

### **LINEAR TIME INVARIANT CONTINUOUS TIME SYSTEMS**

CT systems – Linear Time invariant Systems – Basic properties of continuous time systems – Linearity, Causality, Time invariance, Stability – Frequency response of LTI systems – Analysis and characterization of LTI systems using Laplace transform – Computation of impulse response and transfer function using Laplace transform – Differential equation – Impulse response – Convolution integral and Frequency response.

#### **System**

A system may be defined as a set of elements or functional blocks which are connected together and produces an output in response to an input signal. The response or output of the system depends upon transfer function of the system. Mathematically, the functional relationship between input and output may be written as

$$y(t)=f[x(t)]$$

#### **Types of system**

Like signals, systems may also be of two types as under:

1. Continuous-time system
2. Discrete time system

#### **Continuous time System**

Continuous time system may be defined as those systems in which the associated signals are also continuous. This means that input and output of continuous – time system are both continuous time signals.

For example:

Audio, video amplifiers, power supplies etc., are continuous time systems.

#### **Discrete time systems**

Discrete time system may be defined as a system in which the associated signals are also discrete time signals. This means that in a discrete time system, the input and output are both discrete time signals.

For example, microprocessors, semiconductor memories, shift registers etc., are discrete time signals.

#### **LTI system:-**

Systems are broadly classified as continuous time systems and discrete time systems. Continuous time systems deal with continuous time signals and discrete time systems deal with discrete time system. Both continuous time and discrete time systems have several basic properties. Out of these several basic properties of systems, two properties namely linearity and time invariance play a vital role in the analysis

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of signals and systems. If a system has both the linearity and time invariance properties, then this system is called linear time invariant (LTI) system. **Characterization of Linear Time Invariant (LTI) system**

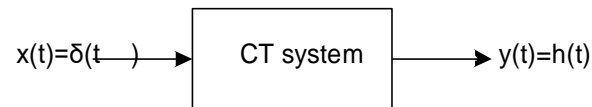
Both continuous time and discrete time linear time invariant (LTI) systems exhibit one important characteristics that the superposition theorem can be applied to find the response  $y(t)$  to a given input  $x(t)$ .

Hence, following steps may be adopted to find the response of a LTI system using super position theorem:

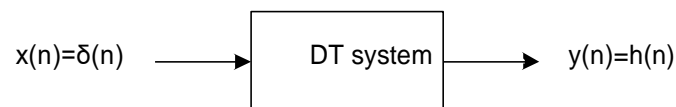
1. Resolve the input function  $x(t)$  in terms of simpler or basic function like impulse function for which response can be easily evaluated.
2. Determine individually the response of LTI system for the simpler input impulse functions.
3. Using superposition theorem, find the sum of the individual responses, which will become the overall response  $y(t)$  of function  $x(t)$ .

From the above discussions, it is clear that to find the response of a LTI system to any given function, first we have to find the response of LTI system input to an unit impulse called unit impulse response of LTI system.

Hence, the impulse response of a continuous time or discrete time LTI system is the output of the system due to an unit impulse input applied at time  $t=0$  or  $n=0$ .



Here,  $\delta(t)$  is the unit impulse input in continuous time and  $h(t)$  is the unit impulse response of continuous time LTI system. Continuous time unit impulse response  $h(t)$  is the output of a continuous time system when applied input  $x(t)$  is equal to unit impulse function  $\delta(t)$



Similarly, for a discrete time system, discrete time impulse response  $h(n)$  is the output of a discrete time system when applied input  $x(n)$  is equal to discrete time unit impulse function  $\delta(n)$ . Here,  $\delta(n)$  is the unit impulse input in discrete time and  $h(n)$  is the unit impulse response of discrete time LTI system. Therefore, any LTI system can be completely characterized in terms of its unit impulse response.

