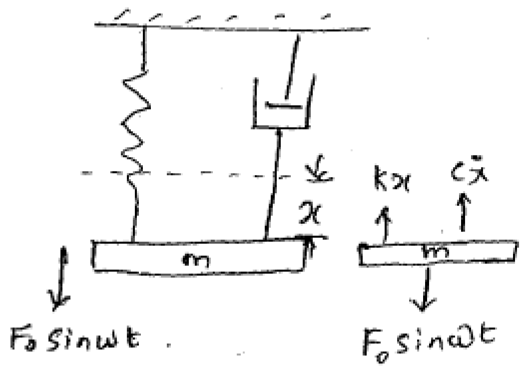
UNIT - IV

Force Vibration with Harmonic Excitation

Consider a spring mass system with viscous damping and subjected to a harmonic excitation $F_0 \sin\omega t$, in which F_0 is constant.Consider the mass to be displaced by a distance ‘x’ downwards with respect to static equilibrium position as the reference line. Selection of static equilibrium position as the reference line eliminates the need to consider the weight of the mass [which is nullified by spring force due to deflection] in the free body diagram.



The type of vibration which occurs under the influence of external force, is called “FORCED VIBRATION”. The external force is called External excitation.

The excitation may be periodic, impulsive or random in nature.

Sources of Excitation

- Thermal effect, (un even expansion of embers give rise to unbalance,
- Resonance (large amplitudes), loose or defective mating part, bent shaft (because of critical speeds)

From Neωton's 2^{nd} law of motion.

Rate of change of momentum = - F_s- F_d + F_o Sinωt

i.e, $m\ddot{x} = -kx - c\dot{x} + F_o \sin\omega t$

$\Rightarrow \boxed{m\ddot{x} + c\dot{x} + kx = F_o \sin\omega t}$ … (1)

⇑ Governing Differential Equation.

Eqn (1) is Non-homogeneous, 2^{nd} order Differential equation of motion. The complete solution consists of two parts (i) complementary function part [x_c] (ii) particular integral part [x_p]

We know, that

$X_c = X_e^{-}\zeta\omega_n t$ .sin [ω_d +ψ]

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$

Due to exponential decay, x_c dies out eventually. This part is called **'transient motion'**

Particular integral part [x_p]

x_p = Xsin [ωt -ψ] …(2)

Where $\boxed{X = \frac{F_o}{\sqrt{[k-m\omega^2]^2+[c\omega]^2}}}$ …(2A)

The amplitude of x of the particular Internal Part Replace ω by does not depend on time. In other words, the amplitude of vibration represented by X_p does not change with time and therefore it is called steady state motion.

Also $\tan\psi = \frac{2\zeta r}{1-r^2}$ ...(3)

Where r = frequency ratio = $\frac{\omega}{\omega_n}$ & ψ = Phase angle

(Note $\omega<\omega_n$ then $r < 1$
$\omega>\omega_n$ then $r > 1$)

Equation (2A) can be written as,

$$\boxed{X = \frac{x_0}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}} \quad ...(4)$$

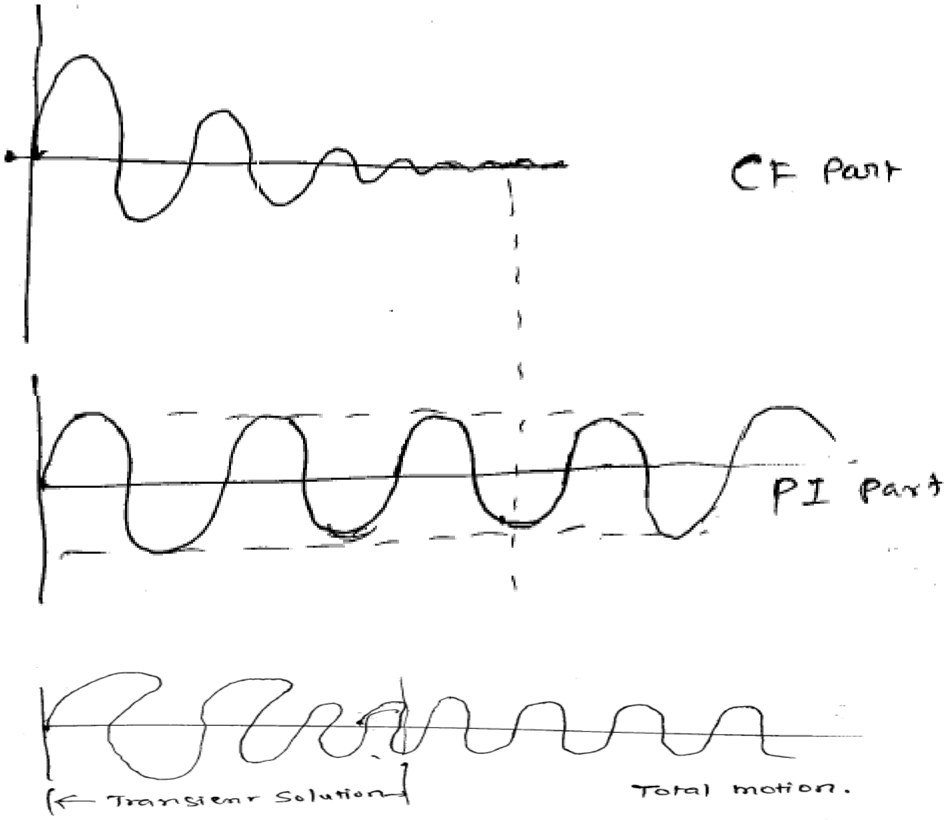
Let M $= \frac{x}{x_o}$ → Magnification Factor

