UNIT 4 FRICTION 12 Hrs.

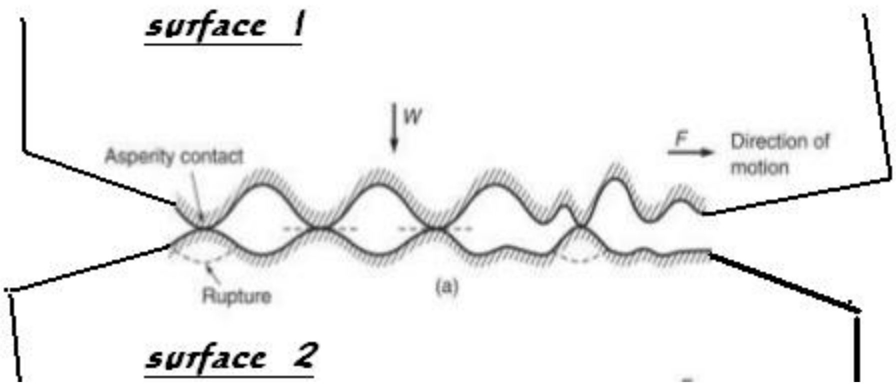
Frictional Force - Laws of Coulomb friction - Cone of friction- Angle of repose- Simple contact friction - Screw - Wedge - Ladder - Rolling resistance - Belt friction.

UNIT – IV

FRICTION

In the previous units, the surfaces in contact have been assumed to be frictionless. But practically, the surfaces are rough in nature.

Friction gets developed because of surface irregularities between the contact surfaces.



Let us say a block of weight W rests on a table. Let us further say a maximum of 20 N force it can generate against the applied load which will always oppose motion.

If we apply say 1N the generated Frictional force is also 1 N.

If we apply say 5N the generated Frictional force is also 5 N.

If we apply say 10 N the generated Frictional force is also 10 N.

If we apply say 20 N the generated Frictional force is also 20 N.

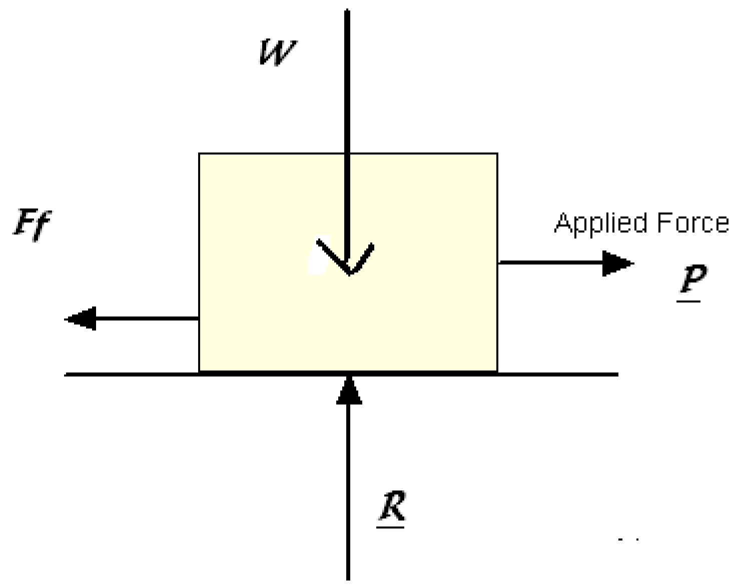
It is sure that it will generate equal frictional force as above. Otherwise because of force imbalance the object will move. It does not happen Hence the applied load= generated Frictional force.

Read the last case discussed above. i.e., **If we apply say 20 N the generated Frictional force is also 20 N. This state is called impending motion state where motion is likely to occur but is in equilibrium(i.e., not moving)**

However if we apply more than to 20 N say 21 N then object can generate only a maximum of 20 N in the opposite direction of applied force. Hence the object will move with 21 N- 20 N= 1N force in the applied force direction.

Evaluation of Frictional force:

Let us consider a block of weight W rests on a table. Let the developed reaction to support the load is R.



Now W=R, Is it not?

Let us assume that an applied load P acts on the block to RHS.

Frictional force F_f will act to the LHS to oppose motion.

The frictional force is directly proportional to the normal reaction

i.e., $F_f \alpha R$

F_f = (a constant) R

The constant is called the coefficient of Friction and is referred as μ.

i.e., $F_f = \mu R$

Coefficient of Friction μ is defined as the ratio of the frictional force to the normal reaction which is dimensionless.

i.e., $\mu = F_f /R$

Types of friction:

1. **Dry or Coulomb Friction:** When friction occurs between two non-lubricated bodies in contact, it is known as dry friction. The two surfaces of bodies may be at rest or one of the bodies is moving and the other is at rest.

2. **Fluid friction:** When adjacent layers in a fluid are moving with different velocities, then the friction is called fluid friction.

