SATHYABAMA UNIVERSITY

SCHOOL OF ELECTRONICS AND ELECTRICAL ENGINEERING

DEPARTMENT OF ELECTRONICS AND INSTRUMENTATION

COURSE MATERIAL

SICX1003	MEASUREMENTS AND INSTRUMENTATION	L	T	P	Credits	Total Marks
	(Common to ECE, ETCE and EEE)	3	0	0	3	100

UNIT I BASIC MEASUREMENTS **8 hrs.**

Functional Elements of measurement system- Examples - Characteristics of instruments: Static characteristics - Dynamic characteristics - Types of errors -sources of errors- methods of elimination-Analysis of data - Limiting errors-Relative limiting error-Combination of Quantities with limiting errors – Statistical treatment of data: Histogram, Mean, Measure of dispersion from the mean, Range, Deviation, Average deviation, Standard Deviation, Variance-Calibration and Standards- Process of Calibration.

UNIT II TRANSDUCERS **10 hrs.**

Transducers –Classification of Transducers, Characteristics - Basic Requirements of a Transducer- Resistive Transducer: Strain Gauge - Inductive Transducer: Linear Variable Differential Transducer - Capacitive displacement transducer, Load cell - Piezoelectric Transducers - Digital transducers – Thermoelectric transducers : Thermocouple, Thermistor, Radiation Pyrometers- Interfacing transducers to measurement systems: IEEE488.

UNIT III ELECTRICAL MEASUREMENTS **10 hrs.**

Measurement of Voltage and Current: D'Arsonval Galvanometer, permanent magnet moving coil, permanent magnet moving iron, Dynamometer - Measurement of Resistance, Inductance and Capacitance: Wheat stone bridge, Kelvin double bridge, Wein Bridge, Hay's bridge, Maxwell bridge, Anderson bridge, Schering bridge – ohmmeter, VOM meter.

UNIT IV ELECTRONIC MEASUREMENTS **10 hrs.**

Signal generators: Function Generators – RF Signal Generators – Sweep generators- Wave Analyzer- Harmonic Distortion Analyzer - Spectrum Analyzer - DC & AC Voltmeters – Digital Voltmeters- Types- Automatic polarity indication, Automatic Ranging, Auto Zeroing- Electronic Multimeters – Digital Multimeters – Frequency Counters- Measurement of Frequency and Time Interval

UNIT V STORAGE AND DISPLAY INSTRUMENTS **12 hrs.**

Cathode Ray Oscilloscopes - Storage Oscilloscope - Digital Storage Oscilloscope – Sampling CRO - Dual Trace Oscilloscopes – Electromechanical servo type XT & XY recorders - LED, LCD, Dot Matrix Display – Magnetic Tape and Disk Recorders/ Reproducers.

TEXT BOOKS:-

1. A.K.Sawhney," Electrical, Electronic measurement & Instrumentation", Dhanpat Rai & sons, 18th Edition, Reprint 2010
2. Doeblien E..O. "Measurement System Applications and Design", McGraw Hill , 5th Edition,2004.
3. Albert.D.Helfrick & William.D.Cooper, "Modern Electronic Instrumentation & Measurement Techniques", PHI , 2003.

REFERENCE BOOKS:

1. E.W. Golding and F.C. Widdis, "Electrical Measurements and measuring Instruments", 5th Edition, Wheeler Publications

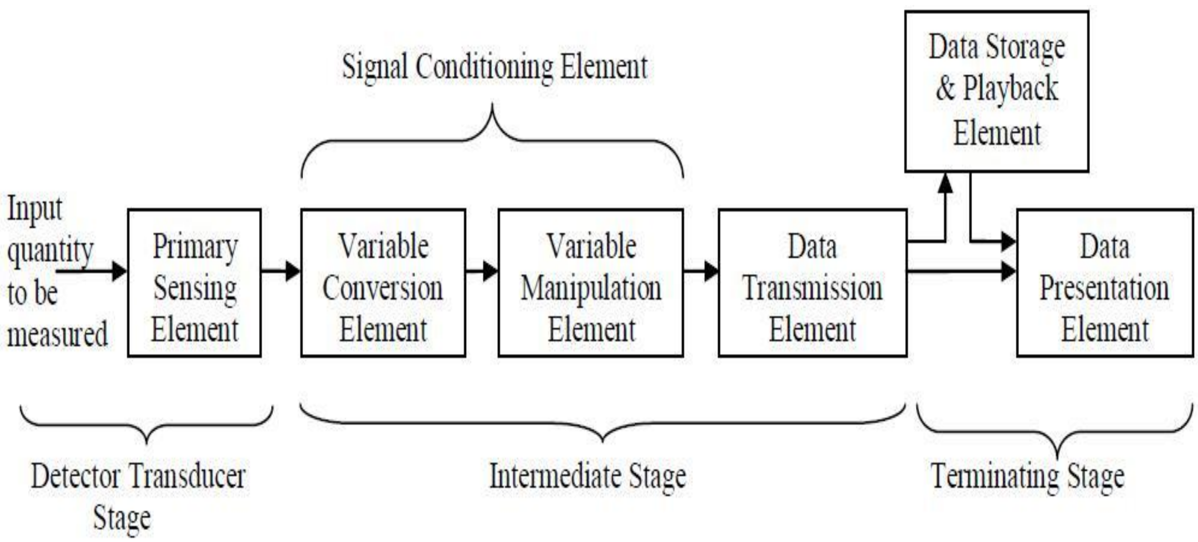
UNIT - I

UNIT I BASIC MEASUREMENTS 8 hrs.

Functional Elements of measurement system- Examples - Characteristics of instruments: Static characteristics - Dynamic characteristics - Types of errors -sources of errors- methods of elimination-Analysis of data - Limiting errors-Relative limiting error-Combination of Quantities with limiting errors – Statistical treatment of data: Histogram, Mean, Measure of dispersion from the mean, Range, Deviation, Average deviation, Standard Deviation, Variance-Calibration and Standards- Process of Calibration.

FUNCTIONAL ELEMENTS OF MEASUREMENT SYSTEM

A systematic organization and analysis are more important for measurement systems. The whole operation system can be described in terms of functional elements. The functional elements of generalized measurement system are shown in figure.



Functional elements of generalized measurement system

Most of the measurement system consists of following functional elements.

1. Primary sensing element
2. Variable conversion element
3. Variable manipulation element
4. Data transmission element
5. Data storage and playback element
6. Data presentation element

1. Primary Sensing Element

The quantity under measurement makes its first contact with primary sensing element of measurement system. The quantity is first sensed or detected by primary sensor. Then detected physical quantity signal is converted into an electrical signal by a transducer.

Transducer is defined as a device which converts a physical quantity into an electrical quantity. Sensor is act as primary element of transducer. In many cases the physical quantity is directly converted into an electrical quantity by a transducer. So the first stage of a measurement system is known as a *detector transducer stage.*

Example, Pressure transducer with pressure sensor, Temperature sensor etc.,

2. Variable Conversion Element

The output of primary sensing element is electrical signal of any form like a voltage, a frequency or some other electrical parameter. Sometime this output not suitable for next level of system. So it is necessary to convert the output some other suitable form while maintaining the original signal to perform the desired function the system.

For example the output primary sensing element is in analog form of signal and next stage of system accepts only in digital form of signal. So, we have to convert analog signal into digital form using an A/D converter. Here *A/D converter* is act as variable conversion element.

3. Variable Manipulation Element

The function of variable manipulation element is to manipulate the signal offered but original nature of signal is maintained in same state. Here manipulation means only *change in the numerical value of signal.*

Examples,

1. *Voltage amplifier* is act as variable manipulation element. Voltage amplifier accepts a small voltage signal as input and produces the voltage with greater magnitude .Here numerical value of voltage magnitude is increased.

