UNIT 5 KINETICS OF RIGID BODIES AND DYNAMICS OF PARTICLES 12 Hrs.

Work, Kinetic Energy, Potential Energy, Power, Impulse, Momentum, Impact - Combination problems. Displacement, Velocity and acceleration their relationship - Relative motion - Curvilinear motion - Newton's Law - D'Alembert's Principle, Work Energy Equation - Impulse and Momentum - Impact of elastic bodies. Translation and rotation of rigid bodies- General plane motion.

UNIT 5

INTRODUCTION

The dynamics of particles deals with the study of forces acting on a body and its effects, when the body is in motion. It is further divided into Kinematics and kinetics.

Kinematics – The study of motion of body without considering the forces which cause the motion of the body.

Kinetics – The study of motion of body with considering the external forces which cause the motion of the body.

<u>Plane motion</u> – If a particle has no size but mass it is considered to have only plane motion, not rotation. In this chapter the study motion of particles with only plane motion is taken without considering force that cause motion i.e., Kinematics.

The plane motion of the body can be sub divided into two types

(i) Rectilinear motion

(ii) Curvilinear motion

1. RECTILINEAR MOTION *(Straight Line Motion)* - It is the motion of the particle along a straight line.

Example: A car moving on a straight road

A stone falls vertically downwards

A ball thrown vertically upwards

This deals with the relationship among displacement, velocity, acceleration and time for a moving particle. The rectilinear motion is of two types as Uniform acceleration and Variable acceleration.

1.1 **Displacement** –The displacement of a moving particle is the change in its position, during which the particle remain in motion. It is the vector quantity, i.e., it has both magnitude and direction. The SI unit for displacement is the metre (m).

1.2 **Velocity** – The rate of change of displacement is velocity. It is the ratio between distances travelled in particular direction to the time taken. It is also a vector quantity, i.e., it has both magnitude and direction. The SI unit for velocity is the metre/second (m/sec) or kilometer/hour (km/h)

1.3 **Acceleration** – The rate of change of velocity is acceleration. It is the ratio between changes in velocity to the time taken. The change in velocity means the difference between final velocity and initial velocity. It is also a vector quantity. The SI unit for acceleration is the metre/second^2 (m/sec^2).

1.4 **Retardation** – The negative acceleration is retardation. It occurs when final velocity is less than initial velocity ($v<u$).

1.5 **Speed** – The distance travelled by a particle or a body along its path per unit time. It is a scalar quantity, i.e., it has only magnitude. The SI unit for speed is the metre/second (m/sec) or kilometer/hour (km/h)

RELATIVE MOTION

A body is said to be in motion if it changes its position with respect to the surroundings, taken as fixed. This type of motion is known as the individual motion of the body. An example of relative motion is how the sun appears to move across the sky, when the earth is actually spinning and causing that apparent motion. Usually, we consider motion with respect to the ground or the Earth. Within the Universe there is no real fixed point. The basis for Einstein's Theory of Relativity is that all motion is relative to what we define as a fixed point.

Relative velocity – Basic concept

Let's consider two motors A and B are moving on a road in same direction moving in uniform speed. Let the uniform velocities of motors A and B be u m/sec and v m/sec respectively (assume $v > u$)

Now, a person standing on the road looks at the motor A and finds that it is going at a speed of u m/sec. Similarly, looks at motor B and finds it is going at a speed of v m/sec separately. But for the driver of motor A, the motor B seems to move faster than him at the rate of only (v - u) m/sec. i.e., the motor A is imagined to be at ret or, the driver of motor A forgets his own motion.

Relative velocity of B with respect to A is (v-u). It is denoted by $V_{B/A}$

$$\therefore \mathbf{V_{B/A} = V_B - V_A = (v - u)\ m/sec}$$

Similarly for the driver of motor B, the motor A seems to move slower (assume $u < v$) than him at the rate of only (u - v) m/sec. i.e., the motor B is imagined to be at ret or, the driver of motor B forgets his own motion.

Relative velocity of A with respect to B is (v - u). It is denoted by $V_{A/B}$

$$\therefore \mathbf{V_{A/B} = V_A - V_B = (u - v)\ m/sec}$$

PROBLEM

Example1. The car A travels at a speed of 30 m/ sec and car B travels at a speed of 20 m/ sec in the same direction. Determine, i) the velocity of car A relative to car B ii) the velocity of car B, relative to car A

Given data

V_A = 30 m/se

V_B = 20 m/sec

Same direction

Solution

Let the cars A and B, travels in the same direction, say towards right.

Now, let's use the sign convention, the RHS velocity is taken as positive, and the LHS velocity is taken as negative. Hence, V_A = 30 m/sec and V_B = 20 m/sec.

Velocity of car A relative to car B

$V_{A/B} = V_A – V_B = 30 – 20 = 10$ m/sec (→) (since due to positive)

Velocity of car B relative to car A

$V_{B/A} = V_B – V_A = 20 – 30 = - 10$ m/sec (←) (since due to negative)

Example2. The car A travels at a speed of 30 m/ sec and car B travels at a speed of 20 m/ sec in the opposite direction. Determine, i) the velocity of car A relative to car B ii) the velocity of car B, relative to car A

Given data

V_A = 30 m/se

V_B = - 20 m/sec (- due to LHS)

Opposite direction

Solution

Let the cars A and B, travels in the opposite direction, say A towards right and towards left.

Velocity of car A relative to car B

$V_{A/B} = V_A – V_B = 30 – (-20) = 50$ m/sec (→) (since due to positive)

Velocity of car B relative to car A

$V_{B/A} = V_B – V_A = - 20 – 30 = - 50$ m/sec (←) (since due to negative)

MATHEMATICAL EXPRESSION FOR VELOCITY AND ACCELERATION

(i) Velocity, $v = ds/dt$

(ii) Acceleration, $a = d^2s/dt^2$

Where, s – distance travelled by a particle in a straight line.

t – time taken by the particle to travel the distance 's'

Equation of motion in straight line

Let, u – initial velocity (m/sec)

v – Final velocity (m/sec)

s – Distance travelled by a particle (m)

t – Time taken by the particle to change from u to v (second)

a – acceleration of the particle (m/sec^2)

$$\boxed{\begin{array}{c} v = u + at \\ s = ut + \tfrac{1}{2}(at^2) \\ v^2 = u^2 + 2as \end{array}}$$

Note: 1) If a body starts from rest, its initial velocity is zero i.e., u=0 2) If a body comes to rest, its final velocity is zero i.e., v=0

PROBLEMS

Example1. A car is moving with a velocity of 20 m/sec. the car is brought to rest by applying brakes in 6 seconds. Find i) retardation ii) distance travelled by the car after applying brakes.

Given data

u = 20 m/s

v = 0 (car is brought to rest) t = 6 sec

Solution

i) Retardation or negative acceleration

Using equation of motion, v = u+at

$0 = 20 + (a*6)$

$a = -3.33\ m/sec^2$

$\boxed{\textbf{Retardation = 3.33 m/sec}^2}$

ii) Distance travelled

Using equation of motion, $s = ut+1/2\ (at^2)$

$= (20*6) + 1/2(3.33*6^2)$

$= 60\ m$

$\boxed{\textbf{Distance, s = 60 m}}$

Example2. A train starts from rest and attains a velocity of 45 kmph in 2 minutes, with uniform acceleration. Calculate i) acceleration ii) distance travelled and iii) time required to reach a velocity of 36n kmph.

Given data

Initial velocity, u = 0 (train starts from rest)

Final velocity, v = 45 kmph = 12.5 m/sec

Time taken , t = 2 minutes = 120 seconds

Solution

i) Acceleration, a

Using equation of motion, v = u+at

$A = 0.104\ m/sec^2$

ii) Distance travelled in 2 minutes, s

Using equation of motion, $s = ut+1/2$

(at^2) S = 748.8 m

iii) Time required to attain velocity of 36 kmph u = 0

v = 36 kmph = 10 m/sec

Using equation of motion, v = u+at t

= 96.15 sec

Example3. A thief's car had a start with an acceleration of 2 m/sec^2**.** A police's car came after 5 seconds and continued to chase the thief's car with a uniform velocity of 20 m/sec. Find the time taken in which the police car will overtake the thief's car?

