UNIT – V

HEAT TRANSFER

Heat Energy and Heat Transfer

Heat is a form of energy in transition and it flows from one system to another, without transfer of mass, whenever there is a temperature difference between the systems. The process of heat transfer means the exchange in internal energy between the systems and in almost every phase of scientific and engineering work processes, we encounter the flow of heat energy.

Importance of Heat Transfer

Heat transfer processes involve the transfer and conversion of energy and therefore, it is essential to determine the specified rate of heat transfer at a specified temperature difference. The design of equipments like boilers, refrigerators and other heat exchangers require a detailed analysis of transferring a given amount of heat energy within a specified time. Components like gas/steam turbine blades, combustion chamber walls, electrical machines, electronic gadgets, transformers, bearings, etc require continuous removal of heat energy at a rapid rate in order to avoid their overheating. Thus, a thorough understanding of the physical mechanism of heat flow and the governing laws of heat transfer are a must.

Modes of Heat Transfer

The heat transfer processes have been categorized into three basic modes: Conduction, Convection and Radiation.

Conduction – It is the energy transfer from the more energetic to the less energetic particles of a substance due to interaction between them, a microscopic activity.

Convection - It is the energy transfer due to random molecular motion a long with the macroscopic motion of the fluid particles.

Radiation - It is the energy emitted by matter which is at finite temperature. All forms of matter emit radiation attributed to changes m the electron configuration of the constituent atoms or molecules The transfer of energy by conduction and convection requires the presence of a material medium whereas radiation does not. In fact radiation transfer is most efficient in vacuum.

All practical problems of importance encountered in our daily life Involve at least two, and sometimes all the three modes occuring simultaneously When the rate of heat flow is constant, i.e., does not vary with time, the process is called a steady state heat transfer process. When the temperature at any point in a system changes with time, the process is called unsteady or transient process. The internal energy of the system changes in such a process when the temperature variation of an unsteady process describes a particular cycle (heating or cooling of a budding wall during a 24 hour cycle), the process is called a periodic or quasi-steady heat transfer process.

Heat transfer may take place when there is a difference In the concentration of the mixture components (the diffusion thermoeffect). Many heat transfer processes are accompanied by a transfer of mass on a macroscopic scale. We know that when water evaporates, the heal transfer is accompanied by the transport of the vapour formed through an air-vapour mixture. The transport of heat energy to steam generally occurs both through molecular interaction and convection. The combined molecular and convective transport of mass is called convection mass transfer and with this mass transfer, the process of heat transfer becomes more complicated.

Mechanism of Heat Transfer by Conduction

The transfer of heat energy by conduction takes place within the boundaries of a system, or a cross the boundary of t he system into another system placed in direct physical contact with the first, without any appreciable displacement of matter comprising the system, or by the exchange of kinetic energy of motion of the molecules by direct communication, or by drift of electrons in the case of heat conduction in metals. The rate equation which describes this mechanism is given by Fourier Law

$$\dot{Q} = -kA dT / dx$$

where $\dot{Q}$ = rate of heat flow in X-direction by conduction in J/S or W,

k = thermal conductivity of the material. It quantitatively measures the heat conducting ability and is a physical property of t he material that depends upon the composition of the material, W/mK,

A = cross-sectional area normal to the direction of heat flow, m^2, dT/dx = temperature gradient at the section, as shown in Fig. 1 I The neganve sign IS Included to make the heat transfer rate Q positive in the direction of heat flow (heat flows in the direction of decreasing temperature gradient).

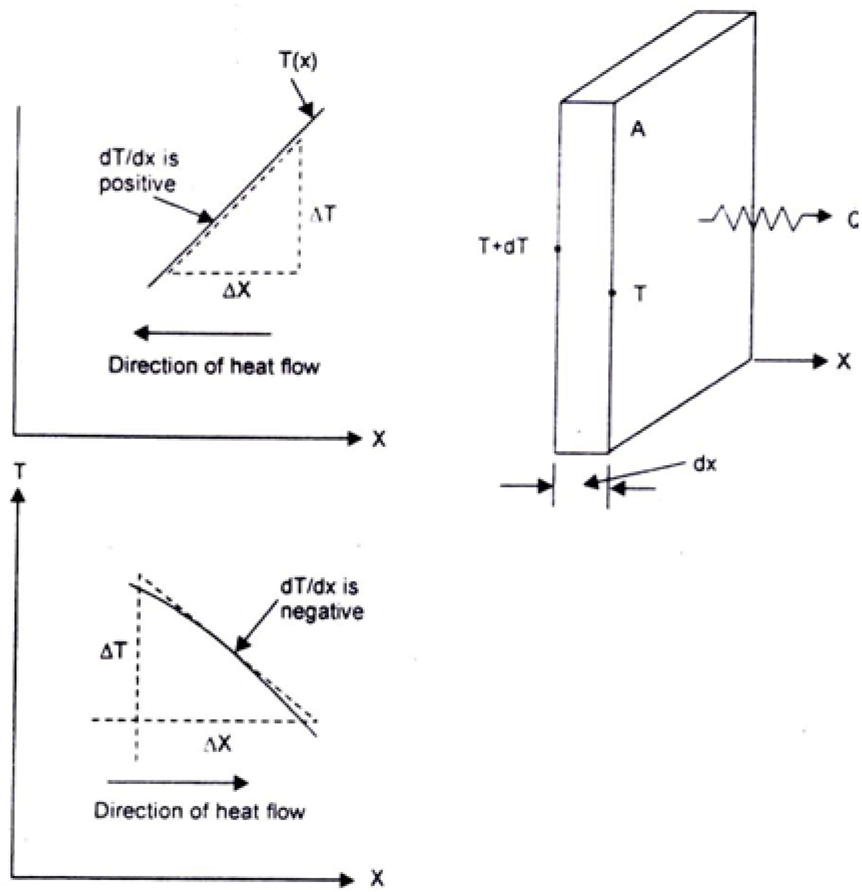


Fig 1.1: Heat flow by conduction

Thermal Conductivity of Materials

Thermal conductivity is a physical property of a substance and In general, It depends upon the temperature, pressure and nature of the substance. Thermal conductivity of materials are usually determined experimentally and a number of methods for this purpose are well known.

Thermal Conductivity of Gases: According to the kinetic theory of gases, the heat transfer by conduction in gases at ordinary pressures and temperatures take place through the transport of the kinetic energy arising from the collision of the gas molecules. Thermal conductivity of gases depends on pressure when very low «2660 Pal or very high ($> 2 \times 10^9$ Pa). Since the specific heat of gases Increases with temperature, the thermal conductivity Increases with temperature and with decreasing molecular weight.

Thermal Conductivity of Liquids: The molecules of a liquid are more closely spaced and molecular force fields exert a strong influence on the energy exchange In the collision process. The mechanism of heat propagation in liquids can be conceived as transport of energy by way of unstable elastic oscillations. Since the density of liquids decreases with increasing temperature, the thermal conductivity of non-metallic liquids generally decreases with increasing temperature, except for liquids like water and alcohol because their thermal conductivity first Increases with increasing temperature and then decreases.

Thermal Conductivity of Solids (i) Metals and Alloys: The heat transfer in metals arise due to a drift of free electrons (electron gas). This motion of electrons brings about the equalization in temperature at all points of t he metals. Since electrons carry both heat and electrical energy. The thermal conductivity of metals is proportional to its electrical conductivity and both the thermal and electrical conductivity decrease with increasing temperature. In contrast to pure metals, the thermal conductivity of alloys increases with increasing temperature. Heat transfer In metals is also possible through vibration of lattice structure or by elastic sound waves but this mode of heat transfer mechanism is insignificant in comparison with the transport of energy by electron gas. (ii) Nonmetals: Materials having a high volumetric density have a high thermal conductivity but that will depend upon the structure of the material, its porosity and moisture content High volumetric density means less amount of air filling the pores of the materials. The thermal conductivity of damp materials considerably higher than the thermal conductivity of dry material because water has a higher thermal conductivity than air. The thermal conductivity of granular material increases with temperature. (Table 1.2 gives the thermal conductivities of various materials at $0^{o}C$.)

STEADY STATE CONDUCTION ONE DIMENSION

The General Heat Conduction Equation

Any physical phenomenon is generally accompanied by a change in space and time of its physical properties. The heat transfer by conduction in solids can only take place when there is a variation of temperature, in both space and time. Let us consider a small volume of a solid element as shown in Fig. 1.2. The dimensions are: Δx, Δy, Δz along the X-, Y-, and Z-coordinates.

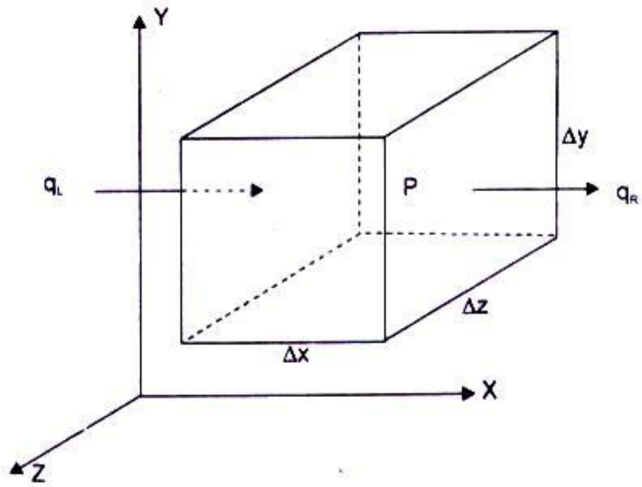


Fig 1.2 Elemental volume in Cartesian coordinates

First we consider heat conduction the X-direction. Let T denote the temperature at the point P (x, y, z) located at the geometric centre of the element. The temperature gradient at the left hand face (x - ~x12) and at the right hand face (x + □x/2), using the Taylor's series, can be written as:

$\partial T/\partial x|_L = \partial T/\partial x - \partial^2 T/\partial x^2 . \Delta x/2$ + higher order terms.

$\partial T/\partial x|_R = \partial T/\partial x + \partial^2 T/\partial x^2 . \Delta x/2 +$ higher order terms.