2. *Attenuator* acts as variable manipulation element. It accepts a high voltage signal and produces the voltage or power with lower magnitude. Here numerical value of voltage magnitude is decreased.

- Linear process manipulation elements: Amplification, attenuation, integration, differentiation, addition and subtraction ect.,

- Nonlinear process manipulation elements: Modulation, detection, sampling, filtering, chopping and clipping ect.,

All these elements are performed on the signal to bring it to desired level to be accepted by the next stage of measurement system. This process of conversion is called *signal conditioning*. The combination of variable conversion and variable manipulation elements are called as *Signal Conditioning Element.*

4. Data Transmission Element

The elements of measurement system are actually physically separated; it becomes necessary to transmit the data from one to another. The element which is performs this function is called as data transmission element.

Example, Control signals are transmitted from earth station to Space-crafts by a telemetry system using radio signals. Here telemetry system is act as data transmission element.

The combination of Signal conditioning and transmission element is known as *Intermediate Stage* of measurement system.

5. Data storage and playback element

Some applications requires a separate data storage and playback function for easily rebuild the stored data based on the command. The data storage is made in the form of pen/ink and digital recording. Examples, magnetic tape recorder/ reproducer, X-Y recorder, X-t recorder, Optical Disc recording ect.,

6. Data presentation Element

The function of this element in the measurement system is to communicate the information about the measured physical quantity to human observer or to present it in an understandable form for monitoring, control and analysis purposes.

Visual display devices are required for monitoring of measured data. These devices may be analog or digital instruments like ammeter, voltmeter, camera, CRT, printers, analog and digital computers. Computers are used for control and analysis of measured data of measurement system. This

Final stage of measurement system is known as *Terminating stage*.

EXAMPLE OF GENERALIZED MEASUREMENT SYSTEM

Bourdon Tube Pressure Gauge:

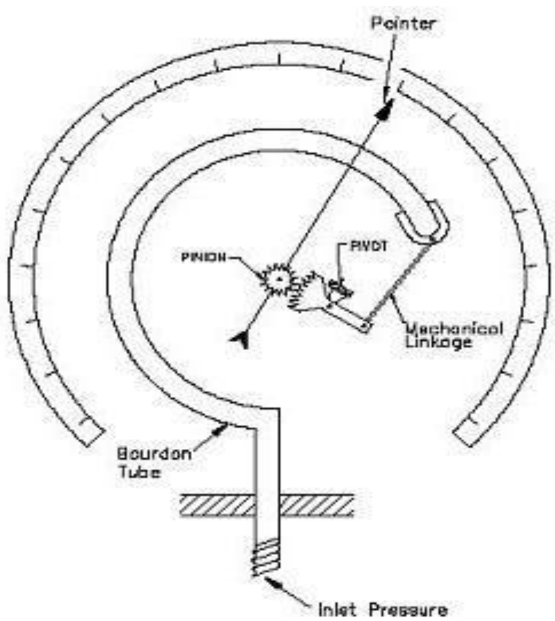
The simple pressure measurement system using bourdon tube pressure gauge is shown in figure.The detail functional elements of this pressure measurement system is given below.

Quantity to be measured : Pressure

Primary sensing element and
Variable conversion element : Bourdon Tube

Data Transmission element : Mechanical Linkages
Variable manipulation Element : Gearing arrangement

Data presentation Element : Pointer and Dial



Bourdon tube pressure gauge

In this measurement system, bourdon tube is act as primary sensing and variable conversion element. The input pressure is sensed and converted into small displacement by a bourdon tube. On account of input pressure the closed end of the tube is displaced. Because of this pressure in converted into small displacement. The closed end of bourdon tube is connected through mechanical linkage to a gearing arrangement.

The small displacement signal can be amplified by gearing arrangement and transmitted by mechanical linkages and finally it makes the pointer to rotate on a large angle of scale. If it is calibrated with known input pressure, gives the measurement of the pressure signal applied to the bourdon tube in measurand.

CHARACTERISTICS OF MEASURING INSTRUMENTS

These performance characteristics of an instrument are very important in their selection.

- *Static Characteristics:* Static characteristics of an instrument are considered for instruments which are used to measure an unvarying process condition. Performance criteria based upon static relations represent the static Characteristics. (The static characteristics are the value or performance given after the steady state condition has reached).

- *Dynamic Characteristics:* Dynamic characteristics of an instrument are considered for instruments which are used to measure a varying process condition. Performance criteria based upon dynamic relations represent the dynamic Characteristics.

STATIC CHARACTERISTICS

1) Accuracy

Accuracy is defined as the degree of closeness with which an instrument reading approaches to the true value of the quantity being measured. It determines the closeness to true value of instrument reading.

Accuracy is represented by percentage of full scale reading or in terms of inaccuracy or in terms of error value.

Example, Accuracy of temperature measuring instrument might be specified by ±3ºC. This accuracy means the temperature reading might be within + or -3ºC deviation from the true value.

Accuracy of an instrument is specified by ±5% for the range of 0 to 200ºC in the temperature scale means the reading might be within + or -10ºC of the true reading.

2) Precision

Precision is the degree of repeatability of a series of the measurement. Precision is measures of the degree of closeness of agreement within a group of measurements are repeatedly made under the prescribed condition.

Precision is used in measurements to describe the stability or reliability or the reproducibility of results.

Comparison between accuracy and precision.

S.No	Accuracy	Precision
1.	It refers to degree of closeness of the measured value to the true value.	It refers to the degree of agreement among group of readings
2.	Accuracy gives the maximum error that is maximum departure of the final result from its true value.	Precision of a measuring system gives its capability to reproduce a certain reading with a given accuracy

3) Bias

Bias is quantitative term describing the difference between the average of measured readings made on the same instrument and its true value (It is a characteristic of measuring instruments to give indications of the value of a measured quantity for which the average value differs from true value).

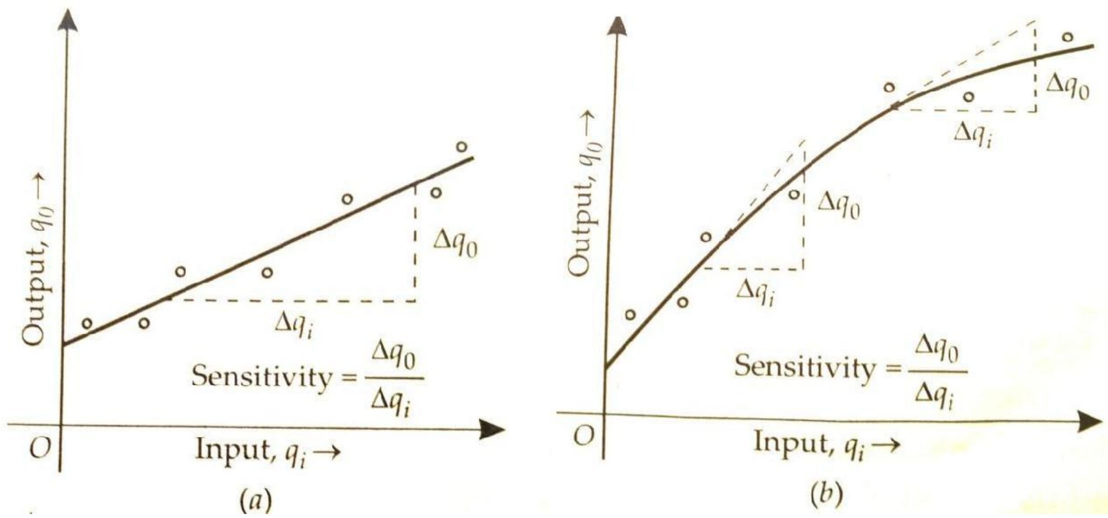
4) Sensitivity

Sensitivity is defined as the ratio of change in output signal (response) to the change in input signal (measurand). It is the relationship indicating how much output changes when input changes.

$$\text{Sensitivity} = \frac{\text{change in output}}{\text{change in input}}$$

$$\text{Sensitivity} = \frac{\Delta q_o}{\Delta q_i}$$

If the sensitivity is constant then the system is said to be linear system. If the sensitivity is variable then the system is said to be non linear system.

