

THERMAL SYSTEMS

SMEX1044

UNIT I :AIR STANDARD CYCLES

DEFINITION OF A CYCLE

A cycle is defined as a repeated series of operations occurring in a certain order. It may be repeated by repeating the processes in the same order. The cycle may be of imaginary perfect engine or actual engine. The former is called ideal cycle and the latter actual cycle. In ideal cycle all accidental heat losses are prevented and the working substance is assumed to behave like a perfect working substance.

AIR STANDARD EFFICIENCY

To compare the effects of different cycles, it is of paramount importance that the effect of the calorific value of the fuel is altogether eliminated and this can be achieved by considering air (which is assumed to behave as a perfect gas) as the working substance in the engine cylinder. The efficiency of engine using air as the working medium is known as an "Air standard efficiency". This efficiency is often called ideal efficiency. The actual efficiency of a cycle is always less than the air-standard efficiency of that cycle under ideal conditions. This is taken into account by introducing a new term "Relative efficiency" which is defined as the ratio of Actual thermal efficiency to Air standard efficiency.

The analysis of all air standard cycles is based upon the following assumptions:

1. The gas in the engine cylinder is a perfect gas i.e., it obeys the gas laws and has constant specific heats.
2. The physical constants of the gas in the cylinder are the same as those of air at moderate temperatures i.e., the molecular weight of cylinder gas is 29. $C_p = 1.005 \text{ kJ/kg-K}$, $C_v = 0.718 \text{ kJ/kg-K}$.
3. The compression and expansion processes are adiabatic and they take place without internal friction, i.e., these processes are isentropic.
4. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with cylinder at appropriate points during the process.
5. The cycle is considered closed with the same 'air' always remaining in the cylinder to repeat the cycle.

CONSTANT VOLUME OR OTTO CYCLE

This cycle is so named as it was conceived by 'Otto'. On this cycle, petrol, gas and many types of oil engines work. It is the standard of comparison for internal combustion engines.

Figs. 1 (a) and (b) shows the theoretical p-V diagram and T-s diagrams of this cycle respectively.

- The point 1 represents that cylinder is full of air with volume V_1 , pressure P_1 and absolute temperature T_1 .
- Line 1-2 represents the adiabatic compression of air due to which P_1 , V_1 and T_1 change to P_2 , V_2 and T_2 respectively.
- Line 2-3 shows the supply of heat to the air at constant volume so that P_2 and T_2 change to P_3 and T_3 (V_3 being the same as V_2).
- Line 3-4 represents the adiabatic expansion of the air. During expansion P_3 , V_3 and T_3 change to a final value of P_4 , V_4 or V_1 and T_4 , respectively.
- Line 4-1 shows the rejection of heat by air at constant volume till original state (point 1) reaches.

Consider 1 kg of air (working substance):

$$\text{Heat supplied at constant volume} = c_v(T_3 - T_2).$$

$$\text{Heat rejected at constant volume} = c_v(T_4 - T_1).$$

$$\begin{aligned} \text{But, work done} &= \text{Heat supplied} - \text{Heat rejected} \\ &= c_v(T_3 - T_2) - c_v(T_4 - T_1) \end{aligned}$$

$$\begin{aligned} \therefore \text{Efficiency} &= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v(T_3 - T_2) - c_v(T_4 - T_1)}{c_v(T_3 - T_2)} \\ &= 1 - \frac{T_4 - T_1}{T_3 - T_2} \end{aligned}$$

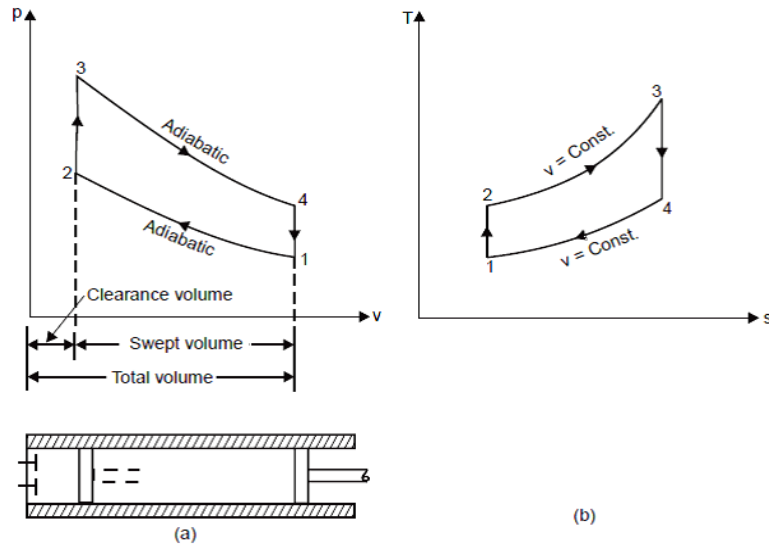


Fig 1

Let compression ratio, $r_c (= r) = \frac{v_1}{v_2}$

and expansion ratio, $r_e (= r) = \frac{v_4}{v_3}$

(These two ratios are same in this cycle)

As $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$

Then, $T_2 = T_1 \cdot (r)^{\gamma-1}$

Similarly, $\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1}$

or $T_3 = T_4 \cdot (r)^{\gamma-1}$

Inserting the values of T_2 and T_3 in equation (i), we get

$$\eta_{otto} = 1 - \frac{T_4 - T_1}{T_4 \cdot (r)^{\gamma-1} - T_1 \cdot (r)^{\gamma-1}} = 1 - \frac{T_4 - T_1}{r^{\gamma-1}(T_4 - T_1)}$$

$$= 1 - \frac{1}{(r)^{\gamma-1}}$$

This expression is known as the air standard efficiency of the Otto cycle. It is clear from the above expression that efficiency increases with the increase in the value of r , which means we can have maximum efficiency by increasing r to a considerable extent, but due to practical difficulties its value is limited to about 8. The net work done per kg in the Otto cycle can also be expressed in terms of p, v . If p is expressed in bar i.e., 10^5 N/m^2 , then work done

$$W = \left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} \right) \times 10^2 \text{ kJ} \quad \dots(13.4)$$

