

UNIT III STABILITY OF VEHICLES

Load distribution

Stability on a slope, curved track and a banked road

Calculation of tractive effort and reactions for different drives

Introduction:

Tractive effort:

- The force available at the contact between the drive wheel tyres and road is known as tractive effort.

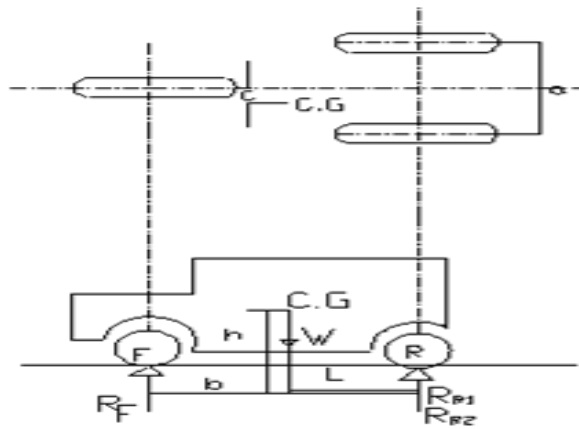
Traction:

- The ability of the drive wheel to transmit this effort without slipping is known as traction.

1. LOAD DISTRIBUTION

THREE WHEELED VEHICLE:

The forces acting on a vehicle at rest are shown in figure.



Where,

W = weight of the wheel, N,

b = wheelbase, m,

l = distance of CG from the rear axle, m,

h = height of CG from road surface, m,

c = distance of CG from the central axis, m,

a = wheel track, m,

R_F = vertical reaction at front wheel, N,

R_{R1}, R_{R2} = vertical reaction at the rear wheels, N

There are three unknowns which can be determined as follows:

Moment about rear axle gives

$$R_F b = Wl$$

Therefore,

$$R_F = \frac{Wl}{b}$$

Moment about central axis gives,

$$(R_{R1} + R_{R2}) \frac{a}{2} = Wc$$

Therefore,

$$R_{R2} - R_{R1} = \frac{2Wc}{a}$$

Moment about central axis of the front wheel gives,

$$(R_{R1} + R_{R2})b = W(b - l)$$

Therefore

$$R_{R1} + R_{R2} = W \frac{(b-l)}{b} = W \left(1 - \frac{l}{b}\right)$$

Addition and subtraction of last two equations respectively gives,

$$R_{R2} = \frac{W}{2} \left(\frac{2c}{a} - \frac{l}{b} + 1 \right)$$

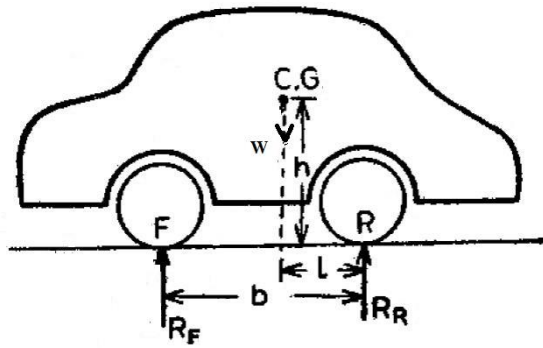
And

$$R_{R1} = \frac{W}{2} \left(1 - \frac{l}{b} - \frac{2c}{a} \right)$$

Also $W = R_F + R_{R1} + R_{R2}$ must be satisfied and server as an extra equation for alternative solution.

FOUR-WHEELED VEHICLE:

Forces acting on a four-wheeled vehicle at rest, we can form three independent equations to take care of four unknown viz., four reactions at the wheels. Thus the problem is simplified by considering it as a two-wheeled vehicle, i.e. the reactions on both rear wheels is equal and also on both front wheels. Let R_F and R_R be vertical reactions at front and rear wheel respectively, The,