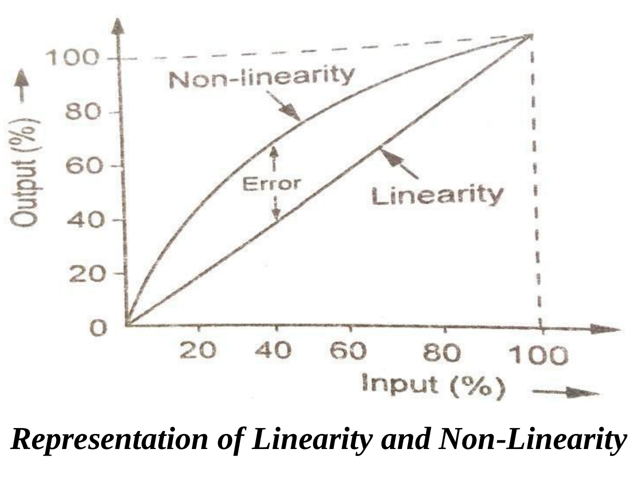


Definition of sensitivity for (a) Linear and (b) Non linear instrument

When the calibration curve is linear as in figure the sensitivity of the instrument can be defined as in slope of the calibration curve. In this case sensitivity is constant over the entire range of instrument. If the curve is not normally straight line or nonlinear instrument sensitivity varies with the input or varies from on range to another as in figure.

4) Linearity

Linearity is the best characteristics of an instrument or measurement system. Linearity of the instrument refers to the output is linearly or directly proportional to input over the entire range of instrument. So the degree of linear (straight line) relationship between the output to input is called as linearity of an instrument.



Representation of Linearity and Non-Linearity of an Instrument

Nonlinearity: The maximum difference or deviation of output curve from the Specified idealized straight line as shown in figure. Independent nonlinearity may be defined as

$$\text{Non linearity} = \frac{\text{Maximum deviation of output from the idealized straight line}}{\text{Actual reading or response}} \text{ X } 100$$

5) Resolution

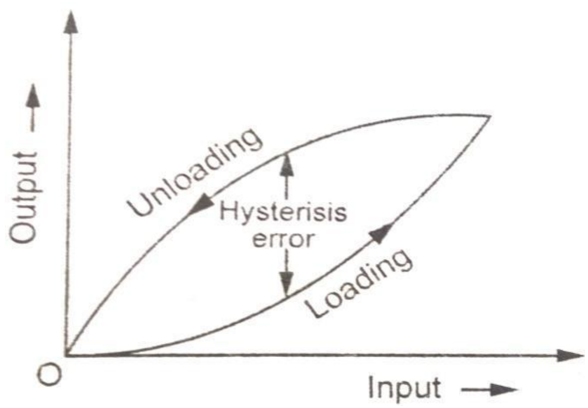
Resolution or Discrimination is the smallest change in the input value that is required to cause an appreciable change in the output. (The smallest increment in input or input change which can be detected by an instrument is called as resolution or discrimination.)

6) Hysteresis

Hysteresis is Non-coincidence of loading and unloading curves on output. Hysteresis effect shows up in any physical, chemical or electrical phenomenon

When input increases, output also increases and calibration curve can be drawn. If input is decreases from maximum value and output also decreases but does not follow the same curve, then there is a residual output when input is zero. This phenomenon is called Hysteresis. The difference between increasing change and decreasing change of output values is known as hysteresis error as shown in figure.

(The different outputs from the same value of quantity being measured are reached by a continuously increasing change or a continuously decreasing change)



Hysteresis Error of an instrument

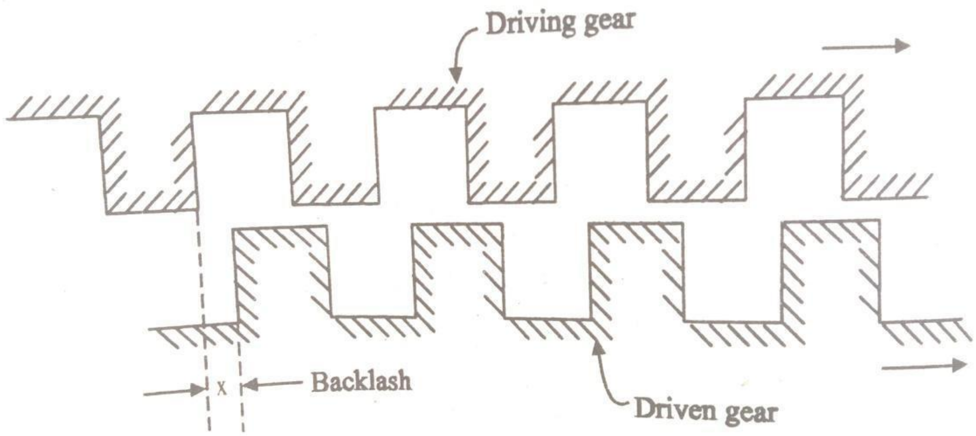
7) Dead Zone

Dead zone or dead band is defined as the largest change of input quantity for which there is no output the instrument due the factors such as friction, backlash and hysteresis within the system.(The region upto which the instrument does not respond for an input change is called dead zone)

Dead time is the time required by an instrument to begin to respond to change in input quantity.

8) Backlash

The maximum distance through which one part of the instrument is moved without disturbing the other part is called as backlash. (Backlash may be defined as the maximum distance or angle through which any part of the instrument can be moved without causing any motion of next part of the system)



Threshold because of backlash

Reasons for the presence of backlash in an instrument include allowing for lubrication, manufacturing errors, deflection under load, and thermal expansion.

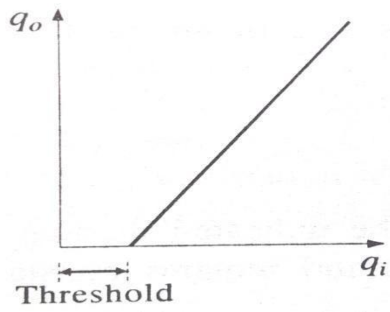
9) Drift

Drift is an undesirable change in output over a period of time that is unrelated to change in input, operating conditions. Drift is occurred in instruments due to internal temperature variations, ageing effects and high stress ect.

Zero drift is used for the changes that occur in output when there is zero output. It is expressed as percentage of full range output.

10) Threshold

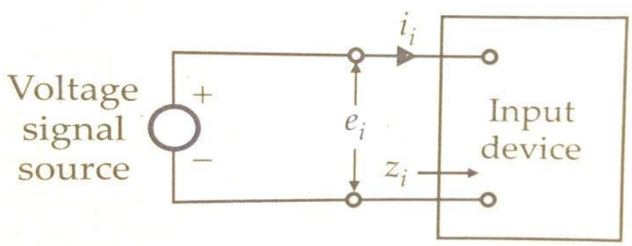
The minimum value of input which is necessary to activate an instrument to produce an output is termed its *threshold* as shown in figure. (Threshold is the minimum value of the input required to cause the pointer to move from zero position).



Threshold effect

11) Input Impedance

The magnitude of the impedance of element connected across the signal source is called Input Impedance. Figure shows a voltage signal source and input device connected across it.



voltage source and input device

The magnitude of the input impedance is given by

$$Z_i = \frac{e_i}{i_i}$$

Power extracted by the input device from the signal source is

$$P = e_i i_i = \frac{e_i^2}{Z_i}$$

From above two expressions it is clear that a low input impedance device connected across the voltage signal source draws more current and more power from signal source than high input impedance device.