The net rate at which heat is conducted out of the element 10 X-direction assuming k as constant and neglecting the higher order terms,

$$\text{we get } -k\Delta y\Delta z\left[\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial x^2}\frac{\Delta x}{2}-\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial x^2}\frac{\Delta x}{2}\right] = -k\Delta y\Delta z\Delta x\left(\frac{\partial^2 T}{\partial x^2}\right)$$

Similarly for Y- and Z-direction,

We have $-k\Delta x\Delta y\Delta z\ \partial^2 T/\Delta y^2$ and $-k\Delta x\Delta y\Delta z\ \partial^2 T/\Delta z^2$.

If there is heat generation within the element as Q, per unit volume and the internal energy of the element changes with time, by making an energy balance, we write

Heat generated within the element	Heat conducted away from the element	Rate of change of internal energy within with the element

or, $\dot{Q}_v(\Delta x \Delta y \Delta z) + k(\Delta x \Delta y \Delta z)\left(\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2\right)$

$$= \rho c(\Delta x \Delta y \Delta z)\, \partial T/\partial t$$

Upon simplification, $\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2 + \dot{Q}_v/k = \dfrac{\rho c}{k}\partial T/\partial t$

or, $\nabla^2 T + \dot{Q}_v/k = 1/\alpha(\partial T/\partial t)$

where $\alpha = k/\rho\,.\,c$, is called the thermal diffusivity and is seen to be a physical property of the material of which the solid is composed.

One-Dimensional Heat Flow

The term 'one-dimensional' is applied to heat conduction problem when:

(i) Only one space coordinate is required to describe the temperature distribution within a heat conducting body;

(ii) Edge effects are neglected;

(iii) The flow of heat energy takes place along the coordinate measured normal to the surface.

Thermal Diffusivity and its Significance

Thermal diffusivity is a physical property of the material, and is the ratio of the material's ability to transport energy to its capacity to store energy. It is an essential parameter for transient processes of heat flow and defines the rate of change in temperature. In general, metallic solids have higher while nonmetallics, like paraffin, have a lower value of . Materials having large respond quickly to changes in their thermal environment, while materials having lower a respond very slowly, take a longer time to reach a new equilibrium condition.

TEMPERATURE DISTRIBUTIONS

A Plane Wall

A plane wall is considered to be made out of a constant thermal conductivity material and extends to infinity in the Y- and Z-direction. The wall is assumed to be homogeneous and isotropic, heat flow is one-dimensional, under steady state conditions and losing negligible energy through the edges of the wall under the above mentioned assumptions the Eq.

$d^2T / dx^2 = 0$; the boundary conditions are: at $x = 0, T = T_1$

Integrating the above equation, $x = L, T = T_2$

$T = C_1x + C_2$, where C_1 and C_2 are two constants.

Substituting the boundary conditions, we get $C_2 = T_1$ and $C_1 = (T_2 - T_1)/L$ The temperature distribution in the plane wall is given by

$$T = T_1 - (T_1 - T_2)\, x/L$$

which is linear and is independent of the material.

Further, the heat flow rate, $\dot{Q}/A = -k\, dT/dx = (T_1 - T_2)k/L$, and therefore the temperature distribution can also be written as

$$T - T_1 = \left(\dot{Q}/A\right)\left(x/k\right)$$

i.e., "the temperature drop within the wall will increase with greater heat flow rate or when k is small for the same heat flow rate,"

A Cylindrical Shell-Expression for Temperature Distribution

In the cylindrical system, when the temperature is a function of radial distance only and is independent of azimuth angle or axial distance, the differential equation would be,

$d^2T /dr^2 + (1/r)\, dT/dr = 0$

with boundary conditions: at $r = r_l$, $T = T_1$ and at $r = r_2$, $T = T_2$.

The differential equation can be written as:

$$\frac{1}{r}\frac{d}{dr}\left(r\, dT/dr\right) = 0, \quad \text{or,} \quad \frac{d}{dr}\left(r\, dT/dr\right) = 0$$

upon integration, $T = C_1 \ln(r) + C_2$, where C_1 and C_2 are the arbitrary constants.

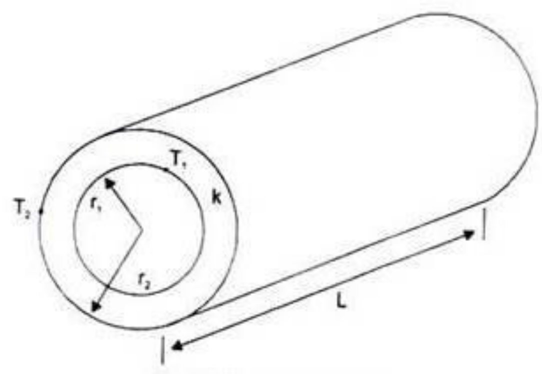


Fig : A Cylindrical shell

By applying the boundary conditions,

$$C_1 = (T_2 - T_1)/\ln(r_2/r_1)$$

and $$C_2 = T_1 - \ln(r_1).(T_2 - T_1)/\ln(r_2/r_1)$$

The temperature distribution is given by

$$T = T_1 + (T_2 - T_1).\ \ln(r/r_1)/\ln(r_2/r_1) \text{ and}$$

$$\dot{Q}/L = -kA\ dT/dr \quad = 2\pi k(T_1 - T_2)/\ln(r_2/r_1)$$

From the above Eqn It can be seen that the temperature varies 10gantJunically through the cylinder wall In contrast with the linear variation in the plane wall .

If we write Eq. (2.5) as $\dot{Q} = kA_m(T_1 - T_2)/(r_2 - r_1)$, where

$$A_m = 2\pi(r_2 - r_1)L/\ln(r_2/r_1) \quad = (A_2 - A_1)/\ln(A_2/A_1)$$

where A_2 and A_1 are the outside and inside surface areas respectively. The term A_m is called 'Logarithmic Mean Area' and the expression for the heat flow through a cylindrical wall has the same form as that for a plane wall.

Spherical and Parallelopiped Shells--Expression for Temperature Distribution

Conduction through a spherical shell is also a one-dimensional steady state problem if the interior and exterior surface temperatures are uniform and constant. The one-dimensional spherical coordinates can be written as

$$\left(1/r^2\right)\frac{d}{dT}\left(r^2 dT/dr\right)=0$$, with boundary conditions,

at $\quad r=r_1,\ T=T_1;\ \text{at } r=r_2,\ T=T_2$

or, $\quad \frac{d}{dr}\left(r^2 dT/dr\right)=0$

and upon integration, T = –C_1/r + C_2, where c_1 and c_2 are constants. substituting the boundary conditions,

$$C_1=\left(T_1-T_2\right)r_1r_2/\left(r_1-r_2\right),\ \text{and}\ C_2=T_1+\left(T_1-T_2\right)r_1r_2/r_1\left(r_1-r_2\right)$$

The temperature distribution m the spherical shell is given by

$$T=T_1-\left\{\frac{\left(T_1-T_2\right)r_1r_2}{\left(r_2-r_1\right)}\right\}\times\left\{\frac{\left(r-r_1\right)}{r\ r_1}\right\}$$

and the temperature distribution associated with radial conduction through a sphere is represented by a hyperbola. The rate of heat conduction is given by

$$\dot{Q}=4\pi k\left(T_1-T_2\right)r_1r_2/\left(r_2-r_1\right)=k\left(A_1A_2\right)^{1/2}\left(T_1-T_2\right)/\left(r_2-r_1\right)$$

where $A_1=4\pi_1^2$ and $A_2=4\pi r_2^2$

If A_l is approximately equal to A_2 i.e., when the shell is very thin,

$$\dot{Q}=kA\left(T_1-T_2\right)/\left(r_2-r_1\right);\ \text{and}\ \dot{Q}/A=\left(T_1-T_2\right)/\Delta r/k$$

which is an expression for a flat slab.

The above equation can also be used as an approximation for parallelopiped shells which have a smaller inner cavity surrounded by a thick wall, such as a small furnace surrounded by a large thickness of insulating material, although the h eat flow especially in the corners, cannot be strictly considered one-dimensional. It has been suggested that for (A_2/A_1) > 2, the rate of heat flow can be approximated by the above equation by multiplying the geometric mean area $A_m = (A_1\ A_2)^{1/2}$ by a correction factor 0.725.]

4.4 Composite Surfaces

There are many practical situations where different materials are placed m layers to form composite surfaces, such as the wall of a building, cylindrical pipes or spherical shells having different layers of insulation. Composite surfaces may involve any number of series and parallel thermal circuits.

4.5 Heat Transfer Rate through a Composite Wall

Let us consider a general case of a composite wall as shown m Fig. 2 There are ‘n’ layers of different materials of thicknesses L_1, L_2, etc and having thermal conductivities k_l, k_2, etc. On one side of the composite wall, there is a fluid A at temperature T_A and on the other side of the wall there is a fluid B at temperature T_B. The convective heat transfer coefficients on the two sides of the wall are h_A and h_B respectively. The system is analogous to a series of resistances as shown in the figure.

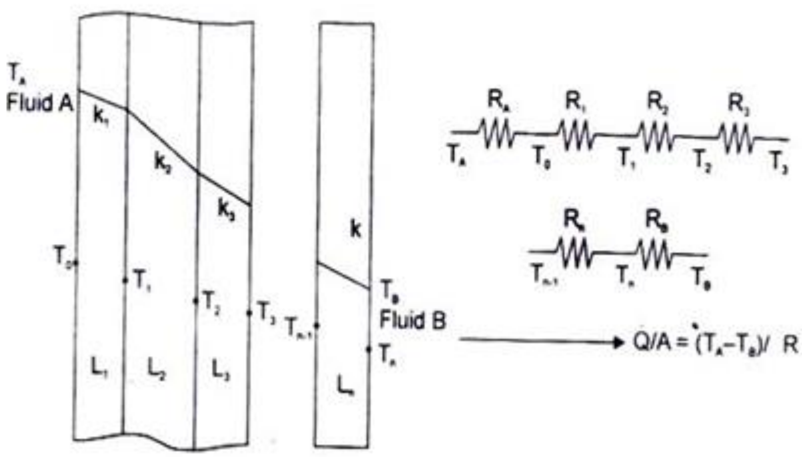


Fig 2: Heat transfer through a composite wall

4.6 The Equivalent Thermal Conductivity

The process of heat transfer through compos lie and plane walls can be more conveniently compared by introducing the concept of 'equivalent thermal conductivity', k_{eq}. It is defined as:

$$k_{eq} = \left(\sum_{i=1}^{n} L_i \right) \Big/ \sum_{i=1}^{n} \left(L_i / k_i \right)$$

$$= \frac{\text{Total thickeness of the composite wall}}{\text{Total thermal resistance of the composite wall}}$$

And, its value depends on the thermal and physical properties and the thickness of each constituent of the composite structure.

