SATHYABAMA UNIVERSITY **FACULTY OF MECHANICAL ENGINEERING**

SME1204	STRENGTH OF MATERIALS (For Mechanical)	L	T	P	Credits	Total Marks
		3	1	0	4	100

COURSE OBJECTIVES

- To gain knowledge of different types of stresses, strain and deformation induced in the mechanical components due to external loads.
- To study the distribution of various stresses in the mechanical elements such as beams, shafts etc.
- To study the effect of component dimensions and shapes on the stresses and deformations.

UNIT 1 STRESS STRAIN DEFORMATION OF SOLIDS 12 Hrs.

Rigid and Deformable bodies - Strength, Stiffness and Stability - Stresses; Tensile, Compressive and Shear - Deformation of simple and compound bars under axial load - Thermal stress - Elastic constants , Strain energy and unit strain energy - Strain energy in uniaxial loads Principal stress and strain, Mohr's stress circle., Theory of failure.

UNIT 2 BEAMS - LOADS AND STRESSES 12 Hrs.

Types of beams: Supports and Loads - Shear force and Bending Moment in beams - Cantilever, Simply supported and Overhanging beams - Stresses in beams - Theory of simple bending - Stress variation along the length and in the beam section - Effect of shape of beam section on stress induced - Shear stresses in beams - Shear flow

UNIT 3 SPRINGS 12 Hrs.

Helical and Leaf Springs- deflection of springs by energy method- helical springs under axial load and under axial twist (respectively for circular and square cross sections) axial load and twisting moment acting simultaneously both for open and closed coiled springs- laminated springs. Application to close-coiled helical springs - Maximum shear stress in spring section including Wahl Factor - Deflection of helical coil springs under axial loads - Design of helical coil springs - stresses in helical coil springs under torsion loads.

UNIT 4 TORSION 12 Hrs.

Analysis of torsion of circular bars - Shear stress distribution - Bars of Solid and hollow circular section - Stepped shaft - Twist and torsion stiffness - Compound shafts - Fixed and simply supported shafts - Columns and Struts: Combined bending and direct stress, middle third and middle quarter rules. Struts with different end conditions - Euler's theory and experimental results - Ranking Gardon Formulae

UNIT 5 CYLINDERS & CURVED BEAMS 12 Hrs.

Thin cylinders & spheres - Hoop and axial stresses and strain.- Volumetric strain.- Thick cylinders- Radial, axial and circumferential stresses in thick cylinders subjected to internal or external pressures - Compound cylinders. Stresses due to interference fits Curved Beams - Bending of beams with large initial curvature, position of neutral axis for rectangular - trapezoidal and circular cross section- stress in crane hooks, stress in circular rings subjected to tension or compression

Max.60 Hours.

TEXT / REFERENCE BOOKS

1. Nash W.A, "Theory and problems in Strength of Materials", Schaum Outline Series,McGraw-Hill Book Co, New York, 1995
2. Kazimi S.M.A, "Solid Mechanics", Tata McGraw-Hill Publishing Co., New Delhi,1981.
3. Ryder G.H, "Strength of Materials, Macmillan India Ltd"., Third Edition, 2002
4. Ray Hulse, Keith Sherwin & Jack Cain, "Solid Mechanics", Palgrave ANE Books,2004.
5. Bansal.R.K., "Strength of Materials", 4th Edition, Laxmi Publications, 2007.

END SEMESTER EXAM QUESTION PAPER PATTERN

Max. Marks : 80 **Exam Duration : 3 Hrs.**
PART A : 2 Questions from each unit, each carrying 2 marks **20 Marks**
PART B : 2 Questions from each unit with internal choice, each carrying 12 marks **60 Marks**

COURSE MATERIAL

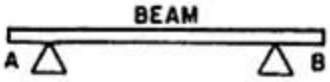
Shear force and Bending Moment diagrams

Types of beams:

There are 5 most important types of beams. They are

- Simple supported beam
- Cantilever beam
- Overhanging beam
- Fixed beam
- Continuous beam

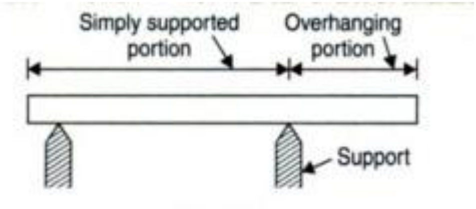
Simple supported beam: A beam supported or resting freely on the supports at its both ends is known as simply supported beam.



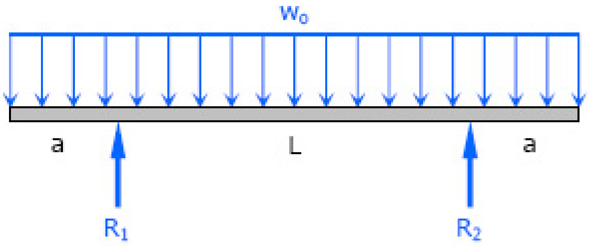
Cantilever beam**:** A beam which is fixed at one end and free at the other end is known as cantilever beam.

The pioneering Junkers J 1 all-metal monoplane of 1915, the first aircraft to fly with cantilever wings

Over hanging beam: If the end portion of a beam is extended beyond the support such beam is known as Overhanging beam. If overhanging is one side, it is called single over hung beam and if overhanging is on the two sides, it is called double over hung beam.

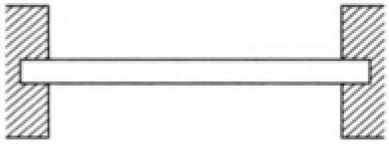


Single over hung beam

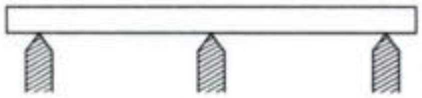


Double over hung beam

Fixed beam: A beam whose both ends are fixed or built in walls is known as fixed beam.



Continuous beam: A beam which is provided more than two supports is known as continuous beam.

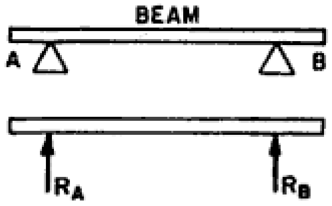


Types of supports:

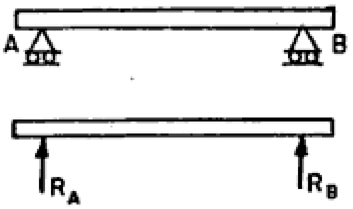
There are 5 most important supports. They are

- Simple supports or knife edged supports
- Roller support
- Pin-joint or hinged support
- Smooth surface support
- Fixed or built-in support

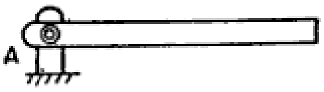
Simple supports or knife edged support: in this case support will be normal to the surface of the beam. If AB is a beam with knife edges A and B, then R_A and R_B will be the reaction.

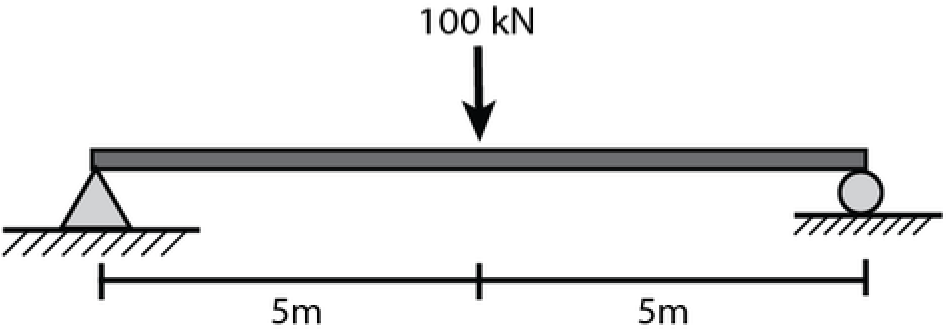


- Roller Support – The roller is used as an external support since it allows rotation and horizontal translation. Therefore it will have a vertical support reaction. Here beam AB is supported on the rollers. The reaction will be normal to the surface on which rollers are placed.

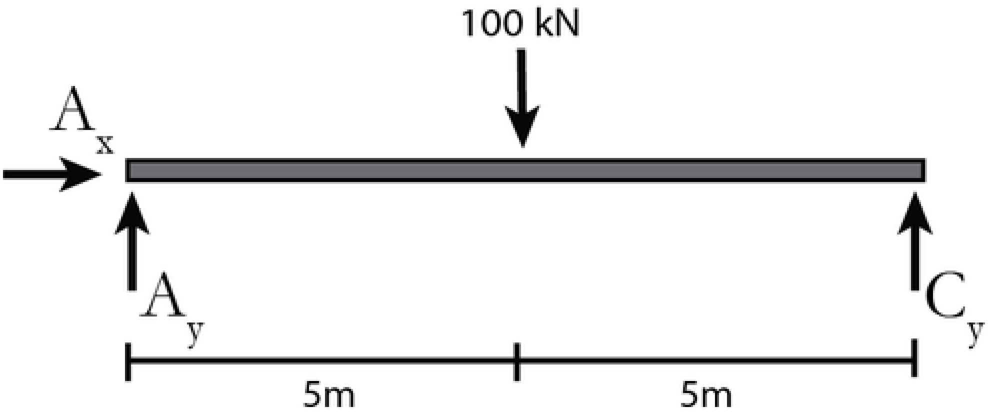


Pin joint (or hinged) support: here the beam AB is hinged at point A. the reaction at the hinged end may be either vertical or inclined depending upon the type of loading. If load is vertical, then the reaction will also be vertical. But if the load is inclined, then the reaction at the hinged end will also be inclined. A hinge resists horizontal and vertical translation but allows rotation. Therefore a hinge consists of horizontal and vertical support reaction





Hinged support **roller support**



Hinged support has both vertical & horizontal direction. For the present example since 100 N acts vertically downwards only vertical reaction exists and the horizontal reaction is zero.

If an inclined load acts on the beam then vertical reaction &horizontal reaction exist.

The Roller support has only vertical reaction.

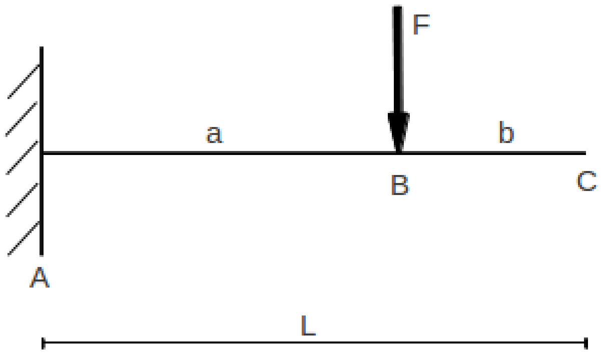
Fixed or built-in support: in this type of support the beam should be fixed. The reaction will be inclined. Also the fixed support will provide a couple.

Types of loading

There are 3 most important type of loading:

- Concentrated or point load
- Uniformly distributed load
- Uniformly varying load

Concentrated or point load: A concentrated load is one which is considered to act at a point. In the following example in a cantilever beam a load F acts at a point.



Uniformly distributed load: A uniformly distributed load is one which is spread over a beam in such a manner that rate of loading is uniform along the length. Usually it refers self weight of the beam.

