UNIT V MECHANISMS FOR CONTROL

Governors - Types - Centrifugal governors – Porter, Proel and Hartnell Governors – Characteristics –Sensitivity– Stability – Hunting – Isochronisms – equilibrium speed - Effect of friction - Controlling Force

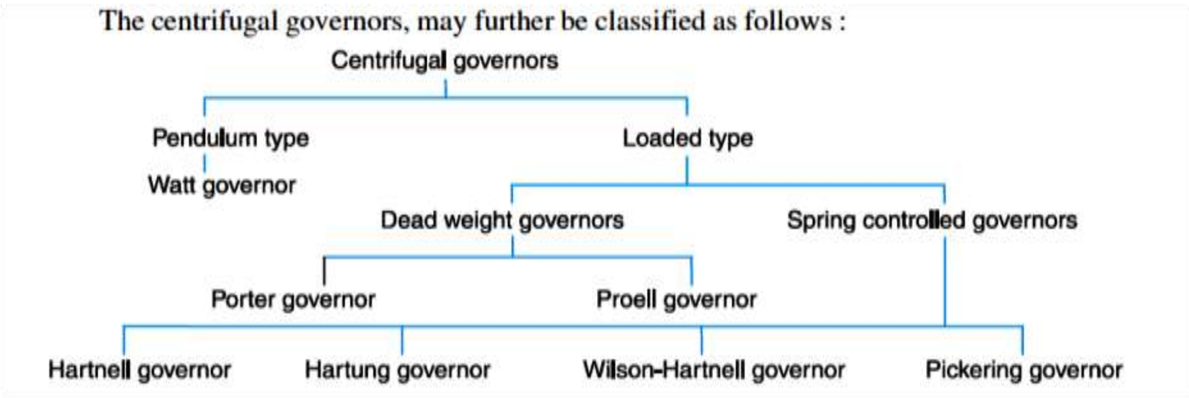
Introduction

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g. when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits. A little consideration will show, that when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid ; **conversely,** when the load decreases, the engine speed increases and the governor decreases the supply of working fluid.

Types of Governors

The governors may, broadly, be classified as

1. Centrifugal governors,
2. Inertia governors.



Centrifugal Governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force********.It consists of two balls of equal mass, which are attached to the arms as shown in Fig. 1. These balls are known as ***governor balls*** **or** ***fly balls*****.** The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops *S*, *S* are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.

Note : When the balls rotate at uniform speed, controlling force is equal to the centrifugal force and they balance each other.

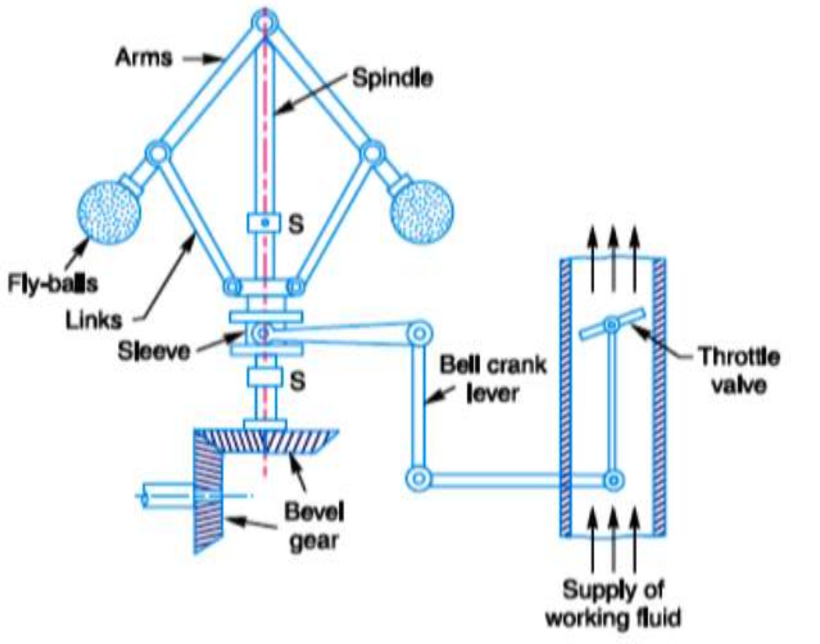


Fig. 1. Centrifugal governor

Terms Used in Governors

The following terms used in governors are important from the subject point of view ;

1. ***Height of a governor*****.** It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by *h.*

2. ***Equilibrium speed*****.** It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. ***Mean equilibrium speed*****.** It is the speed at the mean position of the balls or the sleeve.

4. ***Maximum and minimum equilibrium speeds*****.** The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the minimum equilibrium speeds.

5. ***Sleeve lift*****.** It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Porter Governor

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. 2 (*a*). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Consider the forces acting on one-half of the governor as shown in Fig. 2 (*b*).

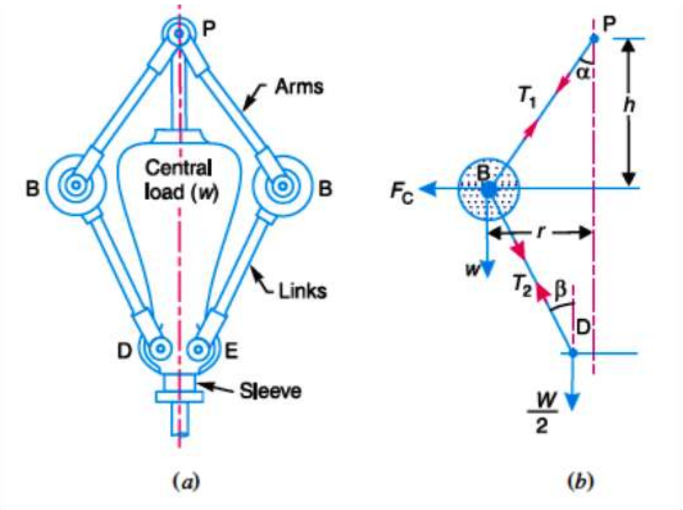


Fig. 2. Porter governor.

Let m = Mass of each ball in kg,

w = Weight of each ball in newtons = $m.g$,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = $M.g$,

r = Radius of rotation in metres,

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω= Angular speed of the balls in rad/s= $2\pi N/60$ rad/s,

FC = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$,

$T1$ = Force in the arm in newtons,

$T2$ = Force in the link in newtons,

α= Angle of inclination of the arm (or upper link) to the vertical, and

β= Angle of inclination of the link (or lower link) to the vertical.

Though there are several ways of determining the relation between the height of the governor (h) and the angular speed of the balls (ω), yet the following two methods are important from the subject point of view :

1. Method of resolution of forces; and

2. Instantaneous centre method.

1. ***Method of resolution of forces***

Considering the equilibrium of the forces acting at D, we have

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2}$$
$$T_2 = \frac{M.g}{2\cos\beta}$$

Again, considering the equilibrium of the forces acting on B. The point B is in equilibrium under the action of the following forces, as shown in Fig. 2 (b).

(i) The weight of ball ($w = m.g$),

(ii) The centrifugal force (FC),

(iii) The tension in the arm (T1), and

(iv) The tension in the link (T2).

Resolving the forces vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \qquad \ldots(ii)$$

$$\ldots\left(\because\ T_2 \cos\beta = \frac{M.g}{2}\right)$$

Resolving the forces horizontally,

$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C \qquad \ldots\left(\because\ T_2 = \frac{M.g}{2\cos\beta}\right)$$

$$T_1 \sin\alpha + \frac{M.g}{2} \times \tan\beta = F_C$$

$$T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta \qquad \ldots(iii)$$

Dividing equation (*iii*) by equation (*ii*),

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

or

$$\left(\frac{M.g}{2} + m.g\right)\tan\alpha = F_C - \frac{M.g}{2} \times \tan\beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

Substituting $\frac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \frac{r}{h}$, we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q \qquad \ldots (\because F_C = m.\omega^2.r)$$

or

$$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$\therefore$

$$h = \left[m.g + \frac{M.g}{2}(1+q)\right]\frac{1}{m.\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2} \qquad \ldots (iv)$$

or

$$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q)\right]\frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

or

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$\therefore$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h} \qquad \ldots (v)$$

$\ldots$ (Taking $g = 9.81$ m/s²)

Notes : 1. When the length of arms are equal to the length of links and the points *P* and *D* lie on the same vertical line, then

tan a = tan b or *q* = tan a / tan b = 1

Therefore, the equation **(*v*)** becomes

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}$$

2. When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.

If *F* = Frictional force acting on the sleeve in newtons, then the equations **(*v*)** and **(*vi*)** may be written as

$$N^2 = \frac{m.g + \left(\frac{M.g \pm F}{2}\right)(1+q)}{m.g} \times \frac{895}{h} \qquad \ldots (vii)$$

$$= \frac{m.g + (M.g \pm F)}{m.g} \times \frac{895}{h} \qquad \ldots (\text{When } q = 1) \ldots (viii)$$

The + sign is used when the sleeve moves upwards or the governor speed increases and negative sign is used when the sleeve moves downwards or the governor speed decreases.

3. Mass of the central load (*M*) increases the height of governor in the ratio (m+M)/m

2. *Instantaneous centre method*

In this method, equilibrium of the forces acting on the link *BD* are considered. The instantaneous centre *I* lies at the point of intersection of *PB* produced and a line through *D* perpendicular to the spindle axis, as shown in Fig. 3. Taking moments about the point *I*,

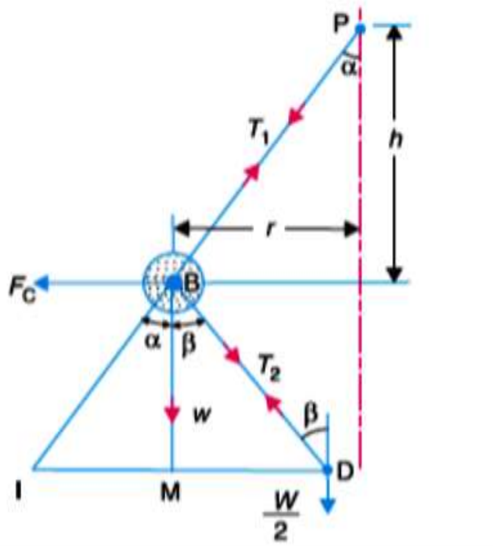


Fig. 3. Instantaneous centre method

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2} \times \frac{ID}{BM}$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= m.g \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)$$

$$\ldots\left(\because \frac{IM}{BM} = \tan\alpha, \text{ and } \frac{MD}{BM} = \tan\beta\right)$$

Dividing throughout by $\tan\alpha$,

$$\frac{F_C}{\tan\alpha} = m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) = m.g + \frac{M.g}{2}(1+q) \qquad \ldots\left(\because q = \frac{\tan\beta}{\tan\alpha}\right)$$

We know that $F_C = m.\omega^2.r$, and $\tan\alpha = \frac{r}{h}$

$$\therefore \quad m\omega^2.r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$$

or

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

. . . (Same as before)

When $\tan\alpha = \tan\beta$ or $q = 1$, then

$$h = \frac{m + M}{m} \times \frac{g}{\omega^2}$$

Proell Governor

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig. 4 (*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively. Consider the equilibrium of the forces on one-half of the governor as shown in Fig. 4 (*b*). The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicualr to the spindle axis. The prependicular *BM* is drawn on *ID*.

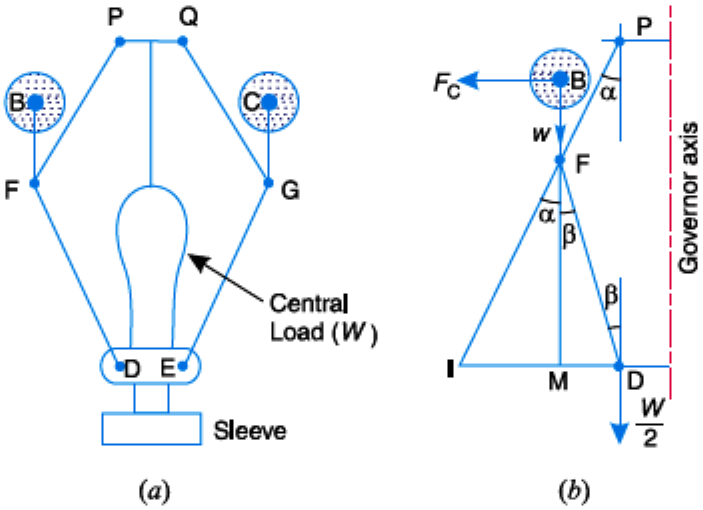


Fig. 4. Proell governor.

