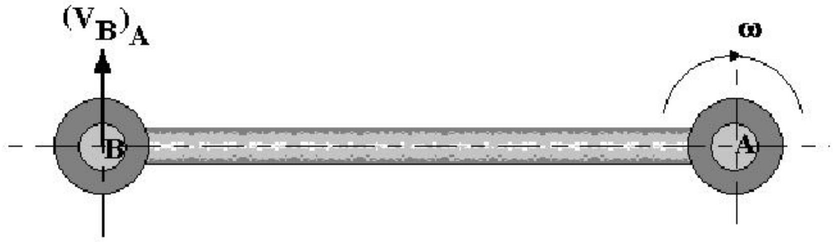
UNIT 2 KINEMATICS OF LINKAGE MECHANISMS

ABSOLUTE AND RELATIVE VELOCITY

An absolute velocity is the velocity of a point measured from a fixed point (normally the ground or anything rigidly attached to the ground and not moving). Relative velocity is the velocity of a point measured relative to another that may itself be moving.

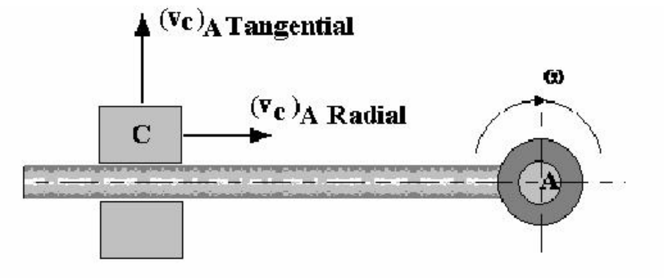
TANGENTIAL VELOCITY

Consider a link A B pinned at A and revolving about A at angular velocity ω. Point B moves in a circle relative to point A but its velocity is always tangential and hence at 90^{o} to the link. A convenient method of denoting this tangential velocity is $(v_B)_A$ meaning the velocity of B relative to A. This method is not always suitable.

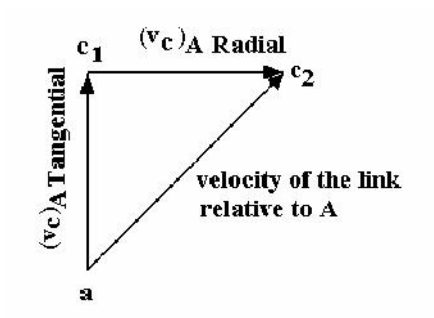


RADIAL VELOCITY

- Consider a sliding link C that can slide on link AB. The direction can only be radial relative to point A as shown.
- If the link AB rotates about A at the same time then link C will have radial and tangential velocities.

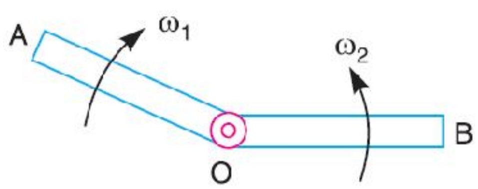


- Note that both the tangential and radial velocities are denoted the same so the tags radial and tangential are added.
- The sliding link has two relative velocities, the radial and the tangential. They are normal to each other and the true velocity relative to A is the vector sum of both added as shown.
- The two vectors are denoted by c_1 and c_2. The velocity of link C relative to point A is the vector a c_2.



Rubbing Velocity at a Pin Joint

The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.



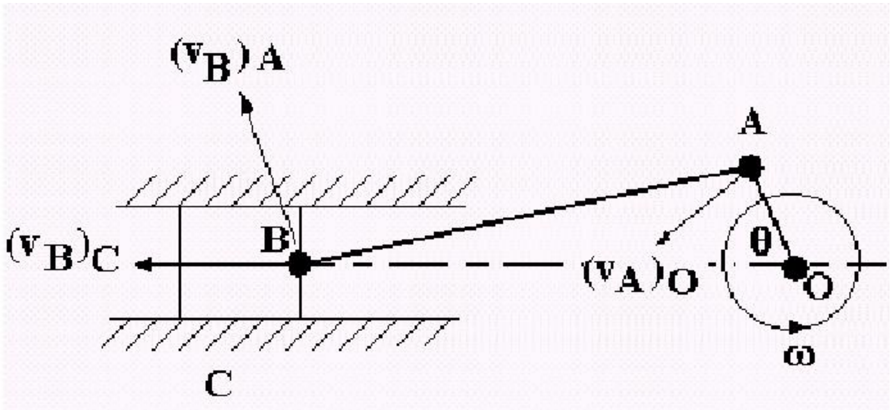
Rubbing velocity at the pin joint *O*

= (ω1 – ω2) *r*, if the links move in the same direction

= (ω1 + ω2) *r*, if the links move in the opposite direction

Slider Crank Chain Mechanism

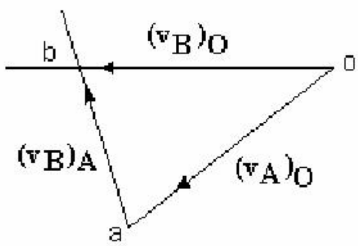
Consider this slider crank chain mechanism, The diagram is called a space diagram.



- Every point on every link has a velocity through space. First we label the centre of rotation, often this is the letter O. Point A can only move in a tangential direction so the velocity of A relative to O is also its absolute velocity and the vector is normal to the crank and it is designated $(v_A)_O$. (Note the rotation is anticlockwise).
- Now suppose that you are sat at point A and everything else moves relative to you. Looking towards B, it would appear the B is rotating relative to you (in reality it is you that is rotating) so it has a tangential velocity denoted $(V_B)_A$.
- The direction is not always obvious except that it is normal to the link. Consider the fixed link OC. Since both points are fixed there is no velocity between them so $(V)_o = 0$.

- Next consider that you at point C looking at point B. Point B is a sliding link and will move in a straight line in the direction fixed by the slider guides and this is velocity $(v_B)_C$. It follows that the velocity of B seen from O is the same as that seen from C so $(v_B)_C = (v_B)_O$.
- The absolute velocity of B is $(v_B)_C = (v_B)_O$ and this must be the vector sum of

$(V_A)_O$ and $(v_B)_A$ and the three vectors must form a closed triangle as shown. The velocity of the piston must be in the direction in which it slides (conveniently horizontal here). This is a velocity diagram.

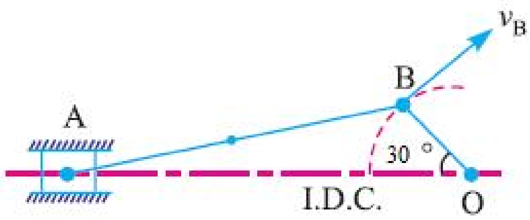


- First calculate the tangential velocity $(v_A)_O$ from $v = \omega \times \text{radius} = \omega \times OA$
- Draw the vector o - a in the correct direction (note lower case letters).

- We know that the velocity of B relative to A is to be added so the next vector ab starts at point a. At point a draw a line in the direction normal to the connecting rod but of unknown length.
- We know that the velocity of B relative and absolute to O is horizontal so the vector ob must start at a. Draw a horizontal line (in this case) through o to intersect with the other line. This is point b. The vectors ab and ob may be measured or calculated. Usually it is the velocity of the slider that is required.
- In a design problem, this velocity would be evaluated for many different positions of the crank shaft and the velocity of the piston determined for each position.
- Remember that the slider direction is not always horizontal and the direction of o - b must be the direction of sliding.

EXAMPLE No.1

The mechanism shown has a crank 50 mm radius which rotates at 2000 rpm. Determine the velocity of the piston for the position shown. Also determine the angular velocity of link AB about A.



SOLUTION

- Suitable scale for example 1 cm = 1 m/s.
- This is important so that the direction at 90^o to the link AB can be transferred to the velocity diagram.
- Angular speed of the crank $\omega = 2\pi N/60 = 2\pi x\ 2000/60 = 209.4$ rad/s
- $(v_A)_O = \omega$ x radius = 209.4 x 0.05 = 10.47 m/s. First draw vector oa. (Diagram a)
- Next add a line in the direction ab (diagram b)

- Finally add the line in the direction of ob to find point b and measure ob to get the velocity. (Diagram C).

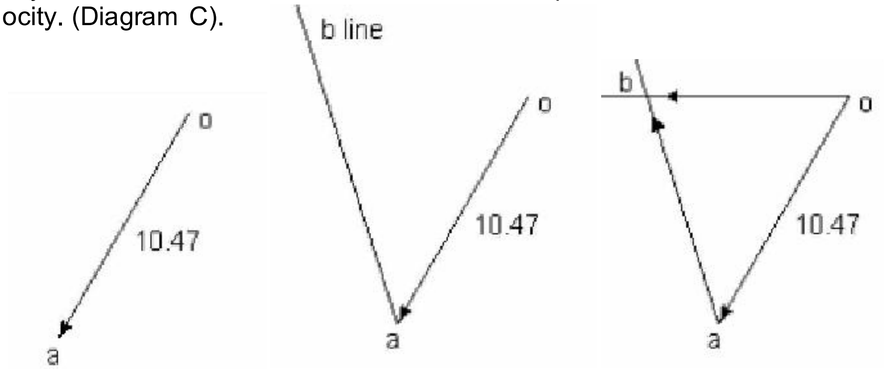
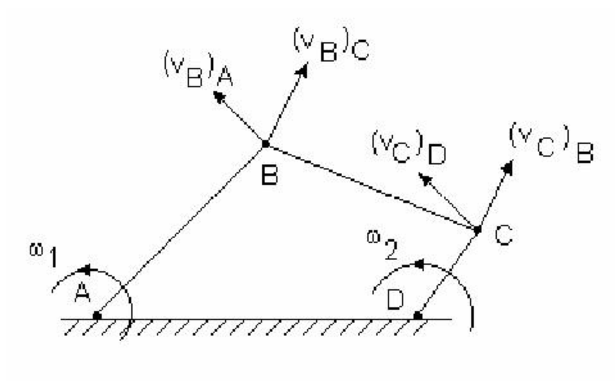


Figure a Figure b Figure c

Diagrams are not drawn to scale.

- The velocity of B relative to O is 7 m/s.
- The tangential velocity of B relative to A is the vector ab and this gives 9.2 m/s.
- The angular velocity of B about A is found by dividing by the radius (length of AB).
- ω for AB is then 9.2/0.09 = 102.2 rad/s. (note this is relative to A and not an absolute angular velocity)

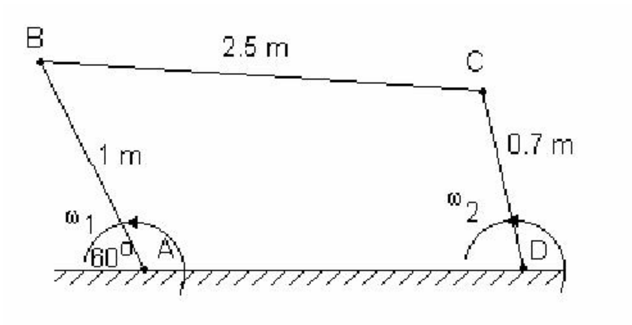
Four Bar Chain Mechanism



- The input link rotates at a constant angular velocity ω_1. The relative velocity of each point relative to the other end of the link is shown.
- Each velocity vector is at right angles to the link. The output angular velocity is ω_2 and this will not be constant. The points A and D are fixed so they will appear as the same point on the velocity diagram.
- The methodology is the same as before and best shown with another example.

EXAMPLE No. 2

Find the angular velocity of the output link when the input rotates at a constant speed of 500 rev/min. The diagram is not to scale.



SOLUTION

First calculate ω_1.

ω_1 = 2∧ x 500/60 = 52.36 rad/s.

Next calculate the velocity of point B relative to A. $(V_B)_A = \omega_1$ x AB = 52.36 x 1 = 52.36 m/s.

Draw this as a vector to an appropriate scale.

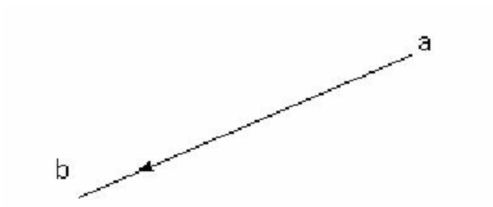
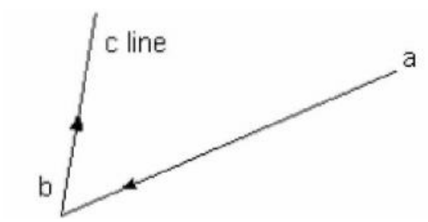


Figure a

Next draw the direction of velocity C relative to B at right angles to the link BC passing through point b on the velocity diagram.



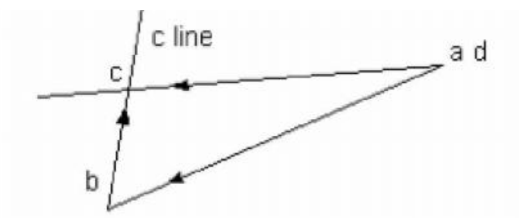


Figure b

Figure c

- Next draw the direction of the velocity of C relative to D at right angles to link DC passing through point a (which is the same as point d). Point c is where the two lines intersect,
- Determine velocity cd by measurement or any other method. The velocity of point C relative to D and is 43.5 m/s.
- Convert this into angular velocity by dividing the length of the link DC into it.

$$\omega_2 = 43.5/0.7 = 62 \text{ rad/s.}$$

4. ACCELERATION DIAGRAMS

- It is important to determine the acceleration of links because acceleration produces inertia forces in the link which stress the component parts of the mechanism.
- Accelerations may be relative or absolute in the same way as described for velocity.
- We shall consider two forms of acceleration, tangential and radial. Centripetal acceleration is an example of radial.

CENTRIPETAL ACCELERATION

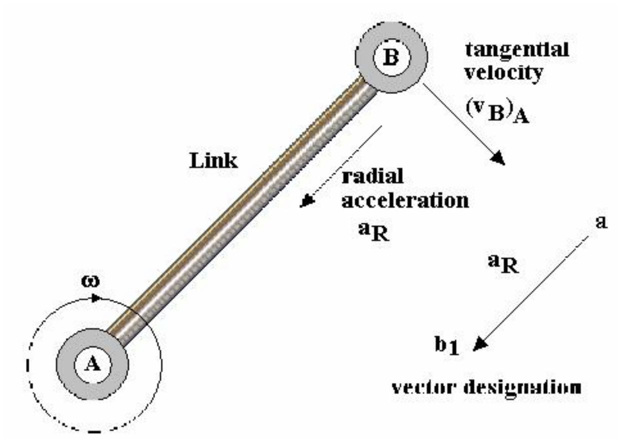
- A point rotating about a centre at radius R has a tangential velocity v and angular velocity ω and it is continually accelerating towards the centre even though it never moves any closer. This is centripetal acceleration and it is caused by the constant change in direction.
- It follows that the end of any rotating link will have a centripetal acceleration towards the opposite end.

The relevant equations are: $v = \omega R$ $a = \omega^2 R$ or $a = v^2/R$.

- The construction of the vector for radial acceleration causes confusion so the rules must be strictly followed. Consider the link AB. The velocity of B relative to A is tangential $(V_B)_A$.

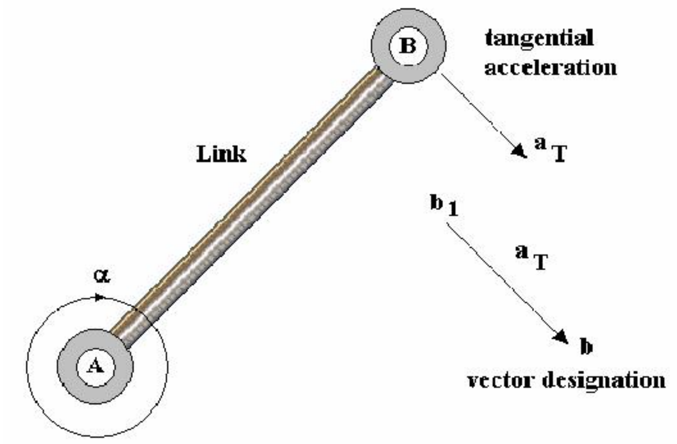
The centripetal acceleration of B relative to A is in a radial direction so a suitable notation might be a_R. It is calculated using $a_R = \omega$ x AB or $a_R = v^2/AB$.

Note the direction is towards the centre of rotation but the vector starts at a and ends at b_1. It is very important to get this the right way round otherwise the complete diagram will be wrong.