Also $\frac{p_3}{p_4} = r^\gamma = \frac{p_2}{p_1}$

$\therefore \frac{p_3}{p_2} = \frac{p_4}{p_1} = r_p$

where r_p stands for *pressure ratio*.

and

$$v_1 = r v_2 = v_4 = r v_3 \quad \left[\because \frac{v_1}{v_2} = \frac{v_4}{v_3} = r \right]$$

$$\begin{aligned} \therefore W &= \frac{1}{\gamma - 1} \left[p_4 v_4 \left(\frac{p_3 v_3}{p_4 v_4} - 1 \right) - p_1 v_1 \left(\frac{p_2 v_2}{p_1 v_1} - 1 \right) \right] \\ &= \frac{1}{\gamma - 1} \left[p_4 v_4 \left(\frac{p_3}{p_4 r} - 1 \right) - p_1 v_1 \left(\frac{p_2}{p_1 r} - 1 \right) \right] \\ &= \frac{v_1}{\gamma - 1} \left[p_4 (r^{\gamma-1} - 1) - p_1 (r^{\gamma-1} - 1) \right] \end{aligned}$$

$$= \frac{v_1}{\gamma - 1} [(r^{\gamma-1} - 1)(p_4 - p_1)]$$

$$= \frac{p_1 v_1}{\gamma - 1} [(r^{\gamma-1} - 1)(r_p - 1)]$$

Mean effective pressure (p_m) is given by :

$$p_m = \left[\left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} \right) \div (v_1 - v_2) \right] \text{ bar}$$

Also

$$p_m = \frac{\left[\frac{p_1 v_1}{\gamma - 1} (r^{\gamma-1} - 1)(r_p - 1) \right]}{(v_1 - v_2)}$$

$$= \frac{\frac{p_1 v_1}{\gamma - 1} [(r^{\gamma-1} - 1)(r_p - 1)]}{v_1 - \frac{v_1}{r}}$$

$$= \frac{\frac{p_1 v_1}{\gamma - 1} [(r^{\gamma-1} - 1)(r_p - 1)]}{v_1 \left(\frac{r-1}{r} \right)}$$

i.e.,

$$p_m = \frac{p_1 r [(r^{\gamma-1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)}$$

MEP may be thought of as the average **pressure** acting on a piston during different portions of its cycle. It is the ratio of the work done to stroke volume of the cycle

CONSTANT PRESSURE OR DIESEL CYCLE

This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle in that heat is supplied at constant pressure instead of at constant volume. Fig. (a and b) shows the p-v and T-s diagrams of this cycle respectively.

This cycle comprises of the following operations:

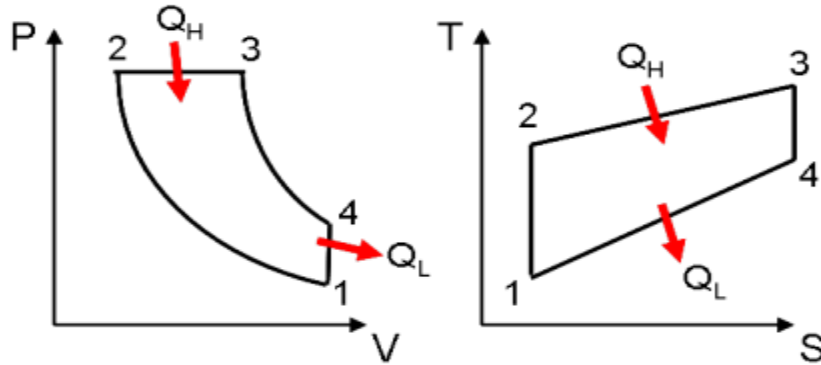


Fig.3. P-V and T-S diagrams of Ideal Diesel Cycle

- (i) 1-2.....Adiabatic compression.
- (ii) 2-3.....Addition of heat at constant pressure.
- (iii) 3-4.....Adiabatic expansion.
- (iv) 4-1.....Rejection of heat at constant volume.

Point 1 represents that the cylinder is full of air. Let P_1 , V_1 and T_1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (i.e., $pV^\gamma = \text{constant}$) till the values become P_2 , V_2 and T_2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure. During this addition of heat let volume increases from V_2 to V_3 and temperature T_2 to T_3 , corresponding to point 3. This point (3) is called the point of cut-off. The air then expands adiabatically to the conditions P_4 , V_4 and T_4 respectively corresponding to point 4. Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state.

Consider 1 kg of air.

Heat supplied at constant pressure = $c_p(T_3 - T_2)$

Heat rejected at constant volume = $c_v(T_4 - T_1)$

Work done = Heat supplied - heat rejected

$$= c_p(T_3 - T_2) - c_v(T_4 - T_1)$$

$$\begin{aligned} \therefore \eta_{\text{diesel}} &= \frac{\text{Work done}}{\text{Heat supplied}} \\ &= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)} \\ &= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \quad \dots(i) \left[\because \frac{c_p}{c_v} = \gamma \right] \end{aligned}$$

Let compression ratio, $r = \frac{v_1}{v_2}$, and cut-off ratio, $\rho = \frac{v_3}{v_2}$ i.e., $\frac{\text{Volume at cut-off}}{\text{Clearance volume}}$

Now, during *adiabatic compression* 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\gamma-1} = (r)^{\gamma-1} \quad \text{or} \quad T_2 = T_1 \cdot (r)^{\gamma-1}$$

During *constant pressure process* 2-3,

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho \quad \text{or} \quad T_3 = \rho \cdot T_2 = \rho \cdot T_1 \cdot (r)^{\gamma-1}$$