Then M $= \frac{x}{x_o} = \frac{1}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$ ...(5)

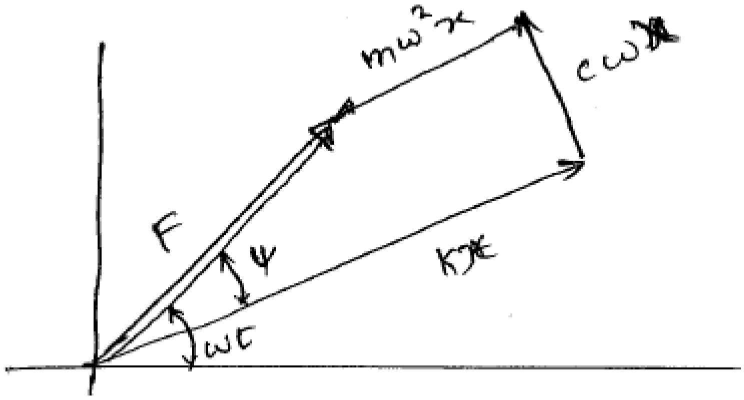
Note 1 : Eqn. (5) has been got from (2A), as follows. In Eqn. 2A ÷ by K (spring stiffness) in both Numerator & Denominator and be simplified.

Note 2 : Magnification factor : (M) : It is the ratio of maximum displacement of forced vibration to the static deflection due to static force.

Note 3 :



Note 4 : Vector representation of forced vibration with damping.

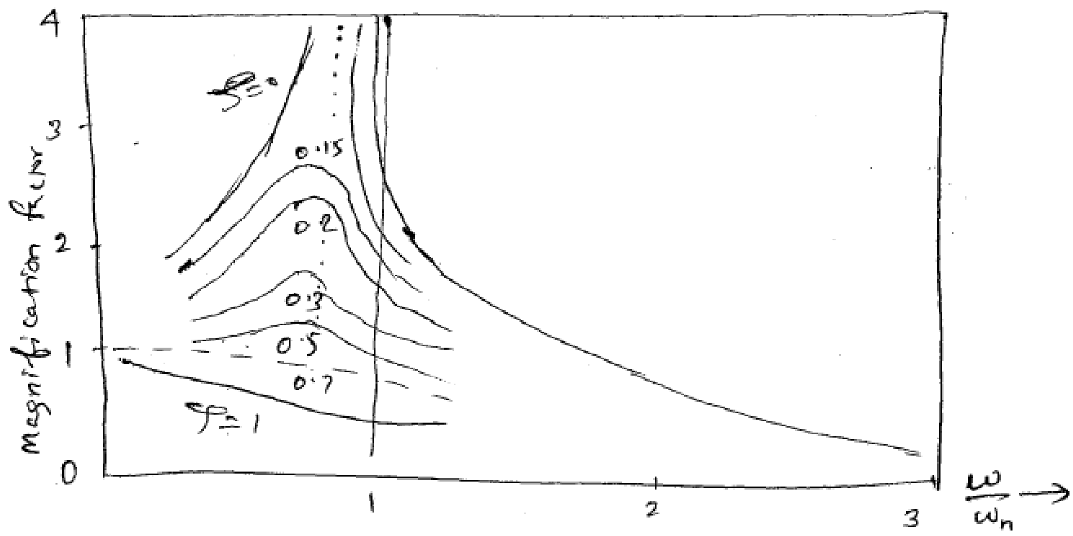


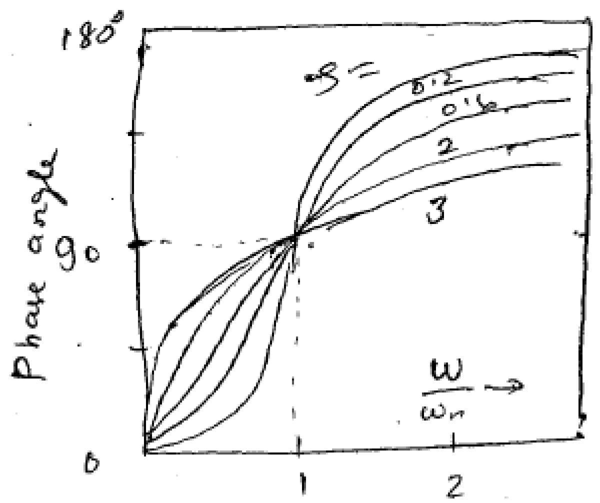
Also $\tan\psi = \frac{2\zeta r}{1-r^2}$

Note 5 : The frequency at which max amplitude occurs is given by

$\frac{\omega_{max}}{\omega_n} = \sqrt{1 - 2\zeta^2}$ where ω_{max} – force corresponding to maximum amplitude.

