Heat Exchangers

In any process industry, we need to transfer heat for different operations (like cooling, heating, vaporizing, or condensing) to or from various fluid streams in various equipment like condensers, water heaters, re-boilers, air heating or cooling devices etc., where heat exchanges between the two fluids. In a chemical process industry, the heat exchanger is frequently used for such applications. A heat exchanger is a device where two fluids streams come into thermal contact in order to transfer the heat from hot fluid to cold fluid stream.

In this chapter, we will discuss about the technical analysis of the heat exchangers along with the method for predicting heat exchanger performance and operational parameters. Moreover the discussion on heat exchanger size will also be discussed. However, we will not discuss the economics (though discuss the heat exchanger size) of the heat exchanger. We will consider that the heat transfer will primarily be taken by conduction and convection only. We will describe the commonly used heat exchangers and their important characteristics.

In general heat exchangers may be categorized into two general classes depending on the relative orientation of the flow direction of the two fluid streams. If the two streams cross one another in space, usually at right angles, the heat exchangers are known as cross flow heat exchanger as shown in the fig.1(a).

In the second class of heat exchanger the two streams move in parallel direction in space. The usual shell and tube heat exchanger or concentric pipe exchanger or double tube exchanger is the most frequently used exchanger in the class. Two situations may arise when the fluid flow in the parallel direction, one in which the fluids flow in same direction and the other in which the fluids flow in opposites direction. "Parallel –flow" or "Co-current flow" is used when the flow is in same direction and counter current is used when the fluid flow is in the opposite direction.

Before understanding the principle of heat exchanger we would first understand it from the point of construction.

Elements of shell and tube heat exchanger

We will discuss shell and tube heat exchanger as they or the most commonly used heat exchangers in the chemical process industries. Schematic of a typical shell and tube heat exchanger is shown in fig.2.



(c)

Fig.1: Orientation of fluid stream in heat exchanger (a) cross flow (b) counter current flow (c) parallel flow

The heat exchanger as shown in fig.2 consists of a bundle of tubes properly secured at either ends in tube sheets. The tube sheets are drilled plates into which the tubes are fixed up using different technique to have leak proof joints. The entire tube bundles shown in the fig.3 is placed inside a closed shell, which seals around the tube sheet periphery to form the two immiscible zones for hot and cold fluids are shown in fig.4.



Fig.2: A schematic of (a) one-shell pass, one tube pass heat exchanger; (b) parallel flow; and (c) counter flow



Fig. 8.3 Tube bundle fitted in two sheets



Fig.4: Tube bundle inside a shell

One fluid flows through the tubes while the other fluid flows around the outside of the tubes, it is the space between the tube sheets and enclosed by the outer shell.

For a thorough distribution of the shell side fluid, baffles are placed normal to the tube bundle. This baffle creates turbulence in the shell side fluid and enhances the transfer coefficients for the shell side flow.

Fig.2 shows the simplified diagram of a shell and tube heat exchanger, showing a few of the important components. Infact, the present heat exchanger used in the process industry are quite complex and have improved design such as factors for thermal expansion stresses, tube fouling due to contaminated fluids, ease of assembly and disassembly, size, weight, etc.

The heat exchanger shows in fig.2 is having one shell and one tube pass since both the shell and tube side fluid make a single traverse through the heat exchanger. Thus, this type of shell-and-tube heat exchangers is designated as 1-1 exchanger. If we desire to pass the tube fluid twice, then it is designated as 1-2 exchangers. Similarly, if there are 2 shell pass and 4 tube pass, the designation will be 2-4 exchanger. The number of pass in tube side is done by the pass partition plate. A pass particular plate or pass divider as shown in fig.5. The shell side pass can be creator by a flat plate as shown in fig.6.



Fig. 8.5: 1-2 exchanger showing pass partition plate



Fig 6: 2-4 exchanger showing shell and tube passes

It can be understood that for a given number of tubes; the area available for flow of the tube side fluid is inversely proportional to the number of passes. Thus, on increasing the pass the area reduces and as a result the velocity of fluid in the tube increases and henceforth the Reynolds number increases. It would result in increased heat transfer coefficient but at the expense of high pressure drop. Generally, even numbers of tube passes are used for the multi-pass heat exchangers.

