UNIT-I.

CLASSIFICATION OF SIGNALS.

Continuous time signal | Discrete Time signal

① Periodic & Non-periodic signal
② Even & Odd signal
③ Energy & power signal
④ Deterministic & Random signal.

Signal: Any physical quantity if it varies with respect to time then it's called a signal.

Egs: Temperature, pressure, voltage, mass, speed, acceleration.

System: Any physical device which performs an operation on the signal.

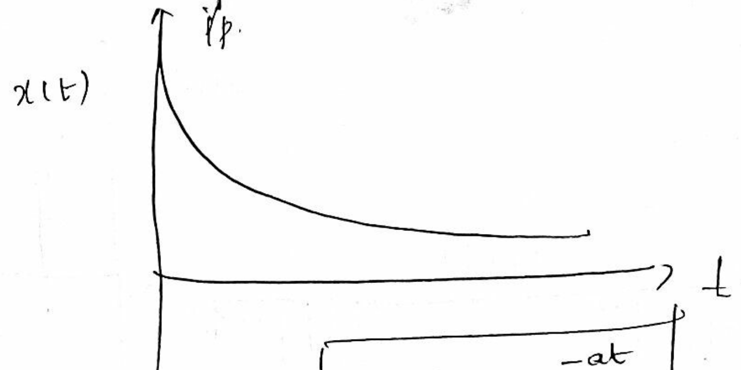
Eg. audio amplifier.

system

low level i/p $\xrightarrow{\text{excitation}}$ [Audio amplifier] $\longrightarrow$ Response amplified o/p.

CT (continuous time signal).

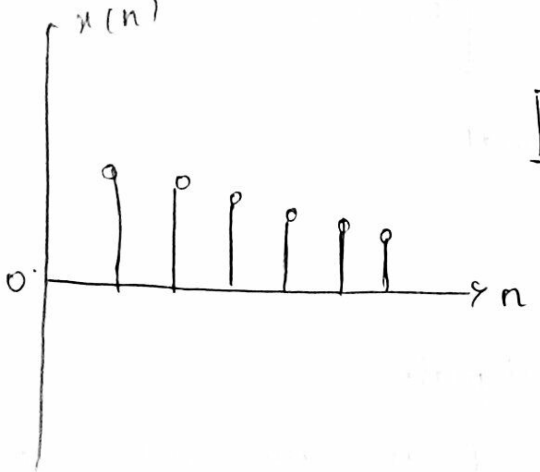
If the amplitude of the signal varies continuously with respect to time then it's called a continuous time signal. It can be represented by,



2(a) decaying exponential signal.

$$\boxed{x(t) = e^{-at}}$$

Discrete Time signal: Has got some discrete set of values for the specific time interval. It can be represented by x(n).

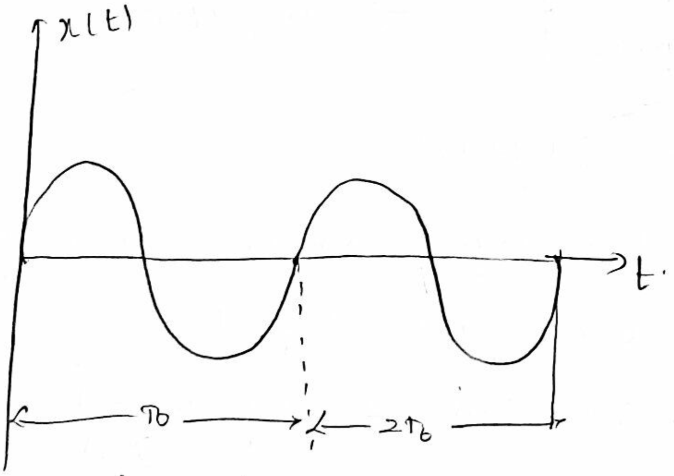


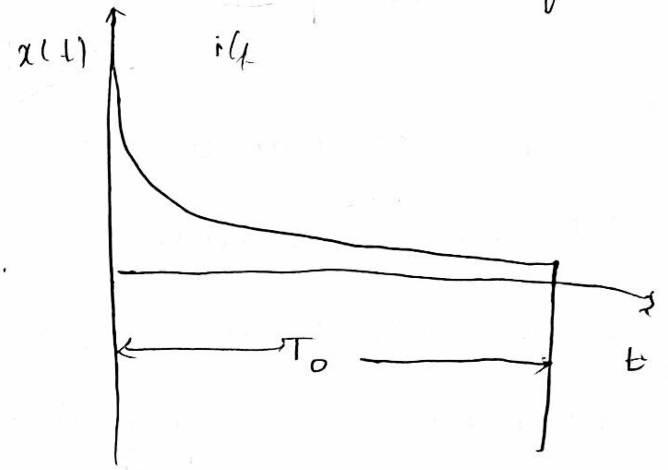
$$\boxed{x(n) = e^{-an}}$$

① Periodic & Non-periodic signal:-

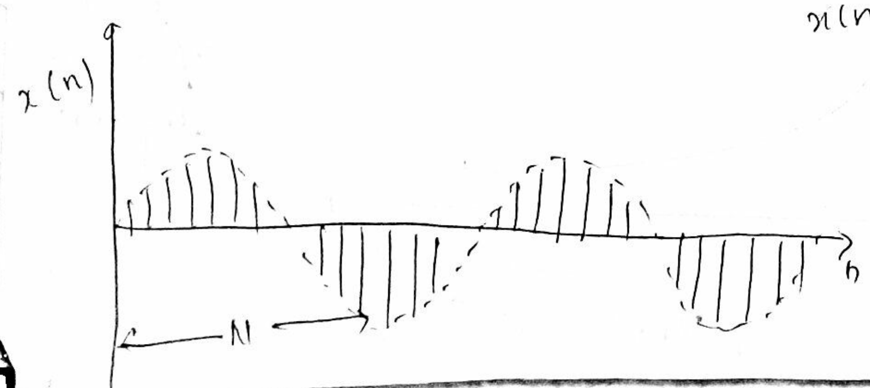
* If a signal is said to be periodic it repeats at regular time intervals.
* Non-periodic signals:- Do not repeat at regular time intervals.

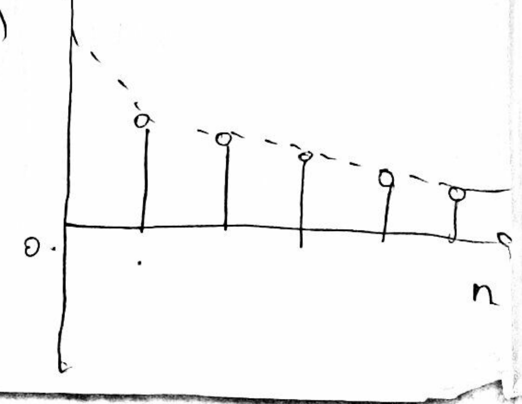
Periodic signal:- Non. periodic signal.





discrete.





CT periodicity: fundamental period.

$x(t) = x(t + T_0)$

DT $x(n) = x(n+N)$

Let us consider DT cosine wave.

$x(n) = \cos(2\pi f_0 n)$ — ①

$x(n+N) = \cos(2\pi f_0 (n+N))$

$= \cos(2\pi f_0 n + 2\pi f_0 N)$ - ②

① - ②.

$\cos(2\pi f_0 n) = \cos(2\pi f_0 n + 2\pi f_0 N)$ → must be integer.

$\cancel{2}\pi f_0 N = 2\pi k$

$f_0 N = k$; $\boxed{f_0 = k/N}$

more than 2.

CT :-

$2\pi f_0 N k = 1/T_0$

$f_0 =$

$x(t) = x_1(t) + x_2(t) + \cdots$

→ $x_1(t) = x_1(t+T_1) = x_2(t+2T_1) = \cdots$

1 cycle, 2nd cycle.

$x_1(t) = x_1(t + mT_1)$

$x_2(t) = x_2(t) + x_2(t) + \cdots$

→ $x_2(t) = x_2(t+T_2) + x_2(t+2T_2) = \cdots$

$x_2(t) = x_2(t+nT_2)$

$T_0 = mT_1 = nT_2$

$y(t) = x(t+T_0)$

$x(n) = x(n+N)$

$$\boxed{T_0 = \frac{T_1}{T_2} = \frac{n}{m}}$$

DT :- $x(n) = x_1(n)$

$$\boxed{N = \frac{N_1}{N_2} = \frac{n}{m}}$$

$T_0 = \frac{T_1}{T_2} = \frac{n}{m}$.

* Even + odd Signals: 4

Even: If a signal is said to be even, the inversion of time axis doesn't change the amplitude of the signal.

Odd Signal: If a signal is said to be odd, the inversion of time axis, also inverts the amplitude of the signal.

Even :- $x(t) = x(-t)$

$x(n) = x(-n).$

Even & odd signal.