$$\sum V = 0$$

Gives,

$$W = R_R + R_F$$

$$\sum M_R = 0$$

Gives,

$$Wl = R_F b$$

$$R_F = \frac{Wl}{b}$$

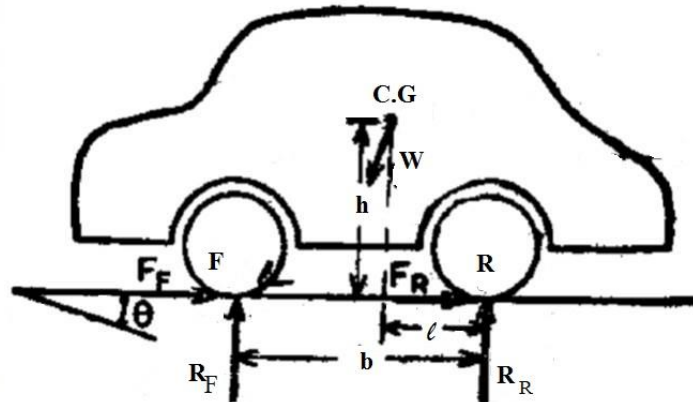
Substituting the value of R_F in the above,

$$R_R = W \left(1 - \frac{l}{b}\right)$$

2. STABILITY ON A SLOPE, CURVED TRACK AND A BANKED ROAD

STABILITY OF A VEHICLE ON A SLOPE:

Let the vehicle rests on a slope of inclination Θ to the horizontal. This alters the distribution of the weight between the front and back axle and gives rise to reaction which can have components along the perpendicular to the inclined plane as shown.



Now, resolving forces parallel and perpendicular to the slope respectively,

$$W \sin \theta = F_F + F_R$$

$$W \cos \theta = R_F + R_R$$

And

$$\sum M_F = 0$$

$$Wh \sin \theta + R_R b = W(b - l) \cos \theta$$

Therefore,

$$R_R = \frac{W}{b} [(b - l) \cos \theta - h \sin \theta]$$

And hence

$$R_F = W \cos \theta - R_R$$

$$\begin{aligned} &= W \cos \theta - W \cos \theta + \frac{Wl}{b} \cos \theta + \frac{Wh}{b} \sin \theta \\ &= \frac{W}{b} [l \cos \theta + h \sin \theta] \end{aligned}$$

If an angle Θ is gradually increased a situation arises when

- (a) Either the vehicle is about to overturn or
- (b) The vehicle is about to slide down the slope

The Instability due to case (a) happens at a point when R_F becomes zero and hence,

$$\frac{W}{b} [(b - l) \cos \theta - h \sin \theta]$$

Thus the limiting angle Θ_L for **overturning** is given by,

$$\tan \theta_L = \left(\frac{b - l}{h} \right)$$

This indicates that at the point of overturning, the lines of action of weights W passes through the contact point F . For the case (b) to arise, the limiting value of Θ is given by

$$W \sin \theta = F_F + F_R$$

The value of $(F_R + F_L)$ can be determined under following condition. Let the brakes be applied to prevent this situation.

Then two cases may arrive:

- (i) The brakes are not efficient enough to prevent the wheels from turning before they slide. In case the limiting value of Θ can be determined by the brake Torques available.

If T_F and T_R are the braking Torques at front and rear wheels respectively then,

$$T_F = F_F r$$

$$T_R = F_R r$$

Where r is the radius of the wheel

Now,

$$F_F + F_R = \frac{T_F + T_R}{r} = W \sin \theta_L$$

Or

$$\sin \theta_L = \frac{T_F + T_R}{Wr}$$

- (ii) The Brakes are Sufficiently powerful for the coefficient of adhesion μ to limit the sliding of the vehicle, then,

$$F_F + F_R = \mu(R_F + R_R) = \mu W \cos \theta$$

$$F_F + F_R = \mu W \sin \theta$$

But

Therefore, θ_L is given by,

$$W \sin \theta_L = \mu W \cos \theta_L$$

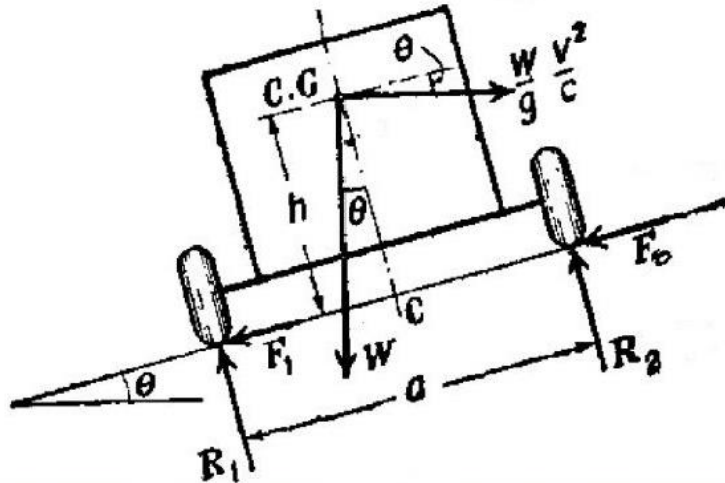
Or

$$\tan \theta_L = \mu$$

It should be noted up that when the vehicle is being driven up, the angle of overturning is in,

