UNIT III ELECTRICAL MEASUREMENTS **10 hrs.**

Measurement of Voltage and Current: D'Arsonval Galvanometer, permanent magnet moving coil, permanent magnet moving iron, Dynamometer - Measurement of Resistance, Inductance and Capacitance: Wheat stone bridge, Kelvin double bridge, Wein Bridge, Hay's bridge, Maxwell bridge, Anderson bridge, Schering bridge – ohmmeter, VOM meter.

MEASUREMENT OF VOLTAGE AND CURRENT

Current

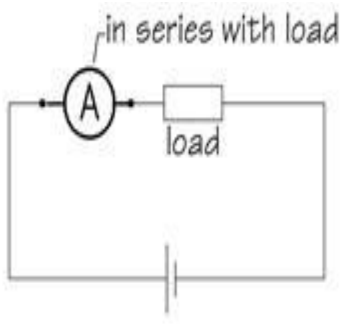
Current is the rate at which electric charge flows past a point in a circuit. Symbol is I and unit is amps (A)

Voltage

Voltage is the electrical force that would drive an electric current between two points. And symbol is V units is volts or voltage (V)

Measuring Current: Ammeters

To measure **current**, the circuit must be broken at the point where we want that current to be measured, and the ammeter inserted at that point. In other words, *an ammeter must be connected in series* with the load under test.



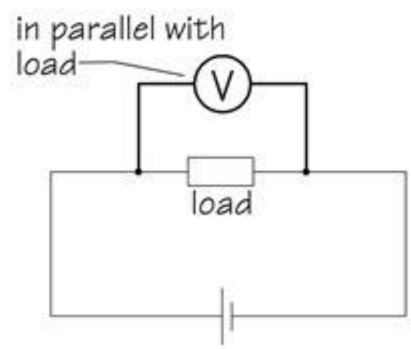
It is very important that the insertion of the ammeter into a circuit has little effect the circuit's existing resistance and, thus, alter the current normally flowing in the circuit, ammeters are manufactured with *very low* values of internal resistance.

Because ammeters have a very low internal resistance, it is vitally important that they are *never* inadvertently connected in parallel with any circuit component —and especially with the supply. Failure to do so will result in a short-circuit current flowing through the instrument which may damage the ammeter (although most ammeters are fused) or even result in personal injury.

> **Ammeters** have a very low internal resistance, and must ***always*** be connected in **series** in a circuit.

Measuring Voltage: Voltmeters

To measure **potential-difference**, or **voltage**, a voltmeter must be connected between*two* points at different potentials. In other words, *a voltmeter must always be connected in parallel* with the part of the circuit under test.

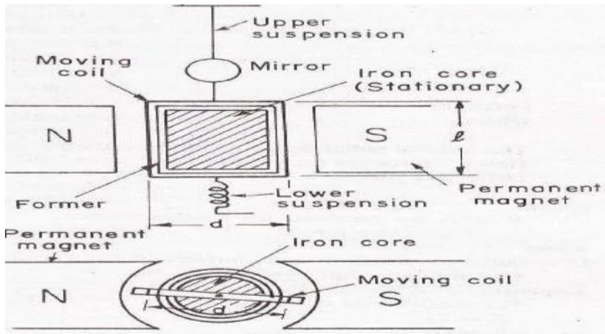


In order to operate, a voltmeter must, of course, draw *some* current from the circuit under test, and this can lead to inaccurate results because it can interfere with the normal condition of the circuit. We call this the 'loading effect' and, to minimise this 'loading effect' (and, therefore, improve the accuracy of a reading), this operating current must be as small as possible and, for this reason, voltmeters are manufactured with a *very high* value of internal resistance —usually many megohms.

> Voltmeters must *always* be connected in **parallel** in a circuit, and have a very high internal resistance.

Principle of D'Arsonval galvanometer

An action caused by electromagnetic deflection, using a coil of wire and a magnetized field. When current passes through the coil, a needle is deflected.Whenever electrons flow through a conductor, a magnetic field proportional to the current is created. This effect is useful for measuring current and is employed in many practical meters.



D'Arsonval Galvanometer

1) Moving Coil

It is the current carrying element. It is either rectangular or circular in shape and consists of number of turns of fine wire. This coil is suspended so that it is free to turn about its vertical axis of symmetry. It is arranged in a uniform, radial, horizontal magnetic field in the air gap between pole pieces of a permanent magnet and iron core. The iron core is spherical in shape if the coil is circular but is cylindrical if the coil is rectangular. The iron core is used to provide a flux path of low reluctance and therefore to provide strong magnetic field for the coil to move in. this increases the deflecting torque and hence the sensitivity of the galvanometer. The length of air gap is about 1.5mm. In some galvanometers the iron core is omitted resulting in of decreased value of flux density and the coil is made narrower to decrease the air gap. Such a galvanometer is less sensitive, but its moment of inertia is smaller on account of its reduced radius and consequently a short periodic time.

2) Damping

There is a damping torque present owing to production of eddy currents in the metal former on which the coil is mounted. Damping is also obtained by connecting a low resistance across the galvanometer terminals. Damping torque depends upon the resistance and we can obtain critical damping by adjusting the value of resistance.

3) Suspension

The coil is supported by a flat ribbon suspension which also carries current to the coil. The other current connection in a sensitive galvanometer is a coiled wire. This is called the lower suspension and has a negligible torque effect. This type of galvanometer must be leveled carefully so that the coil hangs straight and centrally without rubbing the poles or the soft iron cylinder. Some portable galvanometers which do not require exact leveling have ” taut suspensions" consisting of straight flat strips kept under tension for at the both top and at the bottom. The upper suspension consists of gold or copper wire of nearly 0.012-5 or 0.02-5 mm diameter rolled into the form of a ribbon. This is not very strong mechanically; so that the galvanometers must he handled carefully without jerks. Sensitive galvanometers are provided with coil clamps to the strain from suspension, while the galvanometer is being moved.

4) Indication

The suspension carries a small mirror upon which a beam of light is cast. The beam of light is reflected on a scale upon which the deflection is measured. This scale is usually about 1 meter away from the instrument, although ½ meter may be used for greater compactness.

5) Zero Setting

A torsion head is provided for adjusting the position of the coil and also for zero setting.

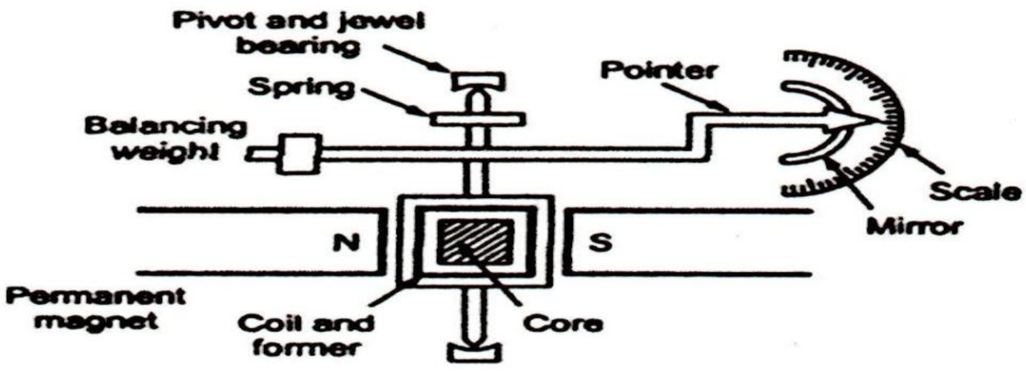
Operation

When a current flows through the coil, the coil generates a magnetic field. This field acts against the permanent magnet. The coil twists, pushing against the spring, and moves the pointer. The hand points at a scale indicating the electric current. Careful design of the pole pieces ensures that the magnetic field is uniform, so that the angular deflection of the pointer is proportional to the current. A useful meter generally contains provision for damping the mechanical resonance of the moving coil and pointer, so that the pointer settles quickly to its position without oscillation

PERMANENT MAGNET MOVING COIL INSTRUMENT

Principle

Moving coil instrument depends on the principle that when a current carrying conductor is placed on a magnetic field, mechanical force acts on the conductor. The coil placed on the magnetic field and carrying operating current is attached to the moving system. With the movement of the coil the pointer moves over the scale.