$x(t) = x(-t)$

$x(n) = x(-n)$

odd:

$x(t) = -x(-t)$

$x(n) = -x(-n)$

$x(t) = x(-t)$

$x(n) = x(-n)$

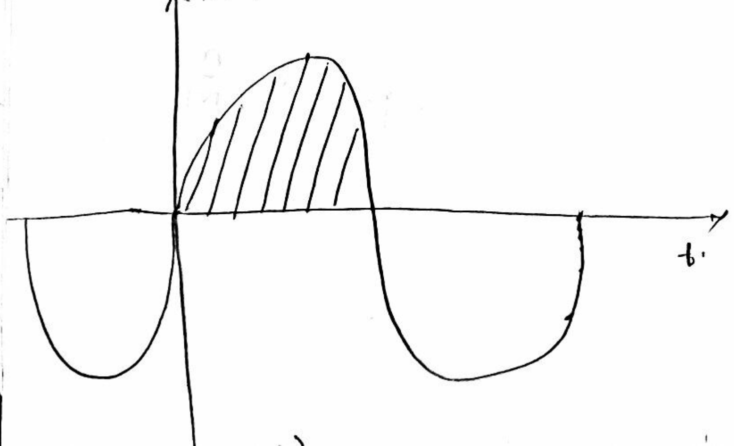
$x(t) = \cancel{-(-t)}$

$x(t) = -x(-t)$

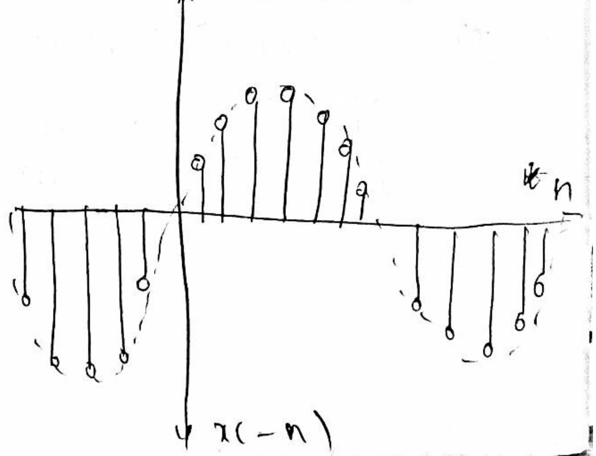
$x(n) = -x(-n)$

~~Even~~ Odd signal:: Eg: sine wave:-

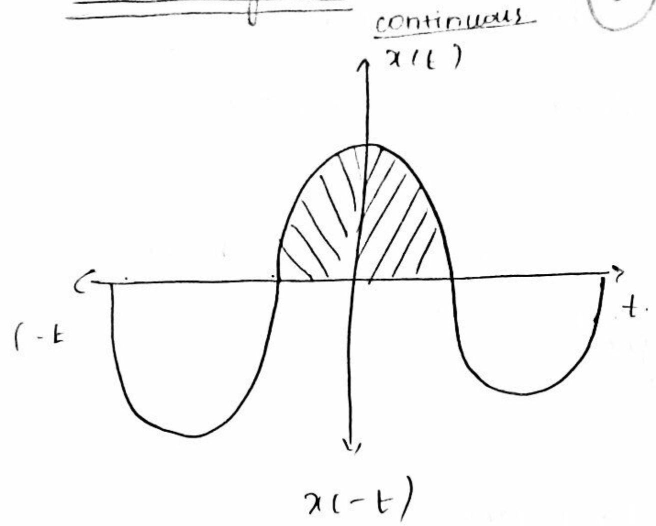
Continuous:

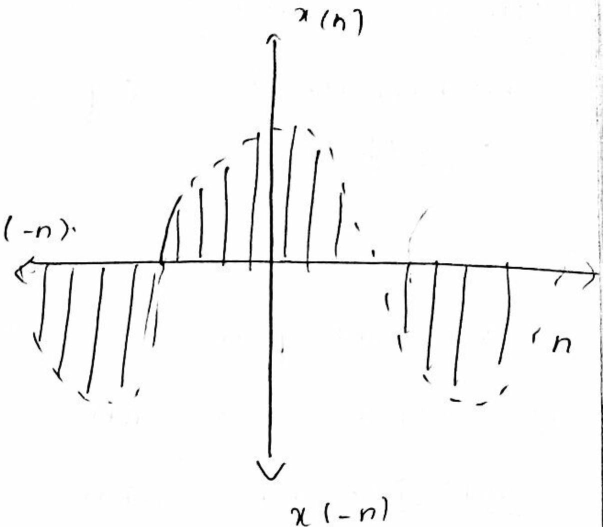


Discrete.



Even signal:





$$x(t) = x_e(t) + x_o(t) \quad -(1).$$

$t = -t$. (invert the time axis.

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \quad -(2)$$

even:

$$(1) + (2) \Rightarrow x(t) + x(-t) = 2x_e(t)$$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right].$$

odd part

$$(1) - (2) \Rightarrow x(t) - x(-t) = 2x_o(t)$$

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right).$$

Discrete even:

$$x_e(n) = \frac{1}{2} \cdot [x(n) + x(-n))$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

* Deterministic & Random signal:-

can be

If any signal can be represented by a mathematical equation, then it's called deterministic signal. eg: sine & cosine wave.

RANDOM SIGNALS: cannot be represented by mathematical representation or equations.

Egs : Noise.

I. Check whether the gn signal is periodic or non-periodic signal.

1) $x(n) = \cos 3\pi n$

Sol: ① compare with std. form.

$x(n) = \cos 2\pi f_0 n.$

$2\cancel{\pi} f_0 \cancel{n} = 3\cancel{\pi} \cancel{n}$

$2f_0 = 3.$

$\boxed{f_0 = 3/2} = K/N.$

$\boxed{N = 2.}$

f_0 → shld be rational or fun of 2.
If it's periodic get Time period.

$2 f_0 \pi n \left(\frac{1}{2}\right)^n$

2) ~~Gin: $x(t) = \sin \omega_0 t$~~

$x(n) = \cos 2\pi n/7 + \cos 5\pi n/7.$

Sol: compare:

$x(t) = \sin \omega_0 t.$

$x(n) = \cos 2\pi f_1 n + \cos 2\pi f_2 n.$

$2\cancel{\pi} f_1 \cancel{n} = \cancel{2}\cancel{\pi} \cancel{n}/7. = \boxed{f_1 = \frac{2}{14}.} = \frac{1}{7}$

$2\cancel{\pi} f_2 \cancel{n} = 5\cancel{\pi} \cancel{n}/7$

$\boxed{f_2 = \frac{5}{14}.}$

N → least common multiples.

$\frac{f_1}{f_2} = \frac{1}{\cancel{7}} \times \frac{\cancel{14}^{2}}{5} = \frac{2}{5}.$

$\frac{f_1}{f_2} = \frac{2}{5} = \frac{N_1}{N_2}$

The gn signal is periodic.

$\boxed{N = 10.}$

3) Gn: $x(n) = \sin(0.08n)$

Sol: $x(n) = \cancel{\cos}\ \sin 2\pi f_0 n.$

π value $180°$.

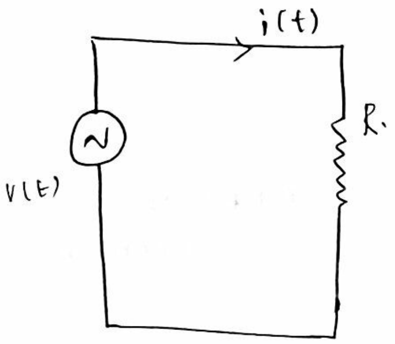
$\sin 2\pi f_0 n = \sin 0.08\, n.$

$2\pi f_0 = 0.08.$

$f_0 = \frac{0.08}{2\pi} = \cancel{0.0127}$

The gn signal is non-periodic since it's not a rational number or functions of 2.

Energy & Power Signal:



Power. $P(t) = \frac{v^2(t)}{R} = i^2(t) R.$

Normalised power:

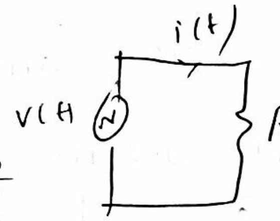
$v(t)$ & $i(t)$ represented by $x(t)$

Power signal: periodi

$0 < P < \infty.$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt.$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$



$p(t) = \frac{v^2(t)}{R} = i^2(t) R.$

$v(t)$ + $i(t)$ → represented by $x(t)$

Energy signal: $0 < E < \infty$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2.$$

- All periodic signals are called power signals
- All non-periodic signals are called energy signal.

Probs: Check whether the gn signal is periodic or non-periodic signal:-.

1) $x(n) = \cos\left(\frac{n}{8}\right) \cdot \cos\frac{n\pi}{8}$

Sol: ① Compare with the standard form;

$x = \cos 2\pi f_0 N$.

$\cos 2\pi f_1 n = \cos\left(\frac{n}{8}\right)$

$\boxed{f_1 = \frac{1}{16\pi}}$

$\cos 2\pi f_2 n = \cos \frac{n\pi}{8}$

$\boxed{f_2 = \frac{1}{16}}$

non perio . peri
= non perio

It's non periodic.

2) $x(n) = \sin(\pi + 0.2n)$

$\theta = \pi$.

Sol: $x(n) = \sin 2\pi f_0 n$.

$2\pi f_0 n = 0.2n$.

$\boxed{f_0 = \frac{0.2}{2\pi}}$ Non. periodic.

3) $x(n) = e^{j(\pi/4)n}$

Sol: $x(n) = \cos\frac{\pi}{4}n + j\sin\frac{\pi}{4}n$.

$= \cos 2\pi f_0 n + j \cdot \sin 2\pi f_0 n$.

$2\pi f_0 n = \frac{\pi}{4}n$ $\qquad$ $2\pi f_0 n = \frac{\pi}{4}n$.

$\boxed{f_0 = \frac{1}{8}}$ $\qquad$ $\boxed{f_0 = \frac{1}{8}}$

(H) Gn: $x(t) = \cos t + \sin\sqrt{2}t$.

wrong.

Sol: compare with ~~std~~ form.

$\cos 2\pi f_1 n = \cos t$.

$$f_1 = \frac{t}{2\pi n}.$$

$\sin 2\pi f_2 n = \sin\sqrt{2}t$.

$$f_2 = \frac{\sqrt{2}.t}{2\pi n}.$$

$$\boxed{N = \frac{f_1}{f_2}} = \frac{t}{\cancel{2\pi n}} \times \frac{\cancel{2\pi n}}{\sqrt{2}.t}$$

$$= \frac{t}{\sqrt{2}.t}$$

4) $x(t) = \cos t + \sin\sqrt{2}t$.

Sol.

$2\pi f_1 t = t$. ; $2\pi f_2 t = \sqrt{2}t$.

$\boxed{2 = \sqrt{2}.\sqrt{2}}$

$\boxed{f_1 = \frac{1}{2\pi}}$ $f_2 = \frac{\sqrt{2}}{2\pi} = \frac{1}{\sqrt{2}\pi}$.

$\boxed{T = \frac{1}{f}}$

$\boxed{T_1 = 2\pi}$ $\boxed{T_2 = \sqrt{2}\pi}$

$$\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2}.$$

It's non periodic.

5) $x(t) = \cos^2(2\pi t)$

$\cos^2\theta = 1 - \sin^2\theta$.