Characteristic curves: A curve between frequency ratio and magnification factor is known as **frequency response curve**. Similarly a curve between phase angle and frequency ratio is known as **phase – frequency response curve**.





The following points are noted:

1. At zero frequency magnification is unity and damping does not have any effect on it.

2. Damping reduces the magnification factor for all values of frequency.
3. The maximum value of amplitude occurs, a little towards left at resonant frequency.
4. At resonant frequency the phase angle is 90^o.
5. The phase angle increases for decreasing value of damping above resonance.
6. The amplitude of vibration is infinite at resonant freq. and zero damping factor.
7. The amplitude ratio is below unity for all values of damping which was more than 0.70.
8. The variation in phase angle is because of damping without damping it is either 180^o or 0^o)

Variation of frequency Ratio ω/ω_n

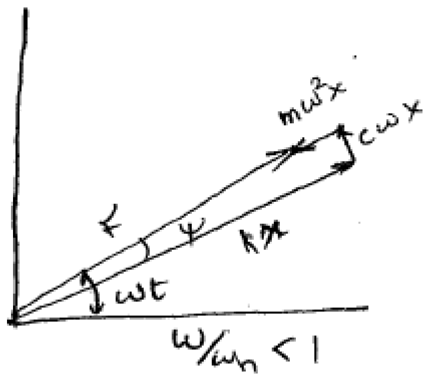
- Three possibilities of ω variation i.e., $\omega<\omega_n$, $\omega = \omega_n$ & $\omega>\omega_n$

Case : i $\quad \frac{\omega}{\omega_n} << 1$

$\therefore \omega$ is very small

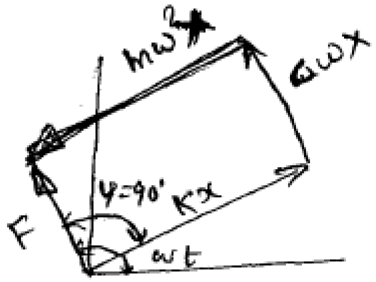
$\therefore \underbrace{m\omega^2 x}_{\substack{\uparrow \\ \text{Inertia force}}}$ & $\underbrace{c\omega x}_{\substack{\uparrow \\ \text{damping force}}}$ get reduced greatly

This results in small value of ψ



Case : ii

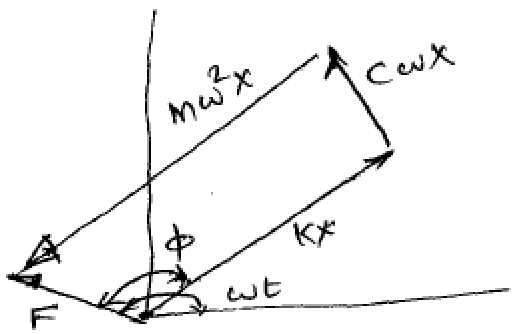
$\frac{\omega}{\omega_n}$ = 1 when $\omega = \omega_n$ i.e, Excitation ω = natural Frequency



Here, inertia force = spring force

Excitation force balances the damping force. $\therefore$ x= $\frac{F_o}{c\omega_n}$

Case : iii $\frac{\omega}{\omega_n}$>> 1



At very high frequencies of ω inertia force increases very rapidly. Damping & spring forces are small in magnitude. For high values of $\frac{\omega}{\omega_n}$ phase angle ψ is close to 180^o.

List of Formulae :

1. $\omega = \frac{2\pi N}{60}$ rad/s

2. K = load/deflection N/m

3. Static deflection $X_o = F_{o/k}$ $\frac{N}{N/m} = \frac{N}{1} x \frac{m}{N} = m$

4. $r = \frac{\omega}{\omega_n}$ → frequency ratio & $\omega = \frac{2\pi N}{60}$ rps

5. $\zeta = \frac{C}{C_c}$;

6. $C_c = 2\sqrt{km} = 2m\omega_n$

7. δ = log, decrement = $\frac{1}{n} \ln\left(\frac{x_o}{x_n}\right); = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$

