UNIT3 PROPERTIES OF SURFACES AND SOLIDS 12 Hrs.

Determination of Areas - First moment of Area and the centroid - simple problems involving composite figures. Second moment of plane area - Parallel axis theorems and perpendicular axis theorems - Polar moment of Inertia - Principal moments of Inertia of plane areas - Principle axes of inertia - relation to area moments of Inertia. Second moment of plane area of sections like C,I,T,Z etc. - Basic Concept of Mass moment of Inertia

First moment of area and the centroid

Centroid, Centre of gravity, Centre of mass and moment of inertia are the important properties of a section which are required frequently in the analysis of many engineering problems.

Centroid

It is the point at which the total area of the plane figure (namely rectangle, square, triangle, circle etc.) is assumed to be concentrated.

Centre of gravity

It is a point through which the resultant of the distributed gravity forces (weights) act irrespective of the orientation of the body.

Centre of mass

It is the point where the entire mass of the body may be assumed to be concentrated.

For all practical purposes the centroid and Centre of gravity are assumed to be the same.

Centroid of one dimensional body (Line)

Let us consider a homogeneous wire which is having length 'L', uniform cross sectional area 'a' and density ρ

The weight of the wire = W= ρga X L

The wire is considered to be made up of a number of elemental lengthsL_1,L_2,...L_n

Then to find the centroid, substitute for W in the following equation

$\bar{x} = \frac{\sum W_{1X_1}}{W}$

$$= \frac{(\rho ga)L_{1X_1} + (\rho ga)L_{2X_2} + (\rho ga)L_{3X_3} \ldots\ldots\ldots\ldots (\rho ga)L_{nX_n}}{(\rho ga)L}$$

$$=\frac{L_1 X_1 + L_2 X_2 + L_3 X_3 + \dots\dots\dots\dots + L_n X_n}{L}$$

$$\bar{X} = \frac{\int X\, dL}{L}$$

Similarly,

$$\bar{Y} = \frac{\int Y\, dL}{L}$$

Centroid of two dimensional body (area)

Let us consider a rectangular plate P,Q,R,S of uniform thickness- t ,density- ρ and area –A

The weight of the plate= W=ρgt x A

This body is considered to be made up of number of imaginary strips or particles of area $A_{1,} A_{2,} A_{3,} \dots \dots \dots \dots A_{n,}$

$$\bar{x} = \frac{\sum W_{1X_1}}{W}$$

$$=\frac{(\rho g t)A_{1X_1} + (\rho g t)A_{2X_2} + (\rho g t)A_{3X_3} \dots\dots\dots\dots (\rho g t)A_{nX_n}}{(\rho g t)A}$$

$$=\bar{X} = \frac{\int X\, dA}{A}$$

Similarly,

$$\bar{Y} = \frac{\int Y\, dA}{A}$$

The integral ∫xdA is known as the first moment of area with respect to the y axis and the integral ∫ydA is known as the first moment of area with respect to the x axis .

Moment of Inertia

the concept which gives a quantitative estimate of the relative distribution of area or mass of a body with respect to some reference axis is termed as the moment of inertia of the body.

The moment of inertia of a body about an axis is defined as the resistance offered by the body to rotation about that axis.it is also defined as the product of the area and the square of the distance of the center of gravity of the area from that axis. Moment of is denoted by I. Hence the moment of inertia about the x axis is represented by I_{xx} and about the y axis is represented by I_{yy}

The moment of inertia of an area is called as the area moment of inertia or the second moment of area .the moment of inertia of the mass of a body is called as the mass moment of inertia

$I_{xx} = \int y^2 dA$

$I_{yy} = \int x^2 dA$

Parallel axis theorem

Parallel axis theorem states that, the moment of inertia of an area with respect to any axis in its plane is equal to the moment of inertia of the area with to a parallel centroidal axis plus the product of the area and the distance between the two axes

Perpendicular axis theorem (polar moment of inertia)

Perpendicular axis theorem states that the moment of inertia of an area with respect to an axis perpendicular to the x-y plane (z axis) and passing through a pole O is equal to the sum of the moment of inertia of the area about the other two axis (x&y axis) passing through the pole. It's also called as polar moment of inertia and is denoted by the letter J.