Example 1. A furnace wall consists of 150 mm thick refractory brick (k = 1.6 W/mK) and 150 mm thick insulating fire brick (k = 0.3 W/mK) separated by an au gap (resistance 0 16 K/W). The outside walls covered with a 10 mm thick plaster (k = 0.14 W/mK). The temperature of hot gases is 1250°C and the room temperature is 25°C. The convective heat transfer coefficient for gas side and air side is 45 W/m2K and 20 W/m^2K. Calculate (i) the rate of heat flow per unit area of the wall surface (ii) the temperature at the outside and Inside surface of the wall and (iii) the rate of heat flow when the air gap is not there.

Solution: Using the nomenclature of Fig. 2.3, we have per m2 of the area, h_A = 45, and $R_A = 1/h_A$ = 1/45 = 0.0222; h_B = 20, and R_B = 1120 = 0.05

Resistance of the refractory brick, $R_1 = L_1/k_1$ = 0.15/1.6 = 0.0937

Resistance of the insulating brick, $R_3 = L_3/k_3$ = 0.15/0.30 = 0.50

The resistance of the air gap, R_2 = 0.16

Resistance of the plaster, R_4 = 0.01/0.14 = 0.0714

Total resistance = 0.8973, m^2K/W

Heat flow rate = $\Delta T/\Sigma R$ = (1250-25)/0.8973= 13662 W/m^2

Temperature at the inner surface of the wall

$= T_A - 1366.2 \times 0.0222 = 1222.25$

Temperature at the outer surface of the wall

$= T_B + 1366.2 \times 0.05 = 93.31$ °C

When the air gap is not there, the total resistance would be

0.8973 - 0.16 = 0.7373

and the heat flow rate = (1250 – 25)/0/7373 = 1661.46 W/m^2

The temperature at the inner surface of the wall

= 1250 – 1660.46 × 0.0222 = 1213.12°C

i.e., when the au gap is not there, the heat flow rate increases but the temperature at the inner surface of the wall decreases.

The overall heat transfer coefficient U with and without the air gap is

$$U = \left(\dot{Q}/A\right) / \Delta T$$

$$= 13662 / (1250 – 25) = 1.115 \text{ Wm}^2 \text{ °C}$$

and 1661.46/l225 = 1356 $W/m^{2o}C$

The equivalent thermal conductivity of the system without the air gap

k_{eq} = (0.15 + 0.15 + 0.01)/(0.0937 + 0.50 + 0.0714) = 0.466 W/mK.

Example 2. A brick wall (10 cm thick, k = 0.7 W/m°C) has plaster on one side of the wall (thickness 4 cm, k = 0.48 W/m°C). What thickness of an insulating material (k = 0.065 W m^oC) should be added on the other side of the wall such that the heat loss through the wall IS reduced by 80 percent.

Solution: When the insulating material is not there, the resistances are:

$R_1 = L_1/k_1 = 0.1/0.7 = 0.143$

and $R_2 = 0.04/0.48 = 0.0833$

Total resistance = 0.2263

Let the thickness of the insulating material is L_3. The resistance would then be

$L_3/0.065 = 15.385\ L_3$

Since the heat loss is reduced by 80% after the insulation is added.

$$\frac{\dot{Q} \text{ with insulation}}{\dot{Q} \text{ without insulation}} = 0.2 = \frac{R \text{ without insulation}}{R \text{ with insulation}}$$

or, the resistance with insulation = 0.2263/0.2 = 01.1315

and, 15385 L_3 = I 1315 – 0.2263 = 0.9052

L_3 = 0.0588 m = 58.8 mm

Example 3. A composite furnace wall is to be constructed with two layers of materials (k_1 =

2.5 W/m°C and k_2 = 0.25 W/m°C). The convective heat transfer coefficient at the inside and outside surfaces are expected to be 250 W/m^2°C and 50 W/m^2°C respectively. The temperature of gases and air are 1000 K and 300 K. If the interface temperature is 650 K, Calculate (i) the thickness of the two materials when the total thickness does not exceed 65 cm and (ii) the rate of heat flow. Neglect radiation.

Solution: Let the thickness of one material (k = 2.5 W / mK) is xm, then the thickness of the other material (k = 0.25 W/mK) will be (0.65 –x)m.

For steady state condition, we can write

$$\frac{\dot{Q}}{A} - \frac{1000-650}{\frac{1}{250}+\frac{x}{2.5}} = \frac{1000-300}{\frac{1}{250}+\frac{x}{2.5}+\frac{(0.65-x)}{0.25}+\frac{1}{50}}$$

$$\therefore \; 700(0.004+0.4x) = 350\{0.004+0.4x+4(0.65-x)+0.02\}$$

(i) 6x = 3.29 and x= 0.548 m.

and the thickness of the other material = 0.102 m.

(ii) $\dot{Q}/A$ = (350) / (0.004 + 0.4 × 0.548) = 1.568 kW/m^2

Example 4. A composite wall consists of three layers of thicknesses 300 rum, 200 mm and 100 mm with thermal conductivities 1.5, 3.5 and is W/mK respectively. The inside surface is exposed to gases at 1200°C with convection heat transfer coefficient as 30W/m^{2}K. The temperature of air on the other side of the wall is 30°C with convective heat transfer coefficient 10 Wm^{2}K. If the temperature at the outside surface of the wall is 180°C, calculate the temperature at other surface of the wall, the rate of heat transfer and the overall heat transfer coefficient.

Solution: The composite wall and its equivalent thermal circuits is shown in the figure.

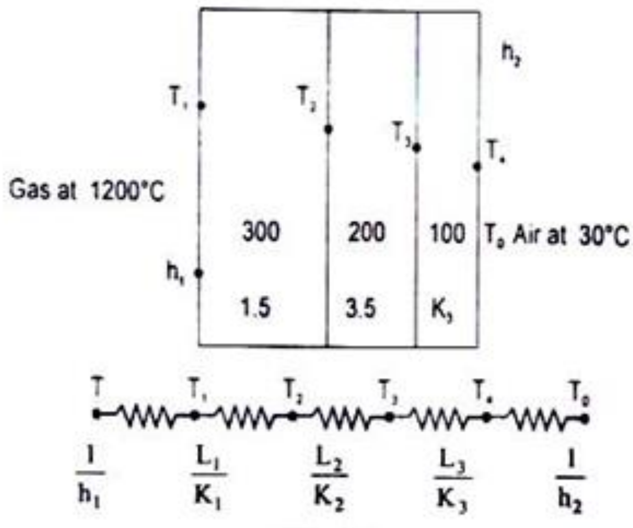


Fig 1.6

The heat energy will flow from hot gases to the cold air through the wall.

From the electric Circuit, we have

$$\dot{Q}/A = h_2(T_4 - T_0) = 10 \times (180 - 30) = 1500\,W/m^2$$

also, $\dot{Q}/A = h_1(1200 - T_1)$

$$T_1 = 1200 - 1500/30 = 1150^{o}C$$

$$\dot{Q}/A = (T_1 - T_2)/L_1/k_1$$

$$T_2 = T_1 - 1500 \times 0.3/1.5 = 850$$

Similarly, $\dot{Q}/A = (T_2 - T_3)/(L_2/k_2)$

$$T_3 = T_2 - 1500 \times 0.2/3.5 = 764.3^{o}C$$

and $\dot{Q}/A = (T_3 - T_4)/(L_3/k_3)$

$L_3/k_3 = (764.3 - 180)/1500$ and $k_3 = 0.256$ W/mK

Check:

$$\dot{Q}/A = (1200 - 30)/\Sigma R;$$

where $\Sigma R = 1/h_1 + L_1/k_1 + L_2/k_2 + L_3/k_3 + 1/h_2$

$$\Sigma R = 1/30 + 0.3/1.5 + 0.2/3.5 + 0.1/0.256 + 1/10 = 0.75$$

and $\dot{Q}/A = 1170/0.78 = 1500\ W/m^2$

The overall heat transfer coefficient, $U = 1/\Sigma R = 1/0.78 = 1.282\ W/m^2K$

Since the gas temperature is very high, we should consider the effects of radiation also. Assuming the heat transfer coefficient due to radiation = 3.0 W/m^2K the electric circuit would be:

The combined resistance due to convection and radiation would be

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{\frac{1}{h_c}} + \frac{1}{\frac{1}{h_r}} = h_c + h_r = 60 W/m^{2o}C$$

$$\therefore\ \dot{Q}/A = 1500 = 60(T - T_1) = 60(1200 - T_1)$$

$$\therefore\ T_1 = 1200 - \frac{1500}{60} = 1175^oC$$

again, $\therefore\ \dot{Q}/A = (T_1 - T_2)/L_1/k_1 \Rightarrow T_2 = T_1 - 1500 \times 0.3/1.5 = 875^oC$

and $T_3 = T_2 - 1500 \times 0.2/3.5 = 789.3^oC$

$$L_3/k_3 = (789.3 - 180)/1500;\ \therefore k_3 = 0.246\ W/mK$$

$$\Sigma R = \frac{1}{60} + \frac{0.3}{1.5} + \frac{0.2}{1.5} + \frac{0.2}{3.5} + \frac{0.1}{0.246} + \frac{1}{10} = 0.78$$

and $U = 1/\Sigma R = 1.282\ W/m^2K$

Example 5. A 20 cm thick slab of aluminium (k = 230 W/mK) is placed in contact with a 15 cm thick stainless steel plate (k = 15 W/mK). Due to roughness, 40 percent of the area is in direct contact and the gap (0.0002 m) is filled with air (k = 0.032 W/mK). The difference in temperature between the two outside surfaces of the plate is 200°C Estimate (i) the heat flow rate, (ii) the contact resistance, and (iii) the drop in temperature at the interface.