General, smaller than in the present case and also the condition of instability becomes different from those discussed above.

STABILITY OF A VEHICLE ON A BANKED TRACK:



The force, which are acting are shown in fig.

Where,

W = weight of vehicle

V = velocity of vehicle on banked track.

C = radius of curved path measured at CG of vehicle.

R_1 and R_2 = normal reaction at inner and outer wheels respectively,

F_1 and F_2 = normal reaction at inner and outer wheels respectively,

μ = coefficient of adhesion between tyre and road surface

a = length of wheel track,

θ = inclination of wheels axes to the horizontal,

$\frac{WV^2}{gC}$ = centrifugal force actign at CG

Then,

Equating sum of vertical force to zero:

$$\sum V = 0$$

$$R_o + R_I = W \cos \theta + \frac{WV^2}{gC} \sin \theta$$

Equating sum of Horizontal force to zero:

$$\sum H = 0$$

$$F_o + F_I = \frac{WV^2}{gC} \cos \theta - W \sin \theta$$

Equating sum of moment about 'o' equal to zero:

$$\sum M_o = 0$$

$$(R_o - R_I) \frac{a}{2} = \frac{wV^2}{gC} h \cos \theta - Wh \sin \theta$$

$$(R_o - R_I) = \frac{wV^2}{gC} \frac{2h}{a} \cos \theta - W \frac{2h}{a} \sin \theta$$

$$2R_o = \frac{WV^2}{gC} \left(\sin \theta + \frac{2h}{a} \cos \theta \right) + W \left(\cos \theta - \frac{2h}{a} \sin \theta \right)$$

And subtracting equation (iii) from equation (i)

$$2R_I = \frac{WV^2}{gC} \left(\sin \theta - \frac{2h}{a} \cos \theta \right) + W \left(\cos \theta + \frac{2h}{a} \sin \theta \right)$$

If the vehicle begins to slide outward along the banking at the limiting speed, V_s , then,

$$F_1 + F_0 = \mu(R_1 + R_0)$$

Substitution of equation (i) and (ii) in equation (iii) gives,

$$\mu W \cos \theta + \mu \frac{WV_s^2}{gC} \sin \theta = \frac{WV_s^2}{gC} \cos \theta - W \sin \theta$$

Or,

Limiting speed:

$$V_s^2 = gC \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta}$$

If V_o is the overturning speed of vehicle then putting $R_I = 0$ in equation (i) and (iii), gives

$$W \cos \theta + \frac{WV_o^2}{gC} \sin \theta = \frac{WV_o^2}{gC} \frac{2h}{a} \cos \theta - W \frac{2h}{a} \sin \theta$$

or,

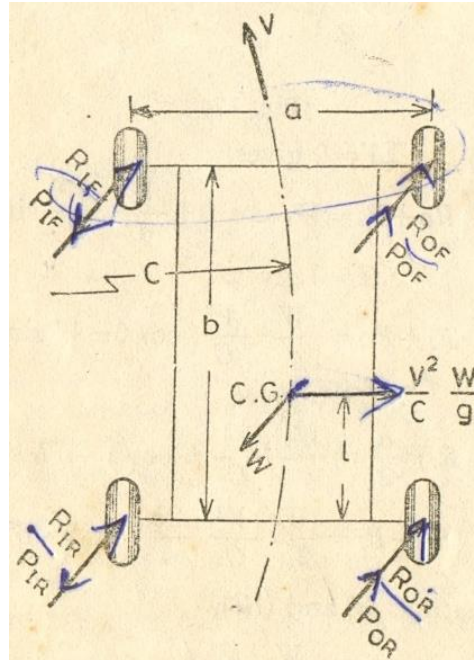
Overturning Speed:

$$V_o^2 = gC \frac{(a \sin \theta + 2h \cos \theta)}{2h \cos \theta - a \sin \theta}$$

CURVED TRACK

Stability of a Vehicle Taking a Turn

a) A four-wheeled vehicle



Let a vehicle take a turn to the left as shown in fig.

