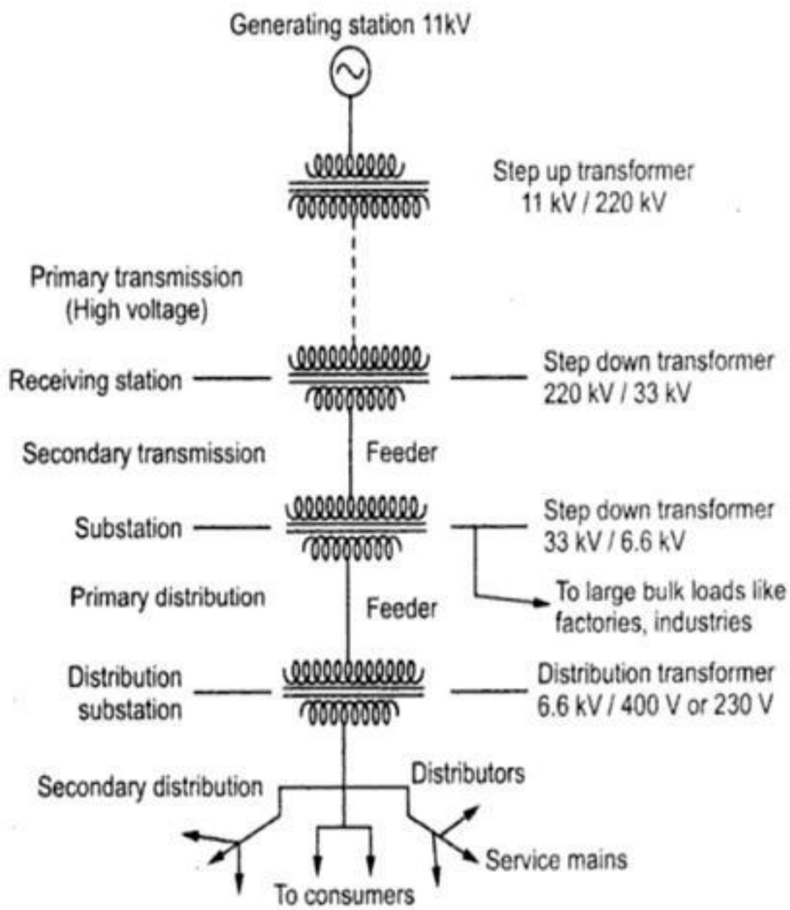
Subject Name: Transmission and Distribution
Subject Code: SEE1206

UNIT 1

STRUCTURE OF ELECTRIC POWER SYSTEM



Single line diagram

In power engineering, a one-line diagram or single-line diagram (SLD) is a simplified notation for representing a three-phase power system.[1] The one-line diagram has its largest application in power flow studies. Electrical elements such as circuit breakers, transformers, capacitors, bus bars, and conductors are shown by standardized schematic symbols.[1] Instead of representing each of three phases with a separate line or terminal, only one conductor is represented. It is a form of block diagram graphically depicting the paths for power flow between entities of the system. Elements on the diagram do not represent the physical size or location of the electrical equipment, but it is a common convention to organize the diagram with the same left-to-right, top-to-bottom sequence as the switchgear or other apparatus represented.

Introduction of Electric power Transmission and Distribution:

For economical generation of power large generating stations are used. Capacities of individual generating sets have gone up recently. Generating sets in the range of 10 MW, 210 MW and 500 MW are being manufactured in many countries. Generating station are now not necessarily located at load centers. In fact other factors like availability of fuel and water play more dominating role in the selection of sites for thermal stations. Hydro stations are obviously located only at the sites where water is available at sufficient head. A vast network of transmission system has been created so that power generated at one station may be fed to grid system and may be distributed over large areas and number of states. The transmission and distribution system comprises a network of three-phase circuits with transforming and or switching substations at the various junctions. The parts of a transmission and distribution network maybe grouped as given below.

Electric power TRANSMISSION:

Several generating stations can be inter connected. The main advantages are :

(i) reduction in the number of spare plants required as one station can assist the other at the time of emergency.

(ii) During light loads one station or some generators can be shut off, thus affecting operational economy.

Primary electric power transmission:

High voltages of the order of 66 kV 132 kV 220 kV and 400 kV are used for transmitting power by 3 phase 3 wire overhead system. This is supplied to substations usually at the out skirts of major distribution center or city.

Secondary electric power transmission:

The primary voltage is reduced to low values of the order of 3.3 kV, 11 kV or 33 kV for secondary transmission.

Primary electric power distribution:

The transmission lines or inner connectors terminate at large main substations from which the power is distributed to small secondary substations scattered throughout the load area. The voltage may range from 11 kV to 132 kV.

Secondary electric power distribution:

This consists of the low-voltage network laid along the streets, localities and over the rural areas. From these sources connections to individual customers are provided. The circuit used for this purpose is 3 phase 4 wire, 440 V/220 V from which either 3 phase 440 V or single phase 220 V supply to the consumers may be provided.

Advantages of AC electric power Transmission:

1. Power can be generated at high voltages as there is no commutation problem.

2. Ac voltages can be conveniently stepped up or stepped down.

3. High voltage transmission of ac power reduces losses.

Disadvantages of AC electric power transmission:

1. Problems of inductances and capacitances exist in transmission lines

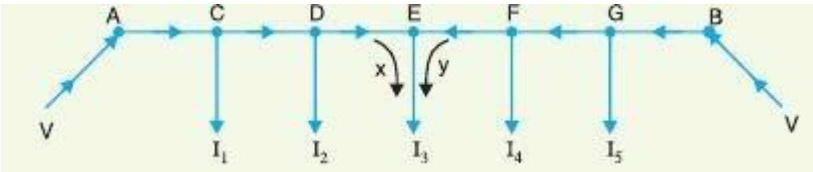
2. Due to skin effect, more copper is required.

3. Construction of AC transmission lines is more complicated as well as costly

4. Effective resistance of ac transmission lines is increased due to skin effect.

Concentrated Loading

Whenever possible, it is desirable that a long distributor should be fed at both ends instead of at one end only, since total voltage drop can be considerably reduced without increasing the cross-section of the conductor. The two ends of the distributor may be supplied with (I) equal voltages (ii) unequal voltages



All the currents tapped off between points A and E (minimum p.d. point) will be supplied from the feeding point A while those tapped off between B and E will be supplied from the feeding point B.

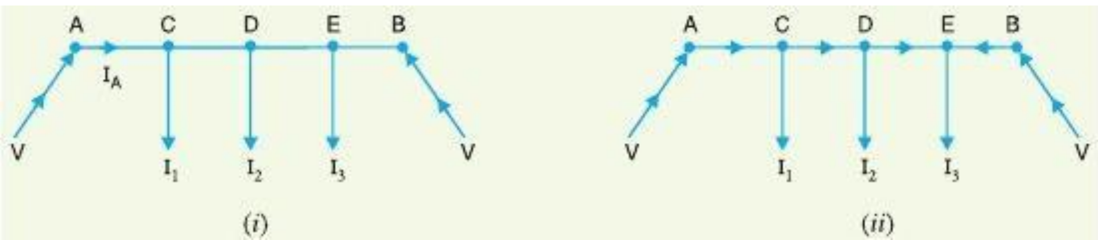
The current tapped off at point E itself will be partly supplied from A and partly from B. If these currents are x and y respectively, then,

$$I3 = x + y$$

Therefore, we arrive at a very important conclusion that at the point of minimum potential, current comes from both ends of the distributor.

Point of minimum potential. It is generally desired to locate the point of minimum potential. There is a simple method for it. Consider a distributor A B having three concentrated loads I1, I2 and I3 at points C, D and E respectively. Suppose that current supplied by feeding end A is IA . Then current distribution in the various sections of the distributor can be worked out as shown in Fig.
(i). Thus

$$IAC = IA ;ICD = IA - I1$$
$$IDE = IA - I1 - I2 ;IEB = IA - I1 - I2 - I3$$



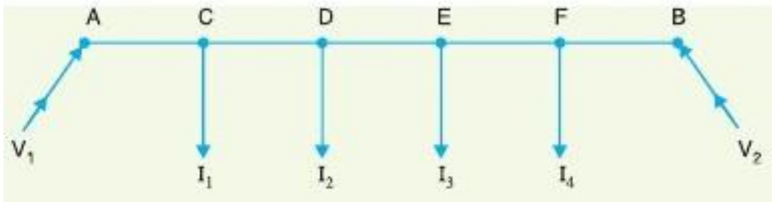
From this equation, the unknown IA can be calculated as the values of other quantities are generally given. Suppose actual directions of currents in the various sections of the distributor are indicated as shown in Fig. 13.15 (ii). The load point where the currents are coming from both sides of the distributor is the point of minimum potential i.e. point E in this case

(ii) Two ends fed with unequal voltages. Fig. 13.16 shows the distributor A B fed with unequal voltages ; end A being fed at V 1 volts and end B at V 2 volts. The point of minimum potential can be found by following the same procedure as discussed above. Thus in this case,

Voltage drop between A and B = Voltage drop over A B

or

$$V_1 - V_2 = \text{Voltage drop over A B}$$



Resistance of 1 m length of distributor

= 2· 1 ⊕ 7 · 10□6· 100□4= 3·4 · 10 &

1

Resistance of section AC, R AC = □4(3·4 · 10) · 100 = 0·034 &

Resistance of section CD, RCD = (3·4 · 10□4) · 150 = 0·051 &

Resistance of section DE, R DE = (3·4 · 10□4) · 150 = 0·051 &

Resistance of section EF, R EF = □4(3·4 · 10) · 100 = 0·034 &

Resistance of section FB, R FB = □4(3·4 · 10) · 100 = 0·034 &

Voltage at B = Voltage at A □ Drop over length A B

orVB = V A □ [IA RAC + (IA □ 20) R CD + (IA □ 60) R DE

+ (IA □ 110) R EF + (IA □ 140) R FB]

220 □ [0·034 IA + 0·051 (IA □ 20) + 0·051 (IA □ 60)

+ 0·034 (IA □ 110) + 0·034 (IA □ 140)]

or 220 = 220 □ [0·204 IA □ 12·58]

12·58

= 12·58/0·204 = 61·7 A

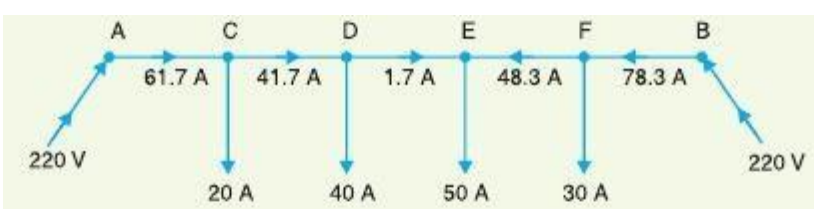
or 0·204 IA =

4 IA =

The *actual distribution of currents in the various sections of the distributor is shown in Fig. It is clear that currents are coming to load point E from both sides i.e. from point D and point F. Hence, E is the point of minimum potential.

∴ □ Minimum consumer voltage,

$$V_E = V_A - [I_{AC} R_{AC} + I_{CD} R_{CD} + I_{DE} R_{DE}]$$



= 220 − [61·7 □ 0·034 + 41·7 □ 0·051 + 1·7 □ 0·051]

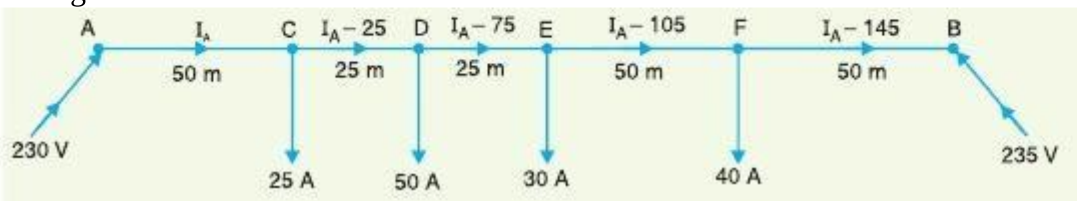
= 220 − 4·31 = 215·69 V

Example 13.11. A 2-wire d.c. distributor AB is fed from both ends. At feeding point A, the voltage is maintained as at 230 V and at B 235 V. The total length of the distributor is 200 metres and loads are tapped off as under :

25 A at 50 metres from A; 50 A at 75 metres from A

30 A at 100 metres from A; 40 A at 150 metres from A
The resistance per kilometre of one conductor is 0·3 Ω. Calculate :
(i) currents in various sections of the distributor
(ii) minimum voltage and the point at which it occurs

Solution. Fig shows the distributor with its tapped currents. Let I_A amperes be the current supplied from the feeding point A . Then currents in the various sections of the distributor are as shown in Fig



Resistance of 1000 m length of distributor (both wires)

$$= 2 \times 0·3 = 0·6\ \Omega$$

Resistance of section AC, $R_{AC} = 0·6 \times 50/1000 = 0·03\ \Omega$
Resistance of section CD, $R_{CD} = 0·6 \times 25/1000 = 0·015\ \Omega$
Resistance of section DE, $R_{DE} = 0·6 \times 25/1000 = 0·015\ \Omega$
Resistance of section EF, $R_{EF} = 0·6 \times 50/1000 = 0·03\ \Omega$
Resistance of section FB, $R_{FB} = 0·6 \times 50/1000 = 0·03\ \Omega$

Voltage at B = Voltage at A – Drop over AB or

$$V_B = V_A - [I_A R_{AC} + (I_A - 25) R_{CD} + (I_A - 75) R_{DE} + (I_A - 105) R_{EF} + (I_A - 145) R_{FB}]$$

or

$$235 = 230 - [0·03\ I_A + 0·015\ (I_A - 25) + 0·015\ (I_A - 75) + 0·03\ (I_A - 105) + 0·03\ (I_A - 145)]$$

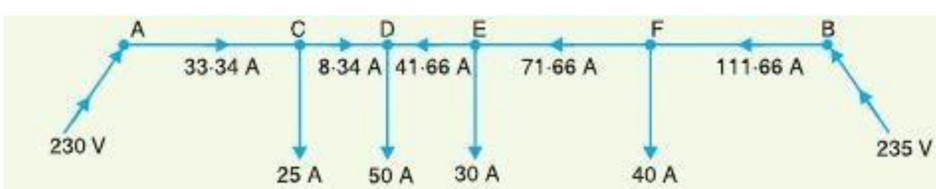
or

$$235 = 230 - [0·12\ I_A - 9]$$

$$\therefore I_A = \frac{239 - 235}{0·12} = 33·34\ A$$

(i) Current in section AC, $I_{AC} = I_A = 33·34$ A
Current in section CD, $I_{CD} = I_A - 25 = 33·34 - 25 = 8·34$ A
Current in section DE, $I_{DE} = I_A - 75 = 33·34 - 75 = -41·66$ A from D to E
$= 41·66$ A from E to D
Current in section EF, $I_{EF} = I_A - 105 = 33·34 - 105 = -71·66$ A from E to F
$= 71·66$ A from F to E
Current in section FB, $I_{FB} = I_A - 145 = 33·34 - 145 = -111·66$ A from F to B
$= 111·66$ A from B to F

(ii) The actual distribution of currents in the various sections of the distributor is shown in Fig. 13.20. The currents are coming to load point D from both sides of the distributor. Therefore, load point D is the point of minimum potential.



Voltage at D, $V_D = V_A - [I_{AC} R_{AC} + I_{CD} R_{CD}]$

$= 230 - [33·34 \times 0·03 + 8·34 \times 0·015]$

$= 230 - 1·125 = 228·875$ V

DC two wire and three wire systems

It is a common knowledge that electric power is almost exclusively generated, transmitted and distributed as a.c. However, for certain applications, d.c. supply is absolutely necessary. For instance, d.c. supply is required for the operation of variable speed machinery (i.e ., d.c. motors), for electrochemical work and for congested areas where storage battery reserves are necessary. For this purpose, a.c. power is converted into d.c. power at the substation by using converting machinery

e.g., mercury arc rectifiers, rotary converters and motor-generator sets. The d.c. supply from the substation may be obtained in the form of (i) 2-wire or (ii) 3-wire for distribution.

(i) 2-wire d.c. system

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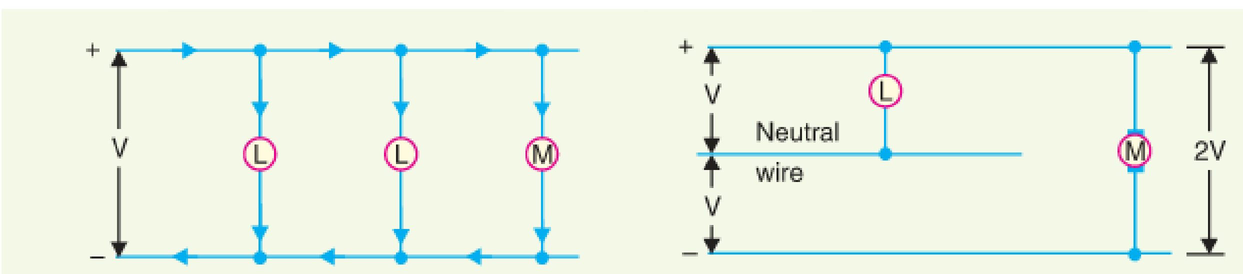
As the name implies, this system of distribution consists of two wires. One is the outgoing or positive wire and the other is the return or negative wire. The loads such as lamps, motors etc. are connected in parallel between the two wires as shown in Fig. 12.4. This system is never used for transmission purposes due to low efficiency but may be employed for distribution of d.c. power.



(ii) 3-wire d.c. system.

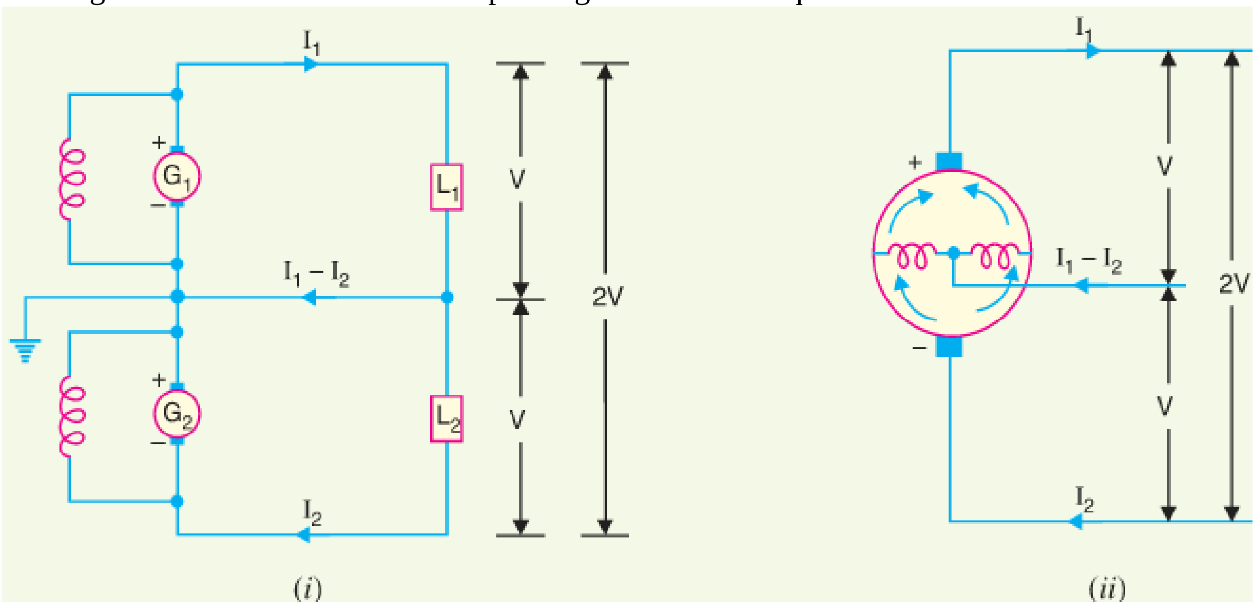
It consists of two outers and a middle or neutral wire which is earthed at the substation. The voltage between the outers is twice the voltage between either outer and neutral wire as shown in Fig. 12.5. The principal advantage of this system is that it makes available two voltages at the consumer terminals viz V between any outer and the neutral and 2V between the outers. Loads requiring high voltage (e.g ., motors) are connected across the outers, whereas lamps and heating circuits requiring less voltage are connected between either outer and the neutral. The methods of obtaining 3-wire system are discussed in the following article

Methods of Obtaining 3-wire D.C. System

There are several methods of obtaining 3-wire d.c. system. However, the most important ones are

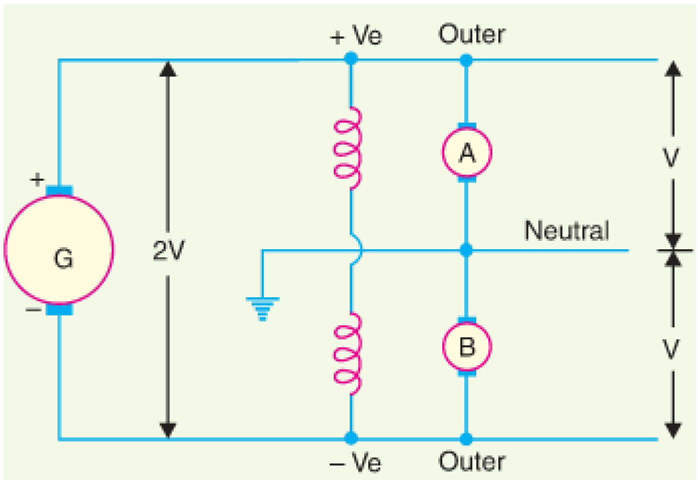
(i) Two generator method.

In this method, two shunt wound d.c. generators G1and G2 Are connected in series and the neutral is obtained from the common point between generators as shown in Fig. (i). Each generator supplies the load on its own side. Thus generator G1 supplies a load current of I1, whereas generator G2 supplies a load current of I 2 The difference of load currents on the two sides, known as out of balance current (I1 I2) flows through the neutral wire. The principal disadvantage of this method is that two separate generators are required.



3-wire d.c. generator.

The above method is costly on account of the necessity of two generators. For this reason, 3-wire d.c. generator was developed as shown in Fig. (ii). It consists of a standard 2-wire machine with one or two coils of high reactance and low resistance,connected permanently to diametrically opposite points of the armature winding. The neutral wire is obtained
from the common point as shown



iii) Balancer set.

The 3-wire system can be obtained from 2-wire d.c. system by the use of balancer set as shown in Fig. G is the main 2-wire d.c. gen erator and supplies power to the whole system. The balancer set consists of two identical d.c shunt machines A and B coupled mechanically with their armatures and field windings joined in series across the outers. The junction of their armatures is earthed and neutral wire is taken out from here. The balancer set has the additional advantage that it maintains the potential difference on two sides of neutral equal to each other.

AC Distributors

Now-a-days electrical energy is generated, transmitted and distributed in the form of alternating current. One important reason for the widespread use of alternating current in preference to direct current is the fact that alternating voltage can be conveniently changed in magnitude by means of a transformer. Transformer has made it possible to transmit a.c. power at high voltage and utilise it at a safe potential. High transmission and distribution voltages have greatly reduced the current in the conductors and the resulting line losses.

There is no definite line between transmission and distribution according to voltage or bulk capacity. However, in general, the a.c. distribution system is the electrical system between the step down substation fed by the transmission system and the consumers' meters. The a.c. distribution system is classified into (I) primary distribution system and (ii) secondary distribution system.

(i) Primary distribution system.

It is that part of a.c. distribution system which operates at voltages somewhat higher than general utilisation and handles large blocks of electrical energy than the average low-voltage consumer uses. The voltage used for primary distribu tion depends upon the amount of power to be conveyed and the distance of the substation required to be fed. The most commonly used primary distribution voltages are 11 kV, 6·6 kV and 3·3 kV. Due to economic considerations, primary distribution is carried out by 3- phase, 3-wire system.

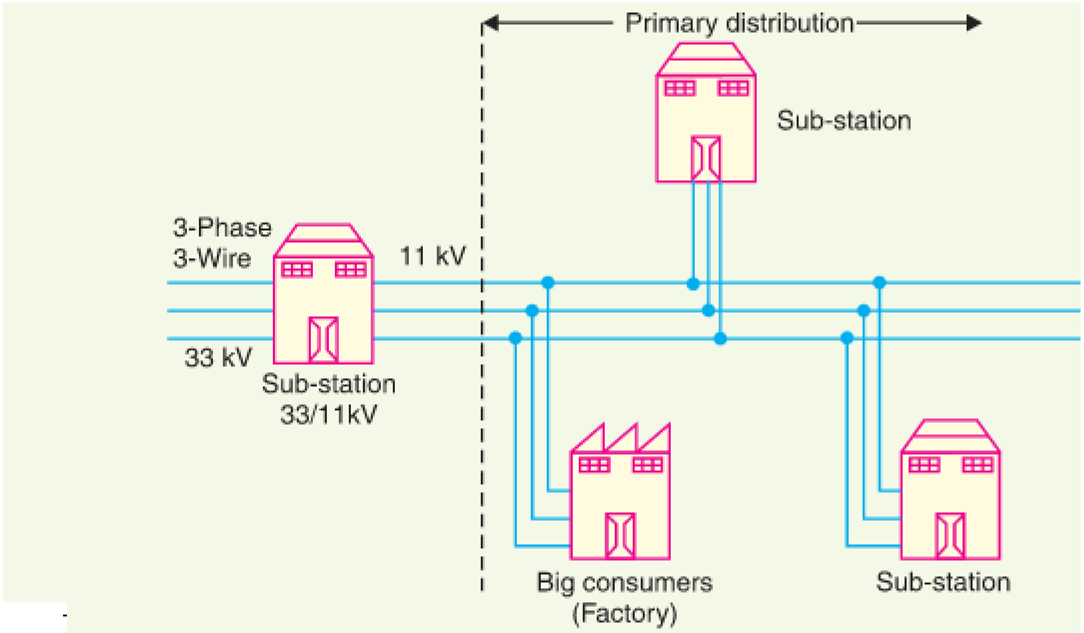


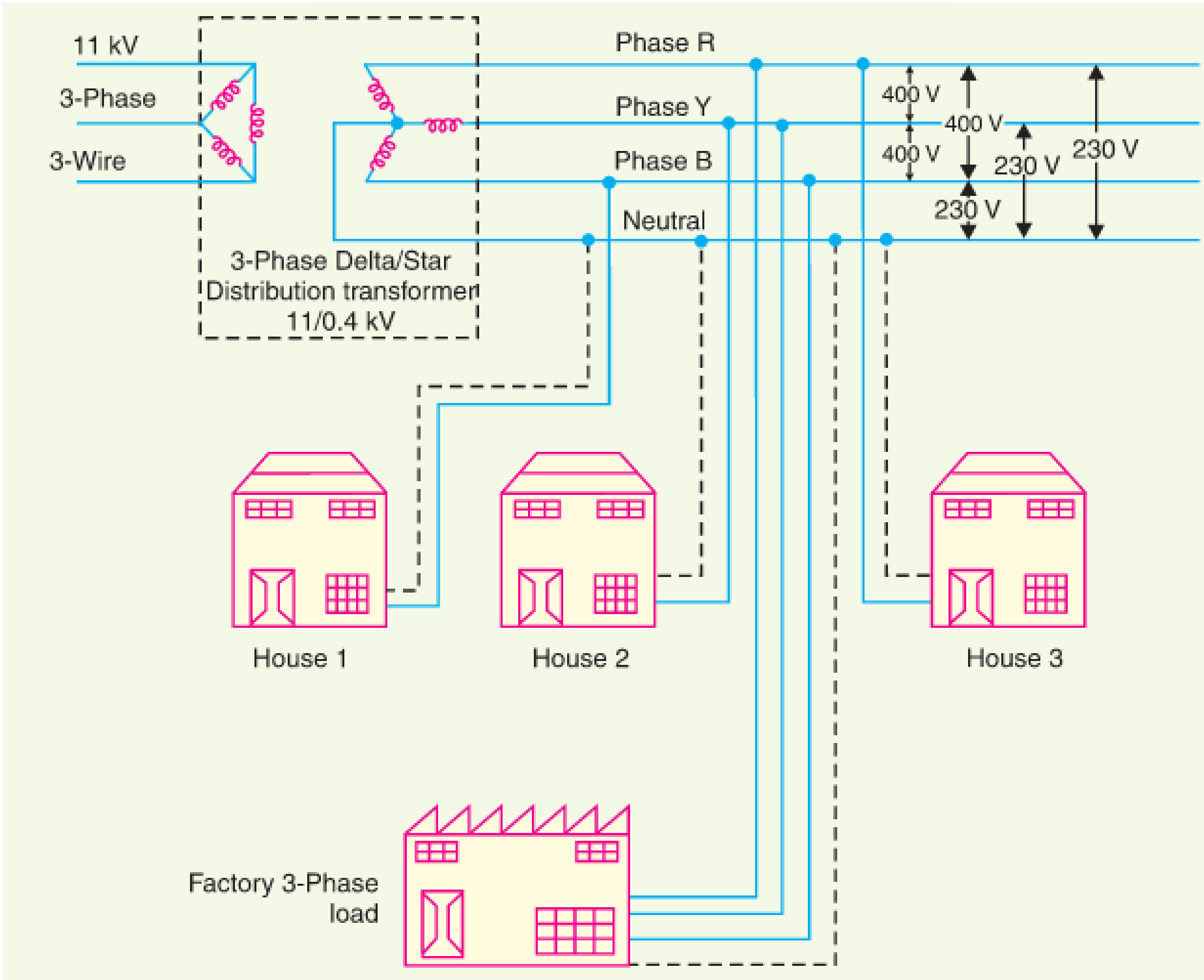
Fig shows a typical primary distribution system. Electric power from the generating station is transmitted at high voltage to the substation located in or near the city. At this substation, voltage is stepped down to 11 kV with the help of step-down transformer. Power is supplied to various substations for distribution or to big consumers at this voltage. This forms the high voltage distribution or primary distribution.

(ii) Secondary distribution system.

It is that part of a.c. distribution system which includes the range of voltages at which the ultimate consumer utilises the electrical energy delivered tohim. The secondary distribution employs 400/230 V, 3-phase, 4-wire system.

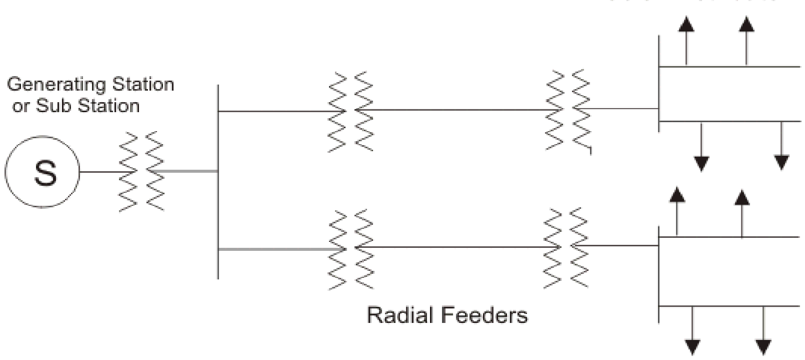
Fig. shows a typical secondary distribution system. The primary distribution circuit delivers power to various substations, called distribution sub stations. The substations are situated near the consumers localities and contain step down transformers. At each distribution substation, the voltage is stepped down to 400V and power is delivered by 3-phase,4-wire a.c. system.

The voltage between any twophases is 400 V and between any phase and neutral is 230 V. The single phase domestic loads are connected between any one phase and the neutral, whereas 3-phase 400 V motor loads are connected across 3 phase lines directly.



Radial Electrical Power Distribution System

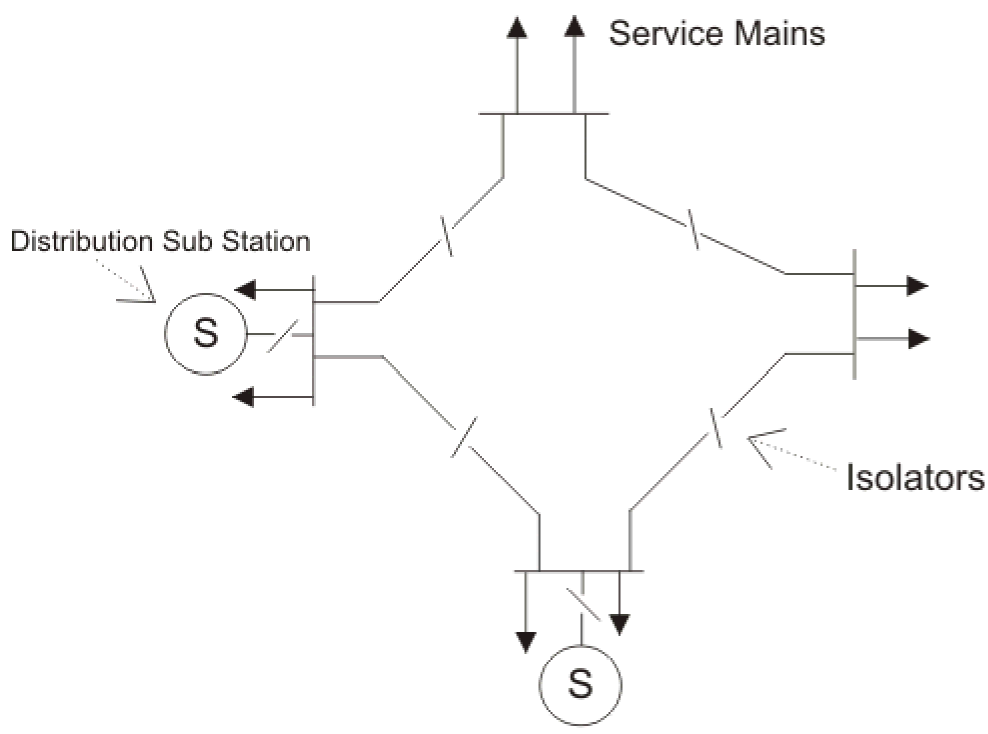
In early days of electrical power distribution system, different feeders were radially come out from the substation and connected to the primary of distribution transformer directly.



But **radial electrical power distribution system** has one major drawback that in case of any feeder failure, the associated consumers would not get any power as there was no alternative path to feed the transformer. In case of transformer failure also, the power supply is interrupted. In other words the consumer in the radial electrical distribution system would be in darkness until the feeder or transformer was rectified.

Ring Main Electrical Power Distribution System

The drawback of **radial electrical power distribution system** can be overcome by introducing a **ring main electrical power distribution system**. Here one ring network of distributors is fed by more than one feeder. In this case if one feeder is under fault or maintenance, the ring distributor is still energized by other feeders connected to it. In this way the supply to the consumers is not affected even when any feeder becomes out of service. In addition to that the ring main system is also provided with different section isolates at different suitable points. If any fault occurs on any section, of the ring, this section can easily be isolated by opening the associated section isolators on both sides of the faulty zone.



In this way, supply to the consumers connected to the healthy zone of the ring, can easily be maintained even when one section of the ring is under shutdown. The number of feeders connected to the **ring main electrical power distribution system** depends upon the following factors.

1. **Maximum demand of the system :** If it is more, then more numbers of feeders feed the ring.

2. **Total length of the ring main distributors :** It length is more, to compensate the voltage drop in the line, more feeders to be connected to the ring system.
3. **Required voltage regulation :** The number of feeders connected to the ring also depends upon the permissible allowable, voltage drop of the line.

The sub distributors and service mains are taken off may be via distribution transformer at different suitable points on the ring depending upon the location of the consumers. Sometimes, instead of connecting service main directly to the ring, sub distributors are also used to feed a group of service mains where direct access of ring distributor is not possible.

Interconnectors

Electricity interconnectors are the physical links which allow the transfer of electricity across borders.

Interconnectors derive their revenues from congestion revenues. Congestion revenues are dependent on the existence of price differentials between markets at either end of the interconnector. European legislation governs how capacity is allocated. It requires all interconnection capacity to be allocated to the market via market based methods, i.e. auctions. It also includes specific conditions on how revenues are used.