8. When damper is not present (ζ=0)

$\frac{X_{\max}}{X_o} = \frac{1}{r^2-1}$ if r < 1

$\frac{X_{\max}}{X_o} = \frac{1}{1-r^2}$ if r >1

9. When damper is present

$\frac{X_{\max}}{X_o} = = \frac{1}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$ & $x_o = \frac{F_o}{K}$

$\omega_n = \sqrt{\frac{k}{m}}$ $\therefore \omega_n^2 = \frac{k}{m}$ & $K = m\omega_n^2$

10. Amplitude at resonance $[X_{max}]_{resonance} = \frac{x_o k}{c\omega_n} = \frac{F_o}{c\omega_n}$

11. Force transmitted to the foundation, F_T,

$$\frac{F_T}{F_o} = \frac{\sqrt{1+(2\,\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

Q. A 300 kg Single cylinder vertical petrol engine is mounted upon chasis frame and cause defection = 1.5mm. Mass of reciprocating parts of engine = 25 kg and it has a stroke of 145mm. A dashpot is provided whose damping resistance = 1.5 KN/(m/s) Determine (1) Amplitude of forced vibration when driving shaft rotates at 480 rpm (2) speed of the driving shaft at which resonance occurs.

m = 300 kg Deflection $\Delta = 1.5\text{mm} = 1.5 \times 10^{-3}\text{m}$ m = 25 kg

$L = 145 \times 10^{-3}\text{m}$ $\therefore$ radius of crank $= \frac{L}{2} = 0.0725$ m

$C = 1.5 \times 10^3 \frac{N}{(\frac{m}{S})}$ Forcing speed N = 480 rpm

Forcing angular speed $\omega = \frac{2\pi N}{60} = 50.3$ rad/s

$K = \frac{load}{\text{deflection}} = \frac{300\ (9.81)}{1.5\ x\ 10^{-3}} = 1.96 \times 10^6$ N/m

$C = 1.5 \times 10^3 \frac{N}{(m/S)}$

$C_c = 2\sqrt{km} = 2\left\{\sqrt{1.96\ x\ 10^6(300)}\right\} = 48497.4\ \frac{N}{(m/S)}$

$\zeta = \frac{C}{C_c}$ = 0.0309

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.96\ x\ 10^6}{300}}$ = 80.83 rad/s

r $= \frac{\omega}{\omega_n} = \frac{50.3}{80.829}$ = 0.622

Let due to reciprocating parts centrifugal force gets developed,

$F_o = m_1$, (eccentric radius)ω^2

= 25 (0.0725) $(50.3)^2$ = 4585.78N

$$X = \frac{X_o}{\sqrt{[(1-r^2)]^2+(2\zeta r)^2}} = \frac{(4585.78/\ 1.96\ x\ 10^6)}{\sqrt{[1-0.622^2]^2+[2\ (0.0309)(0.622)]^2}}$$

$= 3.8\ x\ 10^{-3}$ m = 3.8mm

Speed of driving shaft at which resonance occurs,

when $\omega=\omega_n$ resonance occurs.

$\omega = \omega_n = \sqrt{\frac{K}{m}}$ = 80.83 rps

$\frac{2\pi N}{60}$ = 80.83 $\Rightarrow$ N = 771.86 rpm

Note:1 For $0<\zeta<\frac{1}{\sqrt{2}}$ = 0.707, the max value of M occurs when $\sqrt{1-2\zeta^2} = \frac{\omega}{\omega_n}$

2. The maximum value of x when r $=\sqrt{1-2\zeta^2}$ is given by $\frac{x}{x_0} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$

and the value of x at $\omega = \omega_n$ $\left[\frac{x}{x_0}\right]_{\omega=\omega_n} = \frac{1}{2\zeta}$

3. For small values of damping (ζ< 0.05), we

can take $\left[\frac{x}{x_0}\right]_{max} \cong \left[\frac{x}{x_0}\right]_{\omega=\omega_n} = \frac{1}{2\zeta} = Q$

Q factor or Quality factor of the system

4. $[X_{max}]_{resonance} = \frac{x_o\ K}{C\omega_n} = \frac{F_o}{c\omega_n}$

Q. A machine part having a mass of 2.5kg executes vibration in a viscous damping medium. A harmonic exciting force of 30N acts on the part and causes a resonant amplitude of 14mm, with a period of 0.22S. Find the damping coefficient when the frequency of exciting force is changed to 4 Hz. Determine the increase of forced vibration upon the removal of damper.