Taking moments about *I*,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \quad \ldots(i)$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \quad \ldots(\because \; ID = IM + MD)$$

Multiplying and dividing by *FM*, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

$$\therefore \quad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

and

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h} \qquad \ldots (ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81$ m/s^2, we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h} \qquad \ldots (iii)$$

Notes : 1. The equation *(i)* may be applied to any given configuration of the governor.

2. Comparing equation *(iii)* with the equation *(v)* of the Porter governor (Art. 18.6), we see that the equilibrium speed reduces for the given values of *m*, *M* and *h*. Hence in order to have the same equilibrium speed for the given values of *m*, *M* and *h*, balls of smaller masses are used in the Proell governor than in the Porter governor.

3. When $\alpha = \beta$, then $q = 1$. Therefore equation *(iii)* may be written as

$$N^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h} \qquad (h \text{ being in metres}) \ldots (iv)$$

Hartnell Governor

A Hartnell governor is a spring loaded governor as shown in Fig. 5. It consists of two bell crank levers pivoted at the points *O*, *O* to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm *OB* and a roller at the end of the horizontal arm *OR*. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

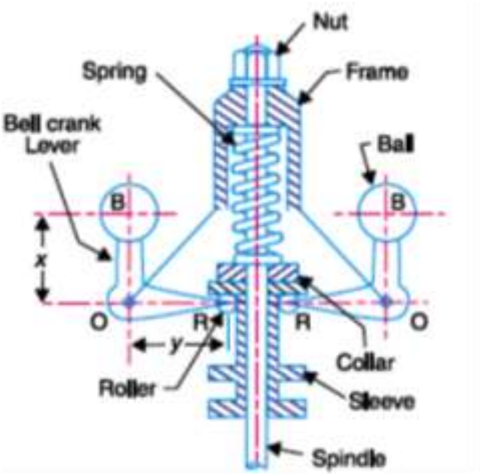


Fig. 5. Hartnell governor.

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

$r1$ = Minimum radius of rotation in metres,

$r2$ = Maximum radius of rotation in metres,

w_1 = Angular speed of the governor at minimum radius in rad/s,

w_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve at w_1 in newtons,

S_2 = Spring force exerted on the sleeve at w_2 in newtons,

$FC1$ = Centrifugal force at w_1 in newtons = $m\ (\omega_1)^2\ r1$,

$FC2$ = Centrifugal force at w_2 in newtons = $m\ (\omega_2)^2\ r2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres,

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

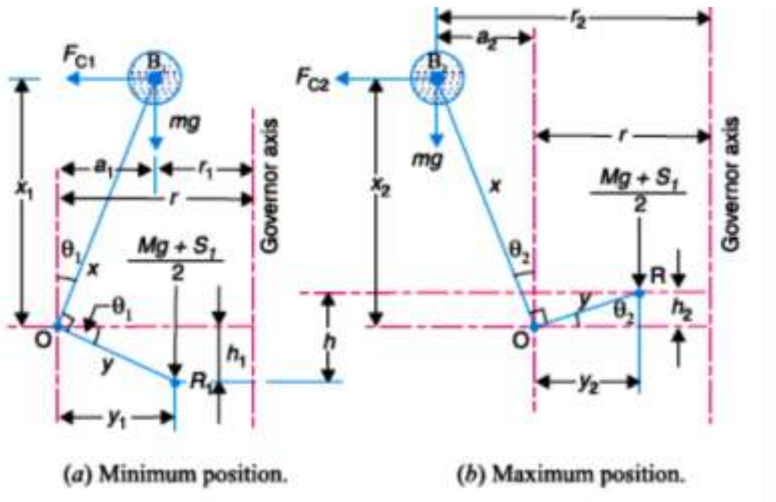


Fig 6

Consider the forces acting at one bell crank lever. The minimum and maximum position is shown in Fig. 6. Let *h* be the compression of the spring when the radius of rotation changes from *r*1 to *r*2.

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \qquad \ldots (i)$$

For the minimum position *i.e*. when the radius of rotation changes from *r* to *r*1, as shown in Fig. 6 (*a*), the compression of the spring or the lift of sleeve *h*1 is given by

Similarly, for the maximum position *i.e*. when the radius of rotation changes from *r* to *r*2, as shown in Fig. 6 (*b*), the compression of the spring or lift of sleeve *h*2 is given by

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \qquad \ldots (ii)$$

Adding equations (*i*) and (*ii*),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x} \qquad \ldots (\because h = h_1 + h_2)$$

$$\therefore \qquad h = (r_2 - r_1)\frac{y}{x} \qquad \ldots (iii)$$

Now for minimum position, taking moments about point O, we get

$$\frac{M.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

or $$M.g + S_1 = \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1) \quad \ldots (iv)$$

Again for maximum position, taking moments about point O, we get

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

or $$M.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \quad \ldots (v)$$

Subtracting equation (*iv*) from equation (*v*),

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1)$$

We know that

$$S_2 - S_1 = h.s, \qquad \text{and} \qquad h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \qquad s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms (*i.e.* $x_1 = x_2 = x$, and $y_1 = y_2 = y$) and the moment due to weight of the balls (*i.e.* $m.g$), we have for minimum position,

$$\frac{M.g + S_1}{2} \times y = F_{C1} \times x \quad \text{or} \quad M.g + S_1 = 2F_{C1} \times \frac{x}{y} \quad \ldots (vi)$$

Similarly for maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad M.g + S_2 = 2F_{C2} \times \frac{x}{y} \qquad \dots(vii)$$

Subtracting equation (*vi*) from equation (*vii*),

$$S_2 - S_1 = 2\,(F_{C2} - F_{C1})\,\frac{x}{y} \qquad \dots(viii)$$

We know that

$$S_2 - S_1 = h.s, \qquad \text{and} \qquad h = (r_2 - r_1)\,\frac{y}{x}$$

$$\therefore \qquad s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \qquad \dots(ix)$$

Notes : 1. Unless otherwise stated, the obliquity effect of the arms and the moment due to the weight of the balls is neglected, in actual practice.

2. When friction is taken into account, the weight of the sleeve ($M.g$) may be replaced by ($M.g. \pm F$).

3. The centrifugal force (F_C) for any intermediate position (*i.e.* between the minimum and maximum position) at a radius of rotation (r) may be obtained as discussed below :

Since the stiffness for a given spring is constant for all positions, therefore for minimum and intermediate position,

$$s = 2\left(\frac{F_C - F_{C1}}{r - r_1}\right)\left(\frac{x}{y}\right)^2 \qquad \dots(x)$$

and for intermediate and maximum position,

$$s = 2\left(\frac{F_{C2} - F_C}{r_2 - r}\right)\left(\frac{x}{y}\right)^2 \qquad \dots(xi)$$

$\therefore$ From equations (*ix*), (*x*) and (*xi*),

$$\frac{F_{C2} - F_{C1}}{r_2 - r_1} = \frac{F_C - F_{C1}}{r - r_1} = \frac{F_{C2} - F_C}{r_2 - r}$$

or

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right) = F_{C2} - (F_{C2} - F_{C1})\left(\frac{r_2 - r}{r_2 - r_1}\right)$$

Sensitiveness of Governors

Consider two governors *A* and *B* running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor *A* is greater than the lift of the sleeve of governor *B*. It is then said that the governor *A* is more sensitive than the governor *B*. In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is the sensitiveness of the governor. It may also be stated in another way that for a given lift of the sleeve, the sensitiveness of the governor increases as the speed range decreases. This definition of sensitiveness may be quite satisfactory when the governor is considered as an independent mechanism. But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a fraction as possible of the mean equilibrium speed. The actual displacement of the sleeve is immaterial, provided that it is sufficient to change the energy supplied to the engine by the required amount. For this reason, the sensitiveness is defined as the ***ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.***

Let N_1 = Minimum equilibrium speed,

N_2 = Maximum equilibrium speed, and

N = Mean equilibrium speed $= \dfrac{N_1 + N_2}{2}$.

$\therefore$ Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2} \qquad \text{. . . (In terms of angular speeds)}$$

Stability of Governors

A governor is said to be ***stable*** when for every speed within the working range there is a definite configuration *i.e*. there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

Note : A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

Isochronous Governors

A governor is said to be ***isochronous*** when the equilibrium speed is constant (*i.e*. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

Let us consider the case of a Porter governor running at speeds $N1$ and $N2$ r.p.m.

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1} \qquad \ldots (i)$$

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2} \qquad \ldots (ii)$$

For isochronism, range of speed should be zero *i.e.* $N2 - N1 = 0$ or $N2 = N1$. Therefore from equations ***(i)*** and ***(ii)***, $h1 = h2$, which is impossible in case of a Porter governor. Hence a ***Porter governor cannot be isochronous.***

Now consider the case of a Hartnell governor running at speeds $N1$ and $N2$ r.p.m.

$$M.g + S_1 = 2\,F_{C1} \times \frac{x}{y} = 2 \times m\left(\frac{2\pi N_1}{60}\right)^2 r_1 \times \frac{x}{y} \qquad \ldots (iii)$$

$$M.g + S_2 = 2\,F_{C2} \times \frac{x}{y} = 2 \times m\left(\frac{2\pi N_2}{60}\right)^2 r_2 \times \frac{x}{y} \qquad \ldots (iv)$$

For isochronism, $N2 = N1$. Therefore from equations ***(iii)*** and ***(iv)***,

$$\frac{M.g + S_1}{M.g + S_2} = \frac{r_1}{r_2}$$

Note : The isochronous governor is not of practical use because the sleeve will move to one of its extreme positions immediately the speed deviates from the isochronous speed.

Hunting

A governor is said to be ***hunt*** if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place. For example, when the load on the engine increases, the engine speed decreases and, if the governor is very sensitive, the governor sleeve immediately falls to its lowest position. This will result in the opening of the control valve wide which will supply the fuel to the engine in excess of its requirement so that the engine speed rapidly increases again and the governor sleeve rises to its highest position. Due to this movement of the sleeve, the control valve will cut off the fuel supply to the engine and thus the engine speed begins to fall once again. This cycle is repeated indefinitely. Such a governor may admit either the maximum or the minimum amount of fuel. The effect of this will be to cause wide fluctuations in the engine speed or in other words, the engine will hunt.

Effort and Power of a Governor

The ***effort of a governor*** is the mean force exerted at the sleeve for a given percentage change of speed* (or lift of the sleeve). It may be noted that when the governor is running steadily, there is no force at the sleeve. But, when the speed changes, there is a resistance at the sleeve which opposes its motion. It is assumed that this resistance which is equal to the effort varies uniformly from a maximum value to zero while the governor moves into its new position of equilibrium.

The ***power of a governor*** is the work done at the sleeve for a given percentage change of speed. It is the product of the mean value of the effort and the distance through which the sleeve moves. Mathematically, Power = Mean effort × lift of sleeve.

Effort and Power of a Porter Governor

The effort and power of a Porter governor may be determined as discussed below.

Let N = Equilibrium speed corresponding to the configuration as shown in Fig. 7 (*a*), and

c = Percentage increase in speed.

Increase in speed = $c.N$

and increased speed = $N + c.N = N(1 + c)$

The equilibrium position of the governor at the increased speed is shown in Fig. 7 (*b*).