$\boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$

Sol:

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= \frac{1}{2} + \frac{1}{2}\cos 2\theta.$$

$\theta = 2\pi t$

$\cos 2\theta =$

$2(2\pi t)$

$4\pi t.$

dont worry abt amplitu

$x(t) = \frac{1}{2} + \frac{1}{2} \cos 4\pi t$

dc → wave shift → amplitude

$2\pi f_1 t = 4\pi t$

$\boxed{f_1 = 2}$

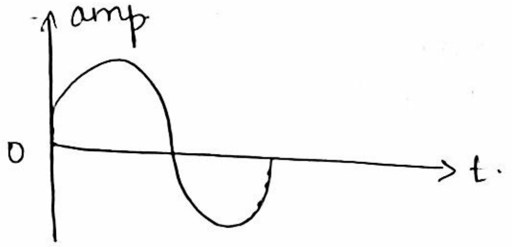
$T = \frac{1}{f} = \frac{1}{2}$ $\boxed{T = \frac{1}{2}}$

. It's a periodic signal.

Representation of signals::

1. Graphical Representation
2. Functional Representation ⇒ $x(n) = \begin{cases} 2 & \text{for } n=0 \\ 3 & \text{for } n=1,2 \\ 1 & \text{for } n=3 \end{cases}$
3. Tubular Representation
4. sequence Representation. ⇒ $x(n) = \{\underset{\uparrow}{2}, 3, 3, 1\}$. ↑ → from origin

Eg ①: ↑ amp



⑤ Tubular:

n	0	1	2	3		
x(n)	2	2	3	1		

k → delay

after time rev

+ → +, −

b4 Time reversal

+ → −, − → +

★ Operations on the signal:

① time shifting → − → +, + → −.

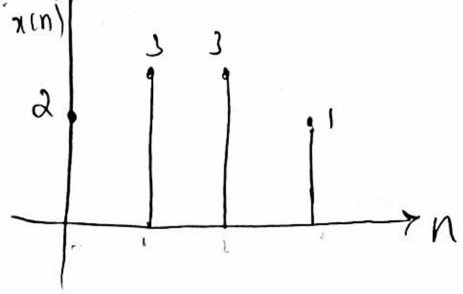
a) $y(n) = x(n-k)$ k → delay.

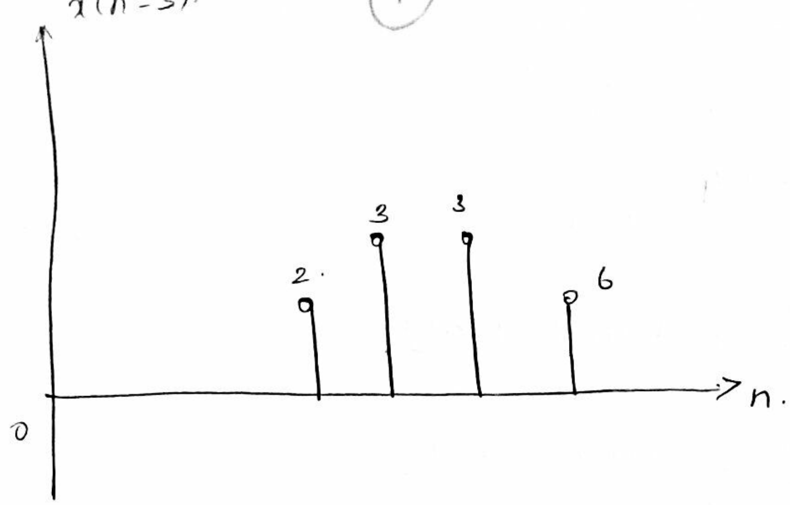
eg: $x(n-3)$ k = −3 → move + side.

TS → multiply

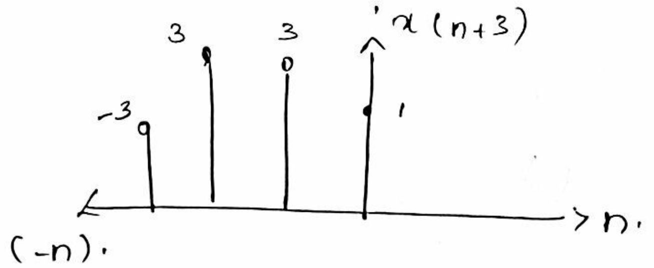
−k → −ve delay

+k → +ve delay





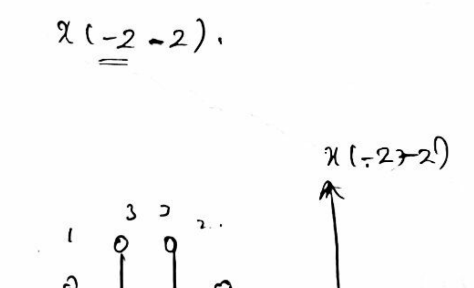
b) x(n+3) move - side.



② Time ~~Shift~~ Reversal (or) folding.

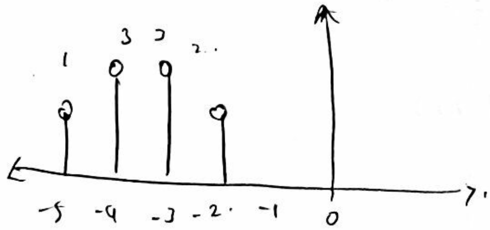
a) $y(n) = x(-n)$.
$x(-n+2)$

time rev.
fold the sig
+ side move -
-side moves → +



after fol → see n.

$x(-2-2)$.



① Time Scaling. $y(n) = x(an)$; $a = 2$

$x(n) = \{\underset{\uparrow}{2}, 3, 3, 1\}$

$y(n) = \{2, 3, 3, 0\}$

Find and sketch the even & odd components for the foll. signals.

i) $x(n) = e^{-(n/4)} u(n)$ — ① $u(n) \to$ unit step

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$ → decaying sig.

$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$

$n=0$ in ①, $x(0) = e^0 = 1$

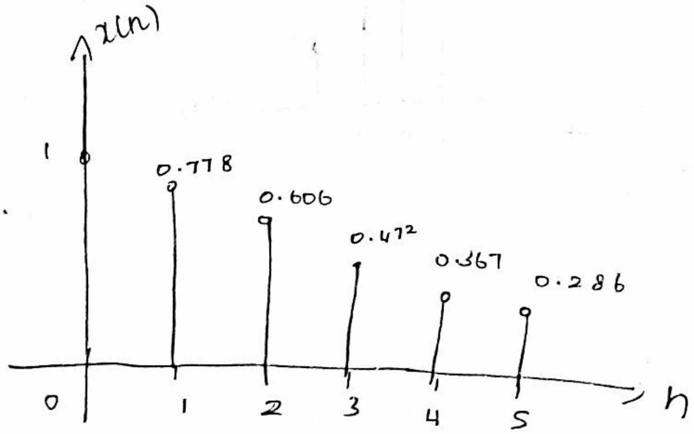
sub $n=1$ in ① ; $x(1) = e^{-(1/4)} = 0.778$

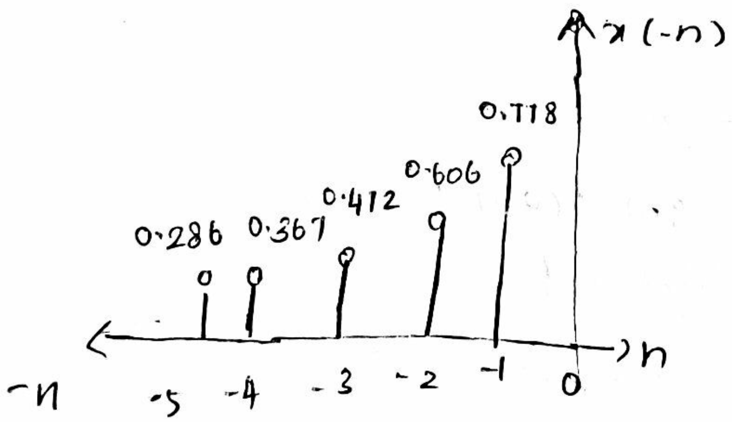
$n=2$ in ① $x(2) = e^{-1/2} = 0.606$ axis $\to$ (+) $= 2$.