Classification of Dry friction:

1. Static Friction (F_s): Frictional force acting between two bodies which are in contact but are not sliding with respect to each other is called static friction.

$$\boldsymbol{F = \mu N}$$

   a) Limiting Friction (F_{max}): The maximum frictional force that a body can exert on the other body having contact with it is known as limiting friction.

$$\boldsymbol{F_{max} = \mu_s N}$$

   where, F_{max} is the maximum possible force of static friction, N is normal force and μ_s is a constant known as coefficient of static friction. Always F_s is smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest. Therefore

$$F_s \leq F_{max} = \mu_s N$$

2. Dynamic or Kinetic Friction(F_k): When two bodies are in contact moving with respect to each other experiences some friction and this force is known as dynamic or kinetic friction(f_k).

$$\boldsymbol{F_k = \mu_k N}$$

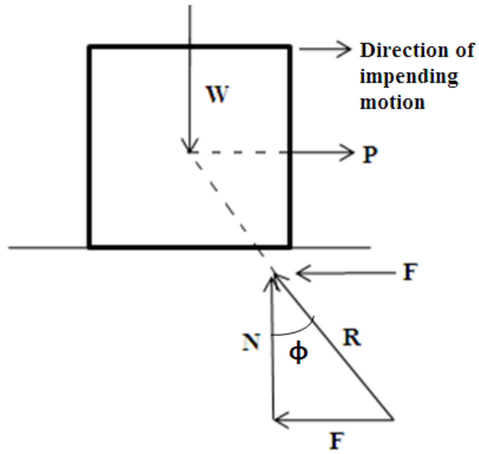
   where, F_k is Kinetic friction, N is normal force and μ_k is a constant known as coefficient of kinetic friction.

   a) Sliding Friction: If a body is moving over or within the other body experiences some frictional force. This force is known as sliding friction.
   b) Rolling Friction: This is the force experienced by a body when it is rolling on the other body.

Laws of Coulomb friction:

1. The frictional force developed is equal to the external force applied to the surface, till the maximum friction.
2. The frictional force is always acting in the opposite direction in which the surface tends to move.
3. The frictional force is independent to the surface area of contact.
4. μ_s and μ_k are not depend upon the area of the surfaces but depends upon the nature of the surfaces which are in contact.
5. μ_s is always greater than μ_k.

Angle of friction (Φ): Angle of friction is defined as the angle made by the resultant and the normal to the surface.

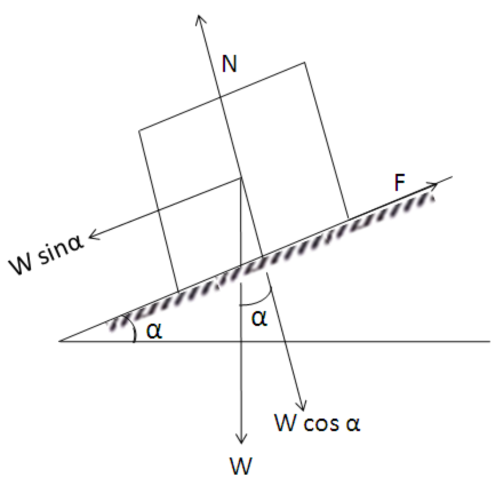


$$\tan \Phi = (F/N) = (\mu N/N) = \mu$$

where, **Φ** is angle of friction and μ is coefficient of friction.

Angle of repose (α): Angle of repose is the maximum angle of inclination of an inclined plane on which a body remains in equilibrium or sleep over the inclined plane by the assistance of friction only.

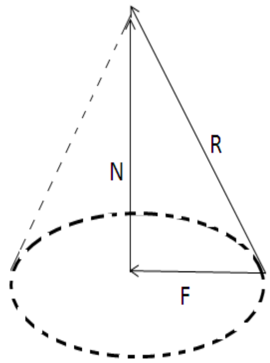
(English Meaning of Repose is Sleep.)



$$\tan \alpha = (W\sin \alpha / W\cos \alpha) = (F/N) = (\mu N/N) = \mu = \tan \Phi$$

(OR) $\alpha = \Phi$

Cone of friction: When the direction of external force is changed gradually through 360°, the resultant generates a right circular cone with semi central angle of cone about normal plane is equal to the angle of friction.