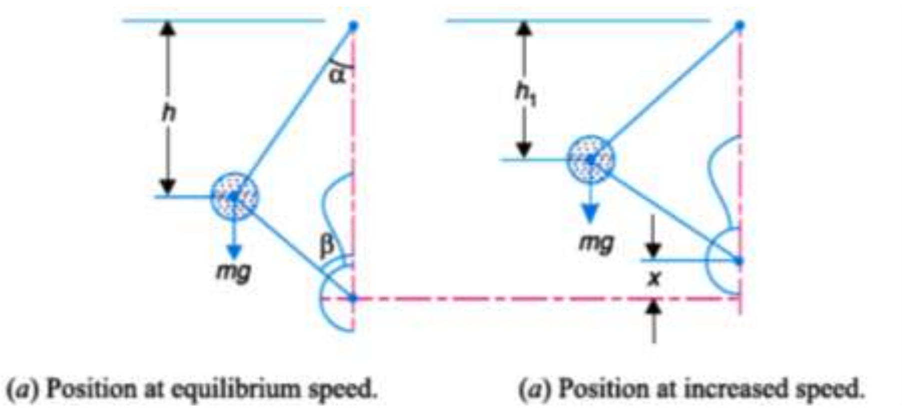


Fig. 7

When the speed is N r.p.m., the sleeve load is $M.g$. Assuming that the angles a and b are equal, so that $q = 1$, then the height of the governor,

$$h = \frac{m + M}{m} \times \frac{895}{N^2} \text{ (in metres)} \quad \ldots (i)$$

When the increase of speed takes place, a downward force P will have to be exerted on the sleeve in order to prevent the sleeve from rising. If the speed increases to $(1 + c)\,N$ r.p.m. and the height of the governor remains the same, the load on the sleeve increases to $M1.g$. Therefore

$$h = \frac{m + M_1}{m} \times \frac{895}{(1 + c)^2 N^2} \text{ (in metres)} \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*), we have

$$m + M = \frac{m + M_1}{(1 + c)^2} \quad \text{or} \quad M_1 = (m + M)(1 + c^2) - m$$

and

$$M_1 - M = (m + M)(1 + c)^2 - m - M = (m + M)\left[(1 + c)^2 - 1\right] \quad \ldots (iii)$$

A little consideration will show that $(M1 - M)g$ is the downward force which must be applied in order to prevent the sleeve from rising as the speed increases. It is the same force which acts on the governor sleeve immediately after the increase of speed has taken place and before the sleeve begins to move. When the sleeve takes the new position as shown in Fig. 7 (*b*), this force gradually diminishes to zero.

Let $\quad P$ = Mean force exerted on the sleeve during the increase in speed or the effort of the governor.

$$\therefore \quad P = \frac{(M_1 - M)\,g}{2} = \frac{(m + M)\,[(1 + c)^2 - 1]\,g}{2}$$

$$= \frac{(m + M)\,[1 + c^2 + 2c - 1]\,g}{2} = c\,(m + M)\,g \qquad \ldots (iv)$$

... (Neglecting c^2, being very small)

If F is the frictional force (in newtons) at the sleeve, then

$$P = c\,(m.g + M.g \pm F)$$

We have already discussed that the power of a governor is the product of the governor effort and the lift of the sleeve.

Let $\quad x$ = Lift of the sleeve.

$\therefore$ Governor power $= P \times x \qquad \ldots (v)$

If the height of the governor at speed N is h and at an increased speed $(1 + c)\,N$ is h_1, then

$$x = 2\,(h - h_1)$$

As there is no resultant force at the sleeve in the two equilibrium positions, therefore

$$h = \frac{m + M}{m} \times \frac{895}{N^2}, \quad \text{and} \quad h_1 = \frac{m + M}{m} \times \frac{895}{(1 + c)^2\,N^2},$$

$$\therefore \quad \frac{h_1}{h} = \frac{1}{(1 + c)^2} \quad \text{or} \quad h_1 = \frac{h}{(1 + c)^2}$$

We know that

$$x = 2\,(h - h_1) = 2\left[h - \frac{h}{(1 + c)^2}\right] = 2\,h\left[1 - \frac{1}{(1 + c)^2}\right]$$

$$= 2\,h\left[\frac{1 + c^2 + 2c - 1}{1 + c^2 + 2c}\right] = 2\,h\left(\frac{2c}{1 + 2c}\right) \qquad \ldots (vi)$$

... (Neglecting c^2, being very small)

Substituting the values of P and x in equation (v), we have

$$\text{Governor power} = c\,(m + M)\,g \times 2\,h\left(\frac{2c}{1 + 2c}\right) = \frac{4c^2}{1 + 2c}\,(m + M)\,g.h \qquad \ldots (vii)$$

Notes : 1. If α is not equal to β, *i.e.* $\tan \beta / \tan \alpha = q$, then the equations (*i*) and (*ii*) may be written as

$$h = \frac{m + \frac{M}{2}(1 + q)}{m} \times \frac{895}{N^2} \qquad \ldots (viii)$$

When speed increases to $(1 + c)\,N$ and height of the governor remains the same, then

$$h = \frac{m + \frac{M_1}{2}(1 + q)}{m} \times \frac{895}{(1 + c)\,N^2} \qquad \ldots (ix)$$

From equations (*viii*) and (*ix*), we have

$$m + \frac{M}{2}(1+q) = \frac{m + \frac{M_1}{2}(1+q)}{(1+c)^2}$$

or
$$\frac{M_1}{2}(1+q) = \left[m + \frac{M}{2}(1+q)\right](1+c)^2 - m$$

$\therefore$
$$\frac{M_1}{2} = \frac{m(1+c)^2}{1+q} + \frac{M}{2}(1+c)^2 - \frac{m}{1+q}$$

or
$$\frac{M_1}{2} - \frac{M}{2} = \frac{m(1+c)^2}{1+q} + \frac{M}{2}(1+c)^2 - \frac{m}{1+q} - \frac{M}{2}$$

$$= \frac{m}{1+q}[(1+c)^2 - 1] + \frac{M}{2}[(1+c)^2 - 1]$$

$$= \left[\frac{m}{1+q} + \frac{M}{2}\right][(1+c)^2 - 1]$$

$\therefore$ Governor effort, $P = \left(\frac{M_1 - M}{2}\right)g = \left[\frac{m}{1+q} + \frac{M}{2}\right][1 + c^2 + 2c - 1]\,g$

$$= \left(\frac{m}{1+q} + \frac{M}{2}\right)(2c)\,g = \left(\frac{2m}{1+q} + M\right)c.g \qquad \text{. . . (Neglecting } c^2\text{)}$$

The equation (*vi*) for the lift of the sleeve becomes,

$$x = (1+q)\,h\left(\frac{2c}{1+2c}\right)$$

$\therefore$ Governor power $= P \times x = \left(\frac{2m}{1+q} + M\right)c.g\,(1+q)\,h\left(\frac{2c}{1+2c}\right)$

$$= \frac{2c^2}{1+2c}[2m + M(1+q)]g.h = \frac{4c^2}{1+2c}\left[m + \frac{M}{2}(1+q)\right]g.h$$

2. The above method of determining the effort and power of a Porter governor may be followed for any other type of the governor.

Controlling Force

We have seen earlier that when a body rotates in a circular path, there is an inward radial force or centripetal force acting on it. In case of a governor running at a steady speed, the inward force acting on the rotating balls is known as ***controlling force.*** It is equal and opposite to the centrifugal reaction.

Controlling force, $FC = m.w^2.r$

The controlling force is provided by the weight of the sleeve and balls as in Porter governor and by the spring and weight as in Hartnell governor (or spring controlled governor).

When the graph between the controlling force (*F*C) as ordinate and radius of rotation of the balls (*r*) as abscissa is drawn, then the graph obtained is known as ***controlling force diagram*.** This diagram enables the stability and sensitiveness of the governor to be examined and also shows clearly the effect of friction.

Controlling Force Diagram for Porter Governor

The controlling force diagram for a Porter governor is a curve as shown in Fig. 8. We know that controlling force,

$$F_C = m.\omega^2.r = m\left(\frac{2\,\pi\,N}{60}\right)^2 r$$

$$N^2 = \frac{1}{m}\left(\frac{60}{2\pi}\right)^2\left(\frac{F_C}{r}\right) = \frac{1}{m}\left(\frac{60}{2\pi}\right)^2 (\tan\phi) \qquad \ldots\left[\because \frac{F_C}{r} = \tan\phi\right]$$

$$N = \frac{60}{2\pi}\left(\frac{\tan\phi}{m}\right)^{1/2} \qquad \ldots(i)$$

where f is the angle between the axis of radius of rotation and a line joining a given point (say *A*) on the curve to the origin *O*.

Notes: 1. in case the governor satisfies the condition for stability, the angle f must increase with radius of rotation of the governor balls. In other words, the equilibrium speed must increase with the increase of radius of rotation of the governor balls.

2. For the governor to be more sensitive, the change in the value of f over the change of radius of rotation should be as small as possible.

3. For the isochronous governor, the controlling force curve is a straight line passing through the origin. The angle f will be constant for all values of the radius of rotation of the governor. From equation **(*i*)**

$$\tan\phi = \frac{F_C}{r} = \frac{m.\omega^2.r}{r} = m.\omega^2 = m\left(\frac{2\pi\,N}{60}\right)^2 = C.N^2$$

where $$C = m\left(\frac{2\pi}{60}\right)^2 = \text{constant}$$

Using the above relation, the angle f may be determined for different values of *N* and the lines are drawn from the origin. These lines enable the equilibrium speed corresponding to a given radius of rotation to be determined. Alternatively, the same results may be obtained more simply by setting-off a speed scale along any arbitrarily chosen ordinate. The controlling force is calculated for one constant radius of rotation and for different arbitrarily chosen values of speed. The values thus obtained are set-off along the ordinate that corresponds to the chosen radius and marked with the appropriate speeds.

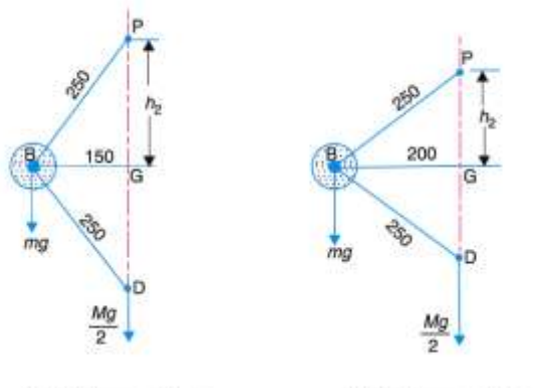
1) The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.

Given : *BP* = *BD* = 250 mm ; *m* = 5 kg ; *M* = 30 kg ; *r*1 = 150 mm ; *r*2 = 200 mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. (a) and (b) respectively.

Let N1 = Minimum speed when r1 = BG = 150 mm, and

N2 = Maximum speed when r2 = BG = 200 mm.



(*a*) Minimum position. (*b*) Maximum position.

Speed range of the governor

From Fig. (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{5 + 30}{5} \times \frac{895}{0.2} = 31\,325$$

$$N_1 = 177 \text{ r.p.m.}$$

Height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

\

$$(N_2)^2 = \frac{m + M}{m} \times \frac{895}{h_2} = \frac{5 + 30}{5} \times \frac{895}{0.15} = 41\,767$$

$$N_2 = 204.4 \text{ r.p.m.}$$

speed range of the governor= $N2 - N1 = 204.4 - 177 = 27.4$ r.p.m.

Speed range when friction at the sleeve is equivalent of 20N of load (i.e. when F = 20 N)

We know that when the sleeve moves downwards, the friction force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2} = 29\,500$$

$$\therefore \quad N_1 = 172 \text{ r.p.m.}$$

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{895}{0.15} = 44\,200$$

$$\therefore \quad N_2 = 210 \text{ r.p.m.}$$

Speed range of the governor = $N2 - N1 = 210 - 172 = 38$ r.p.m.

2)The arms of a Porter governor are 300 mm long. The upper arms are pivoted on the axis of rotation. The lower arms are attached to a sleeve at a distance of 40 mm from the axis of rotation. The mass of the load on the sleeve is 70 kg and the mass of each ball is 10 kg. Determine the equilibrium speed when the radius of rotation of the balls is 200 mm. If the friction is equivalent to a load of 20 N at the sleeve, what will be the range of speed for this position ?

