UNIT 1 BASICS & STATICS OF PARTICLES 12 Hrs.

Introduction - Units and Dimensions - Laws of Mechanics - Vectors - Vectorial representation of forces and moments - Vector operations, Coplanar forces resolution and composition of forces - equilibrium of a particle - Free body diagram - forces in space - equilibrium of a particle in space - equivalent systems of forces - principle of transmissibility - Single equivalent force.

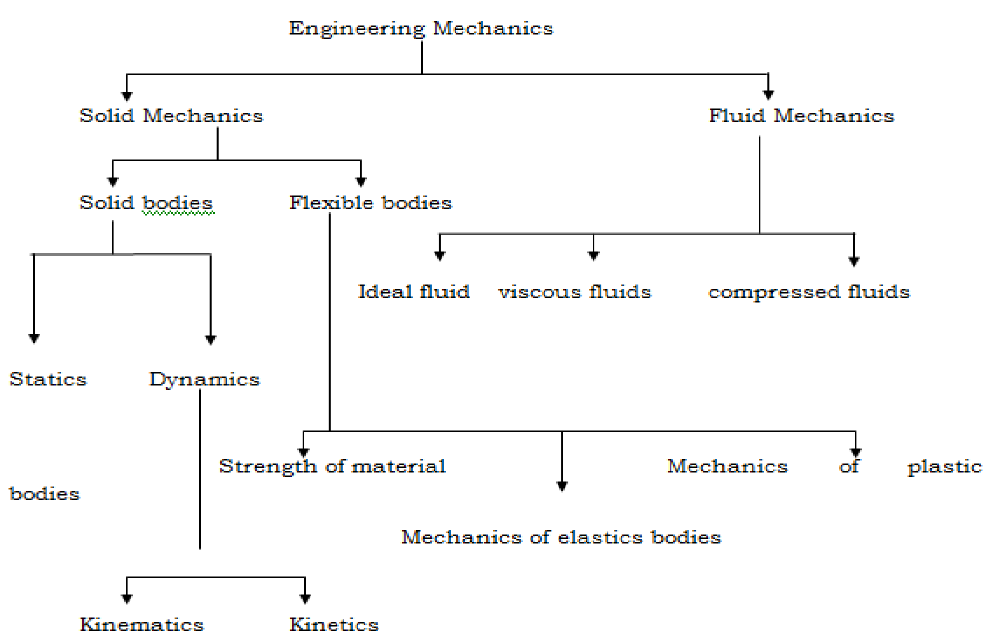
Unit I

BASIC & STATICS OF PARTICLES

INDRODUCTION

Mechanics is that branch of science which deals with the behavior of a body when the body is at root or in motion.

Classification of engineering mechanics on a broad view:



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The engineering mechanics is mainly classified into two branches. They are

1. Statics 2. Dynamics

1. **Statics**: Statics deals with the forces on a body at rest.

2. **Dynamics**: Dynamics deals with the forces acting on a body when the body is in motion.

Dynamics further subdivided in to two sub branches. They are:

(a) **Kinematics**: Deals the motion of a body without considering the forces causing the motion.

(b) **Kinetics**: Deals with the relation between the forces acting on the body and the resulting motion

Rigid body: The rigid body means the body does not deform under the action of force.

Engineering Mechanics deals with Rigid body Dynamics.

Particle: It is an object with its mass concentrated at a point

Force: force is defined as an agency which changes or tends to change the body at rest or in motion . Force is a vector quantity. So we have to specify the magnitude, direction and point of action. The unit of force is Newton.

1 N = 1 kgm/s^2

IMPORTANCE OF MECHANICS TO ENGINEERING:

1) For designing and manufacturing of various mechanical tools and equipments

2) For calculation and estimation of forces of bodies while they are in use.

3) For designing and construing to dams, roads, sheds, structure, building etc.

4) For designing a fabrication of rockets.

Units and dimensions

The following units are used mostly,

1. Centimeter-Gram Second system of unit.
2. Metre-kilogram-second system of units.
3. International system of units.

1. Length is expressed in centimeter, mass in gram and time in second. The unit of force in this system is dyne. Dyne is defined as the force acting on a mass of one gram and producing an acceleration of one centimeter per second square.
2. The length is expressed in metre(m), mass in kilogram and time in second. The unit of force is expressed as kilogram force and is represented as kgf.
3. S.I is abbreviation for "The system International units'. It is also called the international system of units. The length is expressed in metre mass in kilogram and time in second. The unit of force in Newton and is represented N. Newton which is the force acting on a mass of one kilogram and producing as acceleration of one meter per second square. The relation between Newton (N) and dyne is derived as follows,

One Newton = 1 kilogram mass x 1 $meter/S^2$

= 1000gm x100 cm/S^2

=1000x100 x gm x cm/S^2

=10^5 dyne

MKS SYSTEM FORCE Unit is Kgf or kg(wt) or simply Kg. All referring the same.