Screw friction: It is a device used for lifting or lowering heavy load by applying comparatively smaller efforts at the end of the lever. The thread of the screw jack can be considered as an inclined plane.

$$P \times a = F \times r$$

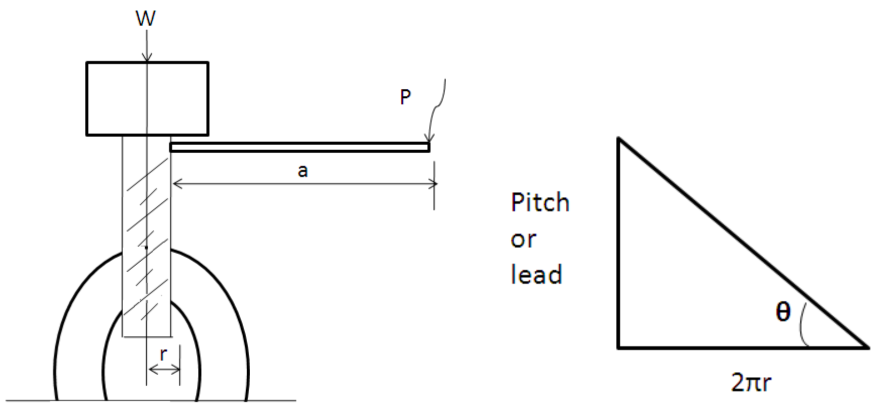
$$F = [(W(\tan\theta + \tan\phi)/(1 - \tan\theta \tan\phi)]$$

For Lifting $$P = [(W r/a)\tan(\theta + \phi)]$$

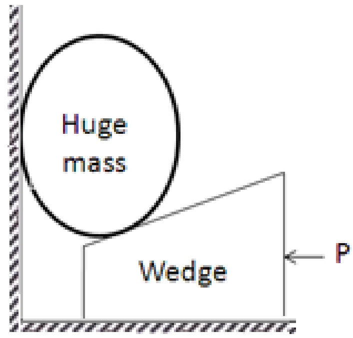
For Lowering $$P = [(W r/a)\tan(\phi - \theta)]$$

$$\tan\theta = \{P \text{ (or) } L/2\pi r\}$$

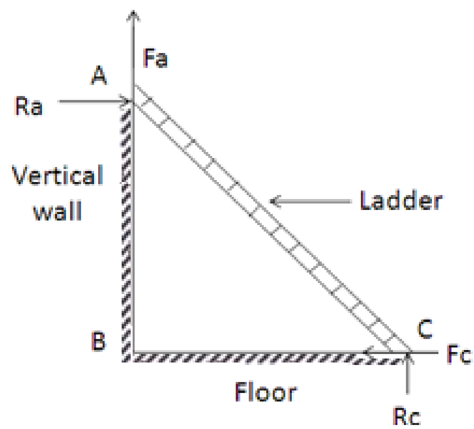
For single start $P = L$ and for multi start $nP = L$.



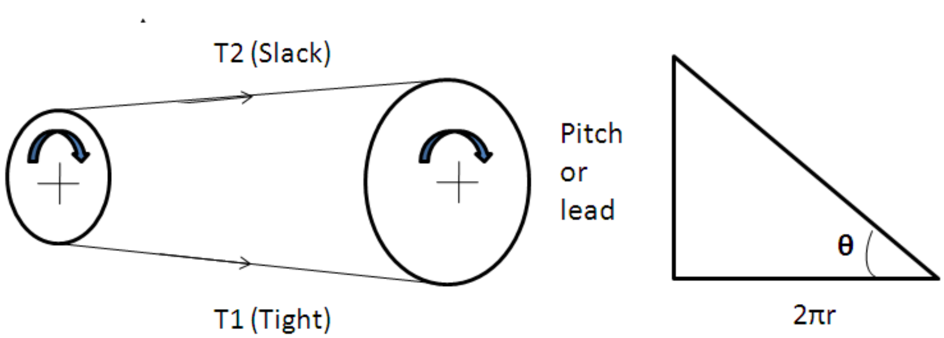
Wedge friction: A wedge is a small wooden or metal piece placed under the huge mass for lifting. This wedge experiences friction at its contact surfaces.



Ladder friction: A ladder placed against a vertical wall and horizontal floor experiences friction at two contact points, one with the wall and the other with the floor. This problem can be solved with equilibrium conditions applicable to non-concurrent and coplanar system of forces ($\sum Fx=0$, $\sum Fy=0$ and $\sum M=0$).



Belt friction: The friction experiences between the pulley and belt is called belt friction.



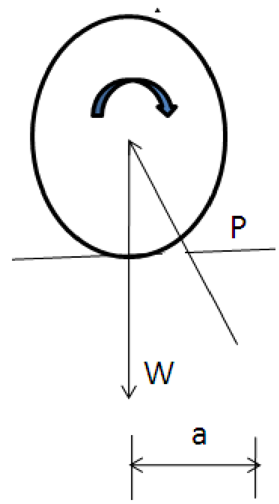
$$(T_1/T_2) = e^{\mu\theta}$$

Torque transmitted, $T = (T_1 – T_2)\, r$

Power transmitted, $P = (T_1 – T_2)\, v$

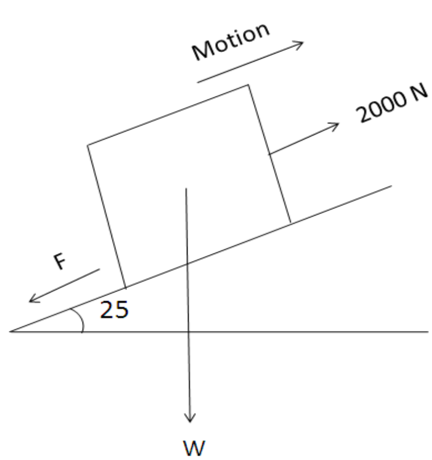
where, T_1 and T_2 are tension in tight and slack sides, r is radius of pulley , v is linear velocity and μ is coefficient of friction.