Solution: Let us assume that out of 40% area m direct contact, half the surface area is occupied by steel and half is occupied by aluminium.

The physical system and its analogous electric circuits is shown in Fig. 2.13.

$$R_1 = \frac{0.2}{230 \times 1} = 0.00087, \qquad R_2 = \frac{0.0002}{230 \times 0.2} = 4.348 \times 10^{-6}$$

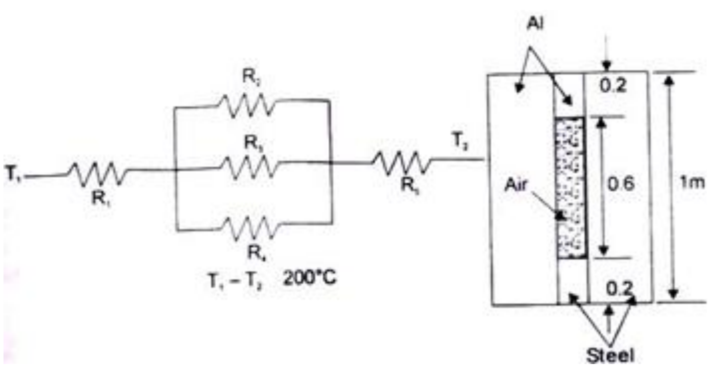
$$R_3 = \frac{0.0002}{0.032 \times 0.6} = 1.04 \times 10^{-2}, \qquad R_4 = \frac{0.0002}{15 \times 0.2} = 6.667 \times 10^{-5}$$

and $R_5 = \dfrac{0.15}{(15 \times 1)} = 0.01$

Again $1/R_{2,3,4} = 1/R_2 + 1/R_3 + 1/R_4$

$= 2.3 \times 10^5 + 96.15 + 1.5 \times 10^4 = 24.5 \times 10^4$

Therefore, $R_{2,3,4} = 4.08 \times 10^{-6}$



Total resistance, $\Sigma R = R_1 + R_{2,3,4} + R_5$

$= 870 \times 10^{-6} + 4.08 \times 10^{-6} + 1000 \times 10^{-6} = 1.0874 \times 10^{-2}$

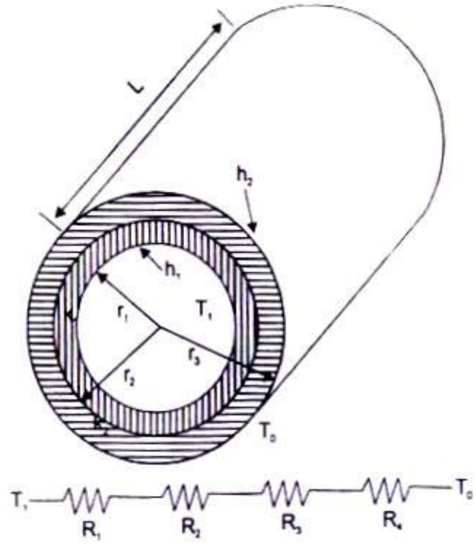
Heat flow rate, $\dot{Q}$ = 200/1.087 × 10^{-2} = 18.392 kW per unit depth of the plate.

Contact resistance, R $R_{2,3,4} = 4.08 \times 10^{-6}$ mK / W

Drop in temperature at the interface, $\Delta T = 4.08 \times 10^{-6} \times 18392 = 0.075$°C

An Expression for the Heat Transfer Rate through a Composite Cylindrical System

Let us consider a composite cylindrical system consisting of two coaxial cylinders, radii r_1, r_2 and r_2 and r_3, thermal conductivities k_l and k_2 the convective heat transfer coefficients at the inside and outside surfaces h_1 and h_2 as shown in the figure. Assuming radial conduction under steady state conditions we have:



$$R_1 = 1/h_1A_1 = 1/2\ \pi_1\ Lh_1$$

$$R_2 = \ln(r_2/r_1)2\pi Lk_1$$

$$R_3 = \ln(r_3/r_2)2\pi Lk_2$$

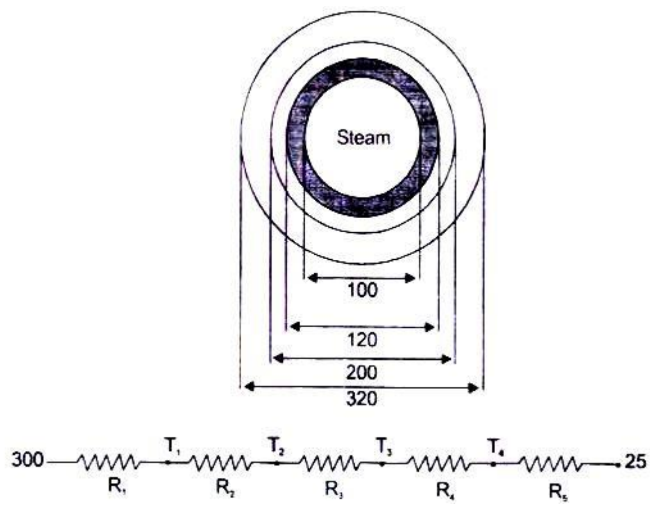
$$R_4 = 1/h_2A_2 = 1/2\pi_3 h_2 L$$

And $\dot{Q}/2\pi L = (T_1 - T_0)/\Sigma R$

$$= (T_1 - T_0)/\left[\left(1/h_1r_1 + \ln(r_2/r_1)/k_1 + \ln(r_3 + r_2)/k_2 + 1/h_2r_3\right)\right]$$

Example 6. A steel pipe. Inside diameter 100 mm, outside diameter 120 mm (k 50 W/mK) IS Insulated with a 40 mm thick high temperature Insulation (k = 0.09 W/mK) and another Insulation 60 mm thick (k = 0.07 W/mK). The ambient temperature IS 25°C. The heat transfer coefficient for the inside and outside surfaces are 550 and 15 W/m^2K respectively. The pipe carries steam at 300^oC. Calculate (1) the rate of heat loss by steam per unit length of the pipe (11) the temperature of the outside surface

Solution: I he cross-section of the pipe with two layers of insulation is shown 111 Fig. 1.16. with its analogous electrical circuit.



Cross-section through an insulated cylinder, thermal resistances in series.

For L = 1.0 m. we have

R_1, the resistance of steam film = 1/hA = 1/(500 × 2 ×3.14× 50 × 10^{-3}) = 0.00579

R_2, the resistance of steel pipe = $\ln(r_2/r_1)$ / 2 π k

= ln(60/50)/2 π × 50 = 0.00058

R_3, resistance of high temperature Insulation

$\ln(r_3/r_2)$ / 2 π k = ln(100/60) / 2 π × 0.09 = 0.903

R_4 = $1n(r_4/r_3)$/2 π k = ln(160/100)/2 π × 0.07 = 1.068

R_5 = resistance of the air film = 1/(15 × 2 π × 160 × 10^{-3}) = 0.0663

The total resistance = 2.04367

and $\dot{Q} = \Delta T / \Sigma R$ = (300 – 25) / 204367 = 134.56 W per metre length of pipe.

Temperature at the outside surface. T_4 = 25 + R_5,

$\dot{Q}$ = 25 + 134.56 × 0.0663 = 33.92° C

When the better insulating material (k = 0.07, thickness 60 mm) is placed first on the steel pipe, the new value of R_3 would be

$R_3 = \ln(120/60) / 2\pi \times 0.07 = 1.576$; and the new value of R_4 will be

$R_4 = \ln(160/120)\ 2\pi \times 0.09 = 0.5087$

The total resistance = 2.15737 and Q = 275/2.15737 = 127.47 W per m length (Thus the better insulating material be applied first to reduce the heat loss.) The overall heat transfer coefficient, U, is obtained as $U = \dot{Q} / A\,\Delta T$

The outer surface area = $\pi \times 320 \times 10^{-3} \times 1 = 1.0054$

and U = 134.56/(275 × 1.0054) = 0.487 W/m^2 K.

Example 7. A steam pipe 120 mm outside diameter and 10m long carries steam at a pressure of 30 bar and 099 dry. Calculate the thickness of a lagging material (k = 0.99 W/mK) provided on the steam pipe such that the temperature at the outside surface of the insulated pipe does not exceed 32°C when the steam flow rate is 1 kg/s and the dryness fraction of steam at the exit is 0.975 and there is no pressure drop.

Solution: The latent heat of vaporization of steam at 30 bar = 1794 kJ/kg.

The loss of heat energy due to condensation of steam = 1794(0.99 – 0.975)

= 26.91 kJ/kg.

Since the steam flow rate is 1 kg/s, the loss of energy = 26.91 kW.

The saturation temperature of steam at 30 bar IS 233.84°C and assuming that the pipe material offers negligible resistance to heat flow, the temperature at the outside surface of the uninsulated steam pipe or at the inner surface of the lagging material is 233.84°C. Assuming one-dimensional radial heat flow through the lagging material, we have

$\dot{Q} = (T_1 - T_2)/[\ln(r_2/r_l)]\ 2\pi Lk$

or, $26.91 \times 1000\ (W) = (233.84 - 32) \times 2\pi \times 10 \times 0.99/1n(r/60)$

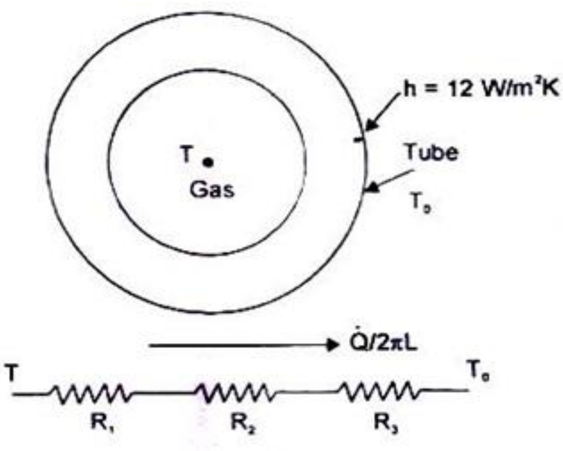
$\ln(r/60) = 0.4666$

$r_2/60 = \exp(0.4666) = 1.5946$

r_2 = 95.68 mm and the thickness = 35.68 mm

Example 8. A Wire, diameter 0.5 mm length 30 cm, is laid coaxially in a tube (inside diameter 1 cm, outside diameter 1.5 cm, k = 20 W/mK). The space between the wire and the inside wall of the tube behaves like a hollow tube and is filled with a gas. Calculate the thermal conductivity of the gas if the current flowing through the wire is 5 amps and voltage across the two ends is 4.5 V, temperature of the wire is 160°C, convective heat transfer coefficient at the outer surface of the tube is 12 W/m^2K and the ambient temperature is 300K.