$J = I_{zz} = I_{xx} + I_{yy}$

Radius of Gyration

$I_{xx} = k_x^2 A$

$k_x = \sqrt{\frac{I_{xx}}{A}}$

k_x is known as the radius of gyration of the area with respect to the X – axis and has the unit of length (m)

Radius of gyration with respect to the Y – axis

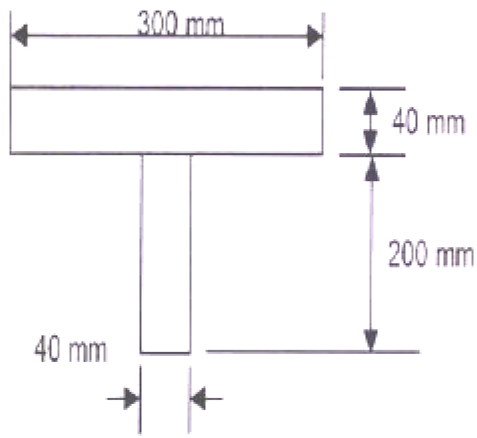
$I_{yy} = k_y^2 A$

$k_x = \sqrt{\frac{I_{yy}}{A}}$

General : The Student is advised to take bottom most line and left most line as reference axes for measuring the CG s of segments.

Problems for finding centroidal axes

1. Locate the centroid of T-section shown in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Bottom rectangle section 2

Section	Area	X in mm	Y in mm
1	300 x 40 =12000	300/2 =150	200 + 40/2 =220
2	40 x 200 = 8000	300/2 =150	200/2 = 100

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{(12000 \ x \ 150) + (8000 \ x \ 150)}{12000 + 8000}$$

$$= \frac{180000 + 1200000}{20000}$$

$$= \frac{1380000}{20000}$$

= 69 mm

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(12000\ x\ 220)+(8000\ x\ 100)}{12000+8000}$$

$$= \frac{2640000+800000}{20000}$$

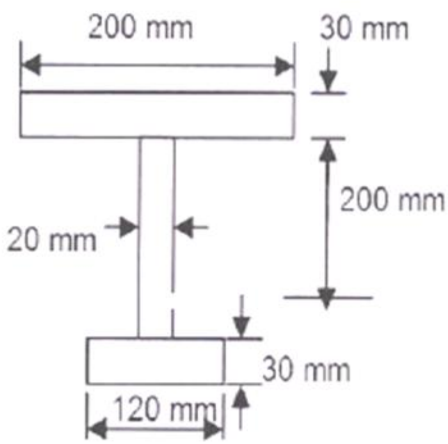
$$= \frac{3440000}{20000}$$

= 172 mm

Result :

The centroid of the given section is (69, 172)

2. Determine the centre of gravity of the I-Section shown in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Middle rectangle section 2

Bottom rectangle section 3

section	Area	X in mm	Y in mm
1	200 x 30 =6000	200/2 =100	30 + 200 + 30/2 =245
2	20 x 200 = 4000	200/2 =100	30 + 200/2 = 130
3	120 x 30 = 3600	200/2 = 100	30/2 =15

$\bar{X} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$

$= \frac{(6000 \ x \ 100) + (4000 \ x \ 100) + (3600 \ x \ 100)}{6000 + 4000 + 3600}$

$= \frac{600000 + 400000 + 360000}{13600}$

$= \frac{1360000}{13600}$

= 100 mm

$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$

$= \frac{(6000 \ x \ 245) + (4000 \ x \ 130) + (3600 \ x \ 15)}{6000 + 4000 + 3600}$