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Solution. Given : BP = BD = 300 mm ; DH = 40 mm ; M = 70 kg ; m = 10 kg ; r = BG = 200 mm

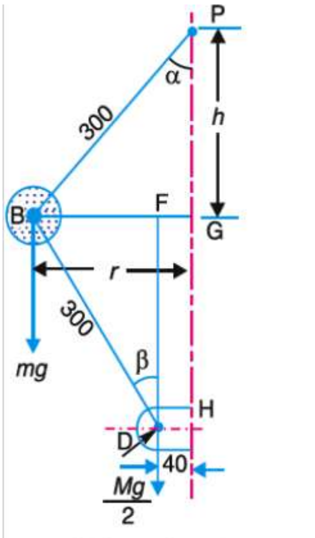
Equilibrium speed when the radius of rotation r = BG = 200 mm

Let N = Equilibrium speed.

The equilibrium position of the governor is shown in Fig. From the figure, we find that height of the governor,

$$h = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm}$$

$$= 0.224\text{m}$$



All dimensions in mm.

BF = BG – FG = 200 – 40 = 160 . . . (FG = DH)

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(300)^2 - (160)^2} = 254 \text{ mm}$$

$\therefore$ $\tan\alpha = BG/PG = 200 / 224 = 0.893$

and $\tan\beta = BF/DF = 160 / 254 = 0.63$

$$\therefore \quad q = \frac{\tan\beta}{\tan\alpha} = \frac{0.63}{0.893} = 0.705$$

We know that

$$N_2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

$$= \frac{10 + \frac{70}{2}(1 + 0.705)}{10} \times \frac{895}{0.224} = 27\,840$$

$\therefore$ $N_2 = 167$ r.p.m. **Ans.**

Range of speed when friction is equivalent to load of 20 N at the sleeve (i.e. when F = 20 N)

Let N1 = Minimum equilibrium speed, and
N2 = Maximum equilibrium speed.

We know that when the sleeve moves downwards, the frictional force (F) acts upwards and the minimum equilibrium speed is given by

$$(N_1)^2 = \frac{m.g + \left(\frac{M.g - F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{10 \times 9.81 + \left(\frac{70 \times 9.81 - 20}{2}\right)(1+0.705)}{10 \times 9.81} \times \frac{895}{0.224} = 27\,144$$

N1 = 164.8 r.p.m.

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum equilibrium speed is given by

$$(N_2)^2 = \frac{m.g + \left(\frac{M.g + F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{10 \times 9.81 + \left(\frac{70 \times 9.81 + 20}{2}\right)(1+0.705)}{10 \times 9.81} \times \frac{895}{0.224} = 28\,533$$

$\therefore \quad N_2 = 169$ r.p.m.

We know that range of speed = $N_2 - N_1$ = – 164.8 = 4.2 r.p.m.

3) All the arms of a Porter governor are 178 mm long and are hinged at a distance of 38 mm from the axis of rotation. The mass of each ball is 1.15 kg and mass of the sleeve is 20 kg. The governor sleeve begins to rise at 280 r.p.m. when the links are at an angle of 30° to the vertical. Assuming the friction force to be constant, determine the minimum and maximum speed of rotation when the inclination of the arms to the vertical is 45°.

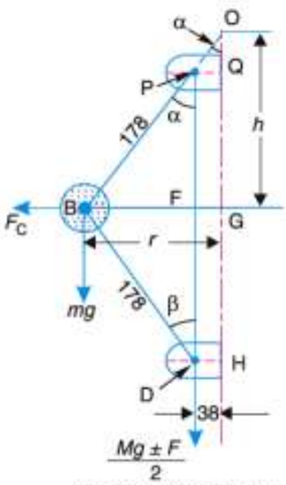
Given: *BP* = *BD* = 178 mm; *PQ* = *DH* = 38 mm; *m* = 1.15 kg; *M* = 20 kg; *N* = 280 r.p.m. ; $\alpha = \beta = 30°$

First of all, let us find the friction force (*F*). The equilibrium position of the governor when the lines are at 30° to vertical, is shown in Fig.. From the figure, we find that radius of rotation,

$r = BG = BF + FG = BP \times \sin\alpha + FG = 178 \sin 30° + 38 = 127$ mm

and height of the governor,

$h = BG / \tan\alpha = 127 / \tan 30° = 220$ mm $= 0.22$ m



All dimensions in mm.

$$N^2 = \frac{m.g + (Mg \pm F)}{m.g} \times \frac{895}{h}$$

$$\ldots (\because \tan\alpha = \tan\beta \text{ or } q = 1)$$

$$(280)^2 = \frac{1.15 \times 9.81 + 20 \times 9.81 \pm F}{1.15 \times 9.81} \times \frac{895}{0.22}$$

or

$$\pm F = \frac{(280)^2 \times 1.15 \times 9.81 \times 0.22}{895} - 1.15 \times 9.81 - 20 \times 9.81$$

$$= 217.5 - 11.3 - 196.2 = 10 \text{ N}$$

We know that radius of rotation when inclination of the arms to the vertical is 45 (*i.e.* when $\alpha = \beta = 45°$),

$r = BG = BF + FG = BP \times \sin\alpha + FG = 178 \sin 45° + 38 = 164$ mm

and height of the governor,

$h = BG / \tan\alpha = 164 / \tan 45° = 164$ mm $= 0.164$ m

Let $N1$ = Minimum speed of rotation, and

$N2$ = Maximum speed of rotation.

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h}$$

$$= \frac{1.15 \times 9.81 + (20 \times 9.81 - 10)}{1.15 \times 9.81} \times \frac{895}{0.164} = 95\,382$$

$\therefore \quad N_1 = 309$ r.p.m. **Ans.**

and

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h}$$

$$= \frac{1.15 \times 9.81 + (20 \times 9.81 + 10)}{1.15 \times 9.81} \times \frac{895}{0.164} = 105\,040$$

$N_2 = 324$ r.p.m. **Ans.**

4) A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

Given :
PF = DF = 300 mm ; BF = 80 mm ; m = 10 kg ; M = 100 kg ; r_1 = 150 mm; r_2 = 200 mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig

Let N_1 = Minimum speed when radius of rotation, r_1 = FG = 150 mm ; and
N_2 = Maximum speed when radius of rotation , r_2 = FG = 200 mm.

From Fig. (a), we find that height of the governor,

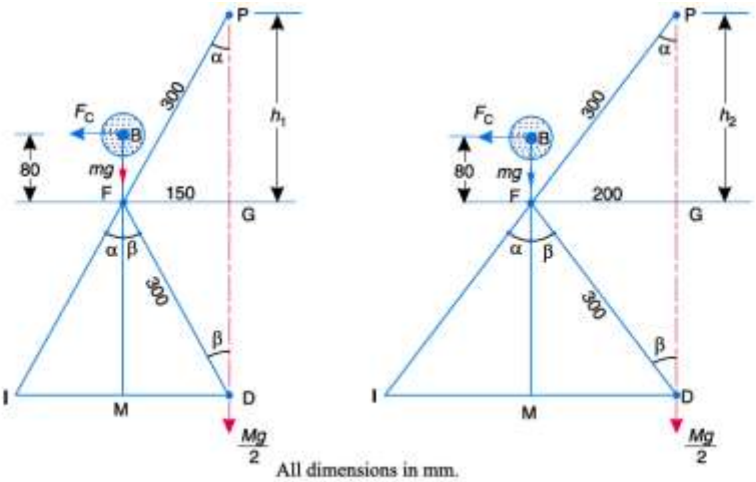
$$h_1 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm} = 0.26 \text{ m}$$

$FM = GD = PG = 260$ mm = 0.26 m
$BM = BF + FM = 80 + 260 = 340$ mm = 0.34 m

$$(N_1)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_1} \qquad \ldots (\because \alpha = \beta \text{ or } q = 1)$$

$$= \frac{0.26}{0.34}\left(\frac{10+100}{10}\right)\frac{895}{0.26} = 28\,956 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$

Now from Fig. *b*, we find that height of the governor,



All dimensions in mm.

(*a*) Minimum position. (*a*) Maximum position.

$$h_2 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm} = 0.224 \text{ m}$$

and $\quad FM = GD = PG = 224 \text{ mm} = 0.224 \text{ m}$

$\therefore \quad BM = BF + FM = 80 + 224 = 304 \text{ mm} = 0.304 \text{ m}$

We know that $(N_2)^2 = \dfrac{FM}{BM}\left(\dfrac{m+M}{m}\right)\dfrac{895}{h_2}$ $\quad \ldots (\because \alpha = \beta \text{ or } q = 1)$

$$= \frac{0.224}{0.304}\left(\frac{10+100}{10}\right)\frac{895}{0.224} = 32\,385 \quad \text{or} \quad N_2 = 180 \text{ r.p.m.}$$

We know that range of speed

$$= N_2 - N_1 = 180 - 170 = 10 \text{ r.p.m. } \textbf{Ans.}$$

Note : The example may also be solved as discussed below :

From Fig. 18.13 (*a*), we find that

$$\sin\alpha = \sin\beta = 150 / 300 = 0.5 \quad \text{or} \quad \alpha = \beta = 30°$$

and $\quad MD = FG = 150 \text{ mm} = 0.15 \text{ m}$

$$FM = FD\cos\beta = 300\cos 30° = 260 \text{ mm} = 0.26 \text{ m}$$

$$IM = FM\tan\alpha = 0.26\tan 30° = 0.15 \text{ m}$$

$$BM = BF + FM = 80 + 260 = 340 \text{ mm} = 0.34 \text{ m}$$

$$ID = IM + MD = 0.15 + 0.15 = 0.3 \text{ m}$$

We know that centrifugal force,

$$F_C = m(\omega_1)^2 r_1 = 10\left(\frac{2\pi N_1}{60}\right)^2 0.15 = 0.0165\,(N_1)^2$$

Now taking moments about point *I*,

$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID$$

or $\quad 0.0165\,(N_1)^2\, 0.34 = 10 \times 9.81 \times 0.15 + \dfrac{100 \times 9.81}{2} \times 0.3$

$$0.0056\,(N_1)^2 = 14.715 + 147.15 = 161.865$$

$\therefore \quad (N_1)^2 = \dfrac{161.865}{0.0056} = 28\,904 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$

Similarly N_2 may be calculated.

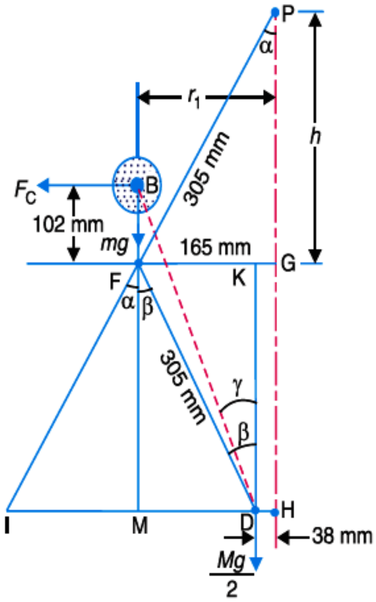
A Proell governor has all four arms of length 305 mm. The upper arms are pivoted on the axis of rotation and the lower arms are attached to a sleeve at a distance of 38 mm from the axis. The mass of each ball is 4.8 kg and are attached to the extension of the lower arms which are 102 mm long. The mass on the sleeve is 45 kg. The minimum and maximum radii of governor are 165 mm and 216 mm. Assuming that the extensions of the lower arms are parallel to the governor axis at the minimum radius, find the corresponding equilibrium speeds.

Given : $PF = DF = 305$ mm ; $DH = 38$ mm ; $BF = 102$ mm ; $m = 4.8$ kg ; $M = 54$ kg

Equilibrium speed at the minimum radius of governor

The radius of the governor is the distance of the point of intersection of the upper and lower arms from the governor axis. When the extensions of the lower arms are parallel to the governor axis, then the radius of the governor (FG) is equal to the radius of rotation (r_1).

The governor configuration at the minimum radius (*i.e.* when $FG = 165$ mm) is shown in Fig.