Solution: Assuming steady state and one-dimensional radial heat flow, we can draw the thermal circuit as shown In Fig.



The rate of heat transfer through the system,

$\dot{Q}/2\,\pi\,L = VI/2\,\pi\,L = (4.5 \times 5)/(2 \times 3.142 \times 0.3) = 11.935$ (W/m)

R_1, the resistance due to gas = $\ln(r_2/r_l)$, k = $\ln(0.01/0.0005)/k = 2.996/k$.

R_2, resistance offered by the metallic tube = $\ln(r_3 / r_2)$ k

= $\ln(1.5 /1.0) / 20 = 0.02$

R_3, resistance due to fluid film at the outer surface

$l/hr_3 = 1/(l2 \times 1.5 \times I0^{-2}) = 5.556$

and $\dot{Q} / 2\,\pi\,L = L/R = [(273 + 160) - 300]/R$

Therefore, R = 133/11.935 = 11.l437, and

R_1 = 2.9996/k = 11.1437 – 0.02 – 5.556 = 5.568

or, k = 2.996/5.568 = 0.538 W/mK.

Example 9. A steam pipe (inner diameter 16 cm, outer diameter 20 cm, k = 50 W/mK) is covered with a 4 cm thick insulating material (k = 0.09 W/mK). In order to reduce the heat loss, the thickness of the insulation is Increased to 8mm. Calculate the percentage reduction in heat transfer assuming that the convective heat transfer coefficient at the Inside and outside surfaces are 1150 and 10 W/m^2K and their values remain the same.

Solution: Assuming one-dimensional radial conduction under steady state,

$$\dot{Q} / 2*3014*L = \Delta T/\Delta R$$

R_1, resistance due to steam film = 1/hr = 1/(1150 × 0.08) = 0.011

R_2, resistance due to pipe material = ln (r_2/r_1)/k = ln (10/8)/50 = 0.00446

R_3, resistance due to 4 cm thick insulation

$= \ln(r_3/r_2)/k = \ln(14/10)/0.09 = 3.738$

R_4, resistance due to air film = 1/hr = 1/(10 × 0.14) = 0.714.

Therefore, $\dot{Q}/2\pi L = \Delta T / (0.011 + 0.00446 + 3.738 + 0.714) = 0.2386\Delta T$

When the thickness of the insulation is increased to 8 cm, the values of R_3 and R_4 will change.

$R_3 = \ln(r_3/r_2)/k = \ln(18/10)/0.09 = 6.53$; and

$R_4 = 1/hr = 1/(10 \times 0.18) = 0.556$

Therefore, $\dot{Q}/2\pi L = \Delta T / (0.011 + 0.00446 + 6.53 + 0.556)$

$= \Delta T / 7.1 = 0.14084\ \Delta T$

Percentage reduction in heat transfer = $\frac{(0.22386 - 0.14084)}{0.22386} = 0.37 = 37\%$

Example 10. A small hemispherical oven is built of an inner layer of insulating fire brick 125 mm thick (k = 0.31 W/mK) and an outer covering of 85% magnesia 40 mm thick (k = 0.05 W/mK). The inner surface of the oven is at 1073 K and the heat transfer coefficient for the outer surface is 10 W/m^{2}K, the room temperature is 20^{o}C. Calculate the rate of heat loss through the hemisphere if the inside radius is 0.6 m.

Solution: The resistance of the fire brick

$$= (r_2 - r_1)/2\pi k r_1 r_2 = \frac{0.725 - 0.6}{2\pi \times 0.31 \times 0.6 \times 0.725} = 0.1478$$

The resistance of 85% magnesia

$$= (r_3 - r_2)/2\pi k r_2 r_3 = \frac{0.765 - 0.725}{2\pi \times 0.05 \times 0.725 \times 0.765} = 0.2295$$

The resistance due to fluid film at the outer surface = 1/hA

$$= \frac{1}{10 \times 2\pi \times (0.765 \times 0.765)} = 0.2295$$

The resistance due to fluid film at the outer surface = 1/hA

$$= \frac{1}{10 \times 2\pi \times (0.765 \times 0.765)} = 0.0272$$

Rate of heat flow, $\dot{Q} = \Delta T / \Sigma R = \dfrac{800 - 20}{0.1478 + 0.2295 + 0.272} = 1930W$

Therefore, 594.44 = (68531.84 + 16825.4) k; or, k = 6.96 × 10^{-3} W/mK.

Example 11. A spherical vessel, made out of2.5 em thick steel plate IS used to store 10m3 of a liquid at 200°C for a thermal storage system. To reduce the heat loss to the surroundings, a 10 cm thick layer of insulation (k = 0.07 W/rnK) is used. If the convective heat transfer coefficient at the outer surface is W/m^{2}K and the ambient temperature is 25°C, calculate the rate of heat loss neglecting the thermal resistance of the steel plate.

If the spherical vessel is replaced by a 2 m diameter cylindrical vessel with flat ends, calculate the thickness of insulation required for the same heat loss.

Solution: Volume of the spherical vessel = $10m^3 = \frac{4\pi r^3}{3} \quad \therefore\ r = 1.336\ m$

Outer radius of the spherical vessle, $r_2 = 1.3364 + 0.025 = 1.361\ m$

Outermost radius of the spherical vessel after the insulation = 1.461 m.

Since the thermal resistance of the steel plate is negligible, the temperature at the inside surface of the insulation is 200°C.

Thermal resistance of the insulating material = $\left(r_3 - r_2\right)/4\pi\ k\ r_3 r_2$

$$= \frac{0.1}{4\pi \times 0.07 \times 1.461 \times 1.361} = 0.057$$

Thermal resistance of the fluid film at the outermost surface = 1/hA

$$= 1/\left[10 \times 4\pi \times (1.461)^2\right] = 0.00373$$

Rate of heat flow = $\Delta T/\Sigma R = (200 - 25)/(0.057 + 0.00373) = 2873.8\ W$

Volume of the insulating material used = $(4/3)\pi\left(r_3^3 - r_2^3\right) = 2.5\ m^3$

Volume of the cylindrical vessel $= 10\ m^3 = \frac{\pi}{4}(d)^2\ L; \quad \therefore\ L = 10/\pi = 3.183\,m$

Outer radius of cylinder without insulation = 1.0 + 0.025 = 1.025 m.

Outermost radius of the cylinder (with insulation) = r_3.

Therefore, the thickness of insulation = $r_3 - 1.025$

Resistance, the heat flow by the cylindrical element

$$= \frac{\ln\left(r_3/1.025\right)}{2\pi Lk} + 1/hA = \frac{\ln\left(r_3/1.025\right)}{2\pi \times 3.183 \times 0.07} + \frac{1}{10 \times 2\pi \times r_3 \times 3.183}$$

= $0.714 \ln (r_3 / 1.025) + 0.005/r_3$

Resistance to heat flow through sides of the cylinder

$$= 2\delta / kA + 1/hA = \frac{2(r_3 - 1.025)}{0.07 \times \pi \times 1} + \frac{1}{10 \times 2 \times \pi}$$

$$= 9.09(r_3 - 1.025) + 0.0159$$

For the same heat loss, $\Delta T / \Sigma R$ would be equal in both cases, therefore,

$$\frac{1}{0.06073} = \frac{1}{0.714 \ln(r_3 / 1.025) + 0.005 / r_3} + \frac{1}{9.09(r_3 - 1.025) + 0.0159}$$

Solving by trial and error, (r – 1.025j) = □ = 9.2 cm.

and the volume of the insulating material required = 2.692 m^3.

CONVECTION

Convection Heat Transfer-Requirements

The heat transfer by convection requires a solid-fluid interface, a temperature difference between the solid surface and the surrounding fluid and a motion of the fluid. The process of heat transfer by convection would occur when there is a movement of macro-particles of the fluid in space from a region of higher temperature to lower temperature.

Convection Heat Transfer Mechanism

Let us imagine a heated solid surface, say a plane wall at a temperature T_w placed in an atmosphere at temperature T_∞, Fig. 2.1 Since all real fluids are viscous, the fluid particles adjacent to the solid surface will stick to the surface. The fluid particle at A, which is at a lower temperature, will receive heat energy from the plate by conduction. The internal energy of the particle would Increase and when the particle moves away from the solid surface (wall or plate) and collides with another fluid particle at B which is at the ambient temperature, it will transfer a part of its stored energy to B. And, the temperature of the fluid particle at B would increase. This way the heat energy is transferred from the heated plate to the surrounding fluid. Therefore the process of heat transfer by convection involves a combined action of heat conduction, energy storage and transfer of energy by mixing motion of fluid particles.

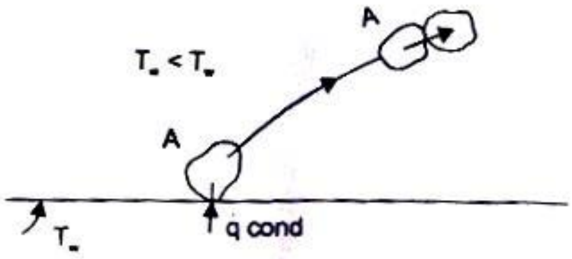


Fig. 2.1 Principle of heat transfer by convection

Free and Forced Convection

When the mixing motion of the fluid particles is the result of the density difference caused by a temperature gradient, the process of heat transfer is called natural or free convection. When the mixing motion is created by an artificial means (by some external agent), the process of heat transfer is called forced convection Since the effectiveness of heat transfer by convection depends largely on the mixing motion of the fluid particles, it is essential to have a knowledge of the characteristics of fluid flow.