Figure 2 shows some baffles. These baffles (or shell-side baffles) are a metal plate usually in the form of the segment of a circular disc having holes to accommodate tubes. Shell-side baffles have two functions. The first is to create turbulence in the shell side fluid by changing the flow pattern parallel or cross flow to the tube bundles and thus increases the shell side heat transfer coefficient. The second major function of these baffles is to support the tube all along its length otherwise the tube may bend. Moreover, these baffles may have horizontal or vertical cuts (segmental baffle) as shown in fig.7.





Fig.7: Baffles; (a) horizontal cut baffles; (b) Vertical cut baffles; (c, d and e)the shaded region show the baffle area

The cut portion of the baffle, which is called baffle window, provides the area for flow of the shell fluid. The baffle window area ranges from 15% to 50%. At 25% cut segmental baffle means that the area of the cut-out portion is 25% of the area of the baffle. The spacing between the baffles is an important aspect. A larger baffles spacing reduces the shell side pressure drop but at the same time decreases the turbulence and heat transfer coefficient. Smaller baffle spacing increases the turbulence and heat transfer coefficient. However, the pressure drop may increase significantly, thus the advantage attained due to the higher heat transfer coefficient may be nullified. Therefore baffle spacing is selected considering the allowable shell side pressure drop and the heat transfer coefficient desired. A rule of thumb is that the minimum spacing of segmental baffles is one by fifth of the shell diameter or 5 cm, whichever is larger.

Thermal design of heat exchangers

The mechanical design is done by the mechanical engineers on the inputs of chemical engineers and using the codes. The most widely used code in Tubular Exchanger Manufactures Associations (TEMA). This USA code along with ASME selection VIII (unfired pressure vessel) code is used together for the mechanical design of the heat exchanger. The Indian code for the heat exchanger design IS 4503.

Here we would discuss about the process design (or thermal design) leading to the sizing of the heat exchanger. Before understanding design steps, it is necessary to understand the following for the heat exchanger.

Overall heat transfer coefficient

As understood by the previous discussion that generally heat exchangers are tubular in nature (Note: we are not discussing about plate type heat exchangers). Thus we can easily find out the overall heat transfer coefficient based on our previous knowledge. Figure.8 shows a simplest form (double pipe heat exchanger) of tubular heat exchanger, where fluid A is being heated by fluid B in a co-current flow pattern. The inside and outside radii of the inner tube is represented r_i and r_0 . The length of the exchanger for heat transfer is considered as L for section 1 to 2.



Fig.8: (a) Schematic of a double pipe heat exchanger (b) thermal resistance network for overall heat transfer

Thus the rate of heat transfer from the hot fluid to the cold fluid will be represented by eq.1,

$$q = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln(r_0/r_i)}{2\pi kL} + \frac{1}{h_0 A_0}}$$
(1)

The overall heat transfer coefficient;

Based on inside area of the inner pipe (eq.2)

$$U_{i} = \frac{1}{\frac{1}{h_{i}} + \frac{A_{i}\ln(r_{o}/r_{i})}{2\pi kL} + \frac{A_{i}}{h_{o}A_{o}}}$$
(2)

Based on outer side area of the outer pipe (eq.3)

$$U_{o} = \frac{1}{\frac{A_{o}}{h_{i}A_{i}} + \frac{A_{o}\ln(r_{o}/r_{i})}{2\pi kL} + \frac{1}{h_{o}}}$$
(3)

Fouling factor or dirt factor

Over a time period of heat exchanger operation the surface of the heat exchanger may be coated by the various deposits present in the flow system. Moreover, the surfaces may become corroded or eroded over the time. Therefore, the thickness of the surface may get changed due to these deposits. These deposits are known as scale. These scales provide another resistance and usually decrease the performance of the heat exchangers. The overall effect is usually represented by dirt factor or fouling factor, or fouling resistance, R_f (Table.1) which must have included all the resistances along with the resistances due to scales for the calculation of overall heat transfer coefficient.

The fouling factor must be determined experimentally using eq.4,

$$R_{f} = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \tag{4}$$

Thus to determine the R_{f} , it is very important to know U_{clean} for the new heat exchanger. The U_{clean} must be kept securely to obtain the R_{f} , at any time of the exchanger's life.