1 Kgf = 9.81 N

The unit of force, kilo-Newton and mega- Newton is used when the magnitude of forces is very large.

1 kN = 10^3 N

And one Mega- Newton = 10^6 Newton

The large quantities are represented as below,

Kilo(K) = 10^3

Mega(M) = 10^6

Giga(G) = 10^9

Tera(T) = 10^{12}

The small quantities are represented are below,

Milli(m) = 10^{-3}

micro(μ)=10^{-6}

nano(n)=10^{-9}

pico(p)=10^{-12}

Basic Units

Physical quantity	**Notation or unit**	**Dimension or symbols**
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Luminous Intensity	Candela	cd

Supplementary units

Plane angle	Radian	rad
Solid angle	Steridian	sr

Derived units

Acceleration	metre/second^2	m/s^2
Angular velocity	radian/second	rad/s
Angular acceleration	radian/second^2	rad/s^2
Force	Newton	Nm
Work, Energy	Joule	J=Nm=kg m^2/s^2
Torque	Newton metre	Nm
Power	Watt	W= J/s
Pressure	Pascal	Pa=N/m^2
Frequency	Hertz	Hz=s^{-1}

Laws of Mechanics:

Newton's first law of Motion:

Everybody continues in a state of root or uniform motion in a straight line unless it is compelled to change that state by some external force acting on it.

Newton's Second Law of Motion:

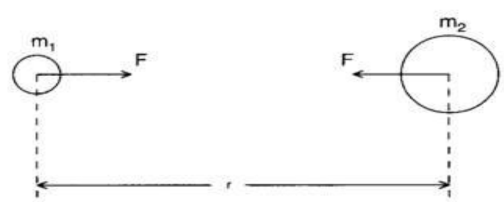
The net external force acting on a body in a direction is directly proportional to the rate of change of momentum in that direction.

Newton's Third law of motion:

To every action there is always equal and opposite reaction.

Law of Gravitation:

It states that two bodies will be attracted towards each other along their connecting line with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between the centers.



According to law of gravitation

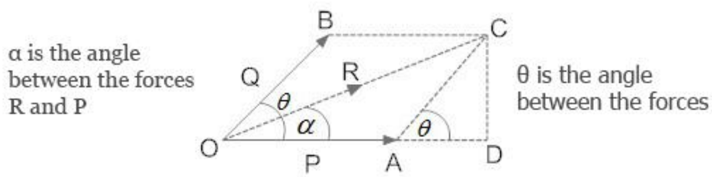
$$F \alpha \frac{m_1 m_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$ where G is the universal gravitational constant

G = 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}

Parallelogram law of forces:

If two forces acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram ram passing through that point.



P and Q are two forces, meet at a point O

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

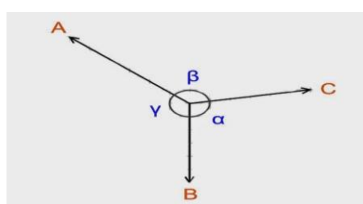
$$\alpha = \tan^{-1} \left[\frac{Q \sin \theta}{P + Q \cos \theta} \right]$$

Triangular law of forces:

If two forces acting at a point are represented by the two sides of a triangle taken in order then their resultant force is represented by the third side taken in opposite order.

Lame's Theorem:

If three forces acting at a point are in equilibrium each force will be proportional to the sine of angle between the other two forces.



Lami's Theorem

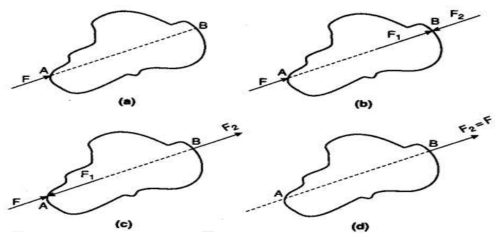
According to Lami's theorem, the particle shall be in equilibrium if

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Principle of transmissibility of forces

It states that " if a force, acting at a point on a rigid body, is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged.