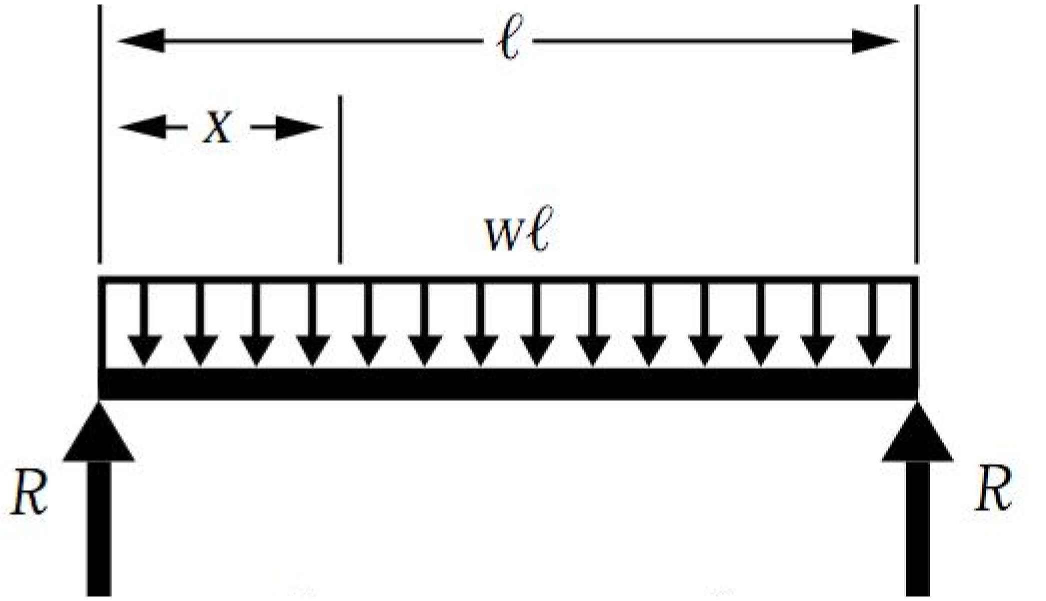


Fig. Uniformly distributed load

Uniformly varying load: A uniformly varying load is one which is spread over a beam in such a manner that rate of loading varies from point to point along the beam. The area of the triangle represents the load intensity and it acts at CG point of the triangle. i.e., bh/2= l*w/2 is the load intensity and it acts at CG point as shown.

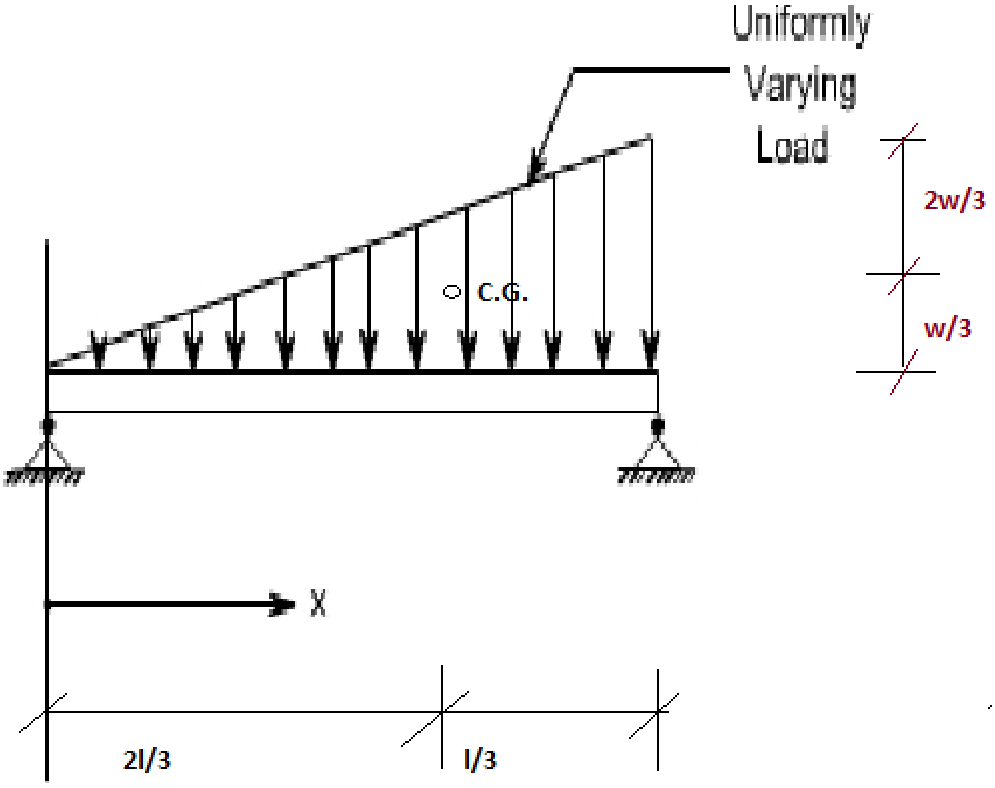


Fig.Uniformly varying load

Trapezium Loading

It is combination of UDL and UVL. This can be understood from the following example. The Trapezium loading = UDL+ UVL.

Note: Principle of Transmissibility of Forces:

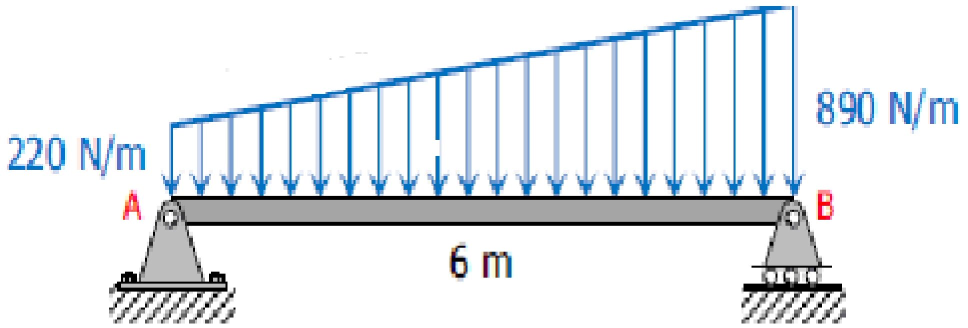
A force can be transferred in the same line of action without altering its value.

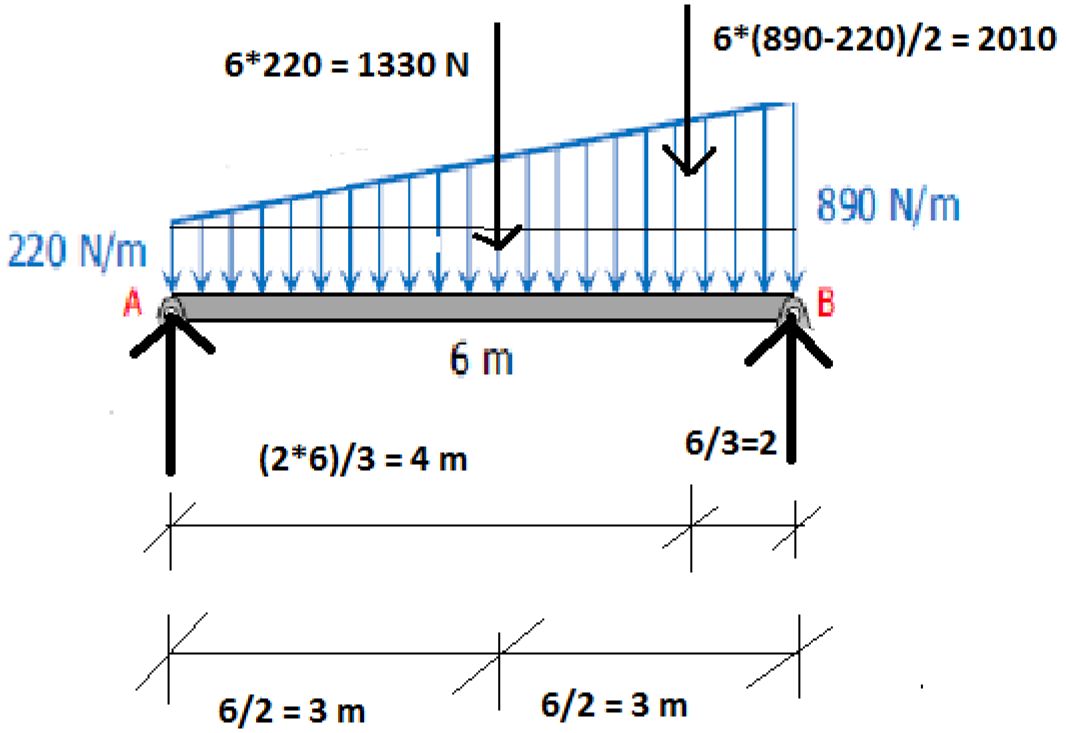
If the force is to be transferred in the different line of action, it should be marked with the force value together with a couple.

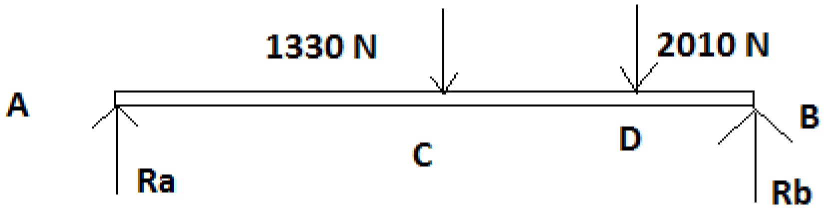
Using the above principle the following last part is drawn such that the two loads act on the beam, even though they are acting in C.G. points. The above approach is a must to find the reaction values.

With reaction only we can draw SFD & BMD.

Hence Reaction calculation is must for simply supported beam and over hanging beam for drawing SFD & BMD. Reaction calculation is not required for drawing SFD & BMD for Cantilever beam.







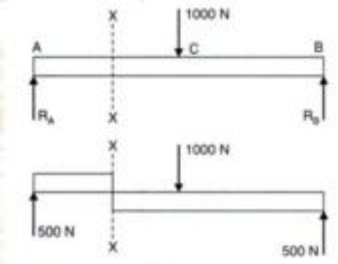
CONCEPT AND SIGNIFICANCE OF SHEAR FORCE AND BENDING MOMENT

SIGN CONVENTIONS FOR SHEAR FORCE AND BENDING MOMENT

(i) Shear force: Fig. 1 shows a simply supported beam AB. carrying a load of 1000 N at its middle point. The reactions at the supports will be equal to 500 N. Hence $R_A = R_B = 500$ N.

Now imagine the beam to be divided into two portions by the section X-X. The resultant of the load and reaction to the left of X-X is 500 N vertically upwards. And the resultant of the load and reaction to the right of X-X is (1000↓ -500 ↑= 500↓N) 500 N downwards. The resultant force acting on any one of the parts normal to the axis of the beam is called the shear force at the section X-X is 500N.

The shear force at a section will be considered positive when the resultant of the forces to the left to the section is upwards, or to the right of the section is downwards. Similarly the shear force at a the section will be considered negative if the resultant of the forces to the left of the section is downward, or to the right of the section is upwards. Here the resultant force to the left of the section is upwards and hence the shear force will be positive.



` Fig 1

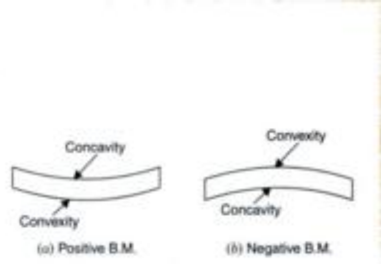


Fig 2

(ii) Bending moment. The bending moment at a section is considered positive if the bending moment at that section is such that it tends to bend the beam to a curvature having concavity at the top as shown in Fig. 2. Similarly the bending moment at a section is considered negative if the bending moment at that section is such that it tends to bend the beam to a curvature haling convexity at the top. The positive B.M. is often called sagging moment and negative B.M. as hogging Moment.

IMPORTANT POINTS FOR DRAWING SHEAR FORCE AND BENDING MOMENT DIAGRAMS

The shear force diagram is one which shows the variation of the shear force along the length of the beam. And a bending moment diagram is one which show the variation of the bending moment along the length of beam. In these diagrams, the shear force or bending moment are represented by ordinates whereas the length of the beam represents abscissa.

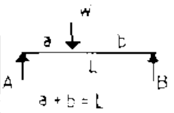
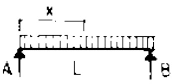
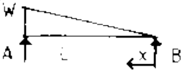
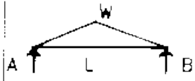
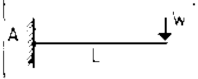
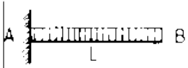
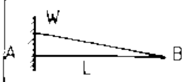
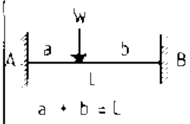
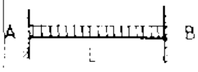
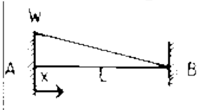
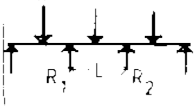
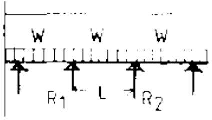
The following are the important points for drawing shear force and bending moment diagrams

1. Consider the left or the right portion of the section.
2. Add the forces (including reaction) normal to the beam on one of the portion. If right portion of the section is chosen, a force on the right portion acting downwards is positive while force acting upwards is negative.
If the left portion of the section is chosen, a force on the left portion acting upwards is positive while force acting downwards is negative.
3. The positive values of shear force and bending moments are plotted above the base line, and negative values below the base line.