Given data

Initial velocity of thief's car = 0

Acceleration of thief's car = 2 m/sec^2

Uniform velocity of police van = 20 m/sec

Police's car came after 5 seconds of the start of thief's car.

Solution

Let us consider that the police's car takes't' seconds to overtake thief's car. Now, the cars are taken separately to solve.

Motion of thief's car

$u = 0$

$a = 2\ m/sec^2$

$t = (t+5)$

Using equation of motion, $s = ut+1/2\ (at^2) = (t+5)^2$ ---------------------- (1)

Motion of police's car

The police's car is moving with an uniform velocity of 20 m/sec.

Therefore, distance travelled by the police's car, from starting point of thief's car and to overtake it

Take, s = uniform velocity * time taken

= 20*t = 20t ---------------------- (2)

The police car overtakes the thief's car. Hence, the distances travelled by both the cars should be equal.

Therefore, equate (1) and (2)

$(t+5)^2 = 20t$

$t^2+25+10t = 20t$

$t^2+25-10t = 0$

Using arithmetical equation, $- \pm\sqrt{\ _2-4}$ where, a = 1, b = 10 and c = 25 $\ _2$

The t is found as 5 seconds.

Conclusion - The time taken by police's car to overtake thief's car is 5 seconds.

2. CURVILINEAR MOTION - It is the motion of the particle along a curved path. It has two dimensions.

Example: A stone thrown into the air at an angle

Throwing paper airplanes in air

There are two systems involved in curvilinear motion. They are

(i) Cartesian systems (rectangular coordinates)

(ii) Polar system (radial coordinates)

CARTESIAN SYSTEMS

It is a rectangular coordinate system which has the horizontal component in X-axis and vertical component in Y-axis.

Horizontal component of velocity, $V_x = dx/dt$

Vertical component of velocity, $V_y = dy/dt$

Therefore, resultant velocity of a particle, $V = \sqrt{(V_x^2+V_y^2)}$

Angle of inclination of velocity with X-axis, $\alpha=\tan^{-1}(V_y/V_x)$

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2$

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2$

Resultant acceleration of a particle, $a = \sqrt{(a_x^2+a_y^2)}$

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1} (a_y/a_x)$

PROBLEMS

Example1. The portion of a particle along a curved path is given by the equations $x=t^2+8t+4$ and $y=t^3+3t^2+8t+4$. Find the i) initial velocity, u ii) velocity of the particle at t=2 sec iii) acceleration of the particle at t=0 and iv) acceleration of the particle at t= 2 sec.

Given data

$x=t^2+8t+4$

$y=t^3+3t^2+8t+4$

Solution

Horizontal component of velocity , $V_x = dx/dt = d(t^2+8t+4)/dt = 2t+8$ -------------- (1)

Vertical component of velocity, $V_y = dy/dt = d(t^3+3t^2+8t+4)/dt = 3t^2+6t+8$ ------- (2)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = d(2t+8)/dt = 2$ --------------- (3)

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2= d(3t^2+6t+8)/dt = 6t+6$ ------ (4)

i) **Initial velocity, u**

Put t = 0 in equation (1) and

(2) $V_x = 2t+8$

Now, $\mathbf{V_x = 8}$ **m/sec**

$V_y = 3t^2+6t+8$

Now, $\mathbf{V_y = 8}$ **m/sec**

Therefore, resultant velocity of a particle, $V =\sqrt{(V_x^2+V_y^2)}$

$= \sqrt{(8^2+8^2)}$

V = 11.31 m/sec

Angle of inclination of velocity with X-axis, $\alpha = \tan^{-1}(V_y/V_x)$

$= \tan^{-1}(8/8)$

α **= 45°**

ii) **Velocity at t= 2 sec**

Put t = 2 seconds in equation (1) and (2)

$V_x = 2t+8$

Now, $\mathbf{V_x = 12}$ **m/sec**

$V_y = 3t^2+6t+8$

Now, $\mathbf{V_y = 32}$ **m/sec**

Therefore, resultant velocity of a particle, $V = \sqrt{(V_x^2 + V_y^2)}$

$= \sqrt{(12^2 + 32^2)}$

$\mathbf{V = 34.17\ m/sec}$

Angle of inclination of velocity with X-axis, $\alpha = \tan^{-1}(V_y/V_x)$

$= \tan^{-1}(32/12)$

$\alpha = \mathbf{69.4^\circ}$

iii) **Acceleration at t=0**

Put t = 0 in equation (3) and (4)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = 2\ m/sec^2$

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2 = 6t+6 = 6\ m/sec^2$

Resultant acceleration of a particle, $a = \sqrt{(a_x^2 + a_y^2)} = \sqrt{(2^2 + 6^2)} = 6.34\ m/sec^2$

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1}(a_y/a_x) = \tan^{-1}(6/2) = 71.56^\circ$

iv) **Acceleration at t = 2 sec**

Put t = 2 sec in equation (3) and (4)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = 2\ m/sec^2$

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2 = 6t+6 = 18\ m/sec^2$

Resultant acceleration of a particle, $a = \sqrt{(a_x^2 + a_y^2)} = \sqrt{(2^2 + 18^2)} = 18.11\ m/sec^2$

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1}(a_y/a_x) = \tan^{-1}(18/2) = 83.66^\circ$

PROJECTILES

The projectile is an example of curvilinear motion of a particle in plane motion. The motion of a particle is neither vertical nor horizontal, but inclined to the horizontal plane.

It is classified under Kinematics since the force which is responsible for motion is left out in the analysis and the rest are considered/

Definitions

Projectile – A particle projected in space at an angle to the horizontal plane.

Angle of projection means the angle to the horizontal at which the projectile is projected. It is denoted by a.

Velocity of projectile means the velocity with which the projectile is thrown into space. It is denoted by u (m/sec)

Trajectory means the path described by the projectile.
Time of flight is the total time taken by the projectile from the instant of projection up to the projectile hits the plane again.

Range is the distance along the plane between the point of projection and the point at which the projectile hits the plane at the end of its journey.

Path of the Projectile

The *horizontal distance* travelled by the projectile in any time t.

X = Velocity * Time taken

Therefore, $\mathbf{X = u \cos \alpha t}$

Or

$$t = X / u \cos \alpha$$

.

Similarly for *vertical distance*,

$$Y = \tan\alpha\, X - \tfrac{1}{2} (g^2 X / u^2 \cos^2 \alpha)$$

From the equation of the trajectory, it is clear that the two variables of projectile motion are initial velocity (u) and the angle of projection (a) to arrive standard results of projectile motion.

Time of flight (T) and time taken to reach highest point (t):

$$T = 2 u \sin \alpha / g \qquad t = u \sin \alpha / g$$

Maximum height attained:

$$h_{max} = u^2 \sin^2 \alpha / 2g$$

Horizontal range:

$$R = u^2 \sin 2\alpha / g$$

PROBLEMS

Example1. A particle is projected with an initial velocity of 60 m/sec, at an angle of 75˚ with the horizontal. Determine i) the maximum height attained by the particle ii) horizontal range of the particle iii) time taken by the particle to reach highest point iv) time of flight

Given data

Initial velocity, u = 60 m/sec

Angle of projection, α = 75˚

Solution

i) the maximum height attained by the particle

$h_{max} = u^2 \sin^2\alpha / 2g = 171.19$ m (take g = 9.81 m/sec^2) ii) horizontal range

$R = u^2 \sin 2\alpha / g = 183.48$ m

iii) time taken to reach highest point $t = u \sin \alpha / g = 5.9$ sec

iv) time of flight

$T = 2 u \sin \alpha / g = 11.8$ sec

Example2. A particle is projected with an initial velocity of 12 m/sec at an angle α with the horizontal. After sometime the position of the particle is observed by its x and y distances of 6m and 4 m respectively from the point of projection. Find the angle of projection?