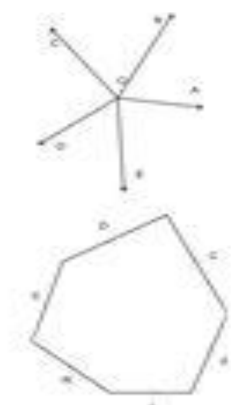
For example a force F is acting at point A on a rigid body along the line of action AB. At point B, apply two equal and opposite forces F_1 and F_2 such that F_1 and F_2 are collinear and equal in magnitude with F. Now, we can transfer $F_1$from B to A such that F and F1 are equal and opposite and accordingly they cancel each other. The net result is force F2 at B. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing the equilibrium of the body.



There is an important observation. If a force is transferred to a different line of action with the force value a couple must be accompanied`

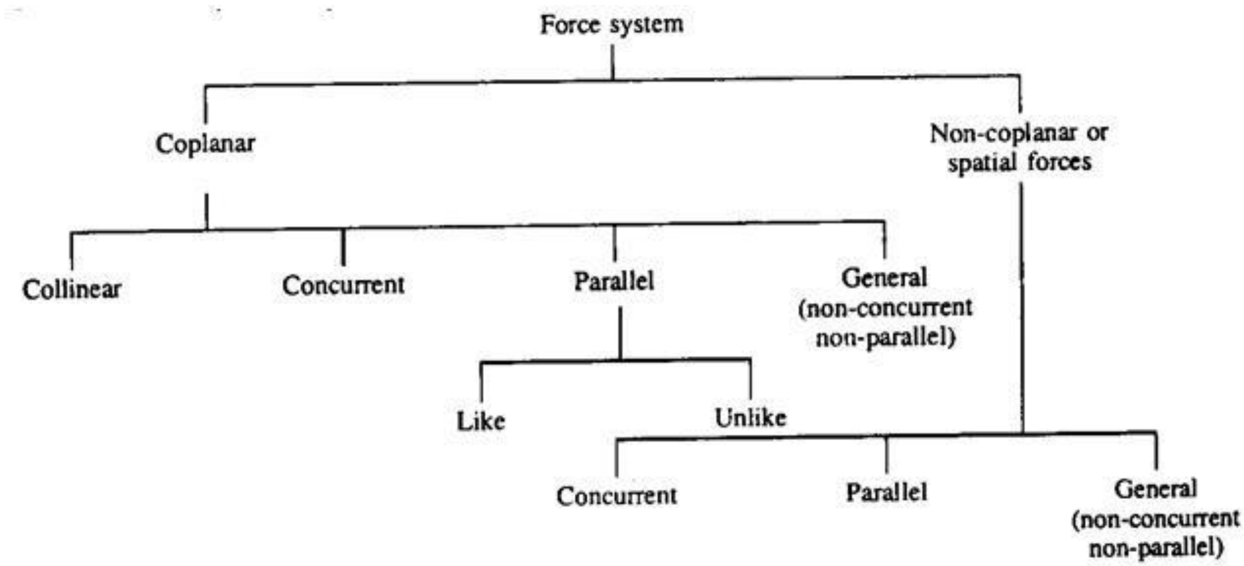
Polygon law of Forces:

If a number of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon taken in order then the resultant of all three forces may be represented in magnitude and direction by the closing side of the polygon taken in opposite order.

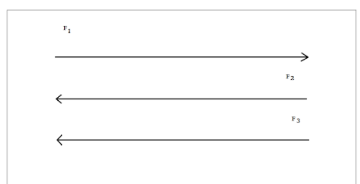


Force and Force system:

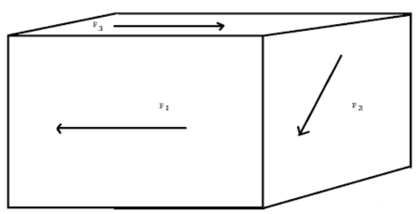
Force is defined as the agency which changes or tends to change the position of rest or motion of the body. The number of forces acting at a point is called force system.



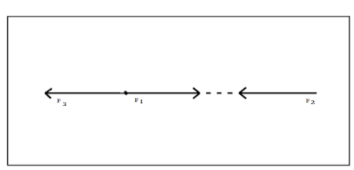
Coplanar force system:- When the lines of action of all forces of a system lie on the same plane then the system is coplanar force system.



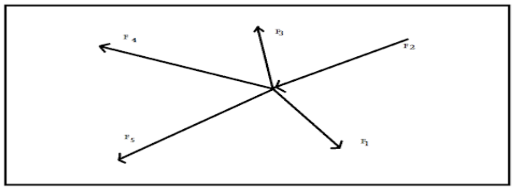
Non coplanar force system:- The system in which the forces do not lie on the same plane is called non coplanar force system.



Collinear forces:- The system in which the forces whose line of action lie on the same line and in same plane is called collinear force system.