$= \frac{1470000 + 520000 + 54000}{13600}$

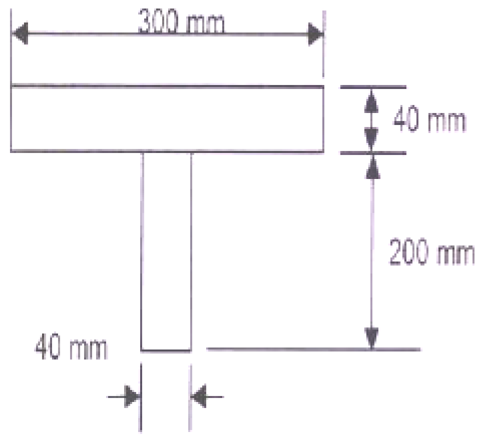
$= \frac{2044000}{13600}$

= 150.294 mm

Result :

The Centre of gravity of the given section is (100, 150.294)

3. Locate the centroid of T-section shown in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Bottom rectangle section 2

Section	Area	X in mm	Y in mm
1	300 x 40 =12000	300/2 =150	200 + 40/2 =220
2	40 x 200 = 8000	300/2 =150	200/2 = 100

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{(12000\ X 150)+(8000\ X\ 150)}{12000+8000}$$

$$= \frac{1800000+1200000}{20000}$$

$$= \frac{3000000}{20000}$$

= 150 mm

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(12000\ x\ 220) + (8000\ x\ 100)}{12000 + 8000}$$

$$= \frac{2640000 + 800000}{20000}$$

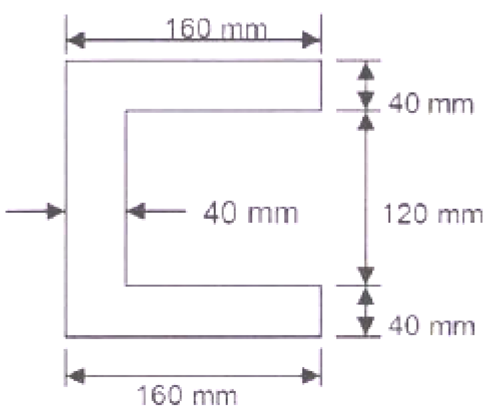
$$= \frac{3440000}{20000}$$

= 172 mm

Result :

The Centre of gravity of the given section is (150, 172)

4. Determine the centre of gravity of the channel section shown in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Middle rectangle section 2

Bottom rectangle section 3

Section	Area	X in mm	Y in mm
1	160 x 40 =6400	160/2 =80	40/2 =20
2	120 x 40 = 4800	40/2 =20	40 + 120/2 = 100
3	160 x 40 = 6400	160/2 = 80	40 +120 +40/2 =180