**Properties of Linear time invariant (LTI) system:-**

The LTI system have a number of properties not exhibited by other systems. Those are as under:

- Commutative property of LTI systems
- Distributive property of LTI systems
- Associative property of LTI systems
- Static and dynamic LTI systems
- Invertibility of LTI systems
- Causality of LTI systems
- Stability of LTI systems
- Unit-step response of LTI system

**Commutative property:**

The commutative property is a basic property of convolution in both continuous and discrete time cases, thus, both convolution integral for continuous time LTI systems and convolution sum for discrete time LTI systems are commutative. According to the property, for continuous time LTI system. The output is given by

Thus, we can say that according to this property, the output of a continuous time LTI system having input  $x(t)$  and unit impulse  $h(t)$  is identical to the output of a continuous time LTI system having input  $h(t)$  and the unit impulse response  $x(t)$ .

**Distributive property:**

The distributive property states that both convolution integral for continuous time LTI system and convolution sum for discrete time LTI system are distributive.

For continuous time LTI system, the distributive property is expressed as

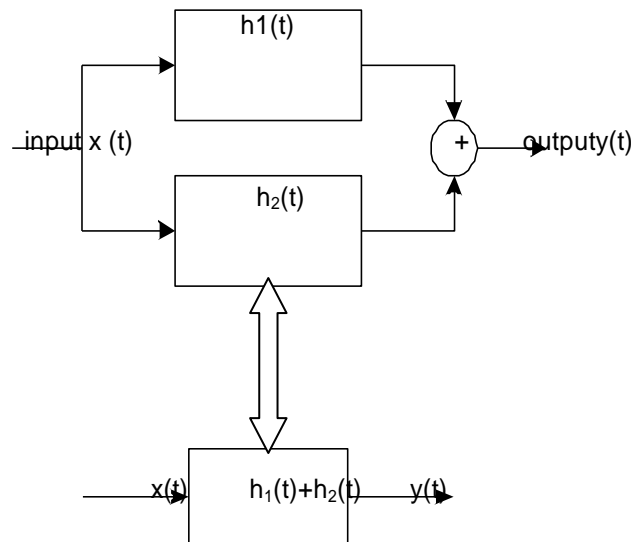


Fig 3.1 The distributive property of convolution integral for a parallel interconnection of continuous time LTI system

**Associative Property of LTI system:**

According to associative property, both convolution integral for continuous time LTI systems and convolution sum for discrete time LTI systems are associative.

For continuous time LTI system, according to associative property,

The output

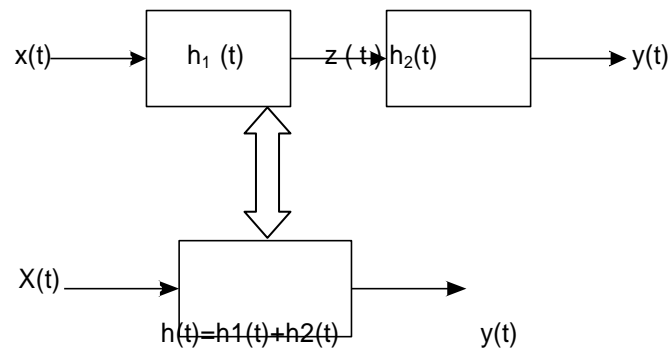


Fig 3.2: The associative property of convolution integral for a cascade interconnection of continuous time LTI systems

Here we have  $y(t) = z(t) * h_2(t)$  But  $z(t) = x(t) * h_1(t)$ . Therefore  $y(t) = [x(t) * h_1(t) * h_2(t)]$