4. The shear force diagram will increase or decrease suddenly i.e., by a vertical straight line at a section where there is a vertical point load.
5. The shear force between any two vertical loads will be constant and hence the shear force diagram between two vertical loads will be horizontal.
6. The bending moment at the two supports of a simply supported beam and at the free end of a cantilever will be zero.

The following table gives about the standard shear force and Bending moment and deflection values.

However SF & BM values will be sufficient for this topic.

Loading Diagram	Shear Force at x: Qx	Bending Moment at x: Mx	Deflection at x: δx
	$Q_A = \frac{Wb}{L}$ $Q_B = -\frac{Wa}{L}$	$M_c = \frac{Wab}{L}$ When a = b $M_c = \frac{WL}{4}$	$\delta_c = \frac{Wa^2b^2}{3EIL}$
Total W = wL 	$Q_A = \frac{W}{2}$ $Q_B = -\frac{W}{2}$	$M_{max} = \frac{WL}{8}$ at $x = \frac{L}{2}$	$\delta_{max} = \frac{5WL^3}{384EI}$ at $x = \frac{L}{2}$
Total $W = \frac{wL}{2}$ 	$Q_A = \frac{2W}{3} = \frac{wL}{3}$ $Q_B = -\frac{W}{3} = -\frac{wL}{6}$	$M_{max} = 0.064\ mL^2$ at x = 0.577L	$\delta_{max} = 0.00652\ \frac{wL^4}{EI}$ at x = 0.519
Total $W = \frac{wL}{2}$ 	$Q_A = \frac{W}{2} = \frac{wL}{4}$ $Q_B = -\frac{W}{2} = -\frac{wL}{4}$	$M_{max} = \frac{wL^2}{12}$ at $x = \frac{L}{2}$	$\delta_{max} = \frac{wL^4}{120EI}$ at $x = \frac{L}{2}$
	$Q_A = Q_B = W$	$M_A = -WL$	$\delta_B = \frac{WL^3}{3EI}$
Total W = wL 	$Q_A = W$ $Q_A = 0$	$M_A = -\frac{WL}{2} = -\frac{wL^2}{2}$	$\delta_B = \frac{WL^3}{3EI} = \frac{wL^4}{3EI}$
Total $W = \frac{wL}{2}$ 	$Q_A \ W$ $Q_A = 0$	$M_A = -\frac{WL}{3} = -\frac{wL^2}{6}$	$\delta_B = \frac{wL^4}{30EI}$
	$Q_A = \frac{Wb}{L}$ $Q_B = -\frac{Wa}{L}$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wa^2b}{L^2}$	$\delta_c = \frac{Wa^3b^3}{3EIL^3}$
Total W = wL 	$Q_A = \frac{W}{2}$ $Q_B = -\frac{W}{2}$	$M_A = M_B = -\frac{WL}{12}$	$\delta_c = \frac{WL^3}{384EI}$
Total $W - \frac{wL}{2}$ 	$Q_A = \frac{2W}{3}$ $Q_B = -\frac{W}{3}$	$M_A = -\frac{WL}{10} = -\frac{wL^2}{20}$ $M_B = -\frac{WL}{15} = -\frac{wL^2}{30}$	$\delta_{max} = \frac{wL^4}{764EI}$ at x = 0.475L
	$Q_A = \frac{W}{2}$	$M_{max} = \frac{WL}{6}$ *	$\delta_{max} = \frac{WL^3}{192EI}$
	$Q_A = \frac{W}{2}$	$M_{max} = \frac{WL}{12}$ *	$\delta_{max} = \frac{WL^3}{384EI}$

SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A CANTILEVER BEAM WITH A POINT LOAD

A cantilever beam of length 2m carries the point loads as shown in fig. draw the shear force and B.M diagrams for the cantilever beam.

Shear force diagram:

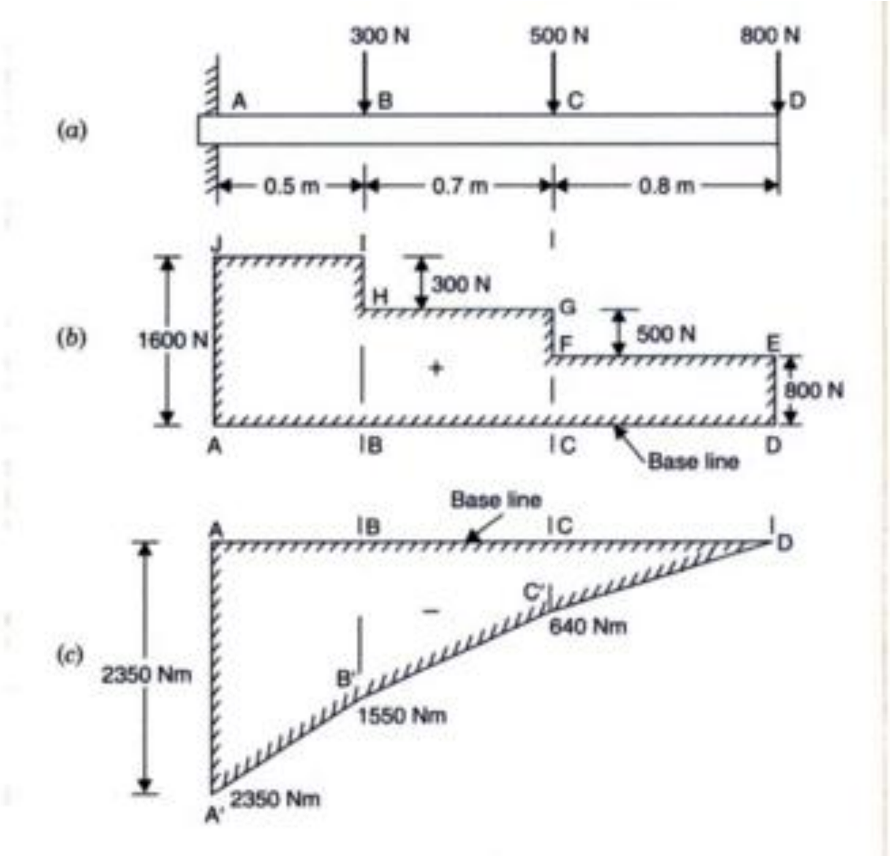
The shear force at D is +800N. this shear force remains constant between D and C. At C, due to point load the force becomes 1300N. between C and D, the shear force remains 1300N. At B again, the shear force becomes 1600N. the shear force between B and A remains constant and equal to 1600N. hence the shear force at different points will be as follows:

S.F. at D, FD= + 800 N
S.F. at C. Fe.=+ 800 + 500= 1300N
S.F. at B, Fa= + 800 + 500 +300 =1600N
S.F. at A, FA = + 1600 N.

The shear force, diagram is shown in Fig. which is drawn as: Draw a horizontal line AD as base line. On the base line mark the points B and C below the point loads. Take the ordinate DE = 800 N in the upward direction. Draw a line EF parallel to AD. The point F is vertically above C. Take vertical line FG is 500 N. Through G, draw a horizontal line GH in which point H is vertically above B. Draw vertical line HI = 300 N. From I, draw a horizontal line IJ. The point J is vertically above A. This completes the shear force diagram.

Bending Moment Diagram

The bending moment at D is zero:



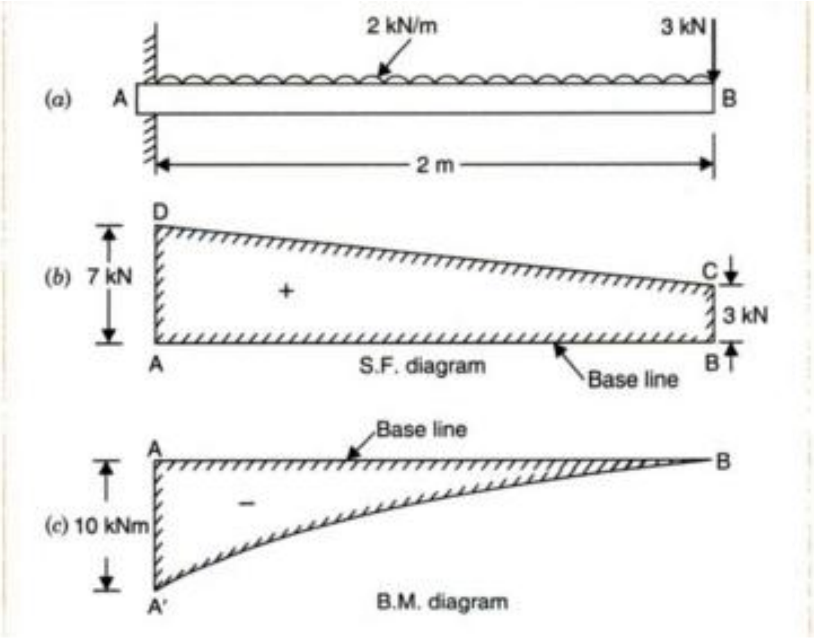
(i) The bending moment at any section between C and Data distance: and D is given by,
Mx = - 800 X x which follows a straight line law.
At C, the value of x = 0.8 m. B.M. at C, = - 800 X 0.8 = - 640 Nm.

(ii) The B.M. at any section between B and C at a distance x from D is given by (At C, x = 0.8 and at B, x = 0.8 + 0.7 = 1.5 m. Hence here varies from 0.8 to 1.5).
Mx = - 800x - 500(x- 0.8)
Bending moment between B and C also varies by a straight line law.
B.M. at B is obtained by substituting x = 1.5 m in equation (i).
M_B = -800 X 1.5 - 500 (1.5 - 0.8)
= 1200 – 350 = 1550 Nm.

(iii) The B.M. at any section between A and B at a distance x from D is given by
(At B, x = 1.5 and at A, x = 2.0 m. Hence here x varies from 1.5m to 2.0 m
Mx = - 800x - 500 (x - 0.8) – 300 (x- 1.5)
Bending moment between A and B varies by a straight line law.
B.M. at A is obtained by substituting x = 2.0 m in equation (ii),
M_A = - 800 X 2 - 500 (2 - 0.8) - 300 (2 - 1.5)
= - 800 X 2 - 500 X 1.2 - 300 X 0.5
= - 1600 - 600 - 160 = - 2350 Nm. Hence the bending moments at different points will be as given below : MD = 0 Mc = - 640 Nm M_B= - 1550 Nm, M_A= - 2350 Nm

SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A CANTILEVER BEAM WITH A UNIFORMLY DISTRIBUTED LOAD

A cantilever beam of length 2m carries a uniformly distributed load of 2kN/m length over the whole length and a point load of 3kN at the free end. draw the shear force and B.M diagrams for the cantilever beam.



Shear Force diagram

The shear force at B = 3 kN

Consider any section at a distance x from the free end B. The shear force at the section is given by.

Fx = 3.0 + w.x (+ve sign is due to downward force on right portion of the section)

= 3.0 + 2 X x

The above equation shows that shear force follows a straight line law.

At B, x = 0 hence F_B = 3.0 kN

At A. x = 2 m hence FA = 3 + 2 x 2 = 7 kN.

The shear force diagram is shown in Fig. 6.18 (b), in which F_B = BC = 3 kN and FA = AD = 7 kN. The points C and D are joined by a straight line.

Bending Moment Diagram

The bending moment at any section at a distance x from the free end B is given by.