N_1 = Equilibrium speed at the minimum radius *i.e.* when $FG = r_1 = 165$ mm.

$$\sin\alpha = \frac{FG}{FP} = \frac{165}{305} = 0.541$$

$$\alpha = 32.75°$$

$$\tan\alpha = \tan 32.75° = 0.6432$$

$$\sin\beta = \frac{FK}{DF} = \frac{FG - KG}{DF}$$

$$= \frac{165 - 38}{305} = 0.4164$$

$$\beta = 24.6°$$

$$\tan\beta = \tan 24.6° = 0.4578$$

$$q = \frac{\tan\beta}{\tan\alpha} = \frac{0.4578}{0.6432} = 0.712$$

From Fig, we find that height of the governor

$$h = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(305)^2 - (165)^2} = 256.5 \text{ mm} = 0.2565 \text{ m}$$

$$MD = FK = FG - KG = 165 - 38 = 127 \text{ mm}$$

$$FM = \sqrt{(DF)^2 - (MD)^2} = \sqrt{(305)^2 - (127)^2} = 277 \text{ mm} = 0.277 \text{ m}$$

$$BM = BF + FM = 102 + 277 = 379 \text{ mm} = 0.379 \text{ m}$$

$$(N_1)^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h}$$

$$= \frac{0.277}{0.379}\left[\frac{4.8 + \frac{54}{2}(1+0.712)}{4.8}\right]\frac{895}{0.2565} = 27\,109$$

$N_1 = 165$ r.p.m. **Ans.**

Note: The valve of *N*1 may also be obtained by drawing the governor configuration to some suitable scale and measuring the distances *BM*, *IM* and *ID*. Now taking moments about point *I*,

$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID,$$

$$F_C = \text{Centrifugal force} = m(\omega_1)^2 r_1 = m\left(\frac{2\pi N_1}{60}\right)^2 r_1$$

Equilibrium speed at the maximum radius of governor

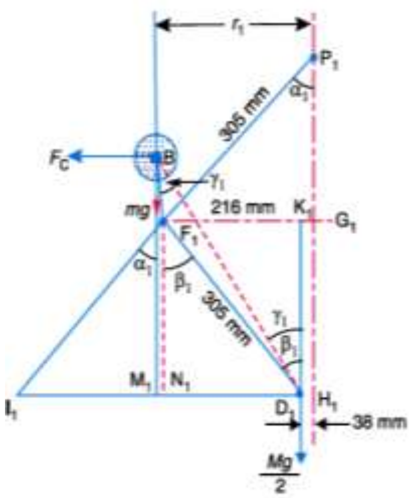
Let *N*2 = Equilibrium speed at the maximum radius of governor, *i.e.* when *F*1*G*1= r_2 = 216 mm.

First of all, let us find the values of *BD* and g in Fig. We know that

$$BD = \sqrt{(BM)^2 + (MD)^2} = \sqrt{(397)^2 + (}$$

$$\tan\gamma = MD/BM = 127/379 = 0.335 \qquad \text{or}$$

The governor configuration at the maximum radius of *F*1*G*1 = 216 mm is shown in Fig. From the geometry of the figure,



$$\sin \alpha_1 = \frac{F_1 G_1}{P_1 F_1} = \frac{216}{305} = 0.7082$$

$$\alpha_1 = 45.1°$$

$$\sin \beta_1 = \frac{F_1 K_1}{F_1 D_1} = \frac{F_1 G_1 - K_1 G_1}{F_1 D_1}$$

$$= \frac{216 - 38}{305} = 0.5836$$

$$\beta_1 = 35.7°$$

Since the extension is rigidly connected to the lower arm (*i.e.* *DFB* or *D*1*F*1*B*1 is one continuous link) therefore *B*1*D*1 and angle *B*1*D*1*F*1 do not change. In other words,

$B_1D_1 = BD = 400$ mm

$\gamma - \beta = \gamma_1 - \beta_1$ ορ $\gamma_1 = \gamma - \beta + \beta_1 = 18.5° - 24.6° + 35.7° = 29.6°$

∴ Radius of rotation,

$$r_2 = M_1D_1 + D_1H_1 = B_1D_1 \times \sin \gamma_1 + 38 \text{ mm}$$

$$= 400 \sin 29.6° + 38 = 235.6 \text{ mm} = 0.2356 \text{ m}$$

From Fig. 18.17, we find that

$$B_1M_1 = B_1D_1 \times \cos \gamma_1 = 400 \times \cos 29.6° = 348 \text{ mm} = 0.348 \text{ m}$$

$$F_1N_1 = F_1D_1 \times \cos \beta_1 = 305 \times \cos 35.7° = 248 \text{ mm} = 0.248 \text{ m}$$

$$I_1N_1 = F_1N_1 \times \tan \alpha_1 = 0.248 \times \tan 45.1° = 0.249 \text{ m}$$

$$N_1D_1 = F_1D_1 \times \sin \beta_1 = 305 \times \sin 35.7 = 178 \text{ mm} = 0.178 \text{ m}$$

$$\therefore \quad I_1D_1 = I_1N_1 + N_1D_1 = 0.249 + 0.178 = 0.427 \text{ m}$$

$$M_1D_1 = B_1D_1 \sin \gamma_1 = 400 \sin 29.6° = 198 \text{ mm} = 0.198 \text{ m}$$

$$\therefore \quad I_1M_1 = I_1D_1 - M_1D_1 = 0.427 - 0.198 = 0.229 \text{ m}$$

We know that centrifugal force,

$$F_C = m(\omega_2)^2 r_2 = 4.8 \left(\frac{2\pi N_2}{60}\right)^2 0.2356 = 0.0124\ (N_2)^2$$

Now taking moments about point I_1,

$$F_C \times B_1M_1 = m.g \times I_1M_1 + \frac{M.g}{2} \times I_1D_1$$

$$0.0124\ (N_2)^2 \times 0.348 = 4.8 \times 9.81 \times 0.229 + \frac{54 \times 9.81}{2} \times 0.427$$

$$0.0043\ (N_2)^2 = 10.873 + 113.1 = 123.883$$

$$(N_2)^2 = \frac{123.883}{0.0043} = 28\,810 \qquad \text{or} \qquad N_2 = 170 \text{ r.p.m.}$$

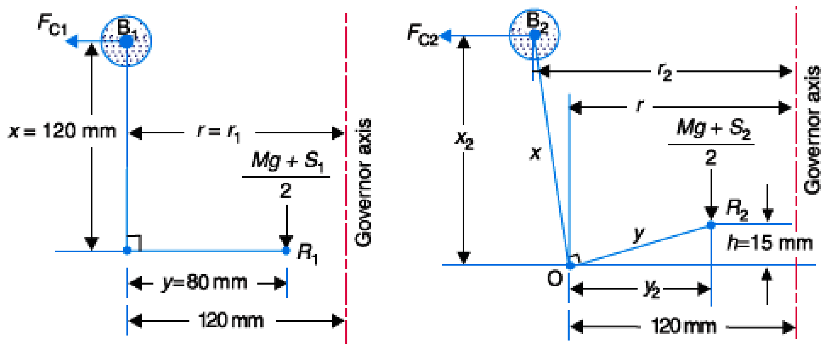
A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine : **1.** loads on the spring at the lowest and the highest equilibrium speeds, and **2.** stiffness of the spring.

Solution. Given : N1 = 290 r.p.m. or ω_1 = 2 _ × 290/60 = 30.4 rad/s ; N2 = 310 r.p.m. or ω_2 = 2 _ × 310/60 = 32.5 rad/s ; h = 15 mm = 0.015 m ; y = 80 mm = 0.08 m ; x = 120 mm = 0.12 m ; r = 120 mm = 0.12 m ; m = 2.5 kg

1. Loads on the spring at the lowest and highest equilibrium speeds

Let S = Spring load at lowest equilibrium speed, and
S2 = Spring load at highest equilibrium speed.

Since the ball arms are parallel to governor axis at the lowest equilibrium speed (i.e. at N1 = 290 r.p.m.), as shown in Fig. (a), therefore r = r1 = 120 mm = 0.12 m



(*a*) Lowest position. (*b*) Highest position.

We know that centrifugal force at the minimum speed,
$FC1 = m\,(\omega_1)^2\, r1 = 2.5\,(30.4)^2\, 0.12 = 277$ N

Now let us find the radius of rotation at the highest equilibrium speed, *i.e.* at *N*2 = 310 r.p.m. The position of ball arm and sleeve arm at the highest equilibrium speed is shown in Fig. (*b*).

Let *r*2 = Radius of rotation at *N*2 = 310 r.p.m.

$$h = (r_2 - r_1)\frac{y}{x}$$

$$r_2 = r_1 + h\left(\frac{x}{y}\right) = 0.12 + 0.015\left(\frac{0.12}{0.08}\right) = 0.1425 \text{ m}$$

Centrifugal force at the maximum speed,

$$FC_2 = m\,(\omega_2)^2\, r_2 = 2.5 \times (32.5)2 \times 0.1425 = 376 \text{ N}$$

Neglecting the obliquity effect of arms and the moment due to the weight of the balls, we have for lowest position,

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 277 \times \frac{0.12}{0.08} = 831 \text{ N}$$

$$\therefore \quad S_2 = 831 \text{ N} \qquad (\because M = 0)$$

and for highest position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 376 \times \frac{0.12}{0.08} = 1128 \text{ N}$$

$$\therefore \quad S_1 = 1128 \text{ N} \qquad (\because M = 0)$$

2. *Stiffness of the spring*

We know that stiffness of the spring,

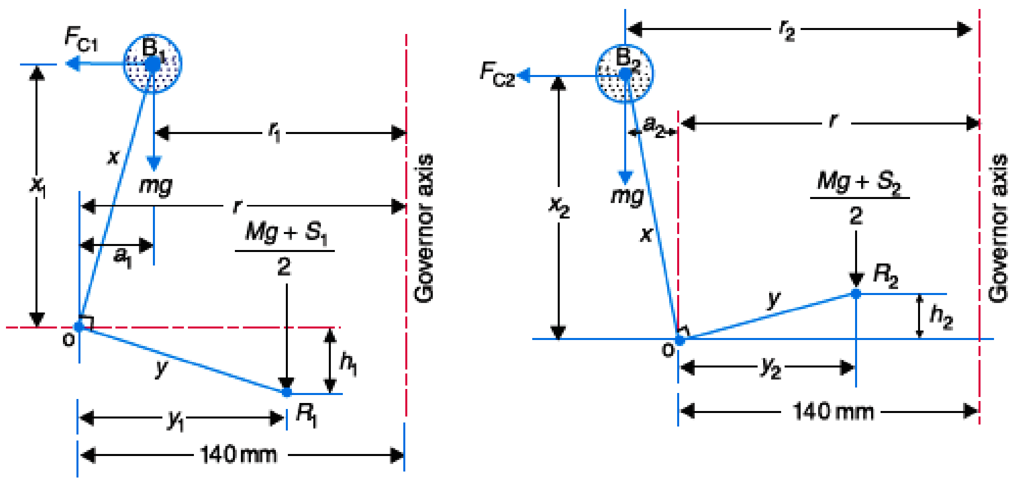
$$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8 \text{ N/mm}$$

In a spring loaded governor of the Hartnell type, the mass of each ball is 5 kg and the lift of the sleeve is 50 mm. The speed at which the governor begins to float is 240 r.p.m., and at this speed the radius of the ball path is 110 mm. The mean working speed of the governor is 20 times the range of speed when friction is neglected. If the lengths of ball and roller arm of the bell crank lever are 120 mm and 100 mm respectively and if the distance between the centre of pivot of bell crank lever and axis of governor spindle is 140 mm, determine the initial compression of the spring taking into account the obliquity of arms. If friction is equivalent to a force of 30 N at the sleeve, find the total alteration in speed before the sleeve begins to move from mid-position.

Solution. Given : m = 5 kg ; h = 50 mm = 0.05 m ; $N1$ = 240 r.p.m. or $\omega_1 = 2\pi \times 240/60$ = 25.14 rad/s ; r_1 = 110 mm = 0.11 m ; x = 120 mm = 0.12 m ; y = 100 mm = 0.1 m ; r = 140 mm = 0.14 m ; F = 30 N

Initial compression of the spring taking into account the obliquity of arms

First of all, let us find out the maximum speed of rotation (ω_2) in rad/s.
We know that mean working speed,



(*b*) Minimum position (*b*) Maximum position

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

and range of speed, neglecting friction

$$= \omega_2 - \omega_1$$

Since the mean working speed is 20 times the range of speed, therefore

$$\omega = 20\,(\omega_2 - \omega_1)$$

or

$$\frac{\omega_1 + \omega_2}{2} = 20\,(\omega_2 - \omega_1)$$

$$25.14 + \omega_2 = 40\,(\omega_2 - 25.14) = 40\,\omega_2 - 1005.6$$

$\therefore$ $\quad 40\,\omega_2 - \omega_2 = 25.14 + 1005.6 = 1030.74$ $\quad$ or $\quad \omega_2 = 26.43$ rad/s

The minimum and maximum position of the governor balls is shown in Fig. (*a*) and (*b*) respectively.

