UNIT 2 EQUILIBRIUM OF RIGID BODIES 12 Hrs.

Types of supports and their reactions - requirements of stable equilibrium - Moments and Couples - Varignon's theorem - Equilibrium of Rigid bodies in two dimensions - Equilibrium of Rigid bodies in three dimensions - Virtual work

FREE BODY DIAGRAM

Free body diagram is a diagram which shows all the forces acting at a rigid body involving 1.self weight, 2. Normal reactions, 3.frictional force, 4. Applied force, 5. External moment applied. In a rigid body mechanics, the concept of Free body diagram is very useful to solve the problems.

PROCEDURE FOR DRAWING A FREE BODY DIAGRAM:

1. Draw outlined shape
   - Isolate rigid body from its surroundings
2. Show all the forces
   - Show all the external forces and couple moments. This includes Applied Loads, Support reactions, the weight of the body, Identify each force.
   - Known forces should be labeled with proper magnitude and direction.
   - Letters are used to represent magnitude and directions of unknown forces.

Equilibrium of rigid bodies:

Static equilibrium:

A body or part of it (shown in Fig.) which is currently stationary or moving with a constant velocity will remain in its status, if the resultant force and resultant moment are zero for all the forces and couples applied on it.

So:

Thus equations of equilibrium for a rigid body are:

$\Sigma F = 0$

$\Sigma M_o = 0$

VARIGNON'S THEOREM (OR PRINCIPAL OF MOMENTS)

Varignon's Theorem states that the moment of a force about any point is equal to the *algebraic sum* of the moments of its components about that point.

Principal of moments states that the moment of the resultant of a number of forces about any point is equal to the *algebraic sum* of the moments of all the forces of the system about the same point.

Proof of Varignon's Theorem

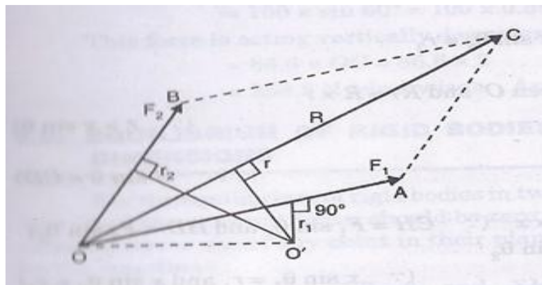


Fig. (a)

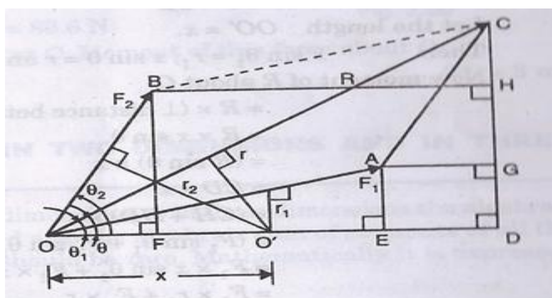


Fig. (b)

Fig *(a)* shows two forces Fj and *F2* acting at point O. These forces are represented in magnitude and direction by *OA* and *OB.* Their resultant R is represented in magnitude and direction by OC which is the diagonal of parallelogram OACB. Let O' is the point in the plane about which moments of F_1, F_2 and R are to be determined. From point *O',* draw perpendiculars

on OA, OC and OB.

Let r_1= Perpendicular distance between F_1 and *O'*.

r_2= Perpendicular distance between R and *O'*.

r_3= Perpendicular distance between F_2 and *O'*.

Then according to Varignon's principle;

Moment *of R* about O' must be equal to algebraic sum of moments of F_1 *and* F_2 about *O'*.

$R \times r = F_1 \times r_1 + F_2 \times r_2$

Now refer to Fig *(b)*. Join *OO'* and produce it to *D*. From points C, *A* and *B* draw perpendiculars on *OD* meeting at *D*, *E* and *F* respectively. From *A* and *B* also draw perpendiculars on *CD* meeting the line *CD* at G and *H* respectively.