$M_x = -(\ 3x + wx\ .\ x/2)$

$= -(\ 3x + 2x^2/2)$

$= -(3x + x^2)$

(The bending moment will be negative as for the right portion of the section. the moment of loads at x is clockwise)

Equation (i) shows that the B. M. varies according to the parabolic law. From equation (i) we have At B. x = 0 hence $M_B = -(3x0 + 0^2) = 0$

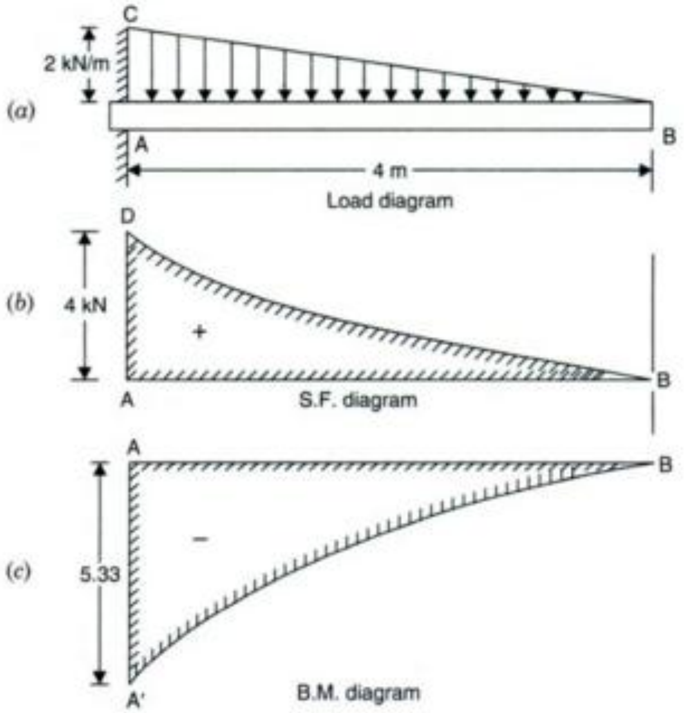
At A, x = 2 m hence $M_A = -(\ 3 \times 2 + 2^2) = -\ 10$ kN/m

Now the bending moment diagram is drawn In this diagram.

AA' = 10 kNm and points A' and B are joined by a parabolic curve.

SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A CANTILEVER CARRYING A GRADUALLY VARYING LOAD

A cantilever of length 4 m carries a gradually varying load, zero at the free end to 2 Kn/m. at the fixed end. Draw the S.F. and B.M. diagrams for the cantilever.



Shear Force Diagram

The shear force is zero at B.

The shear force at C will be equal to the area of load diagram ABC.

Shear force at C = (4 x 2) / 2 = 4 kN

The shear force between A and B varies according to parabolic law.

Bending Moment Diagram

The B.M. at B is zero.

The bending moment at A is equal to $M_A = -w. l^2 / 6 = - 2 \times 4^2 / 6 = - 5.33$ kNm.

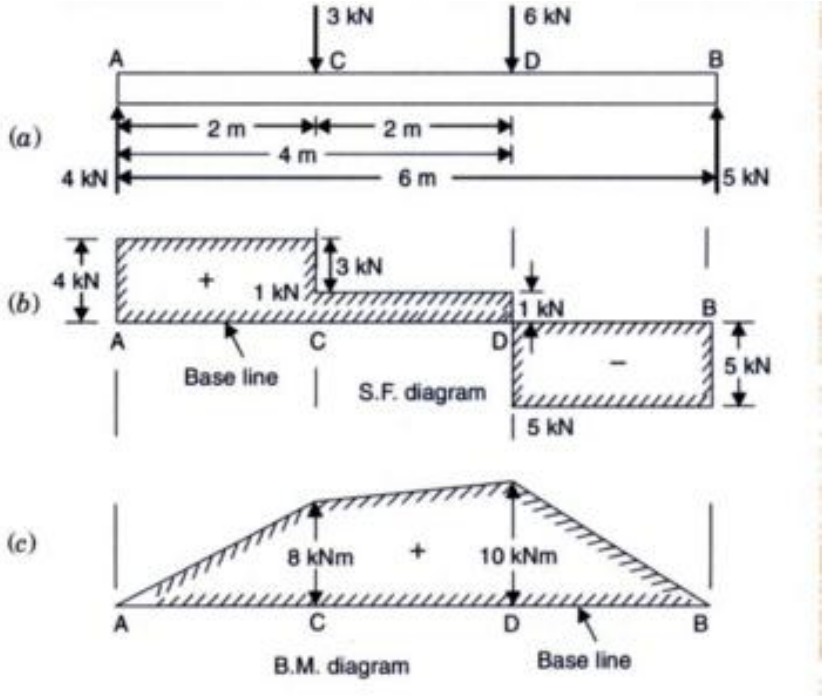
The B.M. between A and B varies according to cubic law.

SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM WITH POINT LOAD

A simply supported beam of length 6 m, carries point load of 3 kN and 6 kN at distances of 2 m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam.

Sol.

First calculate the reactions RA and RB.
Taking moments of the force about A, we get
RB X 6 = 3 X 2 + 6 X 4 = 30
RB = 30/ 6 = 5 kN
RA = Total load on beam - RB = (3 + 6) – 5 = 4 kN



Shear Force Diagram
Shear force at A, FA= + RA= + 4 kN
Shear force between A and C is constant and equal to + 4 kN
Shear force at C, Fc = + 4 - 3.0 = + 1 kN
Shear force between C and D is constant and equal to + 1 kN.
Shear force at D, FD= + 1 - 6 = - 5 kN
The shear force between D and B is constant and equal to - 5 kN.
Shear force at B, FB= - 5 kN
Bending Moment Diagram
B.M. at A, MA = 0
B.M. at C, MC = RA X 2 = 4 X 2 = +8kNm
B.M. at D, MD = RA X 4 -3 x 2 = 4 x 4 -3 x 2 =+ 10 kNm
B.M. at B, MB= 0

SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM WITH A UNIFORMLY DISTRIBUTED LOAD

Draw the S.F. and B.M. diagrams of a simply supported beam of length 7 m carrying uniformly distributed load

Sol. First calculate the reactions RA and RB,
Taking moments of all forces about A, we get
RB X 7 = 10 X 3 X (3/2) + 5 X 2 X (3+2+(2/2)

= 45 + 60 = 105

RB = 105 /7 = 15 kN

and RA =Total load on beam - RB

= (10X 3 +5 X 2) - 15 = 40- 15 = 25kN

S.F. Diagram

The shear force at A is + 25 kN

The shear force at C = RA - 3 X 10 = + 25 - 30 = - 5 kN

The shear force varies between A and C by a straight line law.

The shear force between C and D is constant and equal to - 5 kN.

The shear force at B is - 15 kN The shear force between D and B varies by a straight line law.

The shear force is zero at point E between A and C. Let us find the location of E from A. Let the point E be at a distance x from A.

The shear force at E = RA - 10 x = 25 - 10x

But shear force at E = 0

25-10x = 0 or

10x = 25

x = 2.5m

B.M. Diagram

B.M. at A is zero

B.M. at B is zero

B.M. at C,

$M_C = R_A \times 3 - 10 \times 3 \times 3/2$

= 25 X 3 – 45 = 75 -45 = 30 kNm

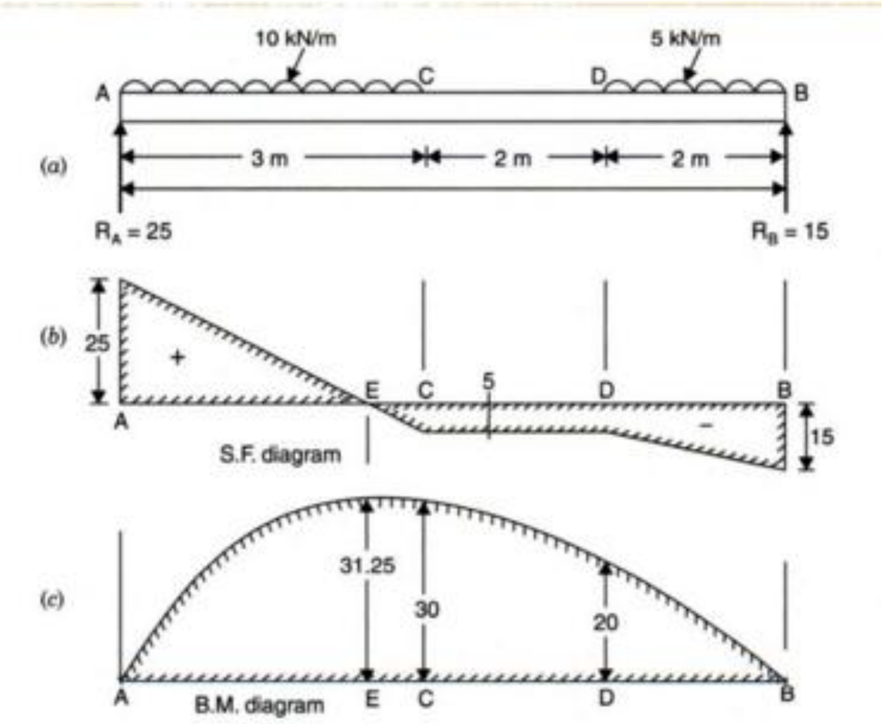
At E, x = 2.5 and hence

B.M. at E, $M_E = R_A \times 2.5 - 10 \times 2.5 \times (2.5/2) = 25 \times 2.5 - 5 \times 6.25 = 62.5 - 31.25$

= 31.25 kNm

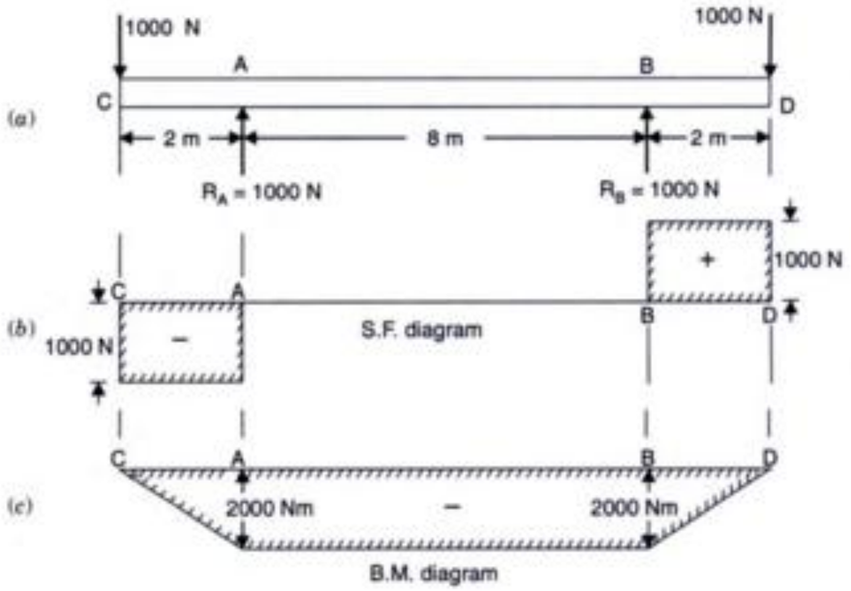
B.M. at D. $M_D = 25(3 + 2) - 10 \times 3 \times ((3/2) + 2) = 125 - 105 = 20$ kNm

The B.M. between AC and between BD varies according to parabolic law. But B.M. between C and D varies according to straight line law.



SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR OVER HANGING BEAM

A beam of length 12 m is simply supported at two supports which are 8m apart, with an overhang of 2 m on each side as shown in Fig. The beam carries a concentrated load of 1000 N at each end. Draw S.F. and B.M. diagrams.



As the loading on the beam is symmetrical. Hence reactions R_A and R_B will be equal and their magnitude will be half of the total load.

$R_A = R_B = (1000 + 1000)/2 = 1000N$

S.F. at C = -1000 N

S.F. remains constant (i.e., = - 1000 N) between C and A

S.F. at A = 1000+R_A = - 1000 + 1000 = 0

S.F. remains constant (i.e., = 0) between A and B

S.F. at B = 0+ 1000 =+ 1000N

S.F. remains constant (i.e., 1000 N) between B and D

B.M. Diagram

B.M. at C = 0

B.M. at A = - 1000 x 2 = - 2000 Nm

B.M. between C and A varies according to straight line law.