Let $\quad r_2$ = Maximum radius of rotation.

We know that lift of the sleeve,

$$h = (r_2 - r_1)\,\frac{y}{x}$$

or

$$r_2 = r_1 + h \times \frac{x}{y} = 0.11 + 0.05 \times \frac{0.12}{0.1} = 0.17 \text{ m}$$

We know that centrifugal force at the minimum speed,

$$F_{C1} = m\,(\omega_1)^2\, r_1 = 5\,(25.14)^2\, 0.11 = 347.6 \text{ N}$$

and centrifugal force at the maximum speed,

$$F_{C2} = m\,(\omega_2)^2\, r_2 = 5\,(26.43)^2\, 0.17 = 593.8 \text{ N}$$

Since the obliquity of arms is to be taken into account, therefore from the minimum position as shown in Fig. (*a*),

$a_1 = r - r_1 = 0.14 - 0.11 = 0.03$ m

$$x_1 = \sqrt{x^2 - (a_1)^2} = \sqrt{(0.12)^2 - (0.03)^2} = 0.1162 \text{ m}$$

and

$$y_1 = \sqrt{y^2 - (h_1)^2} = \sqrt{(0.1)^2 - (0.025)^2} = 0.0986 \text{ m}$$

$\ldots (\because\ h_1 = h/2 = 0.025 \text{ m})$

Similarly, for the maximum position, as shown in Fig. 18.21 (*b*),

$$a_2 = r_2 - r = 0.17 - 0.14 = 0.03 \text{ m}$$

$\therefore$ $\quad x_2 = x_1 = 0.1162$ m $\quad \ldots (\because\ a_2 = a_1)$

and $\quad y_2 = y_1 = 0.0986$ m $\quad \ldots (\because\ h_2 = h_1)$

Now taking moments about point O for the minimum position as shown in Fig. (*a*),

$$\frac{M.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

$$\frac{S_1}{2} \times 0.0968 = 347.6 \times 0.1162 - 5 \times 9.81 \times 0.03 = 38.9 \text{ N} \quad \ldots (\because\ M = 0)$$

$\therefore$ $\quad S_1 = 2 \times 38.9/0.0968 = 804$ N

Similarly, taking moments about point O for the maximum position as shown in Fig. (*b*),

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

$$\frac{S_2}{2} \times 0.0968 = 593.8 \times 0.1162 + 5 \times 9.81 \times 0.03 = 70.47 \text{ N} \quad \ldots (\because\ M = 0)$$

$\therefore$ $\quad S_2 = 2 \times 70.47/0.0968 = 1456$ N

We know that stiffness of the spring

$$s = \frac{S_2 - S_1}{h} = \frac{1456 - 804}{50} = 13.04 \text{ N/mm}$$

$\therefore$ Initial compression of the spring

$$= \frac{S_1}{s} = \frac{804}{13.04} = 61.66 \text{ mm}$$

Total alternation in speed when friction is taken into account

We know that spring force for the mid-position,

$$S = S_1 + h_1.s = 8.4 + 25 \times 13.04 = 1130 \text{ N} \quad \ldots (\because h_1 = h/2 = 25 \text{ mm})$$

and mean angular speed, $\omega = \dfrac{\omega_1 + \omega_2}{2} = \dfrac{25.14 + 26.43}{2} = 25.785 \text{ rad/s}$

or $N = \omega \times 60 / 2\pi = 25.785 \times 60 / 2\pi = 246.2 \text{ r.p.m.}$

$\therefore$ Speed when the sleeve begins to move downwards from the mid-position,

$$N' = N\sqrt{\frac{S-F}{S}} = 246.2\sqrt{\frac{1130-30}{1130}} = 243 \text{ r.p.m.}$$

and speed when the sleeve begins to move upwards from the mid-position,

$$N'' = N\sqrt{\frac{S+F}{S}} = 246.2\sqrt{\frac{1130+30}{1130}} = 249 \text{ r.p.m.}$$

$\therefore$ Alteration in speed $= N'' - N' = 249 - 243 = 6$ r.p.m. **Ans.**

A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the range of speed, sleeve lift, governor effort and power of the governor in the following cases :

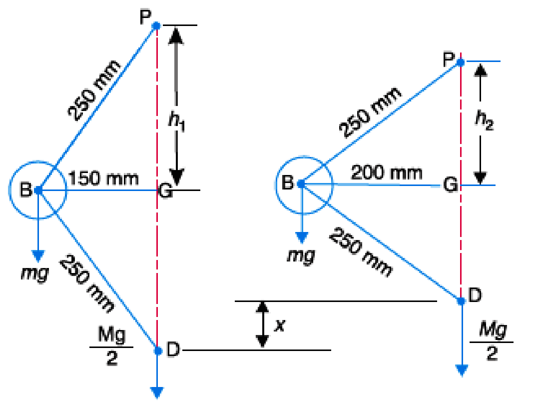
1. When the friction at the sleeve is neglected, and
2. When the friction at the sleeve is equivalent to 10 N.

Given : $BP = BD = 250$ mm ; $m = 5$ kg ; $M = 25$ kg ; $r1 = 150$ mm ; $r2 = 200$ mm ; $F = 10$ N

1. When the friction at the sleeve is neglected

First of all, let us find the minimum and maximum speed of rotation. The minimum and maximum position of the governor is shown in Fig. 18.34 (a) and (b) respectively.

Let $N1$ = Minimum speed, and
N2 = Maximum speed.



(a) Minimum position. (b) Maximum position.

From Fig a

` $h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$

From Fig. (*b*),

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

We know that $(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+25}{5} \times \frac{895}{0.2} = 26\,850$

$\therefore$ $N_1 = 164$ r.p.m.

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+25}{5} \times \frac{895}{0.15} = 35\,800$$

$\because$ $N_2 = 189$ r.p.m.

Range of speed

We know that range of speed = N2 – N1 = 189 – 164 = 25 r.p.m.

Sleeve lift

We know that sleeve lift, x = 2 ($h_1 - h_2$) = 2 (200 – 150) = 100 mm = 0.1 m

Governor effort

Let c = Percentage increase in speed.

We know that increase in speed or range of speed,

c.N1 = $N_2 - N_1$ = 25 r.p.m.

c = 25/N_1 = 25/164 = 0.152

We know that governor effort

P = c (m + M) g = 0.152 (5 + 25) 9.81 = 44.7 N

Power of the governor

We know that power of the governor = P.x = 44.7 × 0.1 = 4.47 N-m

2. When the friction at the sleeve is taken into account

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1} = \frac{5 \times 9.81 + (25 \times 9.81 - 10)}{5 \times 9.81} \times \frac{895}{0.2} = 25\,938$$

$N_1 = 161$ r.p.m.

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2} = \frac{5 \times 9.81 + (25 \times 9.81 + 10)}{5 \times 9.81} \times \frac{895}{0.15} = 37\,016$$

$N_2 = 192.4$ r.p.m.

Range of speed

We know that range of speed = N2 – N1 = 192.4 – 161 = 31.4 r.p.m.

Sleeve lift

The sleeve lift (x) will be same as calculated above.

Sleeve lift, x = 100 mm = 0.1 m

Governor effort

Let c = Percentage increase in speed.

We know that increase in speed or range of speed,

c.N1 = N2 – N1 = 31.4 r.p.m.

c = 31.4/N1 = 31.4/161 = 0.195

We know that governor effort,

P = c (m.g + M.g + F) = 0.195 (5 × 9.81 + 25 × 9.81 + 10) N= 57.4 N

Power of the governor

We know that power of the governor= P.x = 57.4 × 0.1 = 5.74 N-m

Exercise Problems

1) A loaded governor of the Porter type has equal arms and links each 250 mm long. The mass of each ball is 2 kg and the central mass is 12 kg. When the ball radius is 150 mm, the valve is fully open and when the radius is 185 mm, the valve is closed. Find the maximum speed and the range of speed. If the maximum speed is to be increased 20% by an addition of mass to the central load, find what additional mass is required.

[Ans. 193 r.p.m. ; 16 r.p.m.; 6.14 kg]

2) In a Porter governor, the upper and lower arms are each 250 mm long and are pivoted on the axis of rotation. The mass of each rotating ball is 3 kg and the mass of the sleeve is 20 kg. The sleeve is in its lowest position when the arms are inclined at 30° to the governor axis. The lift of the sleeve is 36 mm. Find the force of friction at the sleeve, if the speed at the moment it rises from the lowest position is equal to the speed at the moment it falls from the highest position. Also, find the range of speed of the governor.

[Ans. 9.8 N ; 16 r.p.m.]

3) A Proell governor has all the four arms of length 250 mm. The upper and lower ends of the arms are pivoted on the axis of rotation of the governor. The extension arms of the lower links are each 100 mm long and parallel to the axis when the radius of the ball path is 150 mm. The mass of each ball is 4.5 kg and the mass of the central load is 36 kg. Determine the equilibrium speed of the governor.

[Ans. 164 r.p.m.]

4) A Proell governor has arms of 300 mm length. The upper arms are hinged on the axis of rotation, whereas the lower arms are pivoted at a distance of 35 mm from the axis of rotation. The extension of lower arms to which the balls are attached are 100 mm long. The mass of each ball is 8 kg and the mass on the sleeve is 60 kg. At the minimum radius of rotation of 200 mm, the extensions are parallel to the governor axis. Determine the equilibrium speed of

the governor for the given configuration. What will be the equilibrium speed for the maximum radius of 250 mm?

[Ans. 144.5 r.p.m. ; 158.2 r.p.m.]

5) A spring controlled governor of the Hartnell type with a central spring under compression has balls each of mass 2 kg. The ball and sleeve arms of the bell crank levers are respectively 100 mm and 60 mm long and are at right angles. In the lowest position of the governor sleeve, the radius of rotation of the balls is 80 mm and the ball arms are parallel to the governor axis. Find the initial load on the spring in order that the sleeve may begin to lift at 300 r.p.m. If the stiffness of the spring is 30 kN/m, what is the equilibrium speed corresponding to a sleeve lift of 10 mm?

[Ans. 527 N ; 342 r.p.m.]

6) In a governor of the Hartnell type, the mass of each ball is 1.5 kg and the lengths of the vertical and horizontal arms of the bell crank lever are 100 mm and 50 mm respectively. The fulcrum of the bell crank lever is at a distance of 90 mm from the axis of rotation. The maximum and minimum radii of rotation of balls are 120 mm and 80 mm and the corresponding equilibrium speeds are 325 and 300 r.p.m. Find the stiffness of the spring and the equilibrium speed when the radius of rotation is 100 mm.

[Ans. 18 kN/m, 315 r.p.m.]

7) A Porter governor has all four arms 200 mm long. The upper arms are pivoted on the axis of rotation and the lower arms are attached to a sleeve at a distance of 25 mm from the axis. Each ball has a mass of 2 kg and the mass of the load on the sleeve is 20 kg. If the radius of rotation of the balls at a speed of 250 r.p.m. is 100 mm, find the speed of the governor after the sleeve has lifted 50 mm. Also determine the effort and power of the governor.