Rolling Resistance: When a wheel is made to roll freely with constant angular velocity over a horizontal surface, the wheel slows down due to the deformation of the surface which causes the wheel to have surface contact instead of line contact. This contact surface that resists the motion of wheel called rolling resistance.

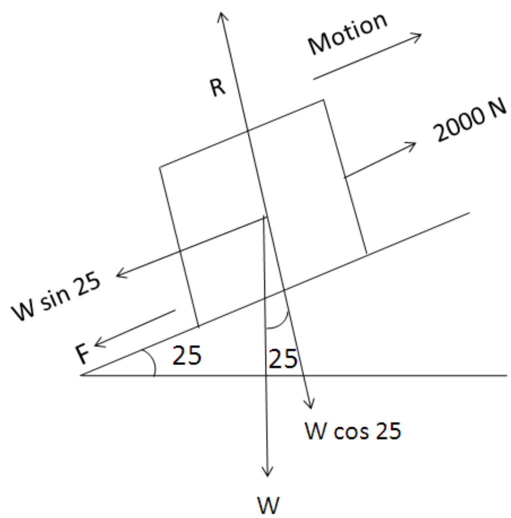


Sample Solved Problems

1. An effort of 2000 N is required to move a certain body up a 25° inclined plane. The force acting parallel to the plane. If the angle of inclination is changed from 25° to 30°, the effort required to move the body increases to 2250 N. Determine the weight of the body and the coefficient of friction.



Free Body Diagram of 2000 N weight:



Case I : When α = 25°

$\sum Fx = 0$:

$2000 - F - W \sin \alpha = 0$

$2000 - \mu R - W \sin \alpha = 0$

$2000 - \mu W \cos 25 - W \sin 25 = 0$

$0.908 \times \mu W + 0.4226 \times W = 2000$

$W (0.908 \times \mu + 0.4226) = 2000$

$W = 2000 / (0.908 \times \mu + 0.4226)$ -------------- 1

Case II : When α = 30°, P = 2250 N

$\sum Fx = 0$:

$2250 - \mu R - W \sin \alpha = 0$

$2250 - \mu W \cos \alpha - W \sin \alpha = 0$

$2250 - \mu W \cos 30 - W \sin 30 = 0$

$0.866 \times \mu W + 0.5 \times W = 2250$

$W \times (0.866 \times \mu + 0.5) = 2250$

$W = 3000 / (0.866 \times \mu + 0.5)$ -------------- 2

Equating Eq. 1 and 2

$2000 / (0.908 \times \mu + 0.4226) = 2250 / (0.866 \times \mu + 0.5)$

$(0.866 \times \mu + 0.5) / (0.908 \times \mu + 0.4226) = 2250 / 2000$

$(0.866 \times \mu + 0.5) / (0.908 \times \mu + 0.4226) = 1.125$

$(0.866 \times \mu + 0.5) = 1.125 \times (0.908 \times \mu + 0.4226)$

$0.866 \times \mu + 0.5 = 1.0215\ \mu + 0.475425$

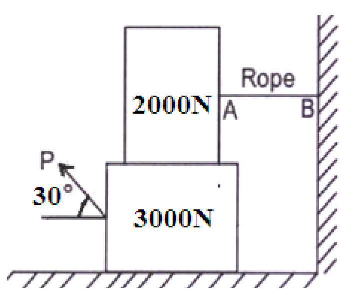
$1.0215\ \mu - 0.866 \times \mu = 0.5 - 0.\ 475425$

$\mu\ (1.0215 - 0.866) = 0.024575$

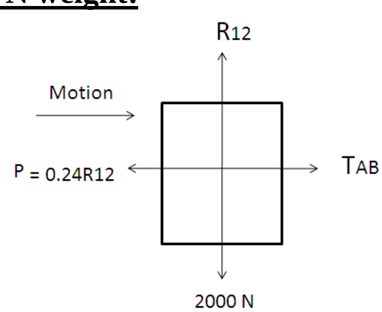
$\mu\ (0.1555) = 0.024575$

$\boldsymbol{\mu = 0.158}$

2. For the blocks shown in Fig. 1 Find the value of pull ‘P’. The coefficient of friction between blocks is 0.24 and the same between block and floor is 0.3.