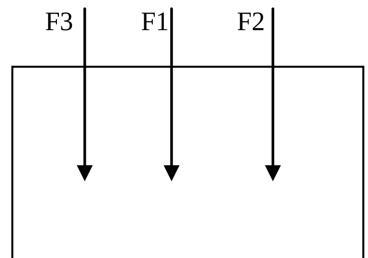


Concurrent force system:- The system in which the forces meet at one point and lie in the same plane is called concurrent force system.

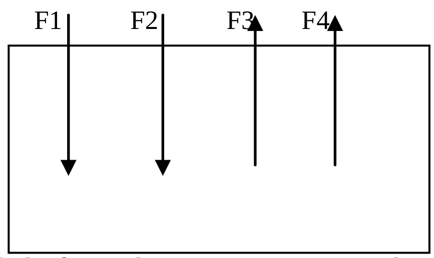


Parallel force system:-

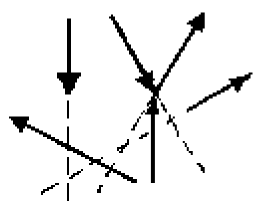
In parallel force system the line of action of forces one parallel to each other.



Parallel forces acting in same direction are called like parallel forces and the parallel forces acting in opposite direction are called unlike parallel force system.



Non concurrent force system:- The system in which the forces do not meet at one point but their lines of action lie on same plane is called non concurrent force system.

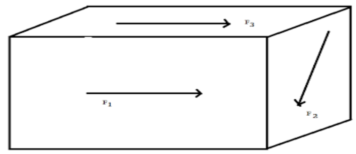


Non-Concurrent
Non-Parallel

Non coplanar force system:-

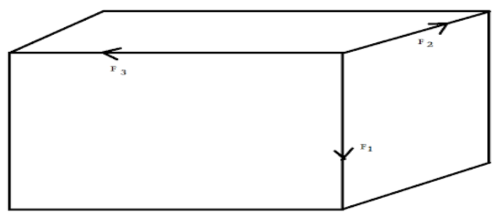
NON COPLANNAR NON CONCURRENT FORCE SYSTEM

The forces which do not meet a point and their lines of action do not lie on the same plane , are called non coplanar non con current force system.



NON COPLANNAR CONCURRENT FORCE SYSTEM

The forces which meet at a point but their lines of action lies on different planes , are known as non coplanar concurrent force system.



Resultant force:

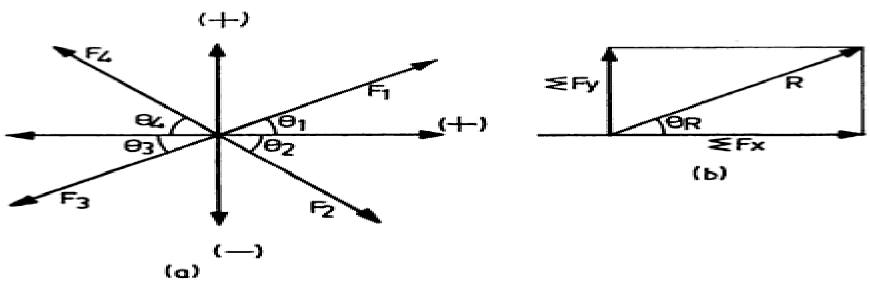
When a number if forces acting on a body are replaced by a single force which has the same effect on the body as that of those number of forces then such a single force is called resultant force.

Composition of forces:

Combining several forces into a single force is called Composition of forces. The single force is called Resultant. The effect by component forces and single force remains the same.

Resolution of a force:

Splitting up of a force into components along the fixed reference axis is called resolution of forces. The effect by single force and component forces remains the same.



Algebraic sum of horizontal components

$\sum Fx = F_1\cos\Theta_1 - F_2\cos\Theta_2 - F_3\cos\Theta_3 + F_4\cos\Theta_4$

Algebraic sum of vertical components

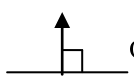
$\sum Fy = F_1\sin\Theta_1 + F_2\sin\Theta_2 - F_3\sin\Theta_3 - F_4\sin\Theta_4$

Resultant $R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

Angle α mode by the resultant with x axis is given by

$\tan\alpha = \sum Fy / \sum Fx$

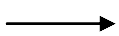
A vertical force has no horizontal component