$n=3$ $x(3) = e^{-3/4} = 0.472$

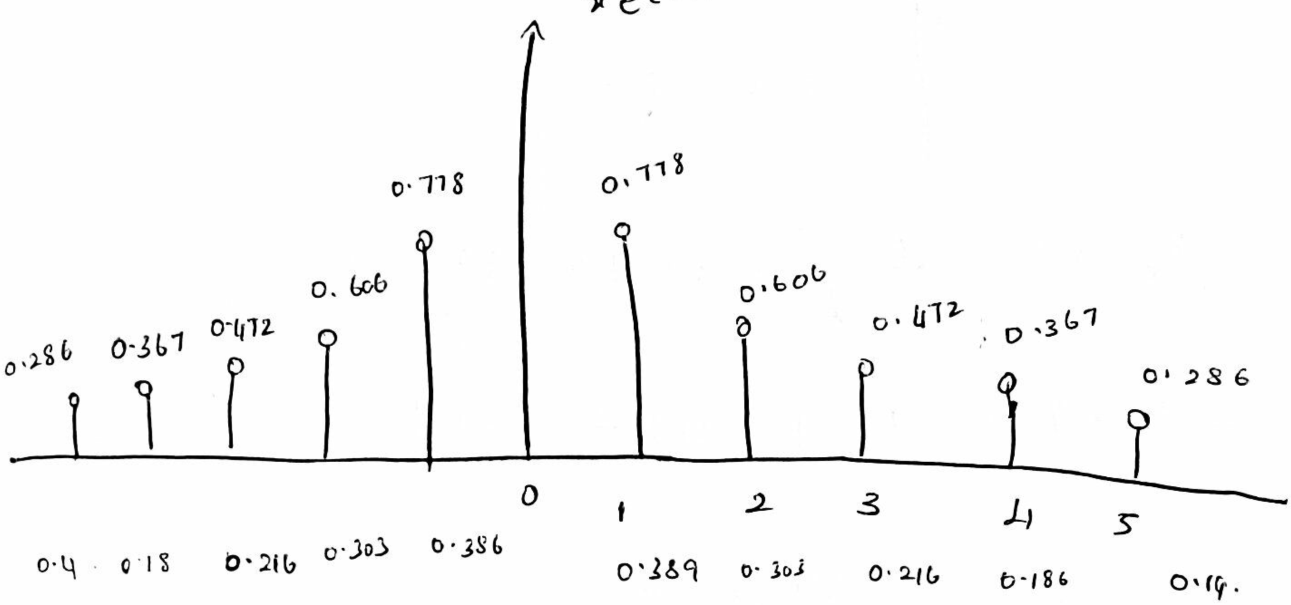
$n=4$ $x(4) = e^{-1} = 0.367$

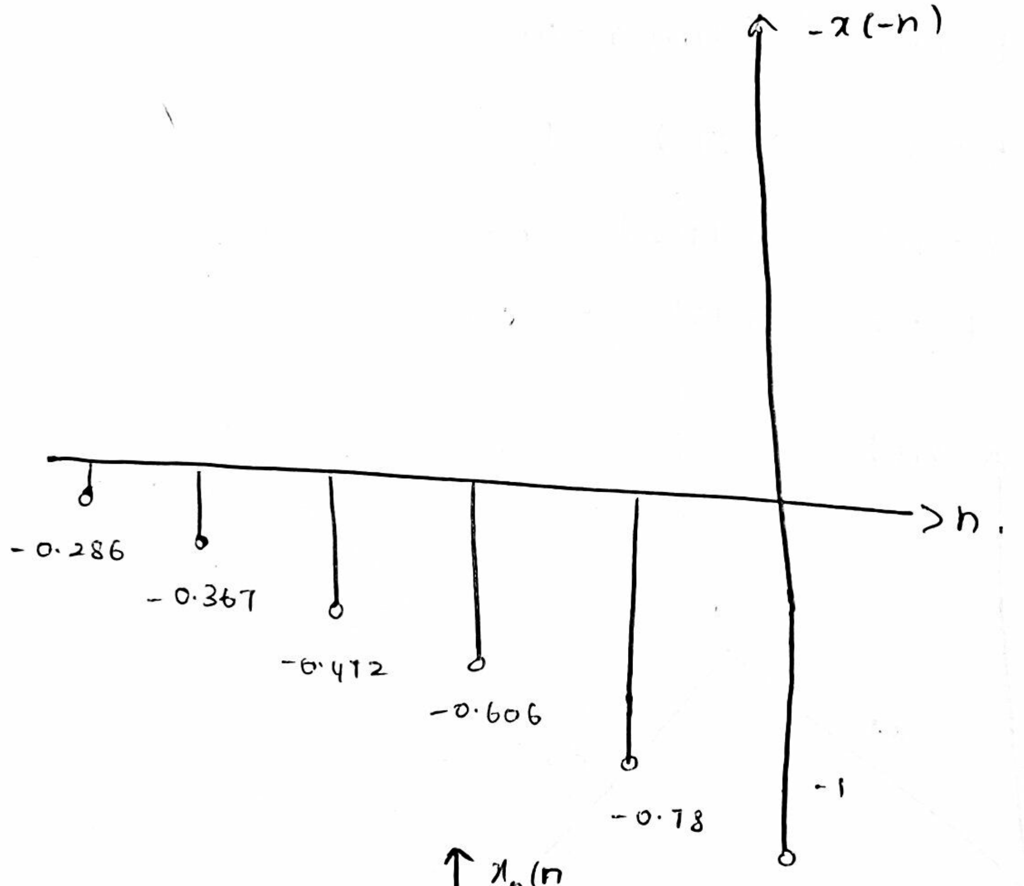
$n=5$ $x(5) = e^{-5/4} = 0.286$

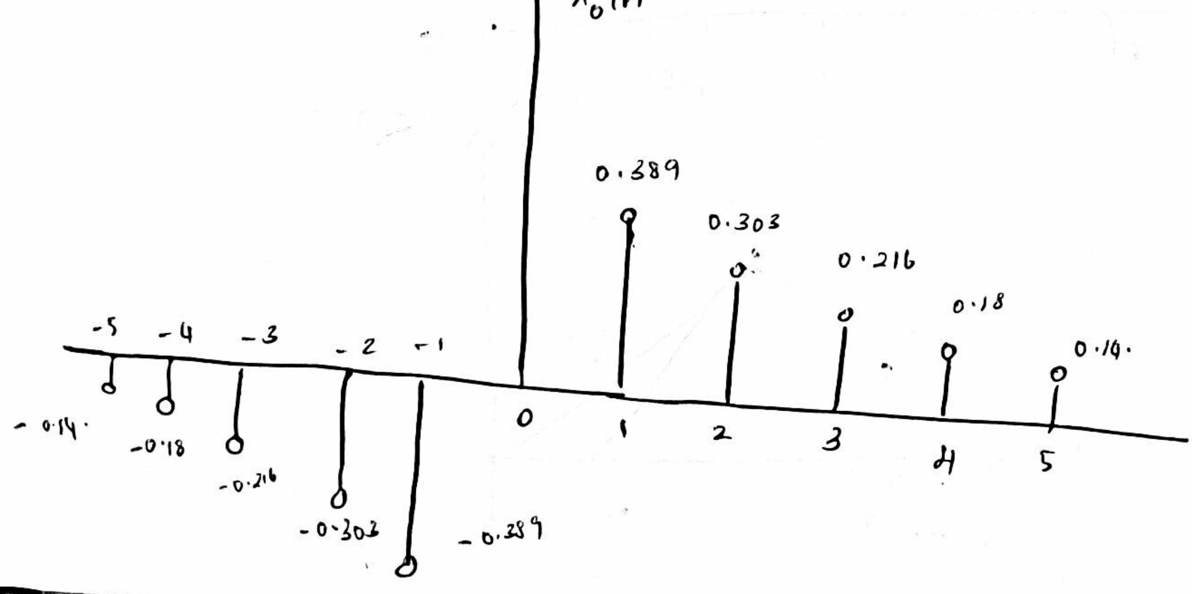




2.


xe(n).
0.778
0.778
0.606
0.606
0.286
0.367
0.472
0.472
0.367
0.286
0
1
2
3
4
5
0.4
0.18
0.216
0.303
0.386
0.389
0.303
0.216
0.186
0.14.


-x(-n)
>n.
-0.286
-0.367
-0.472
-0.606
-0.78
-1


x0(n
0.389
0.303
0.216
0.18
0.14.
-5
-4
-3
-2
-1
0
1
2
3
4
5
-0.14.
-0.18
-0.216
-0.303
-0.389

2) $x(t) = \begin{cases} t & \text{for } 0 \leq t \leq 1 \\ 2-t & \text{for} \end{cases}$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