The B.M. at any section in AB at a distance of x from C is given by,

$M_X = - 1000 X x + R_A (x - 2)$

$= - 1000 X x + 1000(x - 2) = - 2000$ Nm

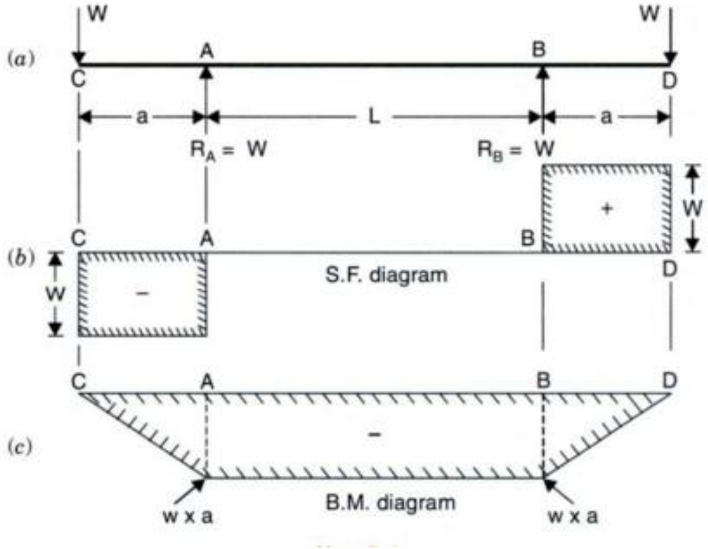
Hence B.M. between A and B is constant and equal to - 2000 Nm.

B.M. at D = 0

STRESSES IN BEAMS

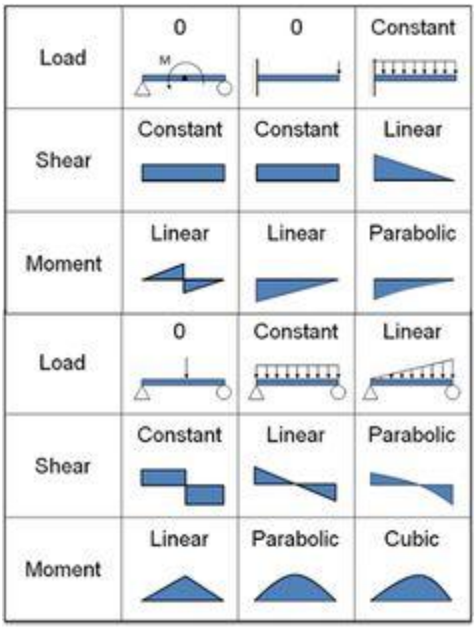
When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam. Due to the shear force and bending moment, the beam undergoes certain deformation. The material of the beam will offer resistance or stresses against these deformations. These stresses with certain assumptions can be calculated. The stresses introduced by bending moment are known as bending stresses.

If a length of a beam is subjected to a constant bending moment and no shear force (i.e., zero shear force), then the stresses will be set up in that length of the beam due to B.M. only and that length of the beam is said to be in pure bending or simple bending. The stresses set up in that length of beam are known as bending stresses.



A beam simply supported at A and B and overhanging by same length at each support is shown in Fig. 7.1. A point load W is applied at each end of the overhanging portion. The S.F. and B.M. for the beam are drawn as shown in Fig. 7.1 (b) and Fig. 7.1 (c) respectively. From these diagrams, it is clear that there is no shear force between A and B but the B.M. between A and B is constant. This means that between A and B, the beam is subjected to a constant bending moment only. This condition of the beam between A and B is known as pure bending or simple bending.

The following diagram gives a general idea of SFD & BMD for different loading cases.



Load	0	0	Constant
Shear	Constant	Constant	Linear
Moment	Linear	Linear	Parabolic
Load	0	Constant	Linear
Shear	Constant	Linear	Parabolic
Moment	Linear	Parabolic	Cubic

THEORY OF SIMPLE BENDING

The beams supporting the multiple overhead cranes system shown in this picture are subjected to transverse loads causing the beams to bend. The normal stresses resulting from such loadings will be determined in this chapter.

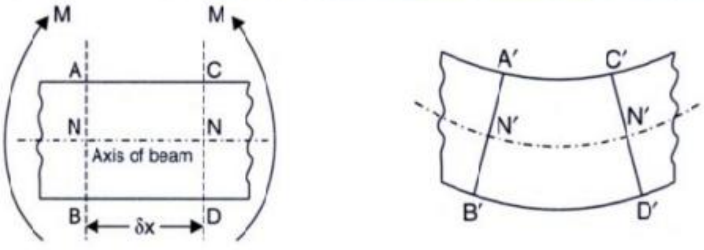
Thanks to : http://www.mhhe.com/engcs/engmech/beerjohnston/mom/samplechap.pdf

THEORY OF SIMPLE BENDING WITH ASSUMPTIONS MADE

Before discussing the theory of simple bending, let us see the assumptions made in the theory of simple bending. The following are the important assumptions:

1. The material of the beam is homogeneous (involving substances in the same phase) and isotropic (property is same in all directions).
2. The value of Young's modulus of elasticity is the same in tension and compression.
3. The transverse sections which were plane before bending, remain plane after bending
4. The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.
5. The radius of curvature is large compared with the dimensions of the cross-section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

Let us consider a beam subjected to simple bending. Consider a small length fit of this part of beam. Consider two sections AB and CD which are normal to the axis of the beam N - N. Due to the action of the bending moment, the part of length &x will be deformed as shown in Fig.(b). From this figure, it is clear that all the layers of the beam, which were originally of the same length, do not remain of the same length any more. The top layer such as AC has deformed to the shape NC. This layer has been shortened in its length. The bottom layer BD has deformed to the shape B'D'. This layer has been elongated. From the Fig. 7.2 (b), it is clear that some of the layers have been shortened while some of them are elongated. At a level between the top and bottom of the beam, there will be a layer which is neither shortened nor elongated. This layer is known as neutral layer or neutral surface. This layer in Fig.(b) is shown by N' — N' and in Fig.(a) by N — N. The line of intersection of the neutral layer on a cross-section of a beam is known as neutral axis (written as N.A.).



(a) Before bending (b) After bending

The layers above N — N (or N' — N') have been shortened and those below, have been elongated. Due to the decrease in lengths of the layers above N— N, these layers will be subjected to compressive stresses. Due to the increase in the lengths of layers below N — N, these layers will be subjected to tensile stresses. We also see that the top layer has been shortened maximum. As we proceed towards the layer N— N, the decrease in length of the layers decreases. At the layer N— N, there is no change in length. This means the compressive stress will be maximum at the top layer. Similarly the increase in length will be maximum at the bottom layer. As we proceed from bottom layer towards the layer N — N, the increase in length of layers decreases. Hence the amount by which a layer increases or decreases in length, depends upon the position of the layer with respect to N - N. This theory of bending is known as theory of simple bending.

Simple Bending Theory OR Theory of Flexure for Initially Straight Beams (The normal stress due to bending are called flexure stresses)

Preamble:

When a beam having an arbitrary cross section is subjected to a transverse loads the beam will bend. In addition to bending the other effects such as twisting and buckling may occur, and to investigate a problem that includes all the combined effects of bending, twisting and buckling could become a complicated one. Thus we are interested to investigate the bending effects alone, in order to do so; we have to put certain constraints on the geometry of the beam and the manner of loading.

Assumptions:

The constraints put on the geometry would form the **assumptions:**

1. Beam is initially **straight,** and has a **constant cross-section.**
2. Beam is made of **homogeneous material** and the beam has a **longitudinal plane of symmetry.**
3. Resultant of the applied loads lies in the plane of symmetry.
4. The geometry of the overall member is such that bending not buckling is the primary cause of failure.
5. Elastic limit is nowhere exceeded and **'E'** is same in tension and compression.
6. Plane cross - sections remains plane before and after bending.

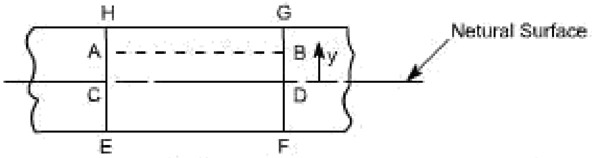


Fig 1(a)

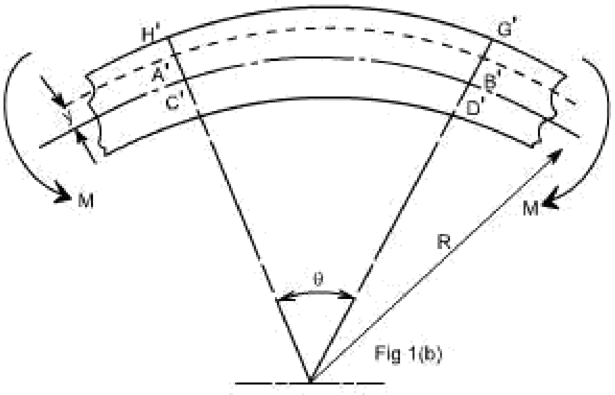


Fig 1(b)

Let us consider a beam initially unstressed as shown in fig 1(a). Now the beam is subjected to a constant bending moment (i.e. „Zero Shearing Force') along its length as would be obtained by applying equal couples at each end. The beam will bend to the radius R as shown in Fig 1(b)

As a result of this bending, the top fibers of the beam will be subjected to tension and the bottom to compression it is reasonable to suppose, therefore, **that somewhere between the two there are points at which the stress is zero. The locus of all such points is known as neutral axis.** The radius of curvature R is then measured to this axis. For symmetrical sections the N. A. is the axis of symmetry but whatever the section N. A. will always pass through the centre of the area or centroid.

The above restrictions have been taken so as to eliminate the possibility of 'twisting' of the beam.

Concept of pure bending: Loading restrictions:

As we are aware of the fact internal reactions developed on any cross-section of a beam may consists of a resultant normal force, a resultant shear force and a resultant couple. In order to ensure that the bending effects alone are investigated, we shall put a constraint on the loading such that the resultant normal and the resultant shear forces are zero on any cross-section perpendicular to the longitudinal axis of the member,

That means F = 0

Since $\frac{dM}{dX} = F = 0$ or M = constant.

Thus, the zero shear force means that the bending moment is constant or the bending is same at every cross-section of the beam. Such a situation may be visualized or envisaged when the beam

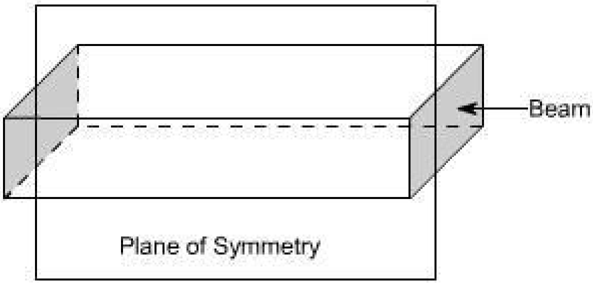


Fig (1)

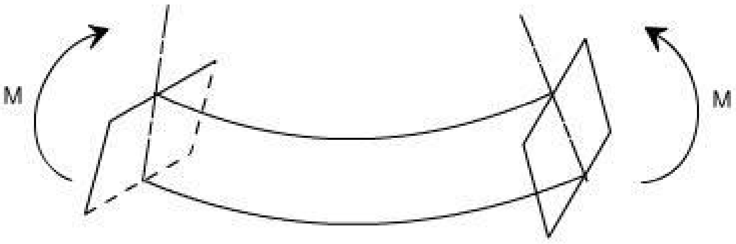
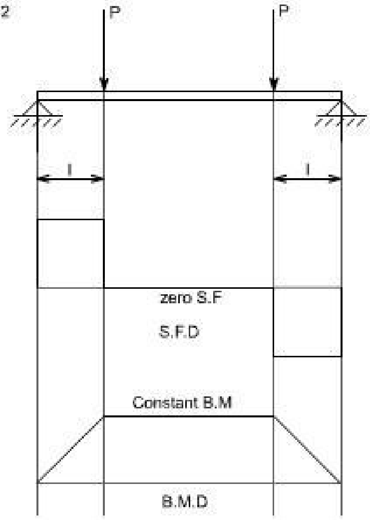
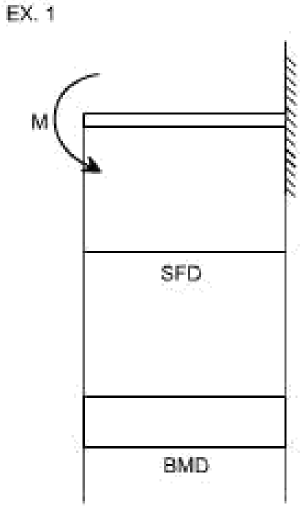


Fig (2)

When a member is loaded in such a fashion it is said to be in **pure bending.** The examples of pure bending have been indicated in EX 1and EX 2 as shown below:

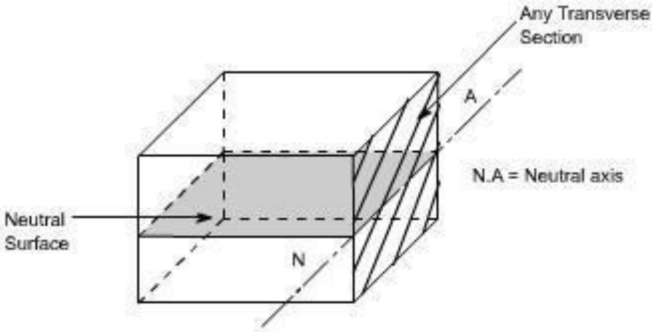
EX .2





When a beam is subjected to pure bending are loaded by the couples at the ends, certain cross-section gets deformed and we shall have to make out the conclusion that,

1. Plane sections originally perpendicular to longitudinal axis of the beam remain plane and perpendicular to the longitudinal axis even after bending , i.e. the cross-section A'E', B'F' (refer Fig 1(a)) do not get warped or curved.
2. In the deformed section, the planes of this cross-section have a common intersection i.e. any time originally parallel to the longitudinal axis of the beam becomes an arc of circle.