Given data

Initial velocity, u = 12 m/sec

Horizontal distance, x = 6 m

Vertical distance, y = 4 m

Solution

If the coordinate points on the projectile path are given, then use equation of trajectory. Equation of path of projectile (trajectory)

$Y = \tan \alpha X – ½ (g^2X/u^2 \cos^2\alpha)$

Put u = 12 m/sec, X = 6 m and Y = 4 m

Take g = 9.81 m/sec^2

We get,

$4 = 6 \tan \alpha - (1.226/ \cos^2 \alpha)$

$1.226 \tan^2 \alpha - 6 \tan \alpha + 5.226 = 0$

Using arithmetical equation, − ±√ 2−4 where, a = 1.226, b = 6 and c = 5.226

Therefore, $\tan \alpha = - 6 \pm \sqrt{(6^2 - (4*1.226*5.226)}/(2*1.226)$

$\boldsymbol{\alpha = 75.1^\circ}$ **or** $\boldsymbol{53.06^\circ}$

Important definitions on kinetics

a) Mass - a fundamental measure of the amount of matter in the object. It is denoted by 'm'. The SI unit of mass is Kilograms (Kg). It's a scalar quantity.

b) Weight - The weight of an object is defined as the force of gravity on the object and may

be calculated as the mass times the acceleration of gravity, w = mg. Since the weight is

a force, its SI unit is the Newton.

Weight = mass * acceleration due to gravity

$$\mathbf{W = mg}$$

c) Momentum - Momentum can be defined as "mass in motion." All objects have mass; so if an object is moving, then it has momentum - it has its mass in motion. It depends upon the variables mass and velocity. In terms of an equation, the momentum of an object is equal to the mass of the object times the velocity of the object. Its SI unit is $kg.m/sec^2$

Momentum = mass • velocity

$$\mathbf{M = mv}$$

LAWS OF MOTION

When a particle / body is at rest, or moving in a straight line (rectilinear motion) or in a curved line (curvilinear motion), the particle / body obeys certain laws of motion. These laws are called Newton's law of motion. These laws are also called the principles of motion, or principles of Dynamics.

First Law

Every body continues to be in its state of rest or of uniform motion in a straight line unless and until it is acted upon some external force to change that state. It is also called ***the law of inertia***, and consists of the following two parts:

1. A body at rest continues in the same state, unless acted upon by some external force. It appears to be self-evident, as a train at rest on a level track will not move unless pulled by an engine. Similarly, a book lying on a table remains at rest, unless it is lifted or pushed.
2. A body moving with a uniform velocity continues its state of uniform motion in a straight line, unless it is compelled by some external force to change its state. It cannot be exemplified because it is, practically, impossible to get rid of the forces acting on a body.

Second Law

The rate of change of momentum of a moving body is directly proportional to the impressed force and takes place in the direction of the force applied.

The change of momentum = final momentum – initial momentum

$= mv - mu = m(v - u)$

The rate of change of momentum = change of momentum / time taken

$= m(v - u) / t = m*a$ (since $(v - u) / t = a$)

Basically, to increase the velocity of the moving body from u to v, there must be some external force to cause this change. Let that external force be 'P'.

As per the law, the external force 'F' is directly proportional to the rate of change of momentum i.e., $F \propto ma \rightarrow F = k * ma$ where, k is the constant of proportionality.

But for a moving body, k and m are constants, and hence it states that, the force acting on the body is directly proportional to the acceleration of the body. From this we can conclude that,

1. For a given body, greater force produces greater acceleration and the lesser force produces the lesser acceleration.
2. The acceleration is zero, if there is no external force on the body which results in u = v.

To find the value of constant 'k' in equation $F = k*ma$ We know that, $1\ N = 1\ kg * 1\ m/sec^2$

That is, the unit force (N) is a force, which produce unit acceleration (1 m/sec^2) on an unit mass (1 kg) hence, by substituting F = 1; m = 1 and a = 1. We get

$$\boxed{\mathbf{F = ma}}$$

Example1. A body of mass 4 kg is moving with a velocity of 2 m/sec and when certain force is applied, it attains a velocity of 8 m/sec in 6 seconds?

Given data

Mass, m = 4 kg

Initial velocity, u = 2 m/sec Final velocity, v = 8 m/sec Time, t = 6 sec

Solution

Acceleration, a = v - u / t = 8-2/6 = 1 m/sec^2

Let, ‘P’ be the force applied to cause this acceleration.

P = ma = 4*1 = 4 N

Example2. A body of mass 4 kg is at rest. What force should be applied to move it to a distance f 2 m in 4 seconds?

Given data

Mass, m = 4 kg

Distance, s = 12 m

Time taken, t = 4 sec

Initial velocity, u = 0

Solution

Using the equation, $s = ut + \frac{1}{2} at^2$

$12 = 0 + 8a$

Therefore, a = 12/8 m/sec^2

The force required to move, P = m*a = 4*(12/8) = 6 N

Therefore, P = 6 N

4. D'ALEMBERT'S PRINCIPLE

It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics."

We know that, that force acting on a body.

$$P = ma \text{ ------- (i)}$$

Where, m = mass of the body, and

a = Acceleration of the body.

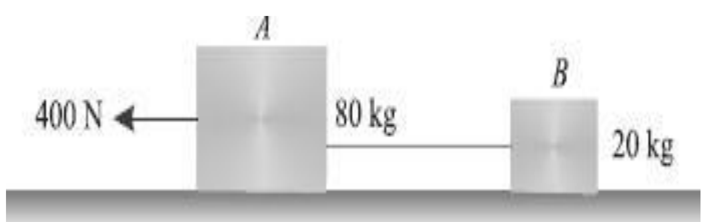
The equation (i) may also be written as:

$$P - ma = 0 \text{ -----(ii)}$$

It may be noted that equation (i) is the equation of dynamics whereas the equation (ii) is the equation of statics. The equation (ii) is also known as the equation of dynamic equilibrium under the action of the real force P. This principle is known as D' Alembert's principle.

PROBLEMS

Example1.Two bodies A and B of mass 80 kg and 20 kg are connected by a thread and move along a rough horizontal plane under the action of a force 400 N applied to the first body of mass 80 kg as shown in Figure. The coefficient of friction between the sliding surfaces of the bodies and the plane is 0.3.Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.



Given data

Mass of body *A* (m_1) = 80 kg

Mass of the body *B* (m_2) = 20 kg

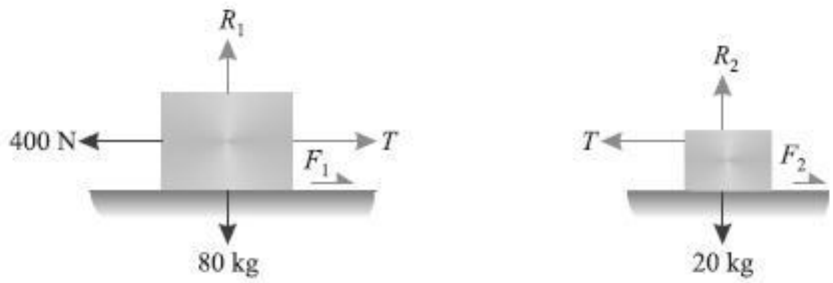
Force applied on first body (P) = 400 N and

Coefficient of friction (μ) = 0.3

Solution

Let a = Acceleration of the bodies, and

T = Tension in the thread.



(*a*) *Body A* (*b*) *Body B*

Consider the body *A*. The forces acting on it are: 400 N forces (acting towards left)

Mass of the body = 80 kg (acting downwards)
Reaction $R_1 = 80 \times 9.8 = 784$ N (acting upwards)

Force of friction, $F_1 = \mu R_1 = 0.3 \times 784 = 235.2$ N (acting towards right) Tension in the thread = T (acting towards right).