12) Loading Effect

Loading effect is the incapability of the system to faith fully measure, record or control the input signal in accurate form.

13) Repeatability

Repeatability is defined as the ability of an instrument to give the same output for repeated applications of same input value under same environmental condition.

14) Reproducibility

Reproducibility is defined as the ability of an instrument to reproduce the same output for repeated applications of same input value under different environment condition.

In case of perfect reproducibility the instrument satisfies no drift condition.

DYNAMIC CHARACTERISTICS

The dynamic behaviour of an instrument is determined by applying some standard form of known and predetermined input to its primary element (sensing element) and then studies the output. Generally dynamic behaviour is determined by applying following three types of inputs.

***l. Step Input*:** Step change in which the primary element is subjected to an instantaneous and finite change in measured variable.

*2. **Linear Input***: Linear change, in which the primary element is, follows a measured variable, changing linearly with time.

*3. **Sinusoidal input***: Sinusoidal change, in which the primary element follows a measured variable, the magnitude of which changes in accordance with a sinusoidal function of constant amplitude.

The dynamic characteristics of an instrument are

(i) Speed of response,
(ii) Fidelity,
(iii) Lag, and
(iv) Dynamic error.

(i) Speed of Response

It is the rapidity with which an instrument responds to changes in the measured quantity.

(ii) Fidelity

It is the degree to which an instrument indicates the changes in the measured variable without dynamic error (faithful reproduction or fidelity of an instrument is the ability of reproducing an input signal faithfully (truly)).

(iii) Lag

It is the retardation or delay in the response of an instrument to changes in the measured variable. The measuring lags are two types:

- ***Retardation type:*** *In this case the response of an instrument begins immediately after a change in measured variable is occurred.*
- ***Time delay type:*** *In this case the response of an instrument begins after a dead time after the application of the input quantity.*

(iv) Dynamic Error

It is the difference between the true values of a quantity changing with time and the value indicated by the instrument, if no static error is assumed. It is also called as *Measurement Error*.

When measurement problems are concerned with rapidly varying quantities, the dynamic relations between the instruments input and output are generally defined by the use of differential equations.

Errors

The difference between the measured value of quantity and true value (Reference Value) of quantity is called as Error.

Error = Measured value - True Value

$\delta A = A_m - A_t$

Classification of Errors

All measurement can be made without perfect accuracy (degree of error must always be assumed). In reality, no measurement can ever made with 100% accuracy. It is important to find that actual accuracy and different types of errors can be occurred in measuring instruments. Errors may arise from different sources and usually classified as follows, Classification of Error

1. Gross Errors

2. Systematic Errors

a) Instrumental errors

i) Inherent shortcomings of instruments
ii) Misuse of instruments
iii) Loading effects

b) Environmental errors

c) Observational errors

3. Random Errors

1. Gross Errors

The main source of Gross errors is human mistakes in reading or using instruments and in recording and calculating measured quantity. As long as human beings are involved and they may grossly misread the scale reading, then definitely some gross errors will be occurred in measured value.

Example, Due to an oversight, Experimenter may read the temperature as 22.7^oC while the actual reading may be 32.7^oC He may transpose the reading while recording. For example, he may read 16.7^oC and record 27.6^oC as an alternative.

The complete elimination of gross errors is maybe impossible, one should try to predict and correct them. Some gross errors are easily identified while others may be very difficult to detect. Gross errors can be avoided by using the following two ways.

Great care should be taken in reading and recording the data.

Two, three or even more readings should be taken for the quantity being measured by using different experimenters and different reading point (different environment condition of instrument) to avoid re-reading with same error. So it is suitable to take a large number of readings as a close agreement between readings assures that no gross error has been occurred in measured values.

2. Systematic Errors

Systematic errors are divided into following three categories.

i. Instrumental Errors
ii. Environmental Errors
iii. Observational Errors

a) Instrumental Errors

These errors are arises due to following three reasons (sources of error).

a) Due to inherent shortcoming of instrument
b) Due to misuse of the instruments, and
c) Due to loading effects of instruments

i) Inherent Shortcomings of instruments

These errors are inherent in instruments because of their mechanical structure due to construction, calibration or operation of the instruments or measuring devices.

These errors may cause the instrument to read too low or too high.

Example, if the spring (used for producing controlling torque) of a permanent magnet instrument has become weak, so the instrument will always read high.

Errors may be caused because of friction, hysteresis or even gear backlash.

Elimination or reduction methods of these errors,

- The instrument may be re-calibrated carefully.
- The procedure of measurement must be carefully planned. Substitution methods or calibration against standards may be used for the purpose.
- Correction factors should be applied after determining the instrumental errors.

Ii) Misuse of Instruments

In some cases the errors are occurred in measurement due to the fault of the operator than that of the instrument. A good instrument used in an unintelligent way may give wrong results.

Examples, Misuse of instruments may be failure to do zero adjustment of instrument, poor initial adjustments, using leads of too high a resistance and ill practices of instrument beyond the manufacturer's instruction and specifications ect.

iii) Loading Effects

The errors committed by loading effects due to improper use of an instrument for measurement work. In measurement system, loading effects are identified and corrections should be made or more suitable instruments can be used.

Example, a well calibrated voltmeter may give a misleading (may be false) voltage reading when connected across a high resistance circuit. The same voltmeter, when connected across a low resistance circuit may give a more reliable reading (dependable or steady or true value).

In this example, *voltmeter has a loading effect* on the circuit, altering the actual circuit conditions by measurement process. So errors caused by loading effect of the meters can be avoided by using them intelligently.

b) Environmental Error

Environmental error occurs due to external environmental conditions of the instrument, such as effects of temperature, pressure, humidity, dust, vibration or external magnetic or electrostatic fields.

Elimination or reduction methods of these undesirable errors are

- Arrangements should be made to keep the conditions as nearly as constant as possible. Example, temperature can be kept constant by keeping the instrument in the temperature controlled region.

- The device which is used against these environmental effects.

  Example, variations in resistance with temperature can be minimized by using very low resistance temperature co-efficient of resistive material.

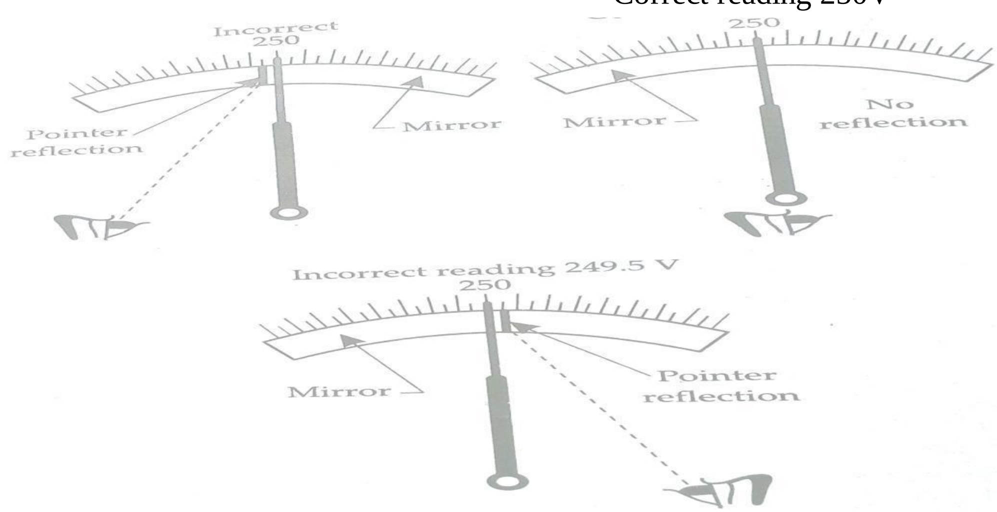
- Employing techniques which eliminate the effects of these disturbances. For example, the effect of humidity dust etc., can be entirely eliminated by tightly sealing the equipment.