During *adiabatic expansion* 3-4

$$\begin{aligned} \frac{T_3}{T_4} &= \left(\frac{v_4}{v_3} \right)^{\gamma-1} \\ &= \left(\frac{r}{\rho} \right)^{\gamma-1} \quad \left(\because \frac{v_4}{v_3} = \frac{v_1}{v_3} = \frac{v_1}{v_2} \times \frac{v_2}{v_3} = \frac{r}{\rho} \right) \end{aligned}$$

$$\therefore T_4 = \frac{T_3}{\left(\frac{r}{\rho} \right)^{\gamma-1}} = \frac{\rho \cdot T_1 (r)^{\gamma-1}}{\left(\frac{r}{\rho} \right)^{\gamma-1}} = T_1 \cdot \rho^\gamma$$

By inserting values of T_2 , T_3 and T_4 in eqn. (i), we get

$$\eta_{\text{diesel}} = 1 - \frac{(T_1 \cdot \rho^\gamma - T_1)}{\gamma(\rho \cdot T_1 \cdot (r)^{\gamma-1} - T_1 \cdot (r)^{\gamma-1})} = 1 - \frac{(\rho^\gamma - 1)}{\gamma(r)^{\gamma-1}(\rho - 1)}$$

or

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}} \left[\frac{\rho^\gamma - 1}{\rho - 1} \right] \quad \dots(13.7)$$

It may be observed that eqn. (13.7) for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because $r > 1$. Hence for a given compression ratio, the Otto cycle is more efficient.

The *net work* for diesel cycle can be expressed in terms of pv as follows :

$$\begin{aligned}
 W &= p_2(v_3 - v_2) + \frac{p_3v_3 - p_4v_4}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1} \\
 &= p_2(\rho v_2 - v_2) + \frac{p_3\rho v_2 - p_4rv_2}{\gamma - 1} - \frac{p_2v_2 - p_1rv_2}{\gamma - 1} \\
 &\quad \left[\begin{array}{l} \because \frac{v_3}{v_2} = \rho \quad \therefore v_3 = \rho v_2 \text{ and } \frac{v_1}{v_2} = r \quad \therefore v_1 = rv_2 \\ \text{But } v_4 = v_1 \quad \therefore v_4 = rv_2 \end{array} \right] \\
 &= p_2v_2(\rho - 1) + \frac{p_3\rho v_2 - p_4rv_2}{\gamma - 1} - \frac{p_2v_2 - p_1rv_2}{\gamma - 1} \\
 &= \frac{v_2[p_2(\rho - 1)(\gamma - 1) + p_3\rho - p_4r - (p_2 - p_1r)]}{\gamma - 1} \\
 &= \frac{v_2 \left[p_2(\rho - 1)(\gamma - 1) + p_3 \left(\rho - \frac{p_4r}{p_3} \right) - p_2 \left(1 - \frac{p_1r}{p_2} \right) \right]}{\gamma - 1} \\
 &= \frac{p_2v_2[(\rho - 1)(\gamma - 1) + \rho - \rho^\gamma \cdot r^{1-\gamma} - (1 - r^{1-\gamma})]}{\gamma - 1} \\
 &\quad \left[\because \frac{p_4}{p_3} = \left(\frac{v_3}{v_4} \right)^\gamma = \left(\frac{\rho}{r} \right)^\gamma = \rho^\gamma r^{-\gamma} \right] \\
 &= \frac{p_1v_1r^{\gamma-1}[(\rho - 1)(\gamma - 1) + \rho - \rho^\gamma r^{1-\gamma} - (1 - r^{1-\gamma})]}{\gamma - 1} \\
 &\quad \left[\because \frac{p_2}{p_1} = \left(\frac{v_1}{v_2} \right)^\gamma \text{ or } p_2 = p_1 \cdot r^\gamma \text{ and } \frac{v_1}{v_2} = r \text{ or } v_2 = v_1 r^{-1} \right] \\
 &= \frac{p_1v_1r^{\gamma-1}[\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma - 1)} \quad \dots(13.8)
 \end{aligned}$$

Mean effective pressure p_m is given by :

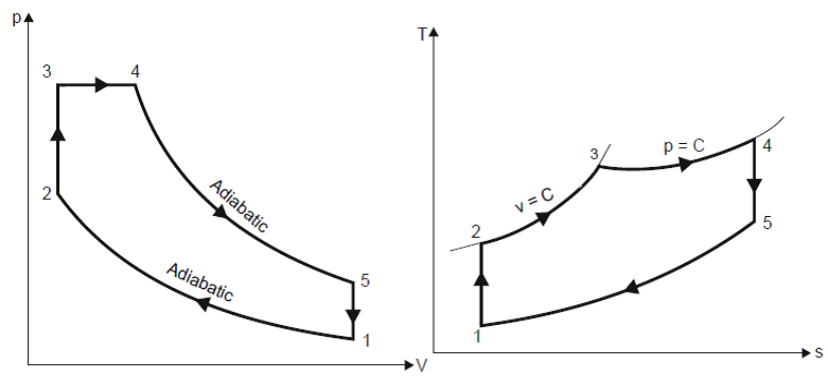
$$\begin{aligned}
 p_m &= \frac{p_1v_1r^{\gamma-1}[\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma - 1)v_1 \left(\frac{r - 1}{r} \right)} \\
 \text{or} \quad p_m &= \frac{p_1r^\gamma[\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)} \quad \dots(13.9)
 \end{aligned}$$

DUAL COMBUSTION CYCLE

This cycle (also called the limited pressure cycle or mixed cycle) is a combination of Otto and Diesel cycles, in a way, that heat is added partly at constant volume and partly at constant pressure ; the advantage of which is that more time is available to fuel (which is injected into the engine cylinder before the end of compression stroke) for combustion. Because of lagging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines.