Let Θ_1 = Angle made by *F;* with *OD,*

Θ = Angle made by *R* with *OD,* and

Θ_2 = Angle made by F_2 with *OD.*

In Fig. *(b), OA* = *BC* and also *OA* parallel *to BC,* hence the projection *of OA and BC* on the same vertical line *CD* will be equal *i.e., GD* = *CH* as *GD* is the projection of *OA* on *CD* and *CH* is the projection of *BC* on *CD.*

Then from Fig. 2.*(b),* we have

$P_1 \sin \Theta_1 = AE = GD = CH$

$F_1 \cos \Theta_1 = OE$

$F_2 \sin \Theta_1 = BF = HD$

$F2 \cos \Theta_2 = OF = ED$

(OB = *AC* and also *OB* || *AC.* Hence projections of *OB* and *AC* on the same horizontal line *OD* will be equal *i.e., OF* = *ED)*

$R \sin \Theta = CD$

$R \cos \Theta = OD$

Let the length *OO'* = *x.*

Then $x \sin \Theta_1 = r$, $x \sin \Theta = r$ and $x \sin \Theta_2 = r_2$

Now moment of *R* about O'

$= R \times$ (distance between O' and $R) = R \times r$

$= R \times x \sin \Theta$ $\quad$ ($r = x \sin \Theta$)

$=(R \sin \Theta) \times x$

$= CD \times x$ $\quad$ ($R \sin \Theta = CD$)

$= (CH + HD) \times x$

$= (F_1 \sin \Theta_1 + F2 \sin \Theta_2) \times x$ $\quad$ ($CH = F_1 \sin \Theta_1$ and $HD = F_2 \sin \Theta_2$)

$= F_1 \times x \sin \Theta_1 + F_2 \times x \sin \Theta_2$

$= F_1 \times r_1 + F_2 \times r_2$ $\quad$ ($x \sin \Theta_1 = r_1$ and $x \sin \Theta_2 = r_2$)

= Moment of F_1 about O' + Moment of F_2 about *O'*.

Hence moment of *R* about any point in the algebraic sum of moments of its components *i.e.,* F_1 and F_2) about the same point. Hence Varignon's principle is proved.

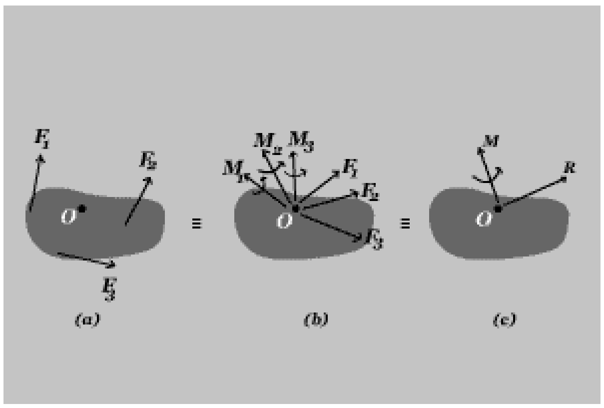
The principle of moments (or Varignon's principle) is not restricted to only two concurrent forces but is also applicable to any coplanar force system, *i.e.,* concurrent or non-concurrent or parallel force system.

Couple

The moment produced by two equal, opposite and non-collinear forces is called a couple.

$\mathbf{M} = \mathbf{r} \times \mathbf{F}$

Resultant of the forces acting on the body



Resultant of the forces acting on a body

Let o the body on which three types of forces are acting. So the resultant force is

$$R = F_1+F_2+F_3+\ldots\ldots.= \Sigma F$$

For each force let the couple introduced to move the forces to the point **O** be M_1, M_2, M_3……… respectively. So the resultant couple is

$$M = M_1+M_2+M_3\ldots\ldots\ldots.$$

$$= (r_1 \times F_1) + (r_2 \times F_2) + (r_3 \times F3)\ldots\ldots.$$

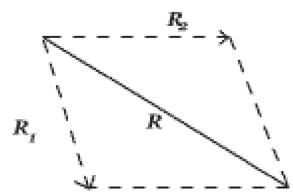
$$= \Sigma(r + F)$$

The point **O** selected as the point of concurrency for the forces is arbitrary, and the magnitude and direction of **M** depend on the particular point **O** selected. The magnitude and direction of **R**, however, are the same no matter which point is selected.

In general, any system of forces may be replaced by its resultant force **R** and the resultant couple **M.** In dynamics we usually select the mass center as the reference point. The change in the linear motion of the body is determined by the resultant force, and the change in the angular motion of the body is determined by the resultant couple. In statics, the body is in *complete equilibrium* when the resultant force **R** is zero and the resultant couple **M** is also zero. Thus, the determination of resultants is essential in both statics and dynamics.

Resolution of forces

Resolution process is the reverse of addition process or resultant process. A force may be result into several parts, such that addition of these forces provide the same force



$$\vec{R}_1 + \vec{R}_2 = R$$

The most common two dimensional resolution of a force vector is into rectangular components.

Let **i** and **j** be the unit vectors in the direction of x and y,

$F = F_x\mathbf{i}+F_y\mathbf{j}$

Hence sum of the resolved components of several forces is equal to the resolved component of the forces.

In general equilibrium means that there is no acceleration i.e., the body is moving with constant velocity but in this special case we take this constant to be zero.