- The external or electrostatic effects can be eliminated by using magnetic or electrostatic shield on the instrument.

- Applying computed corrections: Efforts are normally made to avoid the use of application of computed corrections, but where these corrections are needed and are necessary, they are incorporated for the computations of the results

c) Observational Errors

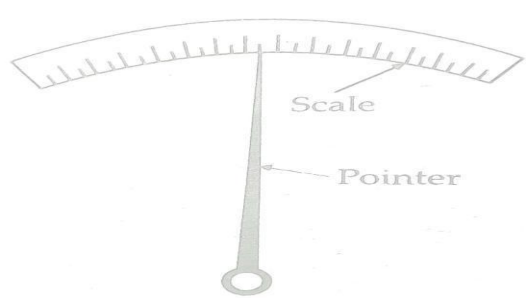
There are many sources of observational errors. As an example, the pointer of a voltmeter rests slightly above the surface of the scale. Thus an error on account of PARALLAX will be acquired unless the line of vision of the observer is exactly above the pointer. To minimize parallax errors, highly accurate meters are provided with mirrored scales as shown in figure.



Errors due to parallax

When the pointer's image appears hidden by the pointer, observer's eye is directly in line with the pointer. Although a mirrored scale minimizes parallax error, an error is necessarily presented through it may be very small.

So we can eliminate this parallax error by having the pointer and scale in the same plane as shown in figure



Arrangements showing scale and pointer in the same plane

The observational errors are also occurs due to involvement of human factors. For example, there are observational errors in measurements involving timing of an event Different observer may produce different results, especially when sound and light measurement are involved.

The complete elimination of this error can be achieved by using digital display of output.

3. Random Errors

These errors are occurred due to unknown causes and are observed when the magnitude and polarity of a measurement fluctuate in changeable (random) manner.

The quantity being measure is affected by many happenings or disturbances and ambient influence about which we are unaware are lumped together and called as *Random* or *Residual*. The errors caused by these disturbances are called *Random Errors*. Since the errors remain even after the systematic errors have been taken care, those errors are called as *Residual (Random) Errors.*

Random errors cannot normally be predicted or corrected, but they can be minimized by skilled observer and using a well maintained quality instrument.

Errors in Measuring Instruments

No measurement is free from error in reality. An intelligent skill in taking measurements is the ability to understand results in terms of possible errors. If the precision of the instrument is sufficient, no matter what its accuracy is, a difference will always be observed between two measured results. So an understanding and careful evaluation of the errors is necessary in measuring instruments. The Accuracy of an instrument is measured in terms of errors.

True value

The true value of quantity being measured is defined as the average of an infinite number of measured values when the average deviation due to the various contributing factors tends to zero.

In ideal situation is not possible to determine the True value of a quantity by experimental way. Normally an experimenter would never know that the quantity being measured by experimental way is the True value of the quantity or not.

In practice the true value would be determined by a “standard method”, that is a method agreed by experts with sufficient accurate.

Static Error

Static error is defined as a difference between the measured value and the true value of the quantity being measured. It is expressed as follows.

$$\delta A = A_m - A_t \quad \text{------------------- (1)}$$

Where, δA= Error, A_m =Measured value of quantity and A_t= True value of quantity.

δA is also called as absolute static error of quantity A and it is expressed as follows.

$$\varepsilon_0 = \delta A \quad \text{------------------------ (2)}$$

Where, ε_0 = Absolute static error of quantity A under measurement.

The absolute value of δA does not specify exactly the accuracy of measurement .so the quality of measurement is provided by relative static error.

Relative static error

Relative static error is defined as the ratio between the absolute static errors and true value of quantity being measured. It is expressed as follows.

$$\varepsilon_r = \frac{\text{Absolute Error}}{\text{True Value}} = \frac{\delta A}{A_t} = \frac{\varepsilon_0}{A_t} \quad \text{-------------------- (3)}$$

Percentage static error= % $\varepsilon_r = \varepsilon_r \times 100$

From equation (1), $A_t = A_m - \delta A$

$$A_t = A_m - \varepsilon_0$$

$$A_t = A_m - \varepsilon_r A_t \quad \text{------------- (4)}$$

$$A_t + \varepsilon_r A_t = A_m$$

$$A_t (1 + \varepsilon_r) = A_m$$

$$A_t = \frac{A_m}{1 + \varepsilon_r}$$

$\varepsilon_0 = \delta A$ is small, which means that the difference between measured value and true values is very small, $A_m - A_t$ = Negligible or small. So Almost

$A_m = A_t$ (that is $\varepsilon_r <<< 1$).

$$\text{From equation (4),} \quad A_t = A_m - \varepsilon_r A_t$$

Substitute $A_t = A_m$ in equation (4),

$$A_t = A_m - \varepsilon_r A_m$$

$$A_t = A_m (1 - \varepsilon_r)$$

Static error Correction or method of Correction:

It is the difference between the true value and the measured value of quantity.

$$\delta C = A_t - A_m \text{ ---------------- (5)}$$

Where, δC= Static Error Correction = - δA

* For Detail Error correction (Rectification or Elimination or Reduction) methods of all categories of errors are discussed in the topic of *classification of errors*.

SOURCES OF ERRORS

The sources of error, other than the inability of a piece of hardware to provide a true measurement are listed below,

1) Insufficient knowledge of process parameters and design conditions.
2) Poor design
3) Change in process parameters, irregularities, upsets (disturbances) ect.
4) Poor maintenance
5) Errors caused by people who operate the instrument or equipment.
6) Certain design limitations.

STATISTICAL EVALUATION OF MEASUREMENT DATA

Statistical Evaluation of measured data is obtained in two methods of tests as shown in below.

- ***Multi Sample Test*:** In multi sample test, repeated measured data have been acquired by different instruments, different methods of measurement and different observer.

- ***Single Sample Test*:** measured data have been acquired by identical conditions (same instrument, methods and observer) at different times.

Statistical Evaluation methods will give the most probable true value of measured quantity. The mathematical background statistical evaluation methods are Arithmetic Mean, Deviation Average Deviation, Standard Deviation and variance.

Arithmetic Mean

The most probable value of measured reading is the arithmetic mean of the number of reading taken. The best approximation is made when the number of readings of the same quantity is very large. Arithmetic mean or average of measured variables X is calculated by taking the sum of all readings and dividing by the number of reading.

The Average is given by,

$$X = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\Sigma x}{n}$$

Where, X= Arithmetic mean, x_1, $x_{2.......}$ x_n = Readings or variable or samples and n= number of readings.

Deviation (Deviation from the Average value)

The Deviation is departure of the observed reading from the arithmetic mean of the group of reading. Let the deviation of reading x_1 be d_1 and that of x_2 be d_2 etc.,

$$d_1 = x_1 - X$$
$$d_2 = x_2 - X$$
..
..

..
$$d_n = x_n - X$$

The algebraic sum deviation is Zero ($d_1 + d_2 + ... + d_n = 0$)

Average Deviation:

Average deviation defined as the average of the modulus (without respect to its sign) of the individual deviations and is given by,

$$D = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n} = \frac{\Sigma|d|}{n}$$

Where, D= Average Deviation.

The average deviation is used to identify precision of the instruments which is used in making measurements. Highly precise instruments will give a low average deviation between readings.