Britain's electricity market currently has 4GW of interconnector capacity:

- 2GW to France (IFA)
- 1GW to the Netherlands (BritNed)
- 500MW to Northern Ireland (Moyle)
- 500MW to the Republic of Ireland (East West).

Under the present regulatory regime based on EU and GB requirements, there are two general routes for interconnector investment:

1. a regulated route under the 'cap and floor' regime. This is a relatively new regime, we decided to roll out the cap and floor regulatory regime to new near-term electricity interconnectors in May 2014.Through the cap and floor approach developers identify, propose and build interconnectors and there is a cap and floor mechanism to regulate how much money a developer can earn once in operation. If applying for a cap and floor regime developers have to comply with all aspects of European legislation on cross border electricity infrastructure. More information about the design of the cap and floor regime can be found in the documents below.

2. as an alternative to the cap and floor model, developers can still seek exemptions from regulatory requirements. Under this route developers would face the full upside and downside of the investment and would usually apply for an exemption from certain aspects of European legislation in order to increase the safeguards for the business case of their investment.

Kelvins Law:

The most economical area of conductor is that for which the total annual cost of transmission line is minimum.
This is called as kelvins law after Lord Kelvin who first stated in 1881.

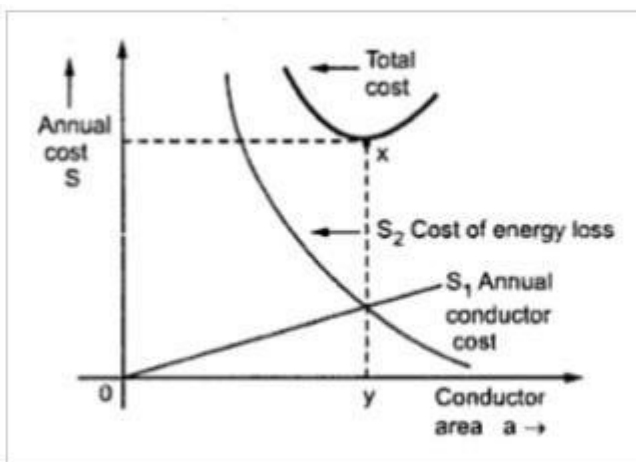
The transmission line cost forms major part in the annual charges of a power system.
The cost is due to

1. Depreciation
2. Repair and maintenance
3. Loss of energy in the line due to its resistance
4. The cost towards the production of the lost energy is considered

- If we decreases the area of the conductor in order to reduce the capital cost, the line losses increase.
- Similarly, if we increase the conductor cross-section to save the cost towards copper loss in the line, the weight of copper increases and hence the capital cost will be more.

Because of the above reasons, it is difficult to find the economical size of the conductor. But it becomes easy with the help of kelvin's law.

In this post we will understand about the kelvin's law and limitations of the kelvin's law.



Assume
A = Cross section of conductor

C = total initial cost towards conductor

C is directly proportional to A

$C \propto A$

$C = PA$

where P is a constant.

Let r be the annual rate of interest and depreciation.
The annual fixed cost $C_1 = C_r = PA_r$
Since line losses are inversely proportional to the area of the conductor
The annual cost on lost energy,

$C_2 = Q/A$ where Q is a constant.

Total annual cost $C = C_1 + C_2$

$= PA_r + Q/A$

For C to be minimum,

$C/dA = 0$

$Pr – Q/A^2 =0$

$Pr = Q/A^2$

$Pr.A^2 = Q$

$A^2 = Q/Pr$

$A = \sqrt{(Q/Pr)}$

The equation shows that
"The economical cross-section of the conductor is that for which the annual charge on the conductor equals the annual charge for the loss of energy in the conductor".
This is known as Kelvin's law.

Limitations of Kelvin's Law
This law has many problems and limits as we are selecting the cross-section from an economical point of view.We did not consider the electrical behaviour of the line.

1. It is not easy to estimate the energy loss in the line without actual load curves, which are not available at the time of estimation.
2. Kelvin's law did not consider many physical factors like voltage regulation, corona loss, temperature rise etc.
3. The assumption that annual cost on account of interest and depreciation on the capital outlay is not 100% true.
4. The conductor size determined by this law may not be always practicable one.
5. The rates of interest and depreciation may vary from time to time.

6. The diameter of the conductor may be so small as to cause high corona loss.
7. The conductor may be too weak to stamp from mechanical point of view.
8. Cost of insulation in cables is assumed to be independent of the cross-section of the conductor which is only an approx. assumption.

UNIT – II

TRANSMISSION LINE PARAMETERS

* Resistance, inductance and capacitance distributed along the transmission line are termed as constants or line parameters.

RESISTANCE:-

* Opposes the flow of current

$$R = \frac{\rho l}{a} \ \Omega$$

INDUCTANCE:-

* An alternating current flowing through a conductor causes a changing flux which links the conductor.
* The conductor posses inductance due to this flux linkage.

$$L = \frac{\psi}{I} \text{ henry}$$

where ψ = flux linkage in weber-turns

I = current in amperes.

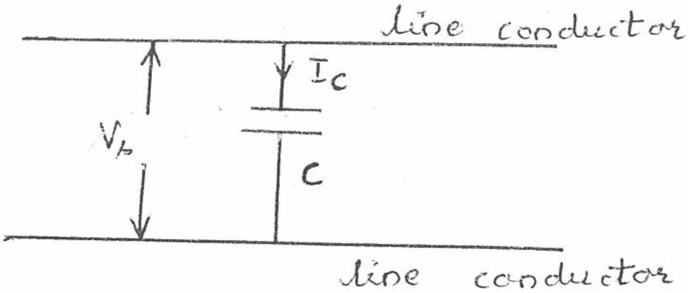
CAPACITANCE:-

* Conductors of overhead line are separated by air which acts as insulation.
* Capacitance exists between the two overhead line conductors.
* Capacitance, $C = q/v$ farad

where q = charge on line (coulomb)

v = potential difference between the conductors (volts).

CHARGING CURRENT FLOWS IN THE OPEN CIRCUITED LINE :- WHY?

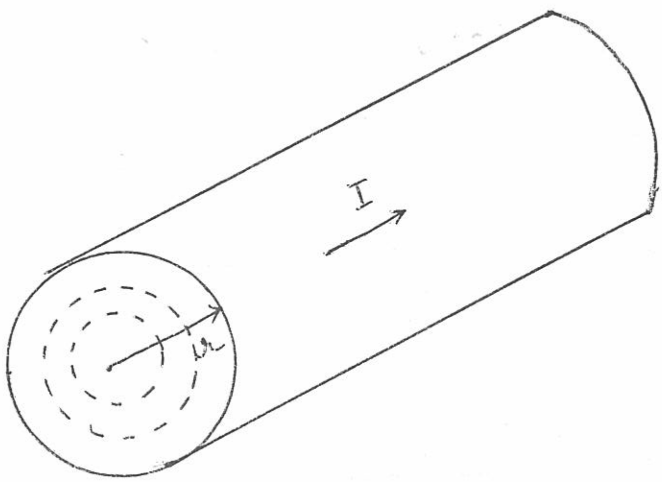


* When V_p increases, the charge on the conductor increases
* This results in a flow of charging current (I_c) between the conductors
* I_c flows in line even when line is open-circuited.
* Flow of I_c affects efficiency, powerfactor & voltage drop in line.

FLUX LINKAGES :-

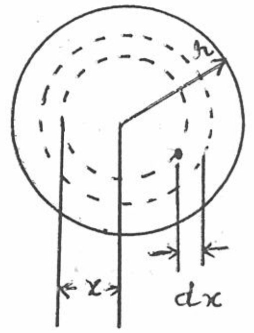
* To find the inductance of a circuit, flux linkage has to be determined.

FLUX LINKAGE DUE TO SINGLE CURRENT CARRYING CONDUCTOR :-



* Consider a long straight conductor of radius 'r' metres and carrying a current of I amperes.
* The current flow causes a flux linkage inside the conductor as well as outside the conductor.

i) FLUX LINKAGE DUE TO INTERNAL FLUX :-



* Consider a small section of thickness dx, length 1m at a distance 'x' from the centre & carrying 'I_x' current.
* NOTE:- Magnetic field intensity in a solid conductor of radius 'r' & carrying a current of 'I' amperes

$$H = \frac{I}{2\pi r} \ AT/m$$

* Magnetic field intensity at a point x metres from centre $H_x = \frac{I_x}{2\pi x}$ — ①
* Current flow in the area $\pi r^2 = I$ amp.
* Current flow in the area πx^2, $I_x = \frac{\pi x^2 \times I}{\pi r^2}$ — ②

sub ② in ①

* $$H_x = \frac{\pi x^2 \times I}{\pi r^2 \times 2\pi x}$$

$$H_x = \frac{xI}{2\pi r^2} \quad AT/m.$$

* Flux density in dx. $(B_x) = \mu_0 \mu_r H_x \quad wb/m^2$

$$= \mu_0 \mu_r \times \frac{xI}{2\pi r^2} \quad wb/m^2.$$

$\mu_r = 1$ for non-magnetic material.

$$B_x = \frac{\mu_0 x I}{2\pi r^2} \quad wb/m^2.$$

* Flux through a cylinder shell of thickness dx & length 1m } $d\phi = B_x \times \text{area}$

$$= B_x \times \text{thickness} \times \text{length}$$

$$d\phi = B_x \times dx \times 1$$

$$d\phi = \left[\mu_0 x I / 2\pi r^2\right] dx \quad \text{weber.}$$

* Flux linkage in area πx^2 } $d\psi = \frac{\pi x^2}{\pi r^2} \times d\phi$

$$= \frac{\pi x^2}{\pi r^2} \times \frac{\mu_0 x I}{2\pi r^2} dx$$

$$d\psi = \frac{\mu_0 I x^3}{2\pi r^4} dx \quad \text{weber-turns}$$

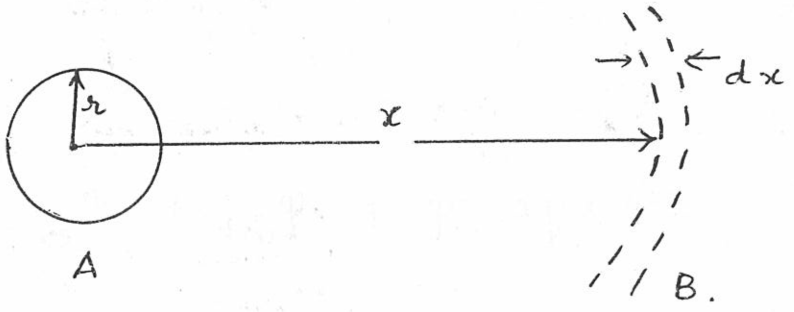
* Total flux linkage in area πr^2 } $= \int_0^r \frac{\mu_0 I x^3}{2\pi r^4} dx$

$$= \frac{\mu_0 I}{2\pi r^4} \left[\frac{x^4}{4}\right]_0^r$$

$$= \frac{\mu_0 I r^4}{8\pi r^4}$$

* Internal flux linkage $\psi_{int} = \frac{\mu_0 I}{8\pi}$ wb-turns/metre length

ii) FLUX LINKAGE DUE TO EXTERNAL FLUX :-



* Consider conductor A of radius r metres.
* Conductor 'B' is a distance 'x' metres from A and carrying a current of 'I' amperes.
* Consider only a section of conductor B having 'dx' thickness & a length of 1metre.
* Conductor A has external flux linking from the conductor surface to infinity due to flux of conductor B.
* Magnetic field intensity in external flux region, H_x $= \frac{I}{2\pi x}$ AT/m.
* Flux density B_x $= \mu_0 \mu_r H_x$

$$= \mu_0 \times \frac{I}{2\pi x} \quad ; \mu_r = 1.$$

$$B_x = \frac{\mu_0 I}{2\pi x} \text{ wb/m}^2$$

* Flux $d\phi$ through section dx = $B_x \times$ area

$$d\phi = B_x \times dx \times 1.$$

$$d\phi = \frac{\mu_0 I}{2\pi x} dx.$$

* Flux $d\phi$ is caused

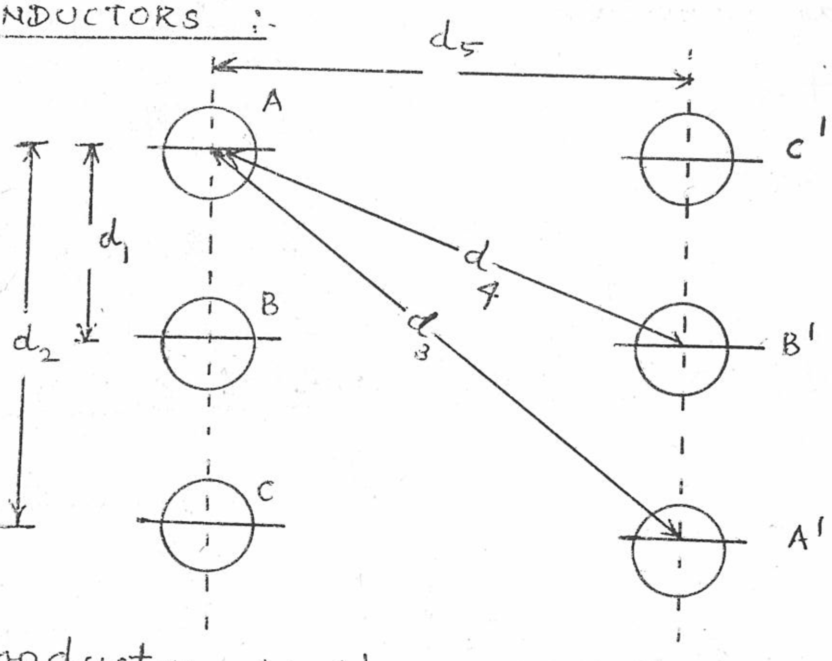
* Flux linkage from surface to infinity $\Big\}\ \psi_{ext} = \int_r^\infty \frac{\mu_0 I}{2\pi x}\,dx$ wb-turns

* Total flux linkage $\psi = \psi_{int} + \psi_{ext}$

$$= \frac{\mu_0 I}{8\pi} + \int_r^\infty \frac{\mu_0 I}{2\pi x}\,dx.$$

$$\boxed{\psi_{tot} = \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right].}$$

2. FLUX LINKAGE IN PARALLEL CURRENT CARRYING CONDUCTORS :-



* Conductor A & A' carry a current of I_A amperes.
* Conductor B & B' carry a current of I_B amperes.
* Conductor C & C' carry a current of I_C amperes.

* Flux linkage with conductor A due to current I_A $\Big\} = \dfrac{\mu_0 I_A}{2\pi}\left[\dfrac{1}{4} + \displaystyle\int_r^{\infty} \dfrac{dx}{x}\right]$ —(1)

* Flux linkage with conductor A due to current I_B of conductor B $\Big\} = \dfrac{\mu_0 I_B}{2\pi} \displaystyle\int_{d_1}^{\infty} \dfrac{dx}{x}$ —(2)

* Flux linkage with conductor A due to current I_c of conductor c $\Big\} = \dfrac{\mu_0 I_c}{2\pi} \displaystyle\int_{d_2}^{\infty} \dfrac{dx}{x}$ —(3)

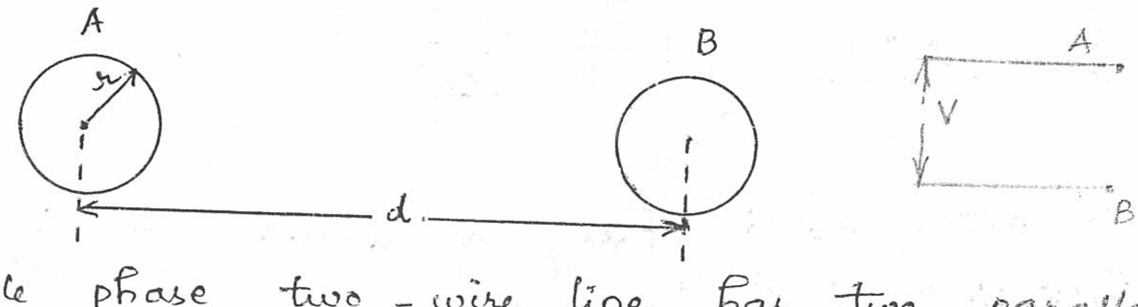
* Flux linkage with conductor A due to current I_A of conductor a' $\Big\} = \dfrac{\mu_0 I_A}{2\pi} \displaystyle\int_{d_3}^{\infty} \dfrac{dx}{x}$ —(4)

* Flux linkage with conductor A due to current I_B of conductor b' $\Big\} = \dfrac{\mu_0 I_B}{2\pi} \displaystyle\int_{d_4}^{\infty} \dfrac{dx}{x}$ —(5)

* Flux linkage with conductor A due to current I_c of conductor c' $\Big\} = \dfrac{\mu_0 I_c}{2\pi} \displaystyle\int_{d_5}^{\infty} \dfrac{dx}{x}$ —(6)

* Total flux linkage with conductor A $\Big\}$ = (1) + (2) + (3) + (4) + (5) + (6).

INDUCTANCE OF A SINGLE PHASE TWO-WIRE LINE :-



* A single phase two-wire line has two parallel conductors. (A & B)
* Distance b/w conductors A & B is 'd' metres.
* Current carried by conductor A is I_A

* Current carried by conductor B (return wire) $= I_B = -I_A$

$\therefore \quad I_A + I_B = 0.$

* Flux linkage with conductor A, ψ_A } = flux linkage due to its own current I_A + flux linkage due to current I_B.

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right] + \frac{\mu_0 I_B}{2\pi}\int_d^{\infty}\frac{dx}{x}$$

↓ internal linkage ↓ external linkage

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right)I_A + I_B\int_d^{\infty}\frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + \left[\log_e x\right]_r^{\infty} I_A + I_B\left[\log_e x\right]_d^{\infty}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left[\frac{1}{4} + \log_e \infty - \log_e r\right]I_A + I_B\left[\log_e \infty - \log_e d\right]\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + \log_e \infty\,(I_A + I_B) - I_A \log_e r - I_B \log_e d\right]$$

$$\psi_A = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + 0 - I_A \log_e r - I_B \log_e d\right]$$

since $I_A + I_B = 0 \quad \therefore \quad I_A = -I_B.$

$$\therefore \quad \psi_A = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \log_e r + I_A \log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \log_e\left(\frac{d}{r}\right)\right]$$

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \text{ wb-turns/m.}$$

* Inductance of conductor A, $L_A = \frac{\psi_A}{I_A}$

$$\boxed{L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m}}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m.}$$

$$L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{d}{r}\right] \text{ H/m.}$$

* Loop inductance $= 2L_A$ H/m

$$= 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right] \text{ H/m}$$

1. A single phase line has two parallel conductors 2m apart. The diameter of each conductor is 1.2cm. Calculate the loop inductance per km of the line.

GIVEN:-

i) $d = 2\text{m} = 200\text{cm}$

ii) dia $= 1.2$ cm ; radius $= \frac{1.2}{2} = 0.6$ cm.

REQUIRED:-

Loop inductance of line.

SOLUTION :-

Loop inductance $= 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right]$ H/m.

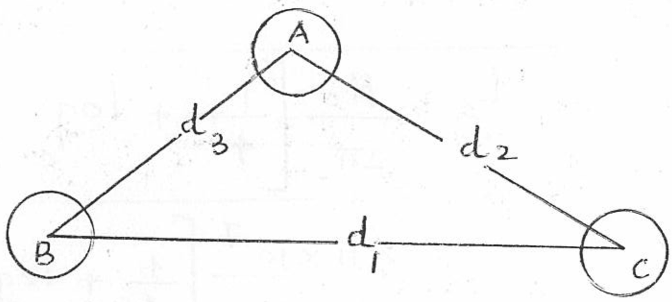
$$= 10^{-7}\left[1 + 4\log_e \frac{200}{0.6}\right] \text{ H/m}$$

$$= 24.23 \times 10^{-7} \text{ H/m.}$$

Loop inductance $= 24.23 \times 10^{-7} \times 1000$ H/km.

$= 2.43$ mH.

INDUCTANCE OF A 3-φ OVERHEAD LINE :-



* Conductors A, B & C of a 3-φ line carry current I_A, I_B & I_C respectively.
* Let d_1, d_2, d_3 be the spacing between the conductors BC, CA & AB respectively.
* Under balanced condition $I_A + I_B + I_C = 0$.
* Flux linkage with conductor A, ψ_A = due to its own current I_A + due to current I_B & I_C.

$$\psi_A = \underbrace{\frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right]}_{\text{internal flux linkage}} + \underbrace{\frac{\mu_0 I_B}{2\pi}\int_{d_3}^\infty \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^\infty \frac{dx}{x}}_{\text{external flux linkage}}$$

* $$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right)I_A + I_B\int_{d_3}^\infty \frac{dx}{x} + I_C\int_{d_2}^\infty \frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \log_e(x)\Big|_r^\infty\right)I_A + I_B\left[\log_e x\right]_{d_3}^\infty + I_C\left[\log_e x\right]_{d_2}^\infty\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left[\frac{1}{4} - \log_e r\right]I_A - I_B\log_e d_3 - I_C\log_e d_2 + \log_e\infty\,(I_A + I_B + I_C)\right]$$

since $I_A + I_B + I_C = 0$

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B\log_e d_3 - I_C\log_e d_2\right] \quad \text{— (I)}$$

INDUCTANCE WHEN CONDUCTORS ARE SYMMETRICALLY SPACED:-

* When $d_1 = d_2 = d_3 = d$, conductors are said to be symmetrically placed. So $L_A = L_B = L_C$

* ∴ Flux linkage in conductor A, ψ_A $\Big\} = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d - I_C \log_e d\right]$

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - (I_B + I_C) \log_e d\right]$$

* Since $I_A + I_B + I_C = 0$; ∴ $I_B + I_C = -I_A$

$$\therefore \psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A + I_A \log_e d\right]$$

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ weber-turns/m.}$$

* Inductance of A, $L_A = \frac{\psi_A}{I_A}$.

$$\therefore \boxed{L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ H/m}}$$

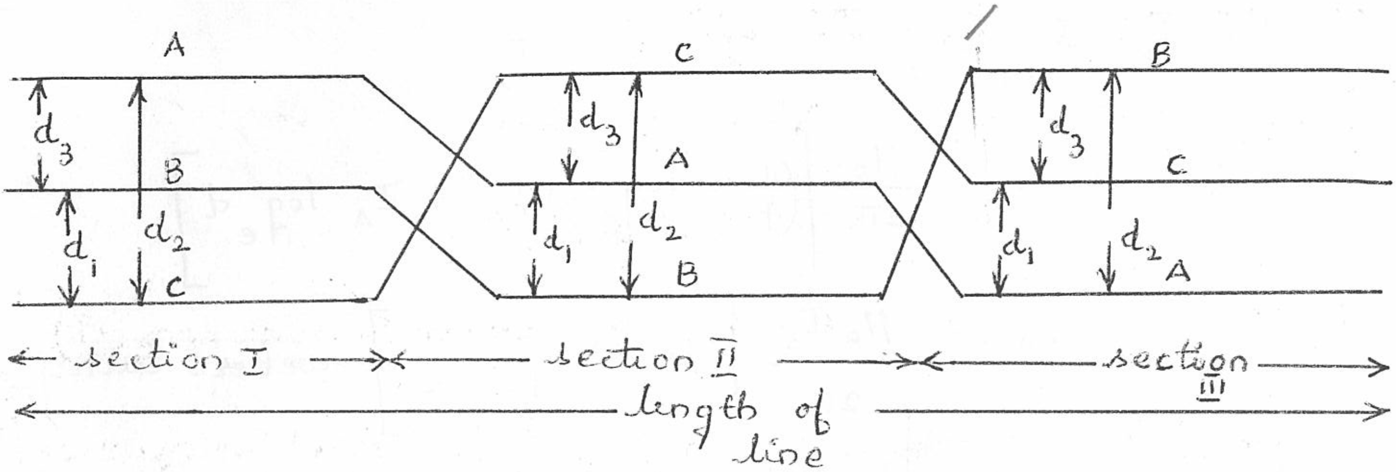
INDUCTANCE OF CONDUCTORS WHEN SPACED UNSYMMETRICALLY:-

* When $d_1 \neq d_2 \neq d_3$, conductors are said to be unsymmetrically spaced.
* $L_A \neq L_B \neq L_C$ at the receiving end.
* To make $L_A = L_B = L_C$, transposition of conductors has to be done.

TRANSPOSITION OF CONDUCTORS:-

* Interchanging the position of conductors at equal intervals along the line so that each conductor occupies the original position of every other conductors over an equal distance. is said to be transposition of conductors.

REASON FOR TRANSPOSITION :-



* Entire length of line is divided into three sections
* Every conductor is made to occupy the original position of every other conductor in each section.
* By transposition, conductor A occupies 1 position in section I, occupies 2nd position in section II & occupies 3rd position in section III
* In I section, conductor A has inductance l_A
* In II section, conductor A has inductance l_B
* In III section, conductor A has inductance l_c.
* ∴ Total inductance of A, $L_A = \frac{1}{3}(l_A + l_B + l_c)$

IIIly Total inductance of B, $L_B = \frac{1}{3}(l_b + l_c + l_a)$

* Total inductance of conductor c, $L_c = \frac{1}{3}(l_c + l_a + l_b)$

* ∴ By transposition of conductors $L_A = L_B = L_C$ although they are unsymmetrically placed.

* From equ I, flux linkage of A in position I is,

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_c \log_e d_2\right] \rightarrow \text{(II)}$$

We know ; $I_A = I(1 + j0)$

$I_B = I(-0.5 - j\,0.866)$

$I_C = I(-0.5 + j\,0.866)$

* Substitute these values of currents in equ (II)

* $$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I - I(-0.5 - j\,0.866)\log_e d_3 - I(-0.5 + j\,0.866)\log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + 0.5 I \log_e d_3 + j0.866 I \log_e d_3 + 0.5 I \log_e d_2 - j0.866 I \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + 0.5 I(\log_e d_3 + \log_e d_2) + j0.866 I(\log_e d_3 - \log_e d_2)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + 0.5 I \log_e d_3 d_2 + j0.866 I \log_e \frac{d_3}{d_2}\right]$$

$$\psi_A = \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_3 d_2}}{r} + j\,0.866 I \log_e \frac{d_3}{d_2}\right]$$

$$\therefore L_A = \frac{\psi_A}{I_A} = \frac{\psi_A}{I}$$

* $l_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j0.866 \log_e \frac{d_3}{d_2}\right]$ H/m.

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j0.866 \log_e \frac{d_3}{d_2}\right] \text{ H/m.}$$

$$\therefore l_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_2 d_3}}{r} + j\,1.732 \log_e \frac{d_3}{d_2}\right] \text{ H/m.}$$

* Inductance of conductor A in II section is

$$l_B = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_3 d_1}}{r} + j1.732 \log_e \frac{d_1}{d_3}\right] \text{ H/m}$$

* Induction of conductor A in III section is

$$l_C = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_1 d_2}}{r} + j1.732 \log_e \frac{d_2}{d_1}\right] \text{ H/m}$$

* Total inductance of A, $L_A = \frac{1}{3}\left[l_A + l_B + l_C\right]$

* $$L_A = \frac{1}{3}\times 10^{-7}\Big[\frac{3}{2} + 2\log_e \frac{\sqrt{d_2 d_3}}{r} + 2\log_e \frac{\sqrt{d_3 d_1}}{r} + 2\log_e \frac{\sqrt{d_1 d_2}}{r} + j1.732 \log_e \frac{d_3}{d_2} + j1.732 \log_e \frac{d_1}{d_3} + j1.732 \log_e \frac{d_2}{d_1}\Big]$$

$$= \frac{1}{3}\times 10^{-7}\left[\frac{3}{2} + 2\log_e \frac{d_1 d_2 d_3}{r^3} + j1.732 \log_e \left(\frac{d_1 d_2 d_3}{d_1 d_2 d_3}\right)\right]$$

$$= 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \text{ H/m.}$$

* $$L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \text{ H/m.}$$

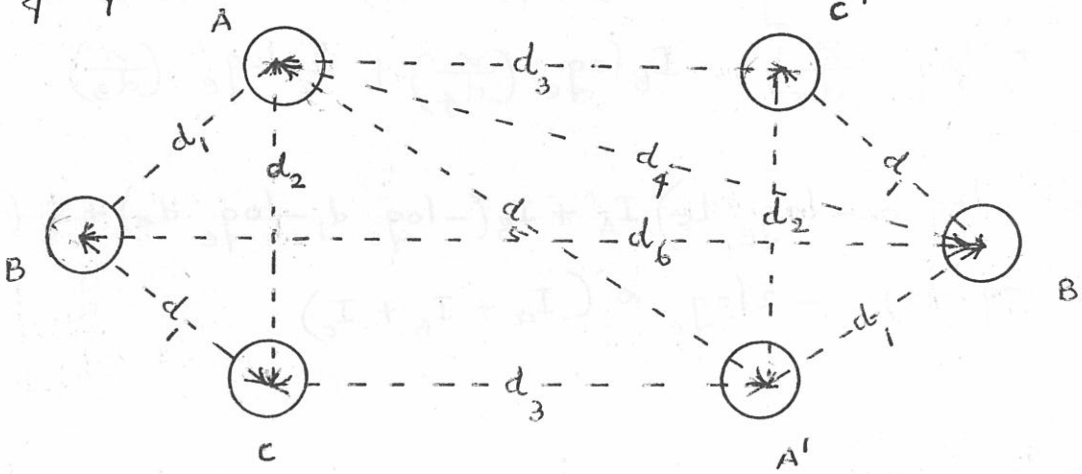
* $L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{D}{r}\right]$ H/m

where $D = \sqrt[3]{d_1 d_2 d_3}$ is equivalent equilateral spacing for unsymmetrically transposed conductors.

INDUCTANCE OF 3-φ DOUBLE CIRCUIT :

DOUBLE CIRCUIT :-

* In double circuit each phase has two conductors carrying the same amount of current.



* Conductors A, B, C form one 3-φ circuit, while conductors A', B', C form another 3-φ circuit.
* Spacing between the conductors is marked in fig. above.
* Conductors A & A' carry current $I_A = I(1+j0)$ —①
* Conductors B & B' carry current $I_B = I(-0.5-j0.866)$ —②
* Conductors C & C' carry current $I_C = I(-0.5+j0.866)$ —③
* Flux linking with conductor A, ψ_A $= \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) I_A$ due to conductor A +

$\frac{\mu_0}{2\pi} I_B \int_{d_1}^{\infty} \frac{dx}{x}$ due to conductor B +

$\frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x}$ due to conductor C +

$$\frac{\mu_0 I_A}{2\pi}\int_{d_5}^{\infty}\frac{dx}{x} \text{ due to conductor } A' +$$

$$\frac{\mu_0 I_B}{2\pi}\int_{d_4}^{\infty}\frac{dx}{x} \text{ due to conductor } B' +$$

$$\frac{\mu_0 I_c}{2\pi}\int_{d_3}^{\infty}\frac{dx}{x} \text{ due to conductor } c'$$

$$\therefore \psi_A = \frac{\mu_0}{2\pi}\left[\left[\frac{1}{4}+\log_e\frac{\infty}{r}\right]I_A + I_B\log_e\left(\frac{\infty}{d_1}\right) + I_c\log_e\left(\frac{\infty}{d_2}\right) + I_A\log_e\left(\frac{\infty}{d_5}\right) + I_B\log_e\left(\frac{\infty}{d_4}\right) + I_c\log_e\left(\frac{\infty}{d_3}\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}-\log_e r-\log_e d_5\right)I_A + I_B(-\log_e d_1-\log_e d_4)+I_c(-\log_e d_2 - \log_e d_3) - 2\log_e\infty\,(I_a+I_b+I_c)\right]$$

since $I_A+I_B+I_C=0$,

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\log_e\frac{1}{r d_5}\right)I_a + I_b\log_e\frac{1}{d_1 d_4} + I_c\log_e\frac{1}{d_2 d_3}\right] \quad -④$$

sub ①, ② & ③ in equ ④

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\log_e\frac{1}{r d_5}\right)I + (-0.5-j0.866)I\log_e\frac{1}{d_1 d_4} + (-0.5+j0.866)I\log_e\frac{1}{d_2 d_3}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left[\frac{1}{4}+\log_e\frac{1}{r d_5}\right]I - 0.5I\left[\log_e\frac{1}{d_1 d_2 d_3 d_4}\right] + j0.866I\left[\log_e\frac{d_1 d_4}{d_2 d_3}\right]\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left[\frac{1}{4}+\log_e\frac{1}{r d_5}\right]I + I\log_e\sqrt{d_1 d_2 d_3 d_4} + j0.866I\log_e\frac{d_1 d_4}{d_2 d_3}\right]$$

* $$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \log_e \frac{\sqrt{d_1 d_2 d_3 d_4}}{r d_5}\right) I + j0.866 I \log_e \frac{d_1 d_4}{d_2 d_3}\right]$$

* $$L_A = \frac{\psi_A}{I_A} = \frac{\psi_A}{I}$$

$$L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_1 d_2 d_3 d_4}}{r d_5} + j0.866 \log_e \frac{d_1 d_4}{d_2 d_3}\right] H/m. \quad (5)$$

|||ly.

$$L_C = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_1 d_2 d_3 d_4}}{r d_5} - j0.866 \log_e \frac{d_1 d_4}{d_2 d_3}\right] H/m. \quad (6)$$

(ii) Conductor C is at the same distance from conductor B & A. So $L_A = L_C$ except the -ve sign for phase shift.

*Flux linkage with conductor B, ψ_B } $= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right] I_B$ due to conductor B +

$\frac{\mu_0}{2\pi} I_A \int_{d_1}^{\infty} \frac{dx}{x}$ due to conductor A +

$\frac{\mu_0 I_C}{2\pi} \int_{d_1}^{\infty} \frac{dx}{x}$ due to conductor C +

$\frac{\mu_0 I_A}{2\pi} \int_{d_4}^{\infty} \frac{dx}{x}$ due to conductor A' +

$\frac{\mu_0 I_B}{2\pi} \int_{d_6}^{\infty} \frac{dx}{x}$ due to conductor B' +

$\frac{\mu_0 I_C}{2\pi} \int_{d_4}^{\infty} \frac{dx}{x}$ due to conductor C'.

$$* \psi_B = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\log_e \frac{\infty}{r}\right) I_b + I_a \log_e \frac{\infty}{d_1} + I_c \log_e \frac{\infty}{d_1} + I_a \log_e \frac{\infty}{d_4} + I_b \log_e \frac{\infty}{d_6} + I_c \log_e \frac{\infty}{d_4}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\log_e \frac{1}{r d_6}\right) I_b + I_a \log_e \frac{1}{d_1 d_4} + I_c \log_e \frac{1}{d_1 d_4} + 2\log_e \infty \,(I_A + I_B + I_C)\right]$$

since $I_A + I_B + I_C = 0$

$$\therefore \psi_B = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\log_e \frac{1}{r d_6}\right) I_b + I_a \log_e \frac{1}{d_1 d_4} + I_c \log_e \frac{1}{d_1 d_4}\right] \quad -(7)$$

* sub ①, ② & ③ in equ ⑦

$$* \psi_B = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\log_e \frac{1}{r d_6}\right)(-0.5-j0.866)I + I \log_e \frac{1}{d_1 d_4} + (-0.5+j0.866)I \log_e \frac{1}{d_1 d_4}\right]$$

$$* \psi_B = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\log_e \frac{1}{r d_6}\right)(-0.5-j0.866)I + I \log_e \frac{1}{d_1 d_4}(0.5+j0.866)\right]$$

$$* \psi_B = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4}+\log_e \frac{1}{r d_6}\right)(-0.5-j0.866)I - (-0.5-j0.866)I \log_e \frac{1}{d_1 d_4}\right]$$

$$= \frac{\mu_0}{2\pi}\left[(-0.5-j0.866)I \left(\frac{1}{4}+\log_e \frac{d_1 d_4}{r d_6}\right)\right]$$

$$* L_B = \frac{\psi_B}{I_B} = \frac{\psi_B}{(-0.5-j0.866)I}$$

$$* L_B = \frac{\mu_0}{2\pi}\left[\frac{1}{4}+\log_e \frac{d_1 d_4}{r d_6}\right] \text{ H/m.}$$

[**ELECTRIC FIELD INTENSITY:-**

* Force experienced by a unit +ve charge

$$E = \frac{Q}{2\pi \varepsilon_0 x} \; V/m.$$]

* Electric field intensity at a distance x from the centre of conductor is

$$E = \frac{Q_A}{2\pi \varepsilon_0 x} \; V/m.$$

where Q_A = charge per metre length.