∴ Resultant horizontal force, $P_1 = 400 - T - F_1 = 400 - T - 235.2 = 164.8 - T$ (acting towards left)

We know that force causing acceleration to the body *A;* $\rightarrow m_1 a = 80\ a$

And according to *D′* Alembert's principle $P_1 - m_1 a = 0 \rightarrow 164.8 - T - 80\ a = 0$

∴ $T = 164.8 - 80a$ ---------- (*i*)

Now consider the body *B*. The forces acting on it are:
Tension in the thread = T (acting towards left)

Mass of the body = 20 kg (acting downwards)
Reaction $R_2 = 20 \times 9.8 = 196$ N (acting upwards)

Force of friction, $F_2 = \mu R_2 = 0.3 \times 196 = 58.8$ N (acting towards right)
∴ Resulting horizontal force, $P_2 = T - F_2 = T - 58.8$

We know that force causing acceleration to the body $B \rightarrow m_2 a = 20\ a$

And according to *D′* Alembert's principle $P2 - m2a = 0 \rightarrow (T - 58.8) - 20\ a = 0$

∴ $T = 58.8 + 20\ a$ ---------- (*ii*)

Now equating the two values of T from equation (i) and (ii),

$$164.8 - 80\,a = 58.8 + 20\,a$$

$$100\,a = 106$$

$$a = 106/100$$

$$\therefore a = 1.06 \text{ m/sec}^2$$

Tension in the thread

Substituting the value of a in equation (ii)

$$T = 58.8 + (20 \times 1.06)$$

$$\therefore T = 80 \text{ N}$$

Third Law

To every action, there is always an equal and opposite reaction

This law appears to be self-evident as when a bullet is fired from a gun, the bullet moves out with a great velocity, and the reaction of the bullet, in the opposite direction, gives an unpleasant shock to the man holding the gun. Similarly, when a swimmer tries to swim, he pushes the water backwards and the reaction of the water pushes the swimmer forward.

Example: When a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of gun.

Let M = Mass of the gun,

V = Velocity of the gun with which it recoils,

m = mass of the bullet, and

v = Velocity of the bullet after explosion.

∴ Momentum of the bullet after explosion = mv ----- (i)

Momentum of the gun = MV ------------------------ (ii)

Equating the equations (i) and (ii), MV = mv

This relation is popularly known as *Law of Conservation of Momentum.*

PROBLEMS

Example1. A machine gun of mass 25 kg fires a bullet of mass 30 gram with a velocity of 250 m/s. Find the velocity with which the machine gun will recoil?

Given data

Mass of the machine gun (M) = 25 kg

Mass of the bullet (m) = 30 g = 0.03kg and

Velocity of firing (v) = 250 m/s.

Solution

Let V = Velocity with which the machine gun will recoil.

We know that $MV = mv$

$25 \times v = 0.03 \times 250 = 7.5 \rightarrow v = 7.5 / 25$

$\therefore v = 0.3$ m/s

Example2. A bullet of mass 20 g is fired horizontally with a velocity of 300 m/s, from a gun carried in a carriage; which together with the gun has mass of 100 kg. The resistance to sliding of the carriage over the ice on which it rests is 20 N. Find (a) velocity with which the gun will recoil, (b) distance, in which it comes to rest, and (c) time taken to do so.

Given data

Mass of the bullet (m) = 20 g = 0.02 kg

Velocity of bullet (v) = 300 m/s

Mass of the carriage with gun (M) = 100 kg and

Resistance to sliding (F) = 20 N

Solution

(a) Velocity, with which the gun will recoil

Let V = velocity with which the gun will recoil.

We know that $MV = mv$

$100 \times V = 0.02 \times 300 = 6 \rightarrow V = 6 / 100 = 0.06$ m/s

$\therefore V = 0.06$ m/sec

(b) Distance, in which the gun comes to rest

Now consider motion of the gun. In this case, initial velocity (u) = 0.06 m/s and final velocity, $v = 0$ (because it comes to rest)

Let a = Retardation of the gun, and

s = Distance in which the gun comes to rest.

We know that resisting force to sliding of carriage (F)

$$20 = Ma = 100\,a \rightarrow a = 20 / 100$$

$\therefore a = 0.2\ \text{m/sec}^2$

We also know that $v^2 = u^2 - 2as$ (Minus sign due to retardation) $0 = (0.06)2 - 2 \times 0.2\,s$

$$= 0.0036 - 0.4\,s \rightarrow s = 0.0036 / 0.4 = 0.009 \text{ m or } 9 \text{ mm}$$

$\therefore s = 9$ mm

(c) *Time taken by the gun in coming to rest*

Let t = Time taken by the gun in coming to rest.

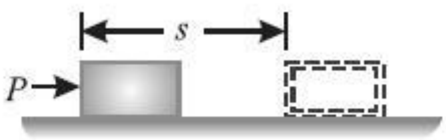
We know that final velocity of the gun (v)

$0 = u + at = 0.06 - 0.2\,t$ (Minus sign due to retardation)
$t = 0.06 / 0.2$ $\therefore t = 0.3$ seconds

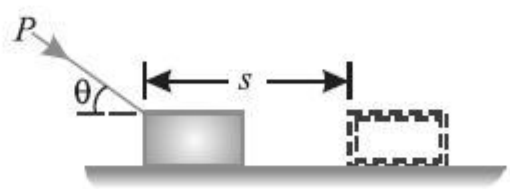
WORK ENERGY EQUATION

Work

Whenever a force acts on a body, and the body undergoes some displacement, then work is said to be done. *e.g.*, if a force P, acting on a body, causes it to move through a distance s as shown in Figure (a).



(a) Body moving in the direction of force



(b) Body not moving in the direction of force

Then work done by the force P = Force × Distance = $P \times s$

Work done by the force = P*S

Sometimes, the force P does not act in the direction of motion of the body, or in other words, the body does not move in the direction of the force as shown in Figure (*b*).

Then work done by the force P = Component of the force in the direction of motion × Distance

$$= P \cos \theta \times s$$

Work done by the force = P cos θ*S

In SI system of units, force is in Newton and the distance is in meters.

∴ Unit of work is (Newton * meter) = 1 Nm = 1 joule

∴ **In SI system of units, unit of work is joule**

PROBLEMS

Example1.A horse pulling a cart exerts a steady horizontal pull of 300 N and walks at the rate of 4·5 kmph. How much work is done by the horse in 5 minutes?

Given data

Pull (*i.e.* force) = 300 N

Velocity (v) = 4·5 kmph. = 75 m/ min

and Time, t = 5 min.

Solution

We know that distance travelled in 5 minutes

$s = 75 \times 5 = 375$ m → ∴ $s = 375$ m

Work done by the horse, W = Force × Distance

$$= 300 \times 375 = 112\,500 \text{ N-m} = 112.5 \text{ kN-m}$$

∴ **W = 112.5 kJ**

Example2. A spring is stretched by 50 mm by the application of a force. Find the work done, if the force required to stretch 1 mm of the spring is 10 N.

Given data

Spring stretched by the application of force (s) = 50
mm Stretching of spring = 1 mm and force = 10 N

Solution

We know that force required stretching the spring by 50 mm = 10 × 50 = 500 N

∴ **Average force = 500 / 2 = 250 N**

Work done = Average **force** × Distance = 250 × 50 = 12 500 N-mm = 12.5 N-m

∴ **Work done = 12.5 J**

Power

The power may be defined as the rate of doing work.

∴ Power = work done / time

= **(Force * Distance) / Time**

Or

∴ Power = Force * (Distance/Time)

= Force * Velocity

In SI systems of units, unit of work is Newton metre, and the unit of time is seconds. Unit of power = Nm / Seconds = 1 watt

∴ **In SI systems, unit of power is watt**

Energy

The energy may be defined as the capacity to do work. It exists in many forms i.e., mechanical, electrical chemical, heat, light etc. the energy is the capacity to do work. Since the energy of a body is measured by the work it can do, therefore the units of energy will be the same as those of the work. Therefore, the SI system of unit of work is joule.

In the study of mechanics, we are concerned only with mechanical energy. Mechanical energy is classified into two types.

1. Potential energy. 2. Kinetic energy.

Potential energy

It is the energy possessed by a body, for doing work, by virtue of its position.

Example1. A body, raised to some height above the ground level, possesses some potential energy; because it can do some work by falling on the earth's surface.

Example2. Compressed air also possesses potential energy; because it can do some work in expanding, to the volume it would occupy at atmospheric pressure.