Standard Deviation

Standard deviation is used to analysis random errors occurred in measurement. The standard Deviation of an infinite number of data is defined as the square root of the sum of individual deviations squared, divided by the number of readings (n).

$$\text{Standard deviation is} \quad S.D = \sigma = \sqrt{\frac{d_1{}^2+d_2{}^2+d_3{}^2+\cdots+d_n{}^2}{n}} = \sqrt{\frac{\Sigma d^2}{n}} \text{ ; for n >20}$$

$$\text{Standard deviation is} \quad S.D = s = \sqrt{\frac{d_1{}^2+d_2{}^2+d_3{}^2+\cdots+d_n{}^2}{n-1}} = \sqrt{\frac{\Sigma d^2}{n-1}} \text{ ; for n <20}$$

Variance

The variance is the mean square deviation, which is the same as S.D except Square root. Variance is Just the squared standard deviation.

$$\text{Variance} \quad V = (\text{Standard deviation})^2$$

$$\text{Variance} \quad V = \sigma^2 = \frac{d_1{}^2+d_2{}^2+d_3{}^2+\cdots+d_n{}^2}{n} = \frac{\Sigma d^2}{n} \text{ ; for n >20}$$

$$\text{Variance} \quad V = s^2 = \frac{d_1{}^2+d_2{}^2+d_3{}^2+\cdots+d_n{}^2}{n-1} = \frac{\Sigma d^2}{n-1} \text{ ; for n <20}$$

Histogram:

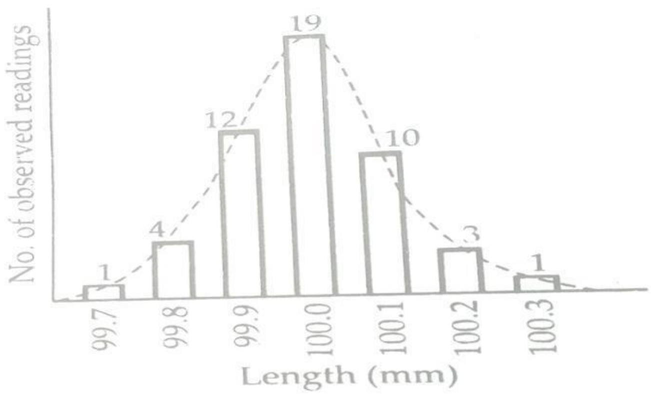
When a number of Multisample observations are taken experimentally there is a scatter of the data about some central value. For representing this results in the form of a *Histogram.* A histogram is also called a *frequency distribution curve.*

Example: Following table3.1 shows a set of 50 readings of length measurement. The most probable or central value of length is 100mm represented as shown in figure Histogram.

Table 3.1

Length (mm)	Number of observed readings (frequency or occurrence)
99.7	1
99.8	4
99.9	12
100.0	19
100.1	10
100.2	3
100.3	1

Total number of readings =50



Histogram

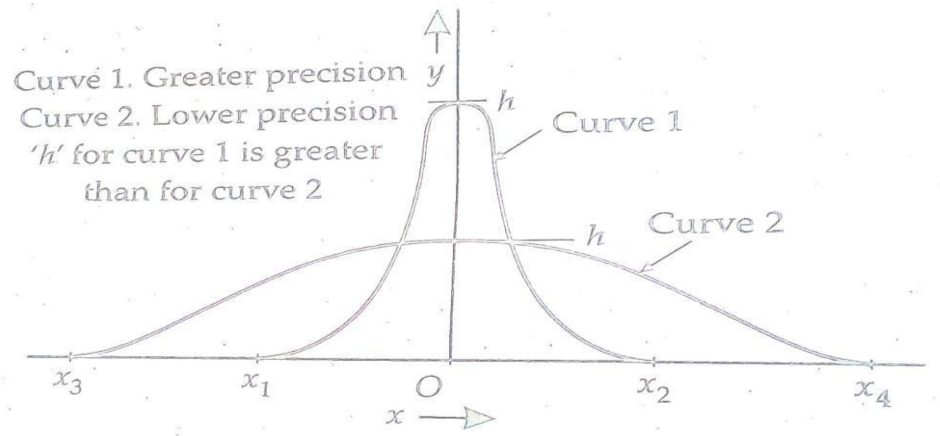
This histogram indicates the number of occurrence of particular value. At the central value of 100mm is occurred 19 times and recorded to the nearest 0.1mm as shown in figure 3.3. Here bell shape dotted line curve is called as normal or Gaussian curve.

Measure of Dispersion from the Mean

The property which denotes the extent to which the values are dispersed about the central value is termed as dispersion. The other name of dispersion is *spread or scatter*.

Measure of dispersion from central value is an indication of the degree of consistency (precision) and regularity of the data.

Example: Figure shows the two sets of data and curve 1 vary from x_1 to x_2 and curve 2 vary from x_3 to x_4. Curve 1 is having smaller dispersion from central value than the curve 2. Therefore curve 1 is having greater precision than the curve 2.



Curves showing different ranges and precision index

Range

The simplest possible measure of dispersion is the range which is the difference between greatest and least values of measured data.

Example: In figure, the range of curve1 is ($x_2 – x_1$) and range of curve 2 is (x_4 - x_3).

Limiting Errors (Guarantee Errors or Limits of errors):

In most of the instruments the accuracy is guaranteed to be within a certain percentage of full scale reading. The manufacturer has to specify the deviations from the *nominal value* of a particular quantity. The limits of these deviations from the specified value are called as *Limiting Errors* or *Guarantee Errors***.**

The magnitude of Limiting Error=Accuracy x Full scale reading. In general the actual value of quantity is determined as follows.

Actual Value of Quantity = Nominal value ± Limiting Error

$$A_a = A_n \pm \delta A$$

Where, A_a = Actual value of quantity; A_n = Nominal value of Quantity; ± δA= Limiting error.

For Example, Nominal magnitude of resister is 1000Ω with a limiting error ±100Ω. Determine the Actual magnitude of the resistance.

Actual value of quantity A_a =1000±100Ω or $A_a \geq 900\Omega$ and $A_a \leq 1100\Omega$.

Therefore the manufacturer guarantees that the value of resistance of resistor lies between 900Ω and 1100Ω.

Relative (Fractional) Limiting Error

The relative limiting error is defined as the ratio of the error to the specified (nominal) magnitude of the quantity.

Relative Limiting Error ε_r=

Then limiting values calculated as follows,

We know that $A_a = A_n \pm \delta A = A_n \pm \varepsilon_r A_n = A_n (1 \pm \varepsilon_r)$

Percentage limiting error $\% \varepsilon_r = \varepsilon_r \times 100$

In limiting errors the nominal value A_n is taken as the true value or quantity, the quantity which has the maximum deviation from A_a is taken as the incorrect quantity.

$$\text{Then } \delta A = A_a - A_n$$

$$\text{Therefore Relative Limiting Error } \varepsilon_r = \frac{A_a - A_n}{A_n} = \frac{\text{Actual value} - \text{nominal value}}{\text{nominal value}}$$

For Example, considered $A_n = 100\Omega$ and $\delta A = \pm 10\ \Omega$;

Relative limiting error $\varepsilon_r = \frac{\delta A}{A_n} = \pm \frac{10}{100} = \pm 0.1$

Percentage Limiting error $\%\varepsilon_r = 0.1 \times 100 = \pm 10\%$

Limiting values of resistance are:

$$A_a = A_n (1 \pm \varepsilon_r) = 100\,(1 \pm 0.1) = 100 \pm 10\Omega$$

Probable error

The most probable or best value of a Gaussian distribution is obtained by taking arithmetic mean of the various values of the variety. A convenient measure of precision is achieved by the quantity r. It is called Probable Error of P.E. It is expressed as follows,

$$\text{Probable Error} = \text{P. E} = r = \frac{0.4769}{h}$$

Where r= probable error and h= constant called precision index

Gaussian distribution and Histogram are used to estimate the probable error of any measurement.

Histogram – Refer the topic of statistical analysis.