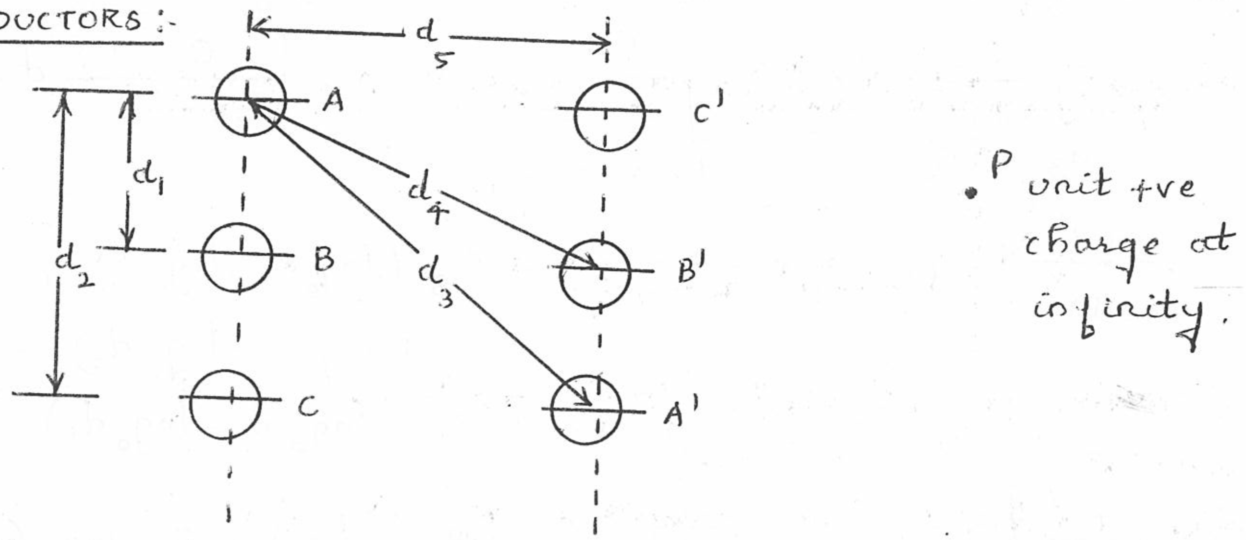
ε_0 = permittivity of free space.

* On integrating E b/w limits r to ∞, we get V_A.

$$V_A = \int_r^\infty \frac{Q_A}{2\pi \varepsilon_0 x} dx.$$

is the electric potential at charged single conductor.

POTENTIAL AT A CONDUCTOR IN A GROUP OF CHARGED CONDUCTORS:-



* On applying V_A, conductors A & A' acquire charge Q_A
* On applying V_B, conductors B & B' acquire charge Q_B

* On applying Q_c, conductors c, c' acquire charge Q_c.

* A unit +ve charge is at infinity.

* Electric potential at point A, V_A } = potential at A due to Q_A
+
potential at A due to Q_B
+
potential at A due to Q_c.
+
potential at A due to Q_A of conductor A'
+
potential at A due to Q_B of conductor B'
+
potential at A due to Q_c of conductor c'.

$$* \quad V_A = \int_{r}^{\infty} \frac{Q_A}{2\pi \varepsilon_0 x} dx + \int_{d_1}^{\infty} \frac{Q_B}{2\pi \varepsilon_0 x} dx + \int_{d_2}^{\infty} \frac{Q_C}{2\pi \varepsilon_0 x} dx +$$

$$\int_{d_3}^{\infty} \frac{Q_A}{2\pi \varepsilon_0 x} dx + \int_{d_4}^{\infty} \frac{Q_B}{2\pi \varepsilon_0 x} dx + \int_{d_5}^{\infty} \frac{Q_C}{2\pi \varepsilon_0 x} dx$$

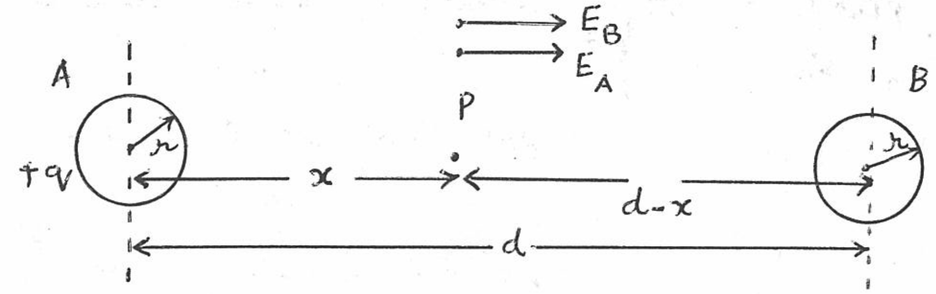
$$= \frac{1}{2\pi \varepsilon_0} \left[Q_A (\log_e \infty - \log_e r) + Q_B (\log_e \infty - \log_e d_1) + Q_C (\log_e \infty - \log_e d_2) + Q_A (\log_e \infty - \log_e d_3) + Q_B (\log_e \infty - \log_e d_4) + Q_c (\log_e \infty - \log_e d_5) \right]$$

$$* \quad V_A = \frac{1}{2\pi \varepsilon_0} \left[Q_A \log_e \left(\frac{1}{r d_3}\right) + Q_B \log_e \left(\frac{1}{d_1 d_4}\right) + Q_c \log_e \left(\frac{1}{d_2 d_5}\right) + 2 \log_e \infty \, (Q_A + Q_B + Q_C) \right]$$

We know $I_A + I_B + I_c = 0 \quad \therefore \quad Q_A + Q_B + Q_C = 0$

* $V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r d_3} + Q_B \log_e \frac{1}{d_1 d_4} + Q_c \log_e \frac{1}{d_2 d_5}\right]$

CAPACITANCE OF A SINGLE PHASE TWO-WIRE LINE:-



* Consider two conductors A & B separated by a distance 'd' metres.
* Let 'r' be the radius of the conductors A & B.
* Conductor A acquires a charge '+q' on application of Vge.
* Conductor B acquires a charge '-q' on application of vge.
* Consider a unit +ve charge at point P at a distance 'x' metres from A.
* On moving +ve charge from P towards conductor A, a repulsive force is experienced

∴ Electric field intensity at A, E_A $= \frac{q}{2\pi\varepsilon_0 x}$.

* Direction of E_A is away from conductor A.
(i.e) from (A to B)
* On moving a +ve charge from P towards conductor B, a attractive force is experienced.

$\therefore$ Electric field intensity at B, E_B $\Big\} = \dfrac{q}{2\pi \varepsilon_0 (d-x)}$

* Direction of E_B is towards the conductor B. ie from A to B.

* Electric field intensity at point P, E_x $\Big\} = E_A + E_B$. (ie) E_A & E_B is direction are same

$$E_x = \frac{q}{2\pi \varepsilon_0 x} + \frac{q}{2\pi \varepsilon_0 (d-x)}$$

* When the unit +ve charge at P is moved between the conductors A & B., potential difference b/w conductors can be obtained.

* Potential difference b/w the conductors A & B, V_{AB} $\Big\} = \displaystyle\int_{r}^{d-r} \frac{q}{2\pi \varepsilon_0 x} + \frac{q}{2\pi \varepsilon_0 (d-x)} \, dx.$

*

* $$V_{AB} = \frac{q}{2\pi \varepsilon_0}\left[\int_{r}^{d-r}\left(\frac{1}{x} + \frac{1}{d-x}\right) dx\right]$$

$$V_{AB} = \frac{q}{2\pi \varepsilon_0}\left[\log_e x - \log_e d-x\right]_{r}^{d-r}.$$

* $$V_{AB} = \frac{q}{2\pi \varepsilon_0}\left[\log_e (d-r) - \log_e (d-d+r) - \log_e r + \log_e (d-r)\right]$$

* $$V_{AB} = \frac{q}{2\pi \varepsilon_0}\left[2 \log_e \left(\frac{d-r}{r}\right)\right]$$

* $d-r \simeq d$

* $$V_{AB} = \frac{q}{\pi \varepsilon_0} \log_e \frac{d}{r}$$

* Potential of each conductor with respect to point P, V $\Big\} = \dfrac{V_{AB}}{2}$

$$V = \frac{q}{2\pi \varepsilon_0} \log_e \frac{d}{r}$$

* Capacitance of each conductor, c $\Big\} = q/v$.

$$C = \frac{q}{\frac{q}{2\pi \varepsilon_0} \log_e d/r}$$

* Capacitance of 1-φ line,

$$\boxed{C = \frac{2\pi \varepsilon_0}{\log_e d/r} \ F/m.}$$

∴ A 3-φ overhead transmission line has its conductors arranged at the corners of an equilateral Δle of 2m side. Calculate the capacitance of each line conductor per km. Given that diameter of each conductor is 1.25cm.

GIVEN :-

* $r = 1.25/2 = 0.625$cm.
* $d = 2m = 200$cm

REQUIRED :-

* C ?? / km

SOLUTION :-

* $$C = \frac{2\pi \varepsilon_0}{\log_e d/r} \ F/m.$$

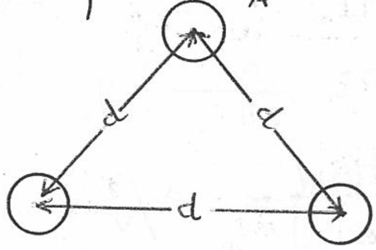
$$= \frac{2\pi \times 8.854 \times 10^{-12}}{\log_e 200/0.625}$$

$$C = 0.0096 \times 10^{-9} \ F/m$$

$$\boxed{C = 0.0096 \times 10^{-6} \ F/km}$$

CAPACITANCE OF 3-φ OVERHEAD LINE :-

i) Symmetrically spaced conductors :-



• P
unit +ve charge at infinity.

* Consider 3-φ conductors ABC equidistant from each other.
* On applying voltages to conductors, conductors A, B & C acquire charges Q_A, Q_B & Q_C respectively.
* Consider a unit +ve charge at infinity.
* Electric potential at conductor A, V_A = Potential due to charge Q_A + potential due to charge Q_B + potential due to charge Q_C

$$V_A = \int_r^{\infty} \frac{Q_A}{2\pi\varepsilon_0 x}dx + \int_d^{\infty} \frac{Q_B}{2\pi\varepsilon_0 x}dx + \int_d^{\infty} \frac{Q_C}{2\pi\varepsilon_0 x}dx$$

* $$V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d}\right]$$

$$V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r} + (Q_B + Q_C) \log_e \frac{1}{d}\right]$$

* Since $I_A + I_B + I_C = 0$; $Q_A + Q_B + Q_C = 0$

$$\therefore Q_A = -(Q_B + Q_C).$$

$$\therefore V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r} - Q_A \log_e \frac{1}{d}\right]$$

$$\therefore V_A = \frac{Q_A}{2\pi\varepsilon_0}\left[\log_e \left(\frac{d}{r}\right)\right] \text{ volts.}$$

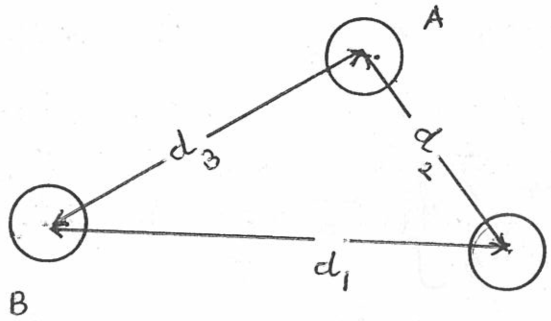
* Capacitance of A, $C_A = Q_A / V_A$

$$\therefore C_A = \frac{Q_A}{\frac{Q_A}{2\pi\varepsilon_0}\log_e d/r} \quad F/m.$$

$$C_A = \frac{2\pi\varepsilon_0}{\log_e d/r} \quad F/m.$$

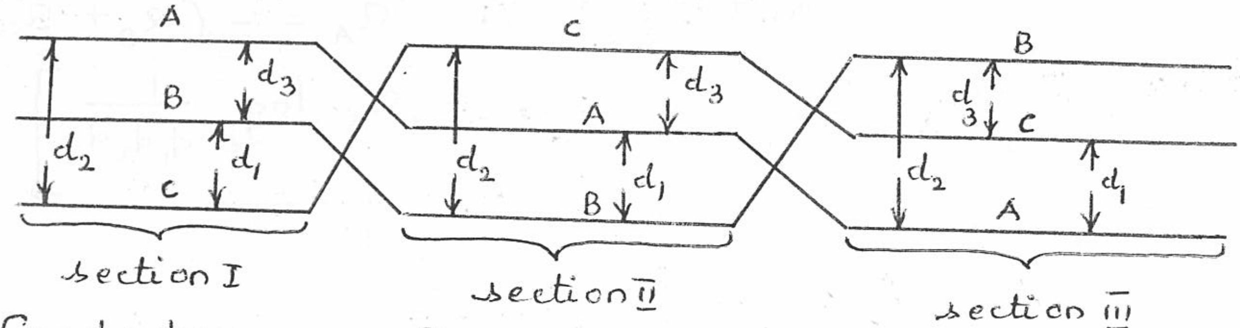
* C_A for 3-φ conductors symmetrically placed } = C_A of 1-φ line.

ii) UNSYMMETRICALLY SPACED 3-φ CONDUCTORS:



* Since the spacing between the conductors is unequal, $C_A \neq C_B \neq C_C$.

* To make $C_A = C_B = C_C$, transposition of conductors have to be done.



* Conductor A has potential V_A in section I.

$$V_A = \frac{1}{2\pi\varepsilon_0}\left[\int_r^{\infty}\frac{Q_A}{x}dx + \int_{d_3}^{\infty} Q_B\frac{dx}{x} + \int_{d_2}^{\infty} Q_C\frac{dx}{x}\right]$$

$$V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A\log_e\frac{1}{r} + Q_B\log_e\frac{1}{d_3} + Q_C\log_e\frac{1}{d_2} + 2(Q_A+Q_B+Q_C)\log_e\infty\right]$$

* since $Q_A + Q_B + Q_C = 0$

$$* \quad v_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_3} + Q_C \log_e \frac{1}{d_2}\right]$$

* Conductor A occupies 'B' conductors position in section II. Its potential in section II is v_B.

$$* \quad v_B = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_3}\right]$$

* Conductor A occupies conductor c's position in section III. Its potential in section III is v_C.

$$* \quad v_C = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_2} + Q_C \log_e \frac{1}{d_1}\right]$$

* Potential on conductor A, V_A $= \frac{1}{3}\left[v_A + v_B + v_C\right]$

$$* \quad V_A = \frac{1}{3\times 2\pi\varepsilon_0}\left[3 Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1 d_2 d_3} + Q_C \log_e \frac{1}{d_1 d_2 d_3}\right]$$

$$* \quad V_A = \frac{1}{3\times 2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r^3} + (Q_B + Q_C) \log_e \frac{1}{d_1 d_2 d_3}\right]$$

We know $Q_A + Q_B + Q_C = 0 \quad \therefore \quad Q_A = -(Q_B + Q_C)$

$$* \quad V_A = \frac{1}{3\times 2\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r^3} - Q_A \log_e \frac{1}{d_1 d_2 d_3}\right]$$

$$= \frac{1}{3\times 2\pi\varepsilon_0}\left[Q_A \log_e \frac{d_1 d_2 d_3}{r^3}\right]$$

$$* \quad V_A = \frac{Q_A}{2\pi\varepsilon_0}\left[\log_e \sqrt[3]{d_1 d_2 d_3} / r\right]$$

* Capacitance of A $= \dfrac{Q_A}{V_A} = \dfrac{1}{2\pi\varepsilon_0} \log_e \dfrac{\sqrt[3]{d_1 d_2 d_3}}{r}$

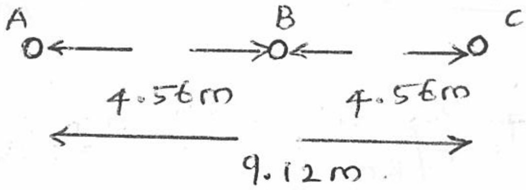
* **Capacitance of A = $\dfrac{2\pi\varepsilon_0}{\log_e\left(\dfrac{\sqrt[3]{d_1 d_2 d_3}}{r}\right)}$ F/m**

* $$C_A = \frac{2\pi\varepsilon_0}{\log_e \frac{D}{r}} \text{ F/m}$$

where $D = \sqrt[3]{d_1 d_2 d_3}$ is equivalent equilateral spacing for unsymmetrically transposed conductor.

1. A 3-phase, 50 Hz, 132kV overhead line has conductors placed in a horizontal plane 4.56m apart. Conductor diameter is 22.4mm. If the line length is 100km, calculate the charging current per phase assuming complete transposition.

GIVEN:-

* 

* dia = 22.4mm ; radius = $\frac{22.4 \times 10^{-3}}{2} = 11.2 \times 10^{-3}$ m.

* l = 100km.

* $V_{line} = 132 \times 10^3$ V

* $V_{ph} = V_L/\sqrt{3} = \frac{132 \times 10^3}{\sqrt{3}} = 76212$ V

REQUIRED:-

* charging current /phase.

SOLUTION :-

* $C = \dfrac{2\pi\varepsilon_0}{\log_e\left(\sqrt[3]{d_1 d_2 d_3}/r\right)}$ F/m.

$d_1 = 4.56m$

$d_2 = 4.56m$

$d_3 = 9.21m.$

$\sqrt[3]{d_1 d_1 d_3} = \sqrt[3]{4.56 \times 4.56 \times 9.21}$

$= 5.76m.$

$C = \dfrac{2 \times \pi \times 8.85 \times 10^{-12}}{\log_e \dfrac{5.76}{11.2 \times 10^{-3}}}$ F/m

$C = \dfrac{5.56 \times 10^{-11}}{6.242} = 8.906 \times 10^{-12}$ F/m.

$C = 8.906 \times 10^{-9}$ F/km.

* capacitance for 100km $= 8.906 \times 10^{-9} \times 100$ F.

$= 8.906 \times 10^{-7}$ F

* charging current $I_c = \dfrac{V_{ph}}{X_c}$

$X_c = \dfrac{1}{2\pi f C}$

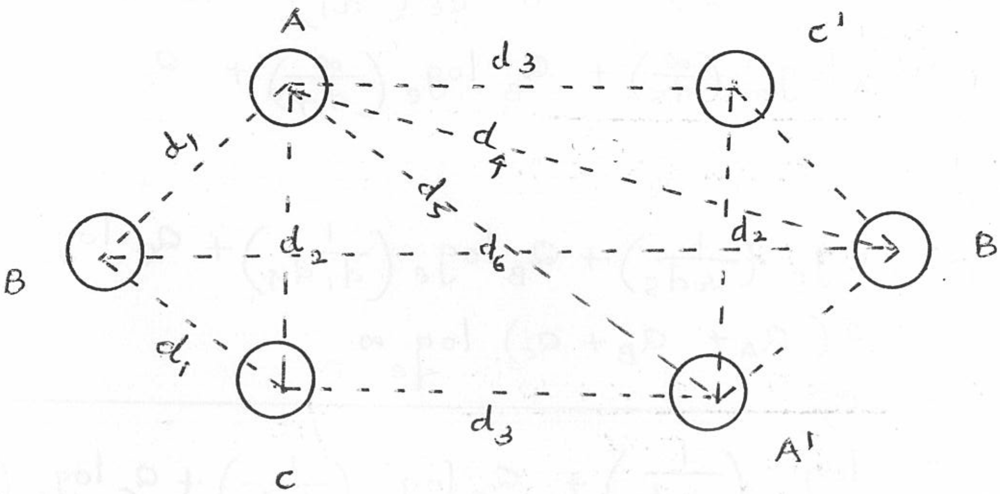
$= \dfrac{1}{2\pi \times 50 \times 8.906 \times 10^{-7}}$

$X_c = 3575.9\,\Omega.$

* $I_c = \frac{76212}{3575.9}$

$$\boxed{I_c = 21.31 \text{ A}}$$

CAPACITANCE OF 3-φ DOUBLE CIRCUIT :-



P
* unit +ve charge at infinity.

* Consider a 3-φ double circuit in which ABC form one 3-φ circuit & conductors A'B'C' form another 3-φ circuit.
* Current carried by conductors A & A' = $I_A = I(1+j0)$
* Current carried by conductors B & B' = $I_B = I(-0.5-j0.866)$
* Current carried by conductors C & C' = $I_C = I(-0.5+j0.866)$

∴ Under balanced condition $I_A + I_B + I_C = 0$

* Conductors A, B & C acquire charges Q_A, Q_B & Q_C on application of voltages.
* IIIly A', B' & C' conductors acquires charges Q_A, Q_B & Q_C respectively.
* Potential of conductor A, V_A } = Potential due to charge Q_A + Potential due to charges Q_B & Q_C.

* $$V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A\int_r^{\infty}\frac{dx}{x} + Q_B\int_{d_1}^{\infty}\frac{dx}{x} + Q_C\int_{d_2}^{\infty}\frac{dx}{x} + Q_A\int_{d_5}^{\infty}\frac{dx}{x} + Q_B\int_{d_4}^{\infty}\frac{dx}{x} + Q_C\int_{d_3}^{\infty}\frac{dx}{x}\right]$$

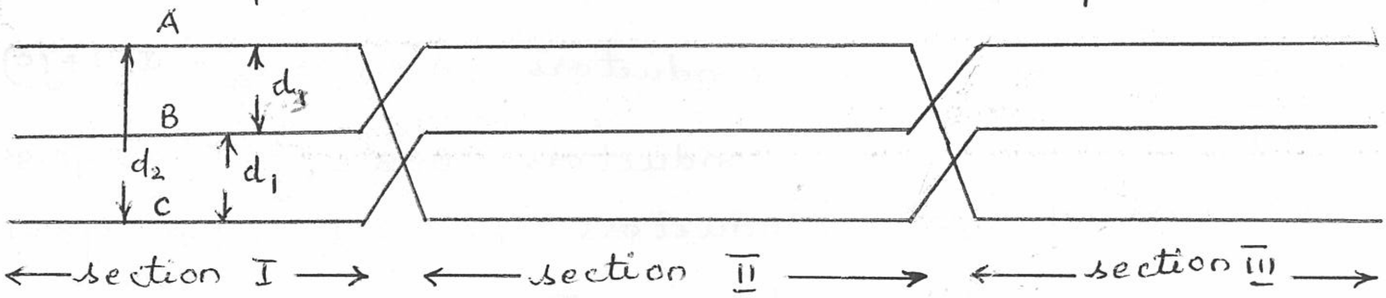
$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A\log_e\left(\frac{\infty}{r}\right) + Q_B\log_e\left(\frac{\infty}{d_1}\right) + Q_C\log_e\left(\frac{\infty}{d_2}\right) + Q_A\log_e\left(\frac{\infty}{d_5}\right) + Q_B\log_e\left(\frac{\infty}{d_4}\right) + Q_C\log_e\left(\frac{\infty}{d_3}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A\log_e\left(\frac{1}{r d_5}\right) + Q_B\log_e\left(\frac{1}{d_1 d_4}\right) + Q_C\log_e\left(\frac{1}{d_2 d_3}\right) + 2(Q_A + Q_B + Q_C)\log_e\infty\right]$$

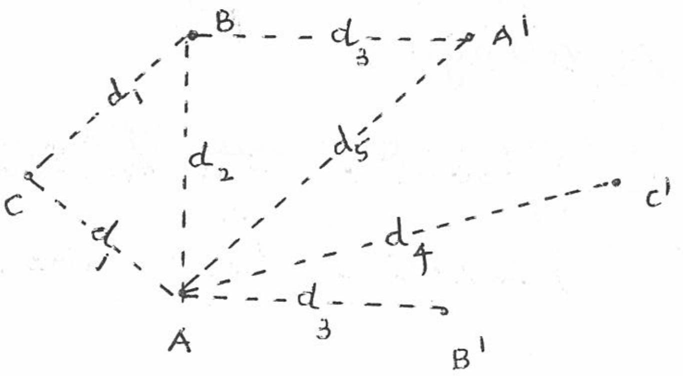
since $Q_A + Q_B + Q_C = 0$;

$$\boxed{V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A\log_e\left(\frac{1}{r d_5}\right) + Q_B\log_e\left(\frac{1}{d_1 d_4}\right) + Q_C\log_e\left(\frac{1}{d_2 d_3}\right)\right]}$$

* Transposition of conductor is done to obtain equal potentials at the receiving end.



POTENTIAL OF CONDUCTOR IN SECTION II :-



* Potential of conductor A in section II, v_B } = Potential of conductor A due to Q_A + Potential due to charges Q_B & Q_C

$$v_B = \frac{1}{2\pi\varepsilon_0}\left[Q_A\int_r^{\infty}\frac{dx}{x} + Q_B\int_{d_2}^{\infty}\frac{dx}{x} + Q_C\int_{d_1}^{\infty}\frac{dx}{x} + Q_A\int_{d_5}^{\infty}\frac{dx}{x} + Q_B\int_{d_3}^{\infty}\frac{dx}{x} + Q_C\int_{d_4}^{\infty}\frac{dx}{x}\right]$$

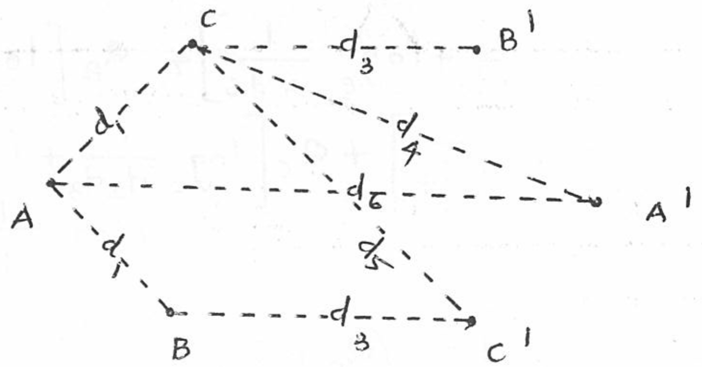
$$v_B = \frac{1}{2\pi\varepsilon_0}\left[Q_A\log_e\left(\frac{\infty}{r}\right) + Q_B\log_e\left(\frac{\infty}{d_2}\right) + Q_C\log_e\left(\frac{\infty}{d_1}\right) + Q_A\log_e\left(\frac{\infty}{d_5}\right) + Q_B\log_e\left(\frac{\infty}{d_3}\right) + Q_C\log_e\left(\frac{\infty}{d_4}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A\log_e\left(\frac{1}{rd_5}\right) + Q_B\log_e\left(\frac{1}{d_2d_3}\right) + Q_C\log_e\left(\frac{1}{d_1d_4}\right) + 2(Q_A+Q_B+Q_C)\log_e\infty\right]$$

* since $Q_A + Q_B + Q_C = 0$,

$$V_B = \frac{1}{2\pi\varepsilon_0}\left[Q_A\log_e\left(\frac{1}{rd_5}\right) + Q_B\log_e\left(\frac{1}{d_2d_3}\right) + Q_C\log_e\left(\frac{1}{d_1d_4}\right)\right]$$

POTENTIAL OF CONDUCTOR A IN SECTION III:



* Potential of conductor A in section III, v_C } = Potential due to charge Q_A + Potential due to charges Q_B & Q_C.

$$* \quad v_C = \frac{1}{2\pi\varepsilon_0}\left[Q_A \int_r^{\infty} \frac{dx}{x} + Q_B \int_{d_1}^{\infty} \frac{dx}{x} + Q_C \int_{d}^{\infty} \frac{dx}{x} + Q_A \int_{d_6}^{\infty} \frac{dx}{x} + Q_B \int_{d_4}^{\infty} \frac{dx}{x} + Q_C \int_{d_4}^{\infty} \frac{dx}{x} \right]$$

$$* \quad v_C = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\left(\frac{\infty}{r}\right) + Q_B \log_e\left(\frac{\infty}{d_1}\right) + Q_C \log_e\left(\frac{\infty}{d}\right) + Q_A \log_e\left(\frac{\infty}{d_6}\right) + Q_B \log_e\left(\frac{\infty}{d_4}\right) + Q_C \log_e\left(\frac{\infty}{d_4}\right) \right]$$

$$* \quad v_C = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\left(\frac{1}{r d_6}\right) + Q_B \log_e\left(\frac{1}{d_1 d_4}\right) + Q_C \log_e\left(\frac{1}{d_1 d_4}\right) + 2(Q_A + Q_B + Q_C)\log_e \infty \right]$$

since $Q_A + Q_B + Q_C = 0$,

$$* \quad v_C = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\left(\frac{1}{r d_6}\right) + Q_B \log_e\left(\frac{1}{d_1 d_4}\right) + Q_C \log_e\left(\frac{1}{d_1 d_4}\right) \right]$$

$\therefore$ Potential at conductor A, $V_A = \frac{1}{3}\left[v_A + v_B + v_C \right]$

$$* \quad V_A = \frac{1}{3} \times \frac{1}{2\pi\varepsilon_0}\left[Q_A\left[\log_e \frac{1}{r d_5} + \log_e \frac{1}{r d_5} + \log_e \frac{1}{r d_6}\right] + Q_B\left[\log_e \frac{1}{d_1 d_4} + \log_e \frac{1}{d_2 d_3} + \log_e \frac{1}{d_1 d_4}\right] + Q_C\left[\log_e \frac{1}{d_2 d_3} + \log_e \frac{1}{d_1 d_4} + \log_e \frac{1}{d_1 d_4}\right] \right]$$

$$* \quad V_A = \frac{1}{3} \times \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\left[\frac{1}{r^3 d_5^2 d_6}\right] + (Q_B + Q_C)\log_e\left[\frac{1}{d_1^2 d_4^2 d_2 d_3}\right] \right]$$

since $Q_B + Q_C + Q_A = 0 \Rightarrow Q_B + Q_C = -Q_A$.

$$\therefore * \quad V_A = \frac{1}{3} \times \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\left[\frac{1}{r^3 d_5^2 d_6}\right] - Q_A \log_e\left[\frac{1}{d_1^2 d_2 d_3 d_4^2}\right] \right]$$

* $$V_A = \frac{1}{3} \times \frac{1}{2\pi \varepsilon_0} \left[Q_A \log_e \left(\frac{d_1^2 d_2 d_3 d_4^2}{r^3 d_5^2 d_6} \right) \right]$$

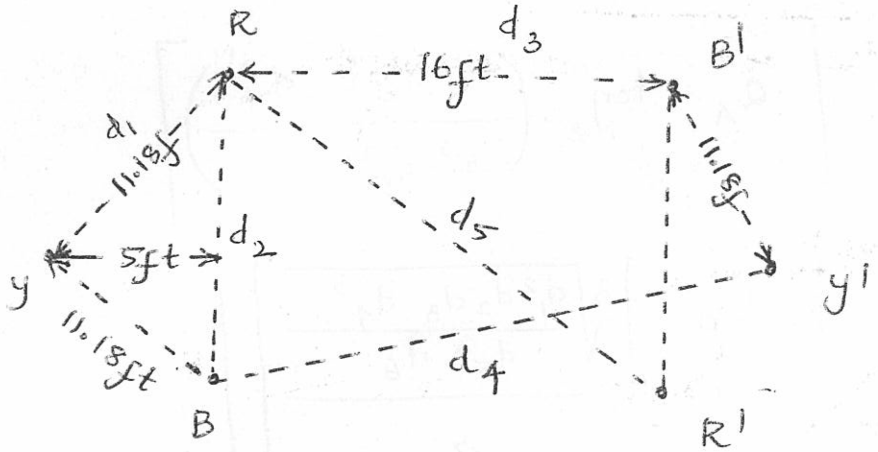
* $$V_A = \frac{Q_A}{2\pi \varepsilon_0} \left[\log_e \left[\frac{\sqrt[3]{\frac{d_1^2 d_2 d_3 d_4^2}{d_5^2 d_6}}}{r} \right] \right]$$

* Capacitance of conductor A $\left.\begin{matrix} \\ C_A \end{matrix}\right\}$ $= \dfrac{Q_A}{V_A}$ F/m.

* $$C_A = \frac{Q_A}{\frac{Q_A}{2\pi\varepsilon_0} \left[\log_e \frac{\sqrt[3]{\frac{d_1^2 d_2 d_3 d_4^2}{d_5^2 d_6}}}{r} \right]} \text{ F/m}$$

* $$\boxed{C_A = \frac{2\pi \varepsilon_0}{\log_e \frac{\sqrt[3]{\frac{d_1^2 d_2 d_3 d_4^2}{d_5^2 d_6}}}{r}} \text{ F/m}}$$

The six conductors of a 3-φ 50 c/s double circuit 110kV line are spaced as shown in fig. and supply a balanced load. If the line are transposed, calculate the inductive and capacitive reactances of each line per mile assuming that a) both lines are present b) one of the circuit is removed. Also determine the percentage change in reactances. The phase sequence is RYB and the conductor radius is ½ inch.



GIVEN DATA:-

* $f = 50$ Hz.
* $V = 110kv$ (line).

$$V_{ph} = \frac{110kv}{\sqrt{3}} = 63,510V.$$

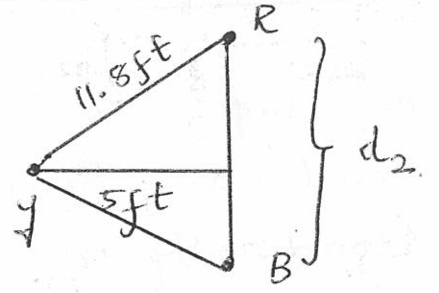
* $r = \frac{1}{2}$ inch.
* $d_1 = 11.18ft$, $d_3 = 16ft$, $d_6 = 26ft$

REQUIRED:-

* To find L & C / mile when both lines are present & when one of the circuit is removed.