Construction of PMMC instrument

Moving coil instrument consists of a powerful permanent magnet with soft iron pieces and light rectangular coil of many turns of fine wire wound on aluminum former inside which is an iron core as shown in the figure. As it uses permanent magnets they are called "***Permanent magnet moving coil instrument***". The purpose of the coil is to make the field uniform. The coil is mounted on the spindle and acts as the moving element. The current is led into and out of the coil by means of the two control hair springs, one above and the other below the coil. The spring also provides the controlling torque. Damping torque is provided by eddy current damping.

Working of PMMC instrument

When the moving coil instrument is connected, the operating current flows through the coil. This current carrying coil is placed in the magnetic field produced by the permanent magnet and therefore, mechanical force acts on the coil. The coil is attached to the moving system, the pointer moves over the scale. It may be noted here that if current direction is reversed the torque will also be reversed since the direction of the field of permanent magnet is same. Hence, the pointer will move in the opposite direction, i.e it will go on the wrong side of zero. In other words, these instruments work only when current in the circuit is passed in a definite direction i.e. for d.c only. So it is called permanent magnet moving coil instruments because a coil moves in the field of a permanent magnet.

Torque Equation for PMMC

The equation for the delevoped torque of the PMMC can be obtained from the basic law of electromagnetic torque. The deflecting torque is given by,

$$\mathbf{Td = NBAI}$$

Where,

Td = deflecting torque in N-m

B = flux density in air gap, Wb/m2

N = Number of turns of the coils

A = effective area of coil m2

I = current in the moving coil, amperes

Therefore, **Td = GI**

Where, G = NBA = constant

The controlling torque is provided by the springs and is proportional to the angular deflection of the pointer.

$$\mathbf{Tc = KØ}$$

Where,

Tc = Controlling Torque

K = Spring Constant Nm/rad or Nm/deg

Ø = angular deflection

For the final steady state position,

$$\mathbf{Td = Tc}$$

Therefore **GI = KØ**

So, Ø = (G/K)I or I = (K/G) Ø

Thus the deflection is directly proportional to the current passing through the coil. The pointer deflection can therefore be used to measure current.

Advantage of PMMC instrument

1. Uniform scale.
2. Very effective eddy current damping
3. Power consumption is low.
4. No hysteresis loss.

5. They are not affected by stray field.
6. Require small operating current.
7. Accurate and reliable.

Disadvantage of PMMC instrument

1. Only used for D.C measurement.
2. Costlier compared to moving iron instrument.
3. Some errors are caused due to the aging of the control springs and the permanent magnets.

Errors in PMMC instrument

The basic sources of errors in PMMC instruments are friction, temperature and aging of various parts. To reduce the frictional errors ratio of torque to weight is made very high. The most serious errors are produced by the heat generated or by changes in the temperature. This changes the resistance of the working coil, causing large errors. In case of voltmeters, a large series resistance of very low temperature coefficient is used. This reduces the temperature errors.

The aging of temperature magnet and control springs also cause errors. The weakening of magnet and springs causes opposite errors. The weakening of magnet causes less deflection while weakening of the control springs cause large deflection, for a particular value of current. The proper use of material and presaging during manufacturing can reduces the errors due to weakening of the control springs.

MOVING IRON INSTRUMENT

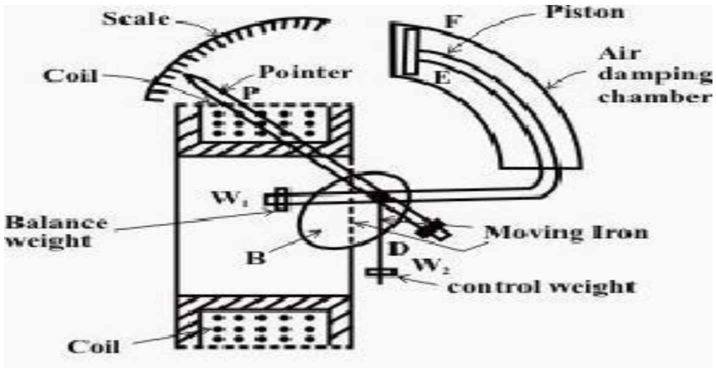
There are classified in to two type

1. Attraction type moving iron instrument
2. Repulsion type moving iron instrument

Attraction type moving iron instrument

Principle

An “attraction type” moving-iron instrument consists of a coil, through which the test current is passed, and a pivoted soft-iron mass attached to the pointer. The resulting magnetic polarity at the end of the coil nearest the iron mass then induces the opposite magnetic polarity into the part of the iron mass nearest the coil, which is then drawn by attraction towards the coil, deflecting the pointer across a scale.



Attraction type

The coil is flat and has a narrow slot like opening. · The moving iron is a flat disc or a sector eccentrically mounted. · When the current flows through the coil, a magnetic field is produced and the moving iron moves from the weaker field outside the coil to the stronger field inside it or in other words the moving iron is attracted in. · The controlling torque is provide by springs hut gravity control can be used for panel type of instruments which are vertically mounted. · Damping is provided by air friction with the help of a light aluminium piston (attached to the moving system) which move in a fixed chamber closed at one end as shown in Fig. or with the help of a vane (attached to the moving system) which moves in a fixed sector shaped chamber as shown.

Operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

The force F, pulling the soft -iron piece towards the coil is directly proportional to;

a) Field strength H, produced by the coil.

b) pole strength “m” developed in the iron piece.

$$F \alpha mH$$

Since $m \alpha H$, $F \alpha H^2$

Instantaneous deflecting torque αH^2

Also, the field strength $H = \mu i$

If the permeability (μ) of the iron is assumed constant,

Then, $H \alpha i$

Where, I is instantaneous coil current, Ampere

Instantaneous deflecting torque αi^2

Average deflecting torque, $T_d \alpha$ mean of i^2 over a cycle.

Since the instrument is spring controlled,

$$T_c \alpha \theta$$

In the steady position of deflection, $T_d = T_c$

$$\theta \alpha \text{ mean of } i^2 \text{ over a cycle}$$

$$\alpha I^2$$

Since the deflection is proportional to the square of coil current, the scale of such instruments is non-uniform (being crowded in the beginning and spread out near the finishing end of the scale).

Moving iron repulsion type instrument

These instruments have two vanes inside the coil, the one is fixed and other is movable. When the current flows in the coil, both the vanes are magnetised with like polarities induced on the same side. Hence due to repulsion of like polarities, there is a force of repulsion between the two vanes causing the movement of the moving van. The repulsion type instruments are the most commonly used instruments.

The two different designs of repulsion type instruments are:

i) Radial vane type and

ii) Co-axial vane type

Radial van repulsion type instrument

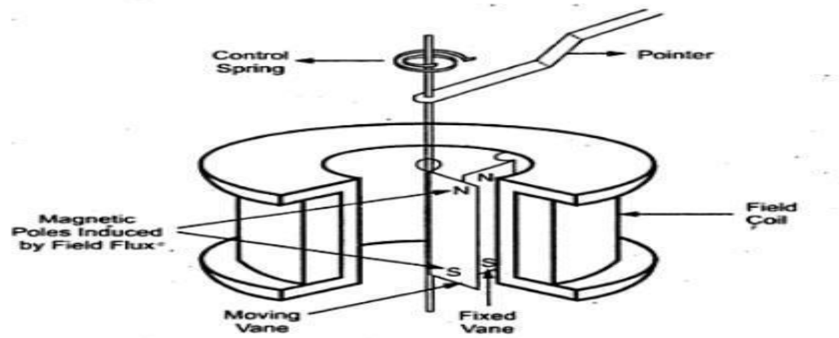


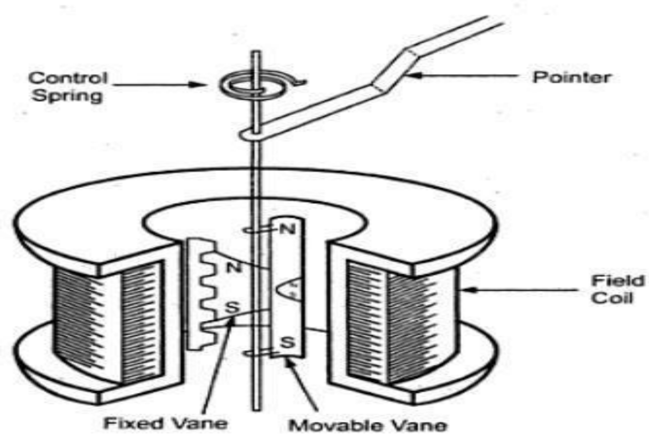
Fig shows the radial vane repulsion type instrument. Out of the other moving iron mechanism, this is the most sensitive and has most linear scale.