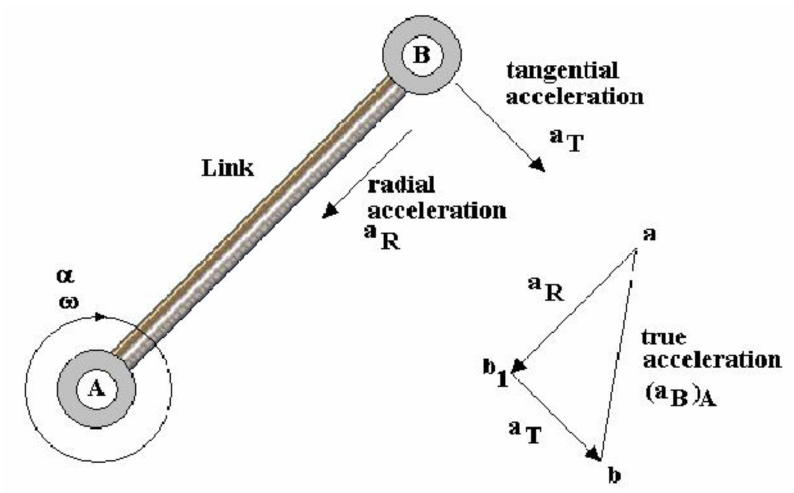


TANGENTIAL ACCELERATION

Tangential acceleration only occurs if the link has an angular acceleration α rad/s^2. Consider a link AB with an angular acceleration about A.

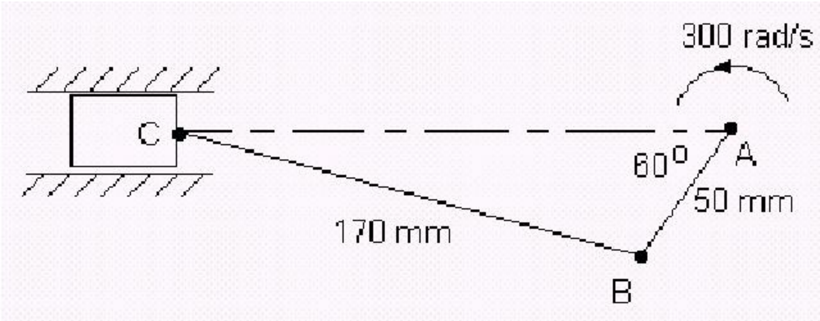


- Point B will have both radial and tangential acceleration relative to point A. The true acceleration of point B relative to A is the vector sum of them. This will require an extra point. We will use b_1 and b on the vector diagram as shown.
- Point B is accelerating around a circular path and its direction is tangential (at right angles to the link). It is designated aT and calculated using $a_T = \alpha x$ AB.
- The vector starts at b1 and ends at b. The choice of letters and notation are arbitrary but must be logical to aid and relate to the construction of the diagram.



EXAMPLE No.3

A piston, connecting rod and crank mechanism is shown in the diagram. The crank rotates at a constant velocity of 300 rad/s. Find the acceleration of the piston and the angular acceleration of the link BC. The diagram is not drawn to scale.



Solution

First calculate the tangential velocity of B relative to A.

$(v_B)_A = \omega \times \text{radius} = 300 \times 0.05 = 15$ m/s.

Next draw the velocity diagram and determine the velocity of C relative to B.

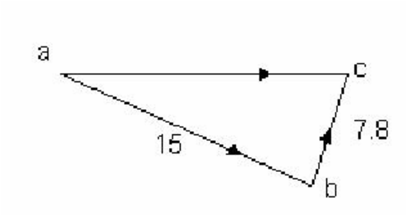


Figure 19

From the velocity diagram $(v_C)_B = 7.8$ m/s

- Next calculate all accelerations possible and construct the acceleration diagram to find the acceleration of the piston.
- The tangential acceleration of B relative to A is zero in this case since the link has no angular acceleration ($\alpha = 0$).
  - The centripetal acceleration of B relative to A

$a_R = \omega^2 \times AB = 300^2 \times 0.05 = 4500$ m/s^2. The tangential acceleration of C relative to B is unknown.

The centripetal acceleration of C to B

$a_R = v^2/BC = 7.8^2 /0.17 = 357.9$ m/s^2.

The stage by stage construction of the acceleration diagram is as follows.

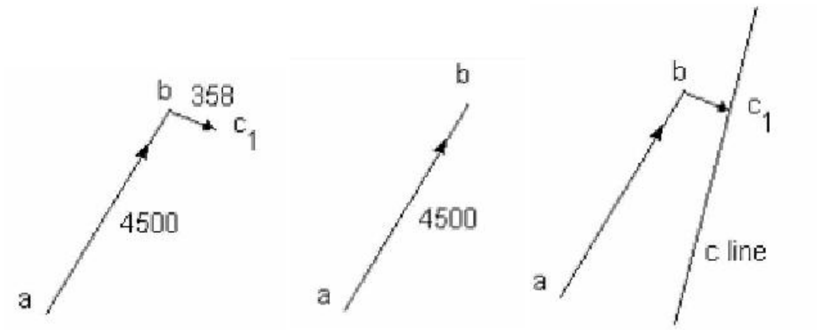


Figure a **Figure b** **Figure c**

- First draw the centripetal acceleration of link AB (Fig.a). There is no tangential acceleration so designate it ab. Note the direction is the same as the direction of the link towards the centre of rotation but is starts at a and ends at b.
- Next add the centripetal acceleration of link BC (Figure b). Since there are two accelerations for point C designate the point c_1. Note the direction is the same as the direction of the link towards the centre of rotation.
- Next add the tangential acceleration of point C relative to B (Figure c). Designate it c_1 c. Note the direction is at right angles to the previous vector and the length is unknown. Call the line a c line.
- Next draw the acceleration of the piston (figure d) which is constrained to be in the horizontal direction. This vector starts at a and must intersect the c line. Designate this point c.

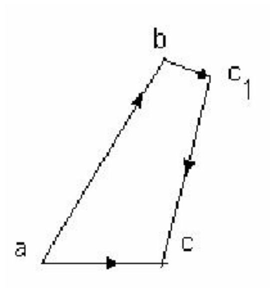


Figure d

- The acceleration of the piston is vector ac so $(a_C)_B = 1505$ m/s^2. The tangential acceleration of C relative to B is c_1 c = 4000 m/s^2.
- At the position shown the connecting rod has an angular velocity and acceleration about its end even though the crank moves at constant speed.
- The angular acceleration of BC is the tangential acceleration divided by the length BC. $\alpha_{(BC)} = 4000 / 0.17 = 23529$ rad/s^2.

EXAMPLE No.4

The diagrams shows a "rocking lever" mechanism in which steady rotation of the wheel produces an oscillating motion of the lever OA. Both the wheel and the lever are mounted in fixed centers. The wheel rotates clockwise at a uniform angular velocity (ω) of 100 rad/s. For the configuration shown, determine the following.

(i) The angular velocity of the link AB and the absolute velocity of point A. (ii) The centrifugal accelerations of BC, AB and OA.

(iii)The magnitude and direction of the acceleration of point A. The lengths of the links are as follows.

BC = 25 mm AB = 100 mm OA = 50 mm OC = 90 mm

SOLUTION

The solution is best done graphically. First draw a line diagram of the mechanism to scale. It should look like this.

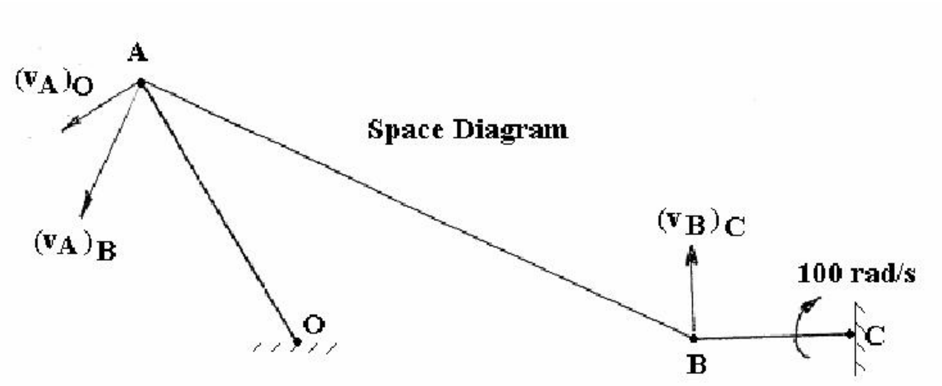
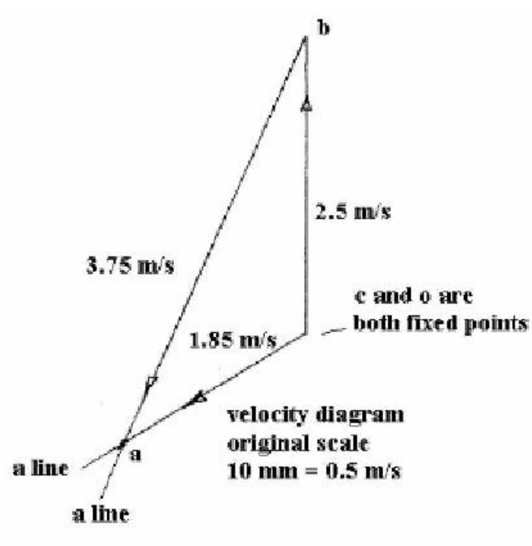


Figure 22

Next calculate the velocity of point B relative to C and construct the velocity diagram.



$(v_B)_C = \omega \times$ radius = 100 x 0.025 = 2.5 m/s

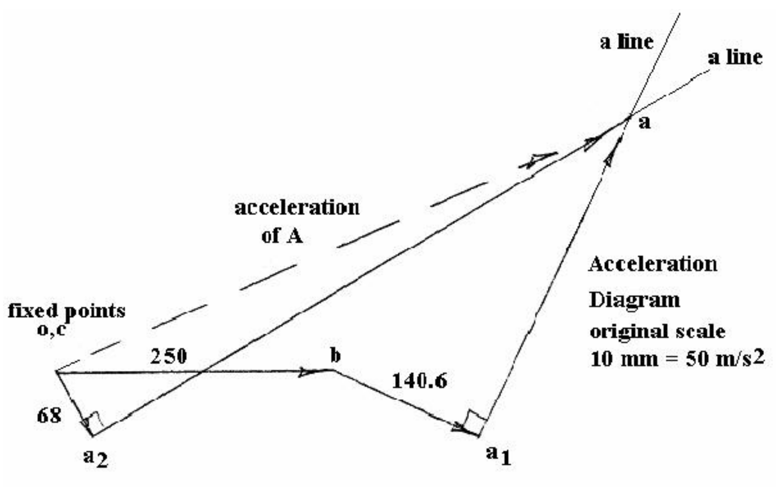
Scale the following velocities from the diagram.

$(v_A)_O$ = 1.85 m/s {answer (i)} $(v_A)_B$ = 3.75 m/s

Angular velocity = tangential velocity/radius

For link AB, Ȧ = 3.75/0.1 = 37.5 rad/s. {answer (i)} Next calculate all the accelerations possible.

- Radial acceleration of BC = $Ȧ^2$ x BC = 100^2 x 0.025 = 250 m/s^2.
- Radial acceleration of AB = v^2/AB = 3.75^2/0.1 = 140.6 m/ s^2.
- Check same answer from $Ȧ^2$ x AB = 37.5^2 x 0.1 = 140.6 m/ s^2.
- Radial Acceleration of OA is v^2/OA = 1.85^2/0.05 = 68.45 m/ s^2. Construction of the acceleration diagram gives the result shown.

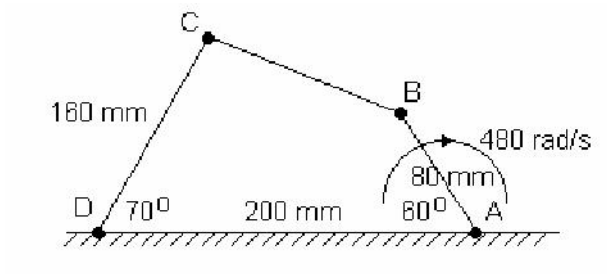


The acceleration of point A is the vector o- a shown as a dotted line.

Scaling this we get 560 m/s^2.

EXAMPLE No. 5

Find the angular acceleration of the link CD for the case shown.

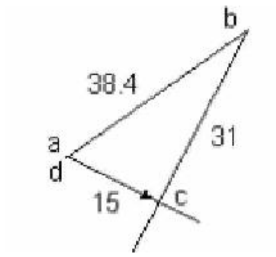


SOLUTION

First calculate or scale the length CB and find it to be 136 mm.

Next find the velocities and construct the velocity diagram. Start with link AB as this has a known constant angular velocity.

$(v_B)_A = \omega$ x radius = 480 x 0.08 = 38.4 m/s



1. Next calculate all the accelerations possible.
2. The centripetal acceleration of B to A is $38.4^2/0.08$ = 18 432 m/s^2
3. The centripetal acceleration of C to D is $15^2/0.16$ = 1406 m/s^2
4. The centripetal acceleration of C to B is $31^2/0.136$ = 7066 m/s^2.
5. We cannot calculate any tangential acceleration at this stage.
6. The stage by stage construction of the acceleration diagram follows.
7. First draw the centripetal acceleration of B to A (Figure a).

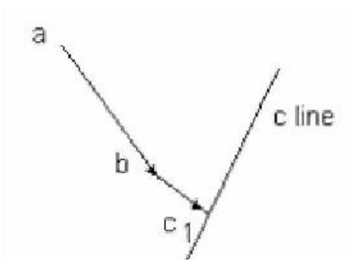


Figure a

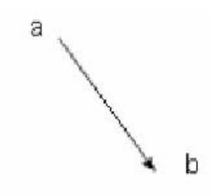


Figure b

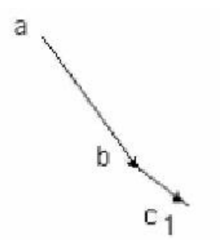
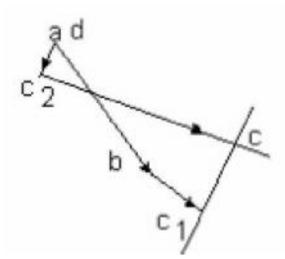


Figure c

8. Next add the centripetal acceleration of C to B (figure b)

9. Next draw the direction of the tangential acceleration of C to B of unknown length at right angles to the previous vector (figure c). Designate it as a c line.

10. We cannot proceed from this point unless we realize that points a and d are the same (there is no velocity or acceleration of D relative to A). Add the centripetal acceleration of C to D (figure d). This is 1406 m/s^2 in the direction of link CD. Designate it d c_2.



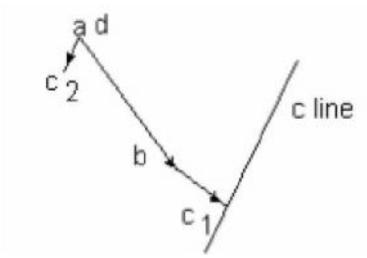


Figure d **Figure e**

Finally draw the tangential acceleration of C to D at right angles to the previous vector to intersect the c line (figure e).

From the diagram determine c_2 c to be 24 000 m/s^2. This is the tangential acceleration of C to D. The angular acceleration of the link DC is then:

$\alpha_{(CD)}$ = 24000/0.16 = 150 000 rad/s^2 in a clockwise direction.

Note that although the link AB rotates at constant speed, the link CD has angular acceleration.

EXAMPLE No. 6

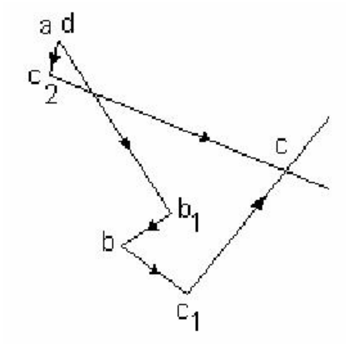
The same arrangement exists as shown for example 5 except that the link AB is decelerating at 8000 rad/s^2 (i.e. in an anticlockwise direction). Determine the acceleration of the link CD.

SOLUTION

The problem is essentially the same as example 5 except that a tangential acceleration now exists for point B relative to point A. This is found from

$a_T = \alpha \times AB = 80000 \times 0.08 = 6400$ m/s^2

- The direction is for an anticlockwise tangent. This is vector b_1 b which is at right angles to a b_1 in the appropriate direction. The new acceleration diagram looks like this.