Free Body Diagram of 2000 N weight:



$\sum Fx = 0$:

$T_{AB} - 0.24\ R_{12} = 0$

$\sum Fy = 0$:

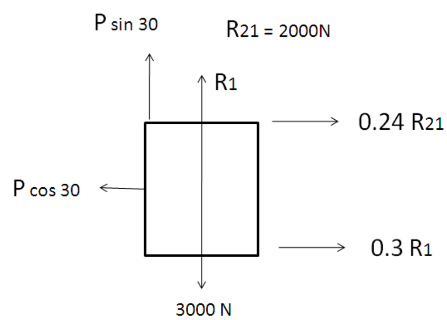
$R_{12} - 2000 = 0$

$\mathbf{R_{12} = 2000\ N}$

Therefore, $T_{AB} = 0.24 \times 2000$

$\mathbf{T_{AB} = 480\ N}$

Free Body Diagram of 3000 N weight:



$\sum Fx = 0$:

$-P \cos 30 + 0.24 \times 2000 + 0.3\ R_1 = 0$

$-P \cos 30 + 0.3\ R_1 = -480$ ------- 1

$\sum Fy = 0$:

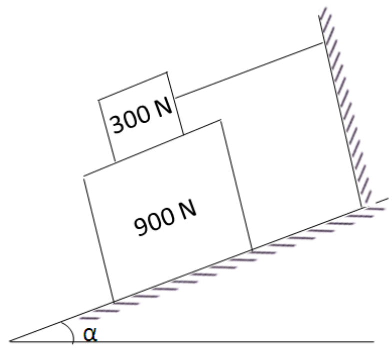
$P \sin 30 + R_1 - 2000 - 3000 = 0$ ------2

Solving equations 1 & 2 we will get

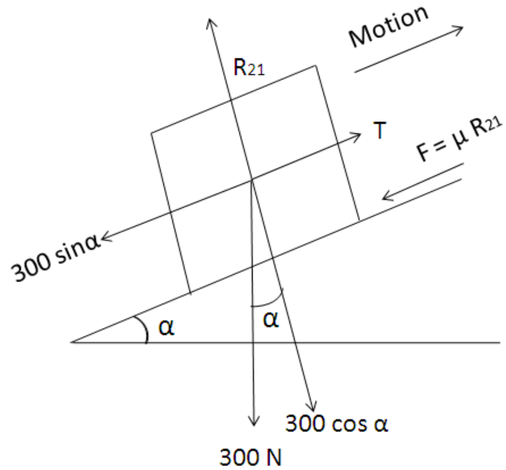
$\mathbf{P = 1948.7\ N}$

$\mathbf{R_1 = 4025.6\ N}$

3. What should be the value of α in Fig. which will make the motion of 900 N blocks down the plane to impend? Take the coefficient of friction for all contact surfaces as 1/3.



Free Body Diagram for 300 N weight:



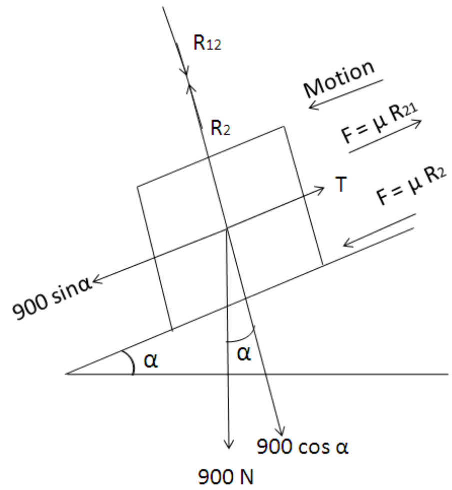
$\sum Fx = 0$:

$T – 0.33\ R_{21} - 300 \sin \alpha = 0$ ------- 1

$\sum Fy = 0$:

$R_{21} = 300 \cos \alpha$ -----------------2

Free Body Diagram For 900N weight:



$\sum Fx = 0$:

$0.33\ R_{21} + 0.33R_2 - 900 \sin\alpha = 0$ ------- 3

$\sum Fy = 0$:

$R_2 - R_{21} - 900 \cos\alpha = 0$

$R_2 - 300 \cos\alpha - 900 \cos\alpha = 0$ [from Eq 2]

$R_2 = 1200 \cos\alpha$ -------------------- 4

Substituting Eq 2 & 4 in Eq 3

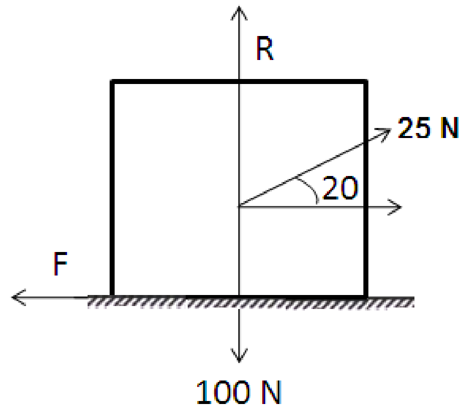
$0.33\ (300 \cos\alpha) + 0.33\ (1200 \cos\alpha) - 900 \sin\alpha = 0$

$0.33 \times 1500 \cos\alpha = 900 \sin\alpha$

or, $\alpha = \tan^{-1} (495/900) = \tan^{-1} 0.55$

Therefore, $\boldsymbol{\alpha = 28.81°}$

4. The force required to pull a block of weight 100 N on a rough plane is 25 N. Find the co-efficient of friction if the force is applied at an angle of 20° with the horizontal.