$$\bar{X} = \frac{A_{1}x_{1} + A_{2}x_{2} + A_{3}x_{3}}{A_{1} + A_{2} + A_{3}}$$

$$= \frac{(6400\ x\ 80)+(4800\ x\ 20)+(6400\ x\ 80)}{6400+4800+6400}$$

$$= \frac{512000+96000+512000}{17600}$$

$$= \frac{1120000}{17600}$$

= 63.636 mm

$$\bar{Y} = \frac{A_{1}y_{1} + A_{2}y_{2} + A_{3}y_{3}}{A_{1} + A_{2} + A_{3}}$$

$$= \frac{(6400\ x\ 20)+(4800\ x\ 100)+(6400\ x\ 180)}{6400+4800+6400}$$

$$= \frac{128000+480000+1152000}{17600}$$

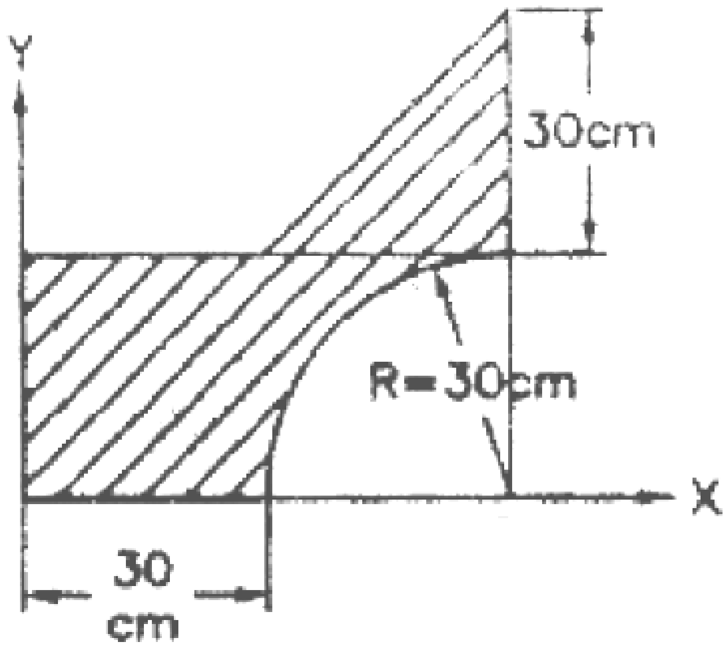
$$= \frac{1760000}{17600}$$

= 100 mm

Result :

The Centre of gravity of the given section is (63.636, 100)

5.Locate the centroid of plane area shaded shown in Fig.



Divide the diagram in to three sections with their individual centroid

Bottom rectangle section 1

Top triangle section 2

Bottom quarter circle section 3

Section	Area in mm^2	X in mm	Y in mm
1	60 x 30 =1800	60/2 =30	30/2 =15
2	½ x 30 x 30 = 450	30 + (2 X 30/3) =50	30 + 1 x 30/3 = 40
3	$\pi X 30^2/4$ = 706.858	60 – (4 x30/3π) = 47.268	(4 x30/3π) =12.732

$$\bar{X} = \frac{A_{1x_1 +} A_2 x_2 - A_{3x_3}}{A_{1+} A_2 - A_3}$$

$$= \frac{(1800\ X\ 30)+(450\ X\ 50)-(706.858\ X\ 47.268)}{1800+450-706.858}$$

$= \frac{54000+22500-33411.764}{1543.142}$

$= \frac{43088.236}{1543.142}$

= 27.922 mm

$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$

$= \frac{(1800\ X\ 15)+(4500\ X\ 40)-(706.858\ X\ 27.922)}{1800+450-706.858}$

$= \frac{27000+18000-19736.889}{1543.142}$

$= \frac{25263.111}{1543.142}$

= 16.371 mm

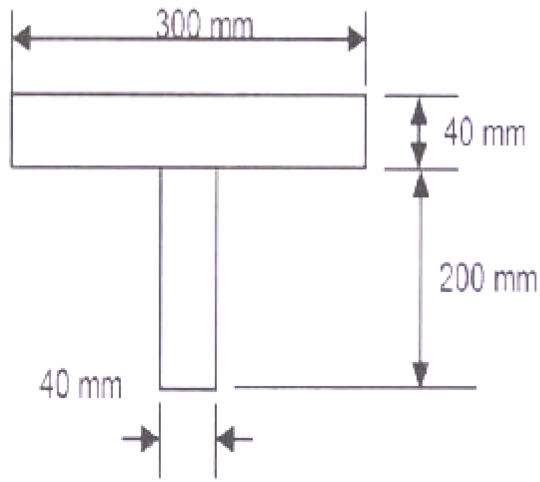
Result :

The Centre of gravity of the given section is (27.922, 16.371)

General : The Student is advised to take bottom most line and left most line as reference axes for measuring the CG s of segments. Finding CG of total fig. is done in accordance to that.