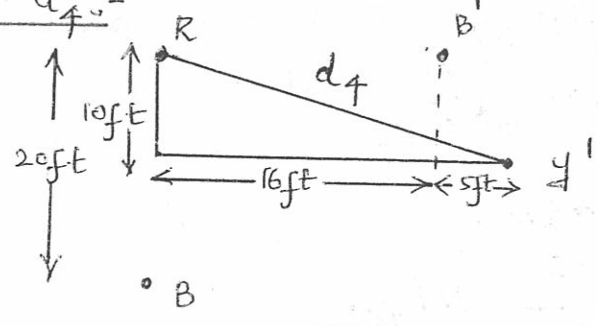
SOLUTION:-

* To find d_2:-



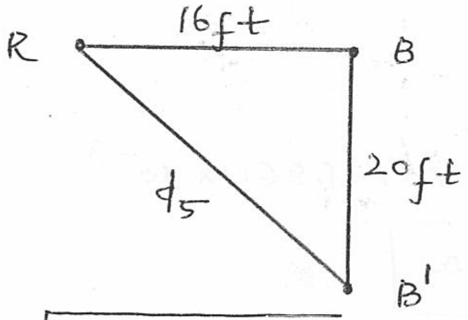
* $d_2 = 2\sqrt{(11.8ft)^2 - (5ft)^2} = 20feet.$

* To find d_4:-



* $d_4 = \sqrt{21^2 + 10^2} = 23.26 ft$

* To find d_5:-



* $d_5 = \sqrt{16^2 + 20^2} = \sqrt{656.} = 25.61 ft.$

a) Inductance & capacitance of 3-ϕ double circuit

$$L = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt[3]{\frac{d_1^2 d_2 d_3 d_4^2}{d_5^2 d_6}}}{r}\right] H/m.$$

$$\sqrt[3]{\frac{d_1^2 d_2 d_3 d_4^2}{d_5^2 d_6}} = \sqrt[3]{\frac{(11.18)^2 \times 20 \times 16 \times (23.26)^2}{(25.61)^2 \times 26}} = 10.81 \text{ feet}.$$

1 feet = 12 inches.

$$\therefore \quad L = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{10.81 \times 12}{0.5}\right)\right] H/m.$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e 259.5\right] H/m.$$

$$= 2 \times 10^{-7}\left[0.25 + 5.56\right] H/m$$

$$= 1.162 \times 10^{-6} \; H/m.$$

$$L = 1.162 \times 10^{-3} \; H/km.$$

We know 1 mile = 1.609 km

$$L = \frac{1.162 \times 10^{-3}}{(1/1.609)} \; H/mile.$$

$$L = 11.62 \times 1.609 \times 10^{-4} \text{ H/mile}$$

$$\boxed{L = 1.861 \text{ mH/mile}}$$

* Inductive reactance, $X_L = 2\pi f L$

$$= 2 \times \pi \times 50 \times 1.861 \times 10^{-3}$$

$$\boxed{X_L = 0.586\ \Omega}$$

* Capacitance $C = \dfrac{2\pi \varepsilon_0}{\log_e \left[\dfrac{\sqrt[3]{\dfrac{d_1^2 d_2 d_3 d_4^2}{d_5^2 d_6}}}{r} \right]}$ F/m.

$$= \frac{2\pi \times 8.854 \times 10^{-12}}{\log_e \left[\dfrac{10.81 \times 12}{0.5} \right]} \text{ F/m.}$$

$$= \frac{5.5 \times 10^{-11}}{5.556} \text{ F/m}$$

$$= 9.89 \times 10^{-12} \text{ F/m.}$$

$$C = 9.89 \times 10^{-9} \text{ F/km}$$

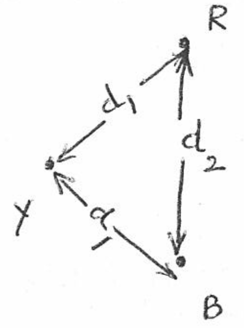
$$C = 9.89 \times 10^{-9} \times 1.609 \text{ F/mile.}$$

$$\boxed{C = 1.588 \times 10^{-8} \text{ F/mile.}}$$

* Capacitance reactance, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times \pi \times 50 \times 1.588 \times 10^{-8}}$

$$\boxed{X_C = 2.00 \times 10^{5}\ \Omega}$$

b). When one circuit is removed, it becomes a 3-φ circuit.



* Removing one circuit means, removing one set of 3-φ circuit (a) R'Y'B'

* $$L = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\left[\frac{\sqrt[3]{d_1 d_1 d_2}}{r}\right]\right] \text{ H/m.}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e\left[\frac{\sqrt[3]{(11.18)^2 \times 20}}{0.5/12}\right]\right] \text{ H/m}$$

$$= 2 \times 10^{-7}\left[0.25 + \log_e\left[\frac{13.57}{0.042}\right]\right] \text{ H/m.}$$

$$= 1.2055 \times 10^{-6} \text{ H/m.}$$

$$L = 1.2055 \times 10^{-3} \text{ H/km.}$$

$$L = 1.2055 \times 10^{-3} \times 1.609 \text{ H/mile}$$

$$\boxed{L = 1.94 \times 10^{-3} \text{ H/mile}}$$

* $$C = \frac{2\pi \varepsilon_0}{\log_e\left[\frac{\sqrt[3]{d_1 d_1 d_2}}{r}\right]} \text{ F/m.}$$

$$= \frac{2 \times \pi \times 8.854 \times 10^{-12}}{\log_e\left[\frac{\sqrt[3]{(11.18)^2 \times 20}}{0.5/12}\right]} \text{ F/m.}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12}}{5.777}$$

$$C = 9.62 \times 10^{-12} \text{ F/m}$$

$$C = 9.62 \times 10^{-9} \text{ F/km.}$$

$$C = 9.62 \times 10^{-9} \times 1.609 \quad F/mile$$

$$\boxed{C = 1.547 \times 10^{-8} \quad F/mile}$$

* Inductive reactance $X_L = 2\pi f L$

$$= 2\pi \times 50 \times 1.93 \times 10^{-3}$$

$$\boxed{X_L = 0.6063\ \Omega}$$

* Capacitive reactance $X_C = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2 \times \pi \times 50 \times 1.547 \times 10^{-8}}$$

$$\boxed{X_C = 2.057 \times 10^{5}\ \Omega.}$$

c) % ge increase in reactances (X_L) $\Big\} = \dfrac{X_L \text{(removed ckt)} - X_L \text{(originalckt)}}{X_L \text{(original ckt)}} \times 100$

$$= \left[\frac{0.6063 - 0.586}{0.586}\right] \times 100$$

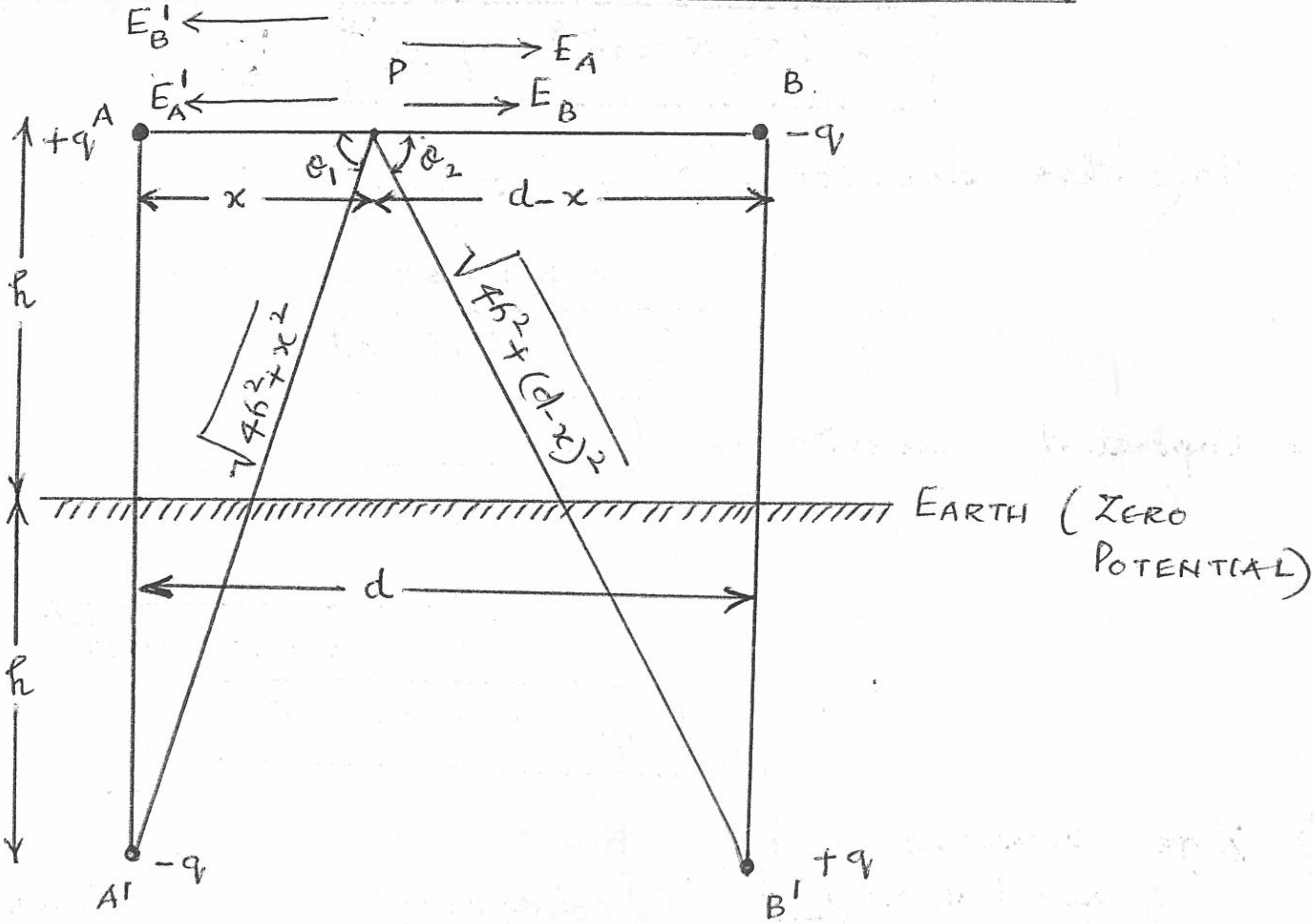
$$= 3.464\%$$

% age increase in reactance X_c $\Big\} = \dfrac{X_c \text{(removedckt)} - X_c \text{(originalckt)}}{X_c \text{(original ckt)}} \times 100$

$$= \left[\frac{2.057 \times 10^{+5} - 2. \times 10^{5}}{2 \times 10^{5}}\right] \times 100$$

$$= 2.85\%$$

EFFECT OF EARTH ON CAPACITANCE OF LINE :-



* Consider a conductor A having charge '+q' at a height of 'h' metres above the earth.
- To have earth at zero potential, an image conductor A' having a charge of '−q' is considered at a height of 'h' metres below the earth.
* ||ly consider a conductor B having charge '−q' at a height of 'h' metres above the earth.
- An image conductor B', having a charge '+q' is a height of 'h' metres below the earth to have zero potential at earth.
- Distance b/w A & B conductors is 'd' metres.
* Consider point P having a unit +ve charge at a distance 'x' metres from A.

* Distance $PA' = \sqrt{4h^2 + x^2}$ & distance $PB' = \sqrt{4h^2 + (d-x)^2}$

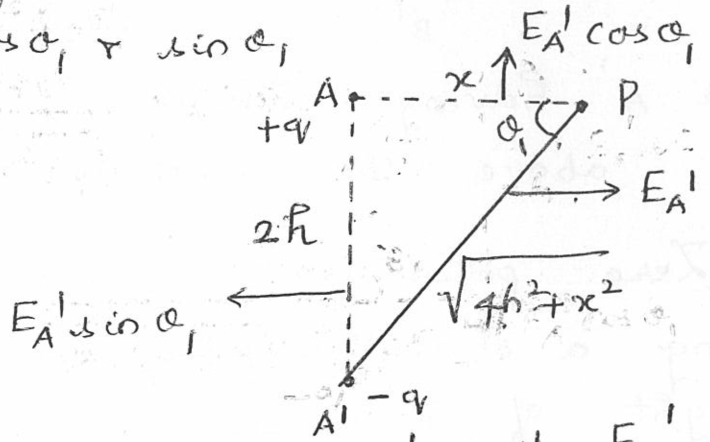
* Electric field intensity at P due to +q at A } $E_A = \dfrac{q}{2\pi \varepsilon_0 x}$ — ①.

* Direction of E_A is from A to B since it is a repulsive force.

* Electric field intensity at P due to −q at B } $E_B = \dfrac{q}{2\pi \varepsilon_0 (d-x)}$ — ②

* Direction of E_B is from P to B since it is an attractive force.

* Electric field intensity at point P due to '−q' at A' will have two component $\cos\theta_1$ & $\sin\theta_1$



• $\cos\theta_1 = \dfrac{x}{\sqrt{4h^2+x^2}}$

• $\sin\theta_1 = \dfrac{2h}{\sqrt{4h^2+x^2}}$

* $\cos\theta_1$ component of E_A' will exist because electric field intensity along the conductor will be zero [(i.e) $\sin\theta_1$ component = Zero]

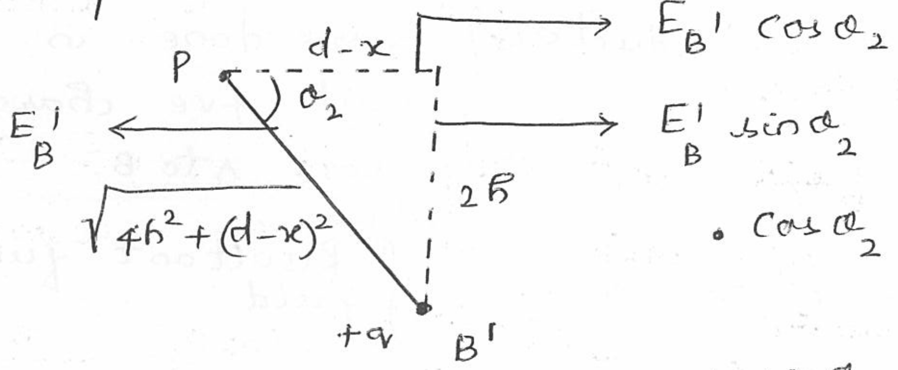
* Electric field intensity at P due to −q at A } $E_A' = \dfrac{q}{2\pi \varepsilon_0 \sqrt{4h^2+x^2}} \times \cos\theta_1$

* Direction of E_A' is from P to A since it is an attractive force.

* $$E_A' = \frac{q}{2\pi\varepsilon_0\sqrt{4h^2+x^2}} \times \frac{x}{\sqrt{4h^2+x^2}}$$

* $$E_A' = \frac{qx}{2\pi\varepsilon_0(4h^2+x^2)} \quad \text{--- (3)}$$

* |||ly electric field intensity at point P due to '+q' at B' will have only $\cos\alpha$ component.
* $\sin\alpha$ component is Zero.



- $$\cos\alpha_2 = \frac{d-x}{\sqrt{4h^2+(d-x)^2}}$$
- $$\sin\alpha_2 = \frac{2h}{\sqrt{4h^2+(d-x)^2}}$$

* Electric field intensity at P due to '+q' at B }
$$E_B' = \frac{q}{2\pi\varepsilon_0\sqrt{4h^2+(d-x)^2}} \times \cos\alpha_2$$

* $$E_B' = \frac{q}{2\pi\varepsilon_0\sqrt{4h^2+(d-x)^2}} \times \frac{d-x}{\sqrt{4h^2+(d-x)^2}}$$

$$E_B' = \frac{q \times d-x}{2\pi\varepsilon_0(4h^2+(d-x)^2)} \quad \text{--- (4)}$$

* Direction of E_B' is from P to A since it is a repulsive force.

* Resultant electric field intensity at point P $\Big\} = E_A + E_B - E_A' - E_B'$

[since E_A' & E_B' are in opposite direction of E_A & E_B, E_A' & E_B' are taken as -ve]

* Resultant field intensity $\Big\} = \dfrac{q}{2\pi\varepsilon_0 x} + \dfrac{q}{2\pi\varepsilon_0 (d-x)} - \dfrac{qx}{2\pi\varepsilon_0 (4h^2+x^2)} - \dfrac{q(d-x)}{2\pi\varepsilon_0 (4h^2+(d-x)^2)}$

* Potential b/w conductor A & B, V $\Big\}$ = Work done in moving a unit +ve charge at P from A to B.

* Potential V $= \displaystyle\int_r^{d-r}$ Resultant field . dx.

* $$V = \frac{q}{2\pi\varepsilon_0}\left[\int_r^{d-r} \frac{1}{x} + \frac{1}{d-x} - \frac{x}{4h^2+x^2} - \frac{d-x}{4h^2+(d-x)^2}\, dx\right]$$

* $$V = \frac{q}{2\pi\varepsilon_0}\left[\log_e x + \log_e (d-x) - \log_e \sqrt{4h^2+x^2} + \log_e \sqrt{4h^2+(d-x)^2}\right]_r^{d-r}$$

(ie) $\displaystyle\int \frac{a}{a^2+b^2}\, dx = \log_e \sqrt{a^2+b^2}$

* $$V = \frac{q}{2\pi\varepsilon_0}\left[\log_e x + \log_e (d-x) - \frac{1}{2}\log_e (4h^2+x^2) + \frac{1}{2}\log_e (4h^2+(d-x)^2)\right]_r^{d-r}$$

$$V = \frac{q}{2\pi \varepsilon_0}\Big[\log_e (d-r) - \log_e r - \log_e (d-d+r) + \log_e (d-r)$$
$$-\frac{1}{2}\log_e\left(4h^2 + (d-r)^2\right) + \frac{1}{2}\log_e\left(4h^2 + r^2\right)$$
$$+\frac{1}{2}\log_e\left(4h^2 + (d-d+r)^2\right) - \frac{1}{2}\log_e\left(4h^2 + (d-r)^2\right)\Big]$$

Take $d-r \simeq d$ & $4h^2 + r^2 \simeq 4h^2$

* $$V = \frac{q}{2\pi \varepsilon_0}\Big[\log_e d - \log_e r - \log_e r + \log_e d \;\; \frac{1}{2}\log_e(4h^2+d^2)$$
$$+\frac{1}{2}\log_e(4h^2) + \frac{1}{2}\log_e(4h^2) - \frac{1}{2}\log_e(4h^2+d^2)\Big]$$

* $$V = \frac{q}{2\pi\varepsilon_0}\left[2\log_e\left(\frac{d}{r}\right) + \log_e(4h^2) - \log_e(4h^2+d^2)\right]$$

* $$V = \frac{q}{2\pi\varepsilon_0}\left[2\log_e\left(\frac{d}{r}\right) + 2\times\frac{1}{2}\log_e(4h^2) - 2\times\frac{1}{2}\log_e(4h^2+d^2)\right]$$

* $$V = \frac{q}{2\pi\varepsilon_0}\left[2\log_e\left(\frac{d}{r}\right) + 2\times\frac{1}{2}\left[\log_e \frac{4h^2}{4h^2+d^2}\right]\right]$$

* $$V = \frac{q}{2\pi\varepsilon_0}\times\frac{1}{2}\left[4\log_e\left(\frac{d}{r}\right) + 2\log_e \frac{4h^2}{4h^2+d^2}\right]$$

* Potential at centre of two conductor $= \frac{V}{2}$

* Potential w.r.t one conductor, u $\Big\} = \frac{q}{2\pi\varepsilon_0}\times\frac{1}{2}\left[2\log_e\frac{d}{r} + \log_e\frac{(2h)^2}{4h^2+d^2}\right]$

$$u = \frac{q}{2\pi\varepsilon_0}\times\frac{1}{2}\left[2\log_e\frac{d}{r} + 2\log_e\frac{2h}{\sqrt{4h^2+d^2}}\right]$$

$$V = \frac{q}{2\pi\varepsilon_0} \log_e \frac{d}{r}\left(\frac{2h}{\sqrt{4h^2+d^2}}\right)$$

* Capacitance of conductor A = $\frac{q}{V}$

$$C_A = \frac{q \times 2\pi\varepsilon_0}{q \log_e \frac{d}{r} \times \left(\frac{2h}{\sqrt{4h^2+d^2}}\right)}$$

$$\boxed{C_A = \frac{2\pi\varepsilon_0}{\log_e \frac{d}{r} \times \left(\frac{2h}{\sqrt{4h^2+d^2}}\right)} \text{ F/m}}$$

is the effect of earth on capacitance of line

1. The conductors in a single phase transmission line are 6m above the ground taking the effect of the earth into account. Calculate the capacitance/km. Each conductor is of 1.5 cm diameter and the conductors are 3m apart.

GIVEN

* $h = 6m$.
* dia = 1.5 cm ; radius = $\frac{1.5 \times 10^{-2}}{2}$ = 7.5×10^{-3} m.
* d = 3m.

REQUIRED:

* C?? C/km???

SOLUTION:

$$C = \frac{2\pi\varepsilon_0}{\log_e \frac{d}{r} \times \left(\frac{2h}{\sqrt{4h^2+d^2}}\right)} \text{ F/m.}$$

$$C = \frac{2\pi \times 8.854 \times 10^{-12}}{\log_e \frac{3}{7.5\times10^{-3}} \times \left(\frac{2\times6}{\sqrt{4\times6^2 + 3^2}}\right)} \; F/m.$$

$$C = \frac{5.563 \times 10^{-11}}{\log_e 400 \times 0.970}$$

$$= \frac{5.563 \times 10^{-11}}{5.961} \; F/m$$

$$= 9.332 \times 10^{-12} \; F/m$$

$$\boxed{C = 9.332 \times 10^{-9} \; F/km}$$

CORONA:-

DEFINITION:-

* Ionisation of air surrounding the conductor is said to be corona.

(or)

* Phenomenon of violet glow, hissing noise and production of ozone gas in an overhead transmission line is known as corona.

THEORY OF CORONA FORMATION:

* Air surrounding the conductors contains some ionised particles such as free electrons +ve ions & neutral molecules
* Apply a voltage b/w the conductors which results in electric field intensity (or) potential gradient around the conductor surfaces.
* Intensity of potential gradient is more near

the conductor surface.

* The potential gradient developed makes free electrons around the conductor to acquire greater velocity for motion.
* Greater the applied voltage, greater the potential gradient & hence greater the velocity of free electrons.
* When applied voltage causes a potential gradient of 30kv/cm, free electrons acquire sufficient velocity to strike a neutral molecule.
* This causes dislodging of electrons from neutral molecule, resulting in ion formation.
* Process of ionisation is cumulative, causing an ionisation of air surrounding the conductor.
* This results in corona formation.

FACTORS AFFECTING CORONA :-

i) ATMOSPHERIC CONDITIONS :-

* Under stormy weather the no. of ions present in air is more than normal & hence corona occurs quickly (ie) at lesser applied voltage.

ii) CONDUCTOR SIZE :-

* Rough & irregular surfaces give rise to more corona.

* Corona in stranded conductor > Corona in solid conductor.

iii) LINE VOLTAGE :-

* If applied voltage is reduced, potential gradient around the conductor will not reach 30kV/cm & hence no corona formation.

iv) SPACING BETWEEN CONDUCTORS :-

* If spacing between conductors is >>>er than diameter of conductor, electrostatic stress at conductor is reduced and hence no corona formation.

TERMS USED IN CORONA ANALYSIS :-

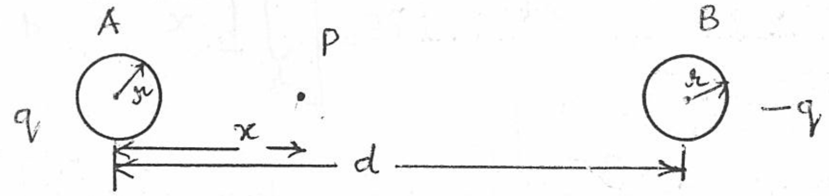
i) CRITICAL DISRUPTIVE VOLTAGE :- V_c

* Minimum phase-neutral voltage at which corona occurs

(OR)

* Value of applied voltage which causes a potential gradient of 30 kV/cm = breakdown strength of air to form corona.

EXPRESSION FOR V_c :-



* Consider conductor A & B separated by a distance 'd' metres.
* Conductor A is having a charge of '+q' & conductor B a charge of '-q' on application of voltages.
* Unit +ve charge is at 'P' at a distance 'x' metres from A.

* Electric field intensity at P due to '+q' at A } $E_A = \dfrac{q}{2\pi \varepsilon_0 x}$ — ①

* Direction of E_A is from P to B since it is a repulsive force.

* Electric field intensity at P due to '-q' at B } $E_B = \dfrac{q}{2\pi \varepsilon_0 \, d-x}$ — ②

* Direction of E_B is from P to B since it is a attractive force

* Resultant field intensity at P } = $E_A + E_B$ (both are in same directn)

$$E = \frac{q}{2\pi \varepsilon_0 x} + \frac{q}{2\pi \varepsilon_0 \, d-x}$$

$$E = \frac{q}{2\pi \varepsilon_0}\left[\frac{1}{x} + \frac{1}{d-x}\right] \quad — ①$$

* Potential developed b/w the conductors } $V_{AB} = \int_{r}^{d-r} {}^{a}E \cdot dx.$

$$V_{AB} = \frac{q}{2\pi \varepsilon_0}\left[\int_{r}^{d-r}\left[\frac{1}{x} + \frac{1}{d-x}\right] dx\right]$$

* $$V_{AB} = \frac{q}{2\pi \varepsilon_0}\left[\Big[\log_e x - \log_e (d-x)\Big]_{r}^{d-r}\right]$$

* $$V_{AB} = \frac{q}{2\pi \varepsilon_0}\left[\log_e (d-r) - \log_e (d-d+r) - \log_e r + \log_e (d-r)\right]$$

* $$V_{AB} = \frac{q}{2\pi \varepsilon_0}\left[\log_e (d-r) - \log_e r - \log_e r + \log_e (d-r)\right]$$

$d-r \approx d$

* $V_{AB} = \frac{q}{2\pi\varepsilon_0}\left[2\log_e d - 2\log_e r\right]$

* $V_{AB} = \frac{q}{\cancel{2}\pi\varepsilon_0}\left[\cancel{2}\log_e\left(\frac{d}{r}\right)\right]$

* $q = \frac{V_{AB} \times \pi\varepsilon_0}{\log_e\left(\frac{d}{r}\right)}$ — ②

sub ② in ①

* $E = \frac{V_{AB} \times \pi\varepsilon_0}{2\pi\varepsilon_0 \times \log_e \frac{d}{r}}\left[\frac{1}{x} + \frac{1}{d-x}\right]$

$E = \frac{V_{AB}}{2} \times \frac{1}{\log_e \frac{d}{r}}\left[\frac{d}{x(d-x)}\right]$

* $\left.\frac{V_{AB}}{2}\right\}$ = Voltage w.r.t to a single conductor
= line to neutral vge is V_{ph}

$E = V_{ph} \times \frac{1}{\log_e \frac{d}{r}}\left[\frac{d}{x(d-x)}\right]$

* When $x \downarrow\downarrow$ es, $E \uparrow\uparrow$ es.

* When $x = r$, E is increased.

* $E = V_{ph} \times \frac{1}{\log_e \frac{d}{r}}\left[\frac{d}{r(d-r)}\right]$

$d-r \approx d$.

* $E = g = V_{ph} \times \frac{1}{\log_e \frac{d}{r}}\left[\frac{\cancel{d}}{r\,\cancel{d}}\right]$

* $g = V_{ph} \times \frac{1}{\log_e \frac{d}{r}}\left[\frac{1}{r}\right]$

* When $g = g_{max} = 30\,kV/cm$. = breakdown strength of air at 76cm of pressure & 25°C of temp.

applied voltage $V_{ph} = V_c$.

* $$g_{max} = \frac{V_c}{r \log_e \frac{d}{r}}$$

* $$V_c = g_{max}\, r \log_e \frac{d}{r}$$

* $g_{max} \propto$ air's pressure & temperature

* For a barometric pressure of 'b' & temp. of t°c } $g_{max} = g_{max}\,\delta$.

where δ = air density factor = $\frac{3.92\,b}{273+t}$.

* $$V_c = g_{max}\,\delta\, r \log_e \frac{d}{r}$$

* Taking irregularity factor m_o into account

(ie) $m_o = 1 \rightarrow$ polished conductors

$= 0.98$ to $0.92 \rightarrow$ dirty conductors

$= 0.87$ to $0.80 \rightarrow$ stranded conductors

$$\therefore V_c = m_o\, g_{max}\, \delta\, r \log_e \frac{d}{r}$$

ii) VISUAL CRITICAL VOLTAGE (V_v):-

* Minimum phase-neutral voltage at which corona glow appears all along the line conductors.

* Empirical formula for V_v is

$$V_v = m_v\, g_{max}\, \delta r \left[1 + \frac{0.3}{\sqrt{\delta r}}\right] \log_e \frac{d}{r} \text{ kV/phase}$$

where irregularity factor $m_v = 1$ for polished conductors

$m_v = 0.72$ to 0.82 for rough conductors.

NOTE:- Corona begins at critical disruptive voltage V_c & corona glow appears at visual critical voltage V_v

$$V_v > V_c.$$

iii) POWER LOSS DUE TO CORONA:-

* Violet glow, hissing sound & ozone formation due to corona results in power loss.

* Power loss is given by.

$$P = 242.2 \left(\frac{f+25}{\delta}\right) \sqrt{\frac{r}{d}}\,(V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

where f = frequency of supply (Hz)

V = applied voltage (r.m.s)

V_c = critical disruptive voltage (r.m.s)

METHODS OF REDUCING CORONA :-

* **By increasing conductor size :**
  - When radius is increased, V_c ↑↑es since $V_c \propto$ radius.
  - ∴ Voltage at which corona occurs ↑↑es, corona effects are reduced.

* **By increasing conductor spacing :-**
  - By increasing 'd', V_c ↑↑es & hence corona effects are reduced because of increase in

ADVANTAGES OF CORONA :-

i) The conduction of air surrounding the conductor due to corona results in increase in diameter of conductor. This results in decrease in electrostatic stress between the conductor.

ii) Reduces effect of transients produced by surges.

DISADVANTAGES OF CORONA :-

i) Corona leads to power loss & affects the transmission efficiency.

ii) Formation of ozone leads to corrosion of conductor.

iii) Due to corona, non-sinusoidal current is drawn by the line & hence non-sinusoidal voltage drop occurs.

1. A 3-ϕ line has conductors 2 cm in diameter spaced equilaterally 1m apart. If the dielectric strength of air is 30kV(max) per cm, find the disruptive critical voltage for the line. Take air density factor $\delta = 0.952$ & irregularity factor $m_0 = 0.9$.

GIVEN :-

dia = 2 cm , $r = \frac{2}{2} = 1$ cm.

$d = 1$m. = 100cm.

$g_{max} = 30$kV/cm $= \frac{30 \text{kV/cm}}{\sqrt{2}} = 21.24$ kV(rms)/cm.

$\delta = 0.952$

$m_0 = 0.9$.

REQUIRED :-

$V_c = ???$ (line Value)

SOLUTION :-

$$V_c = m_0 \, g_{max} \, \delta \, r \, \log_e (d/r) \text{ kV/phase (rms)}$$

$$= 0.9 \times 21.24 \times 0.952 \times 1 \times \log_e \left(\frac{100}{1}\right)$$

$$V_c = 83.80 \text{ kV (rms)/phase.}$$

$$V_{c\,line} = \sqrt{3} \times 83.80 = 145.14 \text{ kV.}$$

$$\boxed{V_{c\,line} = 145.14 \text{ kV}}$$

2. A 132kV line with 1.956cm dia conductors is built so that corona takes place if the line voltage exceeds 210kV (r.m.s). If the value of potential gradient

at which ionisation occurs can be taken as 30kv per cm, find the spacing between the conductors.

GIVEN:

* dia = 1.956cm, $r = 1.956/2 = 0.978$ cm.
* $g_{max} = 30kv/cm = \frac{30}{\sqrt{2}}$ kv (rms)/cm = 21.24 kv(rms)/
* V_c / line = 210kv (rms)
* V_c / phase = $\frac{210kv\ (rms)}{\sqrt{3}}$ = 121.24 kv (rms).

REQUIRED:-

* Spacing b/w conductors.

SOLUTION:-

* $$V_c = m_o \delta g_{max} r \log_e \frac{d}{r} \quad kv/phase$$

Assume conductors to be smooth $m_o = 1$.
At standard temp & pressure $\delta = 1$.

$$121.24 = 1 \times 1 \times 21.24 \times 0.978 \log_e \left(\frac{d}{0.978}\right) \ kv\ rms/phase$$

$$121.25 = 20.77 \log_e \left(\frac{d}{0.978}\right)$$

$$\frac{121.25}{20.77} = \log_e \left(\frac{d}{0.978}\right)$$

$$\log_e \left(\frac{d}{0.978}\right) = 5.837$$

$$2.3 \times \log_{10} \left(\frac{d}{0.978}\right) = 5.837.$$

$$\log_{10} \left(\frac{d}{0.978}\right) = \frac{5.837}{2.3}$$

$$\log_{10} \left(\frac{d}{0.978}\right) = 2.537.$$

$$\frac{d}{0.978} = \text{antilog}(2.537)$$

$$\frac{d}{0.978} = 344.34$$

$$d = 344.34 \times 0.978$$

$$\boxed{d = 336.7 \text{ cm}}$$

2. A 3-φ, 220kV, 50Hz transmission line consists of 1.5cm radius conductor spaced 2m apart in equilateral triangular formation. If the temperature is 40°c and atmospheric pressure is 76cm, calculate the corona loss per km of the line. Take $m_0 = 0.85$.

GIVEN:-

$r = 1.5$ cm

$f = 50$Hz

$m_0 = 0.85$

$b = 76$cm.

$t = 40°c$

$d = 2m = 200$cm.

$V = 220$kv (line)

$V = 220\text{kv}/\sqrt{3} = 127.0$ kv (phase).

REQUIRED:

To find power loss. for 3-φ

SOLUTION:-

$$P = \frac{242.2}{\delta}(f+25)\sqrt{\frac{r}{d}}(V-V_c)^2 \times 10^{-5} \text{ kw/km/phase}$$

$$V_c = m_o\, g_{max}\, \delta r \log_e \frac{d}{r} \text{ kv/phase.}$$

Assume $g_{max} = 30 \text{ kv/cm}^2 \quad 21.2 \text{ kv (rms)/phase.}$

$$\delta = \frac{3.926}{273 + t}$$

$$= \frac{3.92 \times 76}{273 + 40} = 0.9518.$$

$$V_c = 0.85 \times 21.2 \times 0.9518 \times 1.5 \times \log_e \frac{200}{1.5}$$

$$V_c = 125.87 \text{ kv (rms) / phase.}$$

$$P = \frac{242.2}{0.9518} (50+25) \times \sqrt{\frac{1.5}{200}} \times (127 - 125.87)^2 \times 10^{-5}$$

$$= \frac{242.2}{0.9518} \times 75 \times 0.0866 \times 1.2769 \times 10^{-5}$$

$$P = 0.0211 \text{ kw/km/phase.}$$

Power loss for 3-phases $\Big\} = 3 \times 0.0211$ kw/km.

$= 0.0633$ kw/km.

PROXIMITY EFFECT:-

* Every conductor has external flux linkage from a neighbouring current carrying conductor.
* This gives rise to current circulating on outer surface.
* The outer surface has more current distribution than inner surface.
* This non-uniform current distribution gives

an apparent increase in resistance.

* Such an effect is termed as proximity effect.

* Proximity effect is less in overhead lines & pronounced in case of cables where the distance b/w conductors is small.

INDUCTIVE INTERFERENCE WITH COMMUNICATION LINE :-

If the power line is running along the communication line, there will be an interference in the communication line due to both electrostatic and electromagnetic effects.

* Electrostatic effect induces voltage in communication line, which is dangerous to human body, vehicles, buildings and objects of comparable size.

* The electromagnetic effect produces currents, which is superimposed on the true speech currents in the communication signal and cause distortion.

Both the effects depends on the distance between the power and communication lines and the length of the route over which they are parallel.

INTRODUCTION FOR UNIT – II :

CONDUCTORS :-

* A conducting material used for transmission of power.
* Aluminium & copper conductors are used generally.

ADVANTAGES OF ALUMINIUM OVER COPPER CONDUCTORS :

* Low weight.
* low cost.
* low conductivity & less corona loss.

DISADVANTAGES :

* Low tensile strength.
* Large area.

TYPES OF ALUMINIUM CONDUCTOR :

* AAC – all- aluminium conductor.
* AAAC – all- aluminium alloy conductor.
* ACSR – aluminium conductor steel reinforced.
* ACAR – aluminium conductor alloy reinforced.

NOTE : Aluminium conductor is reinforced with steel to increase the tensile strength.

CLASSIFICATION OF CONDUCTORS:

i) Stranded conductors.
ii) Bundled conductors.

i) STRANDED CONDUCTORS:

* They are known as composite conductor.
* They compose of two or more elements (or) strands electrically in parallel.
* Normally stranded conductors are used for overhead transmission systems.

ACSR, ACAR → are of stranded conductors.

ADVANTAGES:

* Corona losses are reduced due to larger diameter.

ii) BUNDLED CONDUCTORS:

* Made up of two or more subconductor forming a single phase conductor.
* Used for ↑↑er voltage power transmission

ADVANTAGES:

*

CLASSIFICATION OF CONDUCTORS :-

i) Stranded conductor.
ii) Bundled conductor.

i) STRANDED CONDUCTOR :

* Also known as composite conductor.
* Composed of two or more or strands electrically in parallel.
* Used for overhead transmission system.
* ACAR, ACSR are stranded conductors.

ADVANTAGES :

* Stranded conductors have larger diameter & hence corona losses are reduced.

ii) BUNDLED CONDUCTOR :

* Made up of two or more subconductors forming a single phase conductor.
* Used for power transmission at higher voltage.