The dual combustion cycle (Fig 3) consists of the following operations :

- (i) 1-2—Adiabatic compression
- (ii) 2-3—Addition of heat at constant volume
- (iii) 3-4—Addition of heat at constant pressure
- (iv) 4-5—Adiabatic expansion
- (v) 5-1—Rejection of heat at constant volume.



Consider 1 kg of air.

Total heat supplied = Heat supplied during the operation 2-3
+ heat supplied during the operation 3-4

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3)$$

Heat rejected during operation 5-1 = $c_v(T_5 - T_1)$

Work done = Heat supplied - heat rejected

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)$$

$$\eta_{\text{dual}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \quad \dots(i) \quad \left(\because \gamma = \frac{c_p}{c_v} \right)$$

Compression ratio, $r = \frac{v_1}{v_2}$

During *adiabatic compression process* 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\gamma-1} = (r)^{\gamma-1} \quad \dots(ii)$$

During *constant volume heating process*,

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

or $\frac{T_3}{T_2} = \frac{p_3}{p_2} = \beta$, where β is known as **pressure or explosion ratio**.

or $T_2 = \frac{T_3}{\beta} \quad \dots(iii)$

During *adiabatic expansion process*,

$$\begin{aligned} \frac{T_4}{T_5} &= \left(\frac{v_5}{v_4} \right)^{\gamma-1} \\ &= \left(\frac{r}{\rho} \right)^{\gamma-1} \quad \dots(iv) \end{aligned}$$

$$\left(\because \frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{v_1}{v_2} \times \frac{v_2}{v_4} = \frac{v_1}{v_2} \times \frac{v_3}{v_4} = \frac{r}{\rho}, \rho \text{ being the cut-off ratio} \right)$$

During *constant pressure heating process*,

$$\begin{aligned} \frac{v_3}{T_3} &= \frac{v_4}{T_4} \\ T_4 &= T_3 \frac{v_4}{v_3} = \rho T_3 \quad \dots(v) \end{aligned}$$

Putting the value of T_4 in the eqn. (iv), we get

$$\frac{\rho T_3}{T_5} = \left(\frac{r}{\rho} \right)^{\gamma-1} \quad \text{or} \quad T_5 = \rho \cdot T_3 \cdot \left(\frac{\rho}{r} \right)^{\gamma-1}$$

Putting the value of T_2 in eqn. (ii), we get

$$\frac{\frac{T_3}{\beta}}{T_1} = (r)^{\gamma-1}$$

$$T_1 = \frac{T_3}{\beta} \cdot \frac{1}{(r)^{\gamma-1}}$$

Now inserting the values of T_1 , T_2 , T_4 and T_5 in eqn. (i), we get

$$\eta_{\text{dual}} = 1 - \frac{\left[\rho \cdot T_3 \left(\frac{\rho}{r} \right)^{\gamma-1} - \frac{T_3}{\beta} \cdot \frac{1}{(r)^{\gamma-1}} \right]}{\left[\left(T_3 - \frac{T_3}{\beta} \right) + \gamma (\rho T_3 - T_3) \right]} = 1 - \frac{\frac{1}{(r)^{\gamma-1}} \left(\rho^\gamma - \frac{1}{\beta} \right)}{\left(1 - \frac{1}{\beta} \right) + \gamma (\rho - 1)}$$

i.e.,

$$\eta_{\text{dual}} = 1 - \frac{1}{(r)^{\gamma-1}} \cdot \frac{(\beta \cdot \rho^\gamma - 1)}{[(\beta - 1) + \beta \gamma (\rho - 1)]} \quad \dots (13.10)$$

Work done is given by,

$$W = p_3(v_4 - v_3) + \frac{p_4 v_4 - p_5 v_5}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

$$= p_3 v_3 (\rho - 1) + \frac{(p_4 \rho v_3 - p_5 r v_3) - (p_2 v_3 - p_1 r v_3)}{\gamma - 1}$$

$$= \frac{p_3 v_3 (\rho - 1)(\gamma - 1) + p_4 v_3 \left(\rho - \frac{p_5}{p_4} r \right) - p_2 v_3 \left(1 - \frac{p_1}{p_2} r \right)}{\gamma - 1}$$

Also

$$\frac{p_5}{p_4} = \left(\frac{v_4}{v_5} \right)^\gamma = \left(\frac{\rho}{r} \right)^\gamma \quad \text{and} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2} \right)^\gamma = r^\gamma$$

also,

$$p_3 = p_4, \quad v_2 = v_3, \quad v_5 = v_1$$

∴

$$W = \frac{v_3 [p_3 (\rho - 1)(\gamma - 1) + p_3 (\rho - \rho^\gamma r^{1-\gamma}) - p_2 (1 - r^{1-\gamma})]}{(\gamma - 1)}$$

$$= \frac{p_2 v_2 [\beta (\rho - 1)(\gamma - 1) + \beta (\rho - \rho^\gamma r^{1-\gamma}) - (1 - r^{1-\gamma})]}{(\gamma - 1)}$$

$$= \frac{p_1 (r)^\gamma v_1 / r [\beta \gamma (\rho - 1) + (\beta - 1) - r^{1-\gamma} (\beta \rho^\gamma - 1)]}{\gamma - 1}$$

$$= \frac{p_1 v_1 r^{\gamma-1} [\beta \gamma (\rho - 1) + (\beta - 1) - r^{\gamma-1} (\beta \rho^\gamma - 1)]}{\gamma - 1} \quad \dots (13.11)$$

Mean effective pressure (p_m) is given by,

$$p_m = \frac{W}{v_1 - v_2} = \frac{W}{v_1 \left(\frac{r-1}{r} \right)} = \frac{p_1 v_1 [r^{1-\gamma} \beta \gamma (\rho - 1) + (\beta - 1) - r^{1-\gamma} (\beta \rho^\gamma - 1)]}{(\gamma - 1) v_1 \left(\frac{r-1}{r} \right)}$$

$$p_m = \frac{p_1 (r)^\gamma [\beta (\rho - 1) + (\beta - 1) - r^{1-\gamma} (\beta \rho^\gamma - 1)]}{(\gamma - 1)(r - 1)} \quad \dots (13.12)$$

COMPARISON OF OTTO, DIESEL AND DUAL COMBUSTION CYCLES

Following are the important variable factors which are used as a basis for comparison of the cycles:

- Compression ratio.
- Maximum pressure
- Heat supplied
- Heat rejected
- Net work

Some of the above mentioned variables are fixed when the performance of Otto, Diesel and dual combustion cycles is to be compared.