So if we take a point particle and apply a force on it, it will accelerate. Thus if we want its acceleration to be zero, the sum of all forces applied on it must vanish. This is the condition for equilibrium of a point particle. So for a point particle the equilibrium condition is

$$\sum_i \vec{F}_i = 0$$

Where $\vec{F}_i$; i =1, 2, 3…….. are the forces applied on the point particle.

Torque

Torque is defined as the vector product of the displacement vector form the reference point where the force applied. Thus

$$\vec{\tau} = \vec{r}_O \times \vec{F}$$

This is also known as the moment of the force

Equilibrium of rigid bodies

Conditions for Equilibrium

- The body should not accelerate(should not move) which, is ensured if $\sum_i \vec{F}_i = 0$ that is the sum of all forces acting on it must be zero no matter at what points on the body they are applied. For example consider the beam in figure given. Let the forces applied by the supports S_1 and S_2 be F_1 and F_2, respectively. Then for equilibrium, it is required that

$$\vec{F}_1 + \vec{F}_2 + \vec{W} + \vec{L} = 0$$

Assuming the direction towards the top of the page to be y-direction, this translates to

$$F_1\hat{j} + F_2\hat{j} - W\hat{j} - L\hat{j} = 0 \text{ or } F_1 + F_2 - W - L = 0$$

The condition is sufficient to make sure that the net force on the rod is zero. But as we learned earlier, and also our everyday experience tells us that even a zero net force can give rise to a turning of the rod. So F_1 and F_2 must be applied at such points that the net torque on the beam is also zero. This is given below as the second rule for equilibrium.

- Summation of moment of forces about any point in the body is zero i.e., $\sum_i \vec{\tau}_{iO} = 0$ where $\vec{\tau}_{iO}$ is the torque due to the force $\vec{F}_i$ about point *O*. One may ask at this point whether $\sum_i \vec{\tau}_{iO} = 0$ should be taken about many different points or is it sufficient to take it about any one convenient point. The answer is that any one convenient point is sufficient because if condition (1) above is satisfied, i.e. net force on the body is zero then the torque as is independent of point about which it is taken. These two conditions are both necessary and sufficient condition for equilibrium.

Torque due to a force

Torque about a point due to a for $\vec{F}$ is obtained

as the vector product

$$\begin{aligned}\vec{\tau}_O &= \vec{r}_O \times \vec{F} \\ &= (yF_z - F_y z)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}\end{aligned}$$

Where $\vec{r}_O$ is a vector from the point *O* to the point where the force is being applied.

Actually $\vec{r}_O$ could be a vector from *O* to any point along the line of action of the force.

The magnitude of the torque is given as

$$|\vec{\tau}_O| = |\vec{F}||\vec{r}_O|\sin\theta$$

The unit of a torque is Newton-meter or simply Nm.

This is known as ***Varignon's theorem***.

Equilibrium of Rigid bodies in two dimensions

Forces are all in *x-y* plane

Thus $\mathbf{F}_2 = 0$ $\mathbf{M}_x = \mathbf{M}_y = 0$ are automatic satisfied

Equations of equilibrium are reduced, i.e.

$$\Sigma \mathbf{F}_x = 0 \qquad \Sigma \mathbf{F}_y = 0 \qquad \Sigma \mathbf{M}_A = 0\ (\Sigma \mathbf{M}_z)$$

Where A is any point in the plane.

Equilibrium of Rigid bodies in three dimensions

Equation of equilibrium

$$\Sigma \mathbf{F} = 0 \qquad \Sigma \mathbf{M}_o = \Sigma(\mathbf{r} \mathrm{x} \mathbf{F}) = 0$$

I.e. $\Sigma \mathbf{F}_x = 0$ $\Sigma F_y = \mathbf{0}$ $\Sigma \mathbf{F}_z = 0$

$\Sigma \mathbf{M}_x = 0$ $\Sigma \mathbf{M}_y = \mathbf{0}$ $\Sigma \mathbf{M}_z = 0$

Equilibrium of rigid bodies

The basics and important condition to be considered for the equilibrium of forces and moments is taken as zero

When a body is acted by upon by some external forces the body starts to rotate or move about any point. If the body does not move or rotate about any point the body said to be in equilibrium.

Moment of force

Moment of a force about a any point is defined as the product of magnitude of force and the perpendicular distance between the forces the point.