The two vanes are radial strips of iron. The fixed vane is attached to the coil. The movable vane is attached to the spindle and suspended in the induction field of the coil. The needle of the instrument is attached to this vane.

Even though the current through the coil is alternating, there is always repulsion between the like poles of the fixed and the movable vane. Hence the deflection of the pointer is always in the same direction. The deflection is effectively proportional to the actual current and hence the scale is calibrated directly to rad amperes or volts. The calibration is accurate only for the frequency for which it is designed because the impedance is different for different frequencies

Concentric vane repulsion type instrument

Fig shows the concentric vane repulsion type instrument. The instrument has two concentric vanes. One is attached to the coil frame rigidly while the other can rotate coaxially inside the stationary vane.



Both the vanes are magnetised to the same polarity due to the current in the coil. Thus the movable vane rotates under the repulsive force. As the movable vane is attached to the pivoted shaft the repulsion results in a rotation of the shaft. The pointer deflection is proportional to the current in the coil. The concentric vane type instrument is moderately sensitive and the deflection is proportional to the square of the current through coil. Thus the instrument said to have square low response. Thus the scale of the instrument is non-uniform in nature. Thus whatever may be the direction of the current in the coil, the deflection in the moving iron instruments is in the same direction. Hence moving iron instruments can be used for both a.c. and d.c. measurements. Due to square low response, the scale of the moving iron instrument is non-uniform.

Torque equation

The deflecting torque results due to repulsion between the similarly charged soft- iron pieces or vanes. If the two pieces develop pole strength of m_1 and m_2 respectively, then;

Instantaneous deflecting torque $\alpha\ m_1 m_2\ \alpha\ H^2$

If the permeability of iron is assumed constant,

Then $H\ \alpha\ i$,

where, i is the coil current

Instantaneous deflecting torque $\alpha\ i^2$

Average deflecting torque, $T_d\ \alpha$ mean of i^2 over a cycle.

Since the instrument is spring controlled, $T_c\ \alpha\ \theta$

In the steady position of deflection, $T_d = T_c$

$\theta\ \alpha$ mean of i^2 over a cycle.

$\alpha\ I^2$

Thus, the deflection is proportional to the square of the coil current. The scale of the instrument is non- uniform; being crowded in the beginning and spread out near the finish end of the scale. However, the non- linearity of the scale can be corrected to some extent by the accurate shaping and positioning of the iron vanes in relation to the operating coil.

Advantages

The various advantages of moving iron instruments are,

1) The instruments can be used for both a.c. and d.c. measurements.

2) As the torque to weight ratio is high, errors due to the friction are very less.

3) A single type of moving element can cover the wide range hence these instruments are cheaper than other types of if instruments.

4) There are no current carrying parts in the moving system hence these meters are extremely rugged and reliable.

5) These are capable of giving good accuracy. Modern moving iron instruments have a d.c. error of 2% or less.

6) These can withstand large loads and are not damaged even under sever overload conditions.

7) The range of instruments can be extended.

Disadvantages

The various disadvantages of moving iron instruments are,

1) The scale of moving iron instruments is not uniform and is cramped at the lower end. Hence accurate readings are not possible at this end.

2) There are serious errors due to hysteresis, frequency changes and stray magnetic fields.

3) The increase in temperature increases the resistance of coil, decreases stiffness of the springs, decreases the permeability and hence affect the reading severely.

4) Due to the non linearity of B-H curve, the deflecting torque is not exactly proportional to the square of the current.

5) There is a difference between a.c. and d.c. calibration on account of the effect of inductance of the meter. Hence these meters must always be calibrated at the frequency at which they are to be used. The usual commercial moving iron instrument may be used within its specified accuracy from 25 to 125 HZ frequency range.

6) Power consumption is on higher side.

Errors in moving iron instrument

The various errors in the moving iron instruments are,

1) Hysteresis error: Due to hysteresis effect, the flux density for the same current while ascending and descending values is different. While descending, the flux density is higher and while ascending it is lesser. So meter reads higher for descending values of current or voltage. So remedy for this is to use smaller iron parts which can demagnetise quickly or to work with lower flux densities.

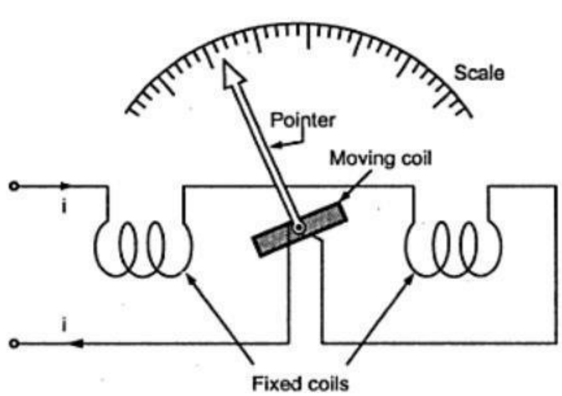
2) **Temperature error**: The temperature error arises due to the effect of temperature on the temperature coefficient of the spring. This error is of the order of 0.02 % per ^{o}C change in temperature. Errors can cause due to self heating of the coil and due to which change in resistance of the coil. So coil and series resistance must have low temperature coefficient. Hence mangnin is generally used for the series resistance.

3) Stray magnetic Field Error: The operating magnetic field in case of moving iron instruments is very low. Hence effect of external i.e. stray magnetic field can cause error. This effect depends on the direction of the stray magnetic field with respect to the operating field of the instrument.

4) Frequency Error: These are related to a.c. operation of the instrument. The change in frequency affects the reactance of the working coil and also affects the magnitude of the eddy currents. This cause error in the instrument.

5) Eddy Current Error: When instrument is used for a.c. measurements the eddy currents are produced in the iron parts of the instrument. The eddy current affects the instrument current causing the change in the deflection torque which produces the error in the meter reading. As eddy current is frequency dependent, frequency changes cause eddy current error.

DYNAMOMETER



Construction

The various type of the electrodynamometer type instrument are:

Fixed Coils: The necessary field required for the operation of the instrument is produced by the fixed coils. A uniform field is obtained near the center of coil due to division of coil in two sections. These coils are air cored. Fixed coils are wound with fine wire for using as voltmeter, while for ammeters and wattmeters it is wound with heavy wire. The coils are usually varnished. They are clamped in place against the coil supports. This makes the construction rigid.

Ceramic is usually used for mounting supports. If metal parts would have been used then it would weaken the field of the fixed coil.

Moving Coil: The moving coil is wound either as a self-sustaining coil or else on a non-metallic former. If metallic former is used, then it would induce eddy currents in it. The construction of moving coil is made light as well as rigid. It is air cored.

Controlling: The controlling torque is provided by springs. These springs act as leads to the moving coil.

Moving System: The moving coil is mounted on an aluminium spindle. It consists of counter weights and pointer. Sometimes a suspension may be used, in case a high accuracy is desired.

Damping: The damping torque is provided by air friction, by a pair of aluminium vanes which are attached to the spindle at the bottom. They move in sector shaped chambers. As operating field would be distorted by eddy current damping, it is not employed.

Shielding: The field produced by these instruments is very weak. Even earth's magnetic field considerably affects the reading. So shielding is done to protect it from stray magnetic fields. It is done by enclosing in a casing high permeability alloy.

Cases and Scales: Laboratory standard instruments are usually contained in polished wooden or metal cases which are rigid. The case is supported by adjustable leveling screws.

A spirit level may be provided to ensure proper levelling.

For using electrodynamometer instrument as ammeter, fixed and moving coils are connected in series and carry the same current. A suitable shunt is connected to these coils to limit current through them upto desired limit.

The electrodynamometer instruments can be used as a voltmeter by connecting the fixed and moving coils in series with a high non-inductive resistance. It is most accurate type of voltmeter.

For using electrodynamometer instrument as a wattmeter to measure the power, the fixed coil acts as a current coil and must be connected in series with the load. The moving coils acts as a voltage coil or pressure oil and must be connected across the supply terminals. The wattmeter indicates the supply power.

Working

When current passes through the fixed and moving coils, both coils produce the magnetic fields. The field produced by fixed coil is proportional to the load current while the field produced by the moving coil is proportional to the voltage. As the deflecting torque is produced due to the interaction of these two fields, the deflection is proportional to the power supplied to the load.