I Impressed force F_o=30N, m = 2.5kg, t_p = 0.22s,

$$f_n = \frac{1}{t_p} = 4.545\text{Hz}$$

$[X_{max}]_{resonance}$ = 14mm = 14 x 10^{-3}m. c = ? when f = 4 Hz

ω_n = $2\pi f_n$ = 2π (4.545) = 28.56 rad/s

ω = $2\pi f$ = 2π (4) = 25.13 rad/s

$r = \frac{\omega}{\omega_n} = \frac{25.13}{28.56} = 0.8817$

Critical damping co eff. $C_c = 2m\omega_n = 2\sqrt{km}$

$= 2\ (2.5)\ (28.56) = 142.8\ \frac{N}{(m/s)}$

$$[X_{max}]_{resonance} = \frac{x_o k}{c\omega_n} = \frac{F_o}{c\omega_n}$$

i.e., $14 \times 10^{-3} = \frac{30}{c(28.56)}$

$$\Rightarrow c = 75.03 \frac{N}{(m/s)}$$

Damping factor, $\zeta = \frac{c}{c_c}$

$$= \frac{75.03}{142.8}$$

$$= 0.525$$

When damper is not removed, $\frac{X}{X_o} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$

$$\frac{X}{X_o} = \frac{1}{\sqrt{[1-0.8817^2]^2+[2(0.525)(0.8817)]^2}} = 1.0502$$

$$X_{max} = \frac{(F_o/k)}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{30}{m\omega_n^2}[1.0502]$$

$$= \frac{30}{(2.5)(28.56)^2}(1.0502)$$

$$= 0.01545 \text{ m}$$

$$= 15.45 \text{ mm}$$

When damper is removed

$$\frac{X_{max}}{X_0} = \frac{1}{1-r^2} \quad r < 1$$

$$X_{max} = \frac{F_o/k}{1-r^2} \qquad \left(\therefore X_o = \frac{F_0}{k}\right)$$

$= \frac{30}{2.5(28.56)^2} \left[\frac{1}{1-(0.8817)^2}\right]^{\& \, k=m\omega_n^2}$

= 0.06609m

= 66.09mm

% increase in amplitude

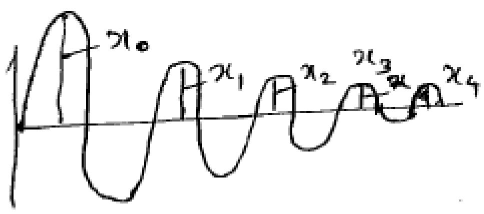
$= \frac{[X_{\max}]_{Nodamping} - [X_{\max}]_{\text{damping}}}{[X_{\max}]_{\text{damping}}}$

= 66.09 = 15.45

= 321.92%

Q. A 12 Kg mass is suspended from end of helical spring, other end is fixed, spring stiffness =15 N/mm. Due to viscous damping, amplitude decreases to 1/10th of initial value in 4 oscillations. If a periodic force 150 cos 50t N is applied at mass in vertical direction, find amplitude of forced vibration. What is its value of resonance?

m = 12 kg   k = $15\frac{N}{\text{mm}}$ x $\frac{1000\text{mm}}{1\text{m}} = 15\, x\, 10^3\,\frac{\text{N}}{\text{m}}$



$x_4 = 0.1\, x_o$

$\therefore \frac{x_o}{x_4} = 10$

$\therefore$ Logarithmic decrement, $\delta = \frac{1}{4}\ln[10]$ = 0.5756

$\zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$ = 0.09122

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{15\ x\ 10^3}{12}}$ = 35.3 rps

$F_x = F_o \cos \omega t$

$= 150 \cos 50t$

$\therefore F_o = 150$ N & $\omega = 50$ rad/s

$\therefore$ frequency ratio $r = \frac{\omega}{\omega_n} = \frac{50}{35.3}$ = 1.416

$C_c = 2\sqrt{km} = 2\sqrt{15\ x\ 10^3\ (12)}$ = 848.52 $\frac{N}{(m/s)}$

$\zeta = \frac{c}{C_c}$

$0.09122 = \frac{c}{848.52} \Rightarrow c = 77.40 = \frac{N}{(m/s)}$

$$\frac{X}{Fo/k} = \frac{1}{\sqrt{[1-1.416^2]^2+[2(0.09122)(1.416)]^2}}$$

$$\frac{X}{[150/15x10^3} = 0.09637$$

$X = \left[\frac{150}{15\ x10^3}\right] 0.9637$

$= 9.637 \times 10^{-3}$m

= 9.637mm

Amplitude at resonance,

$[X_{max}]_{resonance} = \frac{x_o k}{c\omega_n} = \frac{F_o}{c\omega_n}$

$$= \frac{150}{(77.40)(35.3)}$$

$$= 0.05490\text{m}$$

$$= 54.9\text{mm}$$

Note:

Let Force transmitted to the foundation = F_T

$$\frac{F_T}{F_o} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{[1-r^2]^2+(2\zeta r)^2}}$$

$$\text{Also } \frac{X}{X_0} = M = \frac{1}{\sqrt{(1-r^2)^2+(2\varsigma r)^2}}$$

Q. A machine of mass one tonne is acted upon by an external force of 2450N at a frequency of 1500 rpm. To reduce the effects of vibration, isolator of rubber having a static deflection of 2mm under the m/c load and an estimated damping $\zeta = 0.2$ are used. Determine (1) the force transmitted to the foundation (2) the amplitude of vibration of machine (3) The phase lag.