Example3. A compressed spring also possesses potential energy; because it can do some work in recovering to its original shape.

Now consider a body of mass (m) raise through a height (h) above the datum level. We know that work done in raising the body = Weight × Distance = $(mg)\ h = mgh$

Potential Energy, P.E = mg *h

PROBLEM

Example1. A man of mass 60 kg dives vertically downwards into a swimming pool from a tower of height 20 m. He was found to go down in water by 2 m and then started rising. Find the average resistance of the water. Neglect the air resistance.

Given data

Mass of the man (m) = 60 kg and

Height of the tower (h) = 20 m

Solution

Let P = Average resistance of the water

We know that potential energy of the man before jumping

P.E = mg*h = 60 × 9.8 × 20 = 11 760 N-m -------------- (i)

Work done by the average resistance of water = Average resistance of water × Depth of water

= P × 2 = 2 P N-m ------- (*ii*)

Since the total potential energy of the man is used in the work done by the water, therefore equating equations (*i*) and (*ii*),

→ 11 760 = 2 P → P = 11760 / 2 ∴ **P = 5880 N**

Kinetic energy

It is the energy, possessed by a body, for doing work by virtue of its mass and velocity of motion. Now consider a body, which has been brought to rest by a uniform retardation due to the applied force.

Let m = Mass of the body

u = Initial velocity of the body

P = Force applied on the body to bring it to rest, a = Constant retardation, and

s = Distance travelled by the body before coming to rest.

Since the body is brought to rest, therefore its final velocity, $v = 0$

and Work done, W = Force × Distance = $P \times s$ ---------- (i)

Now substituting value of ($P = m.a$) in equation (i),

$W = ma \times s = mas$ ----------- (ii)

We know that $v^2 = u^2 - 2\,as$ (Minus sign due to retardation)

$\therefore 2as = u^2$ (since, $v = 0$)

Now substituting the value of ($a.s$) in equation (ii) and replacing work done with kinetic energy,

$$\textbf{K.E} = \boldsymbol{mu^2/2}$$

In most of the cases, the initial velocity is taken as v (instead of u), therefore kinetic energy,

$$\textbf{K.E} = \boldsymbol{mv^2/2}$$

Kinetic Energy, K.E = ½ (mv^2)

PROBLEM

Example1. A truck of mass 15 tones travelling at 1.6 m/s impacts with a buffer spring, which compresses 1.25 mm per kN. Find the maximum compression of the spring?

Given data

Mass of the truck (m) = 15 t

Velocity of the truck (v) = 1.6 m/s and

Buffer spring constant (k) = 1.25 mm/ kN

Solution

Let x = Maximum compression of the spring in mm.

We know that kinetic energy of the truck = $mv^2/2$ = $(15*1.6^2) / 2$ = 19.2 = 19200 kN - mm

Kinetic Energy, K.E = 19200 kN-mm -------------- (i)

Compressive load = x / 1.25 = 0.8 x kN

Work done in compressing the spring = Average compressive load × Displacement =

(0.8 x / 2) * x = 0.4 x^2 ------------------- (ii)

Since the entire kinetic energy of the truck is used to compress the spring therefore equating equations (i) and (ii),

$$19\,200 = 0.4\,x^2 \rightarrow x^2 = 19200 / 0.4$$

$$= 48000$$

$$\therefore\ \mathbf{x = 219\ mm}$$

Work Energy Equation

The equation of motion in one-dimension (taking the variable to be *x*, and the force to be *F*) is

$$m\frac{d^2x}{dt^2} = F(x)$$

Let us again eliminate time from the left-hand using the technique used above

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\frac{dx}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

To get

$$mv\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}mv^2\right) = F(x)$$

On integration this equation gives

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = \int_{x_0}^{x_1} F(x)\,dx$$

where x_i and x_f refer to the initial and final positions, and v_i and v_f to the initial and final velocities, respectively. We now interpret this result. We define the kinetic energy of a particle of mass *m* and velocity *v* to be

$$Kinetic\ energy = \frac{1}{2}mv^2$$

and the work done in moving from one position to the other as the integral given above

$$Work\ done = \int_{x_i}^{x_f} F\,dx$$

With these definitions the equation derived above tells us that work done on a particle changes its kinetic energy by an equal amount; this known as the *work-energy theorem.*

IMPULSE AND MOMENTUM

Impulse

The impulse of a constant force F is defined as the product of the force and the time t for which it acts. The SI unit of linear impulse is ***N.sec*** $\text{Impulse} = Ft$. -------------- (i)

The effect of the impulse on a body can be found using equation (i) where, a is acceleration, u and v are initial and final velocities respectively and t is time.

$$v = u + at$$

So

$$mat = m(v - u)$$
$$F = ma$$
$$Ft = m(v - u) = \text{change in momentum} \quad \text{------------ (ii)}$$

So we can say that

$$\text{Impulse of a constant force} = Ft = \text{change in momentum produced}$$

Impulse is a vector quantity and has the sane units as momentum, Ns or kg m/s. The impulse of a variable force can be defined by the integral

$$\text{Impulse} = \int_0^t F\,dt$$

Where, t is the time for which F acts.

By Newton's 2^{nd} law

$$F = ma = m\frac{dv}{dt}$$

So impulse can also be written

$$\text{Impulse} = \int_0^t m\frac{dv}{dt}dt$$

$$= \int_u^v m\,dv$$

$$= [mv]_u^v$$

Which for a constant mass

$$\text{Impulse} = m(v-u)$$

In summary

$$\text{Impulse} = \int_0^t F\,dt = \text{change in momentum produced}$$ ----------- (iii)

Impulsive force

Suppose the force F is very large and acts for a very short time. During this time the distance moved is very small and under normal analysis would be ignored. Under these condition the only effect of the force can be measured is the impulse, or change I momentum - the force is called an impulsive force.

In theory this force should be infinitely large and the time of action infinitely small. Some applications where the conditions are approached are collision of snooker balls, a hammer hitting a nail or the impact of a bullet on a target.

PROBLEMS

Example1. A nail of mass 0.02 kg is driven into a fixed wooden block, its initial speed is 30 m/s and it is brought to rest in 5ms. Find a) the impulse b) value of the force (assume this constant) on the nail.

Given data

Mass, m = 0.02 kg

Velocity, v = 30 m/sec

Initial velocity, u = 0

Time, t = 5 minutes

Solution

Using the equation,

$$\begin{aligned}\text{Impulse} &= \text{change in momentum of the nail}\\ &= 0.02(30-0)\\ &= 0.6\ Ns\end{aligned}$$

$$\text{Impulse} = Ft$$

$$F = \frac{\text{Impulse}}{t} = \frac{0.6}{0.005} = 120\ N$$

<u>Momentum</u>

The quantity of motion possessed by the moving body is called momentum. It is the product of mass and velocity.

Momentum = Mass * Velocity

i.e., M = mv

Where, m is mass I kilogram

v = velocity in m/sec

M = Momentum in kg.m/sec

→ mv = (w/g) * v

The SI unit of momentum is also ***N.sec***

Impulse – Momentum equation

The impulse – Momentum equation is also derived from the Newton's secong law,

F = ma = m * (dv/dt) i.e., F dt = m dv

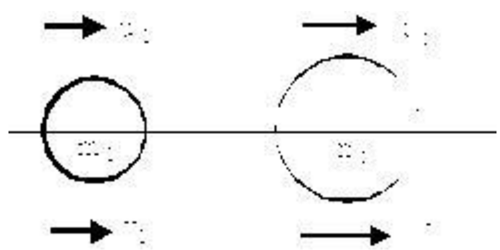
As derived in the impulse, the term $\int_0^t$ F dt is called impulse and m (v-u) is called the change of momentum, i.e., Final momentum – Initial momentum.

Impulse = m (v-u) = (W/g) * (v-u)

Impact of elastic bodies

In the last section the bodies were assumed to stay together after impact. An elastic body is one which tends to return to its original shape after impact. When two elastic bodies collide, they rebound after collision. An example is the collision of two snooker balls.

If the bodies are travelling along the same straight line before impact, then the collision is called a direct collision. This is the only type of collision considered here.