Let i_1 = Instantaneous value of current in fixed coil

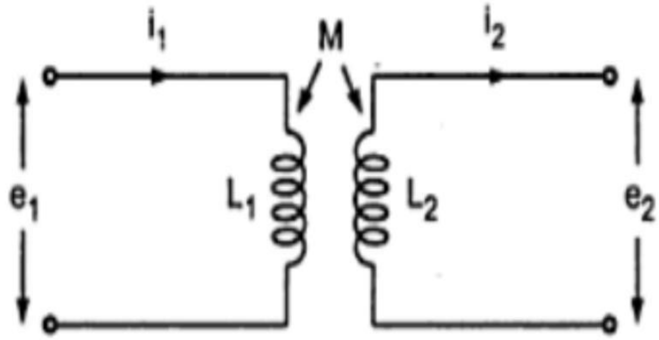
i_2 = Instantaneous value of current in moving coil

L_1 = Self inductance of fixed coil

L_2 = self inductance of moving coil

M = Mutual inductance between fixed and moving coils

The electrodynamometer instrument can be represented by an equivalent circuit as shown in the Fig.2.



Now $$e_1 = \frac{d\phi_1}{dt}$$

and $$e_2 = \frac{d\phi_2}{dt}$$

$$\begin{aligned}\text{Electrical input energy} &= e_1 i_1 dt + e_2 i_2\, dt \\ &= i_1\, d\,\phi_1 + i_2\, d\,\phi_2 \\ &= i_1\, d\,(L_1 i_1 + M\, i_2) + i_2\, d\,(L_2 i_2 + M\, i_1) \\ &= i_1 L_1\, d\, i_1 + i_1^2\, d\, L_1 + i_1\, i_2\, d\, M + i_1\, M\, d\, i_2 + \\ &\quad i_2 L_2 d\, i_2 + i_2^2\, d\, L_2 + i_1\, i_2 d\, M + i_2\, M\, d\, i_1 \qquad \ldots (1)\end{aligned}$$

The energy stored in the magnetic field due to L_1, L_2 and M is given by,

$$\text{Energy stored} = \frac{1}{2} L_1\, i_1^2 + \frac{1}{2} L_2\, i_2^2 + i_1\, i_2\, M$$

$$\begin{aligned}\text{Change in stored energy} &= d\left[\frac{1}{2} L_1\, i_1^2 + \frac{1}{2} L_2\, i_2^2 + i_1\, i_2\, M\right] \\ &= i_1\, L_1\, d\, i_1 + \frac{1}{2} i_1^2\, d\, L_1 + i_2\, L_2\, d\, i_2 + \frac{1}{2} i_2^2\, d\, L_2 + \\ &\quad i_1\, M\, d\, i_2 + i_2\, M\, d\, i_1 + i_1\, i_2\, d\, M \qquad \ldots (2)\end{aligned}$$

From the principle of conversation of energy,

Energy input=Energy stored +Mechanical energy

Mechanical energy=energy input-energy stored

Subtraction (2) from equation (1),

$$\text{Mechanical energy} = \frac{1}{2} i_1^2 \, d L_1 + \frac{1}{2} i_2^2 \, d L_2 + i_1 \, i_2 \, d M$$

The self inductance L_1 and L_2 are constant and hence dL_1 and dL_2 are zero. Mechanical energy=i1i2dm

If Ti is the instantaneous deflecting torque and dθ is the change in the deflection then Mechanical energy = Mechanical work done

$$= T_i \, d\theta$$

$$i_1 \, i_2 \, d M = T_i \, d\theta$$

$$\boxed{T_i = i_1 i_2 \frac{dM}{d\theta}}$$

Advantage of Electrodynamometer instrument

1) As the coils are air cored, these instruments are free from hysteresis and eddy current losses.

2) They have a precision grade security.

3) These instruments can be used on both a.c. and d.c. They are also used as a transfer instruments.

4) Electrodynamometer voltmeter are very useful where accurate r.m.s values of voltage, irrespective of waveforms, are required.

5) Free from hysteresis errors.

6) Low power Consumption.

7) Light in weight.

Disadvantage of electrodynamometer instrument

These instruments have a low sensitivity due to a low torque to weight ratio. Also it introduces increased frictional losses. To get accurate results, these errors must be minimized.

1) They are more expensive than other type of instruments.
2) These instruments are sensitive to overload and mechanical impacts. Therefore must be taken while handling them.
3) They have a non-uniform scale.
4) The operation current of these instruments is large due to the fact that they have weak magnetic field

Error in electrodynamometer instrument

The various errors in electrodynamometer instruments are,

1. Torque to weight ratio: To have reasonable deflecting torque, mmf of the moving coil must be large enough. Thus m.m.f. = NI hence current through moving coil should be high or number of turns should be large. The current cannot be made very high because it may cause excessive heating of springs. Large number of turns hence is the only option but it increases weight of the coil. This makes the system heavy reducing torque to weight ratio. This can cause frictional errors in the reading

2. Frequency errors: The changes in the frequency causes to change self inductances of moving coil and fixed coil. This causes the error in the reading. The frequency error can be reduced by having equal time constants for both fixed and moving coil circuits.

3. Eddy current errors: In metal parts of the instrument the eddy current gets produced. The eddy current interacts with the instrument current, to cause change in the deflecting torque, to cause error. Hnec metal parts should be kept as minimum as possible. Also the resistivity of the metal parts used must be high, to reduce the eddy currents.

4. Stray magnetic field error: Similar to moving iron instruments the operating field in electrodynamometer instrument is very weak. Hence external magnetic field can interact with the operation field to cause change in the deflection, causing the error. To reduce the effect of stray magnetic field, the shields must be used for the instruments.

5. Temperature error: The temperature errors are caused due to the self heating of the coil, which causes change in the resistance of the coil. Thus temperature compensating resistors can be used in the precise instrument to eliminate the temperature errors.

MEASURMENT OF RESISTANC INDUCTANCE AND CAPACITANCE

Classification of Resistances

For the purposes of measurements, the resistances are classified into three major groups based on their numerical range of values as under:

- Low resistance (0 to 1 ohm)
- Medium resistance (1 to 100 kilo-ohm) and
- High resistance (>100 kilo-ohm)

Accordingly, the resistances can be measured by various ways, depending on their range of values, as under:

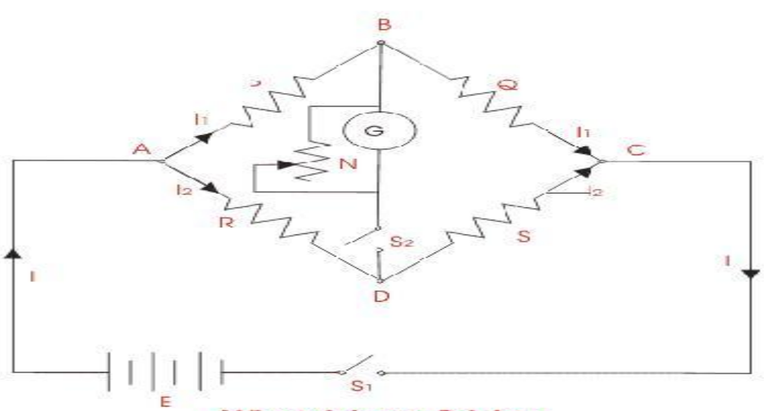
1. Low resistance (0 to 1 ohm): AV Method, Kelvin Double Bridge, potentiometer, doctor ohmmeter, etc.
2. Medium resistance (1 to 100 kilo-ohm): AV method, wheat stone's bridge, substitution method, etc.
3. High resistance (>100 kilo-ohm): AV method, Fall of potential method, Megger, loss of charge method, substitution method, bridge method, etc.