Scaling off the tangential acceleration c_2 c we get 19 300 m/s^2. Converting this into the angular acceleration we get

α = 19 300/0.16 = 120 625 rad/s^2 in a clockwise direction.

MECHANICAL ADVANTAGE

Mechanical advantage is a measure of the force amplification achieved by using a tool, mechanical device or machine system. Ideally, the device preserves the input power and simply trades off forces against movement to obtain a desired amplification in the output force. The model for this is the *law of the lever.* Machine components designed to manage forces and movement in this way are called mechanisms. An ideal mechanism transmits power without adding to or subtracting from it. This means the ideal mechanism does not include a power source, and is frictionless and constructed from rigid bodies that do not deflect or wear. The performance of a real system relative to this ideal is expressed in terms of efficiency factors that take into account friction, deformation and wear.

CORIOLIS COMPONENT OF ACCELERATION

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

In non-vector terms: at a given rate of rotation of the observer, the magnitude of the Coriolis acceleration of the object is proportional to the velocity of the object and also to the sine of the angle between the direction of movement of the object and the axis of rotation. The vector formula for the magnitude and direction of the Coriolis acceleration is

$$a_C = -2\,\boldsymbol{\Omega} \times \boldsymbol{v}$$

where (here and below) a_C is the acceleration of the particle in the rotating system, $\boldsymbol{v}$ is the velocity of the particle with respect to the rotating system, and **Ω** is the angular velocity vector which has magnitude equal to the rotation rate ω and is directed along the axis of rotation of the rotating reference frame, and the × symbol represents the cross product operator.

1. What are the components of acceleration?
2. What is expression for Cariolis component of acceleration?
3. What are the expressions for radial and tangential component of acceleration?
4. How can we represent the direction of linear velocity of any point on a link with respect to another point on the same line?
5. What is the objective of Kinematic analysis?

PROBLEMS

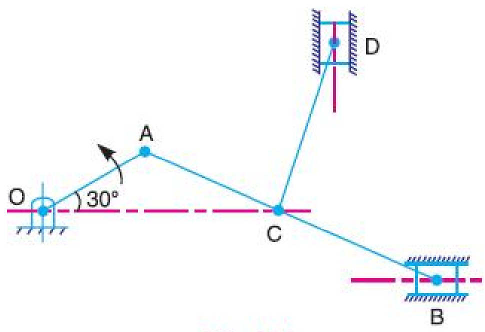
1. The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : **1.** Linear velocity and acceleration of the midpoint of the connecting rod, and **2.** angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

2. The engine mechanism has crank OB = 50 mm and length of connecting rod AB = 225 mm. The centre of gravity of the rod is at G which is 75 mm from B. The engine speed is 200 r.p.m. For the position shown, in which OB is turned 45° from OA, Find 1. the velocity of G and the angular velocity of AB, and 2. the acceleration of G and angular acceleration of AB.

3. In a pin jointed four bar mechanism ABCD, the lengths of various links are as follows: AB = 25 mm ; BC = 87.5 mm ; CD = 50 mm and AD = 80 mm. The link AD is fixed and the angle BAD = 135°. If the velocity of B is 1.8 m/s in the clockwise direction, find 1. velocity and acceleration of the mid point of BC, and 2. angular velocity and angular acceleration of link CB and CD.

4. In a four bar chain ABCD , link AD is fixed and the crank AB rotates at 10 radians per second clockwise. Lengths of the links are AB = 60 mm ; BC = CD = 70 mm ; DA = 120 mm. When angle DAB = 60° and both B and C lie on the same side of AD, find 1. angular velocities (magnitude and direction) of BC and CD ; and 2. angular acceleration of BC and CD.

5. The dimensions of the various links of a mechanism, as shown in Figure, are as follows: OA = 80 mm ; AC = CB = CD = 120 mm If the crank OA rotates at 150 r.p.m. in the anticlockwise direction, find, for the given configuration: 1. velocity and acceleration of B and D ; 2. rubbing velocity on the pin at C, if its diameter is 20 mm ; and 3. angular acceleration of the links AB and CD.



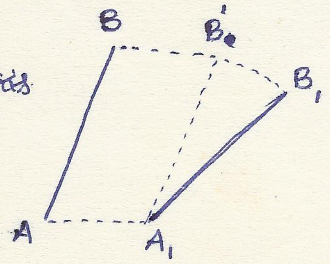
VELOCITY IN MECHANISMS :

Sometimes, a body has simultaneously a motion of rotation as well as translation, such motion will have the combined effect of rotation and translation.

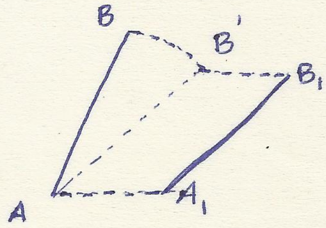
Methods for determining the velocity of a point on a link :

1. Instantaneous centre method. (It is convenient and easy to apply in simple mechanisms)
2. Relative velocity method. (It can be used to any configuration)

~~velocity in mechanisms~~



Motion of translation AB to A_1B'
And then motion of rotation AB' to A_1B_1

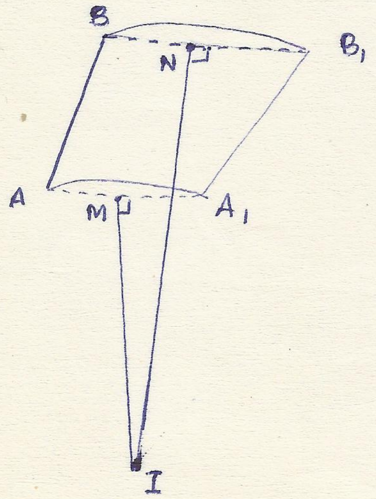


Motion of rotation AB to AB' and then motion of translation AB' to A_1B_1

Such a motion of link AB to A_1B_1 is an example of combined motion of rotation and translation.

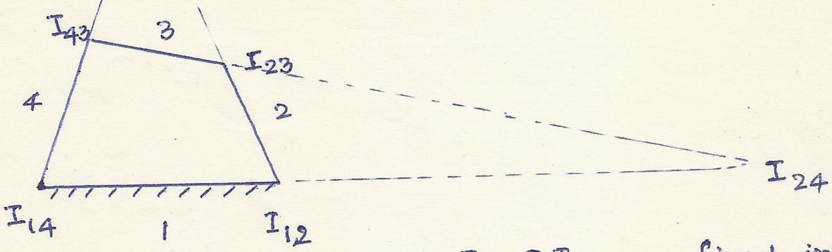
In actual practice, the motion of link AB is so gradual that it is difficult to see the two seperate motions.

The combined motion of rotation and translation of link AB may be assumed to be a motion of pure rotation about some centre I, known as the Instantaneous centre of rotation.



Number of instantaneous centres in a Mechanism.

$$N = \frac{n(n-1)}{2}$$ where n = number of links.



$$N = \frac{4(4-1)}{2} = 6$$

I_{12} & I_{14} - fixed instantaneous centres

I_{23} & I_{43} - permanent instantaneous centres

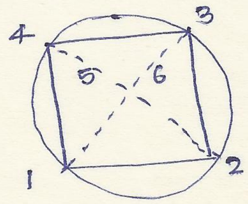
I_{13} & I_{24} - Neither fixed or Permanent instantaneous centres. (vary with configuration)

~~11~~	12	13	14
~~21~~	~~21~~	23	24
~~31~~	32	~~33~~	34
~~41~~	42	43	~~44~~

✓12   13   ✓14

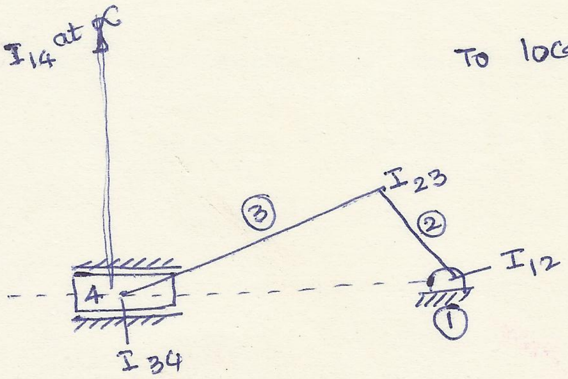
✓23   24

✓34



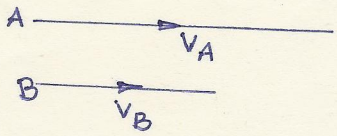
To locate I_{24} - Join 42 $<$ $I_{34}\ I_{23}$ / $I_{14}\ I_{12}$

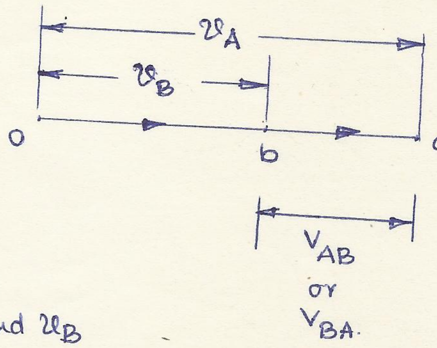
To locate I_{13} - Join 13 $<$ $I_{14}\ I_{43}$ / $I_{12}\ I_{23}$



Velocity in Mechanisms:

(Relative velocity Method):





v_{AB} = vector difference of v_A and v_B

$= \bar{v}_A - \bar{v}_B$

The relative velocity of A with respect to B (ie, v_{AB}) may be written in the vector form,

$$\bar{ba} = \bar{oa} - \bar{ob}$$

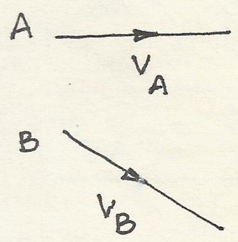
Similarly, the relative velocity of B with respect to A (ie v_{BA}) may be written in the vector form

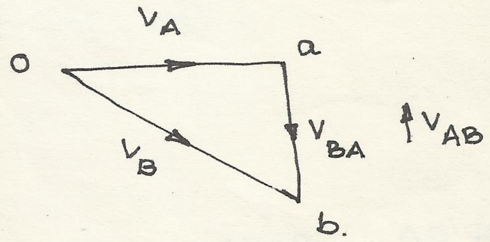
$v_{BA} = \bar{v}_B - \bar{v}_A$

= vector difference of v_B and v_A

$$\bar{ab} = \bar{ob} - \bar{oa}$$

Relative velocity of two bodies moving along inclined lines.





Relative velocity of A with respect to B.

V_{AB} = vector difference of v_A and v_B = $\bar{v}_A - \bar{v}_B$

$\overline{ba} = \overline{oa} - \overline{ob}$ —①

Relative velocity of B with respect to A,

V_{BA} = vector difference of v_B and v_A = $\bar{v}_B - \bar{v}_A$

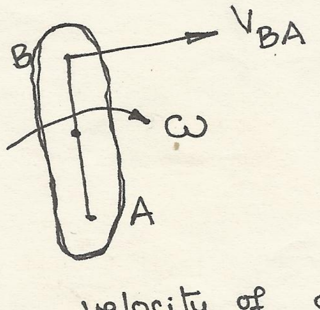
$\overline{ab} = \overline{ob} - \overline{oa}$ ~~②~~ or $= -(\overline{oa} - \overline{ob})$ —②

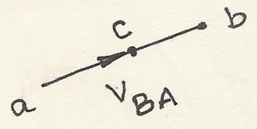
From the above equations ① and ②,

The relative velocity of point A with respect to B (v_{AB}) and the relative velocity of point B with respect to A (v_{BA}) are equal in magnitude but opposite in direction.

ie, $v_{AB} = -v_{BA}$ or $(\overline{ba}) = -(\overline{ab})$

MOTION OF A LINK:





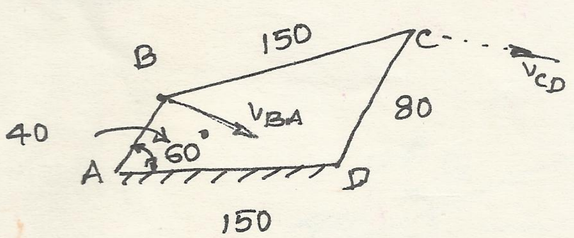
velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the space diagram or configuration diagram.

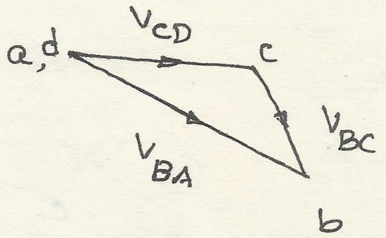
Let ω = angular velocity of the link AB about A (ω_{BA}

$v_{BA} = \omega_{BA} \times$ (length of link AB)

$$\boxed{v_{BA} = \omega_{BA} \times AB}$$

<u>Problem</u>: In a Four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 rpm clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°





$v_{BA} = \omega_{BA} \times AB$

$\omega_{BA} = \dfrac{2\pi N_{BA}}{60}$

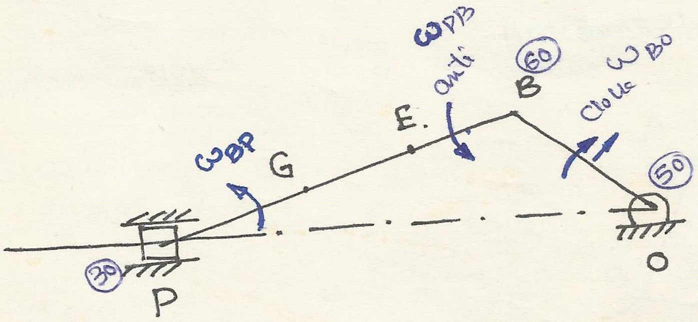
$= 12.568$ rad/sec.

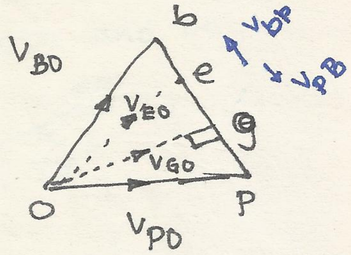
$\omega_{CD} = \dfrac{V_{CD}}{DC}$

$= 4.8$ rad/sec (Clockwise)

Prob. 2 The crank and connecting rod of a steam engine are 0.5 m and 2 m long respectively. The crank makes 180 rpm in the clockwise direction. When it has turned 45° from the inner dead centre position, determine

1. velocity of piston 2. angular velocity of connecting rod.
3. velocity of point E on the C.R. 1.5 m from the gudgeon pin.
4. position ~~of~~ and linear velocity of any point G on the connecting rod which has the least velocity relative to crankshaft.





$$\frac{bg}{bp} = \frac{BG}{BP}$$

$$BG = \frac{bg}{bp} \times BP$$

$\cancel{V_{g0}}$ $V_{GO} = \overline{og}$

$= 8$ m/sec.

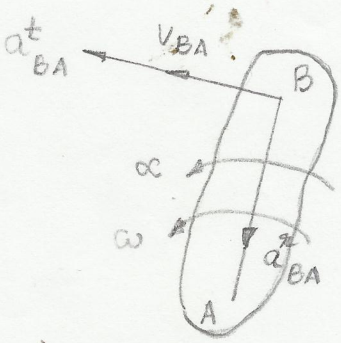
Vel. of rubbing at the pins of the crankshaft, crank & cross-head when dia. of the pins are 50, 60 and 30 mm

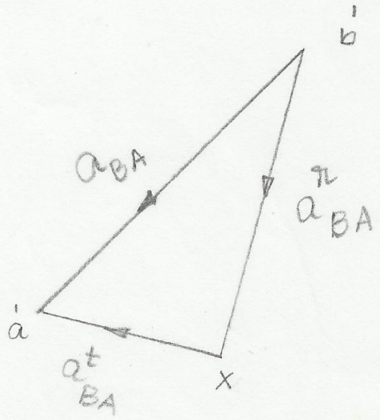
Acceleration in Mechanism:

Consider two points A and B on a rigid link as shown in the diagram.

Let the point B moves with respect to A, with an angular velocity of ω rad/sec.