The moment (M) of the force (F) about 'o' is given by

M = r*F

F-force acting on the body

r- perpendicular distance from the point O on the line of action of force

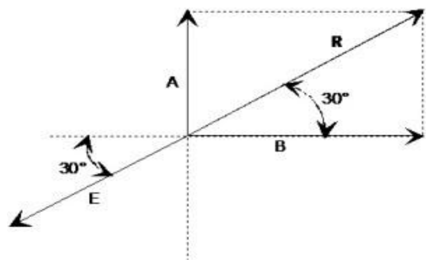
If the moment rotates the body in CW direction about 'O' then it is CW moment(+ve). if the moment rotates the body in CCW direction about 'O' then it is CW moment(-ve)

Equilibrium of a particle:

If the resultant of a number of forces acting on a particle is zero then the particle is in equilibrium

Equilibrium and equilibrant:

The force E which brings the particle to equilibrium is called equilibrant(E).



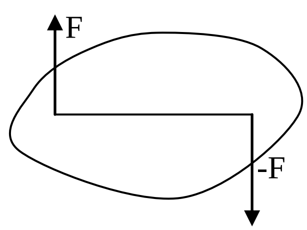
Resultant force and equilibrant force both have same magnitude. But if resultant acts at say θ then Equilibrant acts at $180^0 + \theta$.

Conditions for equilibrium:

1) The algebraic sum of all the external force is zero.$\sum F = 0$ (.$\sum F_x = 0$; .$\sum F_y = 0$; .$\sum F_z = 0$)
2) The algebraic sum of all the external forces about any point in the plane is zero.$\sum M = 0$

Couple:

Two forces F and –F having same magnitude, parallel lines of action and opposite sense are said to form a couple.



When 2 equal and opposite parallel forces act on a body at some distance apart the 2 forces form a couple. Couple has a tendency to rotate the body . The perpendicular distance between the parallel forces is called arm of the couple.

Moment of a couple = forces x Arm of the couple

M = F x a

Equivalent Forces and couples:

The two forces having same magnitude, direction and line of action but acting at different points producing the same external effect on the rigid body are said to be equivalent forces.

If two couples produce the same moment on the rigid body they are called equivalent couples.

Difference between moment and couple:

Couple is a pure turning effect which may be moved anywhere in its own plane or into a parallel plane without change of its effect on the body.

Moment of force must include a description of the reference axis about which the moment is taken.

Resolution of force into a force and a couple at a point.(Force couple system)

Figure

Resultant of coplanar non concurrent force system:-

$R = \sqrt{(\sum H)^2+(\Sigma V)^2}$

$\Theta=\tan^{-}(\Sigma V/\sum H)$

Resultant force vector R= formula

Equilibrium of rigid body in two dimensional

The equilibrium state will be achieved when the summation of all the external forces and the moments of all the forces is zero.

Principle of equilibrium

$\sum F=0$ (force law of equilibrium)

$\sum M=0$(Moment law of equilibrium)

Force Law of equilibrium

$\sum Fx=0$ (with respect to horizontal components)

$\sum Fy=0$(with respect to horizontal components)

Equilibrium of particle in space

In three dimension of space if the forces acting on the particle are resolved into their respective i ,j,k components the equilibrium equation is written as,

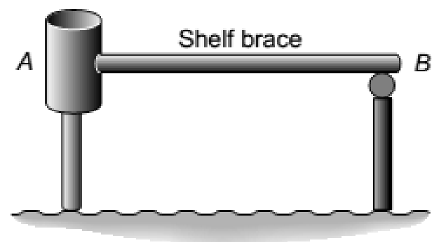
$\sum Fxi+\sum Fj+\sum Fxk=0$

The equation for equilibrium of a particle in space is,

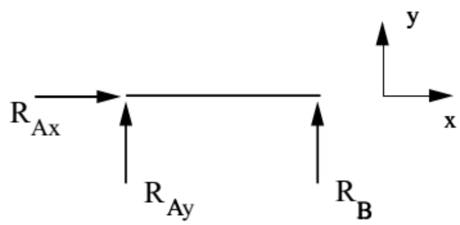
$\sum Fx=0$; $\sum Fy=0$; $\sum Fz=0$;

EXAMPLE 1:

An adjustable shelving system consists of rod mounted to a shaft on the left and a frictionless support on the right. Isolate the shelf brace and make a free body diagram of the system



solution:



Reactions and support reactions

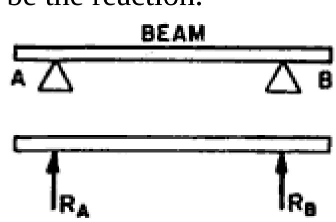
When a number of forces are acting on a body, and the first body is supported on another body, then the second body exerts a force known as reactions on the first body at the points of contact so that the first body is in equilibrium. The second body is known as support and the force, exerted by the second body on the first body, is known as support reactions.