Problems on MI

6.Find the moment of Inertia about the centroidal axes of the section in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Bottom rectangle section 2

Section	Area	X in mm	Y in mm
1	300 x 40 =12000	300/2 =150	200 + 40/2 =220
2	40 x 200 = 8000	300/2 =150	200/2 = 100

$\bar{X} = \frac{A_{1}x_{1} + A_{2}x_{2}}{A_{1} + A_{2}}$

$= \frac{(12000\ X 150)+(8000\ X\ 150)}{12000 + 8000}$

$= \frac{1800000 + 1200000}{20000}$

$= \frac{3000000}{20000}$

= 150 mm

$\bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \frac{(12000 \ x \ 220)+(8000 \ x \ 100)}{12000 + 8000}$

$= \frac{2640000 + 800000}{20000}$

$= \frac{3440000}{20000}$

= 172 mm

Result :

The Centre of gravity of the given section is (150, 172)

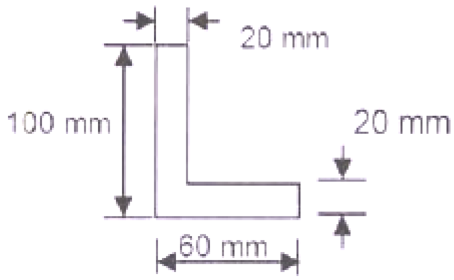
Section	MI about X axis passing through individual centroid I_x	A_1 X $(y_1 - \bar{y})^2$	MI about X axis passing through $\bar{X}$ I_{XX}	MI about y axis passing through individual centroid I_y	A_1 X $(x_1 - \bar{x})^2$	MI about y axis passing through $\bar{Y}$ I_{YY}
1	$\frac{bd^3}{12} = \frac{300 \ X \ 40^3}{12}$ = 1600000	A_1 X $(y_1 - \bar{y})^2$ = 12000 x (220 – 172)^2 =27648000	$I_{X_1} + A_1$ X $(y_1 - \bar{y})^2$ =1600000 + 27648000 =29248000	$\frac{bd^3}{12} = \frac{40 \ X \ 300^3}{12}$ = 90000000	A_1 X $(x_1 - \bar{x})^2$ = 12000 x (150 – 150)^2 =0	$I_{Y_1} + A_1$ X $(x_1 - \bar{x})^2$ =90000000 + 0 =90000000
2	$\frac{bd^3}{12} = \frac{40 \ X \ 200^3}{12}$ = 26666667	A_2X $(\bar{Y} - Y_2)^2$ = 8000 x (172 – 100)^2 =41472000	I_{X_2}+ A_2X$(\bar{Y} - Y_2)^2$ =26666667+ 41472000 = 68138667	$\frac{bd^3}{12} = \frac{200 \ X \ 40^3}{12}$ = 10666667	A_2X $(x_2 - \bar{x})^2$ = 8000 x (150 – 150)^2 =0	$I_{Y_2} + A_2$X $(x_2 - \bar{x})^2$ =10666667 + 0 =10666667
			$\sum I_{XX}$ = 97386667			$\sum I_{YY}$= 100666667

Answer :

Moment of inertia about the centriodal X axis = 97386667 mm^4 = 97.387 X $10^6 mm^4$

Moment of inertia about the centriodal Y axis = 100666667 mm^4 = 100.667 X $10^6 mm^4$

7. .Find the moment of Inertia about the centroidal axes of the section in Fig.



Divide the section in to two rectangles with their individual centroid

left rectangle section 1

Bottom rectangle section 2

Section	Area	X in mm	Y in mm
1	100 x 20 =2000	20/2 =10	100/2 =50
2	40 x 20 = 800	20 + 40/2 =40	20/2 = 10

$$\bar{X} = \frac{A_{1x_1} + A_2 x_2}{A_{1+} A_2}$$

$$= \frac{(2000\ X10) + (800\ X\ 40)}{2000 + 800}$$

$$= \frac{20000 + 32000}{2800}$$

$$= \frac{52000}{2800}$$

= 18.571 mm

$\bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \frac{(2000\ x\ 50)+(800\ x\ 10)}{2000+800}$