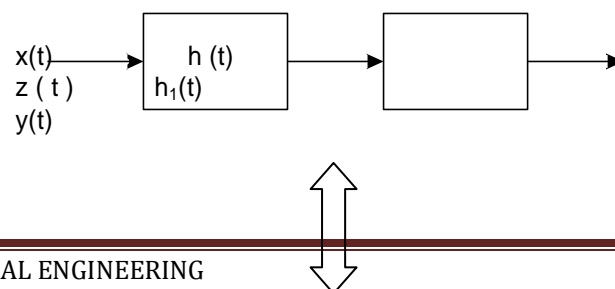
**Static and Dynamic property:**

Static systems are also known as memory less systems. A system is known as static if its output at any time depends only on the value of the input at the same time. A continuous time system is memory less if its unit impulse response  $h(t)$  is zero for  $t \neq 0$ . These memory less LTI systems are characterized by  $y(t) = Kx(t)$  where  $K$  is constant. And its impulse response  $h(t) = K\delta(t)$ . If  $K=1$ , then these systems are called identity systems.

**Invertibility of LTI systems:**

A system is known as invertible only if an inverse system exists which, when cascade with the original system, produces an output equal to the input at first system. If an LTI system is invertible then it will have a LTI inverse system. This means that we have a continuous time LTI system with impulse response  $h(t)$  and its inverse system with impulse response  $h_1(t)$  which results in an output equal to  $x(t)$ . Cascade interconnection of original continuous time LTI system with its inverse system is given as identity system.

Thus, the overall impulse response of a system with impulse response  $h(t)$  cascaded with its inverse system with impulse response  $h_1(t)$  is given as  $h(t) * h_1(t) = \delta(t)$



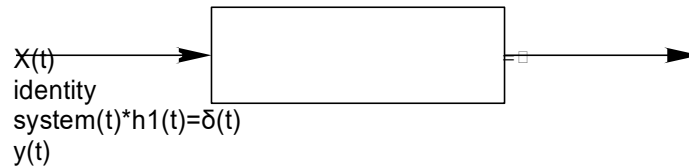


Fig3.3: An inverse system for continuous time LTI system

**Causality for LTI System**

This property says that the output of a causal system depends only on the present and past values of the input to the system.

A continuous time LTI system is

called causal system if its

impulse response  $h(t)$  is zero

$t < 0$ . For a causal continuous

time LTI system, convolution

integral is given as

$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

For pure time shift with unit impulse response  $h(t)=\delta(t-t_0)$  is a causal continuous time LTI system for  $t \geq 0$ . In this case time shift is known as a delay.

**Stability for LTI systems**

A stable system is a system which produces bounded output for every bounded input.

Condition of Stability for continuous time LTI system:

Let us consider an input  $x(t)$  that is bounded in magnitude for all values of  $t$

Now, we apply this input to an continuous time LTI system with unit impulse response  $h(t)$ .

Output of this LTI system is determined by convolution integral and is given by

Magnitude of output  $y(t)$  is given as

Substituting the value for all values of  $\tau$  and  $t$ , we get

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Or

$|h| < \infty$  for all values of  $t$

From the above equation we can conclude that if the impulse response  $h(t)$  is absolutely integrable then output of a continuous time LTI system is bounded in magnitude, and thus, the system is bounded input, bounded output (BIBO) stable.

**Unit step response of an LTI system:**

Unit step response is the output of a LTI system for input is equal to unit step function or sequence. Unit step response of continuous time LTI system is found by convolution integral of  $u(t)$  with unit impulse response  $h(t)$  and is expressed as

$$g(t) = u(t) * h(t) = h(t) * u(t)$$

according to commutative property. Therefore, unit step response  $g(t)$  may be viewed as the response to the input  $h(t)$  of a continuous time LTI system with unit impulse response  $u(t)$ .