Types of supports

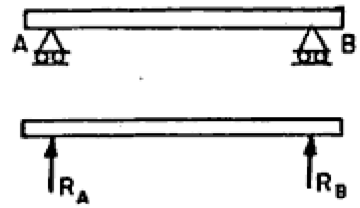
There are 5 most important supports. They are

- Simple supports or knife edged supports
- Roller support
- Pin-joint or hinged support
- Smooth surface support
- Fixed or built-in support

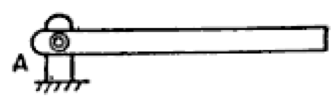
Simple supports or knife edged support: in this case support will be normal to the surface of the beam. If AB is a beam with knife edges A and B, then R_A and R_B will be the reaction.



Roller support: here beam AB is supported on the rollers. The reaction will be normal to the surface on which rollers are placed.



Pin joint (or hinged) support: here the beam AB is hinged at point A. the reaction at the hinged end may be either vertical or inclined depending upon the type of loading. If load is vertical, then the reaction will also be vertical. But if the load is inclined, then the reaction at the hinged end will also be inclined.



Fixed or built-in support: in this type of support the beam should be fixed. The reaction will be inclined. Also the fixed support will provide a couple.

Types of loading

There are 3 most important type of loading:

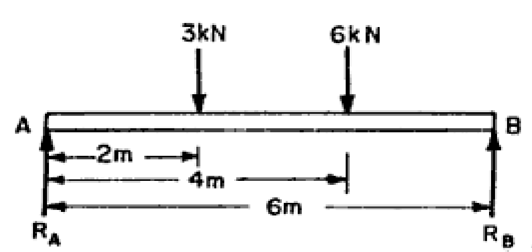
- Concentrated or point load
- Uniformly distributed load
- Uniformly varying load

Concentrated or point load: here beam AB is simply supported at the ends A and B. A load W is acting at the point C. this load is known as point load or Concentrated load. Hence any load acting at a point on a beam, is known as point load.

Problem for point load

- **A simply supported beam AB of span 6m carries point loads of 3kN and 6kN at a distance of 2m and 4m from the left end A as shown in fig. find the reactions at A and B analytically.**

Solution



Given, span of beam = 6m

Let R_A = reaction at A

R_B = reaction at B

As the beam is in equilibrium, the moments of all the forces about any point should be zero.

Now taking the moment of all forces about A and equating the resultant moment to zero, we get

$R_B \times 6 - 3 \times 2 - 6 \times 4 = 0$

$6R_B = 6 + 24 = 30$

$R_B = 30/6 = 5$ kN

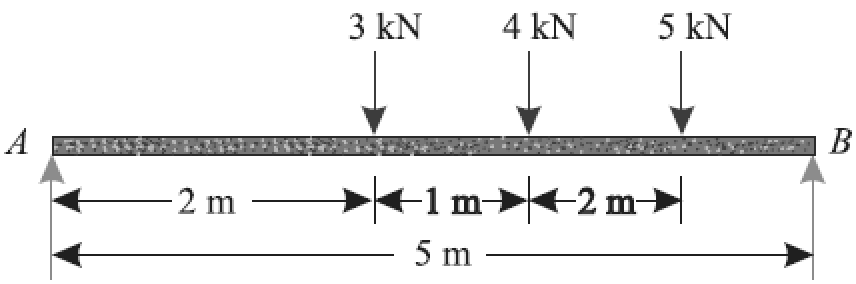
Also for equilibrium, $\Sigma F_y = o$

$\therefore\ R_A + R_B = 3 + 6 = 9$

$\therefore\ R_A = 9 - R_B = 9 - 5 = 4$ kN

EXAMPLE 2:

A simply supported beam AB of span 5 m is loaded as shown in figure. Find the reactions at A and B.



Solution.

Given: Span (l) = 5 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

The example may be solved either analytically or graphically. But we shall solve analytically only.

We know that anticlockwise moment due to R_B about A

$= R_B \times l = R_B \times 5 = 5\ R_B$ kN-m ...(i)

and sum of the clockwise moments about A,

$= (3 \times 2) + (4 \times 3) + (5 \times 4) = 38$ kN-m ...(ii)

Now equating anticlockwise and clockwise moments given in (i) and (ii),

$5\ R_B = 38$

or $R_B = 38/5 = 7.6$ kN

and $R_A = (3+4+5) - 7.6 = 4.4$ kN

EXAMPLE 3:

Problem for Uniformly distributed load

A simply supported beam AB of length 9m, carries a uniformly distributed load of 10 kN/m for distance of 6m from the left end. Calculate the reactions at A and B

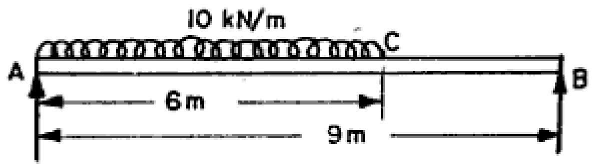
Solution

Given,

Length of beem = 9

Rate of U.D.L = 10 kN/m

Length of U.D.L = 6m



Total load due to U.D.L = (Length of U.D.L) x Rate of U.D.L

= 6 x 10 = 60 kN

This load of 60Kn will be acting at the middle point of AC i.e,at a distance of 6/2= 3 m from A.