$\sum Fx = 0$:

$$\mu R = 25 \cos 20° \qquad \text{------- 1}$$

$\sum Fy = 0$:

$$R + 25 \sin 20° = 100$$

$$\mathbf{R = 91.44\ N} \qquad \text{------- 2}$$

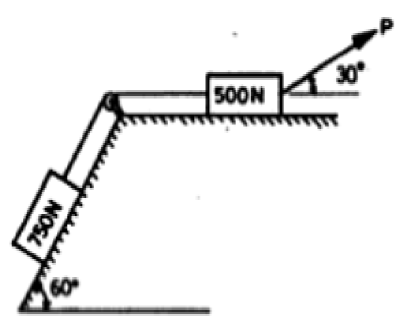
Substituting the value of R in Eq. 1

$$\mu = 25 \cos 20° / R$$

$$\mu = 23.49 / 91.44$$

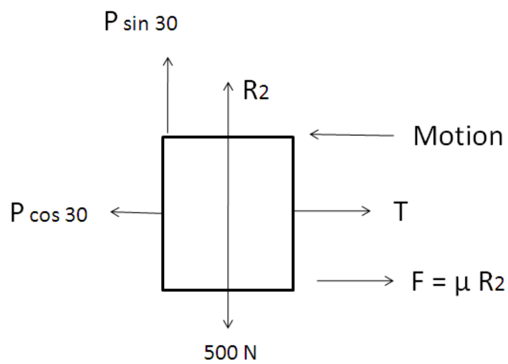
$$\mathbf{\mu = 0.256}$$

5. What is the value of P in the system shown in Fig.4 to cause the motion to impend? Assume the pulley is smooth and coefficient of friction between other contact surfaces is 0.22.



Solution:

FBD For 500N weight:



$\sum Fx = 0$:

$T + \mu R_2 – P \cos 30 = 0$

$T + 0.22 R_2 – P \cos 30 = 0$ ------- 1

$\sum Fy = 0$:

$R_2 + P \sin 30 - 500 = 0$

$R_2 = 500 - P \sin 30$ ------- 2

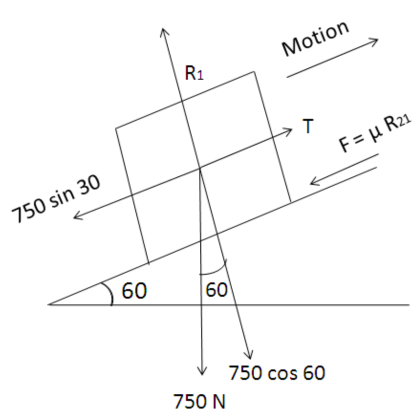
Substituting in Eq. 1

$T + 0.22 (500 – P \sin 30) – P \cos 30 = 0$

$T + 110 – 0.5 P – 0.866 P = 0$

$T + 110 – 1.366 P = 0$ ------- 3

FBD For 750N weight:



$\sum Fx = 0$:

$T - \mu R_1 - 750 \sin 60 = 0$

$T - 0.22 R_1 - 649.52 = 0$ ------- 4

$\sum Fy = 0$:

$R_1 + 750 \cos 60 = 0$

$R_1 = 750 \cos 60$

$\mathbf{R_1 = 375\ N}$

Substituting in Eq. 4

$T = 0.22 \times 375 + 649.52$

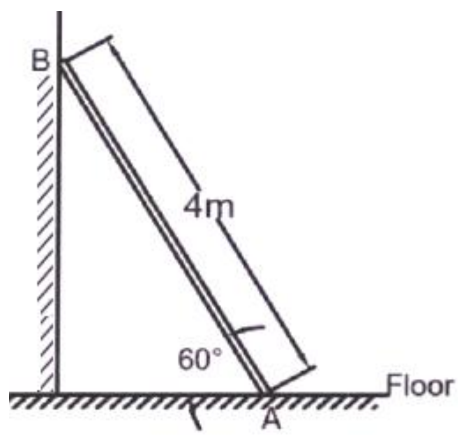
$\mathbf{T = 732\ N}$

Substituting in Eq. 3

$732 + 110 - 1.336\ P = 0$

$\mathbf{P = 630.239\ N}$

6. A ladder of weight 1000 N and length 4m rests as shown in Fig.6. If a 750 N weight is acting a distance of 3m from the bottom of ladder, it is at the point of sliding. Determine the co-efficient of friction between ladder and the floor. Assume the co-efficient of friction is same for all the contacting surfaces.