Let α rad/sec^2 be the angular Acceleration of the link AB





Components of Acceleration

1. Centripetal or radial component, which is perpendicular to the velocity of the particle at the given instant. (Along the ~~crank~~ Link)

2. Tangential Component, which is parallel to the velocity of the particle at the given instant. (⊥er to link)

$$a^r_{BA} = \omega^2 \times (\text{Length of link } AB) = \omega^2_{BA} \cdot AB$$

$$= \frac{v^2_{BA}}{(AB)^2} \times AB = v^2_{BA}/AB \quad - m/sec^2$$

$$a^t_{BA} = \alpha \times \text{link length } AB \quad - m/sec^2$$

Pr.

PQRS is a Four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm; QR = 175 mm; RS = 112 mm; PS = 200 mm; The Crank PQ rotates at 10 rad/sec. clockwise. Draw the velocity and acceleration diagram when angle QPS = 60°, Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.

$\omega_{QP} =$ 10 rad/sec.

$v_{QP} = \omega_{QP} \times PQ = 10 \times 0.062 = 0.62$ m/sec. ($\bar{pq}$)

1. <u>Construct velocity diagram.</u>

2. from vel. diagram measure the following vectors

<u>Measure</u>

ⓐ $\bar{qr} = (\quad) \times$ scale $= v_{RQ}$ m/sec

$$\omega_{RQ} = \omega_{QR} = \frac{v_{RQ}}{QR} = 1.9 \text{ rad/sec (Anticlockwise)}$$

(b) Measure

$\bar{sr} = (\quad)$ scale $= v_{RS}$

$$\omega_{RS} = \frac{v_{RS}}{RS} = 3.78 \text{ rad/sec (Clockwise)}$$

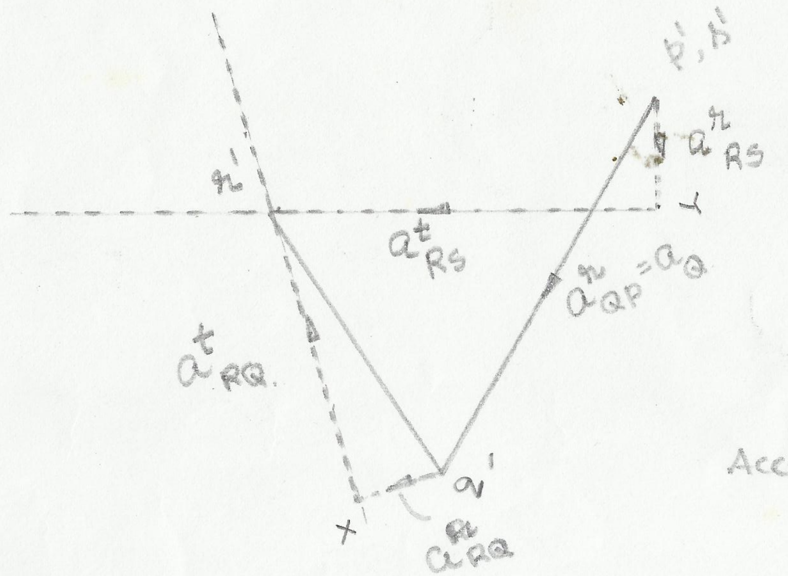
3. Construct Acceleration dia.

$\omega_{QP} =$ 10 rad/sec. $\qquad v_{QP} = \omega_{QP} \times PQ = 0.62$ m/sec.

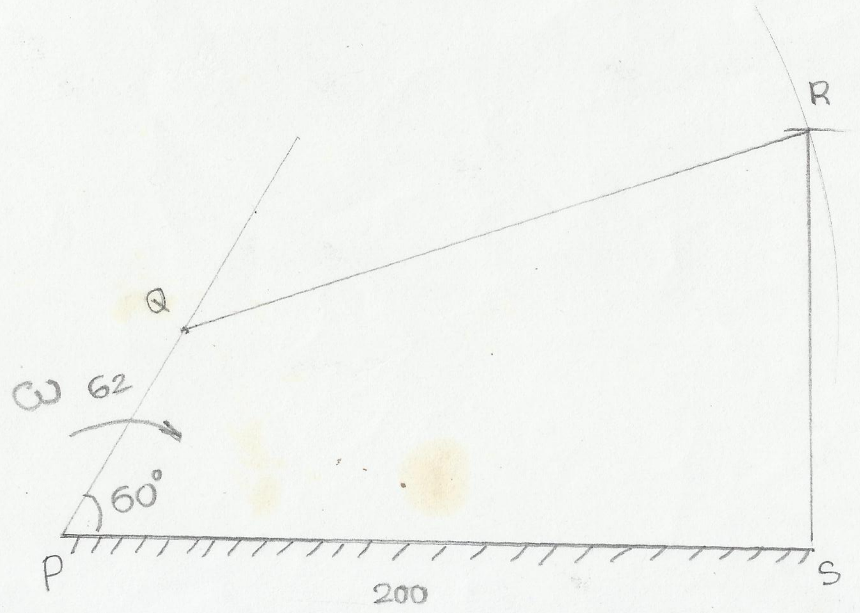
$$a^r_{QP} = \frac{v_{QP}^2}{PQ} = 6.2 \text{ m/sec}^2$$

$$a^r_{RQ} = \frac{v_{RQ}^2}{RQ} = 0.634 \text{ m/sec}^2$$

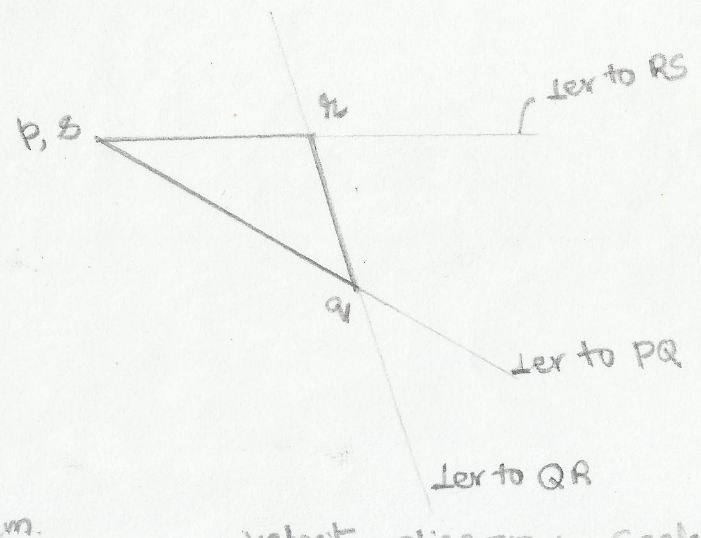
$$a^r_{RS} = \frac{v_{RS}^2}{RS} = 1.613 \text{ m/sec}^2$$



Acceleration diagram
scale :



Scale ~~100~~ 20mm = 1cm.



Velocity diagram : Scale

③

Construction of Accele. dia.

1. Locate $p's'$ (fixed points)

2. from p', draw a vector $p'q'$ parallel to PQ. (which represents a^r_{QP}) with suitable scale.

3. from q' draw a vector $q'x$ parallel to RQ with a suitable scale.

4. from x draw a $\perp$er

5. from s', draw a vector $s'Y$ parallel to RS with a suitable scale.

6. from Y draw a $\perp$er.

7. The two perpendicular lines intersects at r'

8. Join $r'q'$ which represents a_{RQ}.

from Accele. dia. measure the following vectors.

$xr' = a^t_{RQ}$. = ( ) x Scale.

to find $\alpha_{RQ} = \alpha_{QR} = \dfrac{a^t_{RQ}}{RQ}$ rad/sec² ~~24~~ (23.43 rad/sec²)
(Anti-clockwise)

similarly, measure

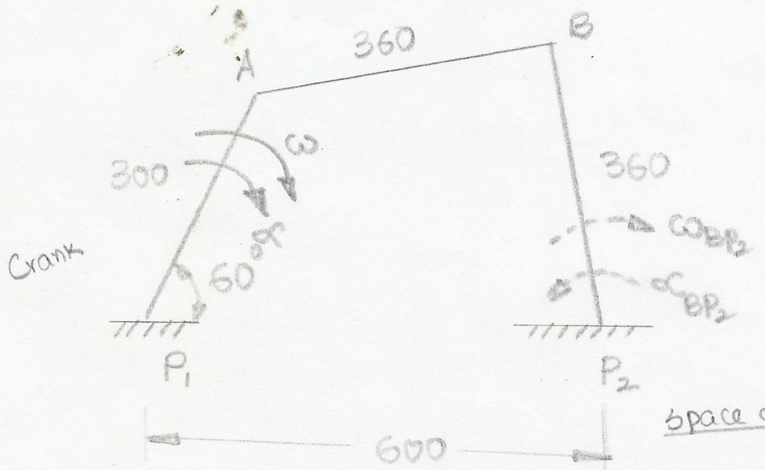
$Yr' = a^t_{RS}$ = ( ) scale.

to find $\alpha_{RS} = \cancel{\alpha_{SR}} = \dfrac{a^t_{RS}}{RS}$ rad/sec² (47.1 rad/sec²)
(Anti-clockwise)

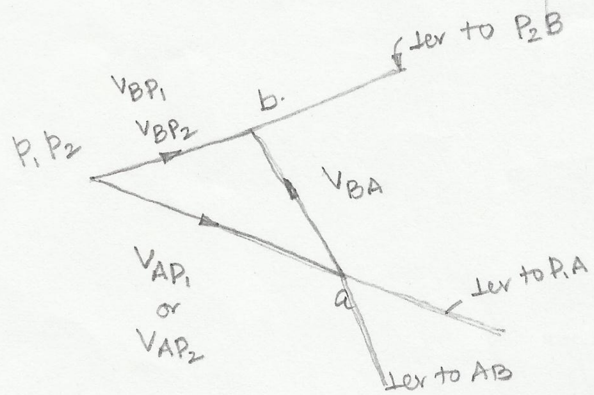
Pr The dimensions and Configuration of the four bar mechanism, shown in the diagram.

$P_1A = 300\,mm$; $P_2B = 360\,mm$; $AB = 360\,mm$; $P_1P_2 = 600\,mm$;

The angle $AP_1P_2 = 60°$; The crank P_1A has an angular velocity 10 rad/sec and an angular acceleration of 30 rad/sec². both Clockwise. Determine the angular velocities and angular accelerations of P_2B and AB



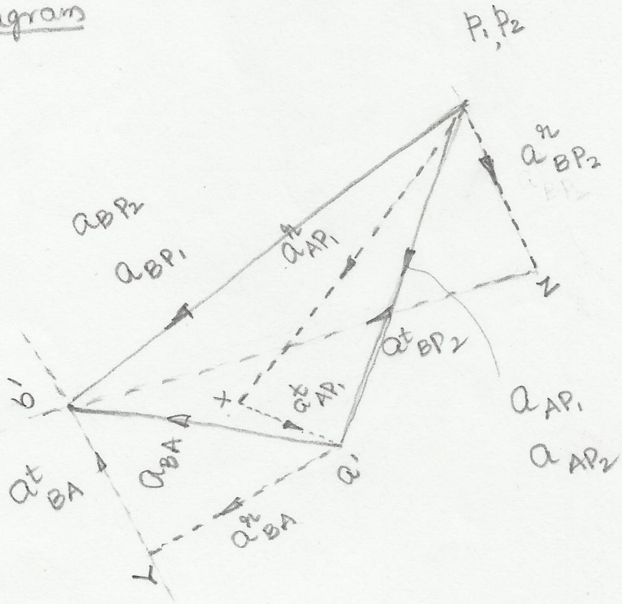
space diagram



velocity diagram

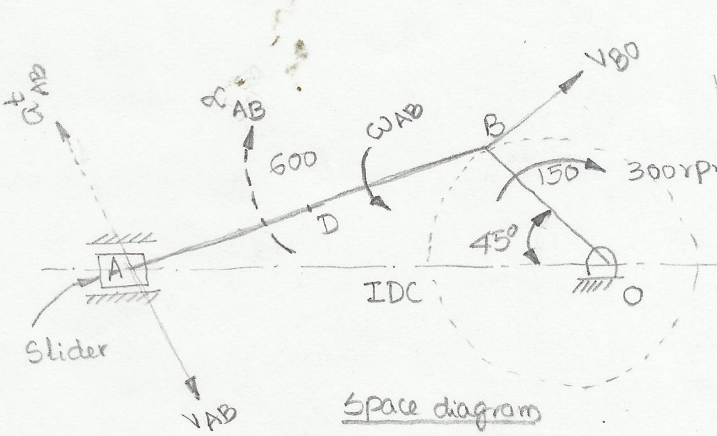
$\alpha_{P_2B} = 73.8$ rad/sec²

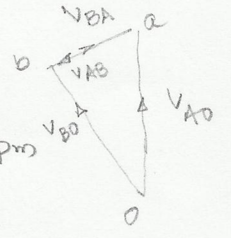
$\alpha_{AB} = 37.8$ rad/sec²



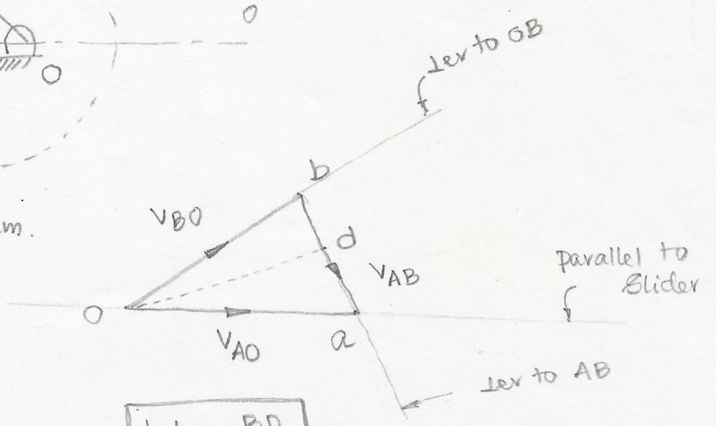
pr.

The Crank of a slider Crank mechanism rotates clockwise at a Constant speed of 300 rpm; The Crank is 150 mm and the Connecting rod is 600 mm long. Determine : ①. Linear velocity and acceleration of the mid point of the Connecting rod and ② Angular velocity and angular acceleration of the Connecting rod, at a Crank angle of 45° from I.D.C. Position.





Space diagram
50 mm = 1 cm.