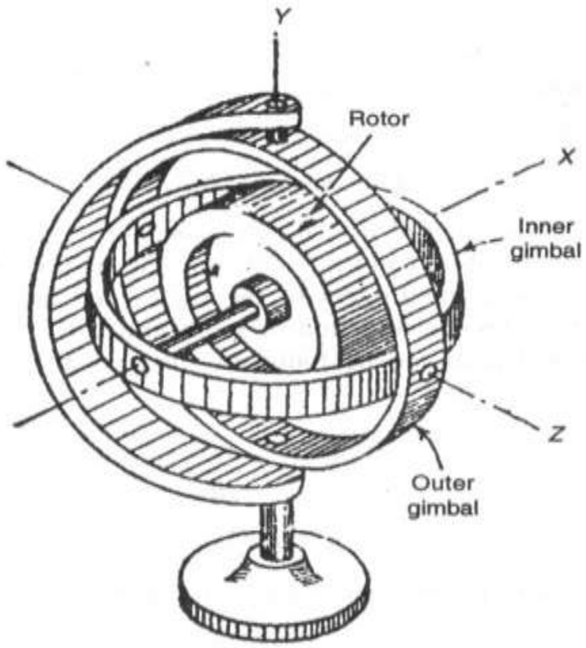
[Ans. 275.6 r.p.m.; 22.4 N; 1.12 N-m]

8) A Porter governor has arms 250 mm each and four rotating flyballs of mass 0.8 kg each. The sleeve movement is restricted to ± 20 mm from the height when the mean speed is 100 r.p.m. Calculate the central dead load and sensitivity of the governor neglecting friction when the flyball exerts a centrifugal force of 9.81 N. Determine also the effort and power of the governor for 1 percent speed change.

[Ans. 11.76 N; 11.12; 0.196 N; 7.7 N-mm]

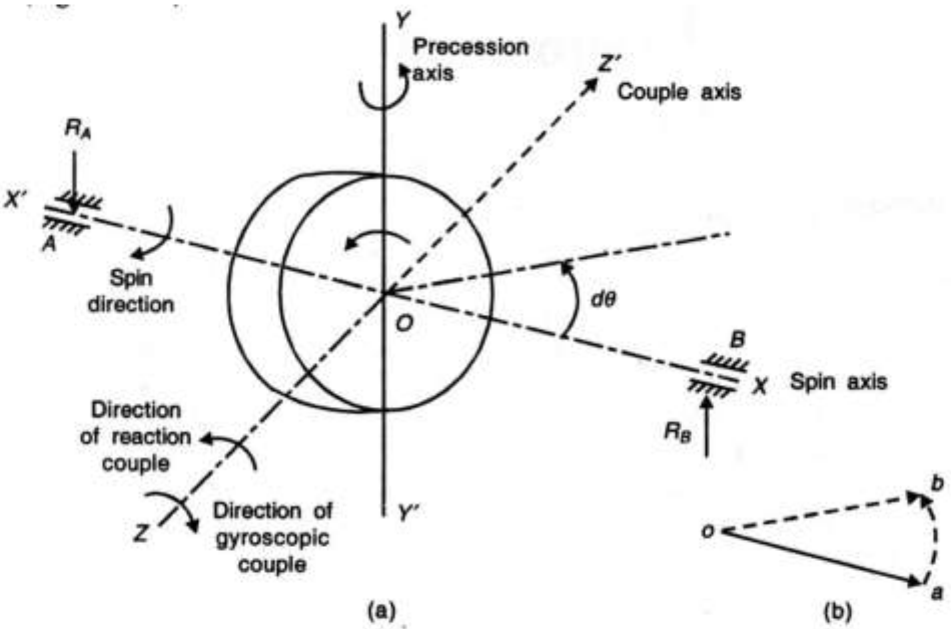
Gyroscopes:

- A gyroscope is a spatial mechanism which is generally employed for the study of precessional motion of a rotary body. Gyroscope finds applications in gyrocompass, used in aircraft, naval ship, control system of missiles and space shuttle. The gyroscopic effect is also felt on the automotive vehicles while negotiating a turn.
- A gyroscope consists of a rotor mounted in the inner gimbal. The inner gimbal is mounted in the outer gimbal which itself is mounted on a fixed frame as shown in Fig.1. When the rotor spins about X-axis with angular velocity ω rad/s and the inner gimbal precesses (rotates) about Y-axis, the spatial mechanism is forced to turn about Z-axis other than its own axis of rotation, and the gyroscopic effect is thus setup. The resistance resistance to change in the direction of rotational axis is called gyroscopic effect.



Gyroscopic couple:

Consider a rotary body of mass *m* having radius of gyration *k* mounted on the shaft supported at two bearings. Let the rotor spins (rotates) about X-axis with constant angular velocity ω rad/s. The X-axis is, therefore, called spin axis, Y-axis, precession axis and Z-axis, the couple or torque axis



The angular momentum of the rotating mass is given by,

$$H = mK^2\omega = I\omega$$

Now, suppose the shaft axis (X-axis) precesses through a small angle δθ about Y-axis in the plane *XOZ,* then the angular momentum varies from *H* to *H* + δ*H,* where δ*H* is the change in the angular momentum, represented by vector ab.For the small value of angle of rotation 50, we can write,

$$ab = oa \times \delta\theta$$

$$\delta H = H \times \delta\theta$$

$$= I\omega\delta\theta$$

However, the rate of change of angular momentum is:

$$C = \frac{dH}{dt} = \lim_{\delta t \to 0}\left(\frac{I\omega\,\delta\theta}{\delta t}\right)$$

$$= I\omega\frac{d\theta}{dt}$$

$$C = I\omega\omega_p$$

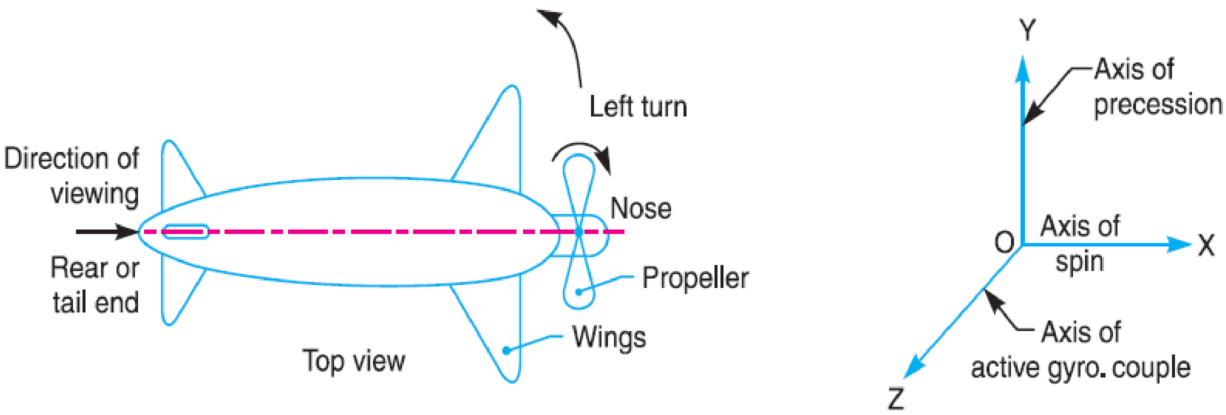
Where,

C = gyroscopic couple (N-m)
ω = angular velocity of rotary body (rad/s)
ωp = angular velocity of precession (rad/s)

The couple *I*.ω.ωp, in the direction of the vector *xx′* (representing the change in angular momentum) is the *active gyroscopic couple*, which has to be applied over the disc when the axis of spin is made to rotate with angular velocity ω_p about the axis of precession. When the axis of spin itself moves with angular velocity ω_p, the disc is subjected to *reactive couple* whose magnitude is same (*i.e. I.* $\omega.\omega_p$) but opposite in direction to that of active couple. This reactive couple to which the disc is subjected when the axis of spin rotates about the axis of precession is known as *reactive gyroscopic couple.*

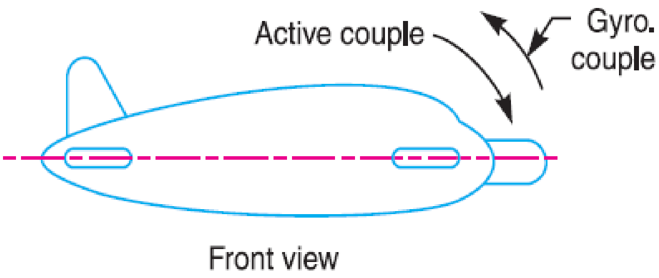
Effect of the Gyroscopic Couple on an Aeroplane:

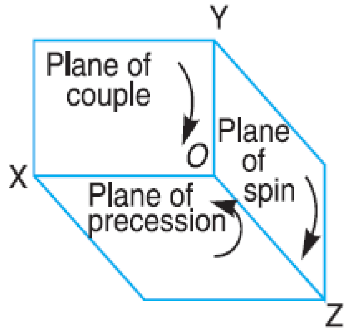
The top and front view of an aeroplane is shown in Fig Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left.



ω = Angular velocity of the engine in rad/s,
m = Mass of the engine and the propeller in kg,
k = Its radius of gyration in metres,
I = Mass moment of inertia of the engine and the propeller in kg-m2
= $m.k^2$,
v = Linear velocity of the aeroplane in m/s,
R = Radius of curvature in metres, and
ω_p = Angular velocity of precession= *v/R* rad/s
∴ Gyroscopic couple acting on the aeroplane,

$C = I.\omega.\omega_p$





1. When the aeroplane takes a right turn under similar conditions as discussed above, the effect of the reactive gyroscopic couple will be to dip the nose and raise the tail of the aeroplane.
2. When the engine or propeller rotates in anticlockwise direction when viewed from the rear or tail end and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to dip the nose and raise the tail of the aeroplane.
3. When the aeroplane takes a right turn under similar conditions as mentioned in note 2 above, the effect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aeroplane.
4. When the engine or propeller rotates in clockwise direction when viewed from the front and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to raise the tail and dip the nose of the aeroplane.
5. When the aeroplane takes a right turn under similar conditions as mentioned in note 4- above, the effect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aeroplane.

GYROSCOPIC EFFECT ON SHIP

Gyroscope is used for stabilization and directional control of a ship sailing in the rough sea. A ship, while navigating in the rough sea, may experience the following three different types of motion

(i) Steering—The turning of ship in a curve while moving forward

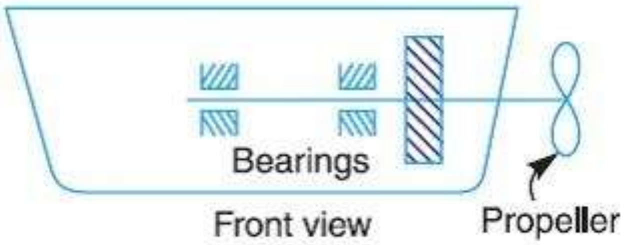
(ii) Pitching—The movement of the ship up and down from horizontal position in a vertical plane about transverse axis

(iii)Rolling—Sideway motion of the ship about longitudinal axis.

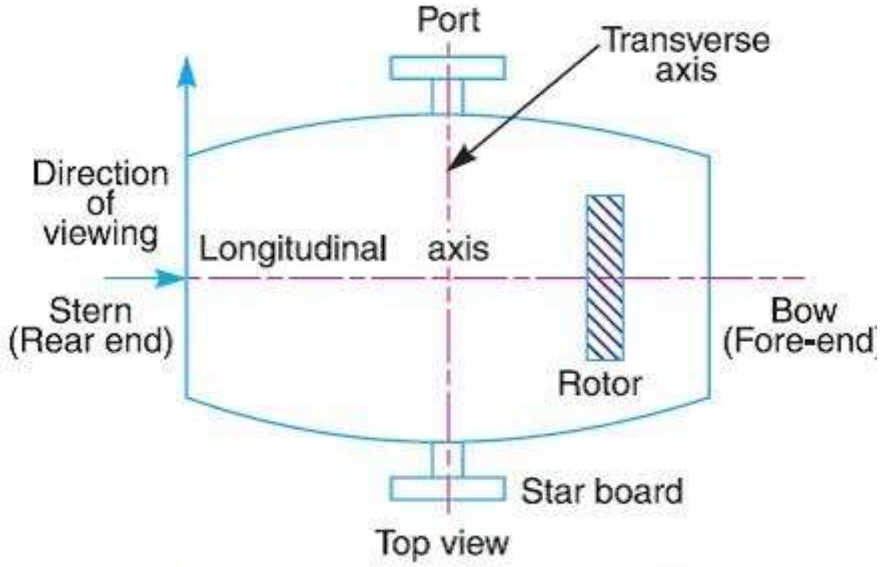
For stabilization of a ship against any of the above motion, the major requirement is that the gyroscope shall be made to precess in such a way that reaction couple exerted by the rotor opposes the disturbing couple which may act on the frame.