ADVANTAGES :-

i) Reduced reactance :-

$$L = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

D_s ↑↑es on bundling & L ↓↓es & X_L ↓↓es.

ii) Reduced surge impedance :

$$\text{Surge impedance} = \sqrt{\frac{L}{C}}$$

* since bundling ↓↓es L, surge impedance ↓↓es.

iii) Reduced voltage gradient :

UNIT - III

PERFORMANCE OF OVERHEAD LINES:

* Performance of overhead line is determined from voltage drop, line losses & efficiency calculated using R, L & C constants.

CLASSIFICATION OF OVERHEAD TRANSMISSION LINE:

Based upon the distribution of capacitance it is classified into.

i) SHORT TRANSMISSION LINE:

* length is upto 80km.
* line voltage < 20kv.
* Capacitance effects are neglected.

ii) MEDIUM TRANSMISSION LINE:

* length 80 to 240km
* line voltage > 20kv but < 100kv.
* Capacitance is lumped in form of condensors & shunted across the line at one or more points.

iii) LONG TRANSMISSION LINE:

* length > 240km.
* line vge > 100kv.
* Capacitance is distributed uniformly throughout the length of the line.

IMPORTANT TERMS FOR DETERMINING THE PERFORMANCE:

i) VOLTAGE REGULATION:-

$$\text{Vge regulation} = \frac{(\text{Receiving end Voltage } V_R)_{\text{at no load}} - (\text{Receiving end Vge } V_R)_{\text{full load}}}{\text{Receiving end Vge at full load}}$$

$$\% \text{ Vge regulation} = \left[\frac{V_S - V_R}{V_R}\right] \times 100$$

* Vge regulation should be low for better operation

ii) TRANSMISSION EFFICIENCY: η_T

$$\% \eta_T = \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100.$$

$$\% \eta_T = \frac{V_R I_R \cos\phi_R}{V_s I_s \cos\phi_s} \times 100$$

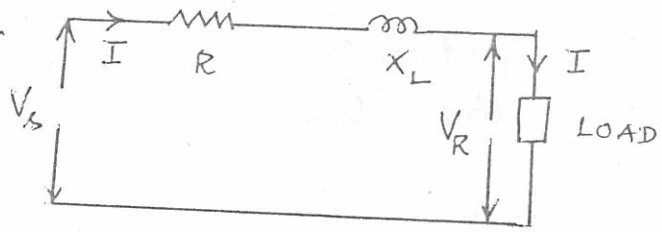
iii) Line Losses:

Line losses $= I^2R$ (1-φ line)

Line losses $= 3I^2R$ (3-φ line)

PERFORMANCE OF 1-φ SHORT TRANSMISSION LINE:-

EQUIVALENT CIRCUIT:-

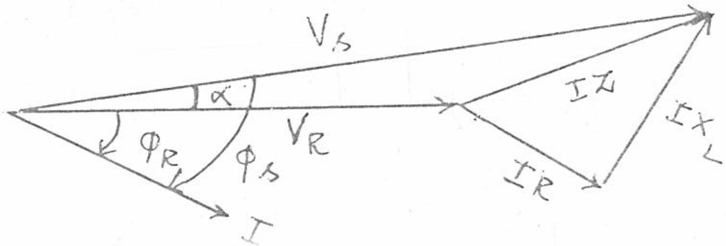


* CAPACITANCE IS NEGLECTED.

* V_s → sending end voltage/phase
* V_R → receiving end voltage/phase
* R → loop resistance (for both conductors)
* X_L → loop reactance
* I → load current lags V_R by an angle ϕ_R.
* $\cos\phi_R$ → receiving end power factor (lagging)

PHASOR DIAGRAM:

* V_R is taken as reference vector.



* Angle b/w V_s & V_R is α
* I lags V_s by an angle ϕ_s

From phasor diagram.

$$\vec{V_R} = V_R + j0 \quad , \quad \vec{Z} = R + jX_L$$

$$\vec{I} = \vec{I} \angle -\phi_R$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$$

$$\vec{V_s} = \vec{V_R} + \vec{I}\vec{Z}$$

$$\vec{V_s} = [V_R + j0] + I[\cos\phi_R - j\sin\phi_R][R + jX_L]$$

$$\vec{V_s} = [V_R + IR\cos\phi_R + IX_L\sin\phi_R] + j[IX_L\cos\phi_R - IR\sin\phi_R]$$

$$V_s = \sqrt{(V_R + IR\cos\phi_R + IX_L\sin\phi_R)^2 + (IX_L\cos\phi_R - IR\sin\phi_R)^2}$$

($IX_L\cos\phi_R - IR\sin\phi_R$ → very small value)

$$\therefore V_s = \sqrt{[V_R + IR\cos\phi_R + IX_L\sin\phi_R]^2}$$

$$\boxed{V_s = V_R + IR\cos\phi_R + IX_L\sin\phi_R.}$$

The above formula is applicable only for lagging power factors.

$$\%\ \text{Vge regulation} = \frac{V_s - V_R}{V_R} \times 100$$

$$= \left[\frac{V_R + IR\cos\phi_R + IX_L\sin\phi_R - V_R}{V_R}\right] \times 100$$

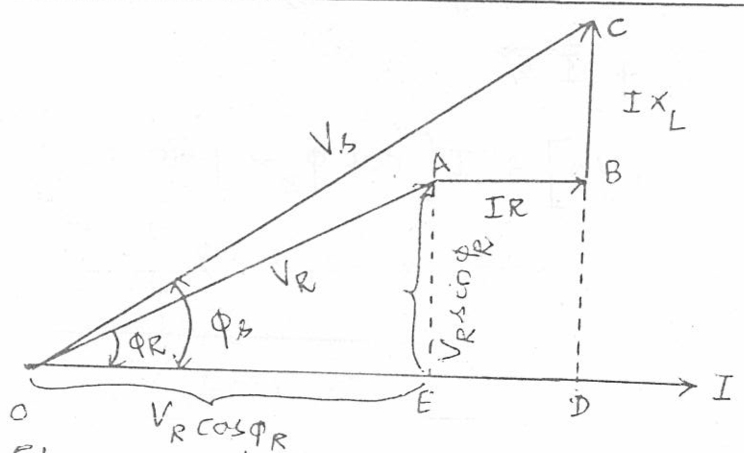
$$\left.\begin{array}{l}\%\ \text{Vge regulation} \\ \text{(lagging power factor)}\end{array}\right\} = \left[\frac{IR\cos\phi_R + IX_L\sin\phi_R}{V_R}\right] \times 100$$

TRANSMISSION η :-

$$\%\ \eta = \frac{\text{Power delivered}}{\text{power delivered} + \text{losses}} \times 100.$$

$$\%\ \eta = \left[\frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I^2 R}\right] \times 100$$

PHASOR DIAGRAM WITH CURRENT AS REFERENCE VECTOR:



From right angled $\triangle$ ODC

$$(OC)^2 = (OD)^2 + (DC)^2$$

$$V_s^2 = (OE + ED)^2 + (DB + BC)^2$$

$$V_s^2 = (V_R\cos\phi_R + IR)^2 + (V_R\sin\phi_R + IX_L)^2$$

$$V_s = \sqrt{(V_R\cos\phi_R + IR)^2 + (V_R\sin\phi_R + IX_L)^2}$$

i) % Vge regulation = $\frac{V_s - V_R}{V_R} \times 100$.

ii) Sending end power factor = $\cos\phi_s = \frac{OD}{OC}$

$= \frac{V_R \cos\phi_R + IR}{V_s}$.

iii) % η = $\frac{\text{Power delivered}}{\text{Power delivered} + \text{losses}} \times 100$

$$\% \eta = \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I^2 R} \times 100$$

1. A single phase overhead transmission line delivers 1100 kW at 33 kV at 0.8 pf lagging. The total resistance & inductive reactance of the line are 10Ω & 15Ω respectively. Determine i) sending end Vge ii) sending end power factor iii) η.

GIVEN :-

i) $P_R = 1100\,kW = 1100 \times 10^3\,W$

ii) $V_R = 33\,kV = 33 \times 10^3\,V$.

iii) $\cos\phi_R = 0.8$ pf (lagging) ; $\phi_R = \cos^{-1}(0.8) = 36.87°$

iv) $R = 10\Omega$ & $X_L = 15\Omega$.

REQUIRED:

i) V_s ??? ii) $\cos\phi_s$ iii) η

SOLUTION:

* $\vec{V_R} = V_R + j0 = 33000V$.

* $I = I_R = \dfrac{P_R}{V_R \cos\phi_R} = \dfrac{1100 \times 10^3}{33 \times 10^3 \times 0.8} = 41.67A$.

* $\vec{I} = I(\cos\phi_R - j\sin\phi_R)$

$\vec{I} = 41.67(0.8 - j0.6)$

$\vec{I} = 33.33 - j25$.

i) sending end voltage $\vec{V_s} = \vec{V_R} + \vec{I}\vec{Z}$

$\vec{V_s} = 33000 + (33.33 - j25)($

$= 33000 + (33.33 - j25)$ $+375$.

$\vec{V_s} = 33708.3 + j250$

$|V_s| = \sqrt{(33708.3)^2 + (250)^2} = 33,709V$.

ii) $\phi_s = \phi_R + \alpha \rightarrow$ (phasor diagram)

α = angle b/w V_s & V_R (phasor diagram)

$V_s = 33708.3 + j250$

$\alpha = \tan^{-1}\left(\dfrac{250}{33708.3}\right) = 0.4249$.

$\phi_R = 36.87°$

$\phi_s = \phi_R + \alpha = 36.87° + 0.4249$

$\phi_s = 37.29°$

$\boxed{\cos\phi_s = 0.7955}$ (lagging).

iii) $\% \eta = \dfrac{\text{Power delivered}}{\text{Power delivered + line losses}} \times 100$.

$$\% \eta = \frac{1100 \times 10^3}{1100 \times 10^3 + (41.67)^2 \times 100} \times 100.$$

$$\% \eta = \frac{1100 \times 10^3}{} \times 100$$

$$\boxed{\% \eta = 98.44 \%}$$

2. A 3-φ line delivers 3600kW at a pf 0.8 lagging on load. If the sending end voltage is 33kV, determine i) receiving end voltage ii) line current iii) η_{trans}. The resistance reactance of each conductor are 5.31Ω & 5.54Ω respectively.

GIVEN :-

i) $R = 5.31\Omega$

ii) $X_L = 5.54\Omega$.

iii) $P_R = 3600\,kW = 3600 \times 10^3\,W$

iv) $\cos\phi_R = 0.8 \quad \therefore \phi_R = 36.87°$.

v) 3-φ system.

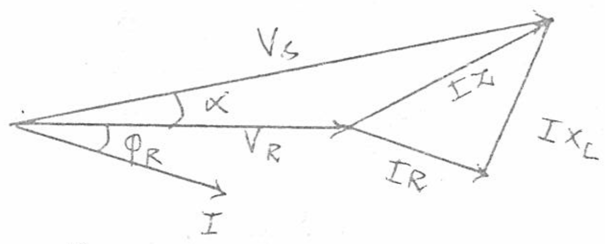
vi) $V_s = 33000$ (Line)

$$= \frac{33000}{\sqrt{3}} = 19{,}052V \text{ (phase)}$$

REQUIRED :

i) V_R ii) line current iii) η

SOLUTION



PHASOR DIAGRAM

From phasor diagram

$$V_s = V_R + IR\cos\phi_R + IX_L\sin\phi_R \quad \text{— ①}$$

$$I_R = I = \frac{P_R}{3 \times V_R \cos\phi_R} = \frac{3600 \times 10^3}{3 \times V_R \times 0.8}$$

$$I = \frac{15 \times 10^5}{V_R} \quad \text{— ②}$$

sub ② in ①

$$19{,}052 = V_R + \left[\frac{15 \times 10^5}{V_R} \times 5.31 \times 0.8\right] + \left[\frac{15 \times 10^5}{V_R} \times 5.54 \times\right.$$

$$V_R^2 - 19{,}052 V_R + 1{,}1358000 = 0.$$

$$V_R = 18435V \text{ or } 661V.$$

V_R can be 18435V only or not 661V.

$$\therefore \boxed{V_R = 18{,}435V}$$

$\therefore$ ② $\Rightarrow$

$$I = \frac{15 \times 10^5}{V_R}$$

$$I = \frac{15 \times 10^5}{18{,}435} = 81.36A$$

$$\boxed{I = 81.36\,A}$$

$$\eta = \frac{P_R}{P_R + \text{line losses}} \times 100$$

$$= \frac{3600 \times 10^3}{3600 \times 10^3 + (3 \times 81.36^2 \times 5.31)} \times 100$$

$3I^2R$.

$$\%\eta = \frac{3600 \times 10^3}{3600 \times 10^3 + 105447.8} \times 100$$

$$\boxed{\eta = 97.15\%}$$

3. A 3-φ, 50Hz, 16km long overhead line supplies 1000kW at 11kV, 0.8pf lagging. The line resistance is 0.03Ω/phase/km & line inductance is 0.7mH/phase/km. Calculate i) the sending end voltage ii) Vge regulation iii) $\eta_{trans.}$

GIVEN:-

i) 3-φ & l = 16km.

ii) f = 50 Hz.

iii) P_R = 1000kW = 1000×10^3 W.

iv) V_R = 11kV

$= \frac{11 \times 10^3}{\sqrt{3}} = 6350.8$ V (phase)

v) $\cos\phi_R = 0.8$ ∴ $\phi_R = 36.87°$

vi) R/phase/km = 0.03Ω

vii) L/phase/km = 0.7mH = 0.7×10^{-3} H.

REQUIRED:

i) V_s ii) Vge regulation iii) η

SOLUTION:-

R/phase = 0.03 × 16 = 0.48Ω.

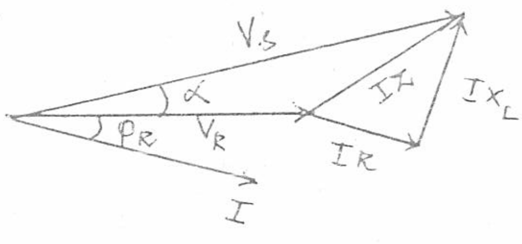
X_L/phase = $2\pi f L$ = $2 \times \pi \times 50 \times 0.7 \times 10^{-3} \times 16$.

= 3.518 Ω.

$$I_R = I = \frac{P_R}{3 \times V_R \times \cos\phi_R} = \frac{1000 \times 10^3}{3 \times 6350.8 \times 0.8}$$

I = 65.50 A

PHASOR DIAGRAM:-



From phasor diagram:

$$V_s = V_R + IR\cos\phi_R + IX_L\sin\phi_R$$

$$= 6350.8 + (65.50 \times 0.48 \times 0.8) + (55.5 \times 8.5 \times 0.6)$$

$$\boxed{V_s = 6514.2\ V}$$

ii) $$\%\ Vge\ regulation = \left[\frac{V_s - V_R}{V_R}\right] \times 100$$

$$= \frac{6514.2 - 6350.8}{6350.8}$$

$$= 2.572\ \%$$

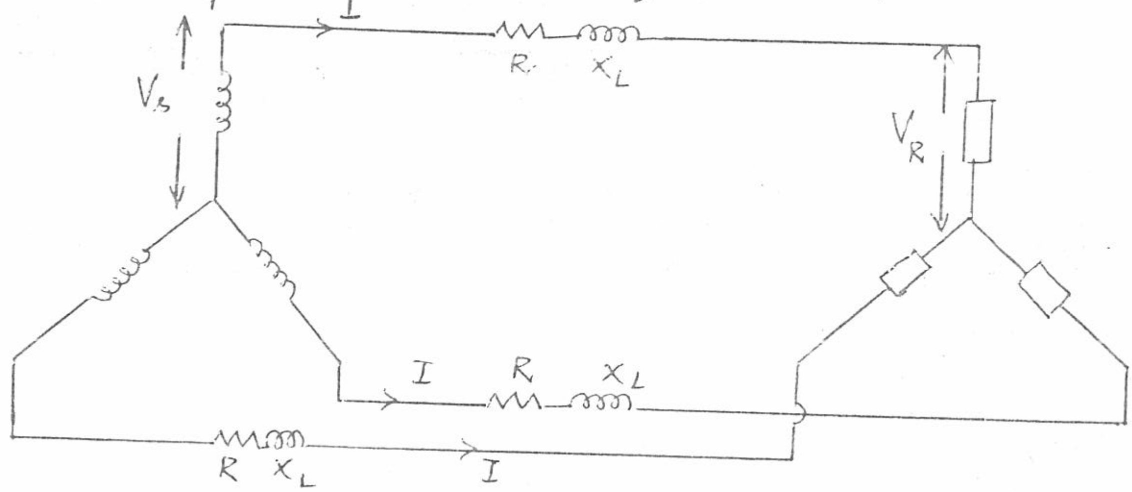
iii) $$\%\ \eta = \frac{P_R}{P_R + 3I^2R} \times 100$$

$$= \frac{1000 \times 10^3}{1000 \times 10^3 + 3 \times (65.5)^2 \times 0.48} \times 100$$

$$\boxed{\%\ \eta = 99.38\%}$$

3-φ SHORT TRANSMISSION LINE:

* Consists of 3 single phase units, each wire transmitting one third of total power.



EFFECT OF LOAD POWER FACTOR ON REGULATION & η IN A SHORT TRANSMISSION LINE:

1. EFFECT ON REGULATION:

% Vge regulation for lagging p.f $= \dfrac{IR\cos\phi_R + IX_L\sin\phi_R}{V_R} \times 100$

* For lagging p.f, $IR\cos\phi_R > IX_L\sin\phi_R$ & vge regulation is +ve.
* For a given V_R & I, Vge regulation ↑↑es with decrease in p.f

% Vge regulation for leading p.f $= \dfrac{IR\cos\phi_R - IX_L\sin\phi_R}{V_R} \times 100$

* For leading p.f, $IX_L\sin\phi_R > IR\cos\phi_R$ & vge regulation is −ve.
* For a given V_R & I, Vge regulation ↓↓es with ↓↓ in power factor.

2. EFFECT ON $\eta_{TRANSMISSION}$:-

$$P_R = V_R I_R \cos\phi_R \qquad (1-\phi)$$

$$P_R = 3V_R I_R \sin\phi_R \qquad (3-\phi)$$

$$I_R = \frac{P_R}{V_R\cos\phi_R} \quad \text{or} \quad \frac{P_R}{3V_R\cos\phi_R}.$$

$$I_R \propto \frac{1}{\cos\phi_R}$$

∴ When $\cos\phi_R$ ↓↓es, I_R ↑↑es, I^2R losses ↑↑es & η ↓↓es.

(18)

MEDIUM TRANSMISSION LINE:-

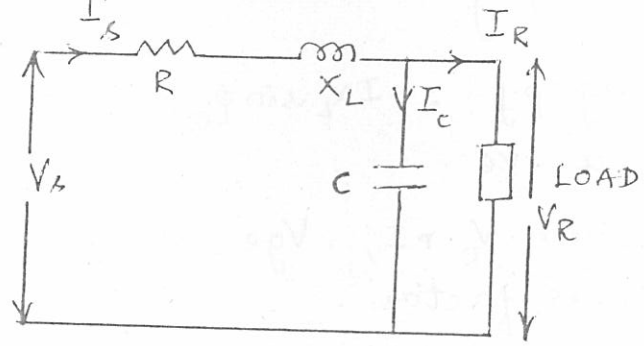
* Line capacitance is assumed to be concentrated in the form of condensors shunted across the line at one or more points for simpler calculations.

PERFORMANCE ANALYSIS IS DONE BY

i) End- condenser method
ii) Nominal T- method
iii) Nominal π- method.

END CONDENSER METHOD OF PERFORMANCE ANALYSIS

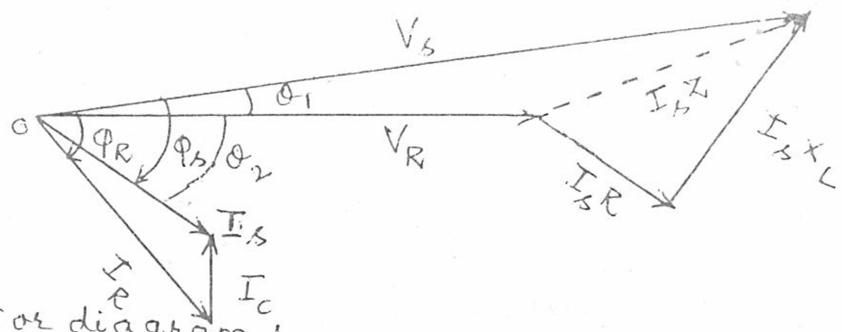
* Capacitance is assumed to be concentrated at receiving end.



* I_R → load current / phase.
* V_s → sending end voltage / phase.
* V_R → receiving end voltage / phase.
* R → resistance / phase.
* X_L → inductive reactance / phase
* C → capacitance / phase.
* I_s → sending end current / phase.
* I_c → current through capacitor.
* $\cos\phi_R$ → receiving end power factor (lagging)
* Z → impedance / phase = $R + jX_L$

PHASOR DIAGRAM:

* V_R is taken as reference vector



From vector diagram :-

Pg (83)

PROCEDURE FOR SKETCHING VECTOR DIAGRAM:

Receiving end voltage (V_R) is taken as reference vector.

Draw $\vec{I_R}$ downwards lagging $\vec{V_R}$ by an angle ϕ_R.

Capacitance current I_C leads V_R by an angle 90° & hence $\vec{I_C}$ is drawn ⊥r to $\vec{V_R}$ & added to $\vec{I_R}$

* Sending end current vector $\vec{I_s} = \vec{I_R} + \vec{I_C}$
* Resistive drop vector $\vec{I_s}\vec{R}$ is drawn ||^l to $\vec{I_s}$ & added to $\vec{V_R}$.
* Reactive drop vector $\vec{I_s} X_L$ is drawn ⊥r to $\vec{I_s}$ & added to $\vec{I_s} R$.
* Addition of $\vec{I_s} R + \vec{I_s} X_L$ gives $\vec{I_s}\vec{Z}$
* Summation of $\vec{V_R}$ & $\vec{I_s} X_L$ gives $\vec{V_s}$
* Angle b/w $\vec{V_s}$ & $\vec{V_R}$ is θ_1
* Angle b/w $\vec{V_R}$ & $\vec{I_R}$ is θ_2.
* ϕ_s be the angle b/w $\vec{V_s}$ & $\vec{I_s}$

$$\% \eta = \frac{P_R}{P_R + \text{line losses}} \times 100$$

$$\% \eta = \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I_s^2 R} \times 100$$

LIMITATIONS OF END CONDENSOR METHOD:

i) Since the distributed capacitance is treated as lumped capacitance we have 10% error

ii) This method overestimates the effect of line capacitance.

1. **A medium single phase transmission line** 100km long has the following constants: resistance/km = 0.25Ω, reactance/km = 0.8Ω, susceptance/km = 14×10^{-6} siemen, receiving end voltage = 66,000V. Assuming that the total capacitance of the line is localised at the receiving end alone, determine i) sending end current ii) sending end voltage iii) regulation iv) supply power factor. The line is delivering 15,000kW at 0.8 p.f lagging. Draw the phasor diagram.

GIVEN:-

i) l = 100km.

ii) r/km = 0.25Ω & X_L/km = 0.8Ω.

iii) Y/km = 14×10^{-6} siemen. ; Y = 14×10^{-4} siemen

iv) V_R = 66000V

v) P_R = 15000kW = 15000×10^3 W

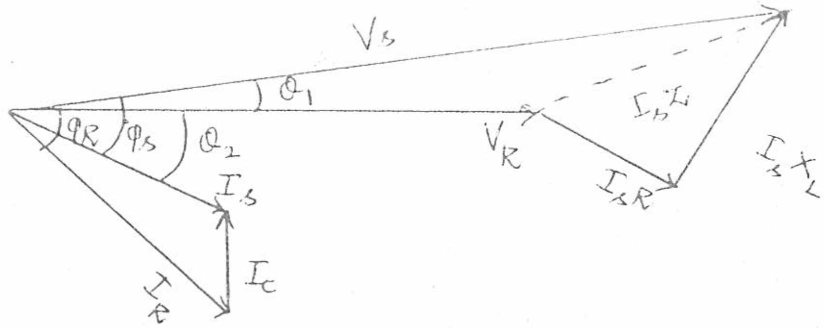
vi) $\cos\phi_R$ = 0.8 pf.

vii) Z = $R + jX_L$ = $0.25 + j0.8$ Ω/km = $(25 + j80)$ Ω.

REQUIRED:

i) I_s, ii) V_s iii) $\cos\phi_s$ & iv) Vge regulation.

SOLUTION:-



* $I_R = \dfrac{P_R}{V_R \cos\varphi_R}$

$= \dfrac{15000 \times 10^3}{66000 \times 0.8}$

$\boxed{I_R = 284\,A}$

* $\vec{V_R} = V_R + j0 = 66{,}000\,V.$

* $\vec{I_R} = I_R(\cos\varphi_R - j\sin\varphi_R)$

$= 284(0.8 - j0.6)$

* $\vec{I_R} = 227.2 - j170.$

* $\vec{I_s} = \vec{I_R} + \vec{I_c}$

$\vec{I_c} = \vec{V_R}\, j\omega C = \vec{V_R}\, jY.$

$= j \times 14 \times 10^{-4} \times 66000$

$\vec{I_c} = j92.4.$

* $\vec{I_s} = (227.2 - j170) + j92.4$

$\vec{I_s} = 227.2 - j78.$

* $|I_s| = \sqrt{227^2 + 78^2} = 240A.$

* $\vec{V_s} = \vec{V_R} + \vec{I_s}\vec{Z}$

$= 66000 + (227.2 - j78)(25 + j80)$

$$\vec{V_s} = 66{,}000 + \left[(240.02 \angle -18.96) \times (83.81 \angle 72.6)\right]$$

$$= 66{,}000 + \left[20116 \angle 53.64\right]$$

$$= 66{,}000 + \left[11925.9 + j\,16199.5\right]$$

$$\vec{V_s} = 77{,}925.9 + j\,16{,}199.5$$

$$|V_s| = \sqrt{(77925.9)^2 + (16{,}199.5)^2}$$

$$\boxed{|V_s| = 79{,}591\ V}$$

$$\%\ \text{Vge regulation} = \frac{V_s - V_R}{V_R} \times 100$$

$$= \frac{79591 - 66000}{66000} \times 100$$

$$\boxed{\%\ \text{Vge reg.} = 20.59\%}$$

To find ϕ_s:

$\phi_s = \theta_1 + \theta_2$ (from phasar diagram)

$\theta_1 \rightarrow$ angle b/w $\vec{V_R}$ & $\vec{I_s}$

$$\vec{I_s} = 227.2 - j78$$

$$\theta_1 = \tan^{-1}\left(\frac{-78}{227.2}\right) = -18.96^\circ$$

$$\boxed{\theta_1 = 18.96^\circ}\ \text{(lagging)}$$

$\theta_2 \rightarrow$ angle b/w $\vec{V_s}$ & $\vec{V_R}$

$$\vec{V_s} = 77{,}925.9 + j\,16{,}199.5$$

$$\theta_2 = \tan^{-1}\left(\frac{16199.5}{77925.9}\right) = 11.74^\circ$$

$$\boxed{\theta_2 = 11.74^\circ}$$

$\phi_s = \theta_1 + \theta_2$

$= 18.96^\circ + 11.74^\circ$

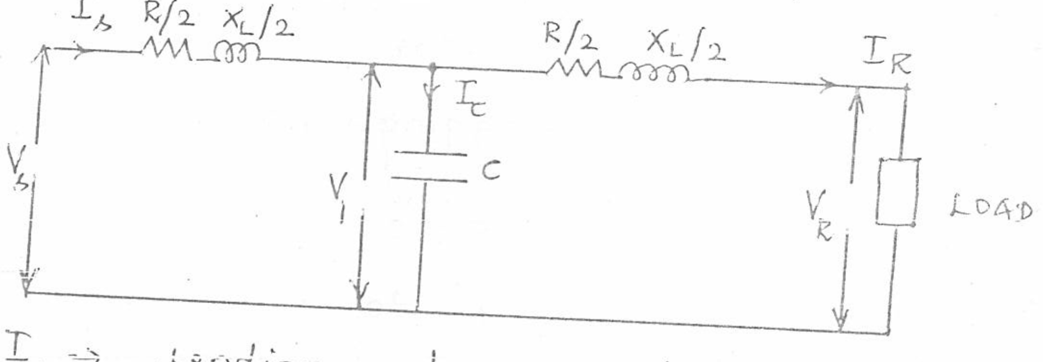
$\phi_s = 30.7^\circ$

$\cos\phi_s = \cos(30.7)^\circ$

$\boxed{\cos\phi_s = 0.8598}$ (lagging)

(ii) NOMINAL T- METHOD :-

* Capacitance is assumed to be concentrated at the middle point of the line & half the line resistance and reactance are lumped on its either side



Let I_s → sending end current/phase

R → resistance/phase

X_L → inductive reactance/phase.

I_R → receiving end current/phase.

C → capacitance/phase.

V_1 → voltage across capacitance/phase.

Z → $R + jX_L$ → impedance/phase.

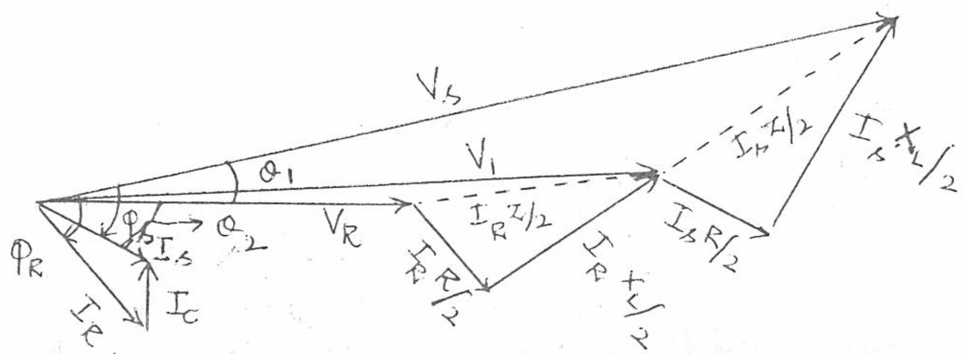
V_s → sending end voltage/phase.

V_R → receiving end voltage/phase.

$\cos\phi_R$ → receiving end power factor (lagging)

PHASOR DIAGRAM :-

* V_R is taken as reference vector



PROCEDURE FOR SKETCHING VECTOR DIAGRAM :

* Take $\vec{V}_R$ as reference vector.
* Draw $\vec{I}_R$ downwards lagging $\vec{V}_R$ by an angle ϕ_R
* Add $\vec{I}_c$ $\perp^r$ to $\vec{I}_R$ because current through capacitance leads $\vec{V}_R$ by 90°.
* $\vec{I}_s = \vec{I}_R + \vec{I}_c$
* Resistance drop vector $\vec{I}_R \frac{\vec{R}}{2}$ is drawn $\parallel^l$ to $\vec{I}_R$ & added to $\vec{V}_R$
* Reactive drop vector $\vec{I}_R X_L/2$ is drawn $\perp^r$ to $\vec{I}_R$ & added to $\vec{I}_R \frac{\vec{Z}}{2}$ vector.
* $\vec{I}_R \frac{\vec{R}}{2} + \vec{I}_R \frac{\vec{X}}{2}$ gives $\vec{I}_R \frac{\vec{Z}}{2}$ vector.
* Addition of $\vec{V}_R$ with $\vec{I}_R \frac{\vec{Z}}{2}$ vector gives $\vec{V}_1$ vector (vge across capacitance).
* Resistive drop vector $\vec{I}_s \frac{\vec{R}}{2}$ is drawn $\parallel^l$ to $\vec{I}_s$ & added to $\vec{V}_1$
* Reactance drop vector $\vec{I}_s \frac{\vec{X}_L}{2}$ is drawn $\perp^r$ to $\vec{I}_s$ & added to $\vec{I}_s \frac{\vec{R}}{2}$.

$$\therefore \vec{V_s} = \vec{I_R}\frac{\vec{Z}}{2} + \vec{V_R} + \vec{I_s}\frac{\vec{Z}}{2}$$

1. A 3-φ, 50 Hz overhead transmission line, 100 km long has the following constants :- r/km/phase = 0.1 Ω, X_L/km/phase = 0.2 Ω, capacitive susceptance/km/phase = 0.04×10^{-4} siemen. Determine i) sending end current ii) sending end vge iii) sending end power factor iv) η_{trans} when supplying a load of 10,000 kW at 66 kV, p.f 0.8 lagging. Use nominal T-method.

GIVEN:- * l = 100 km

* r/phase = 0.1×100 = 10 Ω.
* X_L/phase = 0.2×100 = 20 Ω.
* Y/phase = $0.04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ siemen
* P_R = 10,000 kW = $10{,}000 \times 10^3$ W.
* V_R = 66 kV line = $\frac{66 \times 10^3}{\sqrt{3}}$ phase.

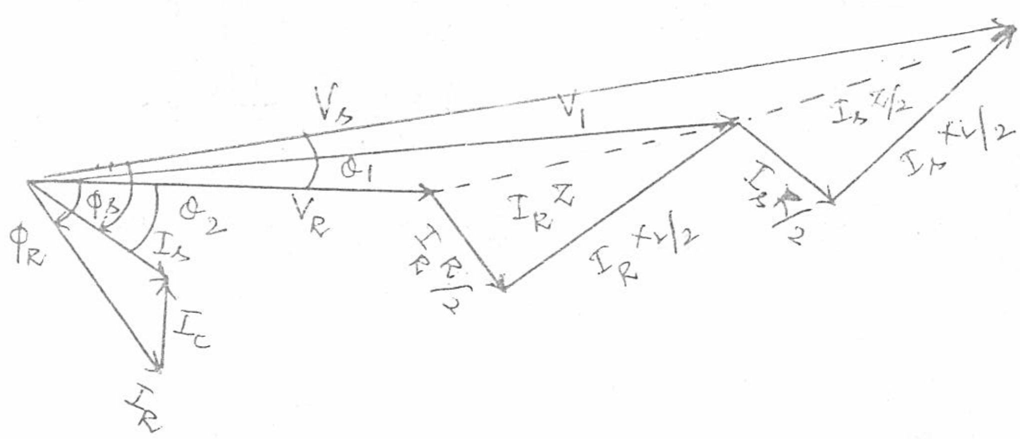
V_R = 38105 V.

* $\cos\phi_R = 0.8$, $Z = 10 + j20$ Ω/phase.

REQUIRED:

i) I_s ii) V_s iii) $\cos\phi_s$ iv) η_{trans}.