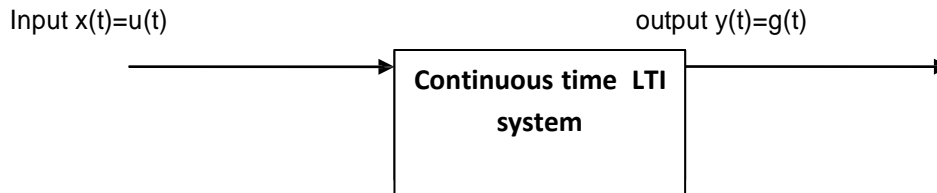


Fig 3.4: Continuous time LTI system

**Classification of CT LTI system:-**

The systems are classified into two types: continuous time and discrete time systems, Now these two broad types of systems are further classified on the basis of system properties as under:

- Causal system and non causal system
- Time invariant and time variant system
- Stable and unstable system
- Linear and Non-linear system
- Static and Dynamic systems
- Invertible and noninvertible system

**Causal systems and Non-causal systems**

A system is causal if the response or output does not begin before the input function is applied. This means that if input is applied at  $t=t_0$ , then for causal system, output will depend on values of input  $x(t)$  for  $t \leq t_0$ .

Mathematically,

$$y(t_0) = f[x(t), t \leq t_0].$$

In other words we can say that, the response or output of the causal system to an input does not depend on future values of that input but depends only on the present or past values of the input. This means that all the real-time systems are also causal systems since these systems cannot know the future values of the input signal when it constructs output signal. Thus, causal systems are physically realizable. For example a resistor is a continuous time causal system because voltage across it is given by the expression  $v(t) = R \cdot i(t)$  and output  $v(t)$ , i.e., voltage depends only on the input  $i(t)$  i.e., current at the present time.

**Time invariant and time variant system**

A system is said to be time invariant if its input-output characteristics do not change with time.  $H\{x(t)\} = y(t)$  implies that  $H\{x(t-t_0)\} = y(t-t_0)$  for every input signal  $x(t)$  and every time shift  $t_0$ . A system is said to be time variant if its input-output characteristics changes with time.

Procedure to Test for Time Invariance:-

1. Delay the input signal by  $t_0$  units of time and determine the response of the system for this delayed input signal. Let this response be  $y(t-t_0)$ .
2. Delay the response of the system for undelayed input by  $t_0$  units of time. Let this delayed response be  $y_d(t)$ .
3. Check whether  $y(t-t_0) = y_d(t)$ . If they are equal then the system is time invariant. Otherwise the system is time variant.

**Stable and unstable system**

A system is called bounded input, bounded output (BIBO) stable if and only if every bounded input results in a bounded output. The output of such a system does not diverge or does not grow unreasonably large.

Condition of Stability for continuous time LTI system:

Let us consider an input  $x(t)$  that is bounded in magnitude

for all values of  $t$

Now, we apply this input to an continuous time LTI system with unit impulse response  $h(t)$ .

Output of this LTI system is determined by convolution integral and is given by

Magnitude of output  $y(t)$  is given as

$= -\infty$

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Substituting the value for all values of and t, we get

$\int$  for all values of t

From the above equation we can conclude that if the impulse response  $h(t)$  is absolutely integrable then output of a continuous time LTI system is bounded in magnitude, and thus, the system is bounded input, bounded output (BIBO) stable.

The systems not satisfying the above conditions are unstable.

#### Linear and nonlinear system

A linear system is one that satisfies the superposition principle. The principles of superposition requires that the response of the system to a weighted sum of the signals is equal to the corresponding weighted sum of the responses of the system to each of the individual input signals.

A system is linear if

$H\{a_1x_1(t)+a_2x_2(t)\}=a_1H\{x_1(t)\}+a_2H\{x_2(t)\}$  for any arbitrary input sequences  $x_1(t)$  and  $x_2(t)$  and for any arbitrary constants  $a_1$  and  $a_2$ .

If a relaxed system does not satisfy the super position principle as given by the above definition, then the system is nonlinear.

#### Static and dynamic system

A continuous time system is called static or memory less if its output at any instant t depends on present input but not on the past or future samples of the input. These systems contain no energy storage elements. This means that the equation relating its output signal to its input signal contains no derivative, integrals or signal delays.