The top and front views of a naval ship are shown in Fig. The fore end of the ship is called bow and the rear end is known as stern or aft. The left hand and right hand sides of the ship, when viewed from the stern are called port and star-board respectively.

1. Steering, 2. Pitching, and 3. Rolling.

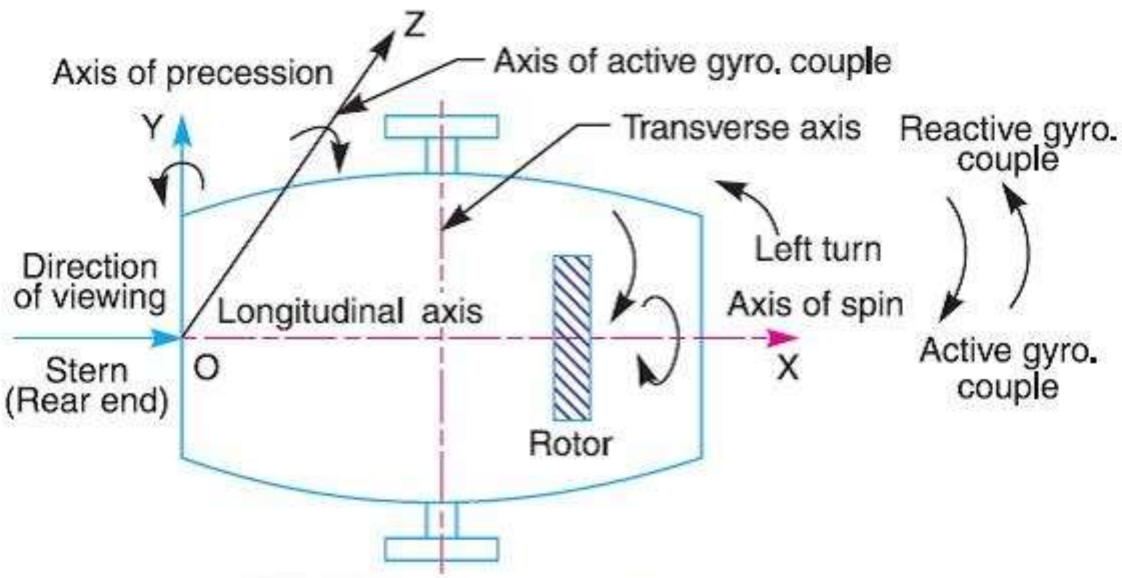


Terms used in a naval ship.



Effect of Gyroscopic Couple on a Naval Ship during Steering

Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. 14.8. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane



Naval ship taking a left turn.

When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction *ox* as shown in Fig. As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector from *ox* to *ox′*. The vector *xx′* now represents the active gyroscopic couple and is perpendicular to *ox*. Thus the plane of active gyroscopic couple is perpendicular to *xx′* and its direction in the axis *OZ* for left hand turn is clockwise as shown. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (*i.e.* in anticlockwise direction). The *effect of this reactive gyroscopic couple is to raise the bow and lower the stern.*

Notes: 1. When the ship steers to the right under similar conditions as discussed above, the effect of the reactive gyroscopic couple, will be to raise the stern and lower the bow.

2. When the rotor rates in the anticlockwise direction, when viewed from the stern and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to lower the bow and raise the stern.

3. When the ship is steering to the right under similar conditions as discussed in note 2 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.

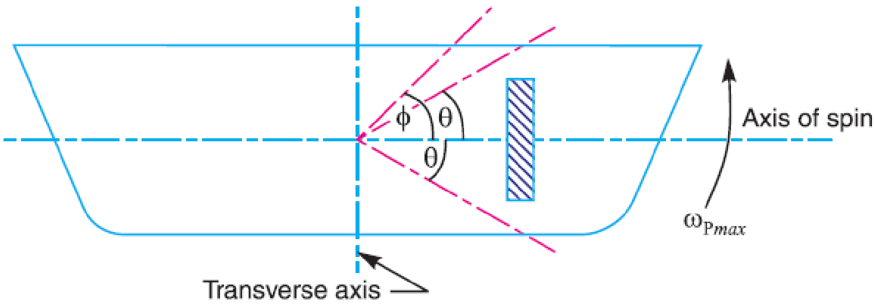
4. When the rotor rotates in the clockwise direction when viewed from the bow or fore end and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to raise the stern and lower the bow.

5. When the ship is steering to the right under similar conditions as discussed in note 4 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.

6. The effect of the reactive gyroscopic couple on a boat propelled by a turbine taking left or right turn is similar as discussed above.

Effect of Gyroscopic Couple on a Naval Ship during Pitching

Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis. In this case, the transverse axis is the axis of precession. The pitching of the ship is assumed to take place with simple harmonic motion *i.e.* the motion of the axis of spin about transverse axis is simple harmonic.



(*a*) Pitching of a naval ship

Notes:

- The effect of the gyroscopic couple is always given on specific position of the axis of spin *i.e.* whether it is pitching downwards or upwards.
- The pitching of a ship produces forces on the bearings which act horizontally and perpendicular to the motion of the ship.
- The maximum gyroscopic couple tends to shear the holding-down bolts.

Effect of Gyroscopic Couple on a Naval Ship during Rolling

- We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

- In case of rolling of a ship, the axis of precession (*i.e.* longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Problems:

1. *A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis*?

   Solution. Given: d = 300 mm or r = 150 mm = 0.15 m ; m = 5 kg ; l = 600 mm = 0.6 m ; N = 300 r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s
   We know that the mass moment of inertia of the disc, about an axis through its centre of
   gravity and perpendicular to the plane of disc,
   $I = m.r2/2 = 5(0.15)2/2 = 0.056$ kg-m2
   and couple due to mass of disc,
   $C = m.g.l = 5 \times 9.81 \times 0.6 = 29.43$ N-m
   Let wP = Speed of precession.
   We know that couple (C),
   $29.43 = I.\omega\, \omega_p = 0.056 \times 31.42 \times \omega P = 1.76\, \omega P$
   $\omega P = 29.43/1.76 = 16.7$ rad/s **Ans.**

2. *An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.*

   Solution:
   Given : R = 50 m ; v = 200 km/hr = 55.6 m/s ; m = 400 kg ; k = 0.3 m ;
   N = 2400 r.p.m. or $\omega = 2\pi \times 2400/60 = 251$ rad/s
   We know that mass moment of inertia of the engine and the propeller,
   $I = m.k2 = 400(0.3)2 = 36$ kg-m2
   and angular velocity of precession,
   $\omega P = v/R = 55.6/50 = 1.11$ rad/s
   We know that gyroscopic couple acting on the aircraft,
   $C = I.\ \omega.\ \omega_p = 36 \times 251.4 \times 1.11 = 100\ 46$ N-m
   $= 10.046$ kN-m Ans.
   when the aeroplane turns towards left, the effect of the gyroscopic couple is to lift the nose upwards and tail downwards.

3. The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.

Solution. Given: $m = 8$ t = 8000 kg ; $k = 0.6$ m ; $N = 1800$ r.p.m. or $\omega = 2\pi \times 1800/60 = 188.5$ rad/s ; $v = 100$ km/h = 27.8 m/s ; $R = 75$ m

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 8000\,(0.6)^2 = 2880 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_P = v / R = 27.8 / 75 = 0.37 \text{ rad/s}$$

We know that gyroscopic couple,

$$C = I.\omega.\,\omega_p = 2880 \times 188.5 \times 0.37 = 200\,866 \text{ N-m}$$

$$= 200.866 \text{ kN-m Ans.}$$

When the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

4. The heavy turbine rotor of a sea vessel rotates at 1500 r.p.m. clockwise looking from the stern, its mass being 750 kg. The vessel pitches with an angular velocity of 1 rad/s. Determine the gyroscopic couple transmitted to the hull when bow is rising, if the radius of gyration for the rotor is 250 mm. Also show in what direction the couple acts on the hull?

Solution. Given: $N = 1500$ r.p.m. or $\omega = 2\pi \times 1500/60 = 157.1$ rad/s; $m = 750$ kg; $\omega_P = 1$ rad/s; $k = 250$ mm = 0.25 m

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 750\,(0.25)^2 = 46.875 \text{ kg-m}^2$$

∴ Gyroscopic couple transmitted to the hull (i.e. body of the sea vessel),

$$C = I.\omega.\omega_p = 46.875 \times 157.1 \times 1 = 7364 \text{ N-m} = 7.364 \text{ kN-m}$$

When the bow is rising i.e. when the pitching is upward, the reactive gyroscopic couple acts in the clockwise direction which moves the sea vessel towards star-board.

5. The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:

1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.
2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum
velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

Solution

Given : $m = 3500$ kg ; $k = 0.45$ m; $N = 3000$ r.p.m. or $\omega = 2\pi \times 3000/60 = 314.2$ rad/s

1. When the ship is steering to the left Given: $R = 100$ m ; $v =$ km/h = 10 m/s

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 3500\,(0.45)^2 = 708.75 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_p = v/R = 10/100 = 0.1 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\,\omega_p = 708.75 \times 314.2 \times 0.1 = 22\,270 \text{ N-m}$$

$$= 22.27 \text{ kN-m Ans.}$$

When the rotor rotates clockwise when looking from the stern and the ship takes a left turn, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

2. When the ship is pitching with the bow falling
Given: tp = 40 s
Since the total angular displacement between the two extreme positions of pitching is 12°
(i.e. $2\varphi = 12°$), therefore amplitude of swing,

$$\varphi = 12 / 2 = 6° = 6 \times \pi/180 = 0.105 \text{ rad}$$

and angular velocity of the simple harmonic motion,

$$\omega1 = 2\pi / tp = 2\pi / 40 = 0.157 \text{ rad/s}$$

We know that maximum angular velocity of precession,

$$\omega_p = \varphi.\omega1 = 0.105 \times 0.157 = 0.0165 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\ \omega_p = 708.75 \times 314.2 \times 0.0165 = 3675 \text{ N-m}$$
$$= 3.675 \text{ kN-m Ans.}$$

When the bow is falling (i.e. when the pitching is downward), the effect of the reactive gyroscopic couple is to move the ship towards port side.

EXERCISES

1. A flywheel of mass 10 kg and radius of gyration 200 mm is spinning about its axis, which is horizontal and is suspended at a point distant 150 mm from the plane of rotation of the flywheel. Determine the angular velocity of precession of the flywheel. The spin speed of flywheel is 900 r.p.m. [Ans. 0.39 rad/s]

2. A horizontal axle AB, 1 m long, is pivoted at the mid point C. It carries a weight of 20 N at A and a wheel weighing 50 N at B. The wheel is made to spin at a speed of 600 r.p.m in a clockwise direction looking from its front. Assuming that the weight of the flywheel is uniformly distributed around the rim whose mean diameter is 0.6 m, calculate the angular velocity of precession of the system around the vertical axis through C.
[Ans. 0.52 rad/s]

3. Each paddle wheel of a steamer have a mass of 1600 kg and a radius of gyration of 1.2 m. The steamer turns to port in a circle of 160 m radius at 24 km / h, the speed of the paddles being 90 r.p.m. Find the magnitude and effect of the gyroscopic couple acting on the steamer. [Ans. 905.6 N-m]

4. An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hour. The rotary engine and the propeller of the plane has a mass of 400 kg with a radius of gyration of 300 mm. The engine runs at 2400 r.p.m. clockwise, when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it. What will be the effect, if the aeroplane turns to its right instead of to the left?
[Ans. 10 kN-m]
5. The rotor of the turbine of a yacht makes 1200 r.p.m. clockwise when viewed from stern. The rotor has a mass of 750 kg and its radius of gyration is 250 mm. Find the maximum gyroscopic couple transmitted to the hull (body of the yacht) when yacht pitches with maximum angular velocity of 1 rad /s. What is the effect of this couple?
[Ans. 5892 N-m]