WHEATSTONE BRIDGE

For measuring accurately any electrical resistance **Wheatstone bridge** is widely used. There are two known resistors, one variable resistor and one unknown resistor connected in bridge form as shown below. By adjusting the variable resistor the current through the Galvanometer is made zero. When the current through the galvanometer becomes zero, the ratio of two known resistors is exactly equal to the ratio of adjusted value of variable resistance and the value of unknown resistance. In this way the value of unknown electrical resistance can easily be measured by using a **Wheatstone Bridge.**

Theory

The general arrangement of **Wheatstone bridge circuit** is shown in the figure below. It is a four arms bridge circuit where arm AB, BC, CD and AD are consisting of electrical resistances P, Q, S and R respectively. Among these resistances P and Q are known fixed electrical resistances and these two arms are referred as ratio arms. An accurate and sensitive Galvanometer is connected between the terminals B and D through a switch S_2. The voltage source of this Wheatstone bridge is connected to the terminals A and C via a switch S_1 as shown. A variable resistor S is connected between point C and D. The potential at point D can be varied by adjusting the value of variable resistor. Suppose current I_1 and current I_2 are flowing through the paths ABC and ADC respectively. If we vary the electrical resistance value of arm CD the value of current I_2 will also be varied as the voltage across A and C is fixed. If we continue to adjust the variable resistance one situation may comes when voltage drop across the resistor S that is $I_2.S$ is becomes exactly equal to voltage drop across resistor Q that is $I_1.Q$. Thus the potential at point B becomes equal to the potential at point D hence potential difference between these two points is zero hence current through galvanometer is nil. Then the deflection in the galvanometer is nil when the switch S_2 is closed.



Wheatstone Bridge

Now, from **Wheatstone bridge circuit**

$$\text{current } I_1 = \frac{V}{P+Q}$$

And

$$\text{current } I_2 = \frac{V}{R+S}$$

Now potential of point B in respect of point C is nothing but the voltage drop across the resistor Q and this is

$$I_1.Q = \frac{V.Q}{P+Q} \text{------------------(i)}$$

Again potential of point D in respect of point C is nothing but the voltage drop across the resistor S and this is

$$I_2.S = \frac{V.S}{R+S} \text{------------------(ii)}$$

Equating, equations (i) and (ii) we get,

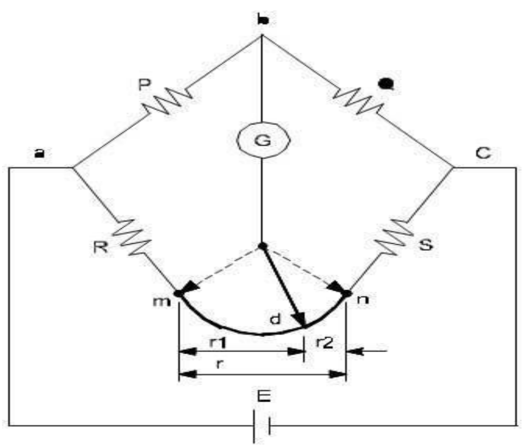
$$\frac{V.Q}{P+Q} = \frac{V.S}{R+S} \Rightarrow \frac{Q}{P+Q} = \frac{S}{R+S}$$
$$\Rightarrow \frac{P+Q}{Q} = \frac{R+S}{S} \Rightarrow \frac{P}{Q} + 1 = \frac{R}{S} + 1 \Rightarrow \frac{P}{Q} = \frac{R}{S}$$
$$\Rightarrow R = SX\frac{P}{Q}$$

KELVIN DOUBLE BRIDGE

The Kelvin double bridge is one of the best devices available for the precise measurement of low resistances. It is the modification of Wheatstone bridge by which the errors due to contact resistance and lead resistances are eliminated. This bridge is named double bridge because it contains a second set of ratio arms. An interesting variation of the Wheatstone bridge is the Kelvin Double Bridge, used for measuring very low resistances (typically less than 1/10 of an ohm)

Theory

Consider the bridge circuit shown in figure below. Here ‘r’ represents the resistance of the lead that connects the unknown resistance ‘R’ to standard resistance ‘S’. Two galvanometer connections indicated by dotted lines are possible. The connection may be either to point 'm' or to point 'n'. When the galvanometer is connected to point ‘m’ the resistance ‘ r’ of the connecting leads is added to the standard resistance ‘S’ resulting in indication of too low an indication for unknown resistance ‘R’. When the connection made to point the resistance ‘r’ is added to the unknown resistance resulting in indication of too high a value for ‘R’. -



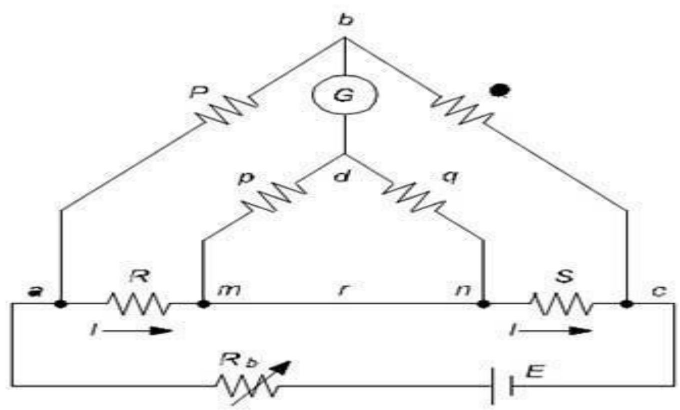
Suppose that instead of using point ‘m’ which gives a low result or ‘n’ which makes the result High, we make the galvanometer connection to any intermediate point‘d’ as shown by full line. If at point‘d’ the resistance ‘r’ is divided into two parts r1, r2 such that r1/r2 = P/Q. Then the presence of r the resistance of connecting leads causes no error in the result. We have,1

$$R + r_1 = \frac{P}{Q}(s + r_2) \text{ but } \frac{r_1}{r_2} = \frac{P}{Q}$$

$$\text{or } \frac{r_1}{r_1 + r_2} = \frac{P}{P+Q} \text{ or } r_1 = \frac{P}{P+Q} r \text{ as } r_1 + r_2 = r \text{ and } r_2 = \frac{Q}{P+Q} r$$

$$\text{We can write eqn above as } \left(R + \frac{P}{P+Q}\right) = \frac{P}{Q}\left(S + \frac{Q}{P+Q}\right) \text{ or } R + \frac{P}{Q} S$$

Therefore we conclude that making the galvanometer connection as at C, the resistance of leads does not affect the result. The process described above is obviously not a practical way of achieving the desired result, as there would certainly be a trouble in determining the correct point for galvanometer connection. It does however suggest the simple modification that two actual resistance units of correct ratio be connected between points ‘m’ and ‘n’ the galvanometer be connected to the junction of the resistors. This is the actual Kelvin bridge arrangement which is shown in figure below. –



The Kelvin double bridge incorporates the idea of a second set of ratio arms, hence the name of double bridge- and the use of four terminal resistors for the low resistance arms. Figure shows the schematic diagram of the Kelvin Bridge. The first of ratio arms is P and Q. The second set of ratio arms, p and q is used to connect the galvanometer to a point ‘d’ at the appropriate potential between points ‘m’ and ‘n’ to eliminate the effect of connecting lead of resistance ‘r’ between the known resistance ‘R’ and the standard resistance ‘S’. The ratio p /q is made equal to P/Q. Under balance conditions there is no current through the galvanometer, which means that the voltage drop between a and b, E is equal to the voltage drop Ed between a and b.

$$Now\ E_{ab}\ \frac{P}{P+Q}\ E_{ac}\ and\ E_{ac} = 1\left[R + S + \frac{(p+q)r}{p+q+r}\right]$$

$$and\ \ E_{amd} = 1\left[R + \frac{P}{p+q}\left\{\frac{(p + +q)r}{p+q+r}\right\}\right] = 1\left[R + \frac{pr}{p+q+r}\right]$$

For zero galvanometer deflection, E = E.

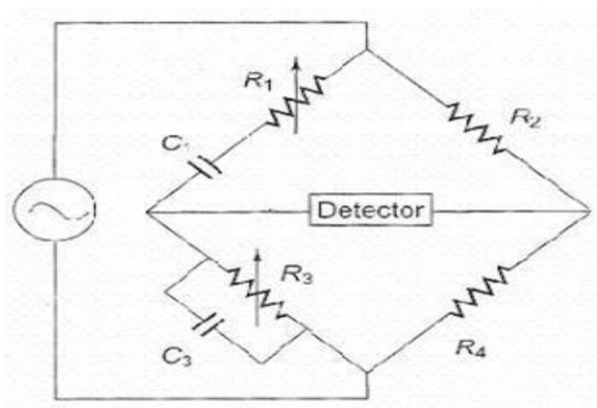
$$or\ \frac{P}{P+Q}\ 1\left[R + S + \frac{(p+q)r}{p+q+r}\right] = 1\left[R + \frac{pr}{p+q+r}\right]$$

$$or\ R + \frac{P}{Q}S + \frac{qr}{p+q+r}\left[\frac{p}{q} - \frac{p}{q}\right]$$

Now if p/q = p/q becomes R = P / Q *S Above equation is the usual working equation for the Kelvin double bridge. It indicates that the resistance of connecting lead ‘r’ has no effect on the measurement provided that the two sets of ratio arms have equal ratios. The above equation is useful however as it shows the error that is introduced in case the ratios are not exactly equal. It is indicated that it is desirable to keep ‘r’ as small as possible in order to minimize the errors in case there is a difference between ratios P / Q and p/q. In a typical Kelvin bridge, the range of resistance calculated is 0.1S to 1.0S.