Normal or Gaussian curve of errors

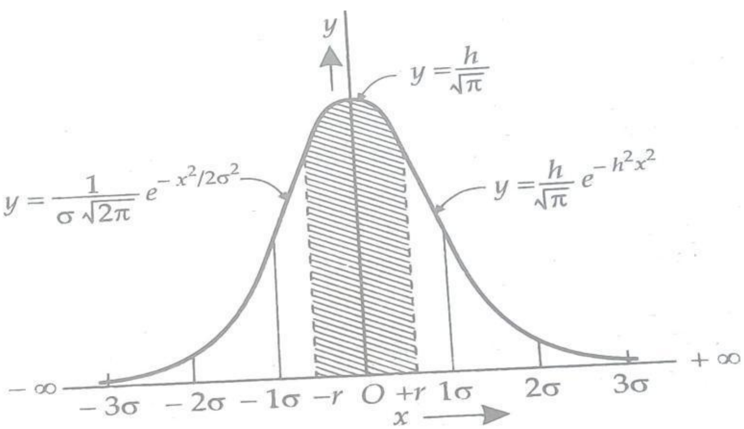
The normal or Gaussian law of errors is the basis for the major part of study of random errors. The law of probability states the normal occurrence of deviations from average value of an infinite number of measurements can be expressed by,

$$y = \frac{h}{\sqrt{\Pi}} \exp(-h^2x^2)$$

Where, x= magnitude deviation from mean

y=Number of readings at any deviation x (the probability of occurrence of deviation x) h= A constant called precision index

The Normal or Gaussian probability curve is shown in figure. In this curve r is the measure of precision quantity (probable error=r). The points –r and +r are locating the area bounded by the Gaussian curve.



The Normal or Gaussian probability curve

Precision index x=0 then, $y = h/\sqrt{\pi}$. The maximum value of y depends upon h. If y is larger, then the corresponding curve is having greater precision. Then the probable is determined using following expression.

$$\text{Probable Error} = r = \frac{0.4769}{h}$$

Instrument Error Combination:

When two or more quantities are measurand, then final result is easily generalised for many measurand by using the combination of these quantities. In many measurand each quantity having limiting error, are combined, it is easy to compute the limiting error of combination. The limiting error can be easily found by considering the relative increment of the function if the final result is in the form of an algebraic equation.

1) Sum of two Quantities (Addition)

Let us consider r_1 and r_2 are measurand quantities and final result is X

Therefore, $X = r_1 + r_2$

The relative increment of the function is given by,

$$\frac{dX}{X} = \frac{d(r_1 + r_2)}{X}$$

$$\frac{dX}{X} = \frac{dr_1}{X} + \frac{dr_2}{X}$$

Expressing the result in terms of relative increment of the component quantities,

$$\frac{dX}{X} = \frac{r_1 dr_1}{X\, r_1} + \frac{r_2 dr_2}{X\, r_2}$$

If the limiting errors of quantities are represented by $\pm\delta r_1$ and $\pm\delta r_2$, then the corresponding relative limiting error in X is given by,

$$\frac{\delta X}{X} = \pm\left[\frac{r_1}{X}\frac{\delta r_1}{r_1} + \frac{r_2}{X}\frac{\delta r_2}{r_2}\right]$$

The above equation shows that the resultant limiting error is equal to the sum of the products formed by multiplying the individual relative limiting errors by ratio of each term to the function.

2) Difference of two Quantities (Subtraction)

Let us consider r_1 and r_2 are measurand quantities and final result is X

Therefore, $X = r_1 - r_2$

The relative increment of the function is given by,

$$\frac{dX}{X} = \frac{d(r_1 - r_2)}{X}$$

$$\frac{dX}{X} = \frac{dr_1}{X} - \frac{dr_2}{X}$$

Expressing the result in terms of relative increment of the component quantities,

$$\frac{dX}{X} = \frac{r_1 dr_1}{X r_1} - \frac{r_2 dr_2}{X r_2}$$

If the limiting errors of quantities are represented by $\pm\delta r_1$ and $\pm\delta r_2$, the signs may be indicated, when the error in r_1 is $+\delta r_1$, the error in r_2 is $-\delta r_2$ and vice versa, then the relative limiting error in X is given by,

$$\frac{\delta X}{X} = \pm\left[\frac{r_1}{X}\frac{\delta r_1}{r_1} + \frac{r_2}{X}\frac{\delta r_2}{r_2}\right]$$

3) Sum of difference of more than two Quantities (Subtraction)

Let us consider r_1, r_2 and r_3 are measurand quantities and final result is X

Therefore, $X = r_1 \pm r_2 \pm r_3$

Then the relative limiting error in X is given by,

$$\frac{\delta X}{X} = \pm\left[\frac{r_1}{X}\frac{\delta r_1}{r_1} + \frac{r_2}{X}\frac{\delta r_2}{r_2} + \frac{r_3}{X}\frac{\delta r_3}{r_3}\right]$$

4) Product of two Quantities (Multiplication)

Let us consider r_1 and r_2 are measurand quantities and final result is X

Therefore, $X = r_1 \; r_2$

$$\log_e X = \log_e r_1 + \log_e r_2$$

Differentiating the above with respect to X,

$$\frac{1}{X} = \frac{1}{r_1}\frac{dr_2}{dX} + \frac{1}{r_2}\frac{dr_2}{dX}$$

$$\frac{1}{X} = \frac{1}{dX}\left[\frac{dr_1}{r_1} + \frac{dr_2}{r_2}\right]$$

$$\frac{dX}{X} = \frac{dr_1}{r_1} + \frac{dr_2}{r_2}$$

If the limiting errors of quantities are represented by $\pm\delta r_1$ and $\pm\delta r_2$, then the corresponding relative limiting error in X is given by,

$$\frac{\delta X}{X} = \pm\left[\frac{\delta r_1}{r_1} + \frac{\delta r_2}{r_2}\right]$$

Therefore the relative limiting error of product of terms is equal to the sum of relative limiting errors of terms.

5) Quotient (Division)

Let us consider r_1 and r_2 are measurand quantities and final result is X

Therefore, $X = \frac{r_1}{r_2}$

$$\log_e X = \log_e r_1 - \log_e r_2$$

Differentiating the above with respect to X,

$$\frac{1}{X} = \frac{1}{r_1}\frac{dr_1}{dX} - \frac{1}{r_2}\frac{dr_2}{dX}$$

$$\frac{1}{X} = \frac{1}{dX}\left[\frac{dr_1}{r_1} - \frac{dr_2}{r_2}\right]$$

$$\frac{dX}{X} = \frac{dr_1}{r_1} - \frac{dr_2}{r_2}$$

If the limiting errors of quantities are represented by $\pm\delta r_1$ and $\pm\delta r_2$ and considering worst condition when $\delta x_1/x_1$ is +ve and $\delta x_2/x_2$ is –ve or vice versa. Then the relative limiting error in X is given by,

$$\frac{\delta X}{X} = \pm\left[\frac{\delta r_1}{r_1} + \frac{\delta r_2}{r_2}\right]$$

The above result is same as the result for product of two quantities.

7) Product or quotient of more than two quantities

Let us consider r_1, r_2 and r_3 are measurand quantities and final result is X

$$\text{Let, } X = r_1 r_2 r_3 \text{ or } X = \frac{r_1}{r_2 r_3} \text{ or } X = \frac{1}{r_1 r_2 r_3}$$

Therefore from multiplication and division result, the relative limiting error in X is

$$\frac{\delta X}{X} = \pm\left[\frac{\delta r_1}{r_1} + \frac{\delta r_2}{r_2} + \frac{\delta r_3}{r_3}\right]$$

7) Power of a factor

$$\text{Let us consider,} \qquad X = r_1^n$$

$$\log_e X = n\log_e r_1$$

Differentiating above with respect to X,

$$\frac{1}{X} = n\frac{1}{r_1}\frac{dr_1}{dX}$$

$$\frac{dX}{X} = n\frac{dr_1}{r_1}$$

Therefore the relative limiting error in X is,

$$\frac{\delta X}{X} = \pm n\frac{\delta r_1}{r_1}$$

It clear from above result under these conditions is magnified n times.