Where

- C = radius of curved path measured at C.G. of the vehicle, m
- r = wheel radius, m
- a = wheel track, m
- b = wheel base, m
- h = height of C.G. of the vehicle from ground, m
- l = distance of C.G. in front of rear axle axis, m
- V = linear speed of the vehicle on the road, m/sec
- W = weight of the vehicle, kgf.

I) Reaction at the wheels due to weight

Let R_{IF} and R_{IR} be the normal reactions at the inner front and inner rear wheels respectively and R_{OF} and R_{OR} be the normal reactions at the outer front and outer rear wheels respectively.

$$R_{IF} + R_{OF} = \frac{Wl}{b}$$

$$R_{IR} + R_{OR} = W \left(1 - \frac{l}{b} \right)$$

Since, $R_{IF} = R_{OF}$

and $R_{IR} = R_{OR}$

therefore, $R_{IF} = R_{OR} = \frac{Wl}{2b}$ kgf

$$R_{IR} = R_{OR} = \frac{W}{2} \left(1 - \frac{l}{b} \right) \text{kgf}$$

ii) Reaction at the wheels due to centrifugal force

The centrifugal force acts outward through C.G of the vehicle with magnitude = $\frac{W}{g} - \frac{V^2}{C}$. This produces a horizontal reaction that constitutes a couple $\left(= \frac{W}{g} \frac{V^2}{C} h \right)$ tending to overturn the vehicle. This couple is balanced by vertical reactions at the wheels which are downward at the two inner wheels and upward at the two outer wheels, as shown in above fig.

If P_{IF} and P_{OF} are the inner and outer normal reactions at front wheels and similarly P_{IR} and P_{OR} for the rear wheels, then

$$P_{IF} + P_{IR} = P_{OF} + P_{OR} = \frac{W}{g} \frac{V^2}{C} \frac{h}{a} \text{kgf}$$

Now $P_{IF} = P_{OF} = \frac{Wl}{2b} \frac{V^2}{gC} \frac{h}{a} \text{kgf}$

and $P_{IR} + P_{OR} = \frac{W}{2b} \left(1 - \frac{l}{b} \right) \frac{V^2}{gC} \frac{h}{a} \text{kgf}$

iii) Reaction at the wheels due to Gyroscopic effect

*If a body revolves about OX and if a couple, called Gyroscopic couple, is applied along OY, then the body tries to process about axis OZ. this is called Gyroscopic effect. The planes of spin, gyroscopic couple and precession are mutually perpendicular. The direction of precession is such that the axis of spin tends to place itself in line with the axis of the applied torque (i.e gyroscopic torque) and in the same sense.

The magnitude of applied gyroscopic couple = $I\omega\omega_p$

Where I = the polar moment of inertia of the body

ω = angular velocity of spin

and ω_p = angular velocity of precession

The reaction couple exerted on the body is equal in magnitude to the applied couple but opposite in direction.

When the vehicle takes a turn the gyroscopic* effect appears

- a) due to the precession of rotating wheels and other parts either rotating at the engine speed or the wheel speed but parallel to the plane of rotation of the wheel.
- b) due to the precession of engine parts and also others rotating either at engine or wheel speed but perpendicular to the plane of rotation of wheel. The axes of rotation in (a) and (b) are horizontal.

Let I_f = moment of inertia of the rotating parts of the engine (faster moving), kgf-m^2

I_s = moment of inertia of the slower rotating parts like wheels, kgf-m^2

ω_f, ω_s = angular velocity of the engine and the wheel respectively (velocity of spin), rad/sec

G = overall gear ration = ω_f / ω_s

N = r.p.m. of the engine = $(60/2\pi) \omega_f$.

Case (a) we have

$$\omega_s = \frac{V}{r}, \quad \omega_f = \pm \frac{GV}{r}$$

Then,
$$\sum I\omega = I_s \frac{V}{r} \pm I_f \frac{GV}{r} = \frac{V}{r} (I_s \pm GI_f)$$

-ve sign has been incorporated to take care of situations where the direction of ω_f is opposite to that of ω_s .