	Fouling factor (or resistance) $\frac{hm^2 \circ C}{k cal} \times 10^3$
	Liquid
Fuel oil	1.024
Refrigerant liquids	0.102
Mono-and di-ethanolamine solution	0.409
Gasoline , naphtha and kerosene	0.205
Light gas oil	0.409
Heavy gas oil	0.615
Gases	and Vapour
Solvent vapour	0.205
Air	0.102-0.205
Flue gases	0.205-0.615
Steam (Saturated, oil free)	0.102-0.307
	Water
River water (treated, velocity > 0.6 m/s)	0.205-0.409
Treated boiler feed water	0.102-0.205
Process water	0.205-0.409

Table-1 Fouling factor of a few of the industrial fluids

Temperature profiles in heat exchangers

Fig.9 shows the temperature profile along the length of a 1-2 exchangers and 2-4 exchangers.





The nomenclature used in the fig.9 is described below

- T_{ha} : Inlet temperature of hot fluid
- T_{hb} : Outlet temperature of hot fluid
- T_{ca} : Inlet temperature of cold fluid
- T_{cb} : Outlet temperature of cold fluid
- T_{ci} : Intermediate temperature of cold fluid

In the above arrangement it is assumed that the hot fluid is flowing in the shell side and cold fluid is flowing in the tube side of the exchangers. The fig.9 (a) shows the 1-2 exchangers in which the hot fluid enter into the exchanger from the left side and exits from the right side. The

cold fluid enters concurrently that is from the left side to the tube of the exchangers and goes up to right end of the exchangers and returns back to make two tube pass, and exits from the left end of the exchangers. The temperature profile all along the length of the exchanger is shown in the corresponding temperature length profile. Figure.9 (b) shows the flow direction and corresponding temperature length profile for 2-4 exchangers. The shell side fluid two passes and the tube side fluid has 4-passes in the exchangers.

It can be easily understood that whenever the number of passes is more than one, the flow cannot be truly co-current or counter current. Thus it will be a mix of co-current and counter current flows in any multi pass heat exchangers.

Though the temperature profile of the hot and cold streams can be easily predictable for single pass heat exchangers but for the complex flow modes, the prediction of temperature distribution will be difficult as shown in fig.9. As can be seen when 1-2 exchangers was (fig.9 (a)) used in co-current mode, the temperature profile was given in the figure. However, if the fluid streams enter in counter current mode a temperature cross may occur sometimes. Temperature cross is described as the positive temperature difference between the cold and the hot fluid, when these fluids leave the exchangers. In that case the cold fluid will attain the maximum temperature inside the exchanger instead of at the exit (fig.10).





At this temperature cross, the cold fluid temperature reaches the maximum at a point inside the exchanger and not at its exits. This temperature cross point also coincides with the point of intersection of the temperature profile of the hot fluid and the co-current zone of the cold fluid. The difference $(T_{c2} - T_{h2})$ is called the temperature cross of the exchanger. However, if the temperature cross does not appear then the $(T_{c2} - T_{h2})$ is called the approach. Moreover, on careful

evaluation it can be seen that for the multi shell side pass a significant length of the exchanger have cross flow pattern in the tube flow when the shell side fluid is migrating from one shell pass to another shell pass. Although the parallel flow or counter flow are quite similar, the parallel flow and counter flow heat exchangers differ greatly in the manner in which the fluid temperatures vary as the fluid pass through. The difference can be understood in the figure 11.

The fig.11 shows an important parameter, mc_p , the product of mass flow rate (m) and the specific heat, c_p , of the fluids. The product mc_p is called the rate of heat capacity. The overall energy balance of the heat exchanger gives the total heat transfer between the fluids, q, expressed by eq.5,

$$q = \dot{m}_{c}c_{pc}(T_{co} - T_{ci}) = \dot{m}_{h}c_{ph}(T_{hi} - T_{ho})$$

$$\Rightarrow \frac{\dot{m}_{c}c_{pc}}{\dot{m}_{h}c_{ph}} = \frac{(T_{hi} - T_{ho})}{(T_{co} - T_{ci})}$$
(5)

The fig.11 shows the relative variation of the two fluid temperatures through the heat exchanger,

 $\dot{m_c}c_{pc}$ $\dot{m_h}c_{ph}$. which is influenced by whether is greater or less than . In particular, for counter flow, examination of the sketches in fig.11 shows that limiting condition

 $\dot{m}_c c_{pc}$ $\dot{m}_h c_{ph}$. for maximum heat transfer is determine by whether is greater or less than .