WIEN’S BRIDGE:

Circuit and derives the expression for the unknown element at balance, Wien Bridge has a series RC combination in one and a parallel combination in the adjoining arm. Wien's bridge is shown in fig. Its basic form is designed to measure frequency. It can also be used for the instrument of an unknown capacitor with great accuracy, The impedance of one arm is



$$Z_1 = R_1 - j/\omega C_1$$

The admittance of the parallelarm is

$$Y_3 = 1/R_3 + j\ \omega\ C_3$$

Using the bridge balance equation,we have

We have

$$Z_1\ Z_4 = Z_2\ Z_3$$

Therefore

$$Z_1 Z_4 = Z_2/Y_3, \text{ i.e. } Z_2 = Z_1 Z_4 Y_3$$

$$R_2 = R_4 \left(R_1 - \frac{j}{\omega C_1}\right)\left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} - \frac{j R_4}{\omega C_1 R_3} + j\,\omega C_3 R_1 R_4 + \frac{C_3 R_4}{C_1}$$

$$R_2 = \left(\frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1}\right) - j\left(\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4\right)$$

Equating the real and imaginary terms we have as,

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} \quad \text{and} \quad \frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 = 0$$

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$$

$$\frac{1}{\omega C_1 R_3} = \omega C_3 R_1$$

$$\omega^2 = \frac{1}{C_1 R_1 R_3 C_3}$$

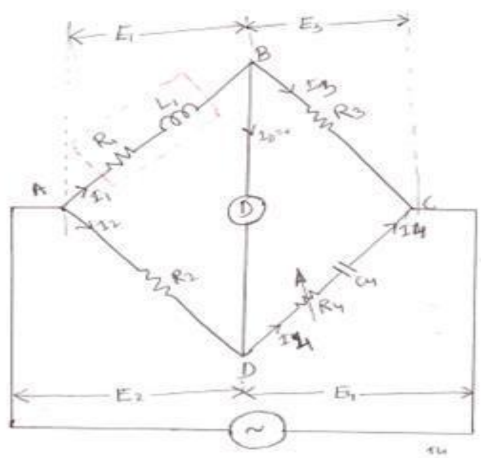
$$\omega = \frac{1}{\sqrt{C_1 R_1 C_3 R_3}}$$

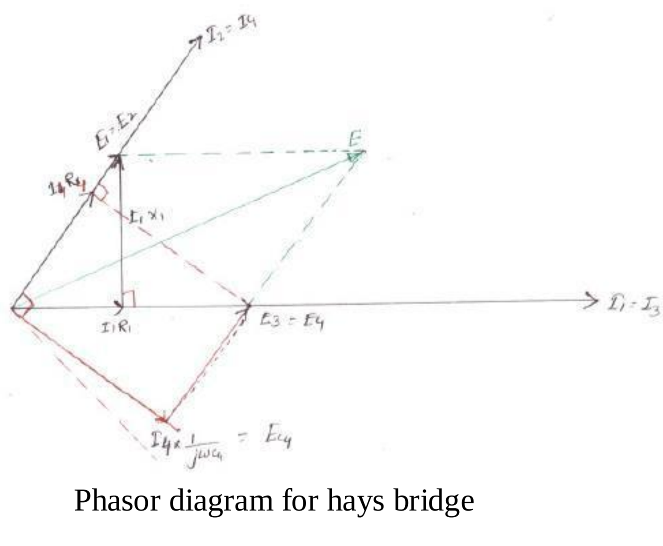
$$\omega = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{C_1 R_1 C_3 R_3}}$$

The bridge is used for measuring frequency in the audio range. Resistances R1 and R3 can be ganged together to have identical values. Capacitors C1 and C3 are normally of fixed values. The audio range is normally divided into 20 - 200 - 2 k - 20 kHz range In this case, the resistances can be used for range changing and capacitors, and C3 for fine frequency control within the range. The bridge can also be use for measuring capacitance. In that case, the frequency of operation must be known. The bridge is also used in a harmonic distortion analyzer, as a Notch filter, an in audio frequency and radio frequency oscillators as a frequency determine element. An accuracy of 0.5% - 1% can be readily obtained using this bridge. Because it is frequency sensitive, it is difficult to balance unless the waveform of the applied voltage is purely sinusoidal.

HAY'S BRIDGE





Phasor diagram for hays bridge

- $\dot{E}1 = I_1 R_1 + jI_1 X_1$
- $\dot{E} = \dot{E}1 + \dot{E}3$
- $\dot{E}_4 = \dot{I}_4 R_4 + \frac{I_4}{jwC_4}$
- $\dot{E}3 = I_3 R_3$

$$Z_4 = R_4 + \frac{1}{jwC_4} = \frac{1 + jwR_4C_4}{jwC_4}$$

Comparing imaginary part

At balance condition, $Z_1Z_4 = Z_3Z_2$

$$(R_1 + jwL_1)(\frac{1 + jwR_4C_4}{jwC_4}) = R_2R_3$$

$$(R_1 + jwL_1)(1 + jwR_4C_4) = jwR_2C_4R_3$$

$$R_1 + jwC_4R_4R_1 + jwL_1 + j^2w^2L_1C_4R_4 = jwC_4R_2R_3$$

$$(R_1 - w^2L_1C_4R_4) + j(wC_4R_4R_1 + wL_1) = jwC_4R_2R_3$$

Comparing the real term,

$$R_1 - w^2L_1C_4R_4 = 0$$

$$R_1 = w^2L_1C_4R_4$$

$$wC_4R_4R_1 + wL_1 = wC_4R_2R_3$$

$$C_4R_4R_1 + L_1 = C_4R_2R_3$$

$$L_1 = C_4R_2R_3 - C_4R_4R_1$$

Substituting the value of R_1 fro eqn. 2.14 into eqn. 2.15, we have,

$$L_1 = C_4R_2R_3 - C_4R_4 \times w^2L_1C_4R_4$$

$$L_1 = C_4R_2R_3 - w^2L_1C_4^2R_4^2$$

$$L_1(1 + w^2L_1C_4^2R_4^2) = C_4R_2R_3$$

$$L_1 = \frac{C_4R_2R_3}{1 + w^2L_1C_4^2R_4^2}$$

Substituting the value of L_1 in eqn. 2.14 , we have

$$R_1 = \frac{w^2C_4^2R_2R_3R_4}{1 + w^2C_4^2R_4^2}$$

$$Q = \frac{wL_1}{R_1} = \frac{w \times C_4R_2R_3}{1 + w^2C_4^2R_4^2} \times \frac{1 + w^2C_4^2R_4^2}{w^2C_4^2R_4R_2R_3}$$

$$Q = \frac{1}{wC_4R_4}$$

Advantage

Fixed capacitor is cheaper than variable capacitor.

This bridge is best suitable for measuring high value of Q-factor

Disadvantage

Equations of $L_1$and R_1 are complicated.

- Measurements of R_1 and L_1 require the value of frequency.
- This bridge cannot be used for measuring low Q- factor.

MAXWELL'S INDUCTANCE BRIDGE

The choke for which R_1 and L_1 have to measure connected between the points 'A' and 'B'. In this method the unknown inductance is measured by comparing it with the standard inductance.

L_2 is adjusted, until the detector indicates zero current.