We know that when a beam is under bending the fibers at the top will be lengthened while at the bottom will be shortened provided the bending moment M acts at the ends. In between these there are some fibers which remain unchanged in length that is they are not strained, that is they do not carry any stress. The plane containing such fibers is called neutral surface.The line of intersection between the neutral surface and the transverse exploratory section is called the neutral axis Neutral axis **(N A).**

Bending Stresses in Beams or Derivation of Elastic Flexural formula :

In order to compute the value of bending stresses developed in a loaded beam, let us consider the two cross-sections of a beam **HE** and **GF** , originally parallel as shown in fig 1(a).when the beam is to bend it is assumed that these sections remain parallel i.e. **H'E'** and **G'F'** , the final position of the sections, are still straight lines, they then subtend some angle q.

Consider now fiber AB in the material, at a distance y from the N.A, when the beam bends this will stretch to A'B'

Therefore,

$$\text{strain in fibre AB} = \frac{\text{change in length}}{\text{orginal length}}$$

$$= \frac{A'B' - AB}{AB} \qquad \text{But } AB = CD \text{ and } CD = C'D'$$

refer to fig1(a) and fig1(b)

$$\therefore \text{strain} = \frac{A'B' - C'D'}{C'D'}$$

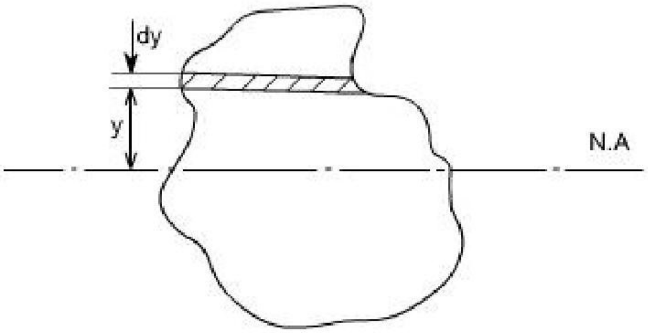
Since CD and C'D' are on the neutral axis and it is assumed that the Stress on the neutral axis zero. Therefore, there won't be any strain on the neutral axis

$$= \frac{(R + y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = \frac{y}{R}$$

$$\text{However } \frac{\text{stress}}{\text{strain}} = E \qquad \text{where } E = \text{Young's Modulus of elasticity}$$

Therefore, equating the two strains as obtained from the two relations i.e,

$$\frac{\sigma}{E} = \frac{y}{R} \text{ or } \frac{\sigma}{y} = \frac{E}{R} \qquad \text{..............(1)}$$



Consider any arbitrary a cross-section of beam, as shown above now the strain on a fibre at a distance „y' from the N.A, is given by the expression

$$\sigma = \frac{E}{R} y$$

if the shaded strip is of area 'dA'

then the force on the strip is

$$F = \sigma\ \delta A = \frac{E}{R} y\ \delta A$$

$$\text{Moment about the neutral axis would be} = F.y = \frac{E}{R}\ y^2 \delta A$$

The toatl moment for the whole cross-section is therefore equal to

$$M = \sum \frac{E}{R} y^2\ \delta A = \frac{E}{R} \sum y^2 \delta A$$

Now the term is the property of the material and is called as a second moment of area of the cross-section and is denoted by a symbol I.

Therefore

$$M = \frac{E}{R} I \qquad \ldots\ldots\ldots(2)$$

combining equation 1 and 2 we get

$$\boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}}$$

This equation is known as the Bending Theory Equation.

The above proof has involved the assumption of pure bending without any shear force being present. Therefore this termed as the pure bending equation. This equation gives distribution of stresses which are normal to cross-section i.e. in x-direction.

Section Modulus:

From simple bending theory equation, the maximum stress obtained in any cross-section is given as

$$\sigma_{\max}{}^{m} = \frac{M}{I} y_{\max}{}^{m}$$

For any given allowable stress the maximum moment which can be accepted by a particular shape of cross-section is therefore

$$M = \frac{I}{y_{\max}{}^{m}} \sigma_{\max}{}^{m}$$

For ready comparison of the strength of various beam cross-section this relationship is sometimes written in the form

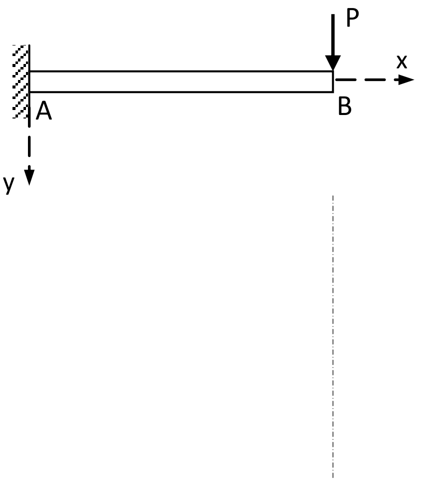
$$M = Z \sigma_{\max}{}^{m} \text{ where } Z = \frac{I}{y_{\max}{}^{m}}$$

Is termed as section modulus

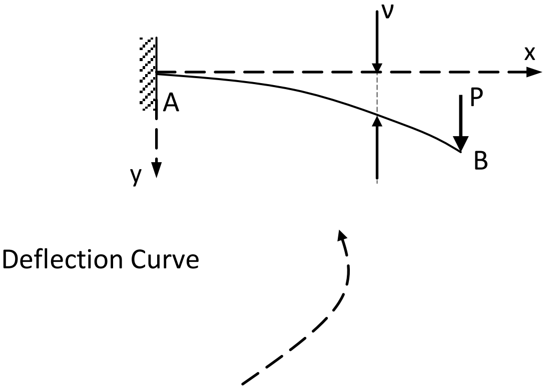
STRESSES IN BEAMS

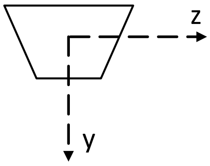
In previous chapter concern was with shear forces and bending moment in beams. Focus in this chapter is on the stresses and strains associated with those shear forces and bending moments.

Loads on a beam will cause it to bend or flex.



xy – plane = plane of bending





Cross –Section assumed symmetric about xy - plane

PURE BENDING AND NONUNIFORM BENDING

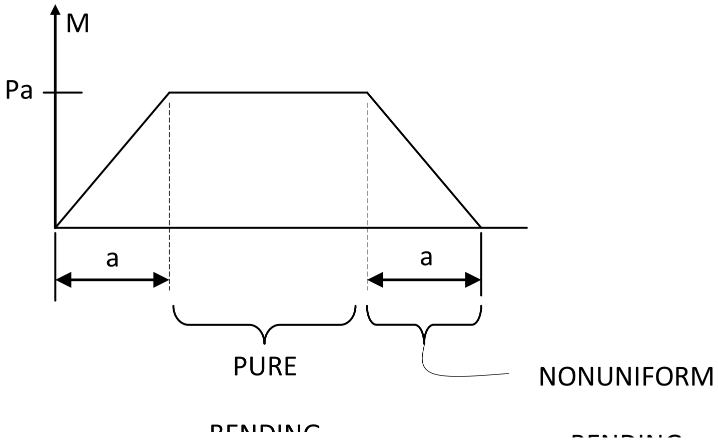
Pure Bending = flexure of a beam under constant bending moment

$\Rightarrow$ shear force = 0 (V = 0 = dM / dx); no change in moment.

Nonuniform Bending = flexure of a beam in the presence of shear forces

$\Rightarrow$ bending moment is no longer constant

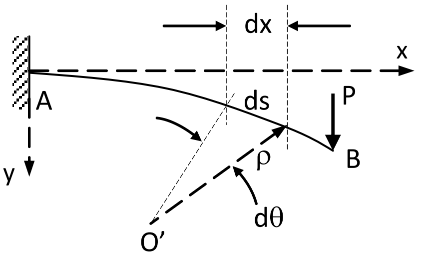
Moment Diagram example:



CURVATURE OF A BEAM

A beam in NONUNIFORM BENDING (V ≠ 0) will have a varying curvature.

ds = curve length



O’ = center of curvature

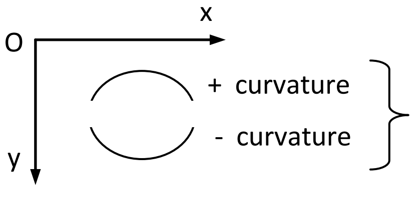
ρ = radius of curvature

κ = curvature = ρ^{-1} = $\frac{1}{\rho}$

$$\rho \, d\theta = ds$$

For small deflections: ds ≈ dx

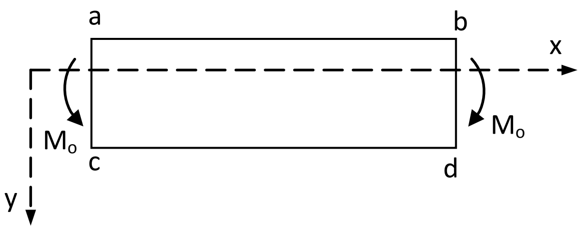
$$\boxed{\kappa = \frac{1}{\rho} = \frac{d\theta}{dx}} \qquad (1)$$

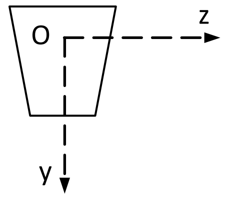


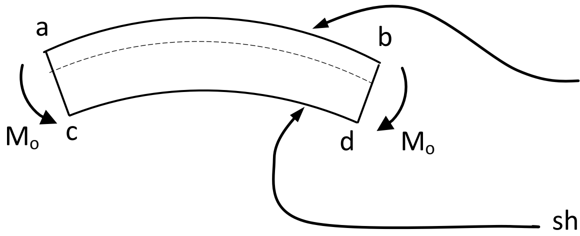
opposite from text because we use + ↓ y

Sign Convention:

NORMAL STRAINS





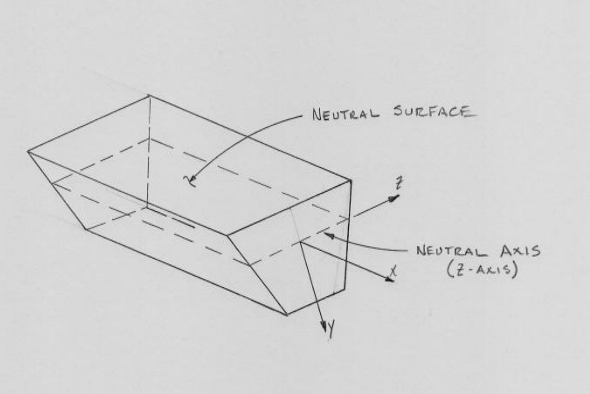


Lengthened ⇒ in tension

shortened ⇒ in compression

Somewhere between the top and bottom of the beam is a place where the fibers are neither in tension or compression.

neutral axis of the cross section



dashed line = neutral surface of the beam

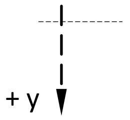
when bent: a b lengthens
c d shortens } causes normal strains, ε_x

The normal strain is: $\boxed{\varepsilon_x = -\frac{y}{\rho} = -\kappa y}$ ___________________ (2)

Where, y = distance from neutral axis

From Eqn (2):

$-y = +\varepsilon_x$ (elongation)



$+y = -\varepsilon_x$ (shortening)

} for + κ

Transverse Strains: $\varepsilon_z = -\nu\varepsilon_x = \nu\kappa y$

Where ν = Poisson's Ratio

NORMAL STRESSES IN BEAMS

If material is elastic with linear stress-strain diagram, THEN:

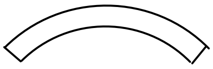
$\sigma = E\varepsilon$ (Hooke's Law)

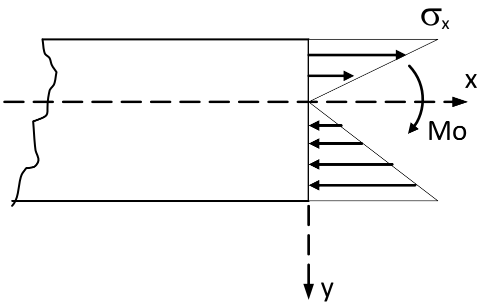
$$\boxed{\sigma_x = E\varepsilon_x = -E\kappa y} \_\_\_\_\_\_\_\_\_\_\_\_\_ \quad (3)$$

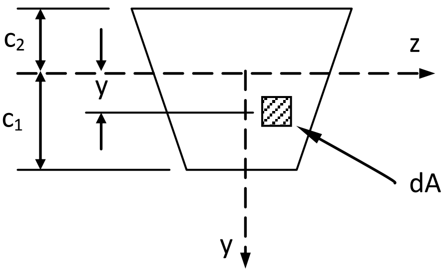
varies
linearly

Where x is longitudinal axis of beam and σ_x is the normal stresses in this direction acting on the cross section. These stresses varies linearly with the distance y from the neutral surface.

