UNIT V
APPLICATIONS OF LAPLACE TRANSFORM

APPLICATIONS OF LAPLACE TRANSFORM:

1) Solve the following using laplace transform

$$\frac{dy}{dt} + 2y + \int_0^t y(t)dt = 0 \text{ given that } y(0) = 0.$$

Solution:

$$y^1(t) + 2y(t) + \int_0^t y(t)dt = 0$$

Taking Laplace Transform

$$L\left[y^1(t) + 2y(t) + \int_0^t y(t)dt\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{We Know That, } L\left[\int_0^t y(t)\,dt\right] = \frac{L[y(t)]}{s} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Using $(2) in (1)$,

$$L\left[y^1(t)\right] + 2L[y(t)] + \frac{L[y(t)]}{s} = 0$$

$$sL[y(t)] - y(0) + 2L[y(t)] + \frac{L[y(t)]}{s} = 0$$

$$sL[y(t)] - 1 + 2L[y(t)] + \frac{L[y(t)]}{s} = 0 \ldots\ldots\ldots(Given, y(0) = 1)$$

$$s^2L[y(t)] - s + 2sL[y(t)] + L[y(t)] = 0$$

$$L[y(t)] = \frac{s}{s^2 + 2s + 1}$$

$$\Rightarrow y(t) = L^{-1}\left[\frac{s}{s^2 + 2s + 1}\right]$$

$$= L^{-1}\left[\frac{s+1-1}{(s+1)^2}\right]$$

$$= L^{-1}\left[\frac{s+1}{(s+1)^2}\right] - L^{-1}\left[\frac{1}{(s+1)^2}\right]$$

$$= e^{-t}L^{-1}\left[\frac{s}{s^2}\right] - e^{-t}\left[\frac{1}{s^2}\right]$$

$$= e^{-t}(1) - e^{-t}t$$

$$\therefore y(t) = e^{-t}[1-t]$$

2) Using Laplace Transform, Solve $x+\int_0^t x\,dt = 1-e^{-t}$

Solution :

$$L\left[x+\int_0^t x\,dt = 1-e^{-t} \right]$$

$$L[x(t)] + L\left[\int_0^t x\,dx\right] = L\left[1-e^{-t}\right]$$

$$L[x] + \frac{L[x]}{s} = L[1] - L\left[e^{-t}\right]$$

$$L[x]\left[1+\frac{1}{s}\right] = \frac{1}{s} - \frac{1}{s+1}$$

$$L[x] = \frac{1}{(s+1)^2}$$

$$x = L^{-1}\left[\frac{1}{(s+1)^2}\right]$$

$$x = e^{-t}L^{-1}\left[\frac{1}{s^2}\right]$$

$$x = e^{-t}.t$$

3)Solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}-5y=5, y(t)=0, \frac{dy}{dt}=2\, when\, t=0;$

$y(0)=0; y^1(0)=2; y(t)=0, y^1(t)=0$

Solution:

$y^{11}(t)+4y^1(t)-5y(t)=5$

$L\left[y^{11}(t)\right]+4L\left[y^1(t)\right]-5L\left[y(t)\right]=5L[1]$

$s^2L\left[y(t)\right]-sy(0)-y^1(0)+4\left[sL\left[y(t)\right]-y(0)\right]-5L\left[y(t)\right]=5L[1]$

$s^2L\left[y(t)\right]-s(0)-2+4\left[sL\left[y(t)\right]-y(0)\right]-5L\left[y(t)\right]=5\left(\frac{1}{s}\right)$

$L\left[y(t)\right]\left[s^2+4s-5\right]-2=\frac{5}{s}$

$y(t)=L^{-1}\left[\frac{5+2s}{s\left[s^2+4s-5\right]}\right]$..(1)

$\frac{5+2s}{s\left[s^2+4s-5\right]}=\frac{5+2s}{s(s+5)(s-1)}=\frac{A}{s}+\frac{B}{s+5}+\frac{C}{s-1}$...............(2)

$$5+2s = A(s+5)(s-1)+Bs(s-1)+cs(s+5)...............(3)$$

We get A = -1, B = $-\frac{1}{6}$, C = $\frac{7}{6}$

Substituting in (2),

$$\frac{5+2s}{s(s+5)(s-1)} = \frac{-1}{s} - \frac{1}{6(s+5)} + \frac{7}{6(s-1)}$$

$$L^{-1}\left[\frac{5+2s}{s(s+5)(s-1)}\right] = L^{-1}\left[\frac{-1}{s}\right] - \frac{1}{6}L^{-1}\left[\frac{1}{(s+5)}\right] + \frac{7}{6}L^{-1}\left[\frac{1}{(s-1)}\right]$$

$$= \frac{7}{6}e^{t} - \frac{1}{6}e^{-5t} - 1....................................(4)$$

substituting (4) *in* (1)

$$\therefore y(t) = \frac{7}{6}e^{t} - \frac{1}{6}e^{-5t} - 1$$

TASK:

1) Solve $\frac{dy}{dt} + 4y + 5\int_0^t y\,dt = e^{-t}, y(0) = 0$

2)Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = 2\cos t, y(0) = 1$

3)Solve $y^{11}(t) - 3y^{1}(t) + 2y(t) = e^{2t}$

4)Solve $y^{11}(t) + 2y^{1}(t) - 3y(t) = \sin t,\ y(0) = 0, y^{1}(0) = 0$

$when\, t = 0, y(t) = 0, y^{1}(t) = 0$