 $\dot{m}_c c_{pc} = \dot{m}_h c_{ph}$. When, > the maximum possible heat transfer is determined by the fact that the hot

 $\dot{m_c}c_{pc}$ $\dot{m_h}c_{ph}$. fluid can be cooled to the temperature of the cold fluid inlet. Thus, for >

$$T_{ho} \rightarrow T_{ci}$$

$$q_{max} = \dot{m}_h c_{ph} (T_{hi} - T_{ho})$$

$$= \dot{m}_h c_{ph} (T_{hi} - T_{ci})$$





 \dot{mc}_p

Fig.11: Temperature profiles of (a) parallel flow, and (b) counter flow, for different inequalities

For the other case when the limit is determined as the cold fluid is heated to the inlet temperature of the hot fluid:

For,

$$\dot{m_c}c_{pc} < \dot{m_h}c_{ph} : T_{co} \rightarrow T_{ho}$$
 $q_{max} = \dot{m_c}c_{pc}(T_{co} - T_{ci})$
 $= \dot{m_c}c_{pc}(T_{ho} - T_{hi})$

Thus for the counter flow exchanger, the above two set equations show that the maximum possible heat exchanger is determined in terms of the inlet parameters. The maximum possible heat exchange may be determined (eq.6) by the fluid stream having low heat capacity rate;

$$q_{max,ccf} = \left(\dot{m}c_p\right)_{min} \left(T_{hi} - T_{ci}\right) \tag{6}$$

 $(\dot{m}c_p)_{min}$

The subscript *ccf* denotes counter current flow. The is for the fluid having lower value

$$(\dot{m}c_p).$$

In case of parallel flow, regardless of the relative sizes of the two stream the limiting heat transfer condition is determine by the fact that the two fluid streams approach the same outlet temperature. Thus,

 $T_{ho} \rightarrow T_{co}$ condition can be found out by the weighted average of the inlet stream.

$$T_{ho} \rightarrow T_{co} \rightarrow \frac{\dot{m_c}c_{pc}T_{ci} + \dot{m_h}c_{ph}T_{hi}}{\dot{m_c}c_{pc} + \dot{m_h}c_{ph}}$$

Thus, the maximum possible heat transfer may be,

$$q_{max} = \dot{m_c} c_{pc} (T_{co} - T_{ci})$$
$$= \frac{1}{\frac{1}{\dot{m_c} c_{pc}} + \frac{1}{\dot{m_h} c_{ph}}} (T_{hi} - T_{ci})$$

or,

$$q_{max,pf} = \frac{(mc_p)_{min}(T_{hi} - T_{ci})}{1 + [(mc_p)_{min}/(mc_p)_{max}]}$$
(7)

The subscript *pf* represents parallel flow (co-current). From the above discussion and q_{max} equations (6 and 7) it can be calculated for a given inlet conditions the counter current flow arrangement always has a better potential for heat transfer as compared to parallel flow arrangement.

Why multi-pass exchangers?

The simplest type of heat exchangers is double pipe heat exchangers, which is inadequate for flow rates that cannot readily be handled in a few tubes. If several double pipes are used in parallel, the metal weight required for the outer tubes becomes so large that the shell and tube construction, such as 1-1 exchanger will be helpful. In that one shell serves for many tubes, is economical. The heat transfer coefficient of tube side and shell side fluid is very important and the individual heat transfer coefficients must be high enough to attain high overall heat transfer coefficient. As the shell would be quite large as compared to the tubes, the velocity and the turbulence of the shell side fluid is important.

In contrast, the 1-1 exchanger has limitations also. When the tube side flow is divided evenly among all the tubes, the velocity may be quite low, resulting in low heat transfer coefficient. There it may be required to increase the area to have the desired heat exchange for this low heat transfer coefficient. The area may be increased by increasing the length of the tube. However, the tube length requirement may be impractical for a given situation. Thus the number of tubes should be increased without increased the tube length. The increased number of tubes would also provide the increased velocity in the shell side resulting in the higher heat transfer coefficient.