Let R_A = Reaction at A and R_B = reaction at B

Taking the moment of all forces about point A, and equating the resultant moment to zero, we get

$R_B \times 9 - (6 \times 10) \times 3 = 0$ or $9R_B - 180 = 0$

$\therefore R_B = 180/9 = 20kN.$

Also for equilibrium, $\Sigma F_y = 0$

Or $R_A + R_B = 6 \times 10 = 60$

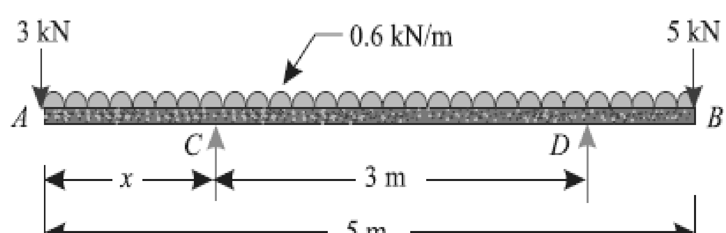
$\therefore R_A = 60 - R_B = 62 - 20 = 40$ kN.

EXAMPLE 4:

Problems for overhanging

- **A beam AB 5 m long, supported on two in termediate supports 3 m apart, carries a uniformly distributed load of 0.6 kN/m. The beam also carries two concentrated loads of 3 kN at left hand end A, and 5 kN at the right hand end B as shown in Figure. Determine the location of the two supports, so that both the reactions are equal.**

Solution.



Given:

Length of the beam AB (L) = 5 m

and span (l) = 3 m.

Let R_C =Reaction at C,

R_D =Reaction at D, and

x =Distance of the support C from the left hand end

We know that total load on the beam

$=3 + (0.6 \times 5) + 5 = 11$ kN

Since the reactions R_C and R_D are equal, therefore reaction at support

$$= \frac{11}{2} = 5.5 \text{ kN}$$

We know that anticlockwise moment due to R_C and R_D about A

$$= 5.5 \times x + 5.5\,(x + 3) = 5.5\,x + 5.5\,x + 16.5 \text{ kN-m}$$

$$= 11x + 16.5 \text{ kN-m} \qquad ...(i)$$

and sum of clockwise moment due to loads about A

$$= (0.6 \times 5)\,2.5 + 5 \times 5 = 32.5 \text{ kN-m} \qquad ...(ii)$$

Now equating anticlockwise and clockwise moments given in (*i*) and (*ii*)

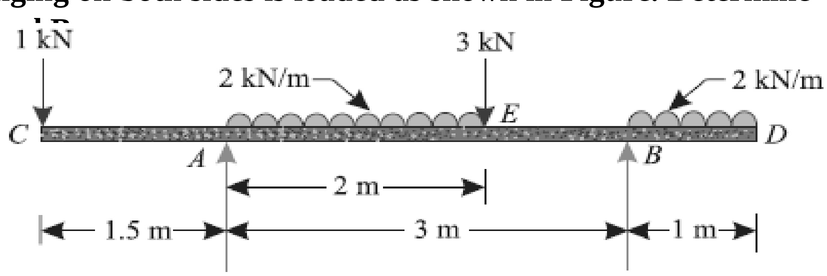
$$11\,x + 16.5 = 32.5 \qquad \text{or} \qquad 11\,x = 16$$

$$\therefore \qquad x = \frac{16}{11} = 1.45 \text{ m}$$

It is thus obvious that the first support will be located at distance of 1.45m from A and second support at a distance of 1.45 + 3 = 4.45 m from A.

EXAMPLE 5:

A beam AB of span 3m, overhanging on both sides is loaded as shown in Figure. Determine the reactions at the support



Solution.