As an example, consider the system described by the following relationship

$Y(t)=x^2(t)$  this system is memory less because the value of the output signal  $y(t)$  at time t depends only on the present value of the input signal  $x(t)$ . In any other case the system is said to be dynamic or to have memory. Dynamic systems have one or more energy storage elements. Input output relationship of a dynamic continuous time system is described by its differential equation.

#### Invertible and non invertible system

A system is said to be invertible if there is a one to one correspondence between its input and output signals. If a system is invertible, then an inverse system exists. The cascading of an invertible system and its inverse system is equivalent to the identity system.



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The frequency response of an inverse system is basically reciprocal of the frequency response of the original system or invertible system.

An example of an invertible continuous time system is given by

$y_1(t)=3x(t)$  and its inverse system will be given by  $y_2(t) = \frac{1}{3}x(t)$  -

#### **Convolution integral:-**

The output of any general input may be found by convolving the given input signal  $x(t)$  with the LTI systems unit impulse response  $h(t)$ .

Properties of convolution integral:

- Commutative property
- Distributive property
- Associative property

#### **Commutative property:**

The commutative property is a basic property of convolution in both continuous and discrete time cases, thus, both convolution integral for continuous time LTI systems and convolution sum for discrete time LTI systems are commutative. According to the property, for continuous time LTI system. The output is given by

Thus, we can say that according to this property, the output of a continuous time LTI system having input  $x(t)$  and unit impulse  $h(t)$  is identical to the output of a continuous time LTI system having input  $h(t)$  and the unit impulse response  $x(t)$ .

#### **The distributive property:**

The distributive property states that both convolution integral for continuous time LTI system and convolution sum for discrete time LTI system are distributive.

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For continuous time LTI system, the distributive property is expressed as