The angular velocity of precession,

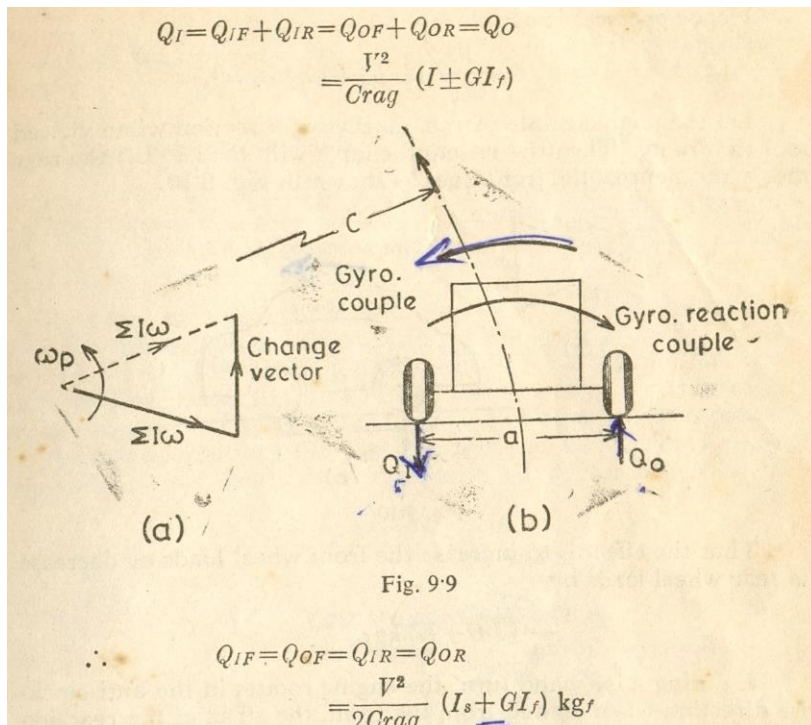
$$\omega_p = \frac{V}{C} \text{ rad/sec} = \frac{2\pi}{60} \frac{Nr}{GC} \text{ rad/sec}$$

Hence the gyroscopic couple

$$= \frac{V^2}{Crg} (I_s \pm GI_f) \text{ kgf-m}$$

The magnitude of vertical reaction due to this couple is same at the outer and inner wheels and its effect (reaction couple is to roll the car in an outward manner similarly to that caused by the centrifugal effect. The situation is shown in fig.

If Q_{IF} and Q_{OF} are respectively inner and outer vertical reactions at the front wheels and Q_{IR} and Q_{OR} are similarly at the rear wheels, then



Where Q_{IF} and Q_{IR} act downward and Q_{OF} and Q_{OR} act upward.

In this analysis, I_s represents the moment of inertia of all the wheels taken together; and the moment of inertia of the individual wheels have been assumed to equal one another. If the moment of inertia of front and rear wheels are different then while calculating individual reactions at the wheels, the respective moment of inertia should be used; but for the calculation of the total gyroscopic torque, individual moment of inertia can be added, giving $I_s = \sum I\omega$, where $I\omega =$ moment of inertia of the wheels. This is because the velocity of spin and the velocity of precession are the same for all the wheels.

Case (b) In this case also we can write

$$\sum I\omega = I_f \frac{Gv}{r} \pm I_s \frac{V}{r}$$

But here I_s stands for the moment of inertia of the slower moving parts rotating in a plane parallel to the plane of the rotation of the engine. The $-ve$ sign takes care of the rotation in a direction opposite to that of the engine.

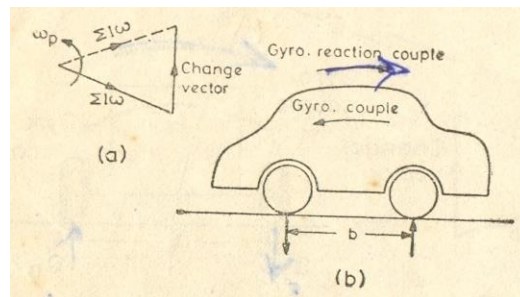
The angular velocity of precession

$$= \frac{V}{C} \text{ rad/sec}$$

Hence gyroscopic couple

$$= \frac{V^2 C r g}{2} [I_f G \pm I_s] \text{ kgf - m}$$

Let the engine rotate in the clockwise direction when viewed from the front. then the reaction couple will tend to lift the rear wheels and depress the front ones as shown in fig.