Given Static deflection $\Delta = 2 \times 10^{-3}$m m= 1000kg F = 2450 N

Forcing frequency $\omega = \frac{2\pi N}{60} = \frac{2\pi(1500)}{60} = 157$ rad/s

Damping factor, $\zeta = 0.2$

$$K = \frac{\text{Force or load}}{\text{Static deflection}} = \frac{mg}{\Delta} = \frac{1000\ (9.81)}{2 \times 10^{-3}} = 49 \times 10^5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{49 \times 10^5}{1000}} = 70 \text{ rad/s}$$

Frequency ratio, $r = \frac{\omega}{\omega_n} = \frac{157}{70} = 2.2428$

Let force transmitted to the foundation, = F_T

$$\frac{F_T}{F_o} = \frac{\sqrt{1+[2\zeta r]^2}}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$$

$$\frac{F_T}{F_o} = \frac{\sqrt{1+[2(0.2)(2.24280]^2}}{\sqrt{[1-2.2428^2)^2+[2(0.2)(2.2428)]^2}}$$

$$\frac{F_T}{2450} = \frac{1.3434}{4.128}$$

$$= 0.3254$$

Also

$$\frac{X}{X_o} = \frac{1}{\sqrt{(1-r)^2+(2\zeta r)^2}}$$

$$X = \frac{F_o/k}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$= \frac{[2450\ /49\text{x}10^5]}{4.128}$$

$$=1.211 \text{ x } 10^{-4}\text{m}$$

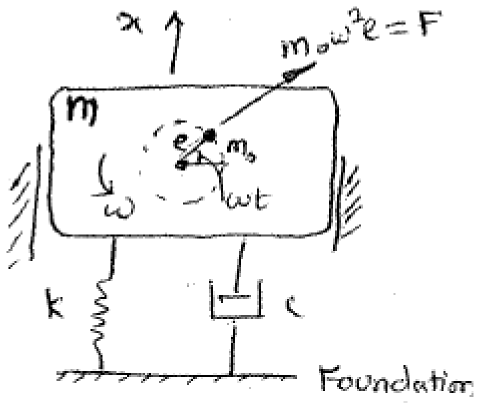
$$= 0.121\text{mm}$$

Let phase lay = ψ

$$\text{Tan } \psi = \frac{2\varsigma r}{1-r^2} = \frac{2(0.2)(2.24)}{1-2.24^2} = -0.223 \qquad \therefore \psi = \tan^{-1}(\psi) = -12.57^o$$

Q. A reciprocating pump 200 kg is driven through a belt by an electric motor at 3000 rpm. The pump is mounted on isolators with total stiffness 5 MN/m and damping 3.125 KN.S Determine (i) the vibratory amplitude of the pump at the running speed due to fundamental harmonic force of excitation 1 KN (ii) Also find the max vibratory amplitude when the pump is switched on and the motor speed passes through resonant condition.

RESPONSE OF A ROTATING AND RECIPRORATING UNBALANCE SYSTEM



Rotating Unbalance

- A machine having rotor as one of its components is called a rotating machine eg. Turbines and I.C. Engines.
- When C.G. of rotor does not coincide with axis of rotation then unbalance occurs.

Let : e = Distance between axis of rotation & C.G. = Eccentric radius

m_o= mass acting at distance 'e' (i.e, Eccentric mass)

Centrifugal force = $m_o e\omega^2$

At any moment vertical displacement = x + esinωt

The centrifugal force $m_o e\omega^2$ has two components vertical and horizontal. The vertical component has the significance and is given by

$m_o e\omega^2$ sinωt.

When we consider single degree Problem(motion in vertical) the excitation is available in vertical direction

i.e., F_o sinωt = $m_o e\omega^2$ sinωt.

$$[m-m_o]\frac{d^2x}{dt^2} + m_o\frac{d^2}{dt^2}(x + e\sin\omega t) + kx + c\frac{dx}{dt} = 0$$

$$(m-m_o)\ \ddot{x} + m_o\ddot{x} - m_o e\omega^2\ e\sin\omega t + kx + c\dot{x} = 0$$

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = m_o e\omega^2\ \sin\omega t$$

Where $m_o e\omega^2 = F_o$

It represents forced vibration.