The output

outputs are added as

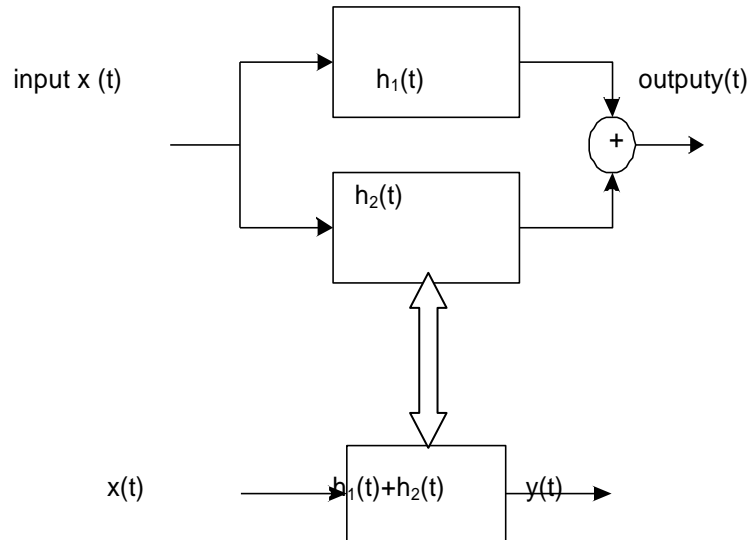


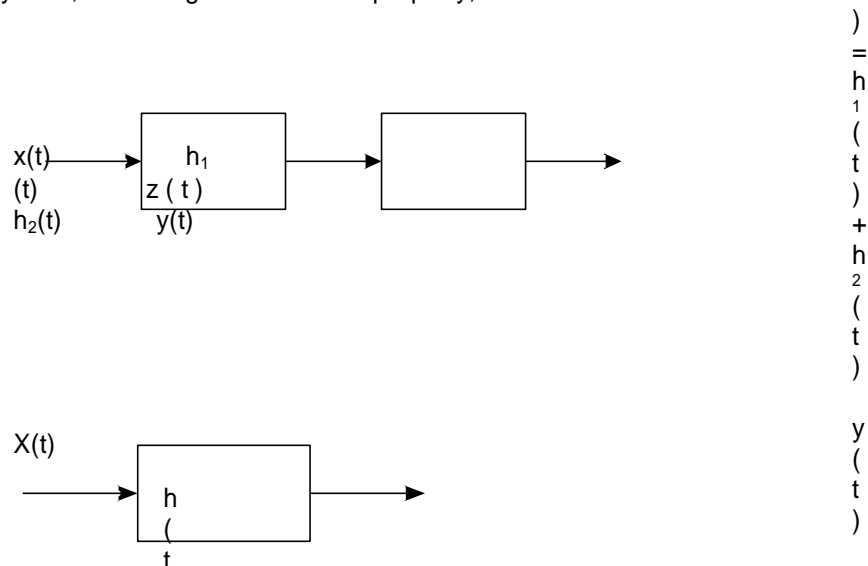
Fig3.1: The distributive property of convolution integral for a parallel interconnection of continuous time LTI system

**Associative Property of LTI system:**

According to associative property, both convolution integral for continuous time LTI systems and convolution sum for discrete time LTI systems are associative.

For continuous time LTI system, according to associative property,

The output



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**Linear constant coefficient differential equation:**

Fig 3.2: The associative property of convolution integral for a cascade interconnection of continuous time LTI

s  
y  
s  
t  
e  
m  
s

The continuous time linear time invariant (LTI) systems are described by their linear constant coefficient

differential equations. For this, let us consider a first order differential equation as under

Where  $x(t)$  and  $y(t)$  are the input and output of the continuous time LTI system.  $A$  is a constant value.

The first order differential equation can be extended for higher order differential equations. A general  $N$ th order

linear constant coefficient differential equation can be given by

Here we have  $y(t) = z(t) * h_2(t)$ . But  $z(t) = x(t) * h_1(t)$ . Therefore  $y(t) = [x(t) * h_1(t) * h_2(t)]$

The complete solution of differential equation consists of the sum of particular solution  $y_p(t)$  and homogenous solution  $y_h(t)$ .

The homogeneous solution of a differential equation is possible by substituting

$$\sum_{n=0}^{\infty} a_n \frac{d^n y}{dt^n} + b_0 y = T$$

This solution to differential equation is also known as natural response of the system.

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A particular case of differential equation is determined by putting  $N=0$ , we obtain

$$\sum_{n=0}^{\infty} x(n) = \dots$$

**Transfer function:**

Transfer functions are commonly used in the analysis of systems such as single-input single-output filters, typically within the fields of signal processing, communication theory, and control theory. The term is often used exclusively to refer to linear, time-invariant systems (LTI). Most real systems have non-linear input/output characteristics, but many systems, when operated within nominal parameters have behavior that is close enough to linear that LTI system theory is an acceptable representation of the input/output behavior.

The descriptions below are given in terms of a complex variable  $s = \sigma + j \cdot \omega$ , which bears a brief explanation. In many applications, it is sufficient to define  $\sigma = 0$  (and  $j \cdot \omega$ ), which reduces the Laplace transforms with complex arguments to Fourier transforms with real argument  $\omega$ . The applications where this is common are ones where there is interest only in the steady-state response of an LTI system, not the fleeting turn-on and turn-off behaviors or stability issues. That is usually the case for signal processing and communication theory.

Thus, for continuous-time input signal  $x(t)$  and output  $y(t)$ , the transfer function  $H(s)$  is the linear mapping of the Laplace transform of the input,  $X(s)=L\{x(t)\}$ , to the Laplace transform of the output  $Y(s)=L\{y(t)\}$ :

$$Y(s)=H(s)X(s)$$

Conditions required for transfer function:

- (i) System should be in unloaded condition (initial conditions are zero)
- (ii) The system should be linear time invariant.