$\boxed{\frac{bd}{ba} = \frac{BD}{BA}}$

Velocity diagram
Scale: 1 m/sec = 1 mm

$N_{BO} = 300\,rpm$

$\omega_{BO} = \frac{2\pi N_{BO}}{OB}$

$= 31.42\ rad/sec$;

$v_{BO} = \omega_{BO} \times OB = 31.42 \times 0.15$

$= 4.713\ m/sec. \approx 4.7\ m/sec\ (\bar{ob})$

Construct velocity diagram,
From the velocity diagram,
measure the following vector

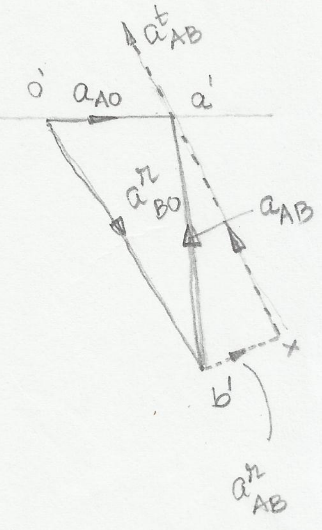
V_{AB} = Measure $(\bar{b}a)$ × Scale = (   )

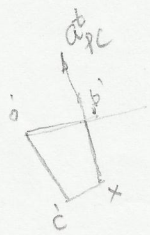
$= 3.4\ m/sec$

V_{AO} = Slider velocity = measure $(\bar{o}a)$ × Scale

$=$ m/sec

$\omega_{AB} = v_{AB}/AB = \frac{3.4}{0.6} = 5.67$ (anti-clockwise).





o'c' = CO
c'x - CQ
xp' - QN -
o'p' - ON

Find vel. of mid-point of connecting rod,

Acceleration diagram:

$$a^r_{BO} = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \ m/sec^2$$

$$a^r_{AB} = \frac{v^2_{AB}}{AB} = \qquad\qquad = \qquad m/sec^2$$

$$\frac{b'd'}{b'a'} = \frac{BD}{BA}$$

Select scale to draw Accel. diagram

to find α^t_{AB}:

from acceleration diagram,

measure the following ~~veloc~~ vectors

$o'a' = a_{AO} = (o'a') \cdot \text{scale}$

Acceleration of Slider $= a_{AO} = \qquad m/sec^2$

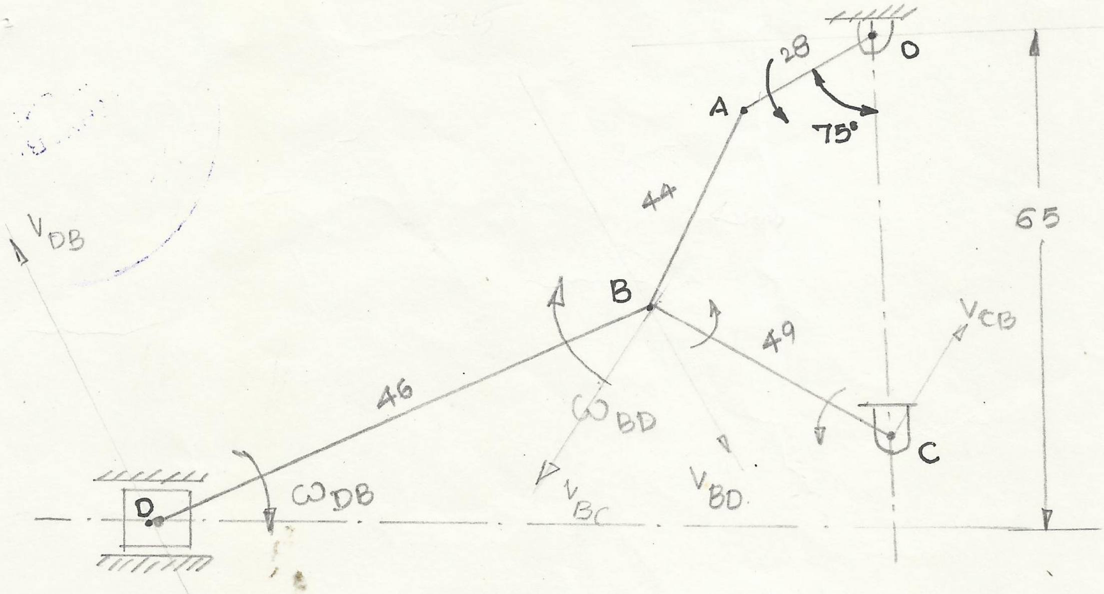
$$\alpha_{AB} = a^t_{AB} \Big/ AB = \frac{\text{Measure } (a'x) \times \text{scale}}{0.6} = \underline{171.67} \ rad/sec^2$$

(Clockwise)

find Accel. of Mid-point:

from accele. diagram, measure $(o'd') \times$ scale

$a_{DO} =$ Acce. of mid. point $= \qquad m/sec^2$



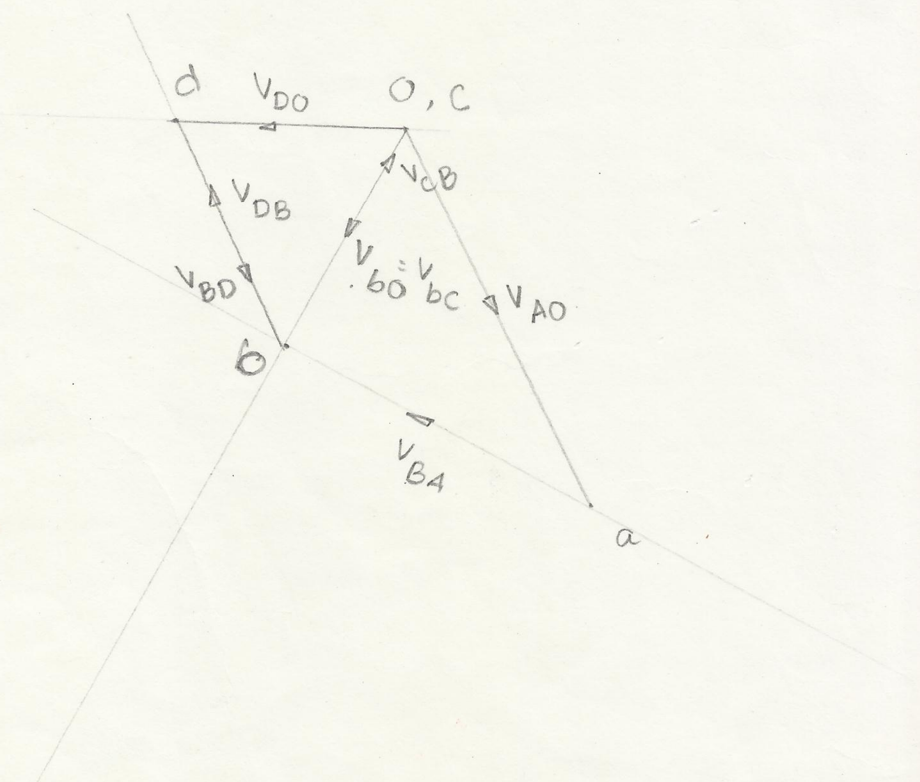
Problem: The angular vel. of crank OA is 600 rpm. Determine the linear velocity of the slider D and the angular vel. of link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are OA = 28 mm; AB = 44 mm; BC = 49 mm; BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.

ω_{BD} = (Clockwise)

ω_{DB} = (Clockwise)

ω_{BC} = anti-clockwise

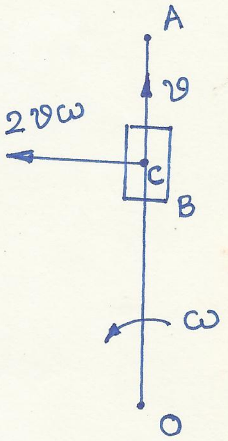
ω_{CB} = anti-clockwise

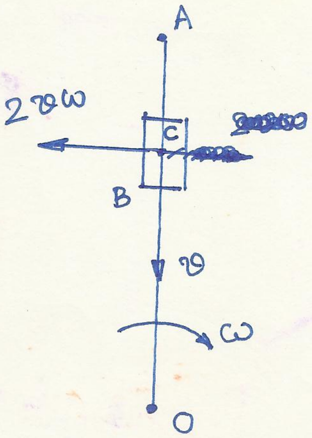


20.1.2003

CORIOLIS COMPONENT OF ACCELERATION:

When a point on one link is sliding along another rotating link, such as in Quick return motion mechanism, then the coriolis component of acceleration must be calculated.



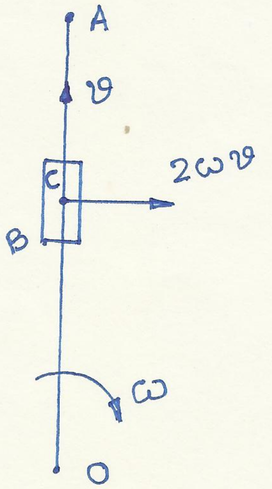


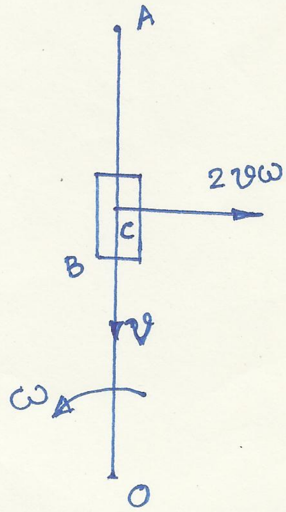
ω - anti-clockwise] ←
v - outwards]

ω - clockwise] →
v - inwards]

The tangential component of acceleration of the slider B w.r.t the coincident point C on the link is known as coriolis component of acceleration and is always perpendicular to the link.

$$\therefore \left.\begin{array}{l}\text{Coriolis component of} \\ \text{accele. of B w.r.t C}\end{array}\right\} = a^c_{BC} = a^t_{BC} = 2.\omega.v.$$





problem: A mechanism of a crank and slotted lever quick return motion is shown in the diagram. If the crank rotates counter clockwise at 120 rpm, determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever. Crank AB = 150 mm; Slotted arm OC = 700 mm and link CD = 200 mm;

N_{BA} = 120 rpm (anti-clockwise)

$$\omega_{BA} = \frac{2\pi N_{BA}}{60} = \frac{2\times\pi\times 120}{60} = 12.56 \text{ rad./sec.}$$

$$v_{BA} = \omega_{BA} \times (AB) = 12.56 \times 0.15 = 1.8849 \text{ m/sec} \quad (\bar{ab})$$

6.3 cm

To locate c;

$$\frac{ob'}{oc} = \frac{OB'}{OC}$$

1 cm = 50 mm

10.4 cm

ob' = ~~5.2 cm~~. 52 mm

oc = ?

OB' = 520 mm

OC = 700 mm

$$\frac{5.2 \cancel{52}}{oc} = \frac{52}{\cancel{10.4}/\cancel{70}}$$

$$\cancel{\therefore oc = \frac{5.2\times 70}{52}}$$

$$\frac{52}{oc} = \frac{520}{700}$$

$$oc = \frac{52\times 700}{520} = 70 \text{ mm}$$

From velocity diagram:

Measure

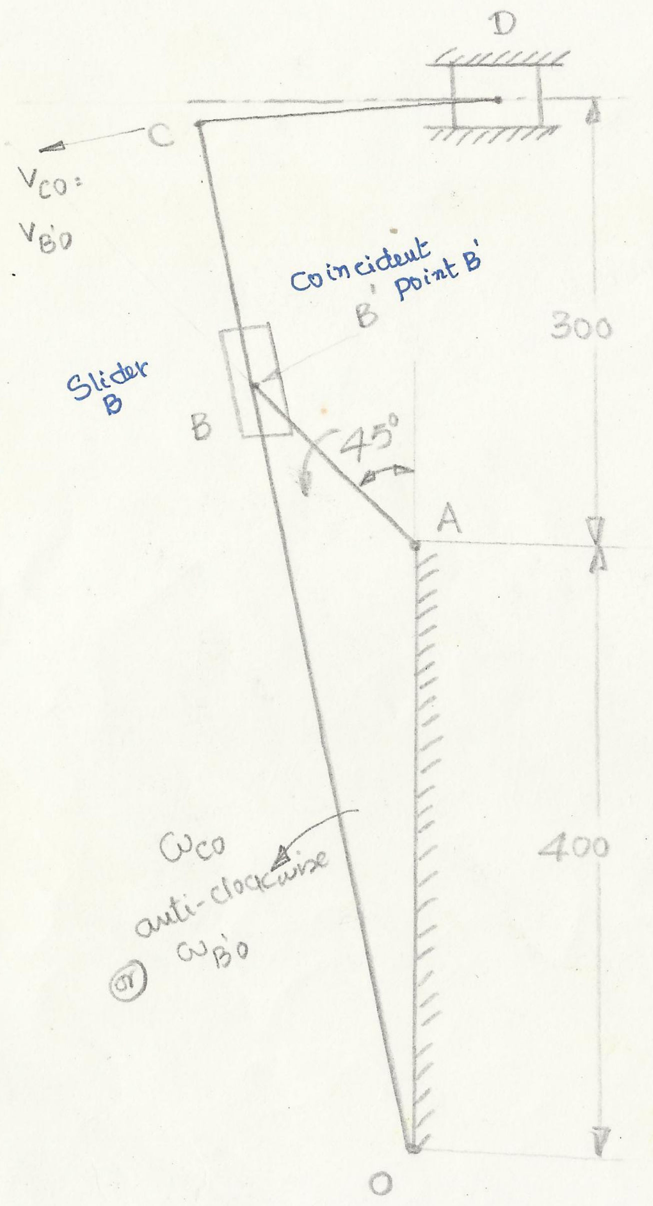
(1) vector $\bar{od}$ = 7 cm × 0.3 = 2.1 m/sec. = v_{DO}

(2) vector $b'b$ = 3.5 cm × 0.3 = 1.05 m/sec = $v_{BB'}$

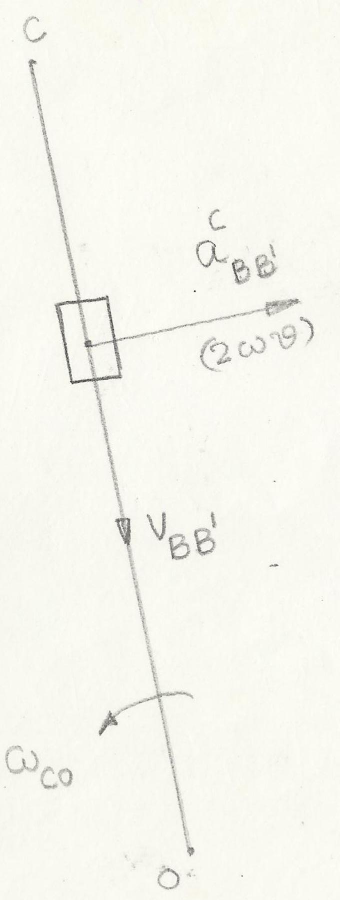
(3) vector cd = 1.4 cm × 0.3 = 0.42 m/sec = v_{DC}

(4) vector ob' = 5.2 cm × 0.3 = 1.56 m/sec = $v_{B'O}$

(5) vector oc = 7 cm × 0.3 = 2.1 m/sec = v_{CO}

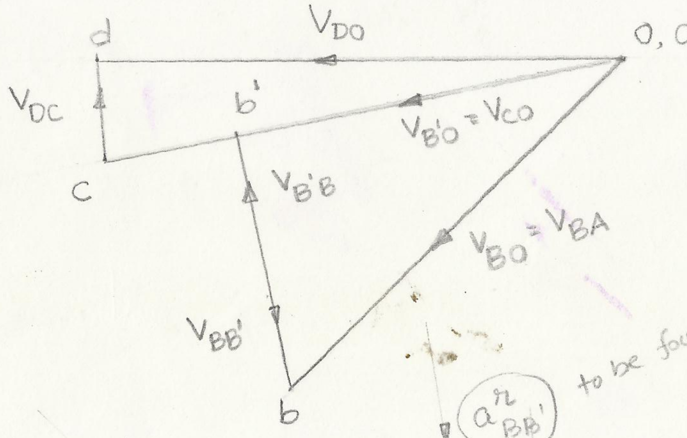


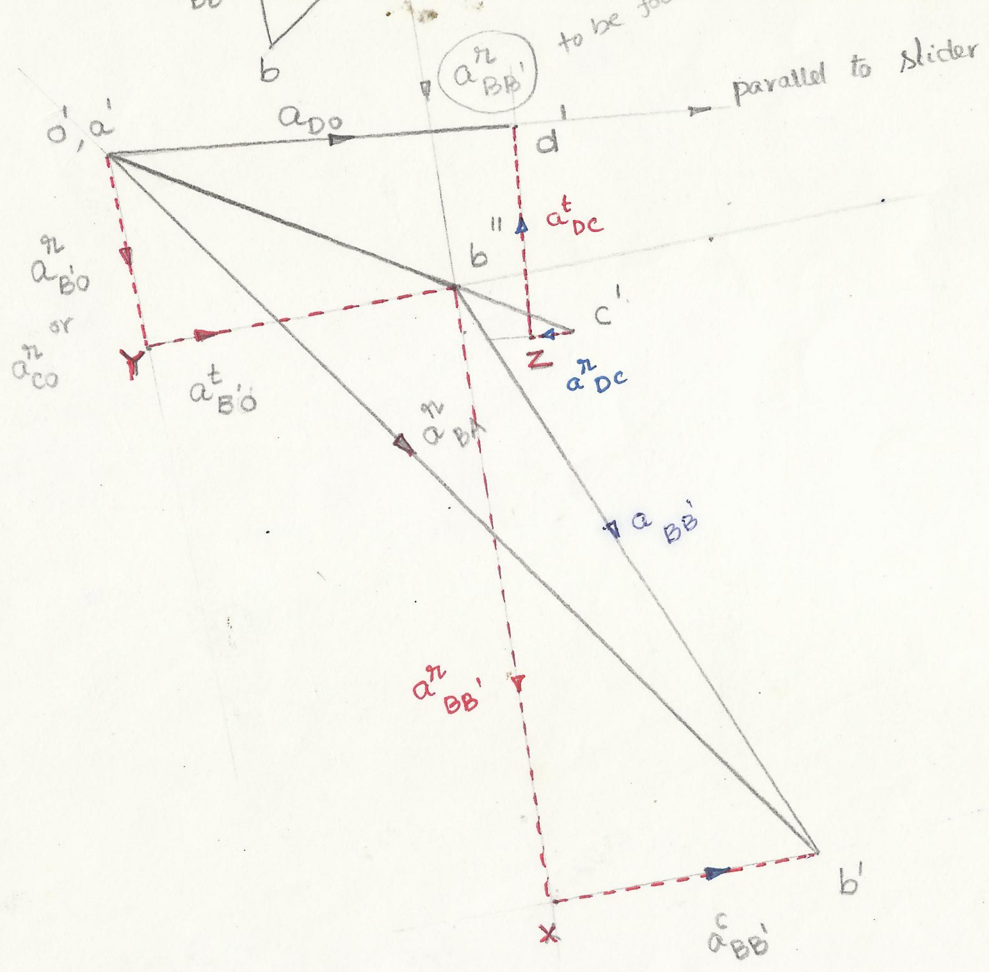
Configuration diagram: scale 50 mm = 1 cm.