Given:

Span (l) = 3m

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that anticlockwise moment due to R_B and load at C about A

$= R_B \times l + (1 \times 1.5) = R_B \times 3 + (1 \times 1.5) = 3R_B + 1.5$ kN ...(i)

and sum of clockwise moments due to loads about A

$= (2 \times 2)\,1 + (3 \times 2) + (1 \times 1)\,3.5 = 13.5$ kN-m ...(ii)

Now equating anticlockwise and clockwise moments given in (i) and (ii),

$$3R_B + 1.5 = 13.5$$

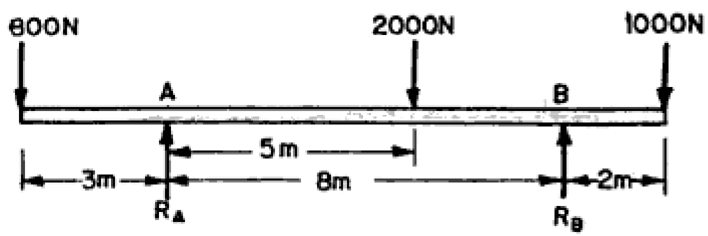
or $R_B = \frac{13.5-1.5}{3} = \frac{12}{3} = 4$ kN

and $R_A = 1 + (2 \times 2) + 3 + (1 \times 1) - 4 = 5$ kN

EXAMPLE 6:

A beam of AB of span 8m, overhanging on both sides is loded as show in figure. Calculate the reactions at both ends.

Solution



Given,

Span of beam = 8m

Let R_A = reaction at A

R_B = reaction at B

Taking the moment of all foreces about point A and equating the resulant moment to zero, we get

$R_B \times 8 + 800 \times 3 - 2000 \times 5 - 1000 \times (8 + 2) = 0$

or $8R_B + 2400 - 10000 - 10000 = 0$

or $8R_B = 2000 - 2400 = 17600$

∴ $R_B = \frac{17600}{8} = 2200$ N.

Also for the equilibrium we have

$$R_A + R_B = 800 + 2000 + 1000 = 3800$$

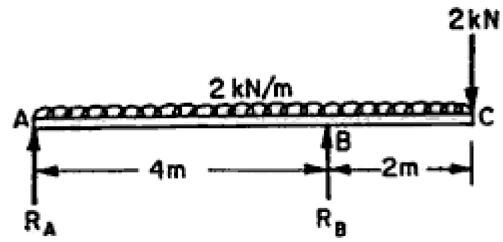
∴ $R_A = 3800 - R_B = 3800 - 2200 = 1600$

EXAMPLE 7:

A beam AB of span 4m, overhanging on one side upto a length of 2m, carries a uniformaly distributed load of 2kN/m over the entire length of 6m and a point load of 2kN/m as shown in figure. Calculate the reactions at A and B.

Solution

Given,



Span of beam = 4m

Total length = 6m

Rate of U.D.L = 2 kN/m

Total load due to U.D.L = 2 x 6 = 12kN

The toad of 12 kN (i.e., due to U.D.L) will act at the middle point of AC, i.e, at a distance of 3m from A.

Let R_A = reaction at A

and R_B = reaction at B

taking the moment of all forces about point A and equating the resultant moment to zero, we get

R_B x 4 – (2 x 6) x 3 – 2 x (4 +2) = 0

Or $4R_B$ = 36 + 12 = 48

$\therefore$ R_B = 48/4 = 12kN.

Also for equilibrium, ΣF_y =0 or $R_A + R_B$ = 12+2 =14

$\therefore R_A = 14 - R_B$ =14 – 12 = 2kN

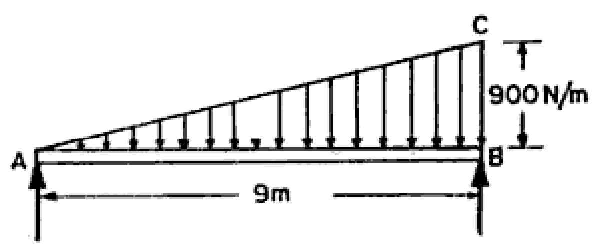
EXAMPLE 8:

Problem for Uniformly varying load

A simply supported beam of span 9m carries a uniformly varying load from zero at end a to 900 N/m at end B. calculate the reactions at end B. calculate the reaction at the two ends of the support.

Solution

Given,



Span of beam = 9m

Load at end A = 0

Load at end B = 900 N/m

Total load the beam = Area of ABC = (AB x BC)/2 = (9x900)/2

= 4050N

Or 5RB – (5 x 800) x 2.5 – {1/2 x 5 x 800} x {2/3 x 5} = 0

Or 5RB – 1000 – 6666.66 = 0

Or 5RB = 1000 + 6666.66 = 16666.66

Or RB = 16666.66/5 = 3333.33 N.