Basic Difference between Laminar and Turbulent Flow

In laminar or streamline flow, the fluid particles move in layers such that each fluid p article follows a smooth and continuous path. There is no macroscopic mixing of fluid particles between successive layers, and the order is maintained even when there is a turn around a comer or an obstacle is to be crossed. If a lime dependent fluctuating motion is observed indirections which are parallel and transverse to the main flow, i.e., there is a random macroscopic mixing of fluid particles across successive layers of fluid flow, the motion of the fluid is called' turbulent flow'. The path of a fluid particle would then be zigzag and irregular, but on a statistical basis, the overall motion of the macro particles would be regular and predictable.

Formation of a Boundary Layer

When a fluid flow, over a surface, irrespective of whether the flow is laminar or turbulent, the fluid particles adjacent to the solid surface will always stick to it and their velocity at the solid surface will be zero, because of the viscosity of the fluid. Due to the shearing action of one fluid layer over the adjacent layer moving at the faster rate, there would be a velocity gradient in a direction normal to the flow.

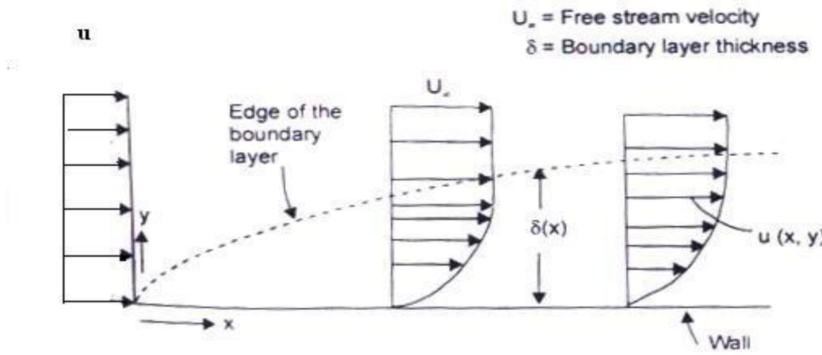


Fig 2.2: sketch of a boundary layer on a wall

Let us consider a two-dimensional flow of a real fluid about a solid (slender in cross-section) as shown in Fig. 2.2. Detailed investigations have revealed that the velocity of the fluid particles at the surface of the solid is zero. The transition from zero velocity at the surface of the solid to the free stream velocity at some distance away from the solid surface in the V-direction (normal to the direction of flow) takes place in a very thin layer called 'momentum or hydrodynamic boundary layer'. The flow field can thus be divided in two regions:

(i) A very thin layer in t he vicinity 0 f t he body w here a velocity gradient normal to the direction of flow exists, the velocity gradient du/dy being large. In this thin region, even a very small Viscosity μ of the fluid exerts a substantial Influence and the shearing stress $\tau = \mu\, du/dy$ may assume large values. The thickness of the boundary layer is very small and decreases with decreasing viscosity.

(ii) In the remaining region, no such large velocity gradients exist and the Influence of viscosity is unimportant. The flow can be considered frictionless and potential.

Thermal Boundary Layer

Since the heat transfer by convection involves the motion of fluid particles, we must superimpose the temperature field on the physical motion of fluid and the two fields are bound to interact. It is intuitively evident that the temperature distribution around a hot body in a fluid stream will often have the same character as the velocity distribution in the boundary layer flow. When a heated solid body IS placed in a fluid stream, the temperature of the fluid stream will also vary within a thin layer in the immediate neighborhood of the solid body. The variation in temperature of the fluid stream also takes place in a thin layer in the neighborhood of the body and is termed 'thermal boundary layer'. Fig. 2.3 shows the temperature profiles inside a thermal boundary layer.

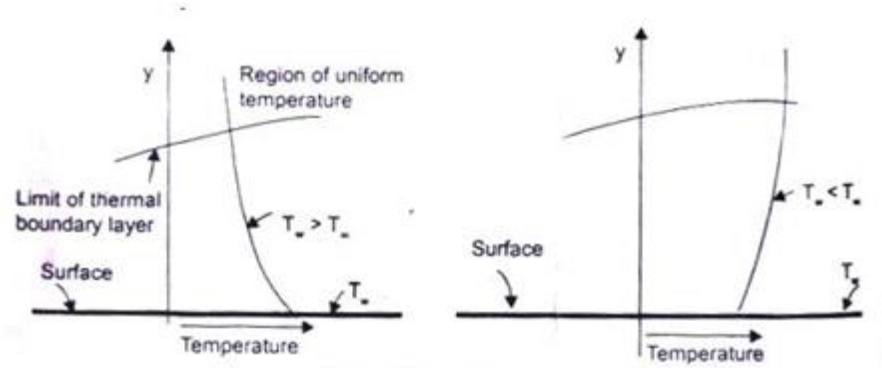


Fig2.3: The thermal boundary layer

Dimensionless Parameters and their Significance

The following dimensionless parameters are significant in evaluating the convection heat transfer coefficient:

(a) *The Nusselt Number (Nu)*-It is a dimensionless quantity defined as hL/ k, where h = convective heat transfer coefficient, L is the characteristic length and k is the thermal conductivity of the fluid The Nusselt number could be interpreted physically as the ratio of the temperature gradient in the fluid immediately in contact with the surface to a reference temperature gradient $(T_s - T_\infty)/L$. The convective heat transfer coefficient can easily be obtained if the Nusselt number, the thermal conductivity of the fluid in that temperature range and the characteristic dimension of the object is known.

Let us consider a hot flat plate (temperature T_w) placed in a free stream (temperature $T_\infty < T_w$). The temperature distribution is shown ill Fig. 2.4. Newton's Law of Cooling says that the rate of heat transfer per unit area by convection is given by

$$\dot{Q}/A = h\left(T_w - T_\infty\right)$$

$$\frac{\dot{Q}}{A} = h(T_w - T_\infty)$$

$$= k\frac{T_w - T_\infty}{\delta_t}$$

$$h = \frac{k}{\delta_t}$$

$$Nu = \frac{hL}{k} = \frac{L}{\delta_t}$$

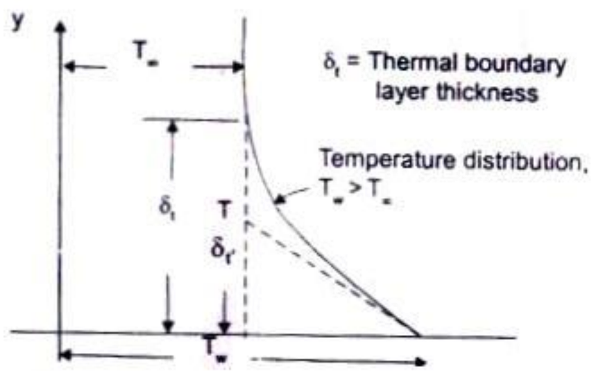


Fig. 2.4 Temperature distribution in a boundary layer: Nusselt modulus

The heat transfer by convection involves conduction and mixing motion of fluid particles. At the solid fluid interface (y = 0), the heat flows by conduction only, and is given by

$$\frac{\dot{Q}}{A} = -k\left(\frac{dT}{dy}\right)_{Y=0} \qquad \therefore h = \frac{\left(-k^{dT}\!/_{dy}\right)_{y=0}}{\left(T_w - T_\infty\right)}$$

Since the magnitude of the temperature gradient in the fluid will remain the same, irrespective of the reference temperature, we can write $dT = d(T - T_w)$ and by introducing a characteristic length dimension L to indicate the geometry of the object from which the heat flows, we get

$$\frac{hL}{k} = \frac{\left({}^{dT}\!/_{dy}\right)_{y=0}}{\left(T_w - T_\infty\right)/L}$$, and in dimensionless form,

$$= \left(\frac{d(T_w - T)/(T_w - T_\infty)}{d\left(y/L\right)}\right)_{y=0}$$

(b) *The Grashof Number (Gr)*-In natural or free convection heat transfer, die motion of fluid particles is created due to buoyancy effects. The driving force for fluid motion is the body force arising from the temperature gradient. If a body with a constant wall temperature T_w is exposed to a qui scent ambient fluid at T_∞, the force per unit volume can be written as $\rho g \beta (t_w - T_\infty)$ where ρ = mass density of the fluid, β = volume coefficient of expansion and g is the acceleration due to gravity.

The ratio of inertia force × Buoyancy force/(viscous force)^2 can be written as

$$Gr = \frac{\left(\rho V^2 L^2\right) \times \rho g \beta \left(T_w - T_\infty\right) L^3}{\left(\mu V L\right)^2}$$

$$= \frac{\rho^2 g \beta \left(T_w - T_\infty\right) L^3}{\mu^2} = g \beta L^3 \left(T_w - T_\infty\right) / \nu^2$$

The magnitude of Grashof number indicates whether the flow is laminar or turbulent. If the Grashof number is greater than 10^9, the flow is turbulent and for Grashof number less than 10^8, the flow is laminar. For $10^8 < Gr < 10^9$, It is the transition range.

(c) *The Prandtl Number (Pr)* - It is a dimensionless parameter defined as

$$Pr = \mu C_p / k = \nu / \alpha$$

Where μ is the dynamic viscosity of the fluid, v = kinematic viscosity and α = thermal diffusivity.

This number assumes significance when both momentum and energy are propagated through the system. It is a physical parameter depending upon the properties of the medium It is a measure of the relative magnitudes of momentum and thermal diffusion in the fluid: That is, for Pr = I, the r ate of diffusion of momentum and energy are equal which means that t he calculated temperature and velocity fields will be Similar, the thickness of the momentum and thermal boundary layers will be equal. For Pr <<I (in case of liquid metals), the thickness of the thermal boundary layer will be much more than the thickness of the momentum boundary layer and vice versa. The product of Grashof and Prandtl number is called Rayleigh number. Or, Ra = Gr × Pr.