Thus the effect is to increase the front wheel loads or decrease the rear wheel loads by

$$\frac{V}{2Crag} [I_f G \pm I_s] \text{ kgf}$$

If during a left-hand turn, the engine rotates in the anti-clock-wise direction when viewed from the front, the effect of the reaction couple due to the engine gyroscopic couple will be just the reverse of the above situation, i.e the reaction couple will tend to lift the front wheels and depress the rear ones.

Now if the vehicle takes a right-hand turn, the above results for the inner and outer wheels will not be affected both in magnitude and direction but the gyroscopic torque due to the engine will be only affected in direction and will be opposite that in the above cases.

When the vehicle takes a turn, the present analysis reveals that precessional (that of wheels) and centrifugal effects are added together and act in a direction opposite to the static distribution of the load at the wheels tending to

overturn it. Considering the sum total of the reactions at the inner and outer wheels it can be stated that the parameters responsible for overturning are as follows.

1. If the vehicle taking a turn at high speed. i.e ω is high.
2. If the loaded vehicle is sufficiently high over the ground making h high.
3. If the vehicle is taking a sharp turn, i.e C is small.
4. If the vehicle is overloaded, i.e W is high.

In this article, the case of the vehicle taking a turn on a level is considered. If the turning of the vehicle on a banked track is considered, then an additional parameter θ , the inclination of the wheel axes to the horizontal will appear. Art 9.10 gives partial treatment of this situation where gyroscopic effects have not been considered.

3. CALCULATION OF TRACTIVE EFFORT AND REACTIONS FOR DIFFERENT DRIVES

Calculation of Maximum Acceleration, Maximum Tractive Effort and Reactions for Different Drives:

Front Wheel Drive:

The forces acting on the vehicle and giving rise to dynamic equilibrium are shown in fig.

If

- b = wheel base,
- h = height of C.G from the road surface,
- l = distance of C.G from rear axle,
- μ = coefficient of adhesion between the tyres and road surface
- R_F and R_R = total normal reactions at front and rear wheels respectively,
- W = weight of the car.

Then the maximum tractive effort, $F_F = \mu R_F$ produces maximum forward acceleration, f and $(W/g)f$ is the inertia force opposite to acceleration, f .

Hence $\sum V = 0$ gives,

$$W = R_F + R_R$$

$\sum H = 0$ gives,

$$F_F = \mu R_F = \frac{W}{g} f$$

And

$\sum MR = 0$ gives,

$$R_F b + \frac{W}{g} f h = W l$$

Substituting the value of R_F , gives

$$\begin{aligned} \frac{W}{g} f \frac{1}{\mu} b + \frac{W}{g} f h &= W l \\ \frac{f}{g} \left(\frac{b}{\mu} + h \right) &= l \end{aligned}$$

Therefore,

$$\frac{f}{g} = \frac{\mu l}{b + \mu h}$$

Solving R_F and R_R become

$$\begin{aligned} R_F &= \frac{l}{b + \mu h} W \\ R_R &= \frac{b - l + \mu h}{b + \mu h} W \end{aligned}$$

Rear Wheel Drive:

Here the tractive effort acts only on rear wheels. Hence eliminating FF and applying FR to the rear wheels in fig., maximum tractive effort becomes,

$$F_R = \mu R_R$$

Now, $\Sigma V=0$ gives, $W=RR+RF$
 $\Sigma H=0$ gives, $FR = \mu RR=(W/g)f$

And

$$\Sigma MF=0 \text{ gives,}$$

$$RR b = W(b-l) + (W/g)fh$$

Substituting the value of RR gives,

$$\frac{Wf}{g\mu}b = W(b-l) + \frac{W}{g}fh$$

Or,

$$\frac{f}{g}\left(\frac{b}{\mu} - h\right) = b - l.$$

Therefore,

$$\frac{f}{g} = \frac{\mu(b-l)}{b-\mu h}$$

Solving of RR and RF gives,

$$RR = \frac{b-l}{b-\mu h} W \quad \text{and,}$$

$$RF = \frac{l-\mu h}{b-\mu h} W$$

FOUR WHEEL DRIVE

This may be with or without third differential

(1) Without third differential:-in this case both FF and FR come into play. Assuming that limiting friction occurs at all the four wheels simultaneously, the maximum tractive effect,