Solution:

$\sum Fx = 0$:

$R_w - \mu_f R_f = 0$

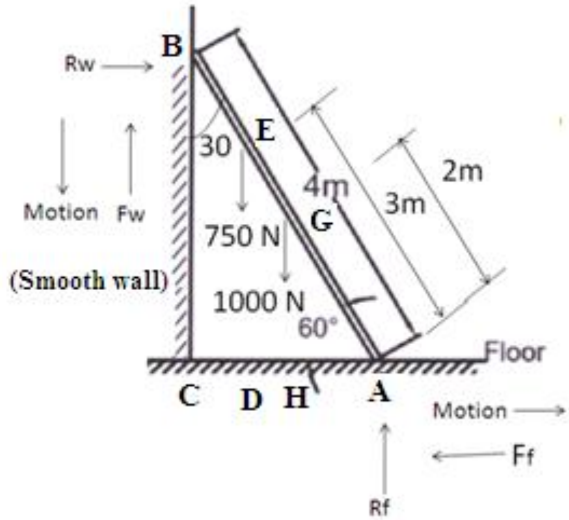
$\mu_f = R_w / R_f$ ------- 1

$\sum Fy = 0$:

$\mu_w R_w - 1000 - 750 + R_f = 0$

$0 - 1000 - 750 + R_f = 0$

$\mathbf{R_f = 1750\ N}$ ------- 2



$\sum M_A = 0$:

$$(1000 \times 2 \times \cos 60) + (750 \times 3 \times \cos 60) - (R_w \times 4 \times \sin 60) - (\mu_w R_w \times 4 \times \cos 60) = 0$$

$$(1000 \times 2 \times \cos 60) + (750 \times 3 \times \cos 60) - (R_w \times 4 \times \sin 60) - (0) = 0$$

$$(R_w \times 4 \times \sin 60) = (1000 \times 2 \times \cos 60) + (750 \times 3 \times \cos 60)$$

$$3.464 \times R_w = 2125$$

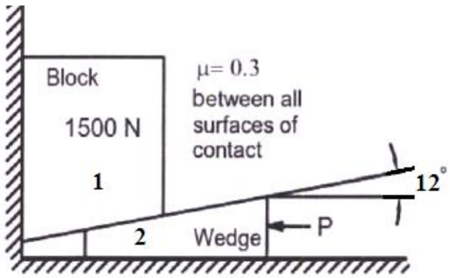
$$\mathbf{R_w = 613.45\ N}$$

From Eq. 1

$$\mu_f = R_w / R_f = 613.45 / 1750$$

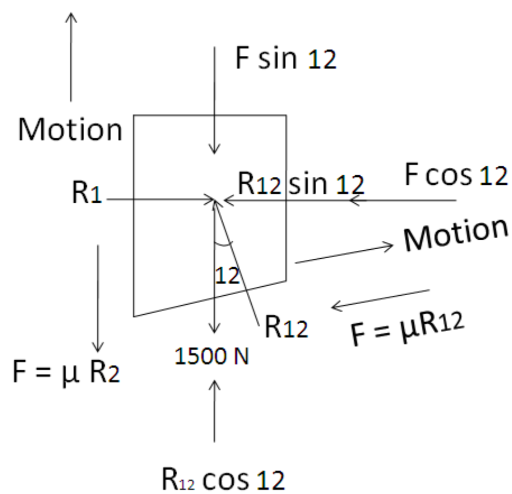
$$\mathbf{\mu_f = 0.35}$$

7. For the block and wedge shown in Fig., determine the value of ‘P’ required for raising the block. Weight of the wedge is 150N.



Solution:

FBD for Block of weight 1500 N:



$\sum Fx = 0$:

$R_1 - R_{21}\sin 12° - F\cos 12° = 0$

$R_1 - R_{21}\sin 12° - \mu R_{21}\cos 12° = 0$

$R_1 - R_{21}\sin 12° - 0.3R_{21}\cos 12° = 0$

$R_1 = 0.501\ R_{21}$ -------- 1

$\sum Fy = 0$:

