of the tube. The properties of air at bulk mean temperature are; Pr = 0.681; μ = 2.57 x 10-5 kg/ms; k = 0.0386 W/m K and Cp = 1.025 kJ/kg K

Solution:

D	-	3×10^{-2} m
u	-	12m/s
Tb	-	200°C
Pr	-	0.681
µ	-	2.57×10^{-5} kg/ms
k	-	0.0386 W/mK
Cp	-	1.025 kJ/kg K

$$N_{\mathrm{Re}} = \frac{D_i \rho u}{\mu} = \mathbf{21137}$$

Dittus-Boelter equation

$$N_{Nu} = 0.023 (N_{\mathrm{Re}})^{0.8} (N_{\mathrm{Pr}})^{0.3} = \mathbf{59.12}$$

$$h = \frac{N_{Nu} k}{D_i} = \mathbf{76.07}\ \mathrm{W/m^2\ K}$$

Newton's law of cooling

$Q = h\ A\ \Delta T = h\ \pi DL\ \Delta T$

Q/L = **143.32 W/m**

3. A horizontal cylinder, 3.0 cm in diameter and 0.8 m length is suspended in water at 20°C. Calculate the rate of heat transfer if the cylinder surface is at 55°C. Given Nu = 0.53 (Gr x Pr) 0.25. The

properties of water at average temperature are as follows: Density = 990 kg/m3, Viscosity = 2.47 kg/hr.m, k = 0.534 kcal/hr.m ·C, Cp = 1 kcal/kg·C,

Solution:

D	-	3×10^{-2}m
L	-	0.8m
T_b	-	20°C
T_s	-	55°C
ρ	-	990 kg/m^3
μ	-	2.47 kg/hr.m
k	-	0.534 kcal/hr.m.°C
Cp	-	1 kcal/kg°C

$$N_{Gr} = \frac{D^3 \rho^2 g \beta \Delta T}{\mu^2} = \mathbf{6.216 \times 10^7}$$

$$N_{Pr} = \frac{\mu C_P}{k} = \mathbf{4.6255}$$

$$Nu = 0.53\,(Gr \times Pr)^{0.25} = \mathbf{69.01}$$

$$h = \frac{N_{Nu} k}{D} = \mathbf{1228.4 kcal/h\ m^2K}$$

Newton's law of cooling

$Q = hAdT = h\prod DL\ dT$ = **3240.03kcal/h**

4. Liquid sodium is to be heated from 120 ºC to 149º C at a rate of 2.3 kg/sec in a 2.5 cm diameter electrically heated tube (constant heat flux). Calculate the heat transfer coefficient. The properties of sodium at 134.5ºC are: density =916 kg/m3, Cp= 1.3565 kJ/kg K, √ = 0.594 x 10-6 m^2/sec, k=84.90 W/m K, Pr= 0.0087. Given $N_{Nu} = 4.82 + 0.0185\, N_{Pe}^{\ 0.827}$

Solution :

1. Calculate h

$h = N_{Nu} * k$
using Peclet number, $N_{Nu} = 14.22$
$N_{Pe} = N_{Pr}\, \rho\, N_{Re}$
$N_{Re} = Di \quad u / \mu = 215067.34$ $N_{Pr} = 0.0087$
$N_{Pe} = 1871.08$ (Reynolds * Prandtl)
Hence h = 48291.12 w/m^2 k.

<u>Reynolds and Colburn Analogy:</u>

Reynolds has taken the following assumptions to find the analogy between heat and momentum transport.

1. Gradients of the dimensionless parameters at the wall are equal.
2. The diffusivity terms are equal. That is

$$(\nu + \epsilon_M) = (\alpha + \epsilon_H)$$

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$