7) Composite factors

Let us consider, $X = r_1^n\, r_1^m$

$\log_e X = n\log_e r_1 + m\log_e r_2$

Differentiating above with respect to X,

$$\frac{1}{X} = \frac{n}{r_1}\frac{dr_1}{dX} + \frac{m}{r_2}\frac{dr_2}{dX}$$

$$\frac{dX}{X} = n\frac{dr_1}{r_1} + m\frac{dr_2}{r_2}$$

Thus the limiting error in X is,

$$\frac{\delta X}{X} = \pm\left[n\frac{\delta r_1}{r_1} + m\frac{\delta r_2}{r_2}\right]$$

Calibration

Calibration is the process of checking the accuracy of instrument by comparing the instrument reading with a standard or against a similar meter of known accuracy. So using calibration is used to find the errors and accuracy of the measurement system or an instrument.

Calibration is an essential process to be undertaken for each instrument and measuring system regularly. The instruments which are actually used for measurement work must be calibrated against some reference instruments in which is having higher accuracy. Reference instruments must be calibrated against instrument of still higher accuracy or against primary standard or against other standards of known accuracy.

The calibration is better carried out under the predetermined environmental conditions. All industrial grade instruments can be checked for accuracy in the laboratory by using the working standard.

Certification of an instrument manufactured by an industry is undertaken by National Physical Laboratory and other authorizes laboratories where the secondary standards and working standards are kept.

Process of Calibration

The procedure involved in calibration is called as process of calibration. Calibration procedure involves the comparison of particular instrument with either

- A primary standard,
- A secondary standard with higher accuracy than the instrument to be calibrated
- An instrument of known accuracy.

Procedure of calibration as follows.

- Study the construction of the instrument and identify and list all the possible inputs.
- Choose, as best as one can, which of the inputs will be significant in the application for which the instrument is to be calibrated.
- Standard and secure apparatus that will allow all significant inputs to vary over the ranges considered necessary.

- By holding some input constant, varying others and recording the output, develop the desired static input-output relations.

Theory and Principles of Calibration Methods

Calibration methods are classified into following two types,

1) Primary or Absolute method of calibration

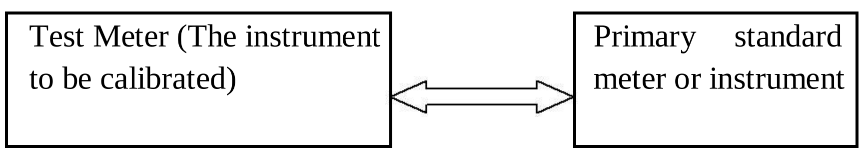
2) Secondary or Comparison method of calibration

    i. Direct comparison method of calibration

    ii. Indirect comparison method of calibration

1) Primary or Absolute method of calibration

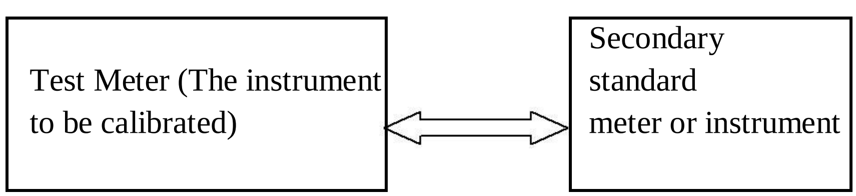
If the particular test instrument (the instrument to be calibrated) is calibrated against primary standard, then the calibration is called as primary or absolute calibration. After the primary calibration, the instrument can be used as a secondary calibration instrument.



Representation of Primary Calibration

2) Secondary or Comparison calibration method

If the instrument is calibrated against secondary standard instrument, then the calibration is called as secondary calibration. This method is used for further calibration of other devices of lesser accuracy. Secondary calibration instruments are used in laboratory practice and also in the industries because they are practical calibration sources.

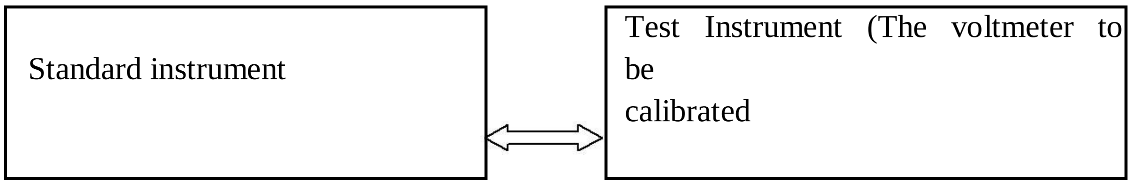


Representation of Secondary Calibration

Secondary calibration can be classified further two types,

i) Direct comparison method of Calibration

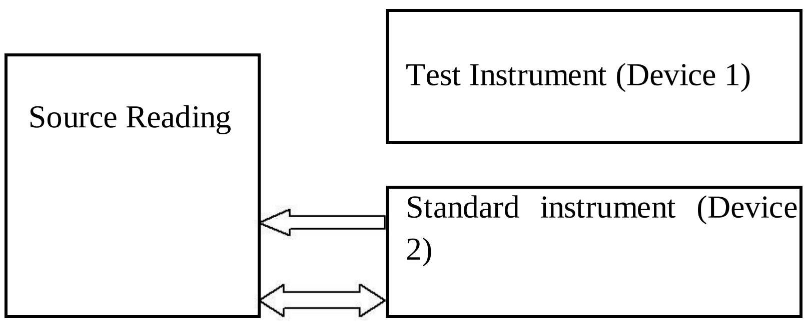
Direct comparison method of calibration with a known input source with same order of accuracy as primary calibration. So the instrument which is calibrated directly is also used as secondary calibration instruments.



Representation of Direct method of Calibration

ii) Indirect comparison method of Calibration

The procedure of indirect method of calibration is based on the equivalence of two different devices with same comparison concept.



Representation of indirect method of Calibration

Standards for calibration

Refer Classification of Standards (4 types).

Standards of measurement:

A standard is a physical representation of a unit of measurement. A known accurate measure of physical quantity is termed as standard. These standards are used to determine the accuracy of other physical quantities by the comparison method.

Example, the fundamental unit of mass in the International System is the Kilogram and defined as the mass of a cubic decimetre of water at its temperature of maximum of density of $4^{o}C$.

Different standards are developed for checking the other units of measurements and all these standards are preserved at the International Bureau of Weight and Measures at Serves, Paris.

Classification of Standards

Standards are classified into four types, based on the functions and applications.

1) International standards
2) Primary standards
3) Secondary standards
4) Working standards

1) International Standard

International standards are defined and established upon internationally. They are maintained at the International Bureau of Weights and measures and are not accessible to ordinary users for measurements and calibration. They are periodically evaluated and checked by absolute measurements in terms of fundamental units of physics.

- **International Ohms** It is defined as the resistance offered by a column of mercury having a mass of 14.4521gms, uniform cross sectional area and length of 106.300cm, to the flow of constant current at the melting point of ice.

2) Primary Standards

- Primary standards are maintained by the National Standards Laboratories (NSL) in different parts of the world.
- The principle function of primary standards is the calibration and verification of secondary standards.
- They are not available outside the National Laboratory for calibration.
- *These primary standards are absolute standards* of high accuracy that can be used as ultimate reference standards.

3) Secondary Standards

- These standards are basic reference standards used in industrial laboratories for calibration of instruments.

- Each industry has its own secondary standard and maintained by same industry.

- Each laboratory periodically sends its secondary standard to the NSL for calibration and comparison against the primary standards.
- Certification of measuring accuracy is given by NSL in terms of primary standards.

4) Working Standards

  - The working standards are used for day-to-day use in measurement laboratories. So this standard is the primary tool of a measurement laboratory.

  - These standards may be lower in accuracy in comparison with secondary standard. It is used to check and calibrate laboratory instruments for accuracy and performance.
  - Example, a standard resister for checking of resistance value manufactured.