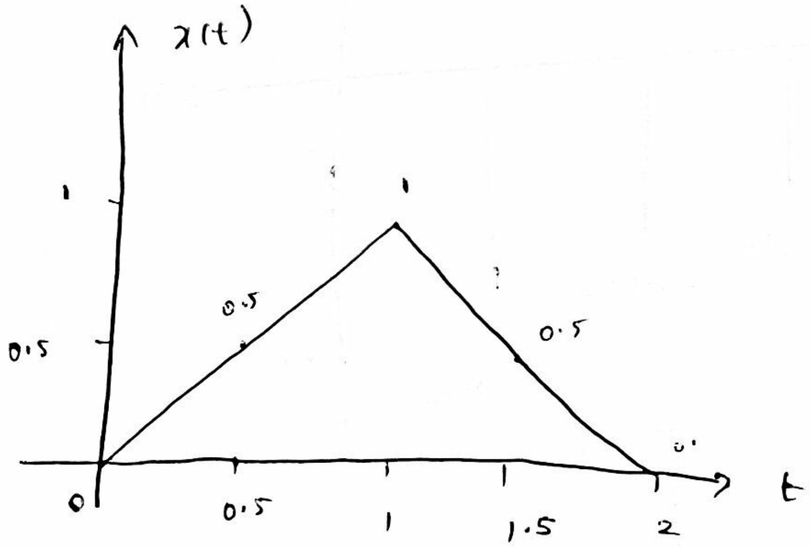
at $t = 0$; $x(0) = 0$

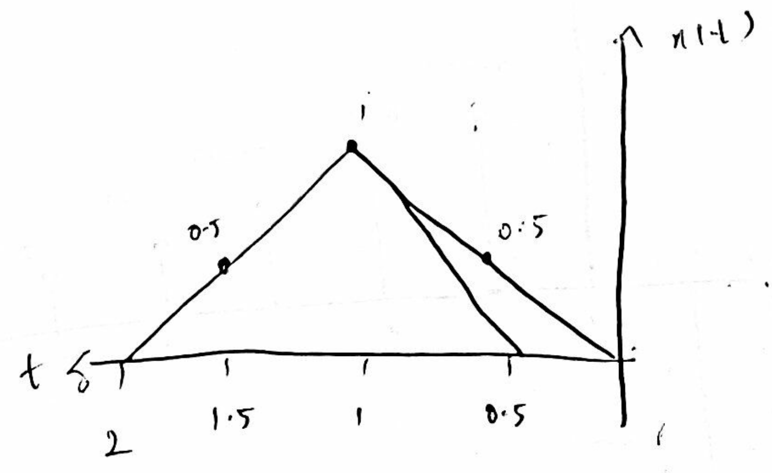
$t = 0.5$; $x(0.5) = 0.5$

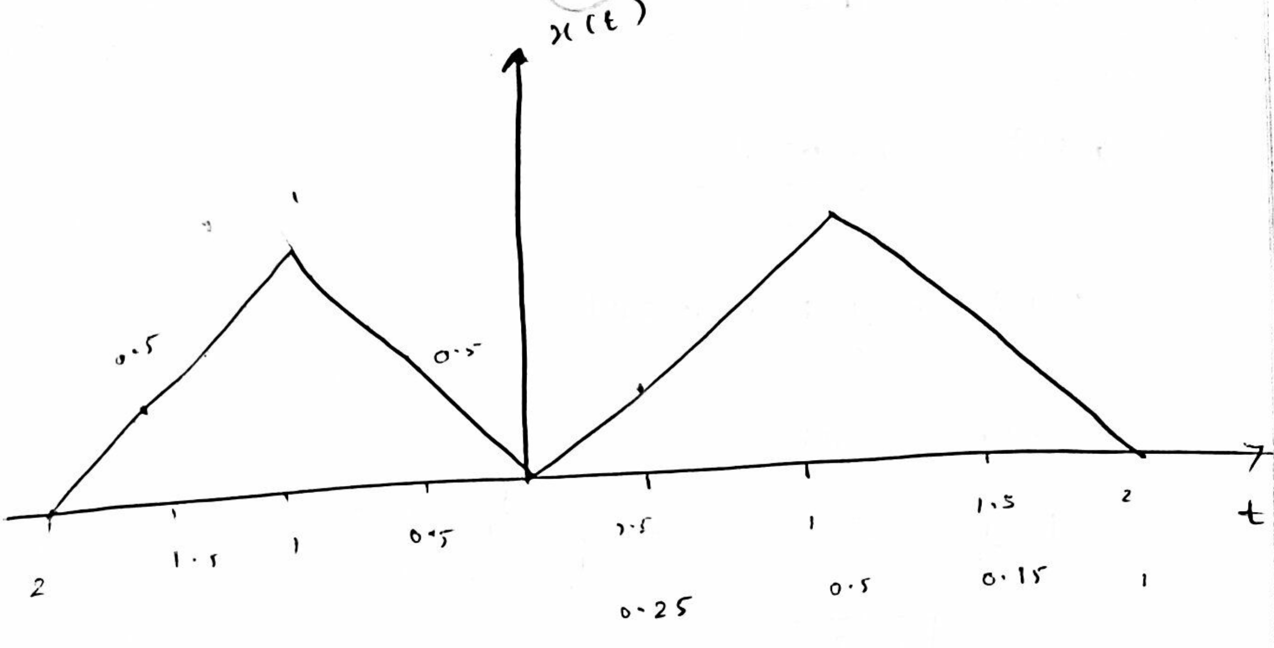
$t = 1$; $x(1) = 1$

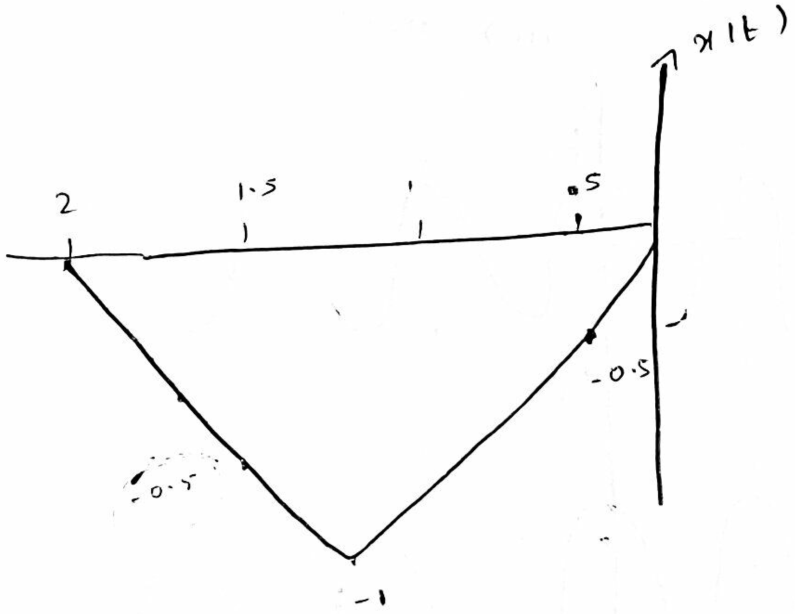
$t = 1.5$ $x(1.5) = 0.5$

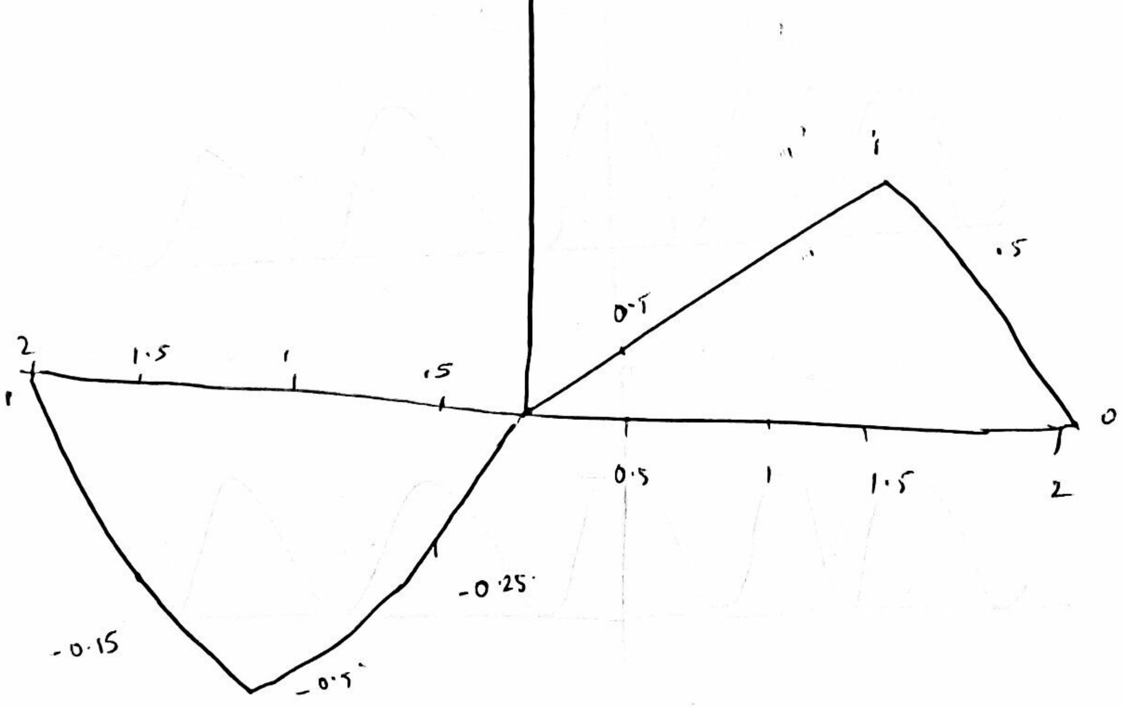
$t = 2$ $x(2) = 0.$






x(t)
t
0.5
0.5
2
1.5
1
0.5
0.5
1
1.5
2
0.25
0.5
0.75
1


x(t)
2
1.5
1
0.5
-0.5
-0.5
-1


x(t)
2
1.5
1
.5
0.5
.5
0
0.5
1
1.5
2
-0.25
-0.15
-0.5

16

3) $x(t) = \cos^2\left(\frac{\pi t}{2}\right)$

peak value
cos, sin → PQ = 1 → amp

$x(t) = \frac{1+\cos \pi t}{2}$

cos → odd → 0
even only

$x(t) = \frac{1}{2} + \frac{1}{2}\cos \pi t$

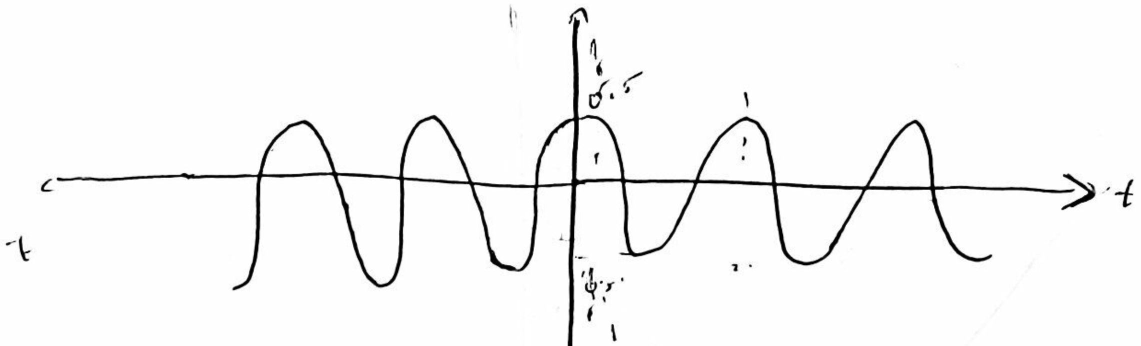
$2\pi f t = \pi t$

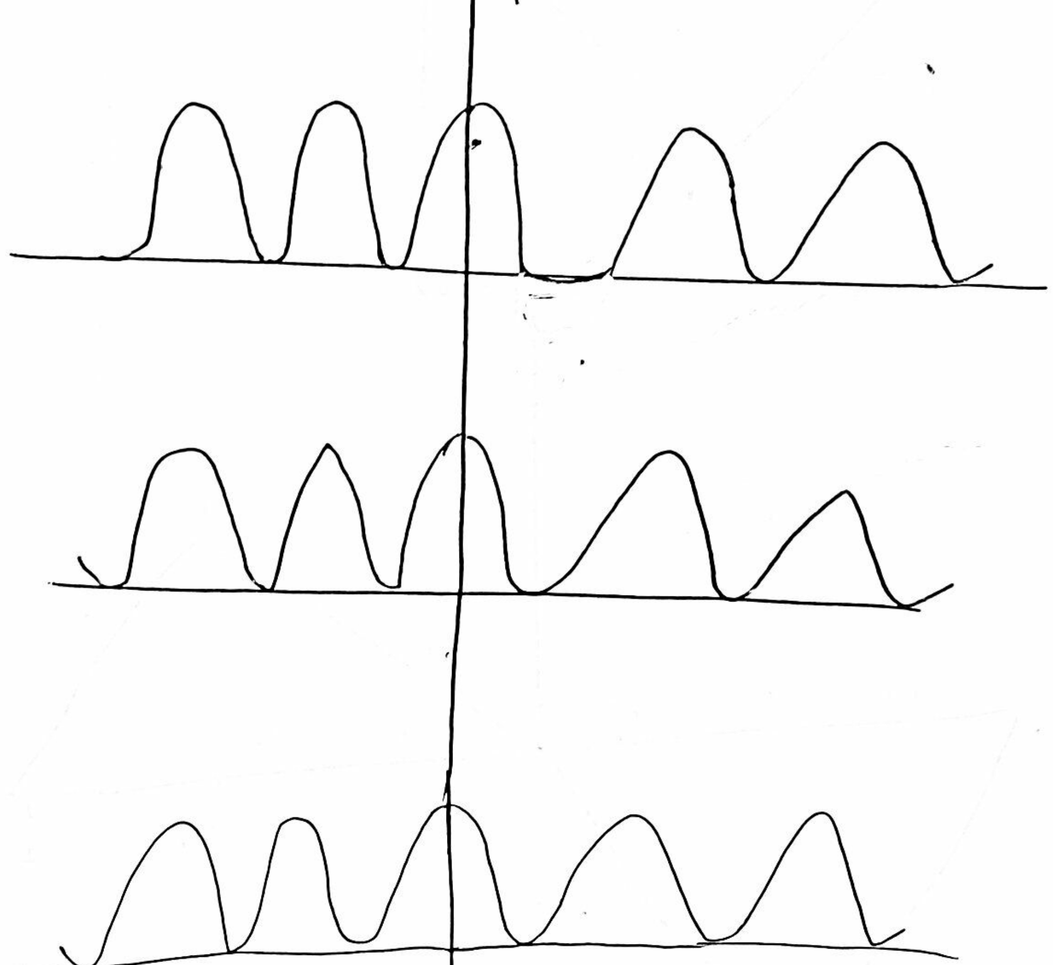
$f = \frac{1}{2} = \frac{1}{T}$

$T = \frac{1}{f} = \frac{1}{\frac{1}{2}} = 2$

$\boxed{T = 2}$