Note -1 :

$$\frac{X}{X_o} = \frac{1}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$$ holds good

Where $X_o = \frac{F_0}{k} = \frac{m_o e\omega^2}{k}$

$$\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{1}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$$

At resonance, $\omega = \omega_n$ $\therefore$ r = 1

Then $\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{1}{2\zeta}$ and share angle tan $\psi = \frac{2\zeta r}{1-r^2}$

The complete solution $x = x_c + x_p$

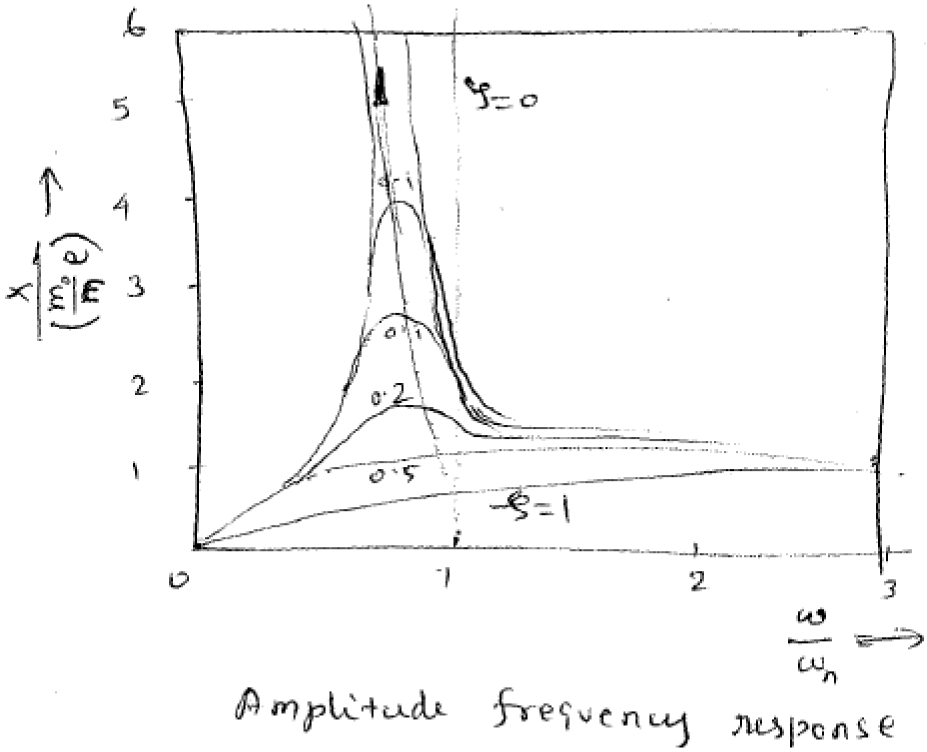
$$x = x_e^{-\zeta w_n t}[\cos(\omega_d t + \psi)] + \frac{(m_o e\, \omega^2/k)}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

Characteristic Curve:

1. Damping plays very important role during resonance.
2. When $\omega \ll \omega_n$ then the system is known as low speed system for a low speed system $\frac{X}{\frac{m_o}{m}e} \rightarrow 0$
3. When $\omega \gg \omega_n$, then the system is called high speed system.

   Here $\frac{X}{\left(\frac{m_o}{m}e\right)} \rightarrow 1$
4. At very high speed effect of damping is negligible.

5. The peak occurs at the left (in comparison to the previous characteristic curve



Amplitude frequency response curve

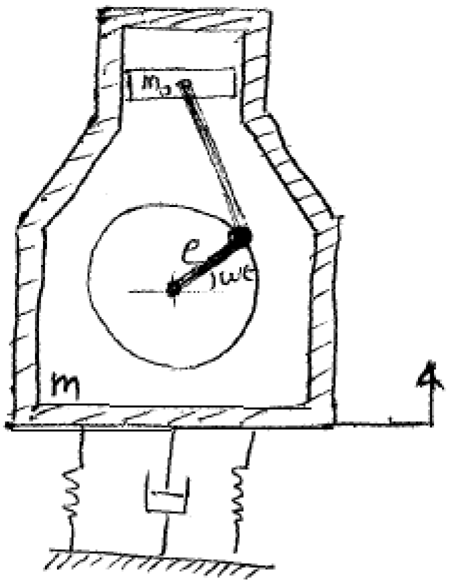
6. At resonance $\omega = \omega_n$&

$$\frac{X}{\left(\frac{m_o}{m}e\right)} = \frac{1}{2\varsigma}$$

Amplitude frequency response curve

Inertia force due to reciprocating mass approximately is equal to

$$F_o = m_o e\omega^2 \left[\sin\omega t + \frac{e}{l}\sin 2\omega t\right]$$



Reciprocating Unbalance

If e is very small as compared to 'l', second term (2nd Harmonic) can be neglected &

$F_o = m_o e\omega^2 \sin\omega t$

Q. In a vibrating system the total mass of the system is 25kg. At speed of 1000 rpm., the system and eccentric mass have a phase difference of 90° and the corresponding amplitude is 1.5cm. The eccentric unbalanced mass of 1 kg has a radius of rotation 4cm. Determine (i) the natural frequency of the system (ii) the damping factor (iii) the amplitude at 1500 rpm and (iv) the phase angle at 1500 rpm.