$\Theta = 90^0$

$Fx = F\cos\Theta$	$Fy = F\sin\Theta$
$= F\cos 90$	$= F\sin 90$
$= 0$	$= F$

A horizontal force has no vertical component



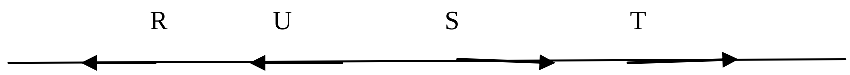
$\Theta = 0^0$

$Fx = F\cos\Theta$	$Fy = F\sin\Theta$
$= F\cos 0$	$= F\sin 0$
$= F$	$= 0$

1. Forces *R*, *S*, *T*, *U* are collinear. Forces *R* and *T* act from left to right. Forces *S* and *U* act from right to left. Magnitudes of the forces *R*, *S*, *T*, *U* are 40 N, 45 N, 50 N and 55 N respectively. Find the resultant of *R*, *S*, *T*, *U*.

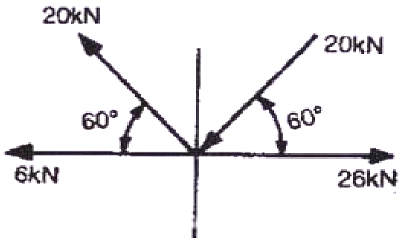
   Given data:

   R=40 N
   S=45 N
   T=50 N
   U=55N



Resultant= -R-U+T=-40-55+45+50=0

2. Find the resultant of the force system shown in Fig



Given data:

F1=20 KN ; Θ1=60°

F2=26 KN ; Θ2=0°

F3=6KN ; Θ3=00°

F4=20KN ; Θ4=60°

Solution:

Resolve the given forces horizontally and calculate the algebraic total of all the horizontal parts or

Σ H=-20cos60°+26cos0°-6cos0°-20cos60°=0

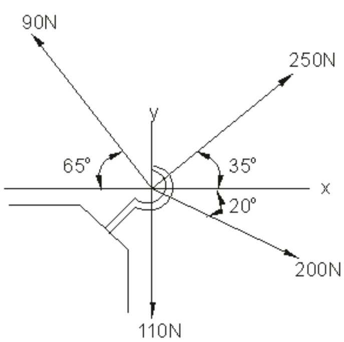
Resolve the given forces vertically and calculate the algebraic total of all the vertical parts or Σ V.

Σ V=-20sin60°±26sin0°±6sin0°+20 sin60=0

R = √((∑H)^2+(V)^2)=0

3.Determine the magnitude and direction of the resultant of forces acting on the hook shown

In fig



Given data:

F1=250 N ; Θ1=35°

F2=200 N ; Θ2=20°

F3=110 N Θ3=90°

$F4=90$ N ; $\Theta 4=65°$

Solution:

Resolve the given forces horizontally and calculate the algebraic total of all the horizontal parts or

$\Sigma H=250\cos35°+200\cos30°\pm110\cos90°-90\cos65°=170.38N$

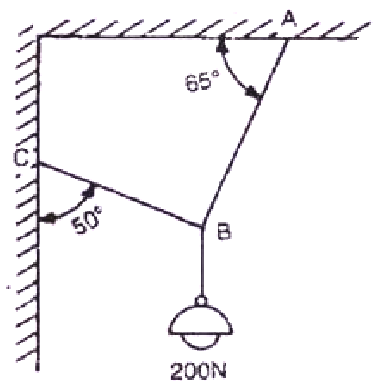
Resolve the given forces vertically and calculate the algebraic total of all the vertical parts or ΣV.

$\Sigma V=250\sin35°-200\sin20°-110\sin90°+90\sin65=46.55N$

$R = \sqrt{(\sum H)^2+(\Sigma V)^2}=176.62N$

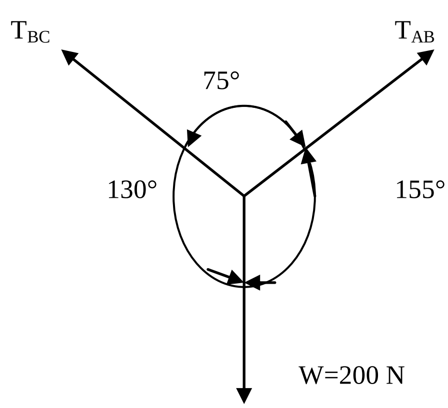
$\Theta=\tan^{-}(\Sigma V/\sum H)=15°$

4.An electric light fixture weighting 200 N is supported as shown in Fig. Determine the tensile forces in the wires and BA and BC



Solution:

Free body diagram(FBD):