Direct collision of two elastic spheres

Consider the two elastic spheres as shown in figure. By the principle of conservation of linear momentum

$$\text{Momentum befor impact} = \text{Momentum after impact}$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Where the *u*'s are the velocities before collision and the *v*'s, the velocities after.

When the spheres are inelastic v_1 and v_2 are equal as we saw in the last section. For elastic bodies v_1 and v_2 depend on the elastic properties of the bodies. A measure of the elasticity is the **coefficient of restitution** *e*, for direct collision this is defined as

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$

This equation is the result of experiments performed by Newton. The values of *e* in practice vary from between 0 and 1. For inelastic bodies *e* = 0, for completely elastic *e* = 1. In this latter case no energy is lost in the collision.

PROBLEMS

***Example*1**. A body of mass 2kg moving with speed 5m/s collides directly with another of mass 3 kg moving in the same direction. The coefficient of restitution is 2/3. Find the velocities after collision.

Solution

$$\text{Momentum befor impact} = \text{Momentum after impact}$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$2\times5 + 3\times4 = 2v_1 + 3v_2$$
$$22 = 2v_1 + 3v_2 \qquad \text{----------- (i)}$$

From equation

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$
$$\frac{2}{3} = -\left(\frac{v_1 - v_2}{5-4}\right)$$
$$-2 = 3v_1 - 3v_2 \qquad \text{--------------- (ii)}$$

Adding [i] and [ii] gives

$$20 = 5v_1$$
$$v_1 = 4\,m/s$$

And by [i]

$$22 = 8 + 3v_2$$
$$v_2 = \frac{14}{3}\,m/s$$

Example2.A railway wagon has mass 15 tones and is moving at 1.0 m/s. It collides with a second wagon of mass 20 tones moving in the opposite direction at 0.5m/s. After the collision the second wagon has changed its speed to 0.4m/s in the opposite direction as before the collision. Find i) the velocity of the 15 tones wagon after the collision ii) the coefficient of restitution and iii) the loss in kinetic energy.

Solution

$$\text{Momentum befor impact} = \text{Momentum after impact}$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$15000 \times 1.0 - 20000 \times 0.5 = 15000v_1 + 20000 \times 0.4$$
$$-3000 = 15000v_1$$
$$v_1 = -0.2\,m/s$$

The negative sign means it has change direction of travel.

Coefficient of restitution is

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$
$$e = -\left(\frac{(-0.2) - 0.4}{1.0 - (-0.5)}\right)$$
$$e = 0.4$$

$$\text{kinetic energy before impact} = \frac{1}{2}15000 \times 1.0^2 + \frac{1}{2}20000 \times 0.5^2$$
$$= 10000\,J$$

$$\text{kinetic energy after impact} = \frac{1}{2}15000 \times 0.2^2 + \frac{1}{2}20000 \times 0.4^2$$
$$= 1900\,J$$

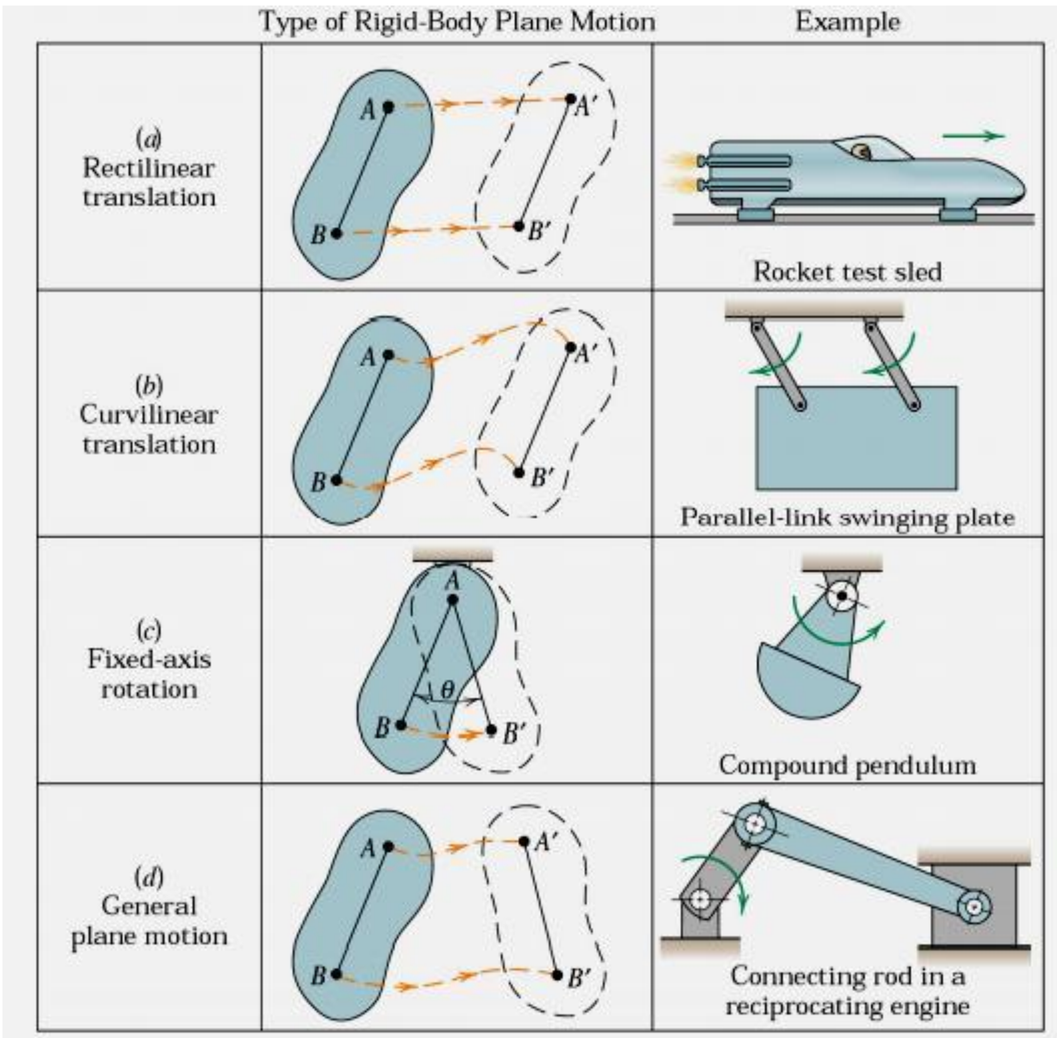
$$\text{loss of kinetic energy} = 10000 - 1900 = 8100\,J$$

Translation, Rotation of rigid bodies and General plane motion

Introduction

Forces acting of rigid bodies can be also separated in two groups: (a) The *external forces* represent the action of other bodies on the rigid body under consideration; (b) The *internal forces* are the forces which hold together the particles forming the rigid body. Only external forces can impart to the rigid body a motion of translation or rotation or both.

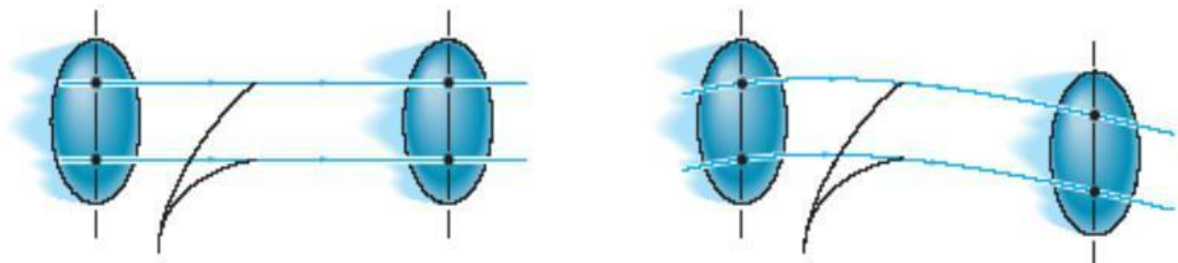
In kinematics the types of motion are *TRANSLATION*, *ROTATION* about a fixed axis and *GENERAL PLANE MOTION*.