Also for the equilibrium of the beam, $\Sigma F_Y = 0$

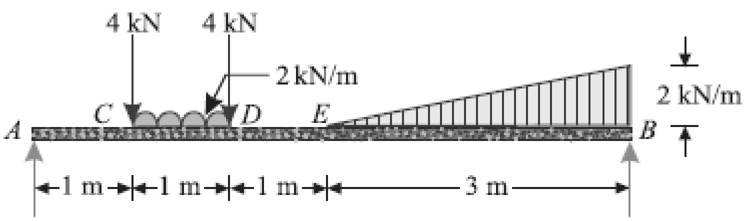
$\therefore$ $R_A + R_B$ = total load of the beam

= 6000 ($\because$ Total load on beam = 6000 N)

$\therefore$ $R_A = 6000 - R_B = 6000 - 3333.33 = 2666.67$ N.

EXAMPLE 9:

A simply supported beam AB of 6 m span is subjected to loading as shown in Figure. Find the support reactions at A and B.



Solution.

Given: Span (l) = 6 m

Let RA = Reaction at A, and

RB = Reaction at B.

We know that anticlockwise moment due to RB about A

= RB × l = RB × 6 = 6 RB kN-m ...(i)

and sum of clockwise moments due to loads about A

= (4 x 1) + (4 x 2) +[(0 + 2)/2] x 3 x 5 = 30 kN-m

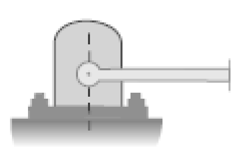
Now equating anticlockwise and clockwise moments given in (i) and (ii),

$6 R_B = 30$

or $R_B = 30/6 = 5$ Kn

and $R_A = (4 + 2 + 4 + 3) - 5 = 8$ kN

Hinged beams



In such a case, the end of a beam is hinged to the support as shown in Figure. The reaction on such an end may be *horizontal*, *vertical* or *inclined*, depending upon the type of loading. All the steel trusses of the bridges have one of their end roller supported, and the other hinged.

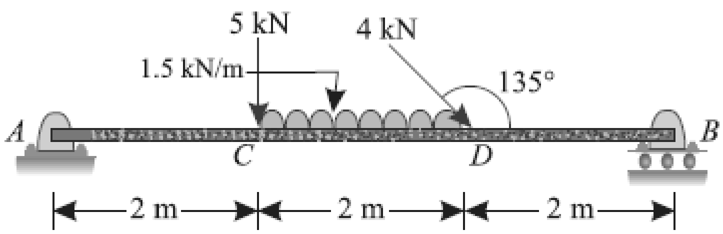
The main advantage of such a support is that the beam remains stable. A little consideration will show that the beam cannot be stable, if both of its ends are supported on rollers. It is thus obvious, that one of the supports is made roller supported and the other hinged.

EXAMPLE 10:

Problem for hinged beam

A beam AB of 6 m span is loaded as shown in Figure. Determine the reactions at A and B.

Solution.



Given:

Span = 6 m

Let R_A = Reaction at *A*, and

R_B = Reaction at *B*.

We know that as the beam is supported on rollers at the right hand support (*B*), therefore the reaction R_B will be vertical (because of horizontal support). Moreover, as the beam is hinged at the left support (*A*) and it is also carrying inclined load, therefore the reaction at this end will be the resultant of horizontal and vertical forces, and thus will be inclined with the vertical.

Resolving the 4 kN load at *D* vertically

$$= 4 \sin 45° = 4 \times 0.707 = 2.83 \text{ kN}$$

and now resolving it horizontally

$$= 4 \cos 45° = 4 \times 0{\cdot}707 = 2{\cdot}83 \text{ kN}$$

We know that anticlockwise moment due to R_B about *A*

$$= R_B \times 6 = 6\, R_B \text{ kN-m} \qquad ...(i)$$

and *sum of clockwise moments due to loads about *A*

$$= (5 \times 2) + (1.5 \times 2)\, 3 + 2.83 \times 4 = 30.3 \text{ kN-m} \qquad ...(ii)$$

Now equating the anticlockwise and clockwise moments in (*i*) and (*ii*),

$$6\, R_B = 30.3$$

or $$R_B = \frac{30.3}{6} = 5.05 \text{ kN}$$

We know that vertical component of the reaction R_A

$$= [5 + (1.5 \times 2) + 2.83] - 5.05 = 5.78 \text{ kN}$$

∴ Reaction at A,

$$R_A = \sqrt{(5.78)^2 + (2.83)^2} = 6.44 \text{ kN}$$

Let θ = Angle, which the reaction at *A* makes with vertical.

∴ $\tan\theta = (2.83)/(5.78) = 0.4896$ or $\theta = 26.1°$