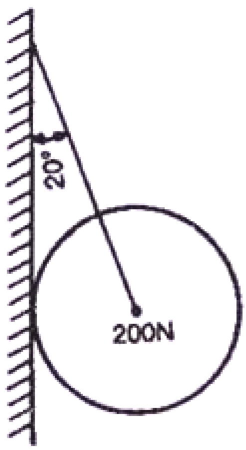
By using lami theorem

$TAB/\sin130°= TBC/\sin155°=200/\sin75°$

$T_{AB=}200/\sin75^\circ * \sin130^\circ=158.61N$

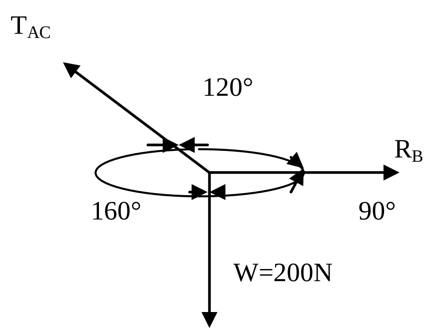
$T_{BC=200/\sin75^\circ * \sin155}{}^\circ=87.50N$

5.A sphere weighing 200 N is tied to a smooth wall by a string as shown in Fig. Find the tension T in the string and reaction R from the wall



Solution:

Free body diagram(FBD):



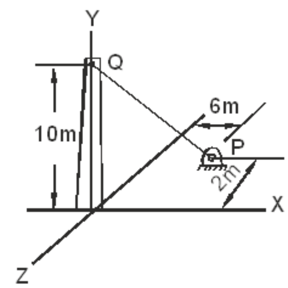
By using lami theorem

$T_{AC}/\sin 90^\circ=R_B/\sin160^\circ=200/\sin120^\circ$

$T_{AC=}200/\mathrm{SIN}120^\circ * \sin 90^\circ=230.94N$

$R_{B=}200/\sin120^\circ * \sin160^\circ=78.98N$

6. A metal guy rope tied to a peg at *P* shown in Fig.12 keeps an electric post in equilibrium. The force in the guy rope is 1.25 kN. Find the components of the force at *P* and the angles of inclination of the force with the three rectangular axes



Given data:

Tension in guy wire is 1250 N

1)Components F_x,F_y,F_z

Consider the tension in guy wire, acting at P. the force I directed from P to Q.

Let it beT_{PQ}

Co ordinates of P $(x_1,y_1,z_1)= (6,0,-2)$

Co ordinates of Q $(x_2,y_2,z_2)= (0,10,0)$

Vector $T_{PQ} = T_{PQ} *\lambda_{PQ}$

$\lambda_{PQ} =PQ/PQ= (X_2-X_1)i+(Y_2-Y_1)j+(Z_2-Z_1)k/\sqrt{}((X_2-X_1)^2+(Y_2-Y_1)^2+(Z_2-Z_1)^2$

$= (0-6)i+(10-0)j+(0-(-2))k/\sqrt{}((6)^2+(10)^2+(2)^2$

$= -6i+10j+2k/11.382$

Vector $T_{PQ} = T_{PQ} *\lambda_{PQ}$

Vector T_{PQ} = 1250 *-6i+10j+2k/11.382=-633i+1056j+211k

from above equation

F_x=633i

F_y=1056j

F_z=211k

2)Angle Θ_X, Θ_Y Θ_Z

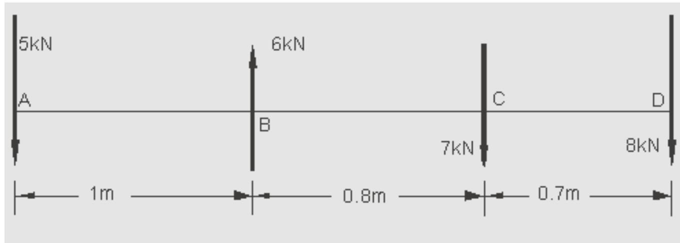
we know force vector F =-633i+1056j+211k

Θ_X =COS¯[Fx/F]=633/1250=59°

Θ_Y =COS¯[FY/F]=1056/1250=32°

Θ_Z = COS¯[Fz/F]=211/1250=80°

7.Find the resultant of the force system shown in Fig.13 and its position from *A*. (Force in '*kN*' and distance in '*m*')



Solution:

Magnitude of resultant of force R =-5+6-7-8 =-14KN

(-) sign shows that the resultant forces acts in the negative direction i.e., downwards

Location of the resultant force:

Lt us locate the resulting force with reference to the point A. Hence , taking the moments of given forces and adding,

Algebraic sum of moments about A,

$\sum M_A =$ -(6*1)+(7*1.8)+(8*2.5)=26.6 KN-m(clockwise)