Direction of Coriolis Component of Acceler.

Velocity diagram | Scale: 0.3 m/sec = 1 cm





Acceleration diagram

Scale 1.8 m/sec² = 1 cm

Angular velocity of link OC or OB'

$$\omega_{CO} \text{ or } \omega_{B'O} = \frac{v_{CO}}{OC} = \frac{2.1}{0.7} = 3 \text{ rad/sec.}$$

(Anti-clockwise direction)

<u>Acceleration</u> <u>diagram</u>:

radial component of Accele of B w.r.t A

$$a^r_{BA} = \frac{v_{BA}^2}{AB} = \frac{1.8849^2}{0.15} = 23.68 \text{ m/sec}^2 \quad (13.2)$$

Coriolis Comp. of accele. of slider B w.r.t Coincident point B'

$$a^c_{BB'} = 2.\underbrace{\omega}_{(\omega_{CO})}.\underbrace{v}_{(v_{BB'})} = 2 \times 3 \times 1.05$$

$$= 6.3 \text{ m/sec}^2 \quad (3.5)$$

radial Comp. of accele. of D w.r.t C

$$a^r_{DC} = \frac{V_{DC}^2}{CD} = \frac{0.42^2}{0.2} = 0.882 \text{ m/sec}^2. \quad (0.49 \simeq 0.5 \text{ cm})$$

radial Comp. of Accele. of the Coincident point B' w.r.t O

$$a^r_{B'O} = \frac{v_{B'O}^2}{OB'} = \frac{1.56^2}{0.52} = 4.68 \text{ m/sec}^2. \quad (2.6 \text{ cm})$$

($a^r_{BB'}$ Cannot cal) To be found

to find : <u>o'c'</u>

$$\frac{o'b''}{o'c'} = \frac{OB'}{OC}$$

$$\frac{4.9 \text{ cm}}{o'c'} = \frac{520}{700} \qquad \therefore o'c' = 65.96 \text{ mm} \simeq 66 \text{ mm}$$

From Acceleration diagram;

Measure vector $o'd' = 5.4 \times 1.8 = 9.72\ m/sec^2$

Angular Accele. of slotted lever :

measure vector $y b'' = 4.2 \times 1.8 = 7.56\ m/sec^2$ $(a^t_{B'O})$

Angular Accele. of slotted lever $= \alpha_{B'O} = \frac{a^t_{B'O}}{OB'}$

$= \frac{7.56}{0.52} = 14.53\ rad/sec^2$

(Anti-clockwise)
(Since $\omega_{B'O}$ is anti-clockwise)

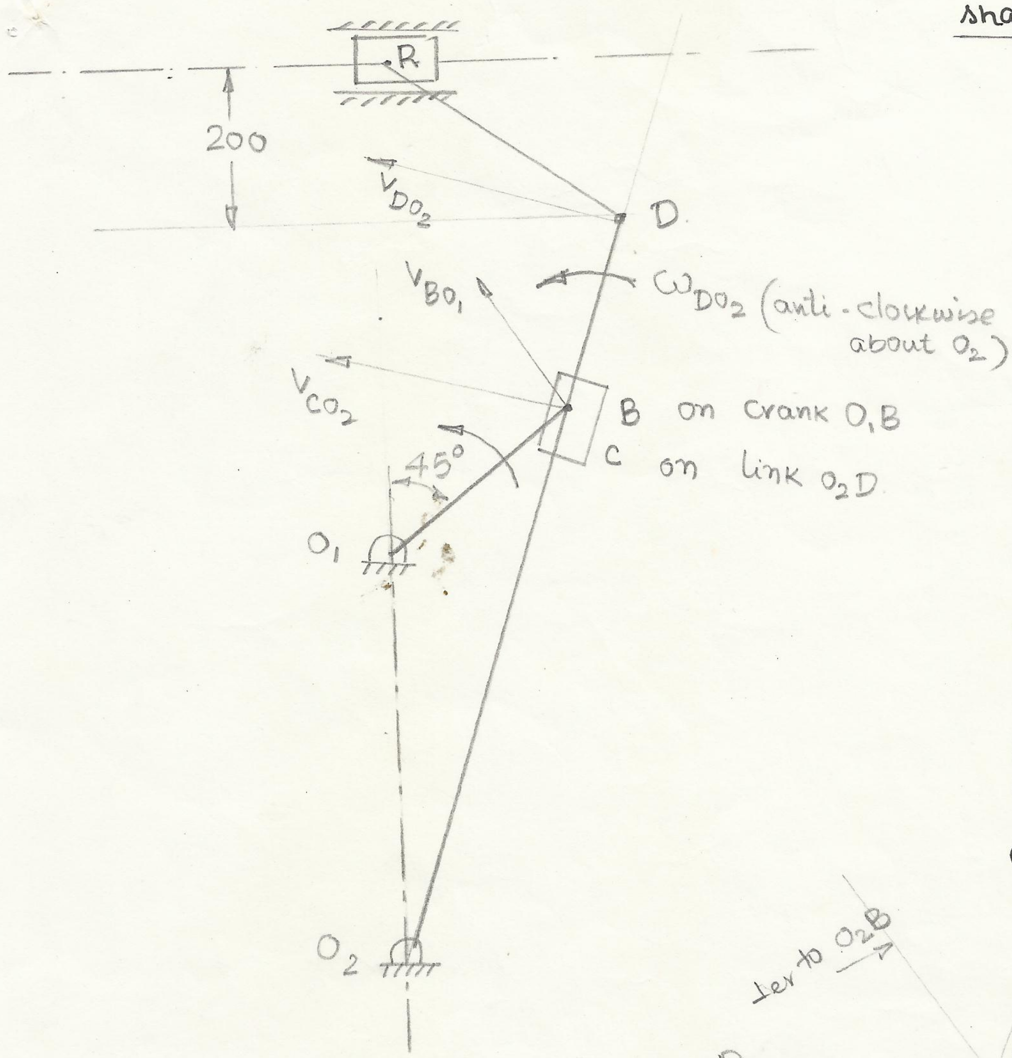
Procedure for constructing Accele. diagram:

1. Locate $o'a'$.
2. Draw a line ||el to link BA (a^r_{BA}) and locate b'
3. From b', draw a line ||el to $a^c_{BB'}$ (Coriolis comp. of accele.) and locate x.
4. from x draw a ⊥er [which represents $a^r_{BB'}$ - (which can't calculate)]
5. from o' draw a line parallel to slotted link $(a^r_{B'O}$ -) and locate Y
6. from Y draw a line ⊥er
7. Line from step 4. and line from step 6 will meet a b''.
8. to locate c' - according to proportion $\frac{o'b''}{o'c'} = \frac{O'B'}{OC}$
9. from c', draw a line parallel to link CD (a^r_{DC}) and locate Z.
10. From Z, draw a perpendicular.
11. Draw a line parallel to slider through o', a'.
12. Line from step 10 and line from step 11 will intersect at d'.
13. Measure the required lengths and find the value of $a^t_{B'O}$ and then calculate $\alpha_{B'O}$.

Accel. of B, wrt B'

Accele. of B' w.r.t O'

Quick-return mechanism, Crank and slotted lever type shaping machine.



$O_2D = 1300$ mm

$O_1O_2 = 800$ mm

$RD = 400$ mm

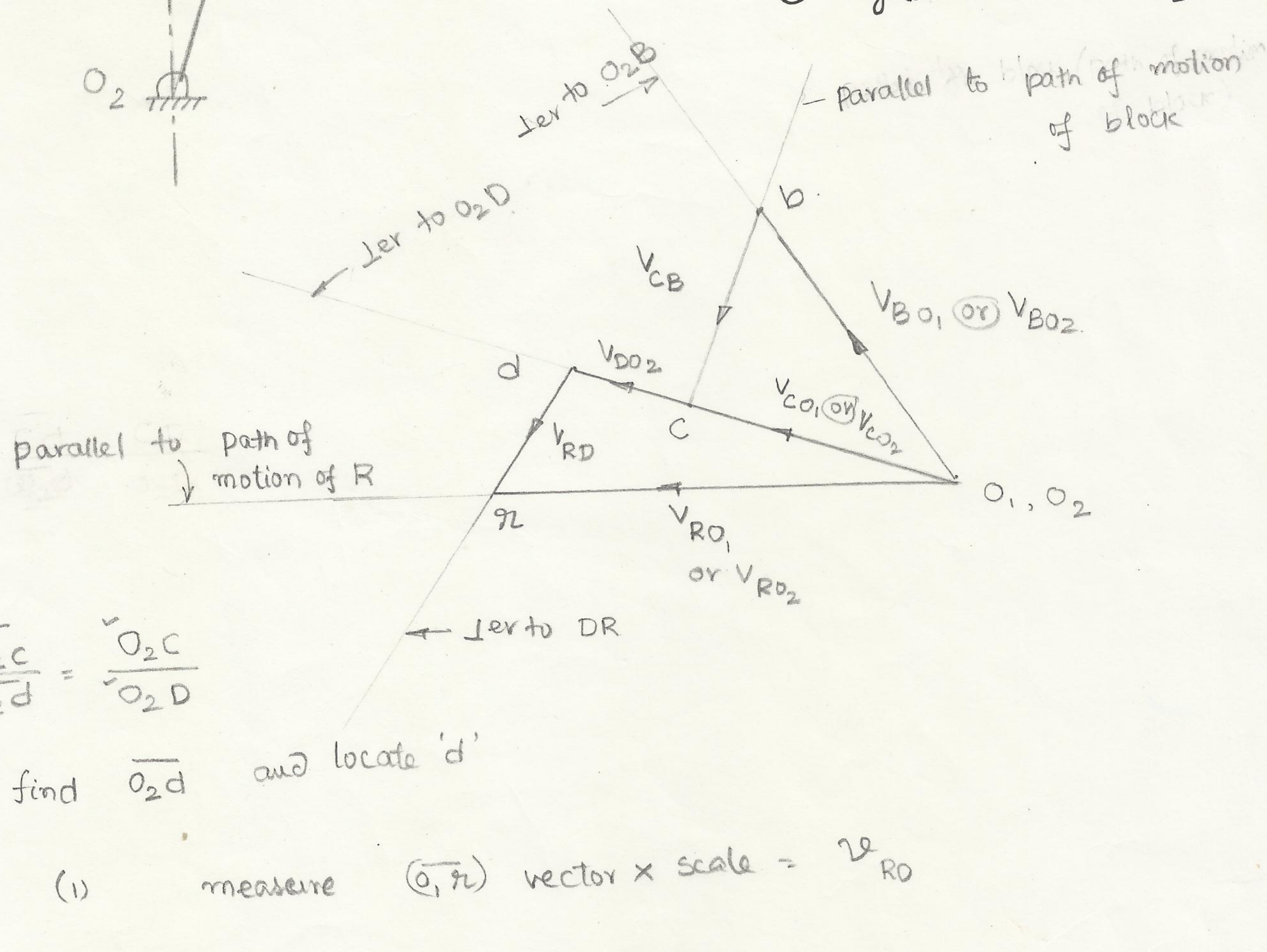
$O_1B = 300$ mm

The crank O_1B makes an angle of 45° with the vertical and rotates at 40 rpm in the counter clockwise direction

Find

① vel. of ram Ⓡ

② Angular vel. of link O_2D



$$\frac{\overline{O_2c}}{\overline{O_2d}} = \frac{O_2C}{O_2D}$$

find $\overline{O_2d}$ and locate 'd'

(1) measure $(\overline{O_1r})$ vector × scale = v_{RO}

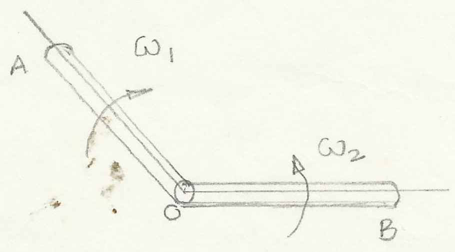
(2) Angular vel. of link O_2D,

measure $\overline{O_2d}$ × scale = v_{DO_2}

$$\omega_{DO_2} = \frac{v_{DO_2}}{\text{link length } DO_2} = \text{rad./sec}$$

Rubbing velocity at a pin Joint :

The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin Joints, multiplied by the radius of the pin.



Rubbing velocity at the pin Joint O

$= (\omega_1 - \omega_2)\, r$ — if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$ - If the links move in the opposite direction

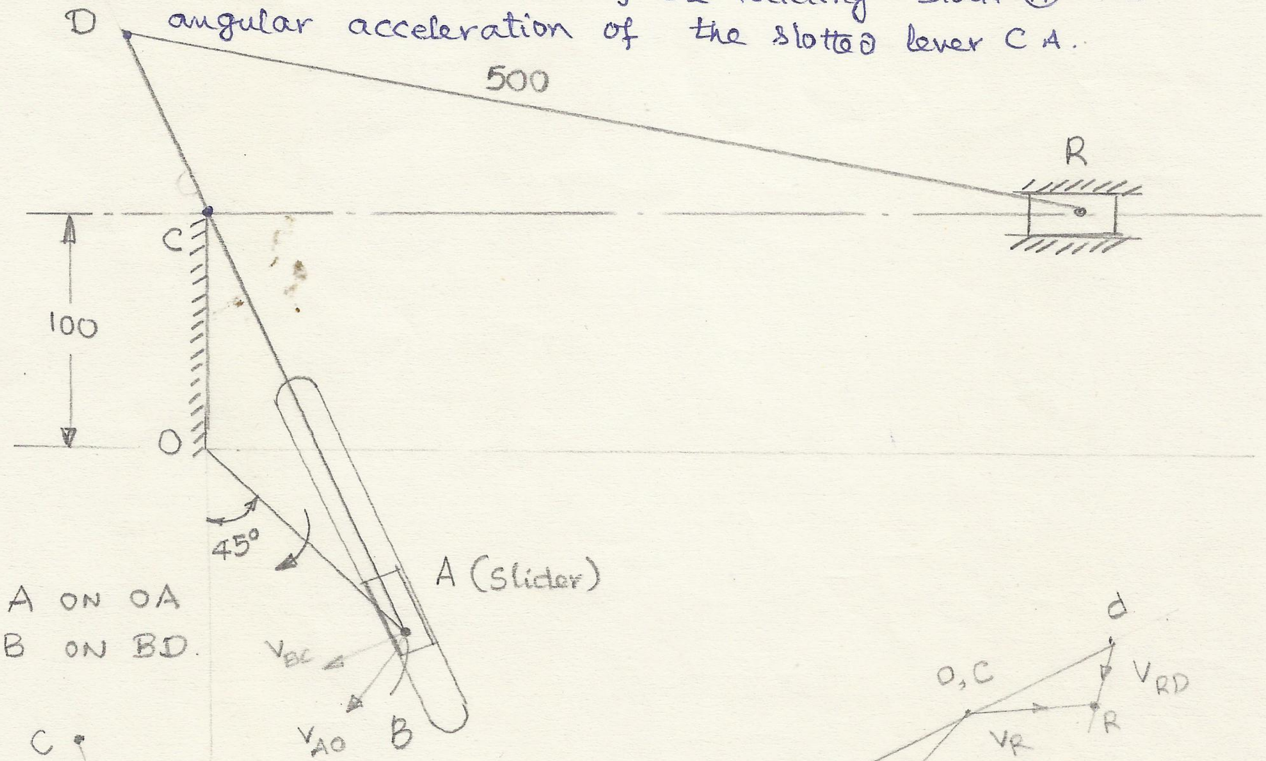
Note : When the pin connects one sliding and the other turning member, the angular velocity of the sliding member is zero. In such cases,

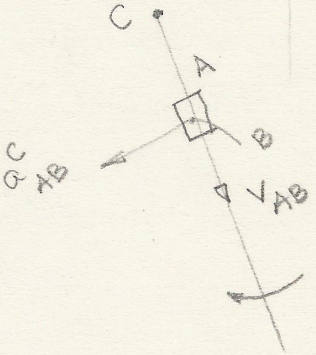
Rubbing velocity at the pin Joint $= \omega . r$

where, ω = Angular velocity of turning member

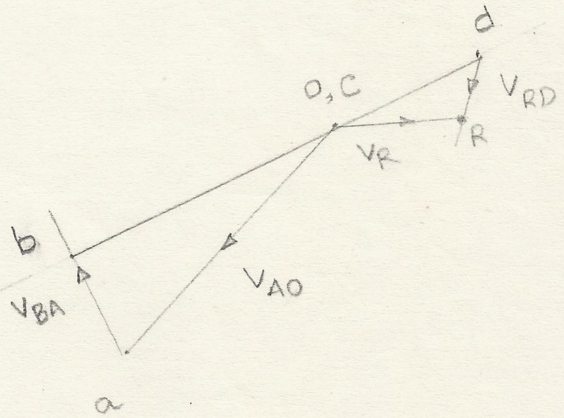
r = radius of the pin.