$x(t) = \cos \pi t$





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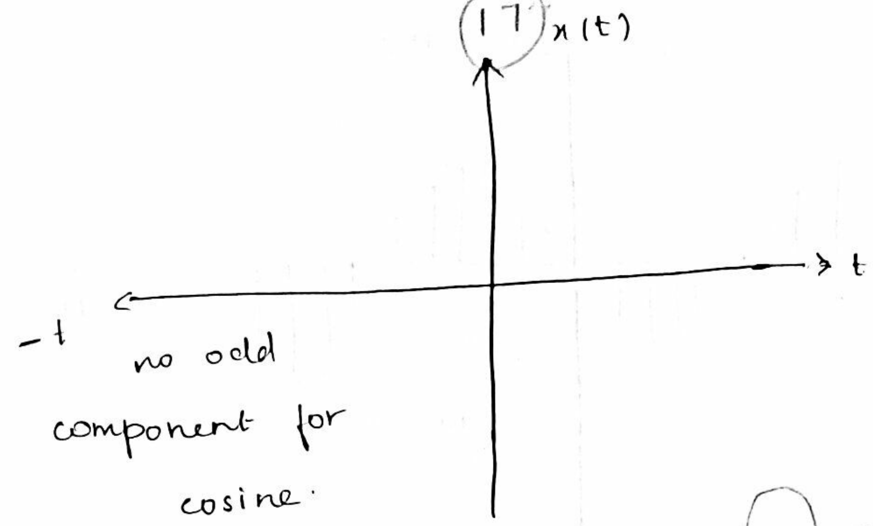
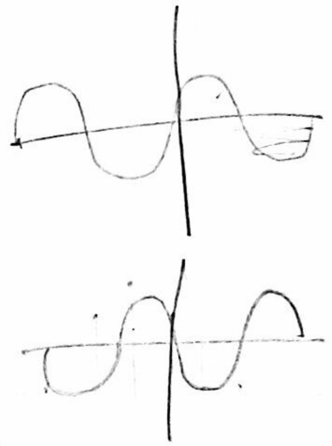


conuda
Im → Imaginary part.
Re → real part

no odd component for cosine.



4) $x(n) = \text{Im}\left[e^{jn\pi/4}\right]$

$e^{jn\pi/4} = \cos\frac{n\pi}{4} + j\sin\frac{n\pi}{4}$

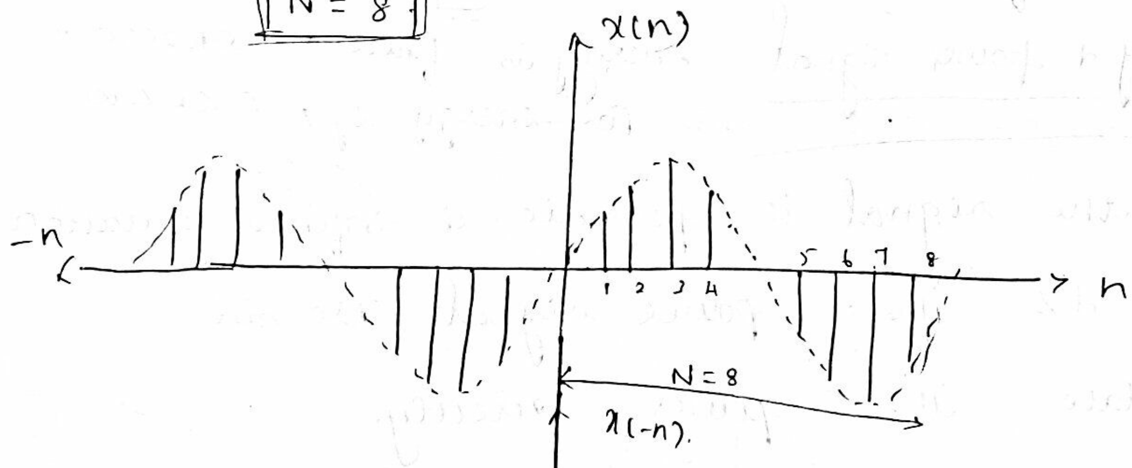
$x(n) = \sin\frac{n\pi}{4}$

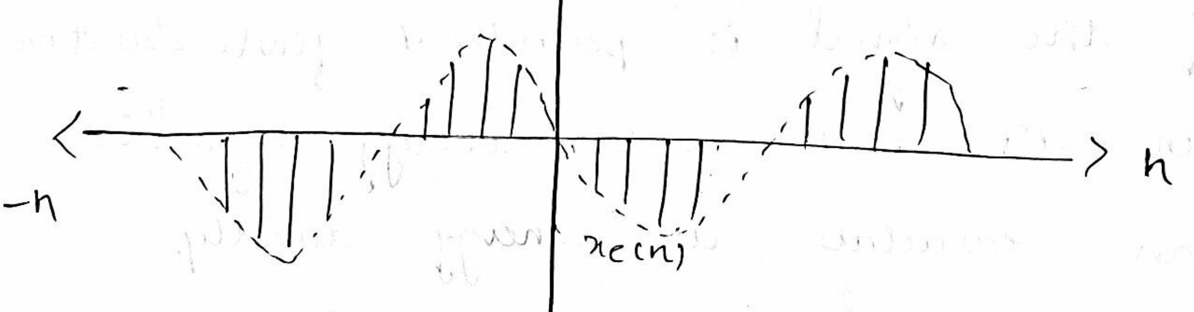
compare with std. form.

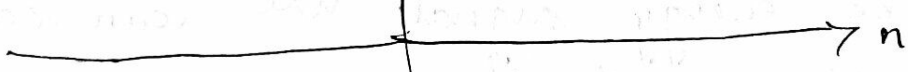
$2\pi f_0 n = \frac{n\pi}{4}$

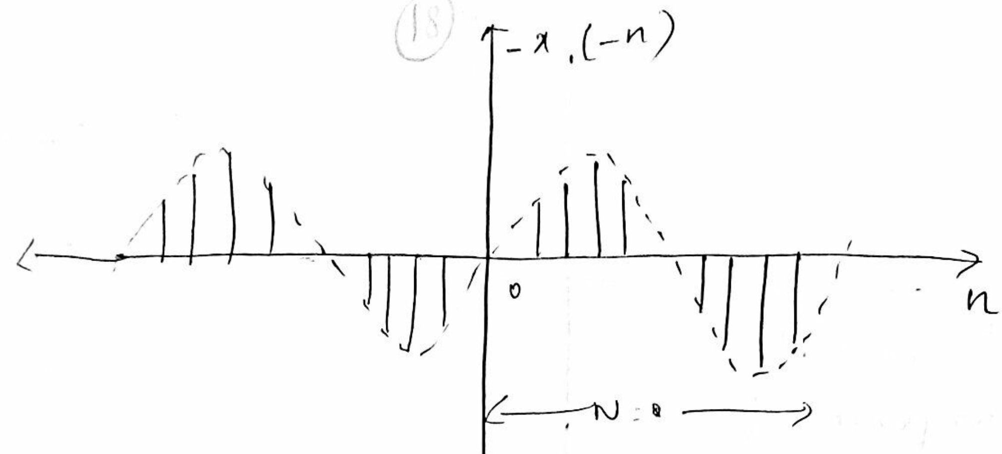
$f_0 = \frac{1}{8} = \frac{k}{N}$

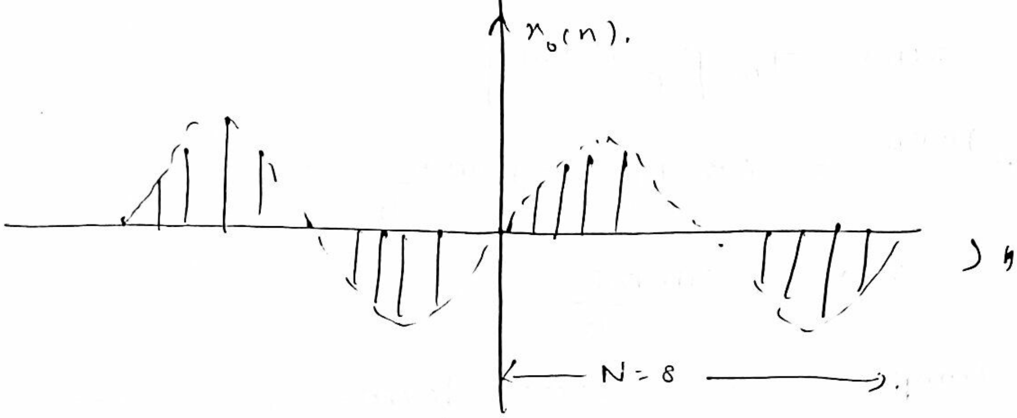
$\boxed{N = 8}$











Power: A signal is said to be power signal if it's normalised power is nonzero and finite. $0 < P < \infty$. Energy sig :- If it's total energy is finite & nonzero...