13.7.1. Efficiency Versus Compression Ratio

Fig. 13.26 shows the comparison for the air standard efficiencies of the Otto, Diesel and Dual combustion cycles at various compression ratios and with given cut-off ratio for the Diesel and Dual combustion cycles. It is evident from the Fig. 13.26 that the air standard efficiencies increase with the increase in the compression ratio. For a given compression ratio Otto cycle is the most efficient while the Diesel cycle is the least efficient. ($\eta_{otto} > \eta_{dual} > \eta_{diesel}$).

Note. The maximum compression ratio for the petrol engine is limited by detonation. In their respective ratio ranges, the Diesel cycle is more efficient than the Otto cycle.

13.7.2. For the Same Compression Ratio and the Same Heat Input

A comparison of the cycles (Otto, Diesel and Dual) on the p-v and T-s diagrams for the same compression ratio and heat supplied is shown in the Fig. 13.27.

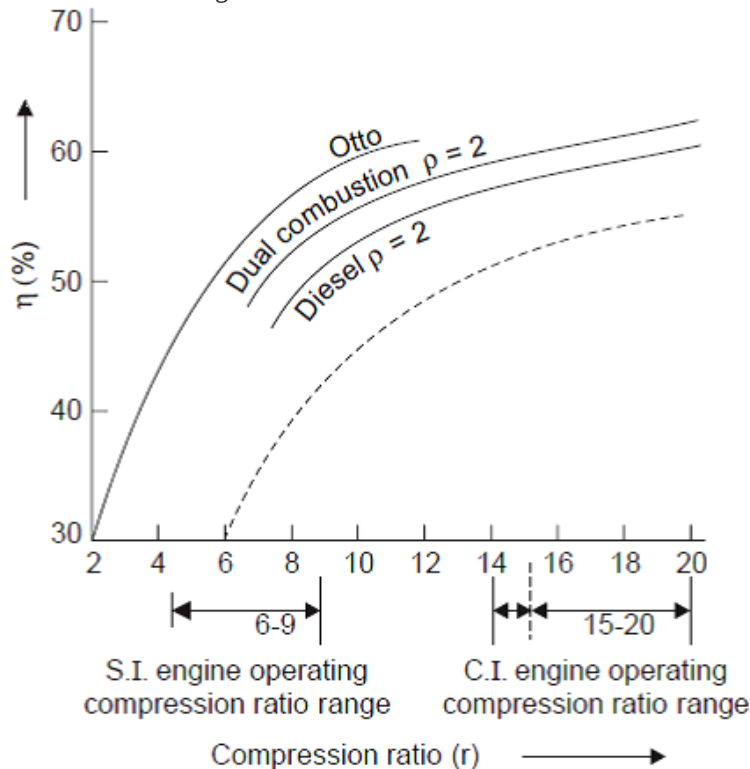


Fig. 13.26. Comparison of efficiency at various compression ratios.

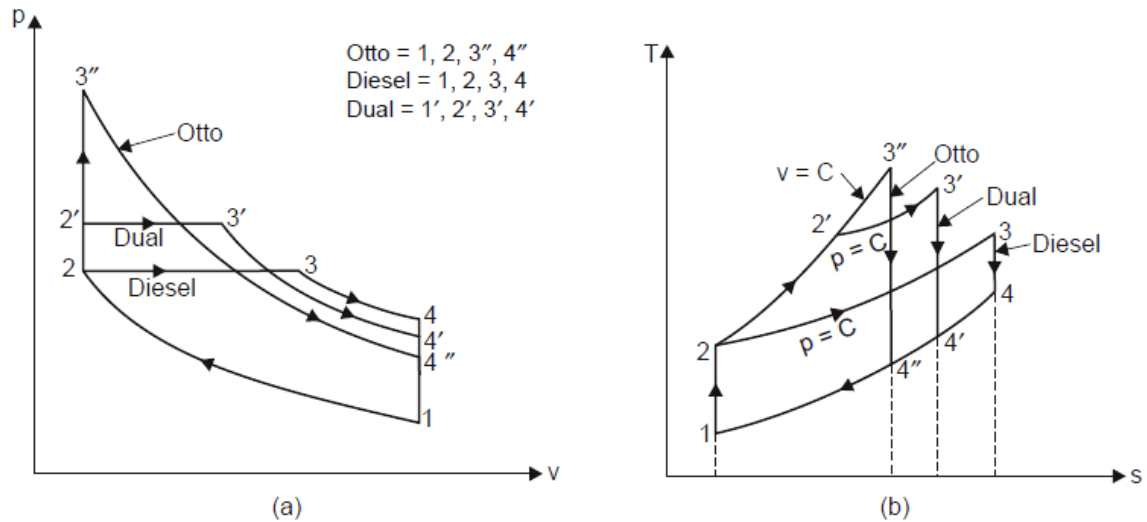


Fig. 13.27. (a) p - v diagram, (b) T - s diagram.

We know that,

$$\eta = 1 - \frac{\text{Heat rejected}}{\text{Heat supplied}} \quad \dots(13.13)$$

Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T - s diagram. As is evident from the eqn. (13.13) the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.

$$\eta_{\text{otto}} > \eta_{\text{dual}} > \eta_{\text{diesel}}$$

For Constant Maximum Pressure and Heat Supplied

Fig. 13.28 shows the Otto and Diesel cycles on p - v and T - s diagrams for constant maximum pressure and heat input respectively.