In a whitworth Quick return motion, as shown in the diagram, OA is a crank rotating at 30 rpm, in a clockwise direction. The dimensions of various links are $OA = 150$ mm ; $OC = 100$ mm ; $CD = 125$ mm ; $DR = 500$ mm. Determine the acceleration of the sliding block (R) and the angular acceleration of the slotted lever CA.

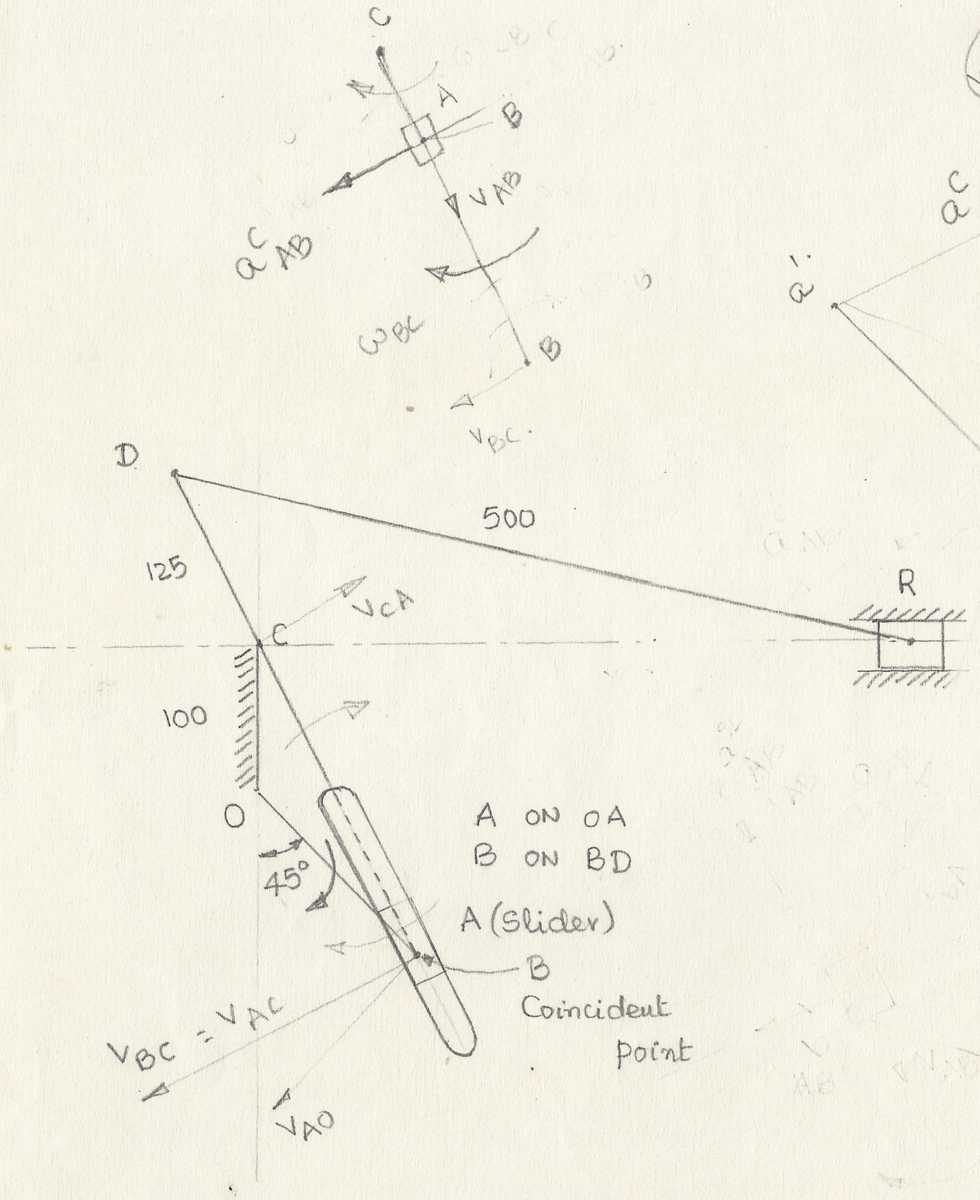




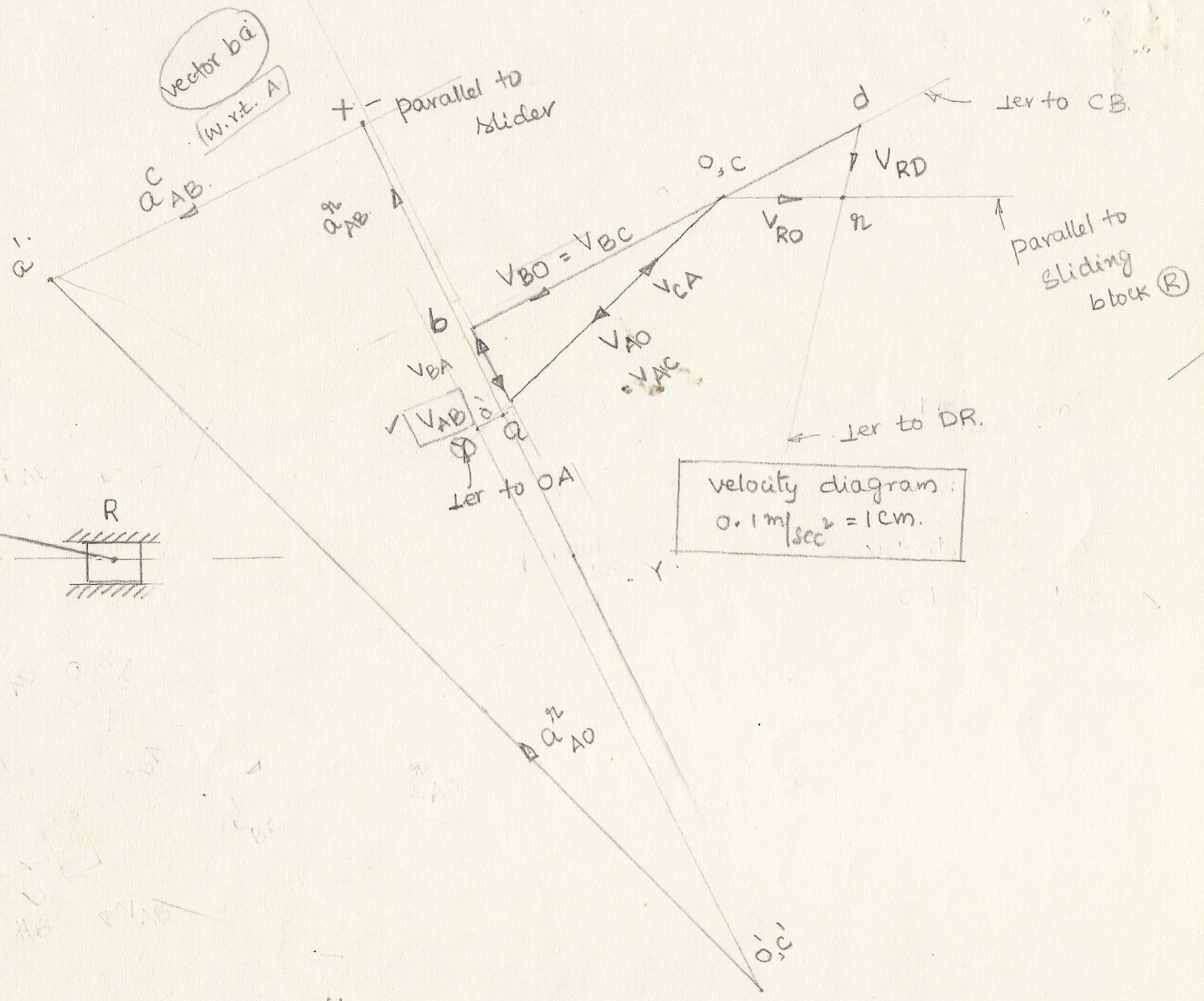
$$\frac{bd}{dC} = \frac{BD}{BC}$$



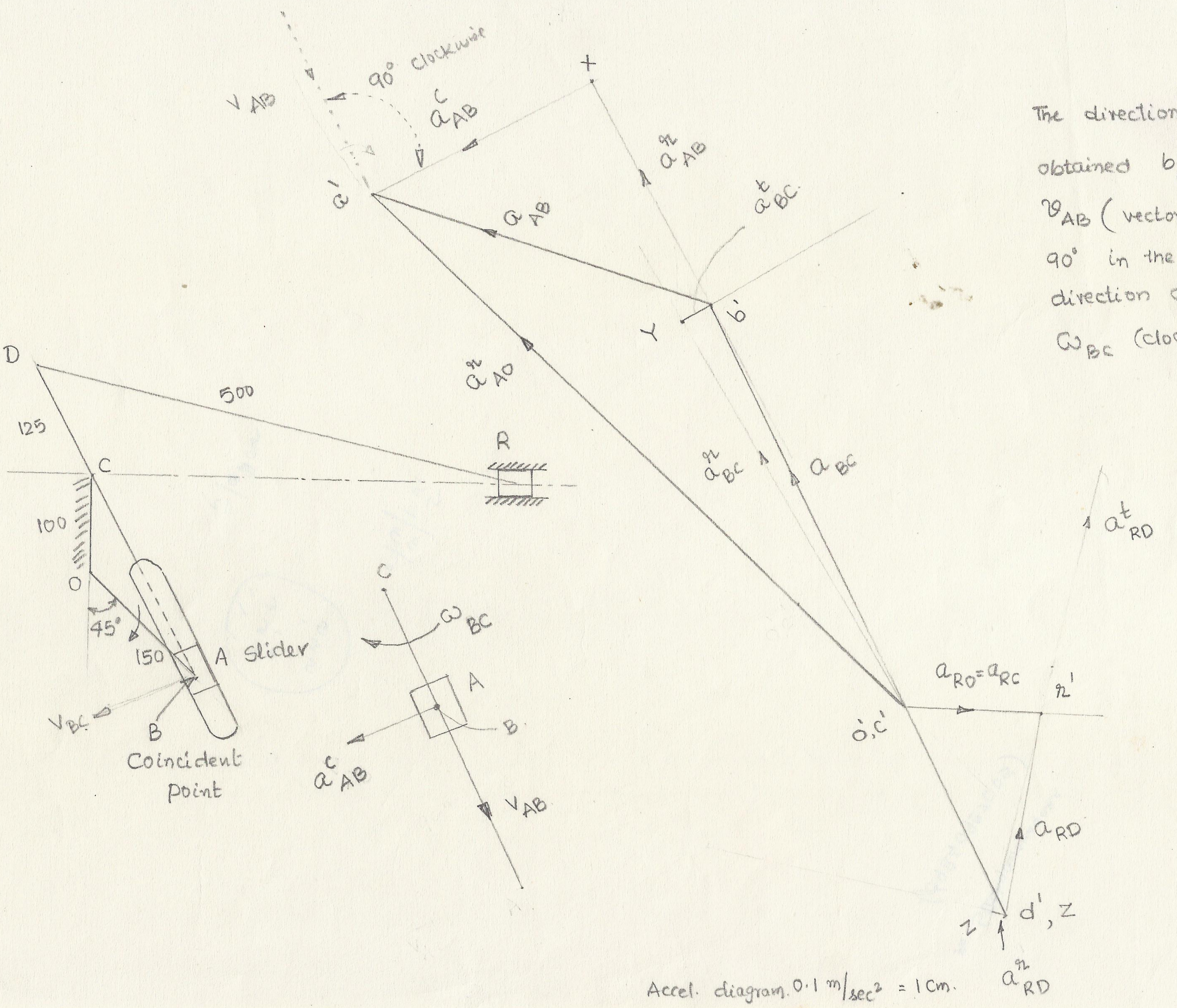
velocity of Ram.



Configuration diagram
Scale : 50mm = 1 cm.



velocity diagram : $0.1\ m/sec^2 = 1\ cm$.



The direction of a^c_{AB} is obtained by rotating v_{AB} (vector ba) through 90° in the same direction as that of ω_{BC} (clockwise)

Accel. diagram. $0.1\ m/sec^2 = 1\ cm$.

$N_{AO} = 30$ rpm ; $\omega_{AO} = \frac{2\pi N_{AO}}{60} = \frac{2\pi \times 30}{60} = 3.142$ rad/sec

OA = 150 mm = 0.15 m ; OC = 100 mm = 0.1 m ; CD = 125 mm = 0.125 mm ;

DR = 500 mm = 0.5 m ;

vel. of A w.r.t O = $V_{AO} = \omega_{AO} \times OA = 3.142 \times 0.15$

$= 0.4713$ m/sec. $(\overline{oa})$

To locate d in the velocity diagram,

$$\frac{\overline{bd}}{\overline{bc}} = \frac{BD}{BC}$$

BD = 360 mm

BC = 235 mm

$\overline{bc}$ = 46 mm

$$\therefore \overline{bd} = \frac{360 \times 46}{235} = 70.46 \text{ mm}$$

$\approx 7.046 \approx 7$ cm.

From velocity diagram,

Measure the following vectors and find velocities.