$$F=FR+FF = \mu RR + \mu RF$$

$$\Sigma V=0,$$

Gives

$$W=RF+RR$$

$$\Sigma H=0,$$

Gives

$$(W/g) f = \mu RR + \mu RF = \mu(RR+RF) = \mu W$$

Hence,

$$(f/g) = \mu$$

(2) With third differential:-The torque at the front and rear wheels becomes equal with the application of third differential. Slip occurs at the wheels where the normal reaction is smaller and thus limits the tractive effect. In case, the load distribution to the front and rear wheels is equal, the slip has to occur first at the front wheels because the static normal reaction at front wheels is reduced due to inertia effect.

Thus,

$$\Sigma V=0$$

Gives

$$W=RR+RF$$

$$\Sigma H=0$$

Gives,

$$(W/g) f = \mu RR + \mu RF$$

And $\mu_s RR = \mu RF$ due to application of third differential, where μ_s is the critical working coefficient of friction being less than μ , the limiting value.

Assuming slip to occur at front wheels first, $R_F < R_R$, then

$$2\mu R_F = (W/g)f$$

$\Sigma MR = 0$ gives,

$$R_F b + (W/g) f h = Wl$$

Substituting the value of

PROBLEMS 1:

A car weighing 2175 kgf has a static weight distribution on the axles of 50 : 50. The wheel base is 3 m and the height of centre of gravity above ground is 0.55 m. if the coefficient of friction on the highway is 0.6, calculate the advantage of having rear wheel drive rather than front wheel drive as far as gradiability is concerned, if engine power is not a limitation.

Solution: C.G. bisects the wheel base as distribution of static weight on the axle is 50 : 50.

$$l = \frac{b}{2} = 1.5 \text{ m.}$$

Hence,

Rear Wheel Drive

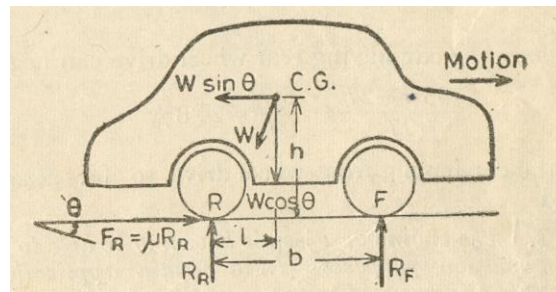
Considering the car moving at constant speed up a grade of angle θ , the forces giving equilibrium are as shown in fig.

Then, we have

$$R_R + R_F = W \cos \theta$$

and

$$F_R = \mu R_R = W \sin \theta$$



$$\tan \theta = \frac{\mu R_R}{R_R + R_F}$$

By division,

Taking moment about G.G

$$R_F \cdot 1.5 + \mu R_R \cdot 0.55 = R_R \cdot 1.5$$

$$\text{or } R_F = R_R \left(1 - \frac{\mu \cdot 0.55}{1.5} \right)$$

$$= R_R \left(1 - \frac{0.6 \times 0.55}{1.5} \right)$$

$$= R_R (1 - 0.2) = 0.78 R_R$$

$$\text{Therefore, } \tan \theta = \frac{0.6 R_R}{R_R + 0.78 R_R} = \frac{0.6}{1.78} = 0.337$$

$$\text{Percentage grade} = -\tan \theta \times 100 = 33.7\%$$

Hence in case of rear wheel drive the maximum grade which the car can negotiate is 33.7%.