Therefore, multi-pass construction is needed, which would permit to use the practical and standard tube lengths. However, the disadvantages are that,

1. The construction of the exchangers become complex.

- 2. Parallel flow cannot be avoided.
- 3. Additional friction losses may occur.

It should be noted that generally even number of tube passes are used in multi pass exchanger.

LMTD correction factor

In the earlier chapter, we have seen for co-current or counter current flow system. The average driving force for heat transfer was defined by log mean temperature difference (LMTD). Thus the LMTD can be used for 1-1 exchangers for co-current and counter current. However, for multi pass exchangers (1-2, 2-4, etc.) the fluids are not always in co-current or counter current flow. The deviation for co-current or counter current flow causes a change in the average driving force. Therefore, in order to use true heat transfer driving force, a correction factor is required into the LMTD. Thus, the heat transfer rate can be written as (eq.8),

$$q = U_d A(F_T \Delta T_m) \tag{8}$$

where,

 U_d = overall heat transfer coefficient including fouling/dirt

A = heat transfer area

 $F_T \Delta T_m$ = true average temperature difference.

 $F_T = LMTD$ correction factor

It is to be noted that the following assumption have been considered for developing LMTD,

1. The overall heat transfer coefficient is constant throughout the exchanger

2. In case any fluid undergoes for phase change (e.g., in condenser), the phase change occurs throughout the heat exchanger and the constant fluid temperature prevails throughout the exchanger.

3. The specific heat and mass flow rate and hence the heat capacity rate, of each fluid is constant.

4. No heat is lost in to the surroundings.

5. There is no conduction in the direction of flow neither in the fluids nor in the tube or shell walls.

6. Each of the fluids may be characterized by a single temperature, at any cross section in the heat exchanger that is ideal transverse mixing in each fluid is presumed.

 F_{T} , the LMTD correction factor can be directly obtained from available charts in the literature. These charts were prepared from the results obtained theoretically by solving the temperature distribution in multi-pass heat exchangers.

Figures 12 and 13 show the two generally used heat exchangers and their corresponding plots for finding F_{T} . It may be noted that the given figures have the representative plots and any standard book on heat transfer may be consulted for the accurate results.



Fig.12: F_T plot for 1-2 exchanger; t: cold fluid in the tube; T: hot fluid in the shell; 1: inlet; 2: outlet



exchanger; t: cold fluid in the tube; T: hot fluid in the shell; 1: inlet; 2: outlet

It should be noted that in case of condensation or evaporation the correction factor becomes unity (F_T =1). While designing a heat exchanger, the rule of thumb is that the F_T should not be less than 0.8.

Individual heat transfer coefficient

we have seen that the overall heat transfer coefficient can be calculated provide the parameters are known including individual heat transfer coefficients. In this, section we will discuss how to find out the individual heat transfer coefficient, which is basically based on the well-established correlations and discussed earlier also.

The heat transfer coefficient (h_i) for the tube side fluid in a heat exchanger can be calculated either by Sieder-Tate equation or by Colburn equation discussed in earlier chapter.

However, the shell side heat transfer coefficient (h_o) cannot be so easily calculated because of the parallel, counter as well as cross flow patterns of the fluid. Moreover, the fluid mass velocity as well as cross sectional area of the fluid streams vary as the fluid crosses the tube bundle. The leakages between baffles and shell, baffle and tubes, short circuit some of the shell fluid thus reduces the effectiveness of the exchanger.

Generally, modified Donohue equation (eq.9) (suggested by D.Q. Kern) is used to predict the h_o,

$$\frac{h_0 D_H}{k_0} = 0.36 \left(\frac{D_H G_s}{\mu}\right)^{0.55} \left(\frac{\mu}{k_0}\right)^{0.33} \left(\frac{\mu}{\mu_w}\right)^{0.14}$$
(9)

where,

 h_0 = shell side heat transfer coefficient

 D_h = hydraulic diameter of the shell side

 k_0 = thermal conductivity of the shell side fluid

 G_s = mass flow rate of the shell side

The D_h and G_s can be easily calculated if the geometry of the tube arrangement in the shell is known. The tubes may be generally arranged as a square or triangular pitch, as shown in figure 14.