$m = 25$ kg $\quad x = 1.5 \times 10^{-2}$m $\quad m_o = 1$ kg $\quad e = 4 \times 10^{-2}$m

At phase angle 90°, the condition of resonance occurs [$\omega = \omega_n$]

(i) $f_n \Rightarrow$ rps

$$f_n = \frac{N}{60} = \frac{1000}{60} = 16.67 \text{ cycles/s}$$

At resonance

ii. $\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{1}{2\zeta}$

$\Rightarrow \zeta = \frac{1}{x}\left(\frac{m_o e}{m}\right)\frac{1}{x} = \frac{m_o e}{2mx}$

$= \frac{(1)(4 \ x \ 10^{-2})}{2(25)(1.5 \ x \ 10^{-2})}$

$= 0.053$

$r = \frac{\omega}{\omega_n} = \frac{f}{f_n} = \frac{N}{N_n} = \frac{1500}{1000} = 1.5$

Amplitude at 1500 rpm, is x

$\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\varsigma r)^2}}$

$\frac{X}{\left(\frac{1}{25}\right)(4x10^{-2})} = \frac{(1.5)^2}{\sqrt{[1-1.5^2]^2+[2(0.053)(1.5)]^2}} \Rightarrow$ x = 0.226 x 10^{-2}m

The phase angle at 1500 rpm.

Now r $= \frac{1500}{1000} = 1.5$

$\tan \psi = \frac{2\varsigma r}{1-r^2}$

$= \frac{2(0.053)(1.5)}{1-1.5^2}$

$\Rightarrow \psi = -7.249^o = 180^o - 7.249^o = 172.75^o$

Q. An electric motor is supported on a spring and a dashpot. The spring has the stiffness 6400 N/m and the dashpot offers resistance of 500N at 4 m/s.The unbalanced mass 0.5kg rotates at 5cm radius and the total mass of vibratory system is 20 kg. The motor runs at 400 rpm. Determine 1) Damping factor, 2) amplitude of vibration& phase angle, 3) resonant speed & resonant amplitude 4) Force exerted by the spring and dashpot on the motor.

K = 6400 N/m

Damping force = 500 N

Velocity = 4 M/s.

$\therefore$ c = $\frac{\text{Damping force}}{\text{velocity}} = \frac{500}{4} = 125 \frac{N}{(m/s)}$

Eccentric mass, m_o = 0.5 kg

Eccentric radius, e = 0.05m

Total mass, m = 20kg

Forcing speed, N = 400 rpm $\therefore \omega = \frac{2\pi N}{60}$ =41.866 rad/s

Determine, (i) Damping factor, ζ = ?

(ii) Amplitude, x & Phase angle, ψ = ?

(iii) Resonant speed= Critical speed = ?

and $X_{resonance}$ = ?

(iv) Resultant force on motor = ?

Now

$C_c = 2\sqrt{km} = 2\sqrt{(6400)(20)} = 715.54 \ \frac{\text{N}}{\text{m/s}}$

(1) Damping ratio, $\zeta = \frac{C}{C_c} = \frac{125}{715.54} = 0.175$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6400}{20}}$ = 17.88 rad/s

Frequency ratio r = $\frac{\omega}{\omega_n} = \frac{41.866}{17.88}$ = 2.342

Let Amplitude = x

$$\therefore \frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\varsigma r)^2}}$$

$$\frac{X}{\left[\frac{0.5}{20}(0.05)\right]} = \frac{(2.342)^2}{\sqrt{(1-2.342^2)^2+[2(0.175)(2.342)]^2}}$$

$X = 1.5 \times 10^{-3}$m

And phase angle = ψ

Tan $\psi = \frac{2\zeta r}{1-r^2} = \frac{2(0.175)(2.342)}{1-2.342^2}$

$\Rightarrow \psi = -10.36^o$ or $180^o - 10.36^o = 169.63^o$

(3) Resonant speed = ω_n = 17.88 rad/s

$\omega_n = \frac{2\pi N}{60} \Rightarrow 17.88 = \frac{2\pi N}{60}$

$\therefore$ N = 170.74 rpm

Let Amplitude at resonance = X_{reso}

$$\therefore \frac{X_{reso}}{\left(\frac{m_o e}{m}\right)} = \frac{1}{\sqrt{2\varsigma}}$$

$\therefore X_{reso} = \frac{m_o e}{2m\varsigma}$ where e = eccentricity

$= \frac{(0.5)(0.05)}{2(0.175)(20)} = 3.57 \times 10^{-3} m = 3.57$mm

4) Force because of dashpot on the motor $F_d = c\omega x$

= 125 [41.866] [1.5 x 10^{-3}]

= 7.85 N

Force because of spring, F_s = Kx

= (6400) (1.5 x 10^{-3})

= 22.4 N

Resultant force, F $= \sqrt{F_d^2 + F_s^2}$

$= \sqrt{7.85^2 + 9.6^2}$

= 12.4N