Which the variations in velocity and temperature would remain confined. The relative thickness of the momentum and the thermal boundary layer strongly depends upon the Prandtl number. Since in natural convection heat transfer, the motion of the fluid particles is caused by the temperature difference between the temperatures of the wall and the ambient fluid, the thickness of the two boundary layers are expected to be equal. When the temperature of the vertical plate is less than the fluid temperature, the boundary layer will form from top to bottom but the mathematical analysis will remain the same.

The boundary layer will remain laminar upto a certain length of the plate ($Gr < 10^8$) and beyond which it will become turbulent ($Gr > 10^9$). In order to obtain the analytical solution, the integral approach, suggested by von-Karman is preferred.

We choose a control volume ABCD, having a height H, length dx and unit thickness normal to the plane of paper, as shown in Fig. 25. We have:

(b) Conservation of Mass:

Mass of fluid entering through face AB = $\dot{m}_{AB} = \int_0^H \rho u dy$

Mass of fluid leaving face CD = $\dot{m}_{CD} = \int_0^H \rho u dy + \frac{d}{dx}\left[\int_0^H \rho u dy\right]dx$

$\therefore$ Mass of fluid entering the face DA = $\frac{d}{dx}\left[\int_0^H \rho u dy\right]dx$

(ii) Conservation of Momentum:

Momentum entering face AB = $\int_0^H \rho u^2 dy$

Momentum leaving face CD = $\int_0^H \rho u^2 dy + \frac{d}{dx}\left[\int_0^H \rho u^2 dy\right]dx$

$\therefore$ Net efflux of momentum in the + x-direction = $\frac{d}{dx}\left[\int_0^H \rho u^2 dy\right]dx$

The external forces acting on the control volume are:

(a) Viscous force = $\mu \left.\frac{du}{dy}\right|_{y=0} dx$ acting in the –ve x-direction

(b) Buoyant force approximated as $\left[\int_0^H \rho g\beta\left(T - T_\infty\right)dy\right]dx$

From Newton's law, the equation of motion can be written as:

$$\frac{d}{dx}\left[\int_0^\delta \rho u^2 dy\right] = -\mu \left.\frac{du}{dy}\right|_{y=0} + \int_0^\delta \rho g\beta\left(T - T_\infty\right)dy \qquad (2.2)$$

because the value of the integrand between δ and H would be zero.

(iii) Conservation of Energy:

$\dot{Q}_{AB}$, convection + $\dot{Q}_{AD}$,convection + $\dot{Q}_{BC}$,conduction = $\dot{Q}_{CD}$ convection

$$\text{or, } \int_0^H \rho uCTdy + CT_\infty\left[\frac{d}{dx}\int_0^H \rho udy\right]dx - k\left.\frac{dT}{dy}\right|_{y=0} dx$$

$$= \int_0^H \rho uCTdy + \frac{d}{dx}\left[\int_0^H \rho uTCdy\right]dx$$

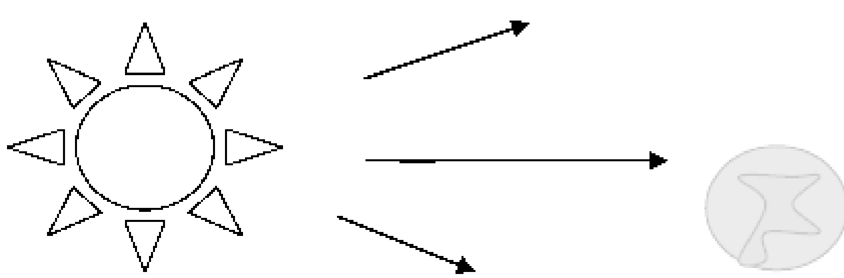
$$\text{or } \frac{d}{dx}\int_0^\delta \rho u\left(T_\infty - T\right)dy \frac{k}{\rho C}\left.\frac{dT}{dy}\right|_{y=0} = \alpha\left.\frac{dT}{dy}\right|_{y=0}$$

RADIATION

Definition:

Radiation is the energy transfer across a system boundary due to a ΔT, by the mechanism of photon emission or electromagnetic wave emission.

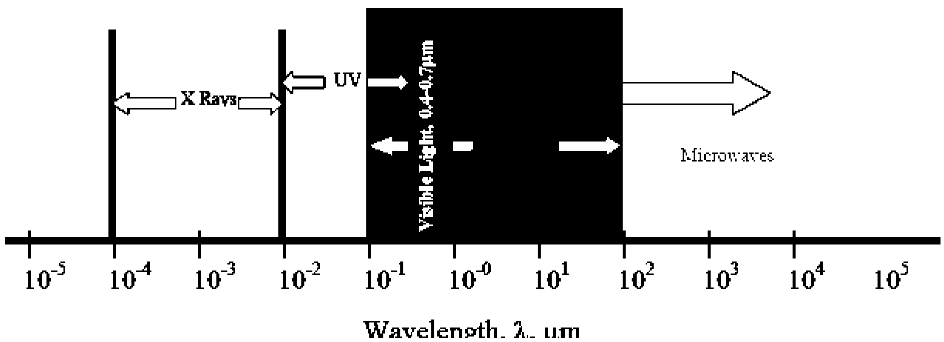
Because the mechanism of transmission is photon emission, unlike conduction and convection, there need be no intermediate matter to enable transmission.



The significance of this is that radiation will be the only mechanism for heat transfer whenever a vacuum is present.

Electromagnetic Phenomena.

We are well acquainted with a wide range of electromagnetic phenomena in modern life. These phenomena are sometimes thought of as wave phenomena and are, consequently, often described in terms of electromagnetic wave length, λ. Examples are given in terms of the wave distribution shown below:



One aspect of electromagnetic radiation is that the related topics are more closely associated with optics and electronics than with those normally found in mechanical engineering courses. Nevertheless, these are widely encountered topics and the student is familiar with them through every day life experiences.

From a viewpoint of previously studied topics students, particularly those with a background in mechanical or chemical engineering will find the subject of Radiation Heat Transfer a little unusual. The physics background differs fundamentally from that found in the areas of continuum mechanics. Much of the related material is found in courses more closely identified with quantum physics or electrical engineering, i.e. Fields and Waves. At this point, it is important for us to recognize that since the subject arises from a different area of physics, it will be important that we study these concepts with extra care.

Stefan-Boltzman Law

Both Stefan and Boltzman were physicists; any student taking a course in quantum physics will become well acquainted with Boltzman's work as he made a number of important contributions to the field. Both were contemporaries of Einstein so we see that the subject is of fairly recent vintage. (Recall that the basic equation for convection heat transfer is attributed to Newton)

$$E_b = \sigma \cdot T_{abs}^4$$

where: E_b = Emissive Power, the gross energy emitted from an ideal surface per unit area, time.

σ = Stefan Boltzman constant, $5.67 \cdot 10^{-8}$ W/m$^2 \cdot$K^4

T_{abs} = Absolute temperature of the emitting surface, K.

Take particular note of the fact that absolute temperatures are used in Radiation. It is suggested, as a matter of good practice, to convert all temperatures to the absolute scale as an initial step in all radiation problems.

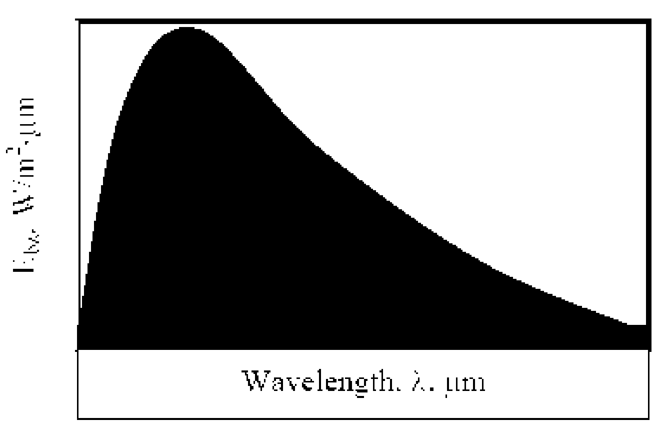
You will notice that the equation does not include any heat flux term, q". Instead we have a term the emissive power. The relationship between these terms is as follows. Consider two infinite plane surfaces, both facing one another. Both surfaces are ideal surfaces. One surface is found to be at temperature, T1, the other at temperature, T2. Since both temperatures are at temperatures above absolute zero, both will radiate energy as described by the Stefan-Boltzman law. The heat flux will be the net radiant flow as given by:

$$q'' = E_{b1} - E_{b2} = \sigma \cdot T_1^4 - \sigma \cdot T_2^4$$

Plank's Law

While the Stefan-Boltzman law is useful for studying overall energy emissions, it does not allow us to treat those interactions, which deal specifically with wavelength, λ. This problem was overcome by another of the modern physicists, Max Plank, who developed a relationship for wave-based emissions.

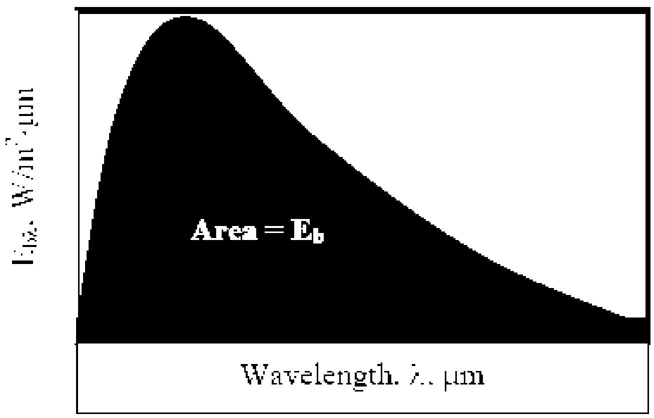
$$E_{b\lambda} = f(\lambda)$$



We haven't yet defined the Monochromatic Emissive Power, $E_{b\lambda}$. An implicit definition is provided by the following equation:

$$E_b = \int_0^{\infty} E_{b\lambda} \cdot d\lambda$$

We may view this equation graphically as follows:



A definition of monochromatic Emissive Power would be obtained by differentiating the integral equation:

$$E_{b\lambda} \equiv \frac{dE_b}{d\lambda}$$

The actual form of Plank's law is:

$$E_{b\lambda} = \frac{C_1}{\lambda^5 \cdot \left[e^{C_2/\lambda \cdot T} - 1 \right]}$$

$C_1 = 2 \cdot \pi \cdot h \cdot c_o^2 = 3.742 \cdot 10^8 \ W \cdot \mu m^4/m^2$
$C_2 = h \cdot c_o/k = 1.439 \cdot 10^4 \ \mu m \cdot K$

Where: h, c_o, k are all parameters from quantum physics. We need not worry about their precise definition here.