Energy & power signal

For energy sig, $0 < E < \infty$

- If the signal is periodic & infinite duration then it's ~~pew~~ power signal. We can calculate it's power directly.
- If the signal is periodic & finite duration then it's said to be energy signal. We can calulate it's energy directly.
- If the signal is non periodic, then it can be energy signal. We can calculate it's energy directly.

. Let us consider. (19)

energy sig $x(t)$

$$P = \lim_{T \to \alpha} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt.$$

$$= \lim_{T \to \alpha} \frac{1}{T} \left[\lim_{T \to \alpha} \int_{-T/2}^{T/2} |x(t)|^2 dt \right]$$

$$= \lim_{T \to \alpha} \frac{1}{T} \left[\int_{-\alpha}^{\alpha} |x(t)^2| dt \right]$$

$$P = \lim_{T \to \alpha} \frac{1}{T} [E]$$

$$P = 0 \times E$$

$$\boxed{P = 0}$$

power = 0
energy = α
$\alpha \cdot \alpha = \alpha$
$\frac{1}{\alpha} = 0$

. Energy of the power signal.

$$E = \int_{-\alpha}^{\alpha} |x(t)^2| dt$$

Let us take the limits from $-\frac{T}{2}$ to $\frac{T}{2}$

$$= \lim_{T \to \alpha} \int_{-T/2}^{T/2} |x(t)^2| dt.$$

(multiply & dividing 2.)

$$= \lim_{T \to \alpha} T \cdot \frac{1}{T} \int_{-T/2}^{T/2} |x(t)^2| dt.$$

$$= \lim_{T \to \alpha} T^{-1/2}$$

$$= \lim_{T \to \alpha} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)^2| dt$$

$$E = \lim_{T \to \alpha} T\,[P].$$

$$\boxed{E = \alpha}$$

Determine whether the foll. signals are energy sig or power signals & calculate their energy & power.

If u(n) is alone it's periodic sig.
$\left(\frac{1}{2}\right)(u(n)) \to$ non periodic

1) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$$E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2.$$

$$= \sum_{n=-\alpha}^{\alpha} \left| \left(\frac{1}{2}\right)^n u(n) \right|^2$$

$$= \sum_{n=0}^{\alpha} \left| \left(\frac{1}{2}\right)^n \right|^2$$

$$E = \sum_{n=0}^{\alpha} \left(\frac{1}{4}\right)^n.$$

$$\sum_{n=0}^{\alpha} a^n = \frac{1}{1-a} \qquad |a| < 1$$

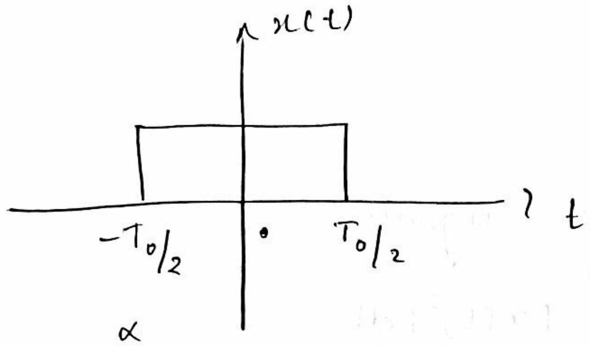
$$= \frac{1}{1-1/4} = \frac{1}{\frac{4-1}{4}} = 1/3/4 = \frac{4}{3}$$

$$\boxed{E = \frac{4}{3}}$$

The given signal is said to be non. periodic & it's energy is $4/3$.

2) Sln :- $x(t) = \text{rect}(t/T_0)$.

$$\text{rect}(t/T_0) = \begin{cases} 1 & \text{for } -T_0/2 \text{ to } T_0/2 \\ 0 & \text{else.} \end{cases}$$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

$$= \int_{-\infty}^{\infty} |\text{rect}(t/T_0)|^2 \, dt$$

$$= \int_{-T_0/2}^{T_0/2} (1)^2 \, dt$$

$$= t \Big]_{-T_0/2}^{T_0/2}$$

upper + low lim lim.
$$= \frac{T_0}{2} + \frac{T_0}{2}$$

$$= \frac{\cancel{2}T_0.}{\cancel{2}}$$

$$\boxed{E. = T_0.}$$

It's a non-periodic signal & it's energy is T_0.

3) $x(t) = \cos^2 \omega_0 t$ (22)

$\int_{-T/2}^{T/2}$ for full cycle $\cos 4\omega_0 t = 0$.

$\boxed{\cos, \sin, u(n) \downarrow \text{periodic.}}$

$\omega = 2\pi f$

Sol:

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= \frac{1}{2} + \frac{1}{2}\cos 2(\omega_0 t)$$

$2\pi f_0 \cancel{t} = \omega_0 \cancel{t}$.

$$f_0 = \frac{\omega_0}{2\pi}$$

It's a periodic signal.

$$P = \lim_{T\to\alpha} 1/T \int_{-T/2}^{T/2} |x(t)^2| \, dt$$

$$= \lim_{T\to\alpha} 1/T \int_{-T/2}^{T/2} |\cos^2 \omega_0 t|^2 \, dt$$

$$= \lim_{T\to\alpha} \frac{1}{T} \int_{-T/2}^{T/2} \left| \frac{1+\cos 2\omega_0 t}{2} \right|^2 dt$$

$(a+b)^2 = a^2 + 2ab + b^2$

$$= \lim_{T\to\alpha} \frac{1}{T} \frac{1}{4} \left| 1 + \cos 2\omega_0 t \right|^2 dt.$$

$$= \lim_{T\to\alpha} \frac{1}{T} \frac{1}{4} \left[1 + \cos^2 2\omega_0 t + 2\cos 2\omega_0 t \right] dt.$$

$$= \lim_{T\to\alpha} \frac{1}{T} \frac{1}{4} \left[1 + \frac{1+\cos 4\omega_0 t}{2} + 2\cos 2\omega_0 t \right] dt$$

(Take $\frac{1}{2}$ out) (LCM)

$$= \lim_{T\to\alpha} \frac{1}{T} \frac{1}{8} \left[2 + 1 + \cos 4\omega_0 t + 4\cos 2\omega_0 t \right] dt$$

$$= \lim_{T\to\alpha} \frac{1}{T} \frac{1}{8} \left[3 + \cos 4\omega_0 t + 4\cos 2\omega_0 t \right] dt.$$

split,

$$= \lim_{T\to\alpha} \frac{1}{T} \int_{-T/2}^{T/2} \frac{3}{8} \, dt + \lim_{T\to\alpha} \frac{1}{8T} \underbrace{\int_{-T/2}^{T/2} \cos 4\omega_0 t \, dt}_{0} + \frac{1}{8T} \underbrace{\int_{-T/2}^{T/2} 4\cos 2\omega_0 t \, dt}_{0}$$

$$= \lim_{T\to\alpha} \frac{1}{T} \frac{3}{8} \left[t \right]_{-T/2}^{T/2}$$

$$= \lim_{T \to \infty} \frac{1}{T} \frac{3}{8} \left[\frac{T}{2} + \frac{T}{2} \right]$$

$$= \lim_{T \to \infty} \frac{1}{\cancel{T}} \frac{3}{8} \left[\frac{2\cancel{T}}{\cancel{2}} \right].$$

$$\boxed{P = \frac{3}{8}}.$$

The gn signal is periodic & has infinite duration & so it's a power signal. & so its the power is $\frac{3}{8}$.

ii) $x(n) = u(n)$.

It's a periodic signal. & infinite duration so it's a power signal.

(0 to N = 1)

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2.$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2$$

$$\frac{N(1+\frac{1}{N})}{N(2+\frac{1}{N})}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)$$

N+1.

(since it's starting from 0 so N+1 if it's only 1 → then N)