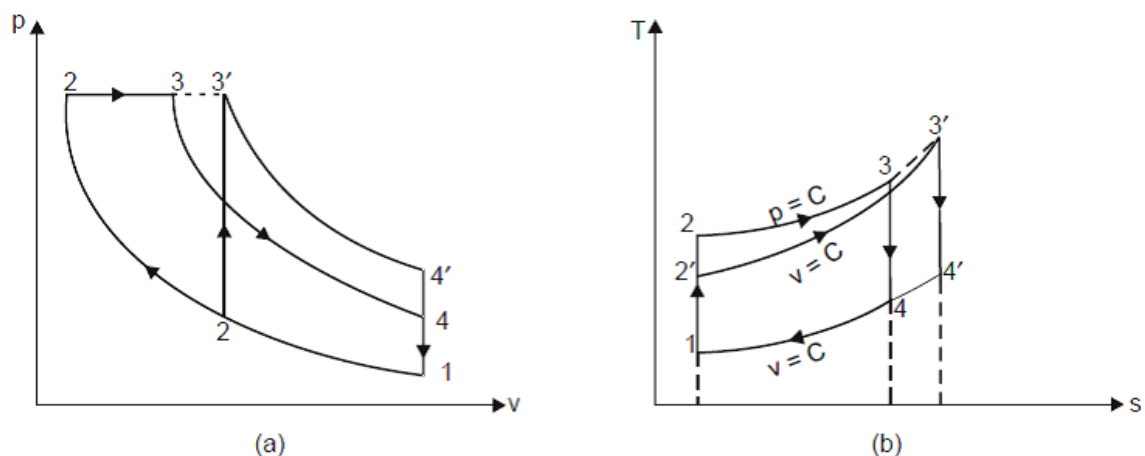


Fig. 13.28. (a) p - v diagram, (b) T - s diagram.

— For the maximum pressure the points 3 and 3' must lie on a constant pressure line.

— On T-s diagram the heat rejected from the Diesel cycle is represented by the area under the line 4 to 1 and this area is less than the Otto cycle area under the curve 4' to 1 ; hence the Diesel cycle is more efficient than the Otto cycle for the condition of maximum pressure and heat supplied.

1. A six cylinder petrol engine has a compression ratio of 5:1. The clearance volume of each cylinder is 110CC. It operates on the four stroke constant volume cycle and the indicated efficiency ratio referred to air standard efficiency is 0.56. At the speed of 2400 rpm. It consumes 10kg of fuel per hour. The calorific value of fuel is 44000KJ/kg. Determine the average indicated mean effective pressure.

Given data:

$$\begin{aligned} r &= 5 \\ V_c &= 110\text{CC} \\ \eta_{\text{relative}} &= 0.56 \\ N &= 2400\text{rpm} \\ m_f &= 10\text{kg} \\ &= 10/3600 \text{ kg/s} \\ C_v &= 44000\text{kJ/kg} \\ Z &= 6 \end{aligned}$$

Solution:

Compression ratio:

$$r = V_s + V_c / V_c \rightarrow 5 = V_s + 110 / 110 \rightarrow V_s = 440\text{CC} = 44 \times 10^{-6} \text{m}^3$$

Air standard efficiency:

$$\eta = 1 - 1 / (r^{\gamma-1}) = 47.47\% \quad (\gamma = 1.4)$$

Relative efficiency:

$$\begin{aligned} \eta_{\text{relative}} &= \eta_{\text{actual}} / \eta_{\text{air-standard}} \rightarrow 0.56 = \eta_{\text{actual}} / 47.47 \\ \eta_{\text{actual}} &= 26.58\% \end{aligned}$$

Actual efficiency = work output / heat input

$$0.2658 = W / m_f C_v \rightarrow W = 0.2658 \times 10/3600 \times 44000$$

$$W = 32.49\text{kw.}$$

The net work output:

$$W = P_m \times V_s \times N/60 \times Z \rightarrow 32.49 \times 10^3 = P_m \times 440 \times 10^{-6} \times 2400/60 \times 6$$

$$P_m = 6.15 \text{ bar}$$

2. One kg of air taken through, a) Otto cycle, b) Diesel cycle initially the air is at 1 bar and 290K. The compression ratio for both cycles is 12 and heat addition is 1.9 MJ in each cycle. Calculate the air standard efficiency and mean effective pressure for both the cycles.

Given data:

$$\begin{aligned} P_1 &= 1 \text{ bar} = 100\text{KN/m}^2 \\ T_1 &= 290\text{K} \\ r &= 12 \\ Q_s &= 1.9\text{MJ} = 1900\text{KJ} \end{aligned}$$

Solution:

a) Otto cycle:

For process 1-2: isentropic compression:

$$P_2/P_1 = (V_1/V_2)^\gamma \rightarrow P_2 = P_1 \times r^\gamma$$

$$P_2 = 3242.3\text{kN/m}^2$$

$$T_2/T_1 = (V_1/V_2)^{\gamma-1} \rightarrow T_2 = T_1 \times (V_1/V_2)^{\gamma-1} = 290 \times (12)^{1.4-1}$$

$$T_2 = 783.55\text{K}$$

Heat supplied:

$$Q = m \times C_v (T_3 - T_2)$$

$$1900 = 1 \times 0.718 \times (T_3 - 783.55)$$

$$T_3 = 3429.79\text{K}$$

For process 2-3 : constant volume process

$$P_3/P_2 = T_3/T_2 \rightarrow P_3 = P_2 \times T_3/T_2 = 3242.3 \times 3429.79/783.55$$

$$P = 14196.7 \text{ KN/m}^2$$

Air standard efficiency:

$$\eta = 1 - 1/(r^{\gamma-1}) = 0.6298$$

$$\eta = 62.98\%$$

$$\text{Pressure ratio, } K = P_3/P_2 = 14196.7/3242.3 = 4.378$$

Mean effective pressure,

$$P_m = p_1 r (k-1/\gamma - 1) (r^{\gamma-1} - 1/r - 1) = 100 \times 12 (4.378 - 1/1.4) [(12^{1.4-1} - 1/12 - 1)]$$

$$P_m = 1567.93 \text{ KN/m}^2$$

b) Diesel cycle:

Consider 1-2 isentropic compression process:

$$T_2 = (V_1/V_2)^{\gamma-1} \times T_1 = (r)^{\gamma-1} \times T_1 = (12)^{1.4-1} \times 290$$

$$T_2 = 783.56 \text{ K}$$

Consider 2-3 constant pressure heat addition:

$$Q_s = C_p (T_3 - T_2)$$

$$1.9 \times 10^3 = 1.005 \times (T_3 - 783.56)$$

$$T_3 = 2674 \text{ K}$$

Cut off ratio:

$$\rho = V_3/V_2 = T_3/T_2 = 2674/783.56 = 3.413$$

Air standard efficiency:

$$\eta = 1 - 1/r^{\gamma-1} \{ \rho^{\gamma-1} - 1 \} = 1 - 1/1.4(12)^{1.4-1} \{ 3.413^{1.4-1} - 1 \}$$

$$= 49.86\%$$

Mean effective pressure:

$$P_m = P_1 r^{\gamma} [\gamma(\rho-1) - r^{\gamma-1} (\rho^{\gamma-1} - 1) / (\gamma-1) (r-1)]$$

$$100 \times (12)^{1.4} [1.4 (3.413-1) - (12)^{1.4-1} (3.413^{1.4-1} - 1)] / (1.4-1) (12-1)$$

$$P_m = 1241 \text{ KN/m}^2$$

3. An air standard dual cycle has a compression ratio of 16 and compression begins at 1 bar and 50°C. The maximum pressure is 70 bar. The heat transformed to air at constant pressure is equal to heat transferred at constant volume. Find the temperature at all cardinal points, cycle efficiency and mean effective pressure take $C_p = 1.005 \text{ KJ/kgK}$, $C_v = 0.718 \text{ KJ/kgK}$.

Given data:

$$r = 16$$

$$P_1 = 1 \text{ bar}$$

$$T_1 = 50^\circ\text{C} = 323 \text{ K}$$

$$P_3 = 70 \text{ bar}$$

$$Q_{s1} = Q_{s2}$$

$$C_p = 1.005 \text{ kJ/kgK}$$

$$C_v = 0.718 \text{ kJ/kgK}$$

Solution:

Specific volume,

$$V_1 = RT_1/P_1 = 287 \times 323/1 \times 10^5$$

$$V_1 = 0.92701 \text{ m}^3/\text{kg}$$

$$V_2 = 0.05794 \text{ m}^3/\text{kg}$$

1-2 isentropic compression process:

$$P_2 = (r)^{\gamma} \times P_1 = (16)^{1.4} \times 1 = 48.5 \text{ bar}$$

$$T_2 = (r)^{\gamma-1} \times T_1 = (16)^{1.4-1} \times 323$$

$$T_2 = 979 \text{ K}$$

2-3 constant volume heat addition process:

$$T_3 = (P_3/P_2) \times T_2 = 70/48.5 \times 979$$

$$T_3 = 1413\text{K}$$

$$Q_{s1} = C_v (T_3 - T_2); 0.718(1413 - 979)$$

$$Q_{s1} = 311.612\text{KJ/kg}$$

3-4 constant pressure heat addition:

$$Q_{s1} = Q_{s2} = C_p (T_4 - T_3)$$

$$311.62 = 1.005 (T_4 - 1413)$$

$$T_4 = 1723\text{K}$$

$$V_4 = T_4/T_3 \times V_3 = 1723/1413 \times 0.05794$$

$$V_4 = 0.070652\text{m}^3/\text{kg}$$

Expansion ratio:

$$r_e = V_4/V_1 = 0.070652/0.92701 = 0.06215$$

4-5 isentropic expansion process:

$$P_5 = (r) \times P_4 = (0.076215)^{1.4} \times 70$$

$$P_5 = 1.9063\text{ bar}$$

$$T_5 = (r)^{\gamma-1} \times T_4$$

$$= (0.076215)^{1.4-1} \times 1723$$

$$= 567\text{ K}$$

Cut off ratio,

$$\rho = V_4/V_3$$

$$= 0.00652/0.05744$$

$$\rho = 1.2194$$

$$\text{Pressure ratio (K)} = (P_3/P_2) = (70/48.5)$$

$$K = 1.4433$$

The cycle efficiency:

$$\eta = 1 - 1/(r)^{\gamma-1} [(k\rho^{\gamma}-1)/(k-1 + K\rho(p-1))]$$

$$= 66.34\%$$

Net heat supplied to the cycle:

$$Q_s = Q_{s1} + Q_{s2}$$

$$= 311.612 + 311.612$$

$$= 623.224\text{ KJ/kg}$$

The mean effective pressure:

$$P_m = W/(V_1 - V_2) = 413.45/(0.92701 - 0.05794)$$

$$P_m = 4.75\text{ bar}$$

4. In a air standard dual cycle, the compression ratio is 12 and the maximum pressure and temperature of the cycle are 1 bar and 300K. Heat added during constant pressure process is upto 3% of the stroke. taking diameter as 25cm and stroke as 30cm, determine.