TRANSLATION

A motion is said to be a translation if any straight line inside the body keeps the same direction during the movement. It occurs if every line segment on the body remains parallel to its original direction during the motion

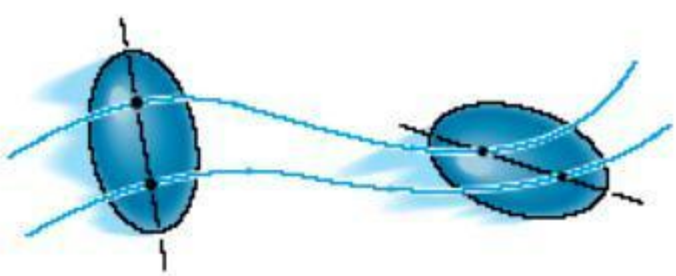
All the particles forming the body move along parallel paths. If these paths are straight lines, the motion is said a ***rectilinear translation***; if the paths are curved lines, the motion is a ***curvilinear motion*** as given below in figure.



Path of rectilinear translation Path of curvilinear translation

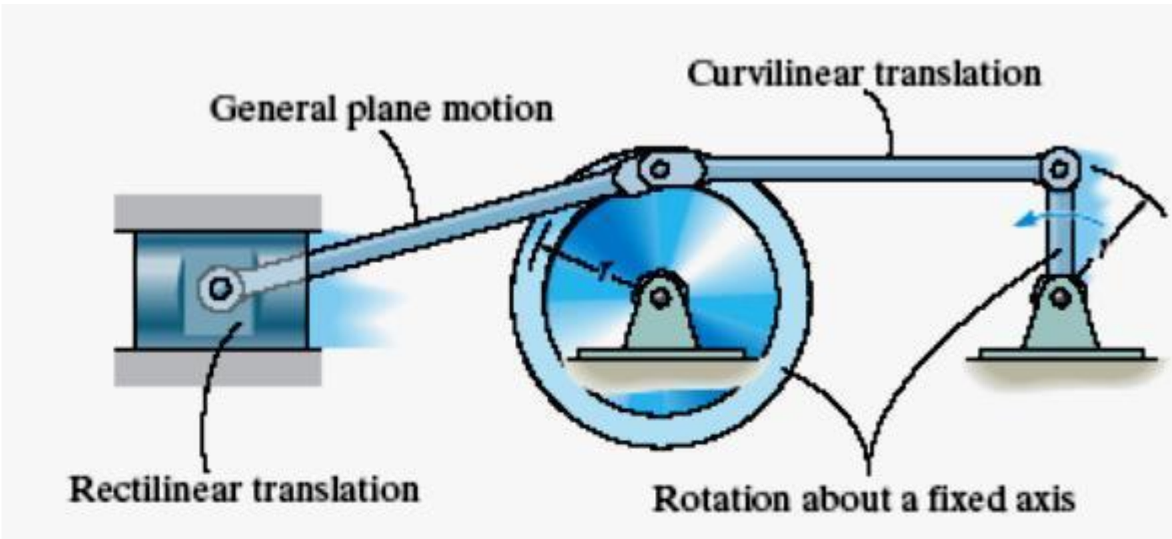
GENERAL PLANE MOTION

Any plane motion which is neither a translation nor a rotation is referred as a general plane motion. Plan motion is that in which all the particles of the body move in parallel planes. Translation occurs within a plane and rotation occurs about an axis perpendicular to this plane.



General plane motion

An example of bodies undergoing the three types of motion is shown in this mechanism. The wheel and crank undergo rotation about a fixed axis. In this case, both axes of rotation are at the location of the pins and perpendicular to the plane of the figure. The piston undergoes rectilinear translation since it is constrained to slide in a straight line. The connecting rod undergoes curvilinear translation, since it will remain horizontal as it moves along a circular path. The connecting rod undergoes general plane motion, as it will both translate and rotate.



ROTATION

Some bodies like pulley, shafts, and flywheels have motion of rotation (*i.e.*, angular motion) which takes place about the geometric axis of the body. The angular velocity of a body is always expressed in terms of revolutions described in one minute, *e.g.*, if at an instant the angular velocity of rotating body in *N* r.p.m. (*i.e*. revolutions per min) the corresponding angular velocity ω (in rad) may be found out as discussed below:

1 revolution/min = 2π rad/min

∴ N revolutions/min = $2\pi N$ rad/min Angular velocity $\omega = 2\pi N$ rad/min

$$\omega = 2\pi N / 60 \text{ rad/sec}$$

Important Terms

The following terms, which will be frequently used in this chapter, should be clearly understood at this stage:

Angular velocity - It is the rate of change of angular displacement of a body, and is expressed in r.p.m. (revolutions per minute) or in radian per second. It is, usually, denoted by ω (omega).

Angular acceleration - It is the rate of change of angular velocity and is expressed in radian per second per second (rad/s^2) and is usually, denoted by α. It may be constant or variable.

Angular displacement - It is the total angle, through which a body has rotated, and is usually denoted by θ. If a body is rotating with a uniform angular velocity (ω) then in t seconds, the angular displacement is $\boldsymbol{\theta = \omega * t}$

Motion of rotation under constant angular acceleration

Consider a particle, rotating about its axis.

Let ω_0 = Initial angular velocity,

ω = Final angular velocity,

t = Time (in seconds) taken by the particle to change its velocity from ω_0 to ω. α = Constant angular acceleration in rad/s^2, and

θ =Total angular displacement in radians.

Since in t seconds, the angular velocity of the particle has increased steadily from ω_0 to ω at the rate of α rad/s^2, therefore

$$\omega = \omega_0 + \alpha t \qquad ...(i)$$

and average angular velocity $= \dfrac{\omega_0 + \omega}{2}$

We know that the total angular displacement,

$$\theta = \text{Average velocity} \times \text{Time} = \left(\frac{\omega_0 + \omega}{2}\right) \times t \qquad ...(ii)$$

Substituting the value of ω from equation (i),

$$\theta = \frac{\omega_0 + (\omega_0 + \alpha t)}{2} \times t = \frac{2\omega_0 + \alpha t}{2} \times t = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad ...(iii)$$

and from equation (i), we find that

$$t = \frac{\omega - \omega_0}{\alpha}$$

Substituting this value of t in equation (ii),

$$\theta = \left(\frac{\omega_0 + \omega}{2}\right) \times \left(\frac{\omega - \omega_0}{\alpha}\right) = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$\therefore \qquad \omega^2 = \omega_0^2 + 2\alpha\theta \qquad ...(iv)$$

Relation between linear motion and angular motion

S. No.	Particulars	Linear motion	Angular motion
1.	Initial velocity	u	ω_0
2.	Final velocity	v	ω
3.	Constant acceleration	a	α
4.	Total distance traversed	s	θ
5.	Formula for final velocity	$v = u + at$	$\omega = \omega_0 + \alpha t$
6.	Formula for distance traversed	$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
7.	Formula for final velocity	$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
8.	Differential formula for velocity	$v = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$
9.	Differential formula for acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

PROBLEMS

Example1. A flywheel starts from rest and revolves with an acceleration of 0.5 rad/ sec^2. What will be its angular velocity and angular displacement after 10 seconds?

Given data

Initial angular velocity ($\omega 0$) = 0 (becasue it starts from rest)

Angular acceleration (α) = 0.5 rad/sec^2 and

Time (t) = 10 sec.

Solution

Angular velocity of the flywheel

We know that angular velocity of the flywheel,

$\omega = \omega 0 + \alpha t = 0 + (0.5 \times 10) = 5$ rad/sec

Angular displacement of the flywheel

We also know that angular displacement of the flywheel,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (0 \times 10) + \left[\frac{1}{2} \times 0.5 \times (10)^2\right] = 25 \text{ rad}$$

Example2. A wheel rotates for 5 seconds with a constant angular acceleration and describes during this time 100 radians. It then rotates with a constant angular velocity and during the next five seconds describes 80 radians. Find the initial angular velocity and the angular acceleration.