Hence acting downwards and to have clock moment, resultant force is taken on the right side of A

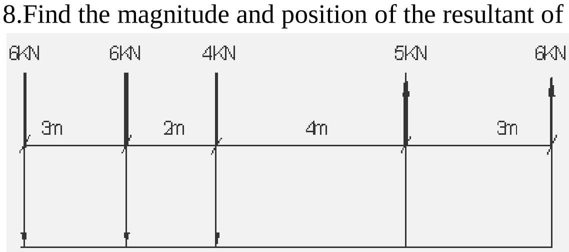
Let resultant force acts at a distance of "x" m from A

$\sum M_A =$ R*x

26.6= 14*x

x =1.9m

8.Find the magnitude and position of the resultant of the system of forces shown in Fig.



Magnitude of resultant of force R =-6-6-4+5+6=-5N

(-) sign how that the resultant forces acts in the negative direction i.e., downwards

Location of the resultant force:

Lt us locate the resulting force with reference to the point A. Hence , taking the moments of given forces and adding,

Algebraic sum of moments about A,

$\sum M_A =$ (6*3)+(4*5)-(5*9)-(6*12)= 38-117=-79 KN-m(Counter clockwise)

Hence acting downwards and to have counter clock moment, resultant force is taken on the right side of A

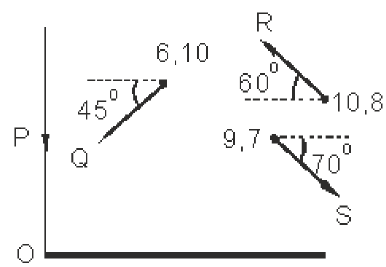
Let resultant force acts at a distance of "x" m from A

$\sum M_A =$ R*x

79= 5*x

x =15.9 m

9. A system of four forces *P*, *Q*, *R* and *S* of magnitude 5 kN, 8 kN, 6 kN and 4 kN respectively acting on a body are shown in rectangular coordinates as shown in Fig.2. Find the moments of the forces about the origin *O*. Also find the resultant moment of the forces about *O*. The distances are in metres.

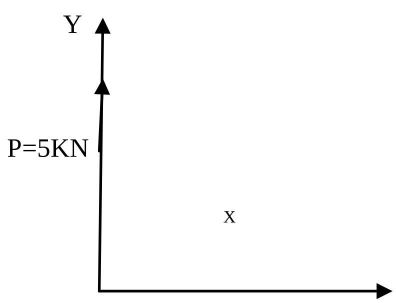


Solution:

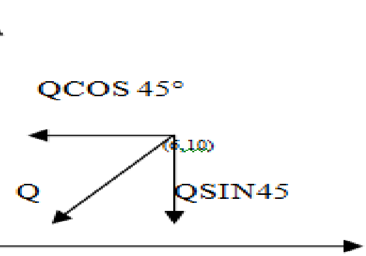
Moment Of P:

Moment of force P about the origin, M_P = Force*perpendicular distance from origin

= 5*0=0



Moment Of Q :



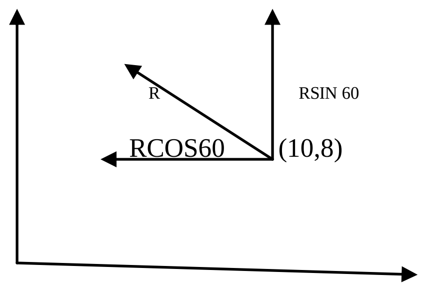
Moment of force Q about the origin, M_Q = sum of the moments of components of the force Q about the origin

= -Q COS 45°*10 +Q SIN 45°*6

=-8COS 45°*10 +8 SIN 45°*6

=-42.025N+40.84=-1.185 KN-m(counter clock wise moment)

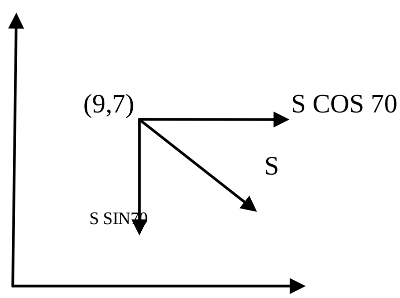
Moment of force R about the origin, M_R = sum of the moments of components of the force R about the origin



= -6 COS 60°*10 –6 SIN 60°*8

=--57.14-14.63=-71.77 KN-m(counter clock wise moment)

Moment of force S about the origin, M_S = sum of the moments of components of the force R about the origin