1) The pressure and temperature at the end of compression

2) The thermal efficiency

3) The mean effective pressure

Take, $C_p = 1.005\text{KJ/kgK}$ $C_v = 0.718\text{KJ/kgK}$, $\gamma = 1.4$

Given data:

$$P_1 = 1\text{ bar}$$

$$r = 12$$

$$T_1 = 300\text{K}$$

$$K = 3\% \text{ of } V_s = 0.03V_s$$

$$P_3 = 70\text{ bar}$$

$$D = 25\text{ cm}$$

$$L = 30\text{cm}$$

Solution :

Specific volumes,:

$$V_1 RT_1/P_1 = 287 \times 300/1 \times 10^5$$

$$= 0.861\text{ m}^3/\text{kg}$$

$$V_3 = V_2 = V_1/r = 0.861/12$$

$$= 0.07175\text{m}^3/\text{kg}$$

$$V_4 - V_3 = 0.03 (V_1 - V_2)$$

$$V_4 = 0.0954275 \text{ m}^3/\text{kg}$$

Cut off ratio:

$$\rho = V_4 / V_3 = 0.054275 / 0.07175$$

$$\rho = 1.33$$

1-2 isentropic compression process:

$$P_2 = (r)^\gamma \times P_1 = (12)^{1.4} \times 1$$

$$= 32.423 \text{ bar}$$

$$V_2 = (r)^{\gamma-1} \times T_1 = (12)^{1.4-1} \times 300$$

$$T_2 = 810.57 \text{ K}$$

2-3 constant volume heat addition process

$$P_3/T_3 = P_2/T_2$$

$$T_3 = (P_3/P_2) \times T_2 = (70 / 32.423) \times 810.57$$

$$T_3 = 1750 \text{ K}$$

3-4 constant pressure heat addition process:

$$T_4 = (V_4/V_3) \times T_3 = (0.0954275 / 0.07175) \times 1750$$

$$T_4 = 2327.5 \text{ K}$$

$$\text{Pressure ratio, } K = (P_3/P_2) = 70/32.423 = 2.159$$

Net heat supplied to the cycle:

$$Q_s = C_v (T_3 - T_2) + C_p (T_4 - T_3)$$

$$= 0.718 (1750 - 810.57) + 1.005 (2327.5 - 1750)$$

$$= 1254.9 \text{ KJ/kg}$$

Efficiency of the cycle:

$$\eta = 1 - 1/(r)^\gamma [(K \times P^\gamma - 1)/(k-1) + K\gamma(p-1)]$$

$$= 61.92\%$$

Net workdone of the cycle:

$$W = \eta \times Q_s$$

$$= 0.6192 \times 1254.9$$

$$= 777.1 \text{ KJ/kg}$$

Mean effective pressure,

$$P_m = W / (V_1 - V_2)$$

$$= 777.1 / (0.361 - 0.07115)$$

$$= 984.6 \text{ Kpa}$$

$$P_m = 9.846 \text{ bar}$$

5. The compression ratio of a dual cycle is 10. The pressure and temperature at the beginning of the cycle are 1 bar and 27°C. the maximum pressure of the cycle is limited to 70 bar and heat supplied is limited to 1675KJ/kg fair find thermal efficiency.

Given data:

$$r = 10$$

$$P_1 = 1 \text{ bar}$$

$$T_1 = 27^\circ\text{C} = 300 \text{ K}$$

$$P_3 = 70 \text{ bar}$$

$$Q_s = 1675 \text{ KJ/kg}$$

Solution:

Specific volumes:

$$V_1 = RT_1/P_1 = 287 \times 300 / 1 \times 10^5$$

$$V_2 = V_1/r$$

$$= 0.861/10$$

1-2 isentropic compression process:

$$P_2 = (r)^\gamma \times P_1 = (10)^{1.4} \times 1 = 25.12 \text{ bar}$$

$$T_2 = (r)^{\gamma-1} \times T_1 = (10)^{1.4-1} \times 300 = 753.57 \text{ K}$$

2-3 constant volume heat addition process:

$$T_3 = (P_3/P_2) \times T_2 = (70 / 25.12) \times 753.37 = 2100 \text{ K}$$

Total heat supplied to the cycle:

$$Q_s = C_v (T_3 - T_2) + C_p (T_4 - T_3)$$

$$1675 = 0.718 (2100 - 753.57) + 1.005 (T_4 - 2100)$$

$$T_4 = 2804.6 \text{ K}$$

Cut off ratio:

$$\rho = V_4/V_3 = T_4/T_3 = 2804.6/2300$$

$$\rho = 1.3356$$

Pressure ratio:

$$K = P_3/P_2 = 70/25.12 = 2.787$$

Efficiency of the cycle:

$$\eta = 1 - \frac{1}{(\rho)^{\gamma-1}} \left[\frac{K \times P^{\gamma-1}}{(k-1)} + K\gamma(P-1) \right]$$

$$= 59.13\%$$

6. In an air standard diesel cycle, the pressure and temperature of air at the beginning of cycle are 1 bar x 40°C. The temperatures before and after the heat supplied are 400°C and 1500°C. Find the air standard efficiency and mean effective pressure of the cycle. What is the power output if it makes 100 cycles / min?

Given data:

$$P_1 = 1 \text{ bar} = 100 \text{ kN/m}^2$$

$$T_1 = 40^\circ\text{C} = 313 \text{ K}$$

$$T_2 = 400^\circ\text{C} = 673 \text{ K}$$

$$T_3 = 1500^\circ\text{C} = 1773 \text{ K}$$

Solution :

1-2 isentropic compression:

$$T_2/T_1 = (\rho)^{\gamma-1}$$

Compression ratio :

$$\begin{aligned} \rho &= V_1/V_2 = (T_2/T_1)^{1/(\gamma-1)} \\ &= (673/313)^{1/1.4-1} \\ &= 6.779 \end{aligned}$$

2-3 constant pressure heating:

$$V_2/T_2 = V_3/T_3$$

$$\text{Cut off ratio, } P = V_3/V_2 = T_3/T_2 = 1773/673 = 2.634$$

Efficiency :

$$\eta = 1 - \frac{1}{\rho^{\gamma-1}} \left(\frac{P^{\gamma-1}}{(\gamma-1)} + \gamma(P-1) \right)$$