Let R_1= unknown resistance

L_1= unknown

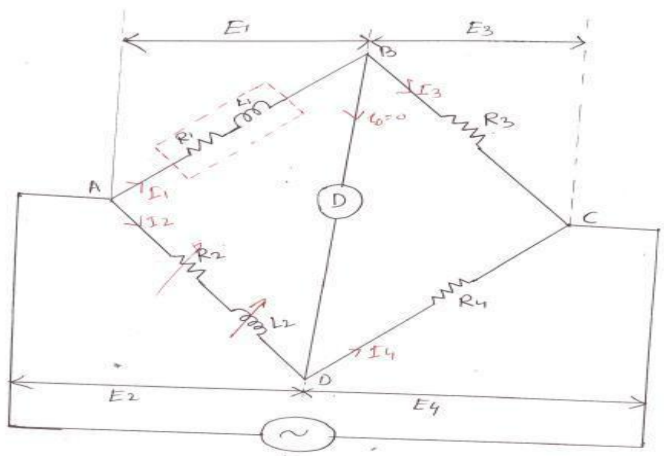
L_2 is adjusted, until the detector indicates zero current.

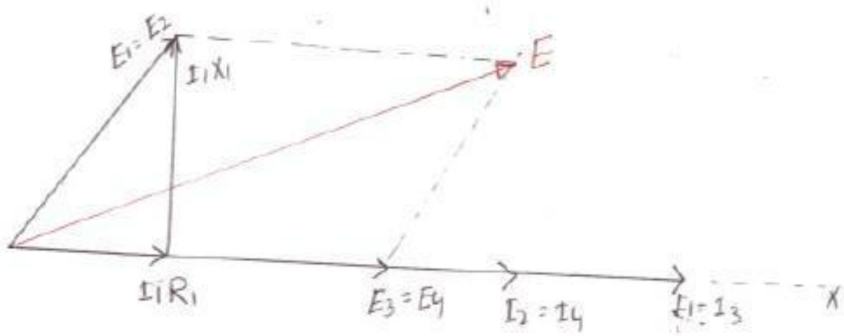
Let R_1= unknown resistance

L_1= unknown inductance of the choke.

L_2= known standard inductance

R_1,R_2,R_4= known resistances





Phasor diagram of maxwell bridge

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$

$$(R_1 + jwL_1)R_4 = (R_2 + jwL_2)R_3$$

$$R_1R_4 + jwL_1R_4 = R_2R_3 + jwL_2R_3$$

Comparing real part,

$$R_1R_4 = R_2R_3$$

$$\therefore R_1 = \frac{R_2R_3}{R_4}$$

Comparing the imaginary parts,

$$wL_1R_4 = wL_2R_3$$

$$L_1 = \frac{L_2R_3}{R_4}$$

Q-factor of choke, $Q = \frac{WL_1}{R_1} = \frac{WL_2R_3R_4}{R_4R_2R_3}$

$$Q = \frac{WL_2}{R_2}$$

Advantage

Expression for R1 and L1 are simple.

Equations area simple

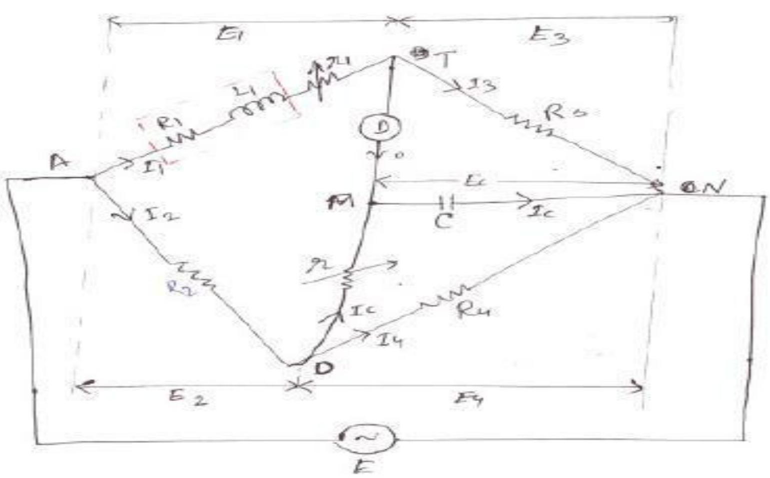
They do not depend on the frequency (as w is cancelled) R1 and L1 are independent of each other

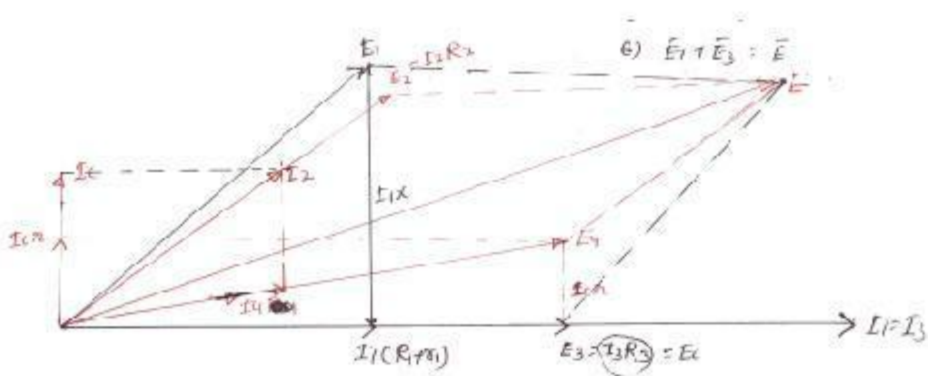
Disadvantage

Variable inductor is costly.

Variable inductor is bulky.

ANDERSON'S BRIDGE





Phasors diagram of Anderson's bridge

Step-1 Take I_1 as references vector .Draw $I_1R_1^1$ in phase with I_1

$R_1^1 = (R_1 + r_1)$, I_1X_1 is $\perp_r$ to $I_1R_1^1$

$E_1 = I_1R_1^1 + jI_1X_1$

Step-2 $I_1 = I_3$, E_3 is in phase with I_3, From the circuit ,

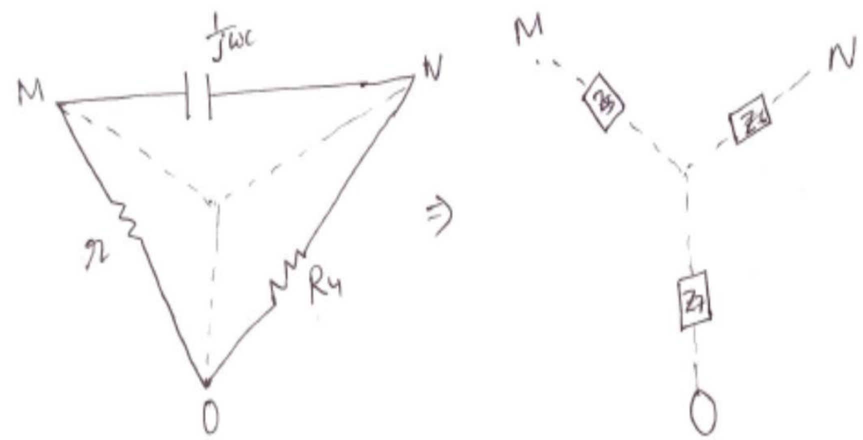
$E_3 = E_C$, I_C leads E_C by 90^0

Step-3 $E_4 = I_C r + E_C$

Step-4 Draw I_4 in phase with E_4 , By KCL , $\bar{I}_2 = \bar{I}_4 + \bar{I}_C$

Step-5 Draw E_2 in phase with I_2

Step-6 By KVL , $\bar{E}_1 + \bar{E}_3 = \bar{E}$ or $\bar{E}_2 + \bar{E}_4 = \bar{E}$



Equivalent delta to star conversion for the loop MON

$$Z_7 = \frac{R_4 \times r}{R_4 + r + \frac{1}{jwc}} = \frac{jwCR_4r}{1 + jwC(R_4 + r)}$$

$$Z_6 = \frac{R_4 \times \frac{1}{jwC}}{R_4 + r + \frac{1}{jwc}} = \frac{R_4}{1 + jwC(R_4 + r)}$$

$$(R_1^1 + jwL_1) \times \frac{R_4}{1 + jwC(R_4 + r)} = R_3(R_2 + \frac{jwCR_4r}{1 + jwC(R_4 + r)})$$

$$\Rightarrow \frac{(R_1^1 + jwL_1)R_4}{1 + jwC(R_4 + r)} = R_3\left[\frac{R_2(1 + jwC(R_4 + r)) + jwCrR_4}{1 + jwC(R_4 + r)}\right]$$

$$\Rightarrow R_1^1R_4 + jwL_1R_4 = R_2R_3 + jCwR_2R_3(r + R_4) + jwCrR_4R_3$$

Comparing Real term

$$R_1^1R_4 = R_2R_3$$

$$(R_1 + r_1)R_4 = R_2R_3$$

$$R_1 = \frac{R_2R_3}{R_4} - r_1$$

Comparing the imaginary term,

$$wL_1R_4 = wCR_2R_3(r + R_4) + wcrR_3R$$

$$L_1 = \frac{R_2R_3C}{R_4}(r + R_4) + R_3rC$$

$$L_1 = R_3C\left[\frac{R_2}{R_4}(r + R_4) + r\right]$$

Advantage

Variable capacitor is not required.