This equation may be solved at any T, λ to give the value of the monochromatic emissivity at that condition. Alternatively, the function may be substituted into the integral $E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda$ to find the Emissive power for any temperature. While performing this integral by hand is difficult, students may readily evaluate the integral through one of several computer programs, i.e. MathCad, Maple, Mathmatica, etc.

$$E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda = \sigma \cdot T^4$$

Emission over Specific Wave Length Bands

Consider the problem of designing a tanning machine. As a part of the machine, we will need to design a very powerful incandescent light source. We may wish to know how much energy is being emitted over the

Ultraviolet band (10^{-4} to 0.4 μm), known to be particularly dangerous.

$$E_b(0.0001 \rightarrow\ 0.4) = \int_{0.001 \cdot \mu m}^{0.4 \cdot \mu m} E_{b\lambda} \cdot d\lambda$$

With a computer available, evaluation of this integral is rather trivial. Alternatively, the text books provide a table of integrals. The format used is as follows:

$$\frac{E_b(0.001 \rightarrow\ 0.4)}{E_b} = \frac{\int_{0.001 \cdot \mu m}^{0.4 \cdot \mu m} E_{b\lambda} \cdot d\lambda}{\int_0^\infty E_{b\lambda} \cdot d\lambda} = \frac{\int_0^{0.4 \cdot \mu m} E_{b\lambda} \cdot d\lambda}{\int_0^\infty E_{b\lambda} \cdot d\lambda} - \frac{\int_{0.}^{0.0001 \cdot \mu m} E_{b\lambda} \cdot d\lambda}{\int_0^\infty E_{b\lambda} \cdot d\lambda} = F(0 \rightarrow\ 0.4) - F(0 \rightarrow\ 0.0001)$$

Referring to such tables, we see the last two functions listed in the second column. In the first column is a parameter, λ·T. This is found by taking the product of the absolute temperature of the emitting surface, T, and the upper limit wave length, λ. In our example, suppose that the incandescent bulb is designed to operate at a temperature of 2000K. Reading from the table:

λ,

λ, μm	T, K	λ·T, μm·K	F(0→λ)
0.0001	2000	0.2	0
0.4	2000	600	0.000014
F(0.4→0.0001μm) = F(0→0.4μm)- F(0→0.0001μm)			**0.000014**

This is the fraction of the total energy emitted which falls within the IR band. To find the absolute energy emitted multiply this value times the total energy emitted:

$$E_{bIR} = F(0.4 \rightarrow 0.0001\,\mu m)\cdot\sigma\cdot T^4 = 0.000014\cdot 5.67\cdot 10^{-8}\cdot 2000^4 = \mathbf{12.7\ W/m^2}$$

Solar Radiation

The magnitude of the energy leaving the Sun varies with time and is closely associated with such factors as solar flares and sunspots. Nevertheless, we often choose to work with an average value. The energy leaving the sun is emitted outward in all directions so that at any particular distance from the Sun we may imagine the energy being dispersed over an imaginary spherical area. Because this area increases with the distance squared, the solar flux also decreases with the distance squared. At the average distance between Earth and Sun this heat flux is 1353 W/m2, so that the average heat flux on any object in Earth orbit is found as:

$$G_{s,o} = S_c\cdot f\cdot \cos\theta$$

Where S_c = Solar Constant, 1353 W/m^2

f = correction factor for eccentricity in Earth Orbit, $(0.97<f<1.03)$

θ = Angle of surface from normal to Sun.

Because of reflection and absorption in the Earth's atmosphere, this number is significantly reduced at ground level. Nevertheless, this value gives us some opportunity to estimate the potential for using solar energy, such as in photovoltaic cells.

Example 12. A large vertical flat plate 3 m high and 2 m wide is maintained at 75°C and is exposed to atmosphere at 25°C. Calculate the rate of heat transfer.

Solution: The physical properties of air are evaluated at the mean temperature. i.e. T = (75 + 25)/2 = 50°C

From the data book, the values are:

$\rho = 1.088\ kg/m^3$; $C_p = 1.00\ kJ/kg.K$;

$\mu = 1.96 \times 10^{-5}$ Pa-s k = 0.028 W/mK.

$Pr = \mu C_p/k = 1.96 \times 10^{-5} \times 1.0 \times 10^3 /0.028 = 0.7$

$$\beta = \frac{1}{T} = \frac{1}{323}$$

$$Gr = \rho^2 g\beta(\Delta T)L^3/\mu^2$$

$$= \frac{(1.088)^2 \times 9.81 \times 1 \times (3)^3 \times 50}{323 \times \left(1.96 \times 10^{-5}\right)^2}$$

$= 12.62 \times 10^{10}$

$Gr.Pr = 8.834 \times 10^{10}$

Since Gr.Pr lies between 10^9 and 10^{13}

We have from Table 2.1

$$Nu = \frac{hL}{k} = 0.1(Gr.Pr)^{1/3} = 441.64$$

$\therefore$ h = 441.64 × 0.028/3 = 4.122 W/m^2K

$$\dot{Q} = hA(\Delta T) = 4.122 \times 6 \times 50 = 1236.6W$$

We can also compute the boundary layer thickness at x = 3m

$$\delta = \frac{2x}{Nu_x} = \frac{2 \times 3}{441.64} = 1.4 \text{ cm}$$

Example 13. A vertical flat plate at 90°C. 0.6 m long and 0.3 m wide, rests in air at 30°C. Estimate the rate of heat transfer from the plate. If the plate is immersed in water at 30°C. Calculate the rate of heat transfer

Solution: (a) *Plate in Air*: Properties of air at mean temperature 60°C

Pr = 0.7, k = 0.02864 W/ mK, v = 19.036 × 10^{-6} m^2/s

Gr = 9.81 × (90 – 30)$(0.6)^3$/ [333 $(19.036 \times 10^{-6})^2$]

= 1.054 × 10^9; Gr × Pr 1.054 ×10^9 × 0.7 = 7.37 × 10^8 < 10^9

From Table 5.1: for Gr × Pr < 10^9, Nu = 0.59 $(Gr. Pr)^{1/4}$

$\therefore$ h = 0.02864 × 0.59 $(7.37 \times 10^8)^{1/4}$/0.6 = 4.64 W/m^2K

The boundary layer thickness, δ = 2 k/h = 2 × 0.02864/4.64 = 1.23 cm

and $\dot{Q}$ = hA (ΔT) = 4.64 × (2 × 0.6 × 0.3) × 60 = 100 W.

Using Eq (2.8). Nu = 0.677 $(0.7)^{0.5}$ $(0.952 + 0.7)^{0.25}$ $(1.054 \times 10^9)^{0.25}$, Which gives h = 4.297 W/m^{2}K and heat transfer rate, $\dot{Q}$ 92.81 W

Churchill and Chu have demonstrated that the following relations fit very well with experimental data for all Prandtl numbers.

For $Ra_L < 10^9$, $Nu = 0.68 + (0.67\, Ra_L^{0.25}) / [1 + (0.492/Pr)^{9/16}]^{4/9)}$ (5.9)

$Ra_L > 10^9$, $Nu = 0.825 + (0.387\, Ra_L^{1/6})/[1 + (0.492/Pr)^{9/16}]^{8/27}$ (5.10)

Using Eq (5.9): $Nu = 0.68 + [0.67(7.37 \times 10^8)^{0.25}] / [1 + (0.492/0.7)^{9/16}]^{4/9}$

= 58.277 and h = 4.07 W /m^{2}k; $\dot{Q}$ = 87.9 W

(b) Plate in Water: Properties of water from the Table

Pr = 3.01, $\rho^2 g \beta C_p/\mu k = 6.48 \times 10^{10}$;

$Gr.Pr = \rho^2 g \beta C_p L^3(\Delta T)/ \mu k = 6.48 \times 10^{10} \times (0.6)^3 \times 60 = 8.4 \times 10^{11}$

Using Eq (5.10): $Nu = 0.825 + [0.387\,(8.4 \times 10^{11})^{1/6}]/ [1+ (0.492/3.01)^{9/16})]^{8/27} = 89.48$ which gives h = 97.533 and Q = 2.107 kW.

Example 14. Glycerine at 35°C flows over a 30cm by 3Ocm flat plate at a velocity of 1.25 m/s. The drag force is measured as 9.8 N (both Side of the plate). Calculate the heat transfer for such a flow system.

Solution: From tables, the properties of glycerine at 35°C are:

ρ = 1256 kg/m^3, C_p = 2.5 kJ/kgK, μ = 0.28 kg/m-s, k = 0.286 W/mK, Pr = 2.4 Re= $\rho VL/\mu$ = 1256 × 1.25 × 0.30/0.28 = 1682.14, a laminar flow.*

Average shear stress on one side of the plate = drag force/area

= 9.8/(2 × 0.3 × 0.3) = 54.4

and shear stress = $C_f \rho U^2/2$

∴ The average skin friction coefficient, $Cr/2 = \dfrac{\tau}{\rho U^2}$

= 54.4/(1256 ×1.25 × 1.25) = 0.0277

From Reynolds analogy, $C_f/2 = St.\, Pr^{2/3}$

or, $h = \rho C_p U \times C_f/2 \times Pr^{-2/3} = \dfrac{1256 \times 2.5 \times 1.25 \times 0.0277}{(2.45)^{0.667}}$ =59.8 kW/m^{2}K.