Fig.14: Tube arrangement in the shell (a) triangular pitch (b) square pitch

$$\frac{4\left(p^2-\pi\frac{d_0^2}{4}\right)}{\pi d_0}$$

The hydraulic diameter (D_h) for tubes on square pitch

$$\frac{4\left[(0.5p)(0.86p) - \pi \frac{d_0^2}{s}\right]}{\pi d_0/2}$$

 D_h For 60° triangular pitch= where,

 d_0 = outer diameter of tube p = tube pitch

$$G_s = \frac{\dot{v}_s}{a_s}$$

where,

 \dot{v}_{s} = flow rate of shell fluid a_{s} = shell side flow area

Shell side flow area can be calculated using baffle information number of tubes in the shell and tube arrangement. If 25% cut baffles are used, that means the shell side flow will be from this 25% area. However we have to reduce the area of the pipes which are accumulated in this opening. So depending upon the information we may determine the shell side fluid flow area. It may also be found out by the following way,

$$a_s = \frac{CBD_s}{p}$$

where,

C = tube clearance B = baffle spacing $D_s =$ inside diameter of shell p = pitch of the tube **Pressure drop in the heat exchanger**

Pressure drop calculation is an important task in heat exchanger design. The pressure drops in the tube side as well as shell side are very important and quite a few co-relations are available in the literature. One such co-relation is given below in the subsequent subsection.

Correlation for tube side pressure drop (eq. 10)

$$\Delta P_{t,f} = \frac{f G_f Ln}{2g\rho_t d_t (\mu/\mu_w)^m} \tag{10}$$

where,

 $\Delta P_{t,f}$ = total pressure drop in the bundle of tube f = friction factor (can be found out from Moody's chart) G_t = mass velocity of the fluid in the tube L = tube length n = no of tube passes g = gravitational acceleration ρ_t = density of the tube fluid d_i = inside diameter of the tube m=0.14 for Re > 2100 0.25 for Re < 2100

The above correlation is for the pressure drop in the tubes owing to the frictional losses. However in case of multi pass flow direction of the flow in the tube changes when flow is from 1-pass to another pass and the pressure losses due to the change in direction is called return-loss. The return-loss ($\Delta P_{t,r}$) is given by eq.11,

$$\Delta p_{t,r} = 4(\frac{v_t^2}{2g})\rho_t \times n \tag{11}$$

n = no of tube pass

 v_t = velocity of the tube fluid

 ρ_t = density of the tube fluid

Therefore, the total tube side pressure drop will be,

 $\Delta p_t = \Delta P_{t,f} + \Delta P_{t,r}$

Correlation for shell side pressure drop

The following correlation (eq.12) may be used for an unbaffled shell,

$$\Delta p_{s} = \frac{f_{s}G_{s}^{2}Ln_{s}}{2g\rho_{s}D_{h}(\mu/\mu_{w})^{0.14}}$$
(12)

The above equation can be modified to the following form (eq.13) for a baffled shell,

$$\Delta p_{s} = \frac{f_{s}G_{s}^{2}D_{si}(n_{b}+1)}{2g\rho_{s}D_{h}(\mu/\mu_{w})^{0.14}}$$
(13)

where

$$L =$$
shell length

 $n_s = no of shell pass$

 $n_b = no of baffles$

 ρ_s = shell side fluid density

 G_s = shell side mass velocity

 D_h = hydraulic diameter of the shell

 D_{si} = inside diameter of shell

 f_s = shell side friction factor

The hydraulic diameter (D_h) for the shell can be calculated by the following equation (eq.14),

$$D_h = \frac{4 \left[\pi D_s^2 / 4 - \pi d_o^2 n_t / 4 \right]}{\pi d_o n_t + \pi D_s} \tag{14}$$

where,

 n_t = number of tubes in the shell

 d_o = outer diameter of the tube

The friction factor (fs) can be obtained by the Moody's chart for the corresponding Reynolds

$$\left(R_{\varepsilon}=\frac{D_{\lambda}G_{s}}{\mu}\right)$$

number

Heat transfer effectiveness and number of transfer units (NTU)

The LMTD is required to be calculated for the evaluation of heat exchanger performance.