$$= \underline{N+1}$$

$$\sum_{n=0}^{N} = 1+1+1+1+1 \; (\ominus(N).$$

(N+1) Times

$$= N+1$$

$\to \frac{1}{2N+1} \; (N+1) \to \infty.$

$$(1)^2 = \frac{1}{2N+1} [N+1]$$

or

Take N common. coz ans must be some value

$$= \lim_{N \to \infty} \frac{N\left(1 + 1/N\right)}{N\left(2 + 1/N\right)}$$

$\frac{1}{N} = \infty$

$\frac{1}{\infty} = 0$

$\frac{N(1+0)}{N(2+0)} = \frac{1}{2}$

$$\boxed{P = \frac{1}{2}}$$

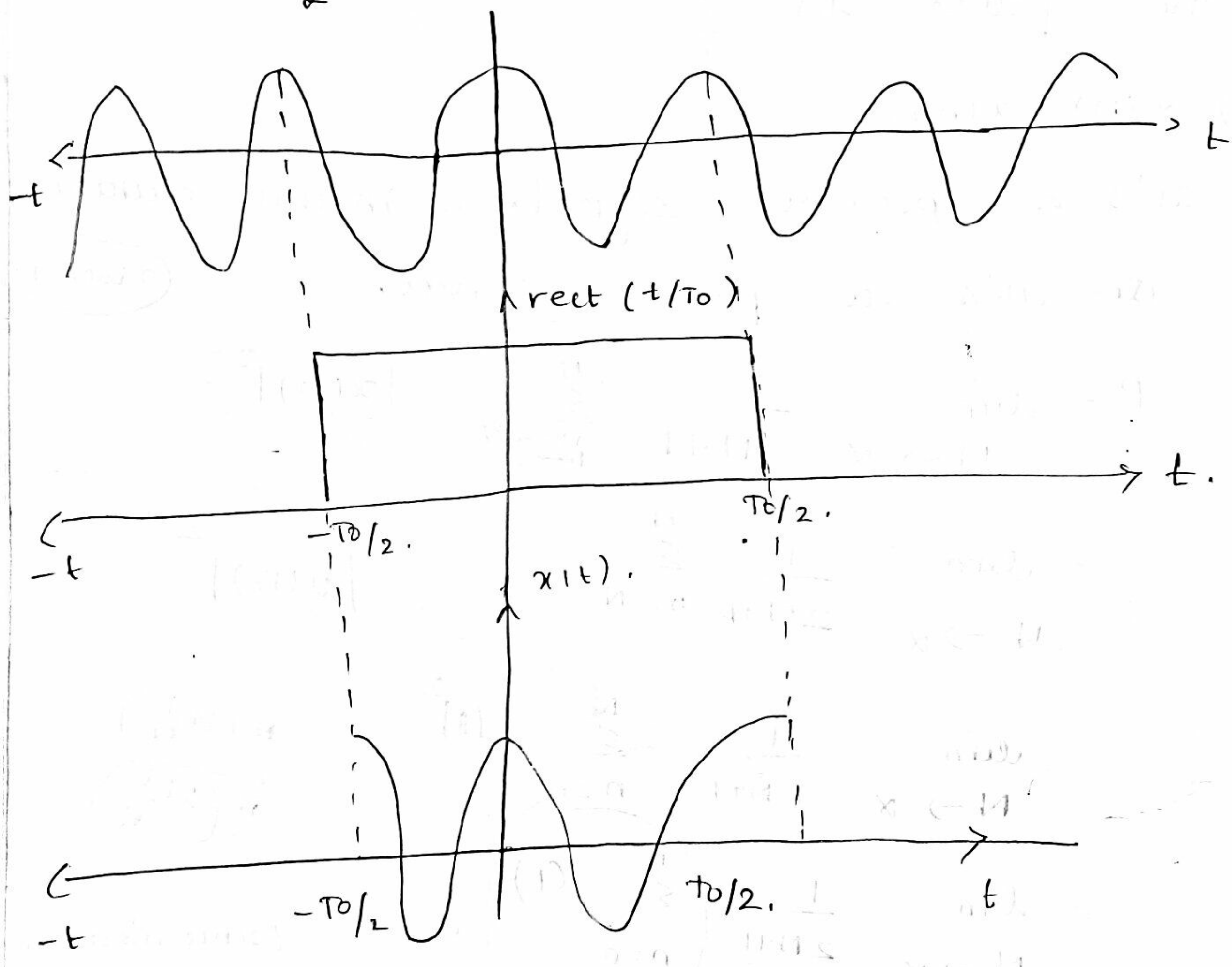
5) $x(t) = \text{rect}(t/T_0) \cos \omega_0 t$

rect → amp → 1

cos → amp → 1

It's a non periodic signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

$$= \int_{-\infty}^{\infty} \left| \text{rect}(t/T_0) \cos \omega t \right|^2 dt$$

$$= \int_{-T_0/2}^{T_0/2} \cancel{(1)^2 \, dt} \; \left|(\cos \omega t)^2\right| \cdot dt$$

$= \frac{1+\cos 2\omega t}{2} = \int_{-T_0/2}^{T_0/2}$ (25) $\Rightarrow \left[\frac{1}{2} + \frac{\cos 2\omega t}{2}\right]_{-T_0/2}^{T_0/2}$

$= \frac{1}{2}\frac{T_0}{\cancel{2}4} + \frac{\cos 2\omega T_0/2}{4} + \frac{-T_0}{4} - \frac{\cos 2\omega t}{4} = \frac{1}{2}\frac{T_0}{2} + \cancel{\frac{T_0}{T_2}} = \cancel{\frac{2T_0}{2}} = T_0$

$e^{-)}$ non periodic

$$\boxed{E = T_0.}$$

It's periodic signal, it's energy signal. it's energy is said to be $\boxed{E = T_0}$

6) Gin : $x(t) = Ae^{-at}u(t)$

It's a non-periodic signal & infinite.

$$E = \int_{-\alpha}^{\alpha} |x(t)|^2 dt.$$

$$= \int_{0}^{\alpha} |Ae^{-at}u(t)|^2 dt.$$

$$= \int_{0}^{\alpha} |Ae^{-at}|^2 dt$$

$$= \int_{0}^{\alpha} A^2 e^{-2at} \cdot dt.$$

$$= A^2 \int_{0}^{\alpha} e^{-2at} \cdot dt.$$

$$= A^2 \left[\frac{e^{-2at}}{-2a}\right]_0^{\alpha}$$

$$= \frac{A^2}{-2a}\left[e^{-\alpha} - e^{0}\right]$$

$$= -\frac{A^2}{2a}(0-1)$$

$$\boxed{E = \frac{A^2}{2a}}$$

$= \frac{1}{2}\int_{-T_0/2}^{T_0/2} 1 \cdot dt + \frac{1}{2}\int_{-T_0/2}^{T_0/2} \cos \omega_0 t \, dt.$ $\Downarrow 0.$

$= \frac{1}{2}[T]_{-T_0/2}^{T_0/2}$

$= \frac{1}{2}\left[\frac{T_0}{2} + \frac{T_0}{2}\right]$

$= \frac{1}{2}\left(\frac{\cancel{2}T_0}{\cancel{2}}\right).$

$$\boxed{E = \frac{T_0}{2}}$$

$e^{-\alpha} = \frac{1}{e^{\alpha}} = \frac{1}{\alpha} = 0$

$e^{0} = 1$

It's a non-periodic signal & infinite, so it's a energy signal, & it's energy is said to be $\boxed{\frac{A^2}{2a}}$

SAMPLING AND ALIASING

Let us define the DT signal as.

$x(nT) = x(n) \quad - \alpha < n < \alpha$

where, $T \Rightarrow$ Sampling period (or) sampling interval

Let sampling frequency be F_s, $\boxed{F_s = 1/T}$

To prove the sampling theorem, we take the CT sinusoidal signal

$x(t) = \sin \Omega t.$

Replace t with nT.

$x(nT) = \sin \Omega nT.$

$\boxed{x(nT) = \sin \omega n.}$

where, $\omega = \Omega T.$

where Ω is the analog range of fre. variables

$-\alpha < \Omega < \alpha.$

ω is digital Range of fre. variables.

$-\pi < \omega < \pi.$

__Sampling Theorem:__

$f_s \geq 2f_m \rightarrow$ Nyquist eqn. $\cancel{\cdot}$

where $f_s \rightarrow$ sampling frequency,

$f_m \rightarrow$ highest frequency present in the signal.

__Aliasing effect:__

ω_s $\omega_s/2$

→ Overlapping of two frequencies.

→ If sampling frequency is ω_s. Then to avoid aliasing i/p should have fre less than $\omega_s/2 \rightarrow$ Nyquist

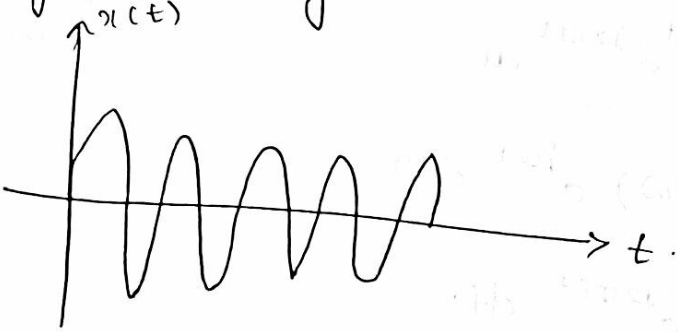
→ Reconstruction of signals:- (27)

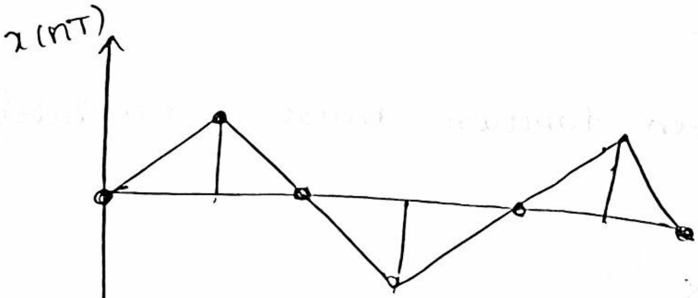
→ If the fre less than the Nyquist Rate $f_s < 2f_m$

→ can't recover all the samples from i/p.

→ $f_s = 2f_m$ & $f_s > 2f_m$, we can recover the original signal.

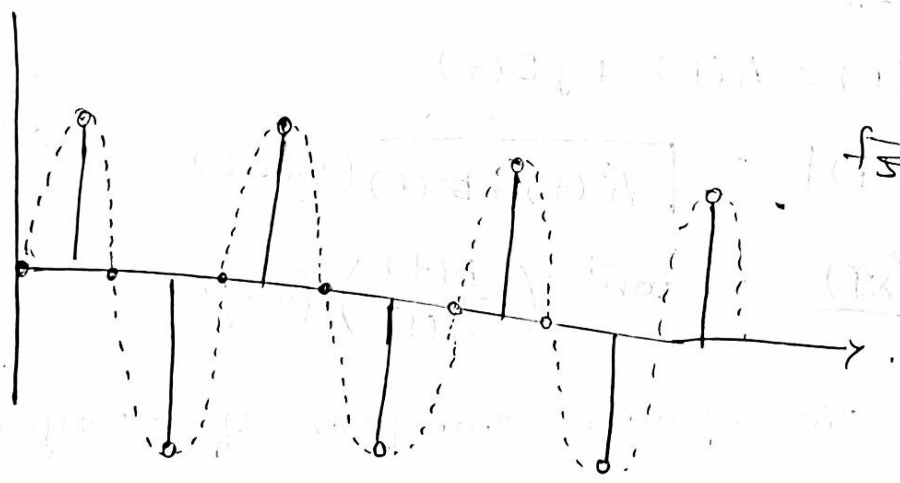
ω = ang. freq





→ $f_s < 2f_m$

$f_s = 2f_m$.



~~$f_s > 2f_m$~~

$f_s > 2f_m$