**Impulse Response:**

In signal processing, the impulse response, of a dynamic system is its output when presented with a brief input signal, called an impulse. More generally, an impulse response refers to the reaction of any dynamic system in response to some external change. In both cases, the impulse response describes the reaction of the system as a function of time. In all these cases, the dynamic system and its impulse response may be actual physical objects, or may be mathematical systems of equations describing such objects. Since the impulse function contains all frequencies, the impulse response defines the response of a linear time-invariant system for all frequencies. The impulse can be modeled as a Dirac delta function for continuous-time systems, or as  $\delta(t)$ . The Dirac delta represents the limiting case of a pulse made very short in time while maintaining its area or integral. While this is impossible in any real system, it is a useful idealization. In Fourier theory, such an impulse comprises equal portions of all possible excitation frequencies, which makes it a convenient test probe. Any system in a large class known as linear, time-invariant (LTI) is completely characterized by its impulse response. That is, for any input, the output can be calculated in terms of the input and the impulse response. The impulse response of a linear transformation is the image of Dirac's delta function under the transformation, analogous to the

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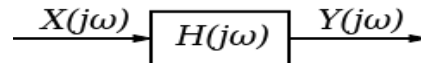
fundamental solution of a partial differential operator. It is usually easier to analyze systems using transfer functions as opposed to impulse responses. The transfer function is the Laplace transform of the impulse response. The Laplace transform of a system's output may be determined by the multiplication of the transfer function with the input's Laplace transform in the complex plane, also known as the frequency domain. An inverse Laplace transform of this result will yield the output in the time domain. To determine an output directly in the time domain requires the convolution of the input with the impulse response. When the transfer function and the Laplace transform of the input are known, this convolution may be more complicated than the alternative of multiplying two functions in the domain. It is obtained by taking inverse Laplace transform of transfer function  $H(s)$ .

**Frequency Response:**

Frequency response is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterize the dynamics of the system. It is a measure of magnitude and phase of the output as a function of frequency, in comparison to the input. It is obtained from the transfer function by substituting  $s=j\omega$  in transfer function.

Systems respond differently to inputs of different frequencies. Some systems may amplify components of certain frequencies, and attenuate components of other frequencies. The way that the system output is related to the system input for different frequencies is called the *frequency response* of the system.

The frequency response is the relationship between the system input and output in the Fourier Domain.



In this system,  $X(j\omega)$  is the system input,  $Y(j\omega)$  is the system output, and  $H(j\omega)$  is the frequency response. We can define the relationship between these functions as:

$$Y(j\omega) = H(j\omega)X(j\omega)$$

Since the frequency response is a complex function, we can convert it to

polar notation in the complex plane. This will give us a magnitude and an angle.

**Amplitude Response:**

For each frequency, the magnitude represents the system's tendency to amplify or attenuate the input signal.

The phase represents the system's tendency to modify the phase of the input sinusoids.

$$\phi(\omega) = \angle H(j\omega)$$

The phase response, or its derivative the group delay, tells us how the system delays the input signal as a function of frequency.

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14. Consider the system  $y(t) = x^2(t) + 2$

Determine whether the system is

(i) Linear (ii) Stable (iii) Causal. Justify your answer.

15. Determine if the following system described by

$y(t) = \sin[x(t+2)]$  is memory less, causal, linear, time invariant, stable.

16. Check whether the following systems are linear or non linear

(a)  $y(t) = tx(t)$

(b)  $y(t) = x^2(t)$

17. Check whether the following systems are time invariant or time variant.

(a)  $y(t) = \sin x(t)$

(b)  $y(t) = tx(t)$

(c)  $y(t) = x(t) \cos 200\pi t$

18.  $\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} = 2x(t)$ . Find the transfer function, impulse response and frequency response.

19. Determine the frequency response and system function of

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} = 2x(t)$$

20.  $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = 2x(t)$ ,  $x(t) = \cos t$ . Find the output  $y(t)$ .