SOLUTION:



$$* \quad I_R = \frac{P_R}{3 \times V_R \cos\phi_R}$$

$$= \frac{10000 \times 10^3}{3 \times 38105 \times 0.8} = 109.3$$

$$\boxed{I_R = 109.3\ A}$$

$$* \quad \vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$$

$$= 109.3(0.8 - j0.6)$$

$$\vec{I_R} = 87.44 - j65.58\ A.$$

$$* \text{ Voltage across } C, \ \vec{V_1} = \vec{V_R} + \vec{I_R}\frac{Z}{2}$$

$$= 38105 + (87.44 - j65.58)(5 + j10)$$

$$= 38105 + [(109.26 \angle -36.88) * (11.18 \angle 63.4$$

$$= 38105 + [1221.5 \angle 26.55]$$

$$= 38105 + (1092.68 + j545.9)$$

$$\boxed{\vec{V_1} = 39{,}197 + j545.9}$$

$$* \quad \vec{I_s} = \vec{I_R} + \vec{I_c}$$

$$\vec{I_c} = jY\vec{V_1} = j \times 4 \times 10^{-4}[39{,}197 + j545.9]$$

$$= [4 \times 10^{-4} \angle 90^\circ][39200.8 \angle 0.7979]$$

$$= 15.680 \angle 90.7979$$

$$\vec{I_c} = (-0.218 + j15.6)\ A$$

$$* \quad \vec{I_s} = \vec{I_R} + \vec{I_c}$$

$$= (87.44 - j65.58) + (-0.218 + j15.6)$$

$$* \quad \vec{I_s} = (87.22 - j49.8)\ A$$

$$|I_s| = 100.4A$$

* $\vec{V_s} = \vec{V_1} + \vec{I_s}\,\frac{\vec{Z}}{2}$

$= (39,197 + j545.9) + [(87.22 - j49.8)\ (5 + j10)]$

$= (39,197 + j545.9) + [(100.43\angle -29.7) \times (11.18\angle 63.4)]$

$= (39,197 + j545.9) + [1122.8\angle 33.7]$

$= (39,197 + j545.9) + (934.1 + j622.97)$

$\vec{V_s} = 40131 + j1168.9\,V$

$|V_s| = \sqrt{(40131)^2 + (1168.9)^2} = 40148\,V$

$\phi_s = \theta_1 + \theta_2$

$\theta_1 \rightarrow$ angle b/w $\vec{V_R}$ & $\vec{V_s}$

* $\vec{V_s} = 40131 + j1168.9$

$\theta_1 = \tan^{-1}\left(\frac{1168.9}{40131}\right) = 1.67°$

$\theta_2 \rightarrow$ angle b/w $\vec{V_R}$ & $\vec{I_s}$

* $\vec{I_s} = 87.22 - j49.8$

$\theta_2 = \tan^{-1}\left(\frac{-49.8}{87.22}\right) = -29.72$

$\theta_2 = 29.72$ (lagging)

* $\phi_s = 1.67° + 29.72 = 31.39$.

* $\cos\phi_s = \cos(31.39)$

$\boxed{\cos\phi_s = 0.853}$ lagging

$$* \quad \eta = \frac{\text{Power delivered}}{\text{Sending end power}} \times 100$$

$$= \frac{10,000 \times 10^3}{3 V_s I_s \cos\phi_s} \times 100$$

$$= \frac{10,000 \times 10^3}{3 \times 40148 \times 100.4 \times 0.853} \times 100$$

$$\boxed{\eta = 96.94\ \%}$$

A 3-ϕ, 50Hz transmission line 100km long delivers 20MW at 0.9pf lagging at 110kv. The resistance & reactance of the line per phase per km are 0.2Ω & 0.4Ω respectively while the capacitance admittance is 2.5×10^{-6} siemen/km/phase. Use nominal T-method to analyse the performance.

GIVEN:-

i) $l = 100km$, 3-ϕ

ii) $P_R = 20MW = 20 \times 10^6 W$.

iii) $V_R = 110kv$ (line)

$$= \frac{110 \times 10^3}{\sqrt{3}} = 63,508.5\ V \text{ (phase)}$$

iv) R/phase/km = 0.2 Ω/km

R/phase = 0.2 × 100 = 20Ω.

v) X_L/phase/km = 0.4 Ω/km

X_L/phase = 0.4 × 100 = 40Ω.

vi) Z/phase = $R + jX_L$ = $(20 + j40)\Omega$

vii) Y/km/phase = 2.5×10^{-6} siemen/km/phase.

= $2.5 \times 10^{-6} \times 100$ siemen/phase.

$Y/\text{phase} = 2.5 \times 10^{-4}\ S/\text{phase}$

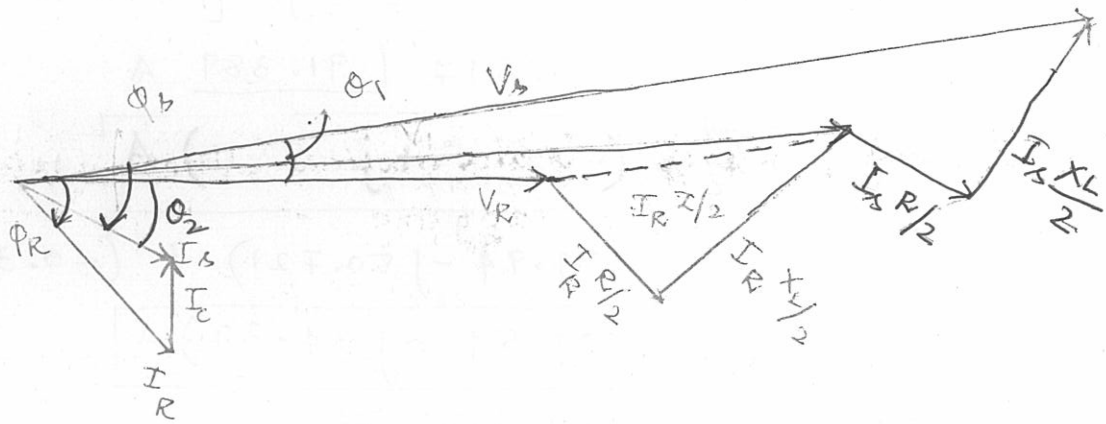
$\cos\phi_R = 0.9$ (lagging) ; $\sin\phi_R = 0.435$

REQUIRED :-

i) V_s ?? ii) line losses iii) η

SOLUTION :-

PHASOR DIAGRAM :-



* $I_R = \dfrac{P_R}{3\, V_R \cos\phi_R} = \dfrac{20 \times 10^6}{3 \times 63508.5 \times 0.9}$

$$\boxed{I_R = 116.6\ A}$$

* $\vec{V_R} = V_R + j0 = 63508.5\,V$

* $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

$= 116.6(0.9 - j0.435)$

$\vec{I_R} = 104.94 - j50.721$

* $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$

$= (63508.5) + (104.94 - j50.721)\dfrac{(20 + j40)}{2}$

$= (63508.5) + (104.94 - j50.721)(10 + j20)$

$= (63508.5) + [(116.5\angle{-25.79}) \times (22.36\angle{63.43})]$

$= 63508.5 + [2604.94\angle{37.64}]$

$$= 63508.5 + (2062.75 + j1590.8)$$

$$\vec{V}_1 = (65,571.2 + j1590.8)\ V$$

* $\vec{I}_s = \vec{I}_R + \vec{I}_c$

* $\vec{I}_c = j\omega c \vec{V}_1 = jY\vec{V}_1$

$$= j \times 2.5\times10^{-4} \times (65,571.2 + j1590.8)$$

$$= [2.5\times10^{-4} \angle 90] \times [65,590.4 \angle 1.389]$$

$$\vec{I}_c = 16.397 \angle 91.389\ A$$

$$\vec{I}_c = (\quad + j \quad)\ A$$

* $\vec{I}_s = (104.94 - j50.721) + (-0.397 + j16.392)$

$$\boxed{\vec{I}_s = (104.54 - j34.32)\ A}$$

* $\vec{V}_s = \vec{V}_1 + \vec{I}_s \vec{Z}/2$

$$= (65,571.2 + j1590.8) + \left[(104.54 - j34.32)\frac{(20+j}{2}\right.$$

$$= [65,571.2 + j1590.8] + [(110.02 \angle -18.17)(22.36 \angle 63.4$$

$$= [65,571.2 + j1590.8] + [2460 \angle 45.26]$$

$$= [65,571.2 + j1590.8] + [1731.57 + j1747.35]$$

$$\vec{V}_s = 67,302.7 + j3338.15$$

$$|V_s| = \sqrt{(67,302.7)^2 + (3338.15)^2}$$

$$\boxed{|V_s| = 67,385.4\ V}$$

* Total line losses for 3 phases $\} = 3I_s^2 \frac{R}{2} + 3I_R^2 \frac{R}{2}$

$$|I_s| = \sqrt{(104.54)^2 + (34.32)^2}$$

$$|I_s| = 110.02\ A$$

* Line losses $= \left(3 \times 110^2 \times \frac{20}{2}\right) + \left(3 \times 116.6^2 \times \frac{20}{2}\right)$

$= 3,63,000 + 4,07,866.8$

Losses $= 770.866 \times 10^3$ W.

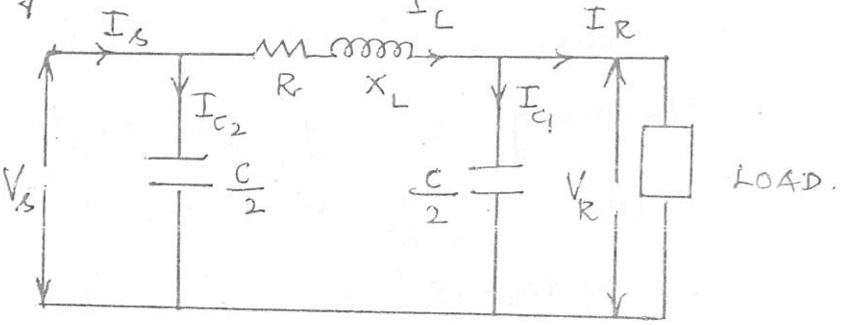
* $\% \eta = \dfrac{P_R}{P_R + \text{losses}} \times 100$

$= \dfrac{20 \times 10^6}{20 \times 10^6 + 770.866 \times 10^3} \times 100$

$\boxed{\% \eta = 96.29\%}$

(iii) NOMINAL π - METHOD OF PERFORMANCE ANALYSIS FOR A MEDIUM TRANSMISSION LINE :-

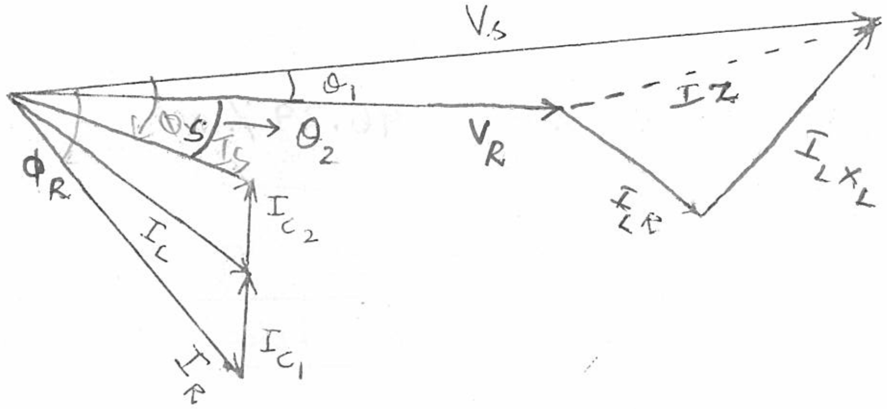
* Capacitance of each conductor is divided into two halves, of which one half is lumped at the sending end and the other half at the receiving end.



* Let V_R → receiving end voltage /phase.
* V_s → sending end voltage /phase.
* I_s → sending end current /phase.
* Z → impedance /phase $= R + jX_L \ \Omega$.
* I_L → current through impedance /phase
* I_R → receiving end current /phase.

* I_{c_1} → current through one capacitance/phase
* I_{c_2} → current through another capacitance/phase
* $\cos\phi_R$ → receiving end lagging power factor

PHASOR DIAGRAM :-



STEPS INVOLVED :-

* Receiving end voltage V_R is taken as reference vector.
* Receiving end current vector $\vec{I_R}$ is drawn downwards lagging $\vec{V_R}$ by an angle $\vec{V_R}$.
* $\vec{I_{c_1}}$ vector leads $\vec{V_R}$ by an angle 90° because the capacitive current leads vge.
  ∴ Draw $\vec{I_{c_1}}$ ⊥r to $\vec{V_R}$ & add to $\vec{I_R}$.
* Addition of $\vec{I_R}$ & $\vec{I_{c_1}}$ gives $\vec{I_L}$
* lly Draw $\vec{I_{c_2}}$ vector ⊥r to $\vec{V_R}$ & add to $\vec{I_L}$.
* Addition of $\vec{I_L}$ & $\vec{I_{c_2}}$ vectors give $\vec{I_s}$.
* Resistance drop vector $\vec{I_L R}$ is drawn ||l to $\vec{I_L}$ & added to $\vec{V_R}$

* Reactance drop vector $\vec{I_L} X_L$ is drawn ⊥r to $\vec{I_L}$ & added to $\vec{I_L} R$.
* Addition of $\vec{I_L} R$ & $\vec{I_L} X_L$ gives $\vec{I_L} Z$.
* Addition of $\vec{V_R}$ & $\vec{I_L} Z$ vector gives $\vec{V_s}$
* $\vec{I_s}$ lags $\vec{V_s}$ by an angle ϕ_s.
* θ_1 be the angle b/w $\vec{V_s}$ & $\vec{V_R}$.
* θ_2 be the angle b/w $\vec{V_R}$ & $\vec{I_s}$
* $\phi_s = \phi_1 + \phi_2$

ANALYSIS :-

* $\vec{V_R}$ is taken as reference vector.
* $\vec{V_R} = V_R + j0.$
* $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$ $\quad\quad \omega = 2\pi f$.
* $\vec{I_{C_1}} = j\omega(C/2)\vec{V_R} = j\pi f C \vec{V_R}$
* $\vec{I_L} = \vec{I_R} + \vec{I_{C_1}}$
* $\vec{V_S} = \vec{V_R} + \vec{I_L} Z$

  $\vec{V_S} = \vec{V_R} + \vec{I_L}(R + jX_L)$
* $\vec{I_{C_2}} = j\omega C/2 \, \vec{V_s} = j\pi f C \vec{V_s}$
* $\vec{I_S} = \vec{I_L} + \vec{I_{C_2}}$

1. A 100km long, 3-φ, 50Hz transmission line has the following line constants R/phase/km = 0.1Ω, X_L/phase/km = 0.5Ω, Y/phase/km = 10×10^{-6}S. If the line supplies load of 20MW at 0.9pf lagging at 66kv at the receiving end, calculate by nominal π-method i) sending end power factor ii) regulation iii) transmission η.

GIVEN:-

* R/phase/km = 0.1Ω

$$R/\text{phase} = 0.1\times100 = 10\Omega$$

* X_L/phase/km = 0.5Ω

* $X_L/\text{phase} = 0.5\times100 = 50\Omega$

* $Z/\text{phase} = R+jX_L = 10+j50\Omega$

* Y/phase/km = 10×10^{-6}S

* $Y/\text{phase} = 10\times10^{-6}\times100$

$= 10\times10^{-4}$ Siemen

* $V_R = 66kv$ (line)

$$V_R = \frac{66\times10^3}{\sqrt{3}} = 38105.1V.$$

* $\cos\phi_R = 0.9$ (lagging) & $P_R = 20MW$

REQUIRED:

i) $\cos\phi_s$ ii) regulation iii) η

SOLUTION:-

* $\vec{V_R} = V_R + j0$

$\vec{V_R} = 38105.1V$

* $\vec{I}_R = I_R(\cos\phi_R - j\sin\phi_R)$

$= 195(0.9 - j\,0.435)$

$\boxed{\vec{I}_R = 175.5 - j\,84.83}$ A

* $\vec{I}_{C_1} = j\omega\frac{C}{2}\vec{V}_R = j\frac{Y}{2}\vec{V}_R$ (a) $j\frac{1}{\frac{X_C}{2}}\vec{V}_R$

$= j \times \frac{10\times10^{-4}}{2} \times 38105$ $j\frac{Y}{2}\vec{V}_R$

$\boxed{\vec{I}_{C_1} = 19.05j}$ A

* $\vec{I}_L = \vec{I}_R + \vec{I}_{C_1}$

$= (175.5 - j\,84.83) + 19.05j$

$\boxed{\vec{I}_L = (175.5 - 65.78j)\text{A}}$

* $\vec{V}_S = \vec{V}_R + \vec{I}_L\vec{Z}$

$= 38105 + (175.5 - 65.78j)(10 + j50)$

$= 38105 + [(187.42\angle{-20.5})(50.99\angle{78.69})]$

$= 38105 + [9556.5\angle{58.19}]$

$= 38105 + (5037.3 + j\,8121.1)$

* $\vec{V}_S = 43142.3 + j\,8121.2$

$|V_s| = \sqrt{(43142.3)^2 + (8121.2)^2}$

$\boxed{|V_s| = 43900\text{ V}}$

* $\vec{I}_{C_2} = j\omega\frac{C}{2}\vec{V}_s = j\frac{Y}{2}\vec{V}_s$

$= j \times \frac{10\times10^{-4}}{2} \times (43142.3 + j\,8121.2)$

$$= \left[5\times10^{-4}\angle 90^\circ\right]\left[43{,}900\angle 10.66\right]$$

$$= 21.95\angle 100.66$$

$$\vec{I}_{C_2} = (-4.06 + j\,21.57)\ A$$

$$\vec{I}_s = \vec{I}_L + \vec{I}_{C_2}$$

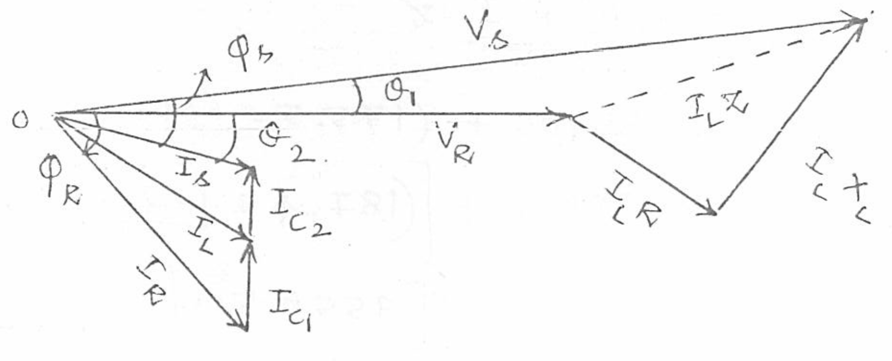
$$= (175.5 - j65.78) + (-4.06 + j\,21.57)$$

$$= (171.44 - j44.21)\ A$$

$$|I_s| = \sqrt{171.44^2 + 44.21^2}$$

$$\boxed{|I_s| = 177.04\ A}$$

PHASOR DIAGRAM :-



$$\phi_s = \theta_1 + \theta_2$$

$\theta_1 \rightarrow$ angle b/w $\vec{V_s}$ & $\vec{V_R}$

$$\therefore\ \vec{V_s} = 43142.3 + j\,8121.2$$

$$\theta_1 = \tan^{-1}\left(\frac{8121.2}{43142.3}\right) = 10.66^\circ$$

$\theta_2 \rightarrow$ angle b/w $\vec{V_R}$ & $\vec{I_s}$

$$\therefore\ \vec{I_s} = 171.44 - j\,44.21$$

$$\theta_2 = \tan^{-1}\left(\frac{-44.21}{171.44}\right) = -14.46^\circ$$

$\theta_2 = 14.46°$ (lagging)

$\phi_s = \theta_1 + \theta_2$

$= 10.66° + 14.46°$

$\phi_s = 25.12$

$\cos\phi_s = \cos(25.12)$

$$\boxed{\cos\phi_s = 0.9054}$$

$$\%\text{ Vge regulation} = \frac{V_s - V_R}{V_R} \times 100$$

$$= \left(\frac{43{,}900 - 38105.1}{38105.1}\right) \times 100$$

$$\boxed{\text{Vge Reg.} = 15.20\%}$$

$$\%\eta = \frac{\text{Receiving end power}}{\text{sending end power}} \times 100$$

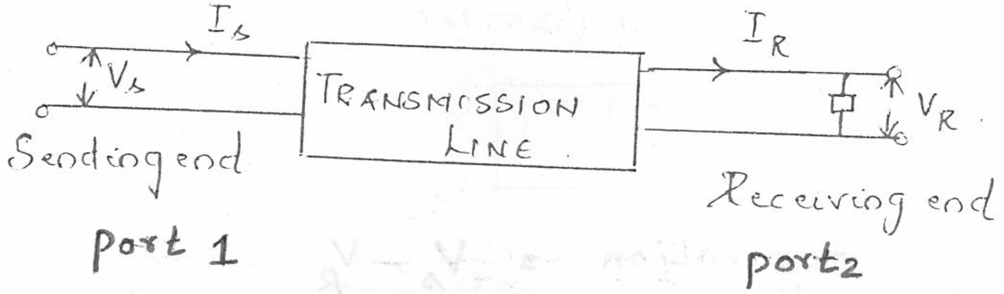
$$= \frac{20 \times 10^6}{3 V_s I_s \cos\phi_s} \times 100$$

$$= \frac{20 \times 10^6}{3 \times 43900 \times 177.04 \times 0.9054} \times 10$$

$$\boxed{\%\eta = 94.73\%}$$

Two Port Network Representation :-

* Every transmission line can be represented as a 2-port network in which the sending end forms one port and the receiving end forms another port.



* Two port n/w equ in terms of sending end Vge & current is

$$V_s = AV_r + BI_r \quad \text{--- ①}$$

$$I_s = CV_r + DI_r \quad \text{--- ②}.$$

where A, B, C & D are constants.

* In any two port network $\boxed{AD - BC = 1. \ \& \ A = D}$

Two Port Network Equation In Terms Of Receiving End Voltage & Current:

We know $V_s = AV_r + BI_r$ --- ①

$I_s = CV_r + DI_r$ --- ②.

x/y ① equ by C & ② equ by A.

$$CV_s = CAV_r + BCI_r \quad \text{--- ③}$$

$$AI_s = CAV_r + ADI_r \quad \text{--- ④}$$

④ − ③ ⇒ $AI_s - CV_s = (AD - BC) I_r$ --- ⑤

since $AD - BC = 1$

$\therefore$ equ (5) $\Rightarrow$ $AI_s - CV_s = I_r$ — (6)

xly (1) equ by D & equ (2) by B.

(1) equ $\Rightarrow$ $DV_s = ADV_r + BDI_r$ — (7)

(2) equ $\Rightarrow$ $BI_s = BCV_r + BDI_r$ — (8)

(7) - (8) $\Rightarrow$ $DV_s - BI_s = (AD - BC)V_r$

since $AD - BC = 1$, above equ becomes

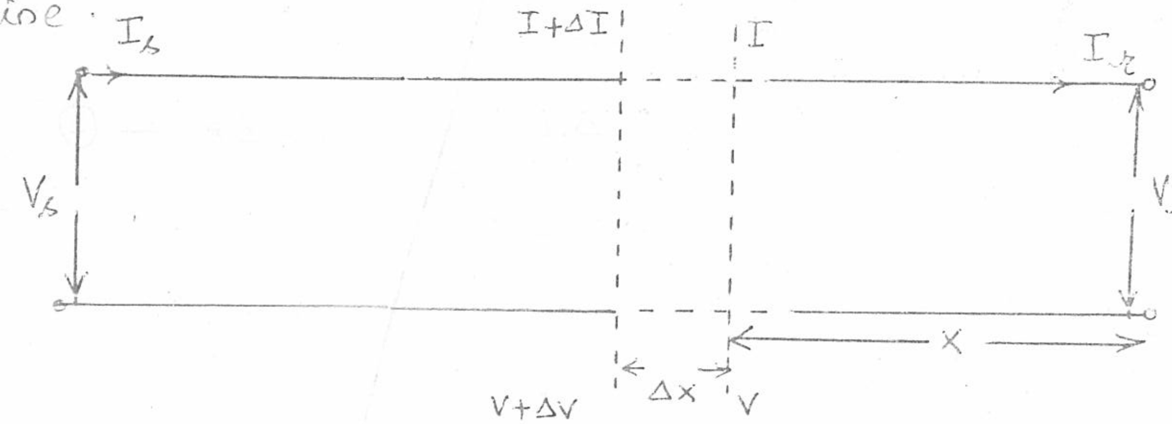
$$V_r = DV_s - BI_s \quad \text{— (9)}$$

$$V_r = DV_s - BI_s$$
$$I_r = AI_s - CV_s$$

* Two port n/w equ in terms of receiving end voltage & current.

PERFORMANCE ANALYSIS OF LONG TRANSMISSION LINE:-

* Analysis is done by rigorous method of analysis
* Capacitance is distributed uniformly along the line.



* Receiving end parameters are taken as reference for analysis.
* Consider an elemental length 'Δx' at a distance 'x' from receiving end.

* Let 'l' → length of the line.
* 'z' → series impedance per unit length.
* 'y' → shunt admittance per unit length.
* $Z = z \times l$ → total series impedance of line.
* $Y = y \times l$ → total shunt admittance of line.
* Let 'V' be voltage & 'I' be the current at a distance 'x' from receiving end.
* |||ly let 'ΔV' be the voltage & ΔI be the
* |||ly '$V+\Delta V$' be the voltage & '$I+\Delta I$' be the current at a distance '$x+\Delta x$' from receiving end.
* Voltage at Δx is $\Delta V = I \times z \times \Delta x$.

$$\frac{dV}{dx} = Iz \quad \text{— (1)}$$

* Current through Δx is $\Delta I = V \times y \times \Delta x$

$$\frac{dI}{dx} = Vy \quad \text{— (2)}$$

* diff (1) w.r.t x

$$\frac{d^2V}{dx^2} = z \cdot \frac{dI}{dx} \quad \text{— (3)}$$

sub (2) in (3)

$$(3) \Rightarrow \frac{d^2V}{dx^2} = z \times V \times y.$$

$$\frac{d^2V}{dx^2} - zy \cdot V = 0. \quad \text{— (4)}$$

* Solution of (4) ⇒ $V = Ae^{\sqrt{yz}\,x} + Be^{-\sqrt{yz}\,x}$ — (5)

[NOTE: $D^2 - zy = 0$; $D = \pm\sqrt{zy}$

Roots are real & unequal

∴ Solution $y = Ae^{m_1 x} + Be^{m_2 x}$]

Differentiate (5) w.r.t x

$$\frac{dv}{dx} = \sqrt{yz}\,Ae^{\sqrt{yz}\,x} - \sqrt{yz}\,Be^{-\sqrt{yz}\,x}$$

$$\frac{dv}{dx} = \sqrt{yz}\left[Ae^{\sqrt{yz}\,x} - Be^{\sqrt{yz}\,x}\right] \quad \text{— (6)}$$

sub equ (1) in equ (6)

$$I \times z = \sqrt{yz}\left[Ae^{\sqrt{yz}\,x} - Be^{-\sqrt{yz}\,x}\right]$$

$$I = \frac{\sqrt{yz}}{z}\left[Ae^{\sqrt{yz}\,x} - Be^{-\sqrt{yz}\,x}\right]$$

$$= \frac{\sqrt{y}\cancel{\sqrt{z}}}{\sqrt{z}\times\cancel{\sqrt{z}}}\left[Ae^{\sqrt{yz}\,x} - Be^{-\sqrt{yz}\,x}\right]$$

$$I = \sqrt{\frac{y}{z}}\left[Ae^{\sqrt{yz}\,x} - Be^{-\sqrt{yz}\,x}\right] \quad \text{— (7)}$$

Let $\gamma = \sqrt{yz} = \alpha + j\beta$ where 'α' → propagation constant

$Z_c = \sqrt{\frac{z}{y}}$ = characteristic impedance.

equ (7) ⇒ $I = \frac{1}{Z_c}\left[Ae^{\gamma x} - Be^{-\gamma x}\right]$ — (8)

equ (5) ⇒ $V = Ae^{\gamma x} + Be^{-\gamma x}$ — (9)

* When $x = 0$, $V = V_r$ & $I = I_r$

$\therefore$ 9 $\Rightarrow$ $V_r = A + B.$ — (10)

(8) $\Rightarrow$ $I_r = \frac{1}{Z_c}[A - B]$ — (11)

Solve (10) & (11) to get A & B.

xly (10) by $\frac{1}{Z_c}$ $\Rightarrow$ $\frac{V_r}{Z_c} = \frac{A}{Z_c} + \frac{B}{Z_c}$

(11) $\Rightarrow$ $\overset{(-)}{I_r} = \overset{(-)}{\frac{A}{Z_c}} \overset{+}{-} \frac{B}{Z_c}$

$$\frac{V_r}{Z_c} - I_r = 2B \times \frac{1}{Z_c}$$

$$\boxed{B = \frac{V_r - I_r Z_c}{2}}$$

sub $B = \frac{V_r - I_r Z_c}{2}$ in equ (10).

$$V_r = A + \frac{V_r - I_r Z_c}{2}$$

$$A = V_r - \left[\frac{V_r - I_r Z_c}{2}\right]$$

$$A = \frac{2V_r - V_r + I_r Z_c}{2}$$

$$\boxed{A = \frac{V_r + I_r Z_c}{2}}$$

sub $A = \frac{V_r + I_r Z_c}{2}$ $B = \frac{V_r - I_r Z_c}{2}$ in equ (8) & (9)

equ (8) $\Rightarrow$ $I = \frac{1}{Z_c}\left[\frac{V_r + I_r Z_c}{2} e^{\gamma x} - \frac{V_r - I_r Z_c}{2} e^{-\gamma x}\right]$

$$* \quad I = \frac{1}{Z_c}\left[\frac{V_r}{2}e^{\gamma x} + \frac{I_r Z_c}{2}e^{\gamma x} - \frac{V_r e^{-\gamma x}}{2} + \frac{I_r Z_c}{2}e^{-\gamma x}\right]$$

$$= \frac{1}{Z_c}\left[V_r\left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right] + I_r Z_c\left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right]\right]$$

$$I = \frac{1}{Z_c}\left[V_r \sinh \gamma x + I_r Z_c \cosh \gamma x\right] \quad \text{— (11)}$$

equ (9) $\Rightarrow$ $V = \frac{V_r + I_r Z_c}{2}e^{\gamma x} + \frac{V_r - I_r Z_c}{2}e^{-\gamma x}$

$$V = V_r\left[\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right] + I_r Z_c\left[\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right].$$

$$V = V_r \cosh \gamma x + I_r Z_c \sinh \gamma x \quad \text{— (12)}$$

$\therefore$ When $x = l$, $V = V_s$ = $I = I_s$.

$\therefore$ equ (11) $\Rightarrow$ $I_s = \frac{1}{Z_c}\left[V_r \sinh \gamma l + I_r Z_c \cosh \gamma l\right]$ — (13)

equ (12) $V_s = V_r \cosh \gamma l + I_r Z_c \sinh \gamma l$. — (14)

Compare (13) & (14) equs with 2-port n/w equs

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + DI_r.$$

$\therefore$

$A = \cosh \gamma l$,	$B = Z_c \sinh \gamma l$.
$C = \frac{\sinh \gamma l}{Z_c}$;	$D = \cosh \gamma l$.

ABCD parameters for long transmission line

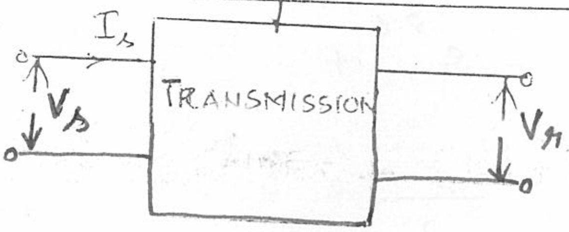
DETERMINATION OF ABCD PARAMETERS FOR SHORT TRANSMISSION LINE:

General two port network equ are

$$V_s = AV_r + BI_r \quad -(1)$$

$$I_s = CV_r + DI_r \quad -(2)$$

* **When the receiving end is open circuited :-**



$I_r = 0$

equ (1) $\Rightarrow$ $A = \dfrac{V_s}{V_r}$ (dimensionless)

* Since $I_r = 0$, $V_s = V_r$

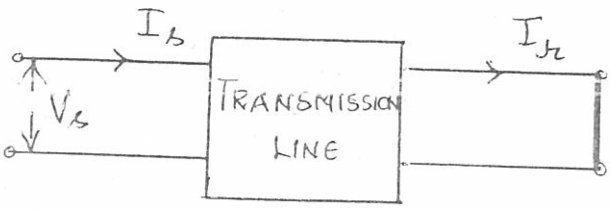
$$\boxed{A = 1}$$

equ (2) $\Rightarrow$ $C = \dfrac{I_s}{V_r}$ (shunt admittance)

* Capacitance is neglected in short transmission line & hence shunt admittance = 0.

$\therefore$ $\boxed{C = 0}$

* **When the receiving end is short circuited:**



$V_r = 0$

equ (1) $\Rightarrow$ $B = \dfrac{V_s}{I_r}$ = impedance.

$$\boxed{B = Z}$$

equ ② ⇒ $D = \frac{I_s}{I_r}$ (dimensionless)

* When receiving end is short circuited, $I_s = I_r$

∴ $\boxed{D = 1}$

$A = D = 1$

$B = Z$

$C = 0.$

Prove $AD - BC = 1$

∴ $1 \times 1 - Z \times 0 = 1.$

∴ $\boxed{AD - BC = 1}$ proved.

% Voltage reg = [(Vge on receiving end at no load − full load receiving end vge) / (full load receiving end Vge)] × 100

—x—

* No load receiving end voltage :

On no load, $I_r = 0$

∴ equ ① ⇒ ∴ $V_s = A V_r$

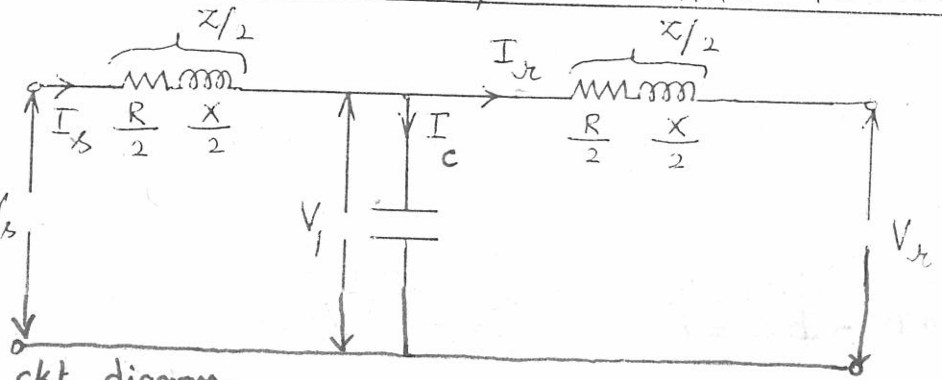
$= \frac{V_s}{A}$

* Full load receiving end vge $= V_r$

$$\% \text{ Vge reg} = \left[\frac{V_s/A - V_r}{V_r}\right] \times 100.$$

$$\% \eta = \left[\frac{P_R}{P_R + \text{line losses}} \right] \times 100$$

Determination of ABCD Parameters for Medium Transmission Line Using Nominal T - Method:



From ckt diagram.

* $I_s = I_r + I_c$.

* $I_c = V_1 Y$

* $I_s = I_r + V_1 Y$. — ①

* $V_s = I_s \frac{Z}{2} + V_1$ — ②

* $V_1 = I_r Z/2 + V_r$ — ③

(or) $I_c = \frac{V_1}{X_c}$

$I_c = X_c^{-1} V_1$

$I_c = Y V_1$

sub ③ in ① & ②

equ ① ⇒ $I_s = I_r + \left[I_r \frac{Z}{2} + V_r \right] Y$.

$I_s = I_r + I_r \frac{Z}{2} Y + V_r Y$.

$$\boxed{I_s = I_r \left[1 + Y\frac{Z}{2} \right] + V_r Y} \quad — ④$$

equ ② ⇒ $V_s = I_s \frac{Z}{2} + I_r \frac{Z}{2} + V_r$ — ⑤

sub equ ④ in ⑤

$$V_s = \left[I_r \left[1 + Y\frac{Z}{2} \right] + V_r Y \right] \frac{Z}{2} + I_r \frac{Z}{2} + V_r$$

$$V_s = I_r \frac{Z}{2} + I_r \frac{YZ^2}{4} + V_r \frac{YZ}{2} + I_r \frac{Z}{2} + V_r$$

$$\boxed{V_s = I_r \left[\frac{YZ^2}{4} + \frac{2Z}{2}\right] + V_r \left[1 + \frac{YZ}{2}\right]} \quad — (5)$$

Compare (5) with $V_s = A.V_r + B I_r$

$$A = 1 + \frac{YZ}{2}.$$

$$B = Z\left[\frac{YZ}{4} + 1\right]$$

compare (4) with $I_s = CV_r + DI_r$

$$C = Y$$

$$D = 1 + \frac{YZ}{2}$$

$\therefore$

$$\boxed{\begin{aligned} A = D &= 1 + \frac{YZ}{2} \\ B &= Z\left[1 + \frac{YZ}{4}\right] \\ C &= Y \end{aligned}}$$

ABCD PARAMETERS FOR MEDIUM TRANSMISSION LINE (T- METHOD)

Show that $AD - BC = 1$.

$$\left[1 + \frac{YZ}{2}\right]\left[1 + \frac{YZ}{2}\right] - Z\left[1 + \frac{YZ}{4}\right] \times Y$$

$$= \left[1 + \frac{YZ}{2}\right]^2 - ZY - \frac{Y^2Z^2}{4}$$

$$= 1 + \frac{Y^2Z^2}{4} + \frac{2YZ}{2} - ZY - \frac{Y^2Z^2}{4}$$

$$= 1$$

$\therefore AD - BC = 1$

1. A balanced 3-ϕ load of 30MW is supplied at 132kV 50Hz and 0.85 pf lagging by means of a transmission line. The series impedance of a single conductor is $(20+j52)\Omega$ and the total phase-neutral admittance is 315×10^{-6} siemen. Using nominal T-method determine i) ABCD constants of the line ii) sending end voltage iii) regulation of the line.