(1) vector $\overline{ba} = 1.3 \times 0.1 = 0.13$ m/sec $= v_{AB}$ (vel. of A w.r.t B)

(2) vector $\overline{ob} = 4.5 \times 0.1 = 0.45$ m/sec $= v_{BO}$ (vel. of B. w.r.t O)
or (or)
(vector $\overline{cb}$) $= v_{BC}$

(3) vector $\overline{dr} = 1.2 \times 0.1 = 0.12$ m/sec $= v_{RD}$ (vel. of R w.r.t D)

(4) vector $\overline{or} = 1.9 \times 0.1 = 0.19$ m/sec $= v_{RO}$ = velocity of slider

To find Acceleration of the sliding block (R) :

1. radial comp. of accele. of A w.r.t O $= a^r_{AO} = \frac{V_{AO}^2}{OA}$

$= \frac{0.4713^2}{0.15} = 1.48\ m/sec^2$ ($o'a'$)

$= 1.5\ m/sec^2$ (15 cm)

2. Coriolis component of accele. of A w.r.t Coincident point B

$a^C_{AB} = 2.\ \omega_{BC}\ v_{AB}$

$= 2 \times 1.9148 \times \cancel{0.15}\ 0.13$

$= 0.4978\ m/sec^2$

$= 0.5\ m/sec^2$ (5cm)

$\omega_{BC} = \frac{v_{BC}}{BC}$

$= \frac{0.45}{0.235}$

$= 1.9148\ rad/sec.$

(Clock-wise direction)

3. radial Component of accele. of B w.r.t. C

$= a^r_{BC} = \frac{v_{BC}^2}{CB} = \frac{0.45^2}{0.235}$

$= 0.8617\ m/sec^2$ ($c'b'$)

$= 0.91\ m/sec^2$ (9 cm)

4. radial Component of accele. of R w.r.t. D

$= a^r_{RD} = \frac{v_{RD}^2}{DR} = \frac{0.12^2}{0.5}$

$= 0.0288\ m/sec^2$ ($d'r'$)

$\simeq 0.03\ m/sec^2$ (0.3 cm)

Scale $0.1\ m/sec^2 = 1cm$

To find d'

$$\frac{\overline{b'd'}}{\overline{b'c'}} = \frac{BD}{BC}$$

$\overline{b'c'} = \cancel{143}\ 90$

$BD = 360\ mm$

$BC = 235\ mm$

$$b'd' = \frac{360 \times \cancel{143}\ 90}{235} = 2\cancel{18}.06\ mm$$

$\simeq$ ~~219 mm~~

137 mm $= 13.7\ cm$

from : Accele. dia :

measure

(1) $\overline{\cancel{b'}yb'} = 0.7\ cm \times \text{scale} \quad a^t_{BC}$

$= 0.7 \times 0.1 = 0.07\ m/sec^2$

$$\alpha_{BC} = \frac{a^t_{BC}}{BC} = \frac{0.07}{0.235} = \quad \alpha_{CA} = 0.2978\ rad/sec^2$$

or

α_{BC}

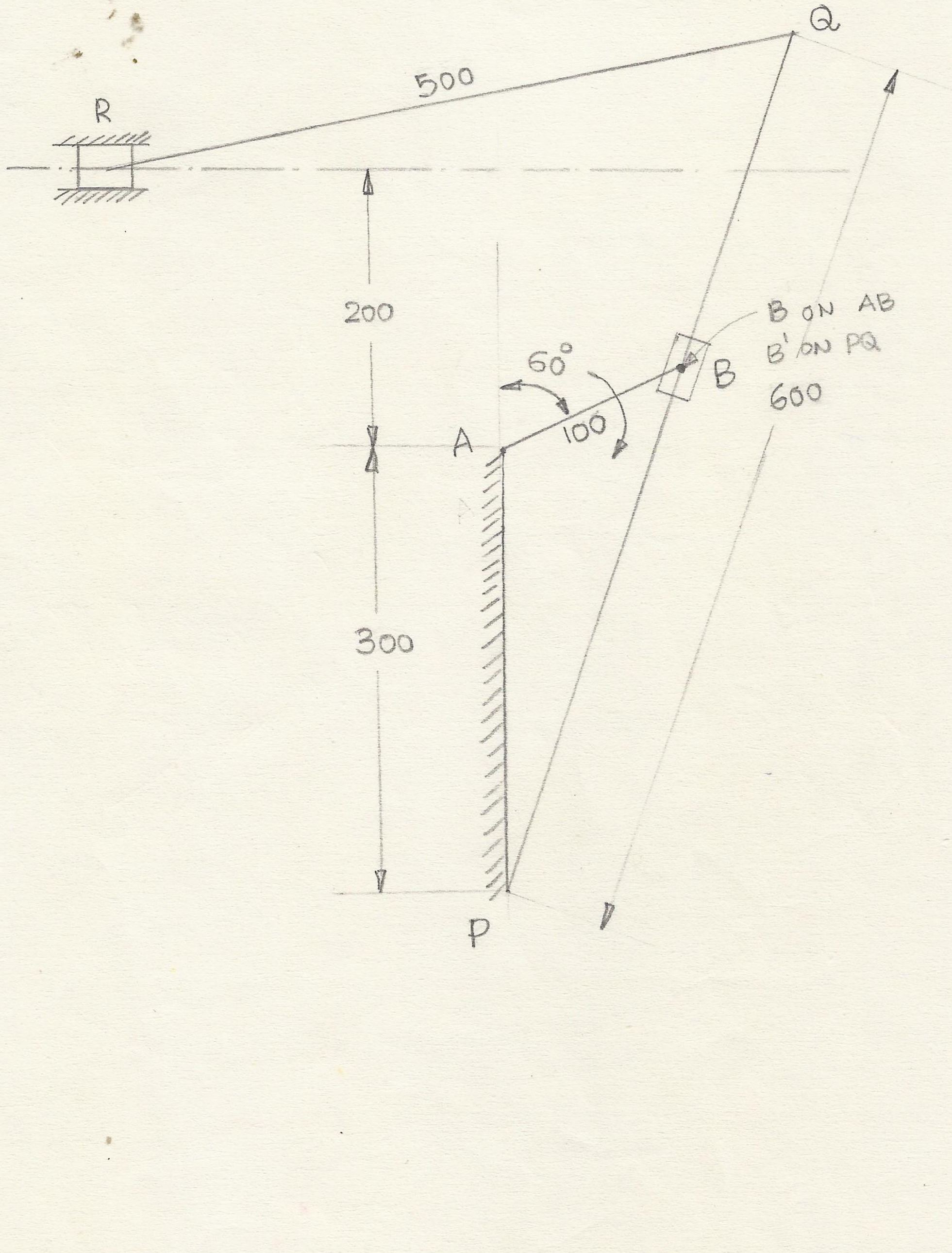
(2) measure $\overline{c'r'} = 2.7 \times 0.1$

$= 0.27\ m/sec^2$

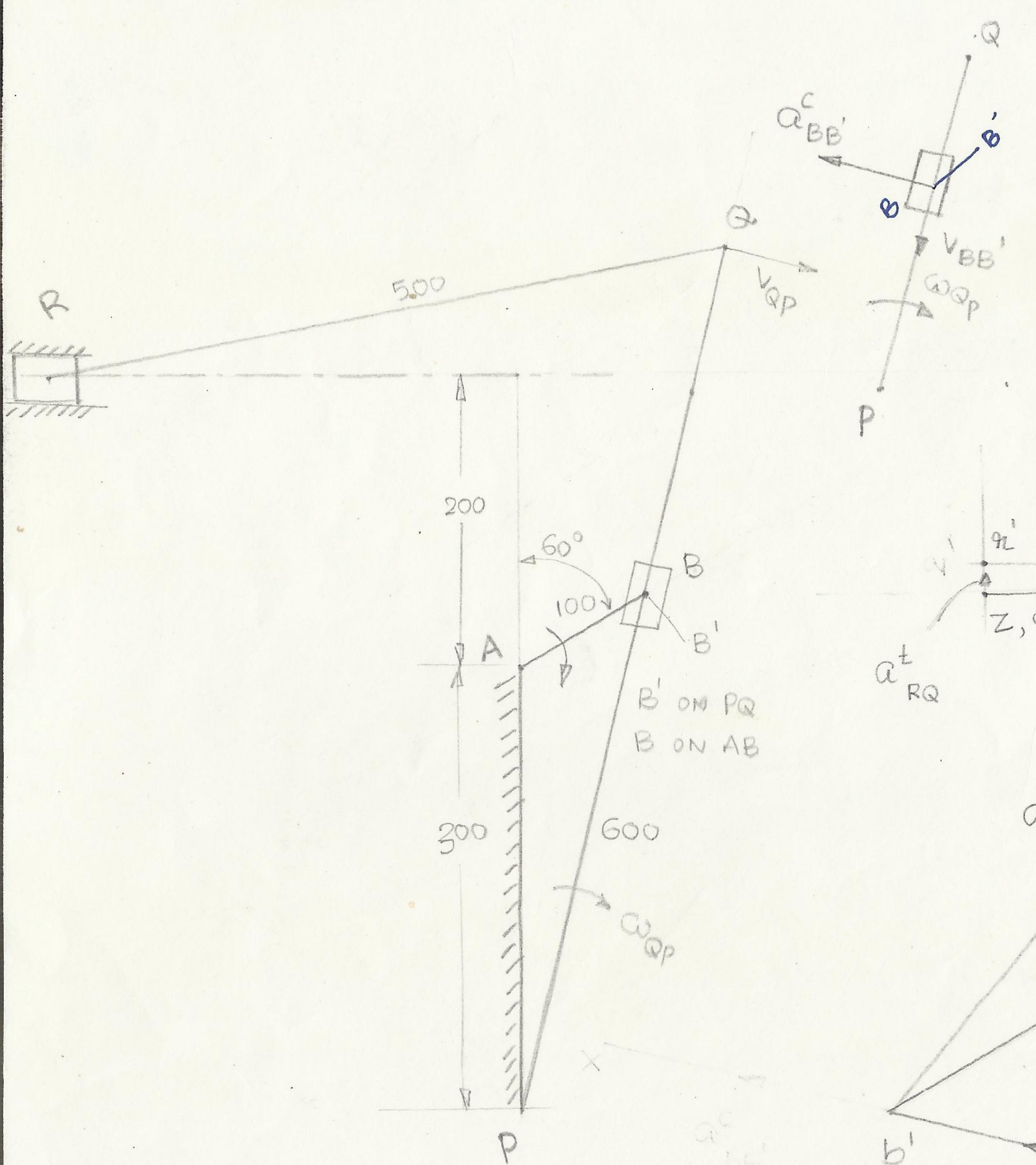
$a_{RC} = 0.27\ m/sec^2$ (clockwise) direction

Problem: (Page No 201)

The driving Crank AB of the Quick-return mechanism, as shown in the diagram, revolves at a uniform speed of 200rpm (clockwise). Find the velocity and Acceleration of the tool-box R, in the position shown, when the crank makes an angle of 60° with the vertical line of Centres PA. What is the acceleration of sliding of the block at B along the slotted lever PQ?

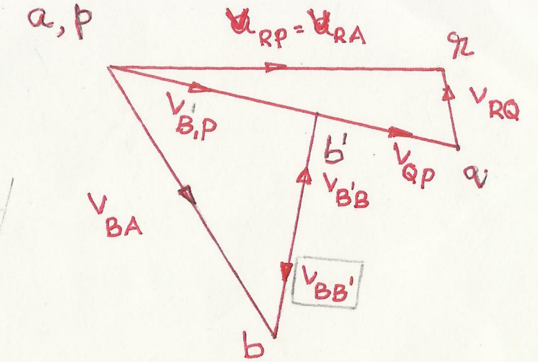


$N_{BA} = 200\ rpm$

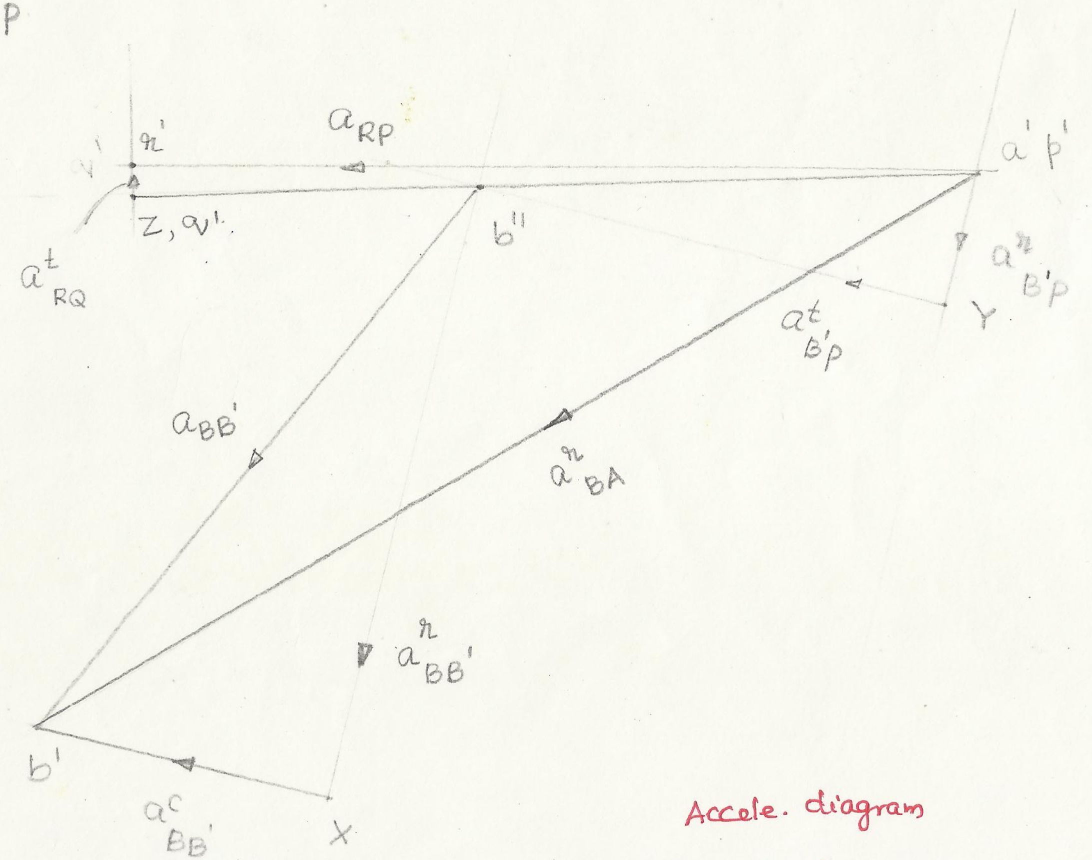


Configuration diagram

50mm = 1cm



velocity diagram
Scale 0.5 m/sec = 1cm.



Accele. diagram

Scale 3 m/sec^2 = 1 cm

1

$N_{BA} = 200$ rpm

$\omega_{BA} = \frac{2\pi \times 200}{60} = 20.94$ rad./sec.

$v_{BA} = \omega_{BA} \times AB = 20.94 \times \cancel{100}\,0.1 = 2.094$ m/sec (4.2)

to locate q :

$\frac{\overline{pb'}}{\overline{pq}} = \frac{PB'}{PQ}$

$pb' = 29$ mm

$PB' = 7.2$ cm. $\times$ 50 scale

$= 360$ mm

$PQ = 600$

$\frac{29}{\overline{pq}} = \frac{360}{600}$

$\overline{pq} = 48.3$ mm

From velocity diagram,

Measure

1. vector ar $= 4.5 \times 0.5 = 2.25$ m/sec $= v_{RA}$ or v_{RP}
2. vector (b'b) $= 3 \times 0.5 = 1.5$ m/sec $= \cancel{v_{BB}}\ v_{BB'}$
3. vector pb' $= 2.8 \times 0.5 = 1.4$ m/sec $= v_{B'P}$
4. vector qr $= 1.1 \times 0.5 = 0.55$ m/sec $= v_{RQ}$
5. vector pq $= 4.8 \times 0.5 = 2.4$ m/sec $= v_{QP}$

To Draw Acceleration diagram,

Calculate

1. radial Component of Accele of B w.r.t A $= a^r_{BA} = \frac{V_{BA}^2}{AB} = \frac{2.094^2}{0.1}$

$= 43.84$ m/sec^2

14.61 cm

2. Coriolis Component of Accele. of B w.r.t B' $= a^c_{BB'} = \cancel{\frac{v^2_{BB'}}{}}$

$= 2\,\omega . v$

$\omega = \omega_{QP} = \frac{v_{QP}}{PQ} = \frac{2.4}{0.6} = 4$ rad/sec

$v = v_{BB'} = 1.5$

$\therefore a^c_{BB'} = 2 \times 4 \times 1.5 = 12$ m/sec^2

(4 cm)

3. Radial Component of Accele. of R w.r.t Q $= a^{r}_{RQ} = \frac{V_{RQ}^2}{QR}$

$= \frac{0.55^2}{0.5}$

$= 0.605\ m/sec^2$

(0.2 cm)

4. Radial Comp. of Accele. of B' w.r.t P $= a^{r}_{B'P} = \frac{v_{B'P}^2}{PB'}$

$= \frac{1.4^2}{0.365}$

$= 5.369\ m/sec^2$

$(1.789 \simeq 1.8\ cm)$

Accele. diagram scale $3\ m/sec^2 = 1\ cm$

to locate q'

$$\frac{p'b''}{p'q'} = \frac{PB'}{PQ}$$

$p'b'' = \cancel{67}\ 67\ mm$

$PB' = 7.2 \times 50 = 360\ mm$

$PQ = 600\ mm$

$$\frac{67}{p'q'} = \frac{360}{600}$$

$p'q' = 111.66\ mm$

$\simeq 112\ mm$

From accele. diagram,

Measure ① $a^{t}_{B'P} =$ vector $rb'' = 6.4 \times 3 = 19.2\ m/sec^2$

Angular Accele of Slotted link $\Big\}$ $\alpha_{B'P} = \frac{a^{t}_{B'P}}{PB'} = \frac{19.2}{0.36} = 53.33\ rad/sec^2$

② accel. of R w.r.t P

vector $\overline{p'r'} = 11.2 \times 3 = 33.6\ m/sec^2$

Accele. of Tool-box $= a_{RP} = 33.6\ m/sec^2$

③ Accel of $B^{\bullet}$ w.r.t $B^{\bullet\prime}$

vector $= b''b^{\bullet} = 9.3 \times 3 = 27.9$

$a_{BB'} = 27.9\ m/sec^2$