Front Wheel Drive

In this case also, similarly as last one, we have

$$R_R + R_F = W \cos \theta$$

$$R_F = \mu R_F = W \sin \theta.$$

Hence,
$$\tan \theta = \frac{\mu R_F}{R_R + R_F}$$

$$\sum M_{c.G.} = 0 \text{ gives,}$$

$$R_F 1.5 + \mu R_F 0.55 = R_R 1.5$$

or,
$$R_R = R_F \left(1 + \frac{0.6 \times 0.55}{1.5} \right) = 1.22 R_F$$

Therefore,
$$\tan \theta = \frac{0.6 R_F}{R_F + 1.22 R_F} = \frac{0.6}{2.22}$$

$$= 0.27 \text{ or } 27\%$$

Therefore, the car having rear wheel drive can negotiate

$$\frac{33.7 - 27}{27} = \frac{6.7}{27} = 24.8\%$$

more than the car having front wheel drive, so far as gradiability is concerned

PROBLEMS 2:

A motor car with wheel base 275 cm with a centre of gravity 85 cm above the ground and 115 cm behind the front axle has a coefficient of adhesion 0.6 between the tyre and the ground. Calculate the maximum possible acceleration when the vehicle, is.

- a) driven on four wheels,
- b) driven on the front wheels only
- c) driven on the rear wheels only

Solution :

$$b = 2.75 \text{ m}$$

$$l = 3.75 - 1.15 = 3.6 \text{ m}$$

$$h = 0.85 \text{ m}$$

$$\mu = 0.6$$

a) **Four wheel drive**

Since it has not been mentioned about the use of third differential we shall take up both the cases.

i) Without third differential ;

$$f = \mu g = 0.6 \times 9.81 = 5.886 \text{ m/sec}^2.$$

ii) With third differential ; assuming that the slip occurs first at the front wheels, then

$$f = \frac{2\mu l g}{b+2\mu h} = \frac{2 \times 0.6 \times 1.6 \times 9.81}{2.75 + 2 \times 0.6 \times 0.85}$$

$$= \frac{18.82}{2.75+1.02} = \frac{18.82}{3.77} = 5 \text{ m/sec}^2$$

check $R_F = \frac{l}{b+2\mu h} W = \frac{1.6}{3.77} W = 0.425 W.$

i.e $R_F < R_R (=0.575 W)$, hence our assumption is correct.

b) Front wheel drive

$$f = \frac{\mu l g}{b+\mu h} = \frac{0.6 \times 1.6 \times 9.81}{2.75 + 0.6 \times 0.85}$$

$$= \frac{9.41}{2.75+0.51} = \frac{9.41}{3.26}$$

$$= 2.89 \text{ m/sec}^2$$

c) Rear wheel drive

$$f = \frac{\mu(b-l) g}{b-\mu h} = \frac{0.6(2.75 - 1.6)9.81}{2.75 - 0.6 \times 0.85}$$

$$= \frac{0.6 \times 1.15 \times 9.81}{2.75-0.51} = \frac{6.765}{2.24}$$

$$= 3.02 \text{ m/sec}^2$$

PROBLEMS 3:

A vehicle of total weight 5000 kg, is held at rest on a slope of 10° . It has a wheel base of 225 cm and its centre of gravity is 100 cm in front of the rear axle and 150 cm above the ground level.

Find:

- What are the normal reactions at the wheels?
- Assuming that sliding does not occur first, what will be the angle of slope so that the vehicle will overturn?
- Assuming all the wheels are to be braked, what will be the angle of the slope so that the vehicle will begin to slide if the co-efficient of adhesion between the tyre and the ground is 0.35?

Solution:

$$\begin{aligned} \text{a) } R_F + R_R &= W \cos \theta \\ &= 5000 \times \cos 10 = 5000 \times 0.9848 \\ &= 4924.0 \text{ kg}^f. \end{aligned}$$

$$\begin{aligned} R_R &= \frac{W}{b} [(b-l) \cos \theta - h \sin \theta] \\ &= \frac{5000}{2.25} [(2.25 - 1.0) \times 0.9848 - 1.5 \times 0.1736] \end{aligned}$$

$$= \frac{5000}{2.25} [1.25 \times 0.9848 - 0.2604]$$

$$= (5000 \times 0.9706) / 2.25 = 21.59 \text{ kgf.}$$

Therefore, $R_F = 4924 - 2159 = 2765 \text{ kgf.}$

b) $\tan \theta_L = (b - l) / h$
 $= (2.25 - 1.0) / 1.5 = 0.8337$

Hence $\theta_L = 39^\circ 49'.$

c) $\tan \theta_L = \mu = 0.35$

Hence $\theta_L = 19^\circ 17'.$