GIVEN:-

i) $P_R = 30MW = 30\times10^6 W$.

line $\frac{132\times10^3}{\sqrt{3}}$ (phase)

$V_R = 76,210.2V$.

iii) $\cos\phi_R = 0.85$ (lagging)

iv) $Z = (20+j52)\Omega$.

v) $Y = j315\times10^{-6}$ siemen.

REQUIRED:

i) ABCD ii) V_s ?? iii) Vge regulation.

$$\% \text{ Vge regulation} = \left[\frac{V_s/_A - V_R}{V_R}\right] \times 100$$

$$\% \eta = \frac{P_R}{P_R + \text{line losses}} \times 100$$

1. A balanced 3-ϕ load of 30MW is supplied at 132kV, 50Hz and 0.8pf lagging by means of a transmission line. The series impedance of a single conductor is $(20+j52)\Omega$ and the total phase neutral admittance ... 315×10^{-6} siemen. Using ... T-method, determine i) A, B, C & D parameters of the line ii) sending end voltage iii) regulation of line.

GIVEN:-

i) $P_R = 30MW = 30\times10^6 W$.

ii) $\cos\phi_R = 0.8$

iii) $Y = j315\times10^{-6}$ Siemen

iv) $V_R = 132kV$ (line) $= \frac{132}{\sqrt{3}} = 76.210\times10^3 V$ (phase).

v) $Z = (20+j52)\Omega$/phase

REQUIRED

i) A, B, C & D constants.

ii) V_s, iii) Vge regulation.

SOLUTION:

* For nominal T-method of medium transmission line $A = D = 1 + \frac{YZ}{2}$, $B = Z\left[1 + \frac{YZ}{4}\right]$, $C = Y$.

* $C = Y = j\,315 \times 10^{-6}$

$= 315 \times 10^{-6} \angle 90^\circ$

$$C = 0.000315 \angle 90^\circ$$

* $A = D = 1 + \frac{YZ}{2}$

$= 1 + \left[(0.000315 \angle 90^\circ)\left(\frac{20 + j52}{2}\right)\right]$

$= 1 + [0.000315 \angle 90^\circ][10 + j26]$

$= 1 + [0.000315 \angle 90^\circ][27.856 \angle 68.96]$

$= 1 + [8.774 \times 10^{-3} \angle 158.96]$

$= 1 + [-8.189 \times 10^{-3} + j\,3.150 \times 10^{-3}]$

$= 0.9918 + j\,3.150 \times 10^{-3}$

$$A = D = 0.9918 \angle 0.1819$$

* $B = Z\left[1 + \frac{YZ}{4}\right]$

$= (20 + j52)\left[1 + \frac{(j315 \times 10^{-6}) \times (20 + j52)}{4}\right]$

$= (20 + j52)\left[1 + \frac{(3.15 \times 10^{-4} \angle 90)(55.71 \angle 68.96)}{4}\right]$

$= (20 + j52)\left[1 + \frac{0.0175 \angle 158.96}{4}\right]$

$= (20 + j52)\left[1 + 4.375 \times 10^{-3} \angle 158.96\right]$

$= (20 + j52)\left[1 + (-4.083 \times 10^{-3} + j\,1.57 \times 10^{-3})\right]$

$= (20+j52)\left[0.9959 + j1.57\times 10^{-3}\right]$

$= (20+j52)\,(0.9959\angle 0.090)$

$= (55.71\angle 68.96)\,(0.9959\angle 0.090)$

$$\boxed{B = 55.48\angle 69.05}$$

(ii) Sending end voltage:-

$V_s = AV_r + BI_r$

$V_r = 76{,}210V$

$I_r = \dfrac{P_R}{3V_r\cos\phi_r} = \dfrac{30\times10^6}{3\times 76210\times 0.85}$

$$\boxed{I_r = 154A}$$

$\vec{I_r} = I_r(\cos\phi_r - j\sin\phi_r)$

$= 154(0.85 - j0.53)$

$\vec{I_r} = (130.9 - j81.62)\,A = 154.26\angle 31.9$

$V_s = AV_r + BI_r$

$= [0.9918\angle 0.1819]\,76{,}210 + [55.48\angle 69.05 \times 154.26\angle 31.9$

$= (75{,}585.0\angle 0.1819) + (8558.34\angle 100.95)$

$= (75{,}584.6 + j239.96) + (-1625.6 + j8402.5)$

$V_s = 73{,}959 + j8642.46$

$|V_s| = \sqrt{(73{,}959)^2 + (8642.46)^2} = 74{,}462.2V$

2. Find the following for a 3-φ transmission line delivering a load of 50MVA at 110kV and power factor 0.8 lagging i) sending end voltage ii) sending end current iii) Sending end power iv) η of transmission. Given that $A = D = 0.98 \angle 3^\circ$, $B = 110 \angle 75^\circ$, $C = 0.0005 \angle 80^\circ$ siemen.

GIVEN:-

* $P_R = 50MVA$
* $\cos\phi_R = 0.8$
* $V_R = 110kV = 110 \times 10^3 V.$ (line)
* $A = D = 0.98 \angle 3^\circ$
* $B = 110 \angle 75^\circ$
* $C = 0.0005 \angle 80^\circ$ siemen.

REQUIRED:

* V_s
* I_s.
* P_s.
* η.

SOLUTION:-

* $P_R = 50MVA$

* $P_R = 50MVA \times 0.8 = 40 MW$

* $V_R / phase = \frac{110 \times 10^3}{\sqrt{3}} = 63508.5V$

* $I_R = \frac{40 \times 10^6}{3 \times 63508.5 \times 0.8} = 262.43A$

$$\boxed{I_R = 262.43A}$$

* $\vec{I_R} = I_R (\cos\phi_R - j\sin\phi_R)$

$= 262.43 (0.8 - j0.6)$

$$\boxed{\vec{I_R} = 209.9 - j157.45 \text{ A}}$$

* $V_s = AV_r + BI_r.$

* $AV_r = (0.98 \angle 3°) \times (63508.5 \angle 0°)$

$= 62238.33 \angle 3°$

* $\boxed{AV_r = 62,153 + j3257.3.}$

* $BI_r = (110 \angle 75°) \times (209.9 - j157.45)$

$= (110 \angle 75°) \times (262.39 \angle -36.87)$

$= 28862.9 \angle 38.13$

* $\boxed{BI_r = 22703.8 + j17821.3.}$

* $CV_r = (0.0005 \angle 80°) \times (63500 \angle 0°)$

$= 31.75 \angle 80°$

* $\boxed{CV_r = 5.513 + j31.26.}$

* $DI_r = (0.98 \angle 3^\circ) \times (209.9 - j157.45)$

$= (0.98 \angle 3^\circ) \times (262.39 \angle -36.87)$

$= 257.14 \angle -33.87$

* $DI_r = 213.50 - j143.30$

$\therefore \quad V_s = AV_r + BI_r$

$= (62{,}153 + j3257.3) + (22703.8 + j17821.3)$

$= 84{,}856.8 + j21078.6$

$V_s = 87.436 \angle 13.9^\circ \text{ V}$

$\therefore \quad I_s = CV_r + DI_r$

$= (5.513 + j31.26) + (213.50 - j143.30)$

$= 219.0 - j112.04$

$I_s = 245.9 \angle -27^\circ \text{ A}$

* Sending end power factor $= \cos\phi_s$.

$\phi_s = \theta_1 + \theta_2$.

* $\theta_1 \rightarrow$ angle b/w V_s & V_r.

$\therefore V_s = 87{,}436 \angle 13.9^\circ$ V.

$\therefore \theta_1 = 13.9^\circ$

* $\theta_2 \rightarrow$ angle b/w V_r & I_s.

$\therefore I_s = 245.9 \angle -27^\circ$.

$\therefore \theta_2 = -27^\circ$

or

$\theta_2 = 27^\circ$ (lagging)

$\therefore \quad \phi_s = \theta_1 + \theta_2 = 13.9^\circ + 27^\circ = 40.9$.

(111)

$$\cos\phi_s = \cos(40.9)$$

$$\boxed{\cos\phi_s = 0.755}$$

* Sending end power(P_s) = $3 V_s I_s \cos\phi_s$.

$= 3 \times 87.436 \times 245.9 \times 0.75$.

$$\boxed{P_s = 48.376 \text{ MW}}$$

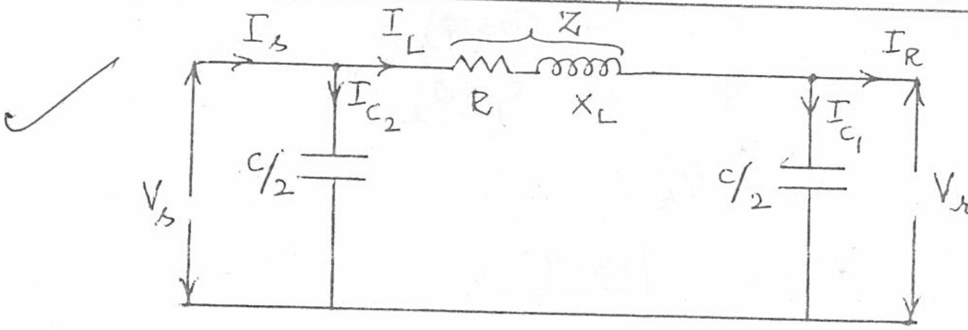
*

$$\eta = \frac{P_R}{\quad} \times 100$$

$$= \frac{40 \times 10^6}{48.376 \times 10^6} \times 100.$$

$$\boxed{\eta = 82.68\%}$$

DETERMINATION OF ABCD PARAMETERS FOR MEDIUM TRANSMISSION LINE USING NOMINAL -π METHOD :-



From circuit diagram.

* $I_{C_1} = V_r Y/2$.

* $I_L = I_{C_1} + I_r$

* $I_L = (V_r Y/2) + I_r$ —— ①

(or) $I_{C_1} = \dfrac{V_r}{X_C/2}$

$= V_r (X_C/2)^{-1}$

$I_{C_1} = V_r Y/2$.

lly * $I_{C_2} = V_s Y/2$. —— ②

* $I_s = I_L + I_{C_2}$ —— ③

sub ② & ① in equ ③

* Equ ③ ⇒ $I_s = \left(V_r Y/2 + I_r\right) + V_s Y/2$ — ④

* |||ly $V_s = I_L Z + V_r$ — ⑤

* sub ① in ⑤.

* ∴ Equ ⑤ ⇒ $V_s = \left(V_r Y/2 + I_r\right) Z + V_r$.

$$V_s = \left[1 + \frac{YZ}{2}\right] V_r + Z I_r \quad — ⑥$$

* sub equ in ④.

* Equ ④ ⇒ $I_s = \left(V_r Y/2 + I_r\right) + \left[\left(1 + \frac{YZ}{2}\right) V_r + Z I_r\right] Y/2$.

$$I_s = V_r Y/2 + I_r + V_r Y/2 + V_r \frac{Y^2 Z}{4} + \frac{ZY}{2} I_r.$$

* $$I_s = V_r \left[\frac{Y}{2} + \frac{Y}{2} + \frac{Y^2 Z}{4}\right] + I_r \left[1 + \frac{ZY}{2}\right].$$

* $$I_s = V_r \left[Y + \frac{Y^2 Z}{4}\right] + I_r \left[1 + \frac{YZ}{2}\right] \quad — ⑦$$

* Compare equ ⑥ with $V_s = A V_r + B I_r$.

$$A = 1 + \frac{YZ}{2}.$$

$$B = Z.$$

* Compare equ ⑦ with $I_s = C V_r + D I_r$.

$$C = Y + \frac{Y^2 Z}{4}$$

$$D = 1 + \frac{YZ}{2}.$$

$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y + \frac{Y^2 Z}{4}$$

$$D = 1 + \frac{YZ}{2}$$

ABCD PARAMETERS FOR MEDIUM TRANSMISSION LINE - NOMINAL Π_METHOD

Show that $AD - BC = 1$.

$$AD - BC = \left[1 + \frac{YZ}{2}\right]\left[1 + \frac{YZ}{2}\right] - Z \times \left[Y + \frac{Y^2 Z}{4}\right]$$

$$= 1 + \frac{YZ}{2} + \frac{YZ}{2} + \frac{Y^2 Z^2}{4} - ZY - \frac{Y^2 Z^2}{4}$$

$$= 1 + YZ - YZ$$

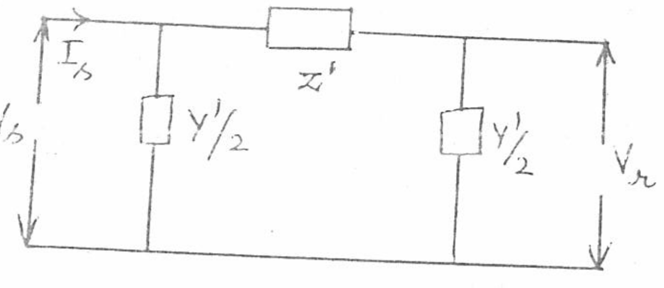
$\therefore AD - BC = 1.$

$$\therefore \%\ V_{ge}\ \text{regulation} = \left[\frac{\frac{V_s}{A} - V_r}{V_r}\right] \times 100.$$

$$\%\ \eta = \frac{P_R}{P_S} \times 100.$$

DETERMINING NOMINAL Π - NETWORK'S PARAMETERS USING ABCD PARAMETERS OF LONG TRANSMISSION LINE:

Nominal Π - n/w of medium transmission line is



where $\boxed{A = 1 + \frac{Y'Z'}{2}, \quad B = Z', \quad C = Y'\left[1 + \frac{Z'Y'}{4}\right]}$ — I

* The ABCD parameters for a long transmission line is

$$\boxed{\begin{aligned} A = D &= \cosh \gamma l \\ B &= Z_c \sinh \gamma l \\ C &= \frac{\sinh \gamma l}{Z_c} \end{aligned}} \quad \text{— II}$$

Compare I & II set of equations.

$$\therefore \quad \cosh \gamma l = 1 + \frac{Y'Z'}{2} \quad \text{—— (3)}$$

$$Z_c \sinh \gamma l = Z' \quad \text{—— (4)}$$

$$\frac{\sinh \gamma l}{Z_c} = Y'\left[1 + \frac{Z'Y'}{4}\right] \quad \text{—— (5)}$$

* Equ (4) $\Rightarrow$ $Z' = Z_c \sinh \gamma l$.

We know $Z_c = \sqrt{\frac{z}{y}}$

$\therefore$ Equ (4) $\Rightarrow$ $Z' = \sqrt{\frac{z}{y}} \sinh \gamma l$.

* X/y & divide RHS of above equ by $\sqrt{zy} \times l$.

* $$\therefore \quad Z' = \sqrt{\frac{z}{y}} \sinh \gamma l \times \frac{\sqrt{zy}}{\sqrt{zy}} \times \frac{l}{l}$$

$$Z' = \frac{z \times l}{\sqrt{zy} \times l} \times \sinh \gamma l.$$

* Sub $\gamma = \sqrt{zy}$ & $Z = z \times l$ in above equ.

* $$\boxed{Z' = \frac{Z \sinh \gamma l}{\gamma l}} \quad \text{--- (6)}$$

* Sub equ (4) in equ (3)

* $$1 + \frac{Y'}{2}\left[Z_c \sinh \gamma l\right] = \cosh \gamma l.$$

* $$\frac{Y'}{2}\left[Z_c \sinh \gamma l\right] = \cosh \gamma l - 1$$

* $$Y' Z_c \left[\frac{\sinh \gamma l}{2}\right] = \cosh \gamma l - 1 \quad \text{--- (7)}$$

We know,

- $\sinh \gamma l = 2 \sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2}$,
- $\cosh \gamma l = \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2}$ &
- $\cosh^2 \frac{\gamma l}{2} - \sinh^2 \frac{\gamma l}{2} = 1$

$\therefore$ Equ (7) $\Rightarrow$
$$Y' Z_c \left[\sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2}\right] = \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2} - \left[\cosh^2 \frac{\gamma l}{2} - \sinh^2 \frac{\gamma l}{2}\right]$$

* $$Y' Z_c \left[\sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2}\right] = \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2} - \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2}$$

* $$Y' Z_c \left[\sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2}\right] = 2 \sinh^2 \frac{\gamma l}{2}.$$

* $$Y' Z_c = \frac{2 \sinh^2 \frac{\gamma l}{2}}{\sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2}}$$

* $$Y' Z_c = 2 \tanh \frac{\gamma l}{2}.$$

* $$Y' = \frac{2 \tanh \frac{\gamma l}{2}}{Z_c}.$$

* Sub $Z_C = \sqrt{\frac{Z}{Y}}$ in above equ

* $$Y' = \frac{2\tanh \gamma l/2}{\sqrt{\frac{Z}{Y}}}.$$

* x/y & ÷ the RHS of above equ by $\sqrt{ZY}$ & $l/2$.

* $$Y' = \sqrt{\frac{Y}{Z}} \times 2\tanh\frac{\gamma l}{2} \times \frac{\sqrt{ZY}}{\sqrt{ZY}} \times \frac{l/2}{l/2}.$$

* $$Y' = \frac{Y \times l}{2} \times 2\tanh\frac{\gamma l}{2} \times \frac{1}{\sqrt{ZY} \times l/2}.$$

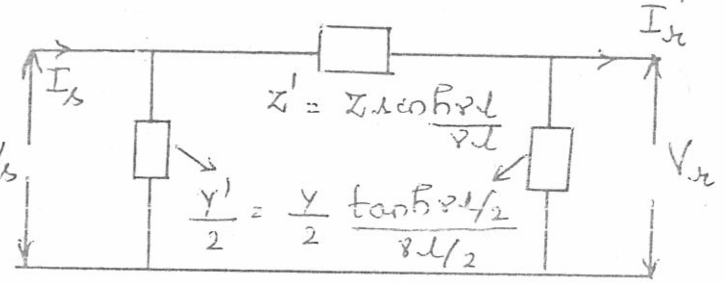
* Sub $\gamma = \sqrt{ZY}$ & $Y \times l = Y$ in above equ.

* $$Y' = \frac{Y}{2} \times 2\tanh\frac{\gamma l}{2} \times \frac{1}{\gamma l/2}.$$

* $$\boxed{\frac{Y'}{2} = \frac{Y}{2} \times \frac{\tanh \gamma l/2}{\gamma l/2}}$$

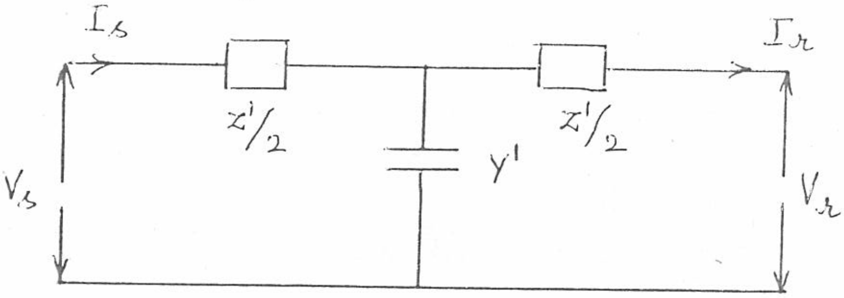
* When Z' is replaced by $Z\frac{\sinh \gamma l}{\gamma l}$ & $\frac{Y'}{2}$ is replaced by $\frac{Y}{2}\frac{\tanh \gamma l/2}{\gamma l/2}$, we can obtain the nominal - π n/w for long transmission line.

NOMINAL π- NETWORK FOR LONG TRANSMISSION LINE.



DETERMINING NOMINAL -T NETWORK'S PARAMETERS USING ABCD PARAMETERS OF LONG TRANSMISSION LINE :-

* Nominal T- network of a medium transmission line is



* ABCD parameters for medium transmission line is

$$A = D = 1 + \frac{Y'Z'}{2}$$

$$B = z'\left[1 + \frac{y'z'}{4}\right] \quad \text{—— ① set}$$

$$C = y'$$

* ABCD parameters for long transmission line is

$$A = D = \cosh \gamma l .$$

$$B = Z_c \sinh \gamma l . \quad \text{—— ② set.}$$

$$C = \frac{\sinh \gamma l}{Z_c .}$$

* Equating set ① & set ②

$$\cosh \gamma l = 1 + \frac{y'z'}{2} \quad \text{—— ③}$$

$$Z_c \sinh \gamma l = z'\left[1 + \frac{y'z'}{4}\right] \quad \text{—— ④}$$

$$\frac{\sinh \gamma l}{Z_c} = y' \quad \text{—— ⑤}$$

* sub $Z_c = \sqrt{\frac{Z}{Y}}$ in equ ⑤

* Equ ⑤ $\Rightarrow$ $\frac{\sinh \gamma l}{\sqrt{\frac{Z}{Y}}} = Y'$

* Mly & ÷ the LHS of above equ by $\sqrt{ZY}$ & l

* $\frac{\sqrt{ZY}}{\sqrt{ZY}} \times \frac{l}{l} \times \sqrt{\frac{Y}{Z}} \sinh \gamma l = Y'$

* $Y' = \frac{Y \times l}{\sqrt{ZY} \times l} \sinh \gamma l.$

* Sub $\gamma = \sqrt{ZY}$ & $Y = Y \times l$ in above equ.

* $$Y' = \frac{Y}{\gamma l} \sinh \gamma l \quad \text{— ⑥}$$

* Sub. ⑤ in ③

* $\cosh \gamma l = 1 + \frac{Z'}{2} \left[\frac{\sinh \gamma l}{Z_c} \right]$ — ⑦

* We know,
  - $\cosh \gamma l = \cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2}$
  - $\cosh^2 \frac{\gamma l}{2} - \sinh^2 \frac{\gamma l}{2} = 1$
  - $\sinh \gamma l = 2 \sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2}$

* ∴ Equ ⑦ $\Rightarrow$ $\cosh^2 \frac{\gamma l}{2} + \sinh^2 \frac{\gamma l}{2} = \cosh^2 \frac{\gamma l}{2} - \sinh^2 \frac{\gamma l}{2} + \frac{Z'}{Z_c} \left[\sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2} \right]$

* $2 \sinh^2 \frac{\gamma l}{2} = \frac{Z'}{Z_c} \left[\sinh \frac{\gamma l}{2} \cosh \frac{\gamma l}{2} \right]$

* $$\frac{2\sinh^2 \gamma l/2}{\sinh\frac{\gamma l}{2}\cosh\frac{\gamma l}{2}} = \frac{Z'}{Z_C}$$

* $$2\tanh\frac{\gamma l}{2} = \frac{Z'}{Z_C}$$

* sub $Z_C = \sqrt{\frac{Z}{Y}}$ & x/y & ÷ the RHS of above equ by $\sqrt{ZY}$ & $l/2$

* $$2\tanh\frac{\gamma l}{2} = \frac{Z'}{\sqrt{\frac{Z}{Y}}} \times \frac{\sqrt{ZY}}{\sqrt{ZY}} \times \frac{l/2}{l/2}$$

* $$2\tanh\frac{\gamma l}{2} = \sqrt{\frac{Y}{Z}} \times \frac{\sqrt{ZY}}{\sqrt{ZY}} \times \frac{l/2}{l/2} \times Z'$$

* $$2\tanh\frac{\gamma l}{2} = \frac{\sqrt{ZY} \times l/2 \times Z'}{z \times l/2}$$

* sub $\gamma = \sqrt{ZY}$ & $z \times l = Z$ in above equ

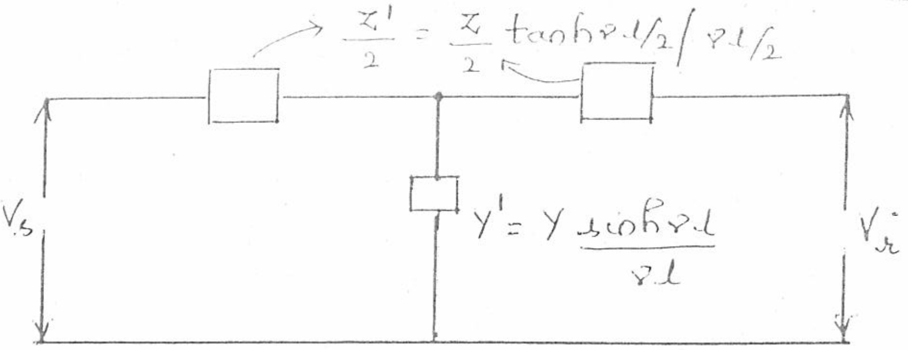
* $$2\tanh\frac{\gamma l}{2} = \frac{\gamma l/2}{Z/2} \times Z'$$

* $$\frac{2\tanh \gamma l/2}{\gamma l/2} \times \frac{Z}{2} = Z'$$

* $$\boxed{\frac{Z'}{2} = \frac{Z}{2}\,\frac{\tanh \gamma l/2}{\gamma l/2}}$$

* On replacing $Y' = Y\frac{\sinh \gamma l}{\gamma l}$ & $\frac{Z'}{2} = \frac{Z}{2}\frac{\tanh \gamma l/2}{\gamma l/2}$ we can obtain the nominal T - n/w of long transmission line.

NOMINAL T- NETWORK FOR A LONG TRANSMISSION LINE :-



1. Determine the sending end voltage current, power & power factor for a 160km section of 3 phase line delivering 50MVA at 132kV and power factor 0.8 lagging. also find the efficiency and regulation of the line. Resistance per line 0.1557 Ω/km, spacing 3.7m, 6.475m, 7.4m transposed. Evaluate A, B, C & D parameters also. Diameter 1.956cm.

GIVEN:

* 3-φ system, $l = 160km$.
* $P_R = 50MVA$
* $V_R = 132kV$ (line)

$$= \frac{132 \times 10^3}{\sqrt{3}} = 76210.2V.$$

* $\cos\phi_R = 0.8$
* $P_R = 50 \times 10^6 \times 0.8 = 40 \times 10^6 W.$
* $R/km = 0.1557 \Omega/km$
* $d_1 = 3.7m$
* $d_2 = 6.475m.$
* $d_3 = 7.4m.$
* $dia = 1.956cm = 1.956 \times 10^{-2} m.$
* $r = \frac{1.956 \times 10^{-2}}{2} = 0.978 \times 10^{-2} m.$

REQUIRED:

* A, B, C, D parameters
* % η
* % regulation.

SOLUTION:

* Since $l = 160 km$, it is a long transmission line.

* $$L/m = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] H/m.$$

$$\sqrt[3]{d_1 d_2 d_3} = \sqrt[3]{[illegible] \times [illegible] 475 \times 7.4}$$

$$= 5.617 m$$

$$\sqrt[3]{d_1 d_2 d_3} = 561. cm.$$

* $$L/m = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{561}{0.978}\right)\right] H/m$$

$$L = 1.32 \times 10^{-6} H/m. = 1.32 \times 10^{-3} H/km$$

* $$X_L = 2\pi f L$$

$$= 2 \times \pi \times 50 \times 1.32 \times 10^{-3}$$

$$X_L = 0.4146 \, \Omega/km.$$

* $$Z = R + jX_L$$

$$= (0.1557 + j\, 0.4146) \, \Omega/km.$$

$$\boxed{Z = 0.4428 \angle 69.416 \, \Omega/km.}$$

* $C = \dfrac{2\pi \varepsilon_0}{\log_e\left[\dfrac{\sqrt[3]{d_1 d_2 d_3}}{r}\right]}$ F/m.

$= \dfrac{2\pi \times 8.854 \times 10^{-12}}{\log_e\left(\dfrac{561}{0.978}\right)}$ f/m

$= 8.758 \times 10^{-12}$ F/m.

$C = 8.758 \times 10^{-9}$ F/km.

* $Y = j\omega C$

$= j \times 2\pi \times f \times C$

$= j \times 2\pi \times 50 \times 8.758 \times 10^{-9}$.

$Y = j\,2.7514 \times 10^{-6}$ ℧/km.

* $Z_c = \sqrt{\dfrac{Z}{Y}}$

$= \sqrt{\dfrac{0.4428 \angle 69.416}{j\,2.7514 \times 10^{-6}}}$

$= \sqrt{\dfrac{0.4428}{2.7514 \times 10^{-6}}\left(\angle 69.416 - \angle 90\right)}$

$= 401.8 \sqrt{\angle -20.584}$ $\left(\sqrt{\text{angle}} = \dfrac{\text{angle}}{2}\right)$

$Z_c = 401.8 \angle -10.29$

* $\gamma = \sqrt{ZY}$

$= \sqrt{0.4428 \angle 69.416 \times j\, 2.7514 \times 10^{-6}}$

$= \sqrt{(0.4428 \times 2.7514 \times 10^{-6})(\angle 69.416 + \angle 90^\circ)}$

$= \sqrt{1.218 \times 10^{-6} \cdot \angle 159.416}$

$\gamma = 1.10 \times 10^{-3} \angle 79.708$ $\qquad \left(\sqrt{\angle 159.4} = \frac{\angle 159.4}{2}\right)$

* $\gamma l = 1.10 \times 10^{-3} \angle 79.708 \times 160$

$= 0.176 \angle 79.708$

$$\boxed{\gamma l = 0.0314 + j\, 0.173.}$$

* $A = \cosh \gamma l.$

$A = \cosh(0.0314 + j\, 0.173).$

$[\cosh(a + jb) = \cosh a \cos b + j \sinh a \sin b].$

* $A = \cosh 0.0314 \cos 0.173 + j \sinh 0.0314 \sin 0.173.$

(0.0314 ↓ rad, 0.173 ↓ rad, 0.0314 ↓ rad, 0.173 ↓ rad.)

* $A = (0.9995 \times 0.985) + j(0.03139 \times 0.172)$

$= 0.9845 + j\, 5.39 \times 10^{-3}.$

$$\boxed{A = 0.9845 \angle 0.3136^\circ = D}$$

* $B = \sinh \gamma l \cdot \times Z_c$

$= \sinh(0.0314 + j\, 0.173). \times Z_c$

$[\sinh(a + jb) = \sinh a \cos b + j \cosh a \sin b]$

* $B = (\sinh 0.0314 \cos 0.173 + j \cosh 0.0314 \sin 0.173)\, Z_c$

(0.0314 ↓ rad, 0.173 ↓ rad, 0.0314 ↓ rad, 0.173 ↓ rad.)

* $B = [(0.03139 \times 0.9995) + j(0.9995 \times 0.172)] \times Z_c$

$= [0.03137 + j\, 0.1719] \times Z_c$

$B = [0.1747 \angle 79.65] \times Z_c$

$= [0.1747 \angle 79.65 \times 401.8 \angle -10.29]$

$$\boxed{B = 70.19 \angle 69.36}$$

$$C = \frac{\sinh \gamma l}{Z_c}$$

$$= \frac{0.1747 \angle 79.65}{401.8 \angle -10.29} = 4.37 \times 10^{-4} (\angle 79.65 + \angle 10.29)$$

$$\boxed{C = 4.37 \times 10^{-4} \angle 89.94}$$

$$\% \text{ Vge reg} = \left[\frac{V_s/A - V_r}{V_r}\right] \times 100$$

* $V_s = A V_r + B I_r$

* $I_r = \dfrac{P_R}{3 V_r \cos\phi_r} = \dfrac{40 \times 10^6}{3 \times 76210 \times 0.8}$

$\boxed{I_r = 218.69\ A}$; $I_r = 218.69 \angle -36.87 \rightarrow \cos\phi_R$

* $V_s = [0.9845 \angle 0.3136^\circ] \times 76210 + [70.19 \angle 69.36] \times [218.69 \angle -36.87]$

$= (75028.7 \angle 0.3136) + (15349.8 \angle 32.49)$

$= (75027.5 + j\, 410.6) + (12947.3 + j\, 8245.18)$

$= 87974.8 + j\, 8655.78$

* $V_s = 88399.5 \angle 5.619$ V.

* $\% \text{reg} = \left[\dfrac{\dfrac{V_s}{A} - V_r}{V_r} \right] \times 100.$

$$= \frac{\left[\dfrac{88399.5}{0.9845}\right] - 76210}{76210} \times 100$$

$$= 89791.2 - 76210$$

* $\boxed{\% \text{ reg} = 17.82\%}$

* $\% \eta = \dfrac{P_R}{3 V_s I_s \cos\phi_s}.$

* $I_s = = C V_r + D I_r.$

$$= [4.37 \times 10^{-4} \angle 89.94] \times 76210 + [0.9845 \angle 0.3136] \cdot \times 218.69 \angle -36.87.$$

$$= (33.303 \angle 89.94) + (215.30 \angle -36.55)$$

$$= (0.0348 + j\,33.30) + (172.95 - j\,128.2)$$

$$= 172.98 - j\,94.9.$$

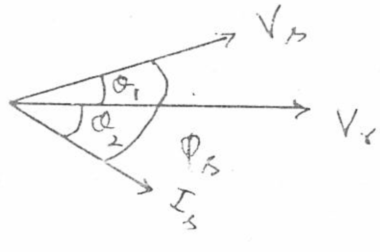
$\boxed{I_s = 197.30 \angle -28.7}$ A

* $\phi_s = \theta_1 + \theta_2$

* $V_s = 88399.5 \angle 5.619$

$\therefore \theta_1 = 5.619$

* $I_s = 197.30 \angle -28.7$



$\theta_2 = -28.7°$

$\theta_2 = 28.7$ (lagging)

* $\phi_s = \theta_1 + \theta_2 = 5.619 + 28.7$

$\phi_s = 34.319$

* $\cos\phi_s = \cos(34.319)$

$$\boxed{\cos\phi_s = 0.8259}$$

* $$\%\eta = \frac{P_R}{3 V_s I_s \cos\phi_s} \times 100$$

$$= \frac{40 \times 10^6}{3 \times 88399.5 \times 197.30 \times 0.8295} \times 100$$

$$\boxed{\%\eta = 92.16\%}$$

Surge Impedance :- Z_0

* Characteristic impedance $Z_c = \sqrt{\frac{z}{y}}$

$$= \sqrt{\frac{z \times l}{y \times l}}$$

$$Z_c = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{r + j\omega l}{g + j\omega c}}$$

* For a lossless line $r = 0$ & $g = 0$.

$$\therefore \quad Z_0 = \sqrt{\frac{l}{c}} = \sqrt{\frac{L}{C}}$$

$Z_0 = \sqrt{\frac{L}{C}} \Rightarrow$ NATURAL OR SURGE IMPEDANE.

Surge Impedance Loading:

* Surge impedance loading of a line is the power transmitted when the line is terminated through a resistance equal to surge impedance.

Ferranti Effect:-

* In medium or long transmission line, when open circuited or loaded lightly, receiving end voltage is found to be more than sending

ferranti effect.

Tuned Power Lines:

* Receiving end voltage & currents are numerically equal to the corresponding sending end current & voltage values so that there is no voltage drop on load. Such a line is called tuned power lines.

POWER CIRCLE DIAGRAM:-

NEED:-

* To provide a convenient graphical method for studying the performance of transmission

USES:

* In system stability studies.
* reactive VA calculations.
* system design & operation.