$= \frac{100000+8000}{2800}$

$= \frac{108000}{2800}$

= 38.571 mm

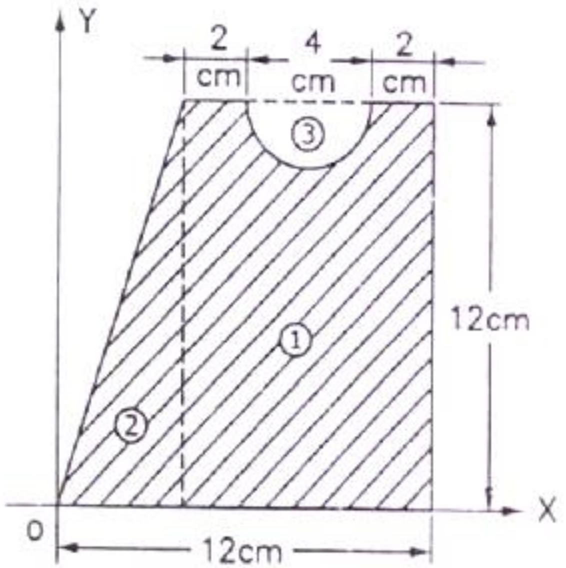
Section	MI about X axis passing through individual centroid I_x	A_1 X $(y_{1-\bar{y}})^2$	MI about X axis passing through $\bar{X}$ I_{XX}	MI about y axis passing through individual centroid I_y	A_1 X $(x_{1-\bar{x}})^2$	MI about y axis passing through $\bar{Y}$ I_{YY}
1	$\frac{bd^3}{12} = \frac{20\ X\ 100^3}{12}$ = 1666667	A_1 X $(y_{1-\bar{y}})^2$ = 2000 x $(50 - 38.571)^2$ =261244	$I_{X_1} + A_1$ X $(y_{1-\bar{y}})^2$ =1666667 + 261244 =1927911	$\frac{bd^3}{12} = \frac{100\ X\ 20^3}{12}$ = 66667	A_1 X $(x_{1-\bar{x}})^2$ = 2000 x $(18.571 - 10)^2$ =146924	$I_{Y_1} + A_1$ X $(x_{1-\bar{x}})^2$ =66667+ 146924 =213951
2	$\frac{bd^3}{12} = \frac{40\ X\ 20^3}{12}$ = 26667	A_2 X $(\bar{Y} - Y_2)^2$ = 800 x $(38.571 - 10)^2$ =653042	I_{X_2}+ A_2 X$(\bar{Y} - Y_2)^2$ =26667+ 653042= 679709	$\frac{bd^3}{12} = \frac{20\ X\ 40^3}{12}$ = 106667	A_2 X $(x_{2-\bar{x}})^2$ = 800 x $(40 - 18.571)^2$ =367362	$I_{Y_2} + A_2$ X $(x_{2-\bar{x}})^2$ =106667 + 367362 =474029
			$\sum I_{XX}$ = 2607620			$\sum I_{YY}$= 687980

Answer :

Moment of inertia about the centriodal X axis = 2607620mm^4 = 2.608 X $10^6 mm^4$

Moment of inertia about the centriodal Y axis = 687980mm^4 = 6.88 X $10^5 mm^4$

8. Find the MI about the horizontal axes of the section shown in Fig.



Moment of inertia of section 1

$I_{X_1} = \frac{bd^3}{12} = \frac{6\ X\ 12^3}{12}$

= 864

Moment of inertia of section 2

$I_{X_2} = \frac{bh^3}{36} = \frac{6\ X\ 12^3}{36}$

= $288cm^4$

Moment of inertia of section 3

= MI of semi-circle about its Centre+ A_3X$(\bar{Y} - Y_3)^2$

= $0.1097r^4$ + ($\frac{\pi\ X\ r^2}{2}$) X $(\bar{Y} - Y_3)^2$ (find $\bar{Y}$ and Y_3 like the previous problems)

= 0.1097 X 2^4) +($\frac{\pi\ X\ 2^2}{2}$) X $(11.151 - 4.43)^2$

= 1.7552 + 283.82

= 285.578 cm^4

Moment of inertia of whole section

= $I_{X_1} + I_{X_2} - I_{X_3}$

= 864 +288 – 285.578

= 866.422 cm^4