$- F\sin 12° - 1500 + R_{21}\cos 12° - \mu R_2 = 0$

$- \mu R_{21}\sin 12° - 1500 + R_{21}\cos 12° - \mu R_2 = 0$

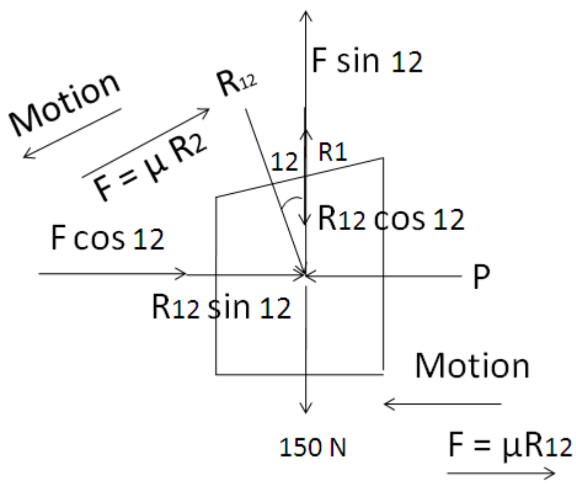
$- 0.3\ R_{21}\sin 12° + R_{21}\cos 12° - \mu R_2 = 1500$ -------- 2

Substituting R_1 value in Eq. 2 we will get

$\mathbf{R_{21} = 1959.75\ N}$

$$\mathbf{R_1 = 981.8\ N}$$

FBD for Wedge of weight 150 N:



$\sum Fx = 0$:

$$\mu R_{21} \cos 12° + R_{21} \sin 12° - P + \mu R_2 = 0$$

$$0.3 \times 1959.75 \cos 12° + 1959.75 \sin 12° - P + 0.3\ R_2 = 0$$

$$P - 0.3\ R_2 = 982.5 \qquad \text{--------}\ 3$$

$\sum Fy = 0$:

$$- R_{21} \cos 12° + R_2 + 0.3\ R_{21} \sin 12° - 150 = 0$$

$$- 1959.75 \cos 12° + R_2 + 0.3 \times 1959.75 \sin 12° - 150 = 0$$

$$\mathbf{R_2 = 1944.68\ N}$$

Substituting R_2 value in Eq. 3 we will get

$$\mathbf{P = 1565.9\ N}$$

8. The pitch of a single threaded screw jack is 6 mm and its mean diameter is 60 mm. If μ is 0.1, determine the force required at the end of lever 250 mm long measure from the axis of screw to a) raise a 65 kN load b) lower the same load.

Given:

$P = L = 6$ mm

$D = 60$ mm: $r = 30$ mm

$\mu = 0.1$

$a = 250$ mm

$W = 65$ KN

Solution:

For Raising

$$P = [(W\ r/a) \tan(\theta + \phi)]$$

$$\tan\theta = (P/2\pi r)$$

$$\theta = \tan^{-1}(P/2\pi r)$$

$$\boldsymbol{\theta} = \tan^{-1}(6/2\pi x 30)$$

$$\boldsymbol{\theta = 1.82°}$$

WKT, $\phi = \tan^{-1}\mu = \tan^{-1}(0.1)$

$$\boldsymbol{\phi = 5.71°}$$

Substituting in Eq. 1

$$P = \{[(65 \text{ x } 30)/250)] \text{ x } \tan(1.82 + 5.71)\}$$

$$\mathbf{P = 1.031\ KN}$$

For Lowering

$$P = [(W\ r/a) \tan(\phi - \theta)]$$

$$P = \{[(65 \text{ x } 30)/250)] \text{ x } \tan(5.71 - 1.82)\}$$

$$\mathbf{P = 0.5302\ KN}$$

9. A belt is running over a pulley of diameter 1 m at 300 rpm. The angle of contact is 160^0 and coefficient of friction is 0.25. If the maximum tension in the belt is 1200 N, determine the power transmitted by it.

Given:

$D = 1$ m

$N = 300$ rpm

$\Theta = 160^0$

$\mu = 0.25$

$T_1 = 1200$ N

Solution:

Power transmitted, $P = (T_1 - T_2)\ v$

$V = (\pi DN/60) = \pi \times 1 \times 300 / 60 = 15.7$ m/s

$(T_1/T_2) = e^{\mu\theta}$

$(1200/T_2) = e^{0.25 \times 160 \times \pi / 180}$

$T_2 = 1200 / e^{0.25 \times 160 \times \pi / 180}$

$T_2 = 334.83$ N

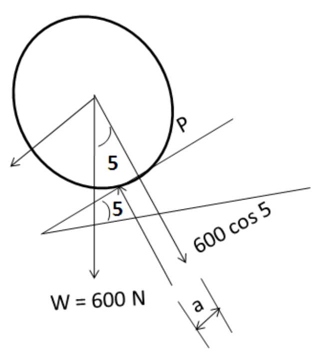
Power transmitted, $P = (T_1 - T_2)\ v$

$P = (1200 - 334.83) \times 15.7$

$\mathbf{P = 13583.16}$ **watts**

10. A wheel of weight 600 N and radius 350 mm rolls down a 5 inclined plane. Find the coefficient of rolling resistance.

Solution:



$\sum Mp = 0$

$-(600 \cos 5 \times a) + (600 \sin 5 \times 0.35) = 0$

$a = (600 \sin 5 \times 0.35) / (600 \cos 5)$

$\mathbf{a = 0.0306\ m}$