However, the LMTD cannot be directly calculated unless all the four terminal temperatures ($T_{c,i}$,

 $T_{c,o}$, $T_{h,i}$, $T_{h,o}$) of both the fluids are known.

Sometimes the estimation of the exchanger performance (q) is required to be calculated on the given inlet conditions, and the outlet temperature are not known until q is determined. Thus the problem depends on the iterative calculations. This type of problem may be taken care of using performance equivalent in terms of heating effectiveness parameter (η) , which is defined as the ratio of the actual heat transfer to the maximum possible heat transfer. Thus,

$$\eta = \frac{q}{q_{\text{max}}} \tag{15}$$

For an infinite transfer area the most heat would be transferred in counter-current flow and the q_{max} will be dependent on the lower heat capacity fluid as such,

$$\begin{split} q_{max} &= \dot{m_c} c_{pc} (T_{hi} - T_{ci}) \text{for } \dot{m_c} c_{pc} < \dot{m_h} c_{ph} \\ q_{max} &= \dot{m_h} c_{ph} (T_{hi} - T_{ci}) \text{for } \dot{m_c} c_{pc} > \dot{m_h} c_{ph} \end{split}$$

The actual heat transfer

$$q = \dot{m}_c c_{pc} (T_{co} - T_{ci}) = \dot{m}_h c_{ph} (T_{ho} - T_{hi})$$
$$\eta = \frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})} \text{for } \dot{m}_c c_{pc} < \dot{m}_h c_{ph}$$
$$\eta = \frac{(T_{hi} - T_{ho})}{(T_{hi} - T_{ci})} \text{for } \dot{m}_c c_{pc} > \dot{m}_h c_{ph}$$

The capacity ratio, which is the relative thermal size of the two fluid streams, is defined as,

$$C_{R} = \frac{\left(\dot{m}c_{p}\right)_{min}}{\left(\dot{m}c_{p}\right)_{max}}$$

On careful analysis, we can say that

 $U \cdot A$: Heat exchange capacities per unit temperature difference.

This thermal sizing (U:A) can be non-dimensionalised by dividing it to the storage capacity of $(\dot{m}c_p)_{min}$

one of the fluid streams. Given limits the maximum heat transfers. The nondimensional term obtained is known as the number of transfer units (NTU)

$$NTU = \frac{UA}{\left(\dot{m}c_p\right)_{min}}$$

It should be noted that

$$\eta = \eta(C_R NTU)$$

The actual determination of this function may be done using heat balances for the streams. For a parallel flow exchanger the relation is shown below

$$\eta = \frac{1 - exp[-(C_R + 1) NTU]}{C_R + 1}$$
$$NTU = \frac{-\ln[1 - (C_R + 1)\eta]}{C_R + 1}$$

(i)
$$\dot{m_c} c_{pc} < \dot{m_h} c_{ph}$$
 (ii) $\dot{m_c} c_{pc} > \dot{m_h} c_{ph}$

The above relation is true for both the condition

Similarly the functional relationship for counter -current exchanger is

$$\eta = \frac{1 - exp[-(1 - C_R) NTU]}{1 - C_R exp[-(1 - C_R) NTU]}$$
(16)

$$NTU = \frac{\ln[(1-\eta)/(1-\eta C_R)]}{C_R - 1}$$
(17)

The previous relation (eq.16 and 17) were for 1-1 exchanger. The relation for 1-2 exchanger (counter current) is given by eq.18, 19),

$$\eta = 2 \left[(1 + C_R) + (1 + C_R^2)^{1/2} \frac{1 + exp[-(1 + C_R^2)^{1/2} NTU]}{1 - exp[-(1 - C_R^2)^{1/2} NTU]} \right]^{-1}$$
(18)

$$NTU = -(1 + C_R^2)^{-1/2} \ln \left[\frac{2/\eta - 1 - C_R - (1 + C_R^2)^{1/2}}{2/\eta - 1 - C_R + (1 + C_R^2)^{1/2}} \right]$$
(19)

When the fluid streams are condensing in a 1-1 pass exchanger (fig.15) as shown below,



Fig.15: Condenser with the temperature nomenclature the following relation arrives.

$$\eta = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} = 1 - \exp(-NTU)$$
$$NTU = -\ln(1 - \eta)$$