REF: + CURVATURE = + STRESSES above







$$\int \sigma_x \, dA \;=\; -\int E\kappa y \, dA = 0$$

must equal ZERO because there is NO resultant normal force that acts on the ENTIRE cross section

$$\boxed{\int y\,dA = 0} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \quad (4)$$

Eqn (4) is the 1st Moment of the Area of the cross section w.r.t. z-axis and it is zero

⇒ z-axis must pass thru the centroid of the cross section.

$\Rightarrow$ z-axis is also the neutral axis

$\Rightarrow$ neutral axis passes thru the centroid of the cross section

Limited to beams where y-axis is the axis of symmetry.

y, z –axes are the PRINCIPAL CENTROIDAL AXES.

Consider the Moment Resultant of σ_x :

RECALL Eqn (3):

$$\sigma = -E\kappa y$$

$$dM_O = -\sigma_x y dA$$

$$M_O = -\int \sigma_x y dA = \kappa E \int y^2 \, dA$$

M = -M$_0$

$$M = -\kappa E I$$

$$\boxed{I = \int y^2 \, dA}$$

$\Leftarrow$ where, I = Moment of Inertia of cross sectional area w.r.t.

$$\kappa = \frac{1}{\rho} = -\frac{M}{EI}$$

$\Leftarrow$ EI = FLEXURAL RIGIDITY

$$-E\kappa = \frac{M}{I}$$

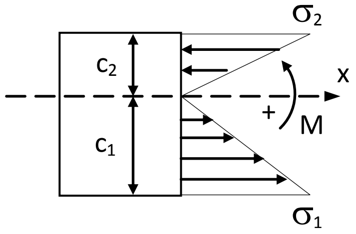
$\Leftarrow$ substitute into Eqn (3)

$$\sigma_x = \left(\frac{M}{I}\right) y$$

$$\boxed{\sigma_x = \frac{My}{I}}$$

$\Leftarrow$ Flexure Formula

σ_x = Bending Stress



MAXIMUM STRESSES:

$$\boxed{\sigma_1 = \frac{Mc_1}{I}} \qquad \boxed{\sigma_2 = -\frac{Mc_2}{I}}$$

Text defines Section Moduli as:

$$S_1 = \frac{I}{c_1} \qquad S_2 = \frac{I}{c_2}$$

$$\sigma_1 = \frac{M}{S_1} \qquad \sigma_2 = -\frac{M}{S_2}$$

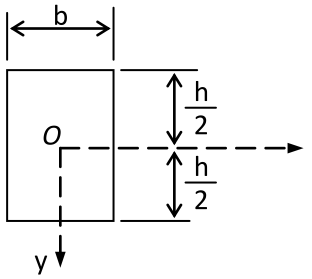
Section Modulus is handy to use when evaluating bending stress w.r.t. to moment which varies along length of a beam.

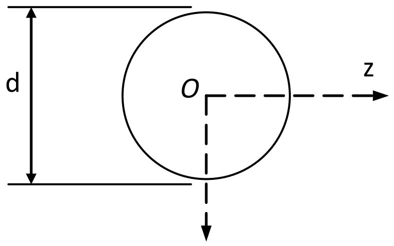
If cross section is symmetrical w.r.t. z-axis, then:

$c_1 = c_2 = c$

$$\sigma_1 = -\sigma_2 = \frac{Mc}{I}$$

Moments of Inertia to know:





$$I = \frac{bh^3}{12}$$

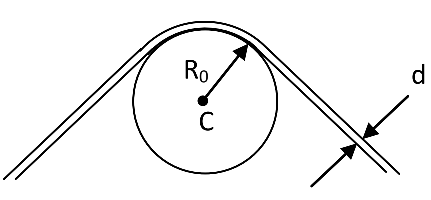
$$I = \frac{\pi d^4}{64}$$

Problems for Practice

A high-strength steel wire of diameter d = 4 mm, modulus of elasticity E = 200 GPa, proportional limit σ_{pl} = 1200 MPa is bent around a cylindrical drum of radius R_0 = 0.5 m .

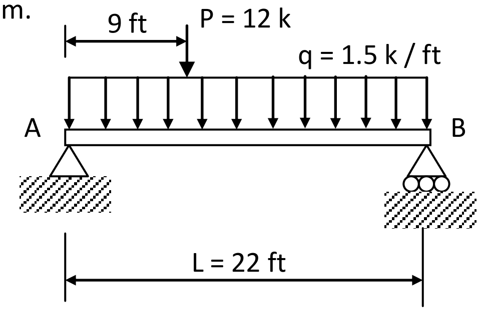
FIND:

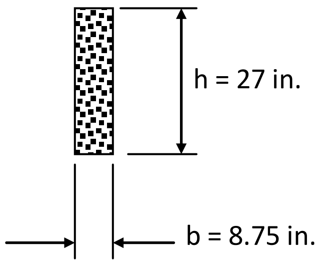
a. bending moment, M
b. maximum bending stress, σ_{max}



Problems for Practice

The beam shown which is constructed of glued laminated wood. The uniform load includes the weight of the beam.





FIND:

a. Maximum Tensile Stress in the beam due to bending.
b. Maximum compressive stress in the beam due to bending.

DESIGN of BEAMS for BENDING STRESSES

After all factors have been considered (i.e., materials, environmental conditions) it usually boils down to

$$\sigma_{Allow} > \sigma_{Beam}$$

$$\sigma_{Allow} > \frac{M_{\max} c}{I}$$

Here is where the section modulus is useful.

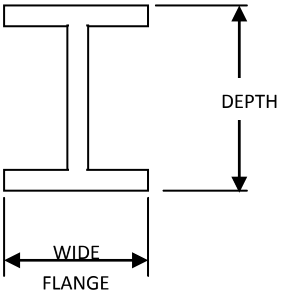
RECALL: $\sigma = \frac{M}{S}$ thus, $S = \frac{M_{\max}}{\sigma_{allow}}$

Appendix E and F give properties of beams.

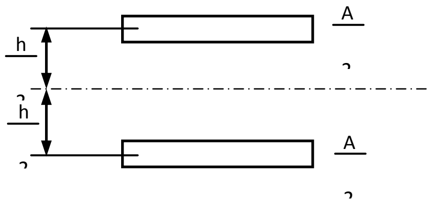
i.e., W8 x 15 Weight per foot (lbs / ft)

Nominal Depth (in)

W Shape = wide flange



Wood Beams - 2 x 4 ⇒ really is: 1.5” x 3.5” net dimensions (should always use net dims.)



$$I = \frac{A}{2}\left(\frac{h}{2}\right)^2 + \frac{A}{2}\left(\frac{h}{2}\right)^2$$

$$I = \frac{Ah^2}{4}$$

$$S = \frac{I}{c} = \frac{\frac{Ah^2}{4}}{\frac{h}{2}} = \frac{1}{2}Ah$$

For W Shapes; ***S ≈ 0.35 Ah***

You want as much material as possible, as far from the neutral axis as possible because this is where the greatest stress is occurring.

However, you have to be careful because if the web is too thin, it could fail by:

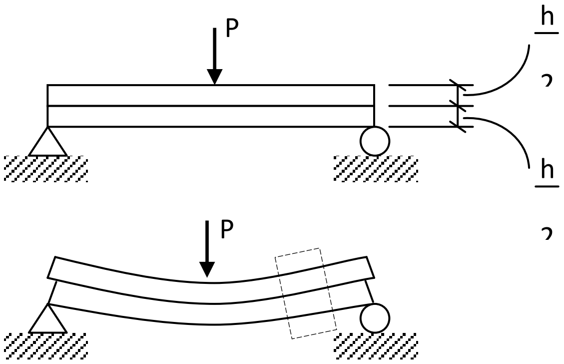
1.) being overstressed in shear
2.) buckling

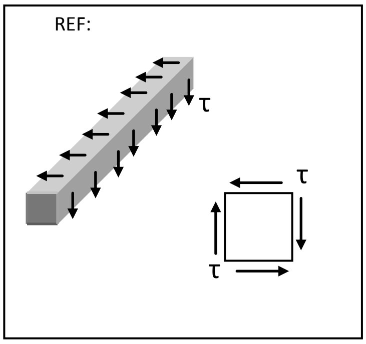
SHEAR STRESSES IN BEAMS (RECTANGULAR CROSS-SECTIONS)

Assumptions: 1. τ acts parallel to V (shear force, also, y – axis)

2. τ is uniform across cross-section

Consider a system of 2 beams

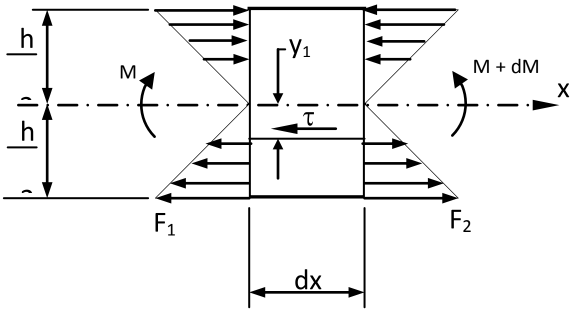


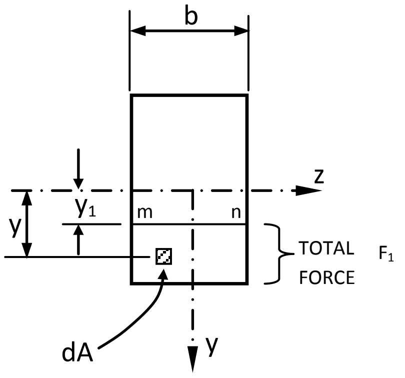


Assuming there is no (or very little) friction, the top beam can slide w.r.t. to the bottom beam.

Thus, there must be shear stresses present that prevent sliding.

Looking at the dotted rectangle:



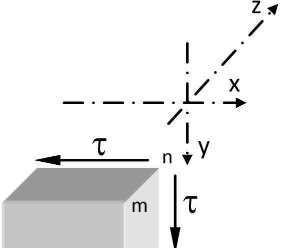


CROSS SECTION OF BEAM AT SUBELEMENT

y = distance at which

▨ Normal Force: $\sigma_x dA = \frac{My}{I} dA$
(left side)

REF:



Total Force:
(left side) $F_1 = \int \frac{My}{I} dA$ ________________ (1)

similarly,

Total Force:
(right side) $F_2 = \int \frac{(M + dM)y}{I} dA$ ___________ (2)

Shear Force: $F_3 = \tau$ b dx _____________ (3)

$$\sum F_x = 0$$
$$F_2 - F_1 - F_3 = 0$$
$$F_3 = F_2 - F_1 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (4)}$$

Sub Eqns (1), (2), and (3) into (4):

$$\tau b dx = \int \frac{(M + dM)y}{I} dA - \int \frac{My}{I} dA$$
$$\tau b dx = \int \frac{dM\ y}{I} dA$$
$$\tau = \frac{dM}{dx}\left(\frac{1}{Ib}\right)\int y\, dA$$
$$\tau = \frac{V}{Ib}\int y\, dA$$

RECALL:

$$\frac{dM}{\ } = V$$

y = distance at which the shear force acts

b = thickness of the cross-sectional area where the stress is to be evaluated.