Inductance can be measured accurately.

R1 and L1 are independent of frequency.

Accuracy is better than other bridges.

SCHERING BRIDGE

$E1 = I1r1 - jI1X4$

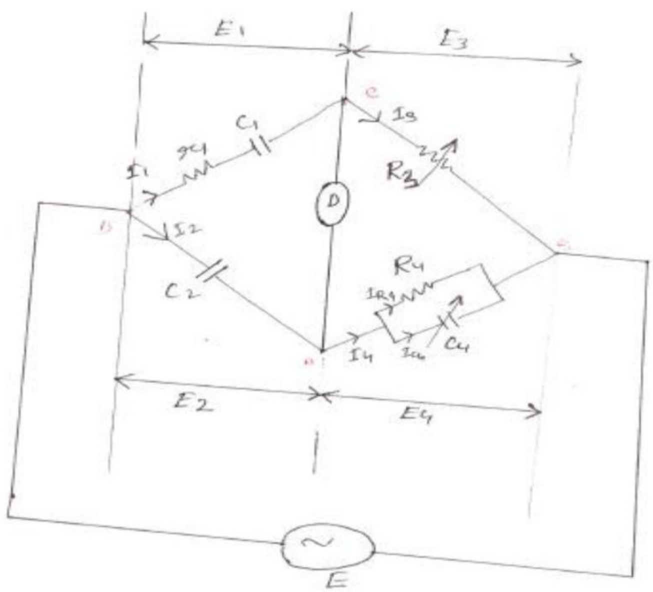
C2 = C4= Standard capacitor (Internal resistance=0)

C4= Variable capacitance.

C1= Unknown capacitance.

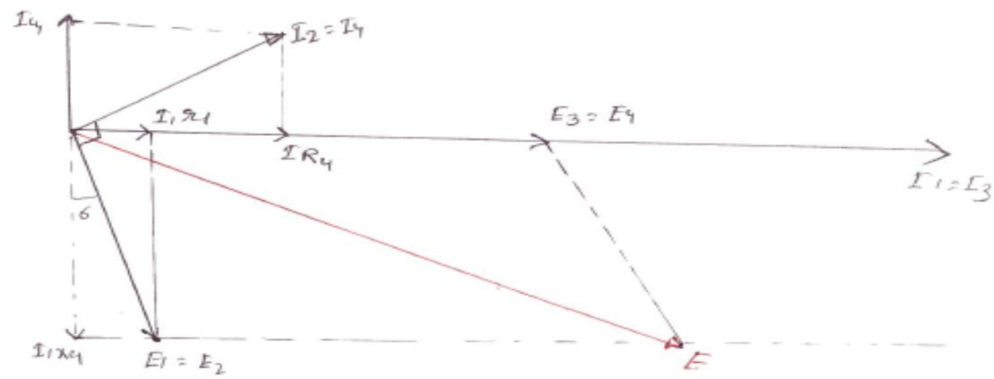
r1= Unknown series equivalent resistance of the capacitor

R3=R4= Known resistor.



$$Z_1 = r_1 + \frac{1}{jwC_1} = \frac{jwC_1r_1 + 1}{jwC_1}$$

$$Z_4 = \frac{R_4 \times \frac{1}{jwC_4}}{R_4 + \frac{1}{jwC_4}} = \frac{R_4}{1 + jwC_4R_4}$$



At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{1+jwC_1r_1}{jwC_1} \times \frac{R_4}{1+jwC_4R_4} = \frac{R_3}{jwC_2}$$

$$(1+jwC_1r_1)R_4C_2 = R_3C_1(1+jwC_4r_4)$$

$$R_2C_2 + jwC_1r_1R_4C_2 = R_3C_1 + jwC_4R_4R_3C_1$$

Comparing the real part,

$$\therefore C_1 = \frac{R_4C_2}{R_3}$$

Comparing the imaginary part,

$$wC_1r_1R_4C_2 = wC_4R_3R_4C_1$$

$$r_1 = \frac{C_4R_3}{C_2}$$

Dissipation factor of capacitor,

$$D = wC_1r_1 = w \times \frac{R_4C_2}{R_3} \times \frac{C_4R_3}{C_2}$$

$$\therefore D = wC_4R_4$$

Advantage

In this type of bridge, the value of capacitance can be measured accurately. It can measure capacitance value over a wide range.

It can measure dissipation factor accurately

Disadvantage

It requires two capacitors. Variable standard capacitor is costly.

TYPES OF DETECTOR

The following types of instruments are used as detector in A.C. bridge.

- Vibration galvanometer
- Head phones (speaker)
- Tuned amplifier

Vibration galvanometer

Between the point 'B' and 'D' a vibration galvanometer is connected to indicate the bridge balance condition. This A.C. galvanometer which works on the principle of resonance. The A.C. galvanometer shows a dot, if the bridge is unbalanced.

Head phones

Two speakers are connected in parallel in this system. If the bridge is unbalanced, the speaker produced more sound energy. If the bridge is balanced, the speaker do not produced any sound energy.

Tuned amplifier

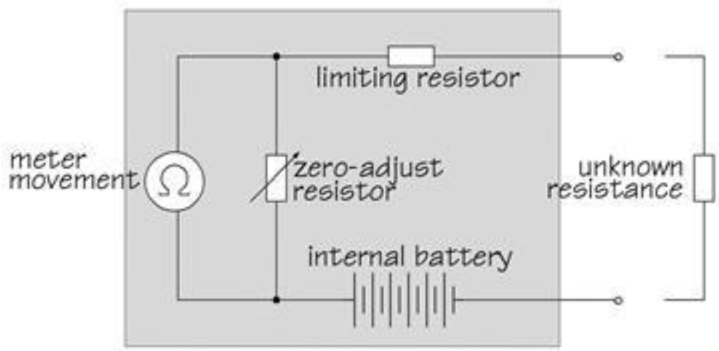
If the bridge is unbalanced the output of tuned amplifier is high. If the bridge is balanced, output of amplifier is zero

Measuring Resistance: Ohmmeters

To measure the *resistance* of a circuit or of a circuit component, we use an instrument called an **ohmmeter**. Ohmmeters also provide a convenient way in which to check *continuity* — that is, to find out whether there are any breaks in a circuit. When checking continuity, we are usually only interested in observing a deflection, and *not* necessarily the value of the resistance reading.

An ohmmeter works by using its internal battery to pass a small test current through the unknown resistance, and measuring the value of that current: the higher the resulting current, of course, the lower the resistance and *vice-versa*. Its scale, of course, is graduated in ohms and kilohms.

The following schematic diagram shows the basic internal circuit for a typical ohmmeter:



ohmmeter interal circuit

The ohmmeter's moving-coil movement is connected in *series* with a **battery**, a fixed-value **limiting resistor**, and a pair of **terminals** across which the unknown resistance will be connected.

Connected in *parallel* with the movement is a variable '**zero-adjust' shunt resistor**, which is used to zero the instrument in order to compensate for any changes in the battery's voltage. Using this variable resistor to obtain a full-scale deflection is called '**zero-ohms adjustment**', and *this action must be carried out prior to taking any resistance measurement*. In the case of a multiple-range ohmmeter, the zero-ohms adjustment must also be made *after* changing the range, but *before* taking a new measurement. This compensates for any variations in the voltage of the instrument's built-in battery; if a zero-ohms adjustment *cannot* be achieved, then the voltage is too low and the battery must be replaced.

The function of the limiting resistor is to protect the movement from burning out, by preventing the current that flows during the zeroing process from significantly-exceeding the movement's full-scale deflection current.

The scale of an ohmmeter differs from that of an ammeter or voltmeter, in *two* very important ways. Firstly, its scale is *reversed* —i.e. it reads from right to left— with 'zero ohms' corresponding to its full-scale deflection. Secondly, the scale is *non-linear*, with its graduations becoming closer and closer together and, therefore, more difficult to read, at the higher values of resistance (i.e. towards the left-hand end of the scale).