Given data

Time (t) = 5 sec and

Angular displacement (θ) = 100 rad

Solution

Initial angular velocity

Let ω_0 = Initial angular velocity in rad/s,

α = Angular acceleration in rad/s^2, and

ω = Angular velocity after 5 s in rad/s.

First of all, consider the angular motion of the wheel with constant acceleration for 5 seconds.

We know that angular displacement (θ),

$$100 = \omega_0 t + \frac{1}{2}\alpha t^2 = \omega_0 \times 5 + \frac{1}{2} \times \alpha (5)^2 = 5\omega_0 + 12.5\alpha$$

$$\therefore \quad 40 = 2\omega_0 + 5\alpha \quad \text{...}(i)$$

and final velocity, $\omega = \omega_0 + \alpha t = \omega_0 + \alpha \times 5 = \omega_0 + 5\alpha$

Now consider the angular motion of the wheel with a constant angular velocityof $(\omega_0 + 5\alpha)$ for 5 seconds and describe 80 radians. We know that the angular displacement,

$$80 = 5(\omega_0 + 5\alpha)$$

or

$$16 = \omega_0 + 5\alpha \quad \text{...}(ii)$$

Subtracting equation (*ii*) from (*i*),

$$24 = \omega_0 \quad \text{or} \quad \omega_0 = 24 \text{ rad/s} \quad \textbf{Ans.}$$

Angular acceleration

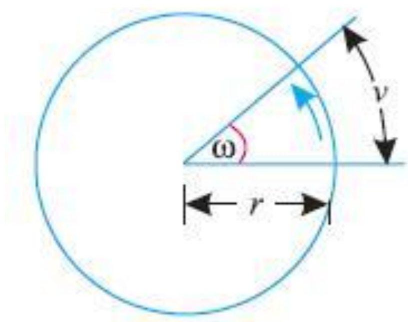
Substituting this value of ω_0 in equation (*ii*),

$$16 = 24 + 5\alpha \quad \text{or} \quad \alpha = \frac{16 - 24}{5} = -1.6 \text{ rad/s}^2 \quad \textbf{Ans.}$$

...(Minus sign means retardation)

Linear (Or Tangential) Velocity of a Rotating Body

Consider a body **rotating about its axis** as shown in Figure.



Let ω = Angular velocity of the body in rad/s,

r = Radius of the circular path in meters, and

v = Linear velocity of the particle on the periphery in m/s.

After one second, the particle will move *v* meters along the circular path and the angular displacement will be ω rad.

We know that length of arc = Radius of arc × Angle subtended in rad.

$\therefore v = r\omega$

PROBLEMS

Example1. A wheel of 1.2 m diameter starts from rest and is accelerated at the rate of 0.8 rad/s2 . Find the linear velocity of a point on its periphery after 5 seconds.

Given data

Diameter of wheel = 1.2 m or radius (r) = 0.6 m

Initial angular velocity (ω_0) = 0 (becasue, it starts from rest)

Angular acceleration (α) = 0.8 rad/s2 and

Time (t) = 5 s

Solution

We know that angular velocity of the wheel after 5 seconds,

$$\omega = \omega_0 + \alpha t = 0 + (0.8 \times 5) = 4 \text{ rad/s}$$

∴ Linear velocity of the point on the periphery of the wheel, $v = r\omega = 0.6 \times 4 = 2.4$ m/s

Example2. A pulley 2 m in diameter is keyed to a shaft which makes 240 r.p.m. Find the linear velocity of a particle on the periphery of the pulley.

Given data

Diameter of pulley = 2 m or radius (r) = 1 m and

Angular frequency (N) = 240 r.p.m.

Solution

We know that angular velocity of the pulley,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.1 \text{ rad/s}$$

$\therefore$ Linear velocity of the particle on the periphery of the pulley,

$$v = r\omega = 1 \times 25.1 = 25.1 \text{ m/s}$$

Linear (Or Tangential) Acceleration of a Rotating Body

Consider a body rotating about its axis with a constant angular (as well as linear) acceleration. We know that linear acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}(v) \qquad ...(i)$$

We also know that in motion of rotation, the linear velocity,

$$v = r\omega$$

Now substituting the value of v in equation (i),

$$a = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$

Where α = Angular acceleration in rad/sec^2 and is equal to $d\omega/dt$.

PROBLEMS

Exapmle1. A car is moving at 72 kmph. If the wheels are 75 cm diameter, find the angular velocity of the tyre about its axis. If the car comes to rest in a distance of 20 meters, under a uniform retardation, find angular retardation of the wheels.

Given data

Linear velocity (v) = 72 kmph = 20 m/s

Diameter of wheel (d) = 75 cm or radius (r) = 0.375 m and

Distance travelled by the car (s) = 20 m.

Solution

Angular retardation of the wheel

We know that the angular velocity of the wheel,

$$\omega = \frac{v}{r} = \frac{20}{0.375} = 53.3 \text{ rad/sec}$$

Let a = Linear retardation of the wheel.

We know that $v^2 = u^2 + 2as$

$\therefore$ $0 = (20)^2 + 2 \times a \times 20 = 400 + 40a$

or $a = -\dfrac{400}{40} = -10 \text{ m/sec}^2$ ...(Minus sign indicates retardation)

We also know that the angular retardation of the wheel,

$$\alpha = \frac{a}{r} = \frac{-10}{0.375} = -26.7 \text{ rad/sec}^2$$

...(Minus sign indicates retardation)

Example2. The equation for angular displacement of a body moving on a circular path is given by θ = 2t3 + 0.5 where θ is in rad and t in sec. Find angular velocity, displacement and acceleration after 2 sec.

Given data

Equation for angular displacement θ = 2t3 + 0.5 ----- (i)

Solution

Angular displacement after 2 seconds

Substituting $t = 2$ in equation (i),

$$\theta = 2\,(2)^3 + 0.5 = 16.5 \text{ rad}$$

Angular velocity after 2 seconds

Differentiating both sides equation (i) with respect to t,

$$\frac{d\theta}{dt} = 6\,t^2 \qquad ...(ii)$$

velocity, $$\omega = 6\,t^2 \qquad ...(iii)$$

Substituting $t = 2$ in equation (iii),

$$\omega = 6\,(2)^2 = 24 \text{ rad/sec}$$

Angular acceleration after 2 seconds

Differentiating both sides of equation (iii) with respect to t,

$$\frac{d\omega}{dt} = 12t \text{ or Acceleration } \alpha = 12t$$

Now substituting $t = 2$ in above equation,

$$\alpha = 12 \times 2 = 24 \text{ rad/sec}^2$$

Example3. The equation for angular displacement of a particle, moving in a circular path (radius 200 m) is given by $\theta = 18t + 3t^2 - 2t^3$ where θ is the angular displacement at the end of t sec. Find (i) angular velocity and acceleration at start, (ii) time when the particle reaches its maximum angular velocity; and (iii) maximum angular velocity of the particle.

Given data

Equation for angular displacement $\theta = 18t + 3t2 - 2t^3$ ------------- (i)

Solution

(i) Angular velocity and acceleration at start

Differentiating both sides of equation (i) with respect to t, $d\theta / dt = 18 + 6t - 6t^2$ i.e. angular velocity, $\omega = 18 + 6t - 6t^2$ --------------(ii)

Substituting t = 0 in equation (ii),

$$\omega = 18 + 0 - 0 = 18 \text{ rad/s}$$

Differentiating both sides of equation (ii) with respect to t, $d\omega / dt = 6 - 12t$ i.e. angular acceleration, $\alpha = 6 - 12t$ ---------------- (iii)

Now substituting t = 0 in equation (iii),

$$\alpha = \mathbf{6\ rad/s^2}$$

(ii) Time when the particle reaches maximum angular velocity

For maximum angular velocity, take equation (iii) and equate it to zero 6 – 12t = 0 or t = 6 / 12

t = 0.5 seconds.

(iii) Maximum angular velocity of the particle

The maximum angular velocity of the particle may now be found out by substituting t = 0.5 in equation (ii),

$$\omega_{max} = 18 + (6 \times 0.5) - 6\,(0.5)^2$$

$$\boldsymbol{\omega_{max} = 19.5\ rad/s}$$