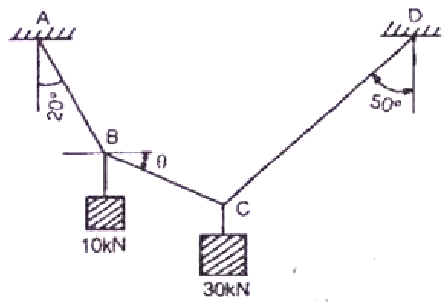


= S COS 70°*7 +S SIN 45°*9

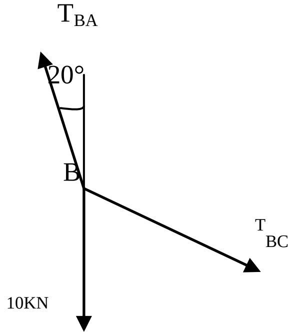
=4COS 70*7 +4 SIN 70°*9

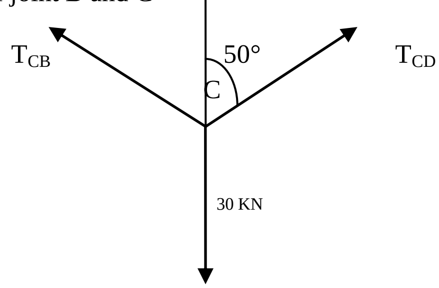
=17.73+27.86=45.59 KN-m(clock wise moment)

10. A wire is fixed at two points A and D as shown in Fig.20.Two weights 10 kN and 30 kN are is $20^0$and that of CD is 50^0 to the vertical. Determine the tension in the segments AB, BC and CD of the wire and also the inclination of BC to the vertical. Take Θ = 30 ^{0}



Free body diagram of joint B and C





At joint B

$\sum H = -T_{BA}Cos70+T_{BC}Cos\ \Theta = 0$

$T_{BC}Cos\ \Theta = T_{BA}Cos70$

$T_{BA} = T_{BC}Cos\ \Theta\ /\ Cos70$

$T_{BA}\ = 2.92\ T_{BC}Cos\ \Theta$ -------------(1)

$\sum V = -T_{BA}Sin70 - 10 - T_{BC}Sin\ \Theta = 0$

$T_{BA}Sin70 - T_{BC}Sin\ \Theta = 10$ -----------(2)

Substitute T_{BA} we get

$2.92\ T_{BC}Cos\ \Theta\ Sin70 - T_{BC}Sin\ \Theta = 10$

$2.74\ \ Cos\ \Theta\ T_{BC\ -}\ T_{BC}Sin\ \Theta = 10$

$T_{BC}(2.74\ Cos\ \Theta - \ Sin\ \Theta) = 10$ --------------(3)

$T_{BC\ =}\ \ 10/2.74\ Cos\ \Theta - \ Sin\ \Theta$

At Joint C

$\sum H = -T_{CD}Cos40 - T_{CB}Cos\Theta = 0$

$T_{CD}Cos40 = T_{CB}Cos\Theta$

$T_{CD} = 1.30\ T_{CB}Cos\Theta$ ------------(4)

$\sum V = -\ T_{CD}Sin40 + T_{CB}Sin\Theta - 30 = 0$

Substitute T_{CD} we get

$1.30\ T_{CB}Cos\Theta\ Sin40 + T_{CB}Sin\Theta = 30$

$0.835\ \ T_{CB}Cos\Theta + T_{CB}Sin\Theta = 30$

$T_{CB}(0.835\ Cos\Theta + \ Sin\Theta) = 30$ --------------(5)

$T_{CB} = \ \ 30/0.835\ Cos\Theta + \ Sin\Theta$

Divide (3) / (5)

$T_{BC}(2.74\ Cos\Theta - \ Sin\Theta) = 10\ /\ T_{CB}(0.835\ Cos\Theta + \ Sin\Theta) = 30$

$(2.74\ Cos\Theta - \ Sin\Theta)\ 30 = (0.835\ Cos\Theta + \ Sin\Theta)\ 10$

$82.2\ Cos\Theta - \ 30Sin\Theta = 8.35\ Cos\Theta + 10\ Sin\Theta$

$73.85\ Cos\Theta = 40\ Sin\Theta$

$Sin\Theta\ /\ Cos\Theta = 1.846$

$tan\Theta = 1.846$

$\Theta = 61.54^0$

$T_{BC} = 10/\ (2.74\ Cos\Theta - \ Sin\Theta)$

$T_{BC} \ = 23.44$ N

$T_{BA} \ = 2.92\ T_{BC}\ Cos\Theta$

= 32.61 KN

$T_{CD} \ = 1.30\ T_{CB}\ Cos\Theta$

= 14.52 KN