∫ y dA = first moment of that portion of the cross-sectional area between the transverse line (m – n)

where the stress is to be evaluated AND the extreme fiber of the beam.

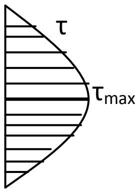
LET: $Q = \int y\, dA$

$\tau = \frac{VQ}{Ib}$ SHEAR FORMULA

For **RECTANGULAR** cross-sections, (see text pg 339 for derivation)

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

Max Shear occurs at the neutral axis, $y_1 = 0$



$$\boxed{\tau_{\max} = \frac{Vh^2}{8I} = \frac{3V}{2A}} \text{ ——— } (5)$$

WHERE:

A = bh

= TOTAL CROSS-SECTIONAL AREA

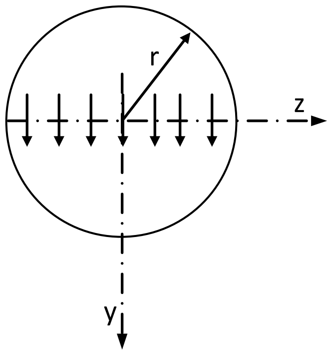
$\overline{bh^3}$

<u>NOTE</u>:

Shear Eqn (5) is limited to Cross-Sectional shapes that have sides parallel to the y-axis.

SHEAR STRESSES IN BEAMS (CIRCULAR CROSS – SECTIONS)

Largest Shear Stresses occur at neutral axis.



We can assume with good accuracy that:

1. τ acts parallel to V (shear force, also, y – axis)
2. τ is uniform across cross-sections

These assumptions are the same used when we developed the shear formula:

$$\tau = \frac{VQ}{Ib}$$

Therefore, we can use this to find τ at the N.A. (Neutral Axis) which is $\tau_{\max}$

$$I = \frac{\pi r^4}{4} \qquad b = 2r$$

$$Q = \frac{A}{2}(\bar{y}) \qquad \Rightarrow \qquad \frac{A}{2} = \frac{\pi r^2}{2} \qquad and \qquad \bar{y} = \frac{4r}{3\pi}$$

$$= \frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)$$

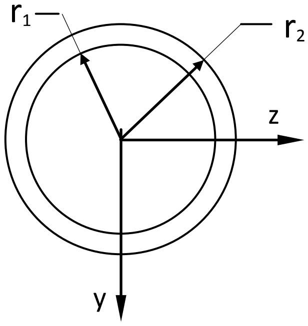
$$Q = \frac{2r^3}{3}$$

$$\boxed{\tau_{\max} = \frac{4V}{3A}}$$

$$\tau_{\max} = \frac{VQ}{Ib} = \frac{V\left(\frac{2r^3}{3}\right)}{\left(\frac{\pi r^4}{4}\right)(2r)} = \frac{4V}{3\pi r^2}$$

WHERE: ⇒ TOTAL CROSS-SECTIONAL AREA A = **πr^2**

HOLLOW CIRCULAR CROSS SECTIONS



$$I = \frac{\pi}{4}(r_2^4 - r_1^4)$$

$$b = 2(r_2 - r_1)$$

$$Q = \frac{2}{3}(r_2^3 - r_1^3)$$

$$\boxed{\tau_{\max} = \frac{4V}{3A}\left(\frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^2 + r_1^2}\right)}$$

WHERE:

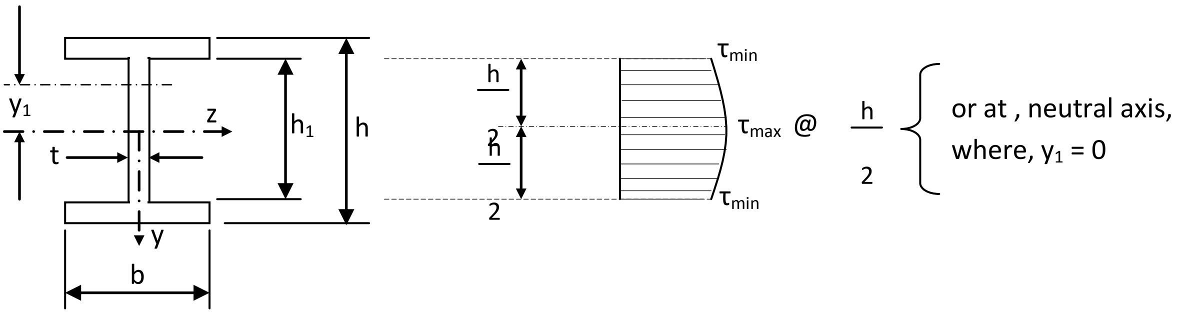
$$A = \pi\left(r_2^2 - r_1^2\right)$$

NOTE:

If $r_1 = 0$ we get our previous equation for solid circular cross-section.

SHEAR STRESSES IN THE WEB OF BEAMS WITH FLANGES

The shear formula $\tau = \dfrac{VQ}{Ib}$ still applies because the same assumptions are made.



We will use: $\boxed{\tau = \dfrac{VQ}{It}}$ where: t = web thickness

$$Q = \frac{b}{8}(h^2 - h_1^2) + \frac{t}{8}(h_1^2 - 4y_1^2)$$

$$I = \frac{1}{12}\left(bh^3 - bh_1^3 + th_1^3\right)$$

where: b = width of flange

h = height of beam

h_1 = web height (inside flanges)

y_1 = distance from N.A.

For Wide-Flange Beams, we use the AVERAGE Shear Stress in the web:

$$\boxed{\tau_{aver} = \frac{V}{th_1}}$$

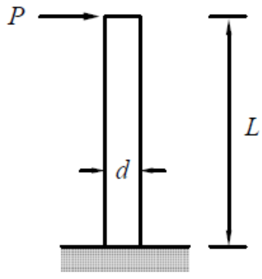
where: t = web thickness

h_1 = web height (inside flanges)

***Example* 1** A circular pole with diameter *d* under a concentrated load *P*. Determine

(a) The maximum shear stress $\tau_{\max}$

(b) The maximum bending stress $\sigma_{\max}$



L=2 m
d=150 mm
P=2.5 kN

Constant shear *V*=*P*

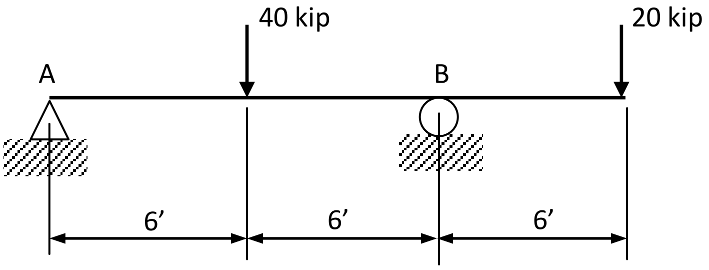
Maximum moment at clamped end $M_{\max} = PL$

$$\tau_{\max} = \frac{4V}{3A} = \frac{4P}{3\pi d^2/4} = \frac{4(2.5\,\text{kN})}{3\pi(150\,\text{mm})^2/4} = 189\,\text{kPa}$$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{PL}{\pi d^3\!/_{32}} = \frac{(2.5\,\text{kN})(2\,\text{m})}{\pi(150\,\text{mm})^3/32} = 15.1\text{MPa}$$

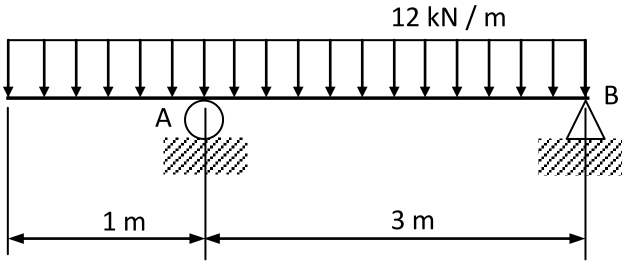
Problems for Practice

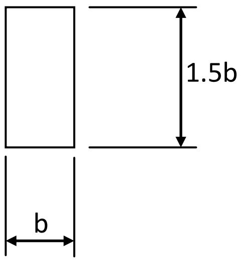
1. A beam is to be made of steel that has an allowable bending stress of σ_{allow} = 24 ksi and an allowable shear stress of τ_{allow} = 14.5 ksi. Select an appropriate W shape that will carry the loading shown



2. The laminated beam shown supports a uniform load of 12 kN/m. If the beam is to have

a height – to – width ratio of 1.5, determine the smallest width. σ_{allow} = 9 MPa, τ_{allow} = 0.6 MPa. Neglect the weight of the beam.





SHEAR FLOW IN BEAMS

The topic of shear flow frequently occurs when dealing with "built-up" beams. These are beams fabricated with several pieces joined by glue, nails, bolts, or welds. These fasteners must be sufficiently strong to withstand the lateral (transverse) or longitudinal shear. It is common to describe the load by the term, "shear flow" given by the following relation:

$$\boxed{q = VQ/I}$$

where q is the shear flow in (lb/in), (lb/ft), (N/mm), (N/m)

V is the value of the shear force at the section

Q is the first moment of the area between the location where the shear stress is being calculated and the location where the shear stress is zero about the neutral (centroidal) axis;

I is the moment of inertia of the entire cross-section about the neutral axis

The shear flow may be used to calculate the shear stress (in the case of continuous joints) by dividing by the width of the beam supporting the stress.

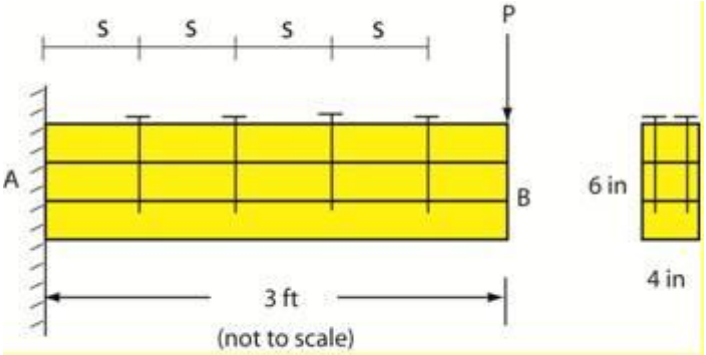
$$\boxed{\tau = VQ/It}$$

Where, t is the width of the cross-section at the location where the shear stress is being calculated

If the joints are not continuous such as in nails, screws, and bolts, then it is more convenient to use q as force per unit length along the beam.

In such a case q (lb/in) = F(lb/nail) / s(in/nail)

Here F = s q and F is the force across one nail and s is the nail spacing.



Strategy for Analyzing Shearing Flow in Beams

Shear flow, q , depends directly on the shear force, V, at the section of the beam where it is to be calculated, on the first moment of area, Q, at the location of the shear flow, and inversely on the moment of inertia of area, I, of the entire cross section about its neutral (centroidal) axis.

$$\mathbf{q = VQ/I}$$

Common units for shear flow are lb/in, lb/ft, N/mm, N/m (English/Metric)

Strategy for analyzing shearing stress involves five key steps as detailed in the table below.	
Step 1	Determine the centroid of the beam's cross-section.
Step 2	Determine the moment of inertia of area of the beam's entire cross-section about its neutral (centroidal) axis. This step may involve use of the parallel axis theorem. Recall for a rectangular cross-section of width b and height h the moment of inertia of area, I, about its centroidal axis is (1/12) bh^3 . Voids of area contribute negative values of moments of inertia.
Step 3	Determine the distribution of shear force across the length of the beam. This step involves plotting the shear force distribution along the length of the beam. Pick the maximum shear force, V.
Step 4	Calculate, Q, the first moment of area at the location where the shear flow is to be calculated. Q is the product of the area, A, from where the shear flow is zero to the location where the shear flow is to be calculated times the distance, y_{bar}, from the neutral axis to the centroid of this area, A.
Step 5	Finally, calculate the shear flow, q, using: $q = VQ/I$ **Note**: Shear flow is independent of the width of the beam's cross-section.

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