RECEIVING END PHASOR DIAGRAM.

$$V_s = AV_r + BI_r$$

where $A = A\angle\alpha$ — 'α' is a +ve angle

$B = B\angle\beta$ — 'β' leads by 90°.

$$\therefore V_s = A\angle\alpha \; V_r\angle 0^\circ + B\angle\beta \; I_r\angle -\phi_r$$

I_r lags V_r by ϕ_r.

$V_s = V_s\angle\delta$ → V_s leads V_r by δ

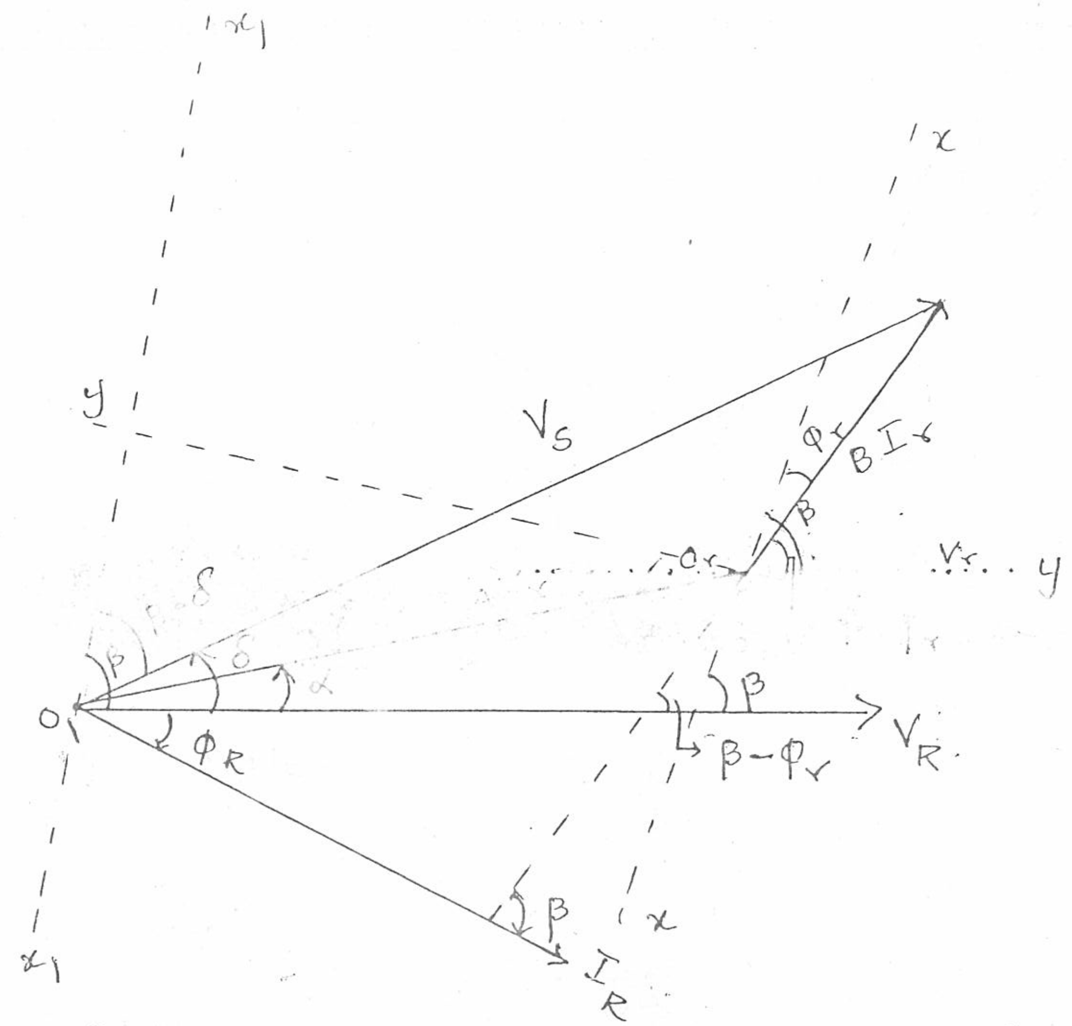
where δ → torque angle.

$$V_s\angle\delta = AV_r\angle\alpha + BI_r\angle(\beta - \phi_r)$$

AV_r vector leads V_r by α

BI_r vector leads V_r by $\beta - \phi_r$.

V_s leads V_r by δ.



STEPS FOR DRAWING PHASOR DIAGRAM :-

* Take V_R as reference vector.
* I_R vector lags V_R by an angle 'ϕ_R'.
* AV_r vector leads V_R by an angle 'α'
* To draw. BI_r vector, draw a line leading by an angle of β (nearer to 90°) from I_R. Join this vector to the arrow head of AV_r to get V_S. [(i.e) to the pt O_r]
* Draw a line xx through O_r such that it makes an angle ϕ_r with BI_r

* xx line makes an angle β with V_R.
* Draw a ⊥r to xx (or) yy
* Draw x_1x_1 line through q such that it makes an angle 'β' with V_R.

RECEIVING END POWER CIRCLE DIAGRAM:

$$P_R = V_R \times I_R$$
$$= V_R \times \left(\frac{V_R}{B}\right).$$

* To get power vectors multiply all the voltage vectors by $\frac{V_R}{B}$.

$$\therefore V_R \Rightarrow V_R \times \frac{V_R}{B} = \frac{V_R^2}{B} \angle -\beta.$$

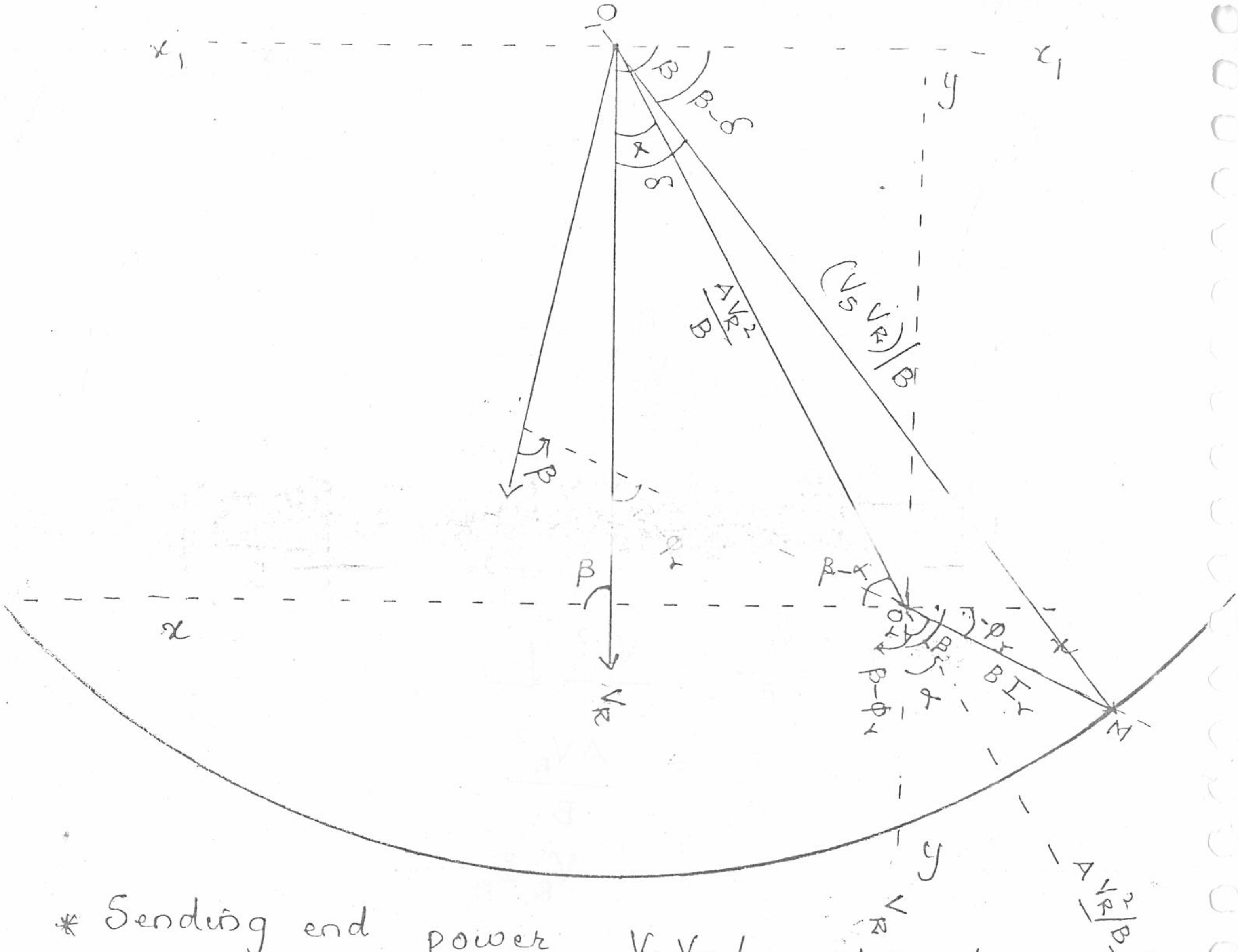
$$AV_R \Rightarrow AV_R \times \frac{V_R}{B} = \frac{AV_R^2}{B} \angle \alpha - \beta.$$

$$BI_R \Rightarrow BI_R \times \frac{V_R}{B} = V_R I_R . \angle -\phi_R .$$

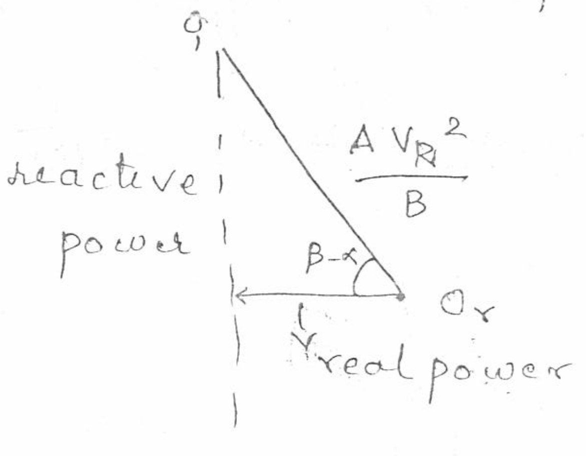
$$V_S \Rightarrow V_S \times \frac{V_R}{B} = \frac{V_S V_R}{B} \angle \delta - \beta.$$

When comparing all the terms with the vge vector terms, ∴ xly all the terms by $\frac{V_R}{B}$ & rotate by an angle $\angle -\beta$ to get the power circle diagram.

* Rotate phasor diagram by $-\beta$ in the clockwise direction.



* Sending end power $V_S V_R / B$ depends mainly on $\frac{A V_R^2}{B}$ vector. ∴ $\frac{AV_R^2}{B}$ → decides the centre of circle whose radius is $V_S V_R/B$.
* To find the centre for the power circle diagram, resolve $\frac{AV_R^2}{B}$ into real & reactive component.



Realpower = $\frac{AV_R^2}{B} \cos \beta$

Reactive power = $\frac{AV_R^2}{B} \sin \beta$

* With O_1 as centre & OM as radius draw the circle.
* This gives receiving end power circle diagram.

∴ EQUATION OF CIRCLE IS

$$(x-x_1)^2 + (y-y_1)^2 = (radius)^2$$

$$\left[P_R + \underbrace{\frac{AV_R^2}{B} \cos(\beta-\alpha)}_{watt}\right]^2 + \left[Q_R + \underbrace{\frac{AV_R^2}{B} \sin(\beta-\alpha)}_{Var}\right]^2 = \left[\frac{V_S V_R}{B}\right]^2$$

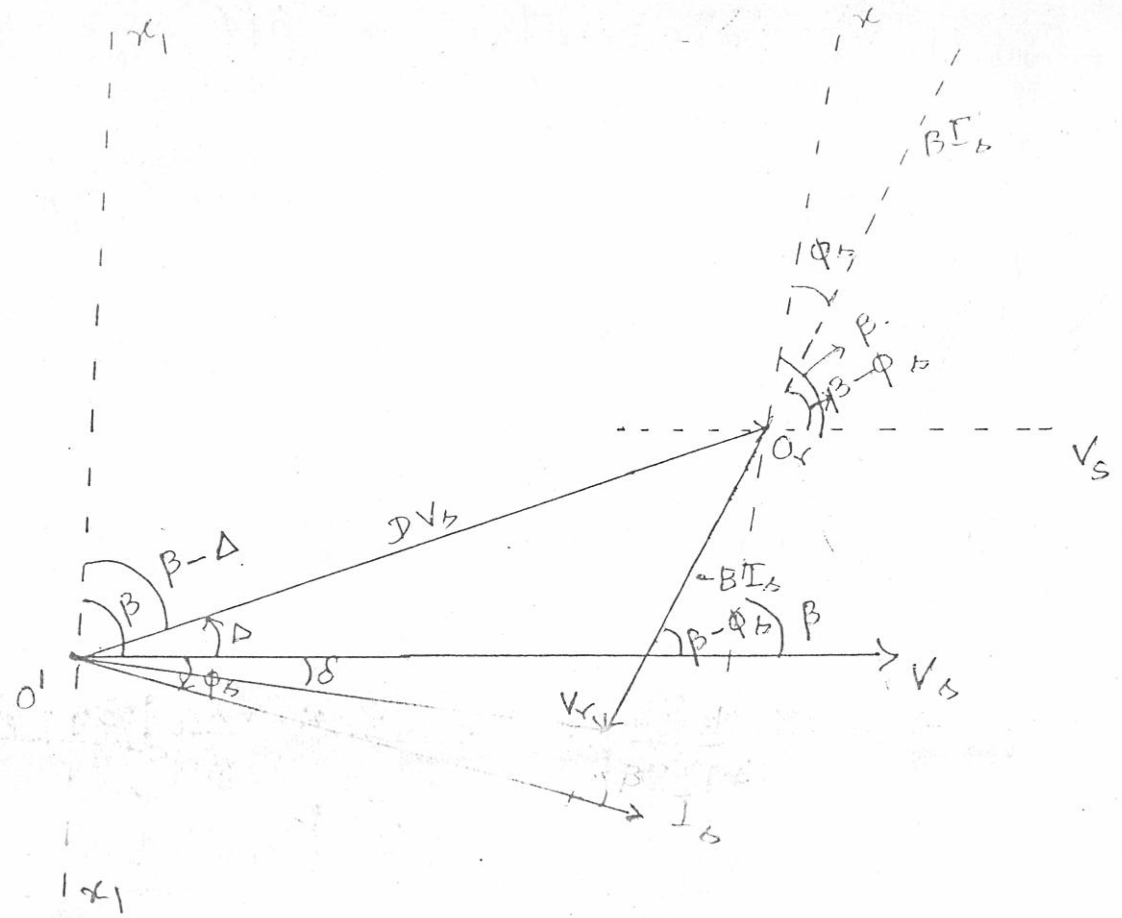
SENDING END PHASOR DIAGRAM:

$$V_R = DV_s - BI_s$$

where: $V_s = V_s\angle 0°$

$DV_s = D\angle\Delta \; V_s\angle 0° = DV_s\angle\Delta$

* Phasor DV_s leads V_s by an angle 'Δ'
* $BI_s = B\angle\beta \; I_s\angle -\phi_s = BI_s\angle\beta-\phi_s$
* Phasor BI_s leads V_s by an angle $\angle\beta-\phi_s$
* $V_R = V_r\angle -\delta$.



STEPS TO DRAW THE PHASOR DIAGRAM :-

* Take V_s as reference vector
* I_s vector lags V_s by angle 'ϕ_s'
* DV_s vector leads V_s by an angle 'Δ'.
* To draw BI_s vector, draw a line leading by an angle of β (nearer to 90°) from I_s.
* Join this vector to the arrow head of DV_s (i.e. to the pt O_r) to get V_s
* Draw a line xx through O_r such that it makes an angle ϕ_s with BI_s.
* xx line makes an angle β with V_s.
* Draw x_1x_1 line thro' O' such that it makes an angle β with V_s.

$$P_s = V_s I_s$$
$$= V_s \times \frac{-V_s}{B}.$$

* To obtain the power vectors, multiply all the vge vectors by $-V_s/B$

$$\frac{-V_s}{B} = \frac{V_s \angle 180^\circ}{B \angle \beta} = \frac{V_s}{B} \angle 180^\circ - \beta.$$

$$V_r = V_r \times \frac{-V_s}{B} = V_r \angle -\delta \times \frac{V_s}{B} \angle 180^\circ - \beta$$

$$V_r = \frac{V_r V_s}{B} \angle 180^\circ - \beta - \delta.$$

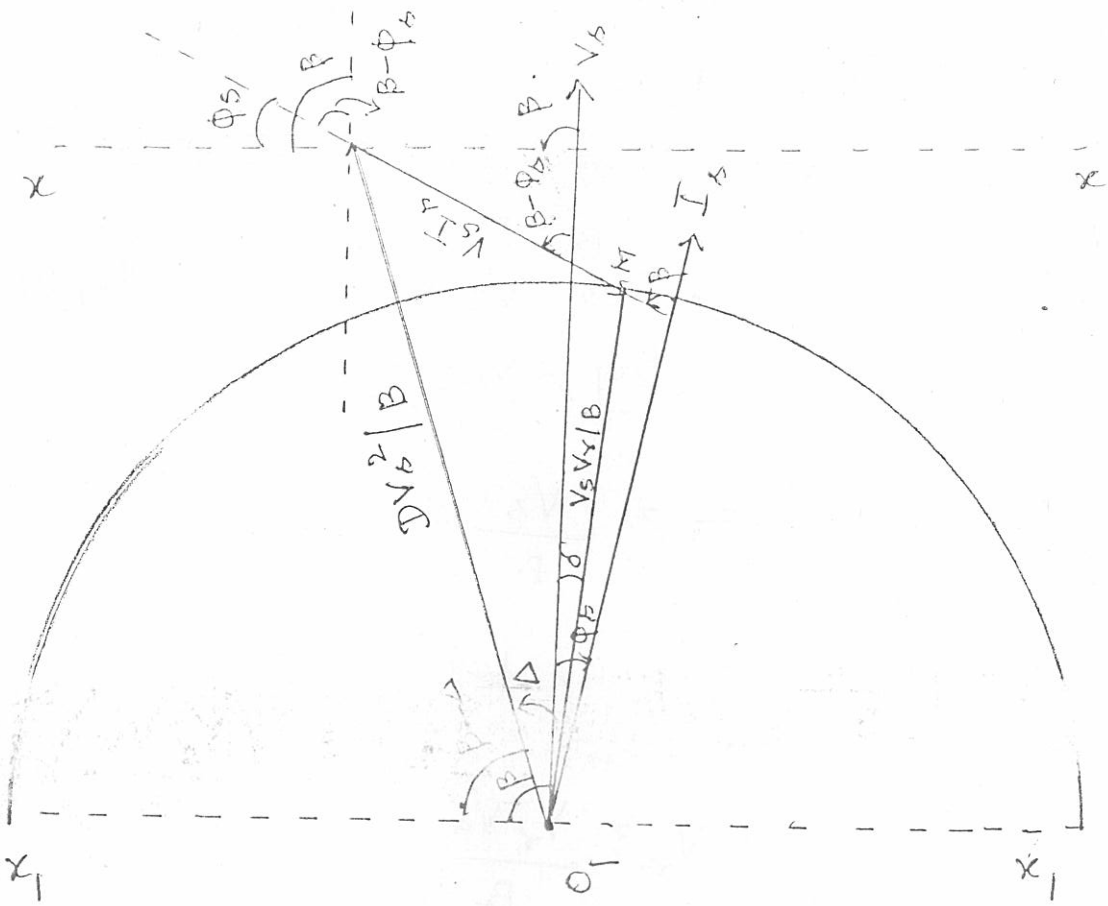
$$DV_s = DV_s \times \frac{-V_s}{B} = D \angle \Delta \times V_s \angle 0^\circ \times \frac{V_s}{B} \angle 180^\circ - \beta$$

$$= \frac{DV_s^2}{B} \angle 180^\circ + \Delta - \beta.$$

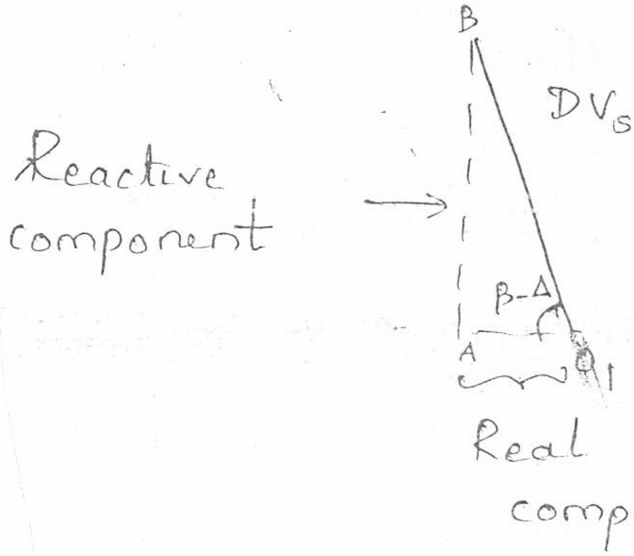
$$-BI_s = -BI_s \times \frac{-V_s}{B} = I_s V_s.$$

* When comparing all the terms with the Vge vector terms all the terms should be xlybed by $\frac{-V_s}{B}$ & rotate by an angle of $\angle 180^\circ - \beta$ to get the power circle diagram

* Rotating by 90° in the anticlock wise direction will give the power circle diagram



* Receiving end power V_sV_r/B depends mainly on DV_s^2/B rather than V_sI_s.
* $DV_s^2/B \rightarrow$ decides the centre of circle whose radius is O_1M.
* To find the centre for the power circle diagram, resolve DV_s^2/B into two compon



$O_1A = \frac{DV_s^2}{B}\cos(\beta-\Delta)$

$AB = \frac{DV_s^2}{B}\sin(\beta-\Delta)$

* With O_1 as centre and O_1M as radius draw the circle
* This gives sending end power circle diagram.

EQU OF CIRCLE

$$(x-x_1)^2 + (y-y_1)^2 = (\text{radius})^2$$

$$\left[P_s - \left(\frac{DV_s^2}{B}\cos\beta-\Delta\right)\right]^2 + \left[Q_s - \left[\frac{DV_s^2}{B}\sin\beta-\Delta\right]\right]^2 = \left(\frac{V_s V_r}{B}\right)^2$$

CHARACTERISTIC IMPEDANCE (Z_c):

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{z \times l}{y \times l}} = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{r + j\omega l}{g + j\omega c}}$$

For a lossless line $r = 0$, $g = 0$.

$$Z_0 = \sqrt{\frac{l}{c}} = \sqrt{\frac{L}{C}}$$

$Z_0 = \sqrt{\frac{L}{C}}$ is said to be surge impedance (or) natural impedance.

UNIT-4

INSULATORS, CABLES AND OVERHEAD LINES

Insulators - Types and Construction - Voltage Distribution in String Insulator - string Efficiency - Methods of Improving String Efficiency - Cables - types - Capacitance of Cables - Insulation Resistance - Dielectric Stress and Grading - Dielectric Loss - Thermal Characteristics - capacitance of Three Core Cables - Stress and Sag Calculations - Effect of Wind and Ice - Supports at Different Levels - stinging Chart.

Types of Insulators

Pin Type Insulator

- Voltages upto 33kV
- >33kV too bulky and uneconomical

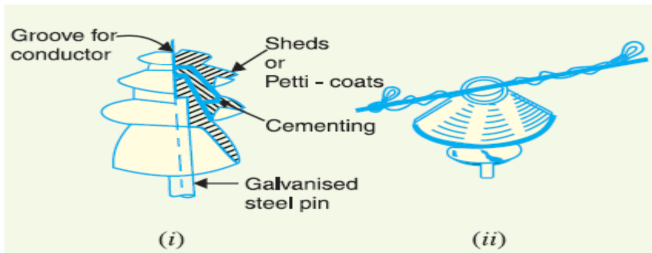


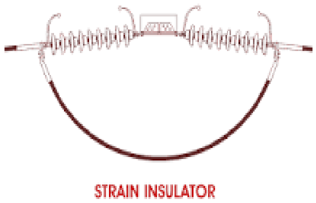
Fig. 8.5. Pin-type insulator

Suspension Type

- Voltage > 33kV
- No.of procelain discs connected in series by metal links.
-

Strain Type

- When there is a dead end of the line or there is corner or sharp curve or at intermediate towers, strain insulators are used.



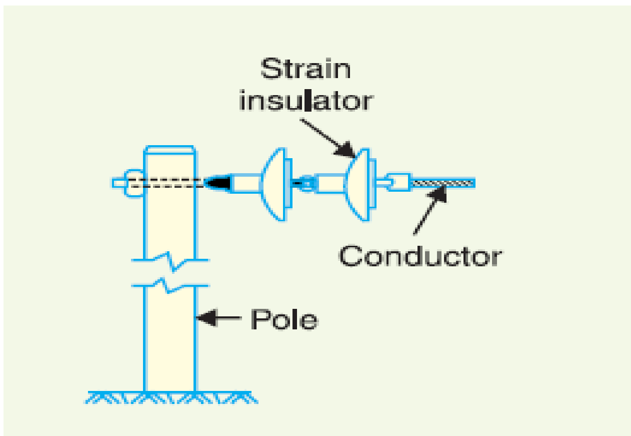


Fig. 8.8. Strain insulator.

Shackle Type

- In early days, shackle insulators are used as strain insulators
- Used for low voltage distribution lines.
- Either in horizontal or vertical position

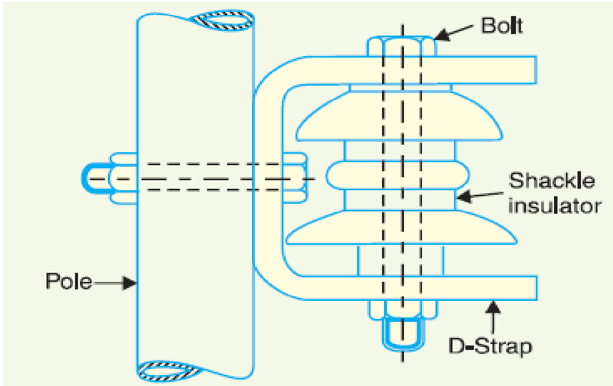
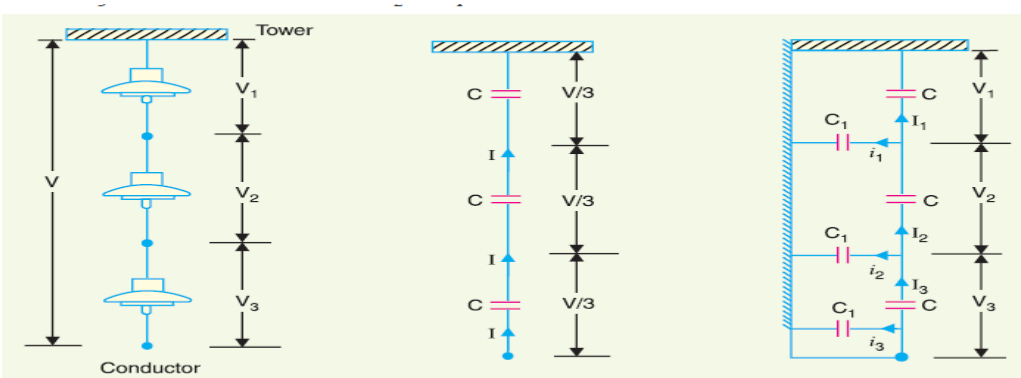


Fig. 8.9

Properties of insulator

- High mechanical strength in order to withstand conductor load, wind load etc.
- High electrical resistance of insulator material in order to avoid leakage currents to earth.
- High relative permittivity of insulator material in order that dielectric strength is high.
- The insulator material should be non-porous, free from impurities and cracks otherwise the permittivity will be lowered

Potential Distribution over Suspension Insulator String



terms of string efficiency.

The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor is known as **string efficiency** *i.e.,*

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

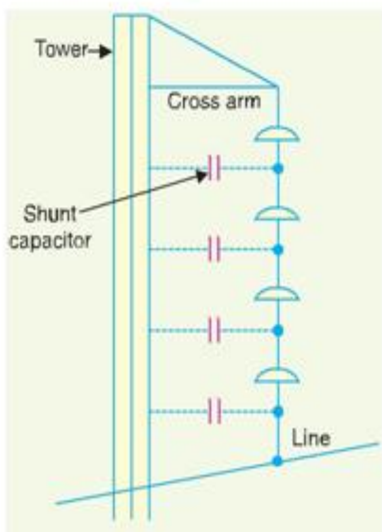
where n = number of discs in the string.

Methods of Improving String Efficiency

By using cross arms

- The value of *K* can be decreased by reducing the shunt capacitance. In order to reduce shunt capacitance, the distance of conductor from tower must be increased *i.e.,* longer cross-arms should be used.
- Limitations-cost and strength of tower do not allow the use of very long cross-arms.
- In practice, $K = 0{\cdot}1$ is the limit that can be achieved by this method

By using longer cross-arms

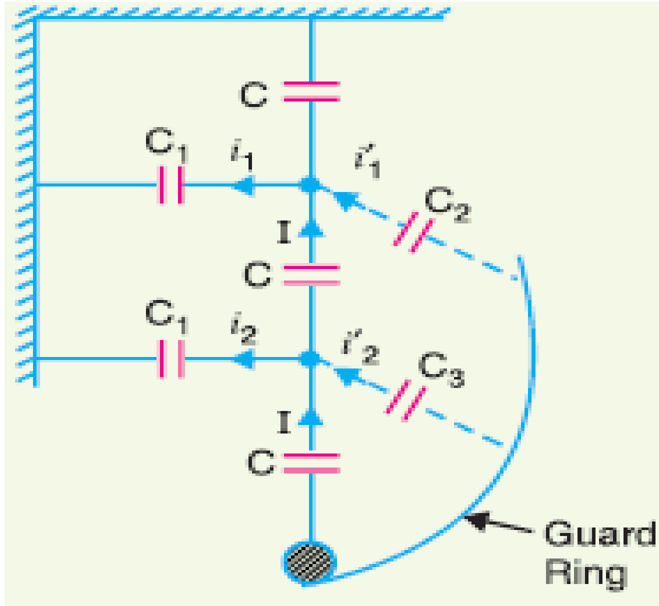


By grading the insulators

- The top unit has the minimum capacitance, in-creasing progressively as the bottom unit (*i.e.,* nearest to conductor) is reached.
- Voltage is inversely proportional to capacitance, this method tends to equalize the potential distribution across the units in the string.

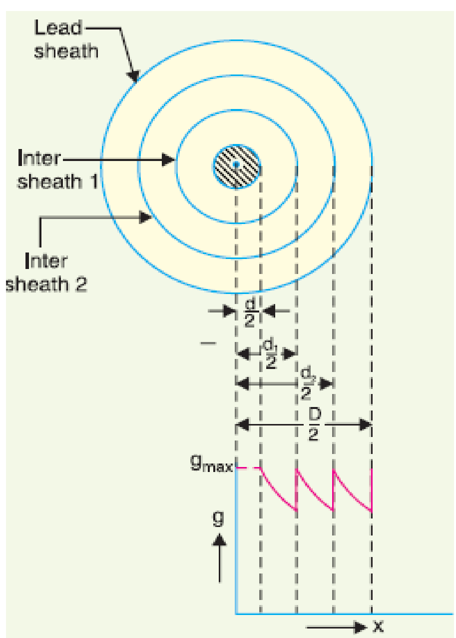
Disadvantage -that a large number of different-sized insulators are required

By using a guard ring



Inter sheath Grading

- In this method of cable grading, a homogeneous dielectric is used, but it is divided into various layers by placing metallic inter sheaths between the core and lead sheath.
- The inter sheaths are held at suitable potentials which are in between the core potential and earth potential.



Maximum stress between core and intersheath 1 is

$$g_{1max} = \frac{V_1}{\frac{d}{2}\log_e \frac{d_1}{d}}$$

Similarly,

$$g_{2max} = \frac{V_2}{\frac{d_1}{2}\log_e \frac{d_2}{d_1}}$$

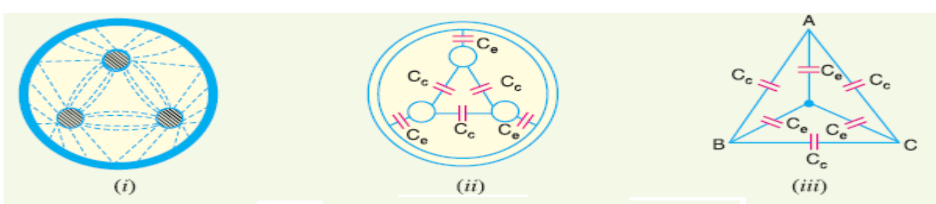
$$g_{3max} = \frac{V_3}{\frac{d_2}{2}\log_e \frac{D}{d_2}}$$

Since the dielectric is homogeneous, the maximum stress in each layer is the same *i.e.*,

$$g_{1max} = g_{2max} = g_{3max} = g_{max} \text{ (say)}$$

$$\therefore \quad \frac{V_1}{\frac{d}{2}\log_e \frac{d_1}{d}} = \frac{V_2}{\frac{d_1}{2}\log_e \frac{d_2}{d_1}} = \frac{V_3}{\frac{d_2}{2}\log_e \frac{D}{d_2}}$$

Capacitance of 3-Core Cables



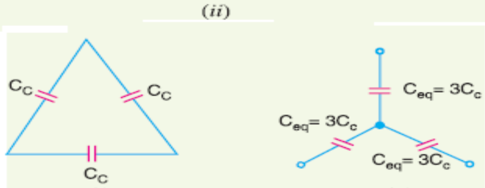
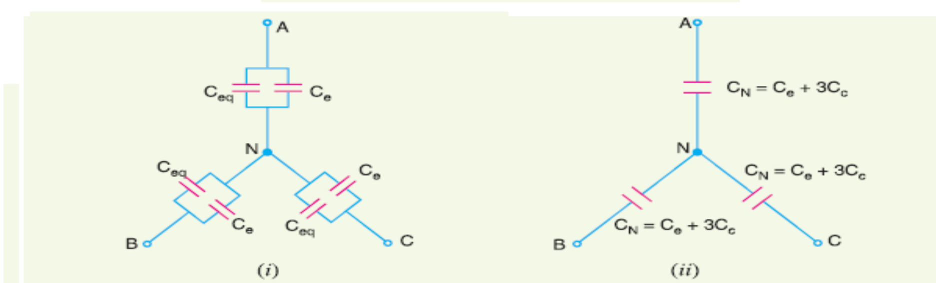


Fig. 11.19



$$C_N = C_e + C_{eq}$$
$$= C_e + 3C_c$$

If V_{ph} is the phase voltage, then charging current I_C is given by ;

$$I_C = \frac{V_{ph}}{\text{Capacitive reactance per phase}}$$
$$= 2\pi f V_{ph} C_N$$
$$= 2\pi f V_{ph} (C_e + 3C_c)$$

Thermal Characteristics

- Current-Carrying Capacity of Underground Cables
  - The safe current-carrying capacity of an underground cable is determined by the maximum permissible temperature rise. The cause of temperature rise is the losses that occur in a cable which appear

(i) *Copper losses in the conductors*

(*ii) Hysteresis losses in the dielectric*

(*iii) Eddy current losses in the sheath*

- The safe working conductor temperature is
  - 65ºC for armoured cables and
  - 50ºC for lead-sheathed
- The maximum steady temperature conditions prevail when the heat generated in the cable is equal to the heat dissipated.
- The heat dissipation of the conductor losses is by conduction through the insulation to the sheath from which the total losses (including dielectric and sheath losses) may be conducted to the earth.
- Therefore, in order to find permissible current loading, the thermal resistivities of the insulation, the protective covering and the soil must be known.

1. Thermal Resistance

The thermal resistance between two points in a medium (*e.g.* insulation) is equal to temperature difference between these points divided by the heat flowing between them in a unit time *i.e.*

$$\text{Thermal resistance, } S = \frac{\text{Temperature difference}}{\text{Heat flowing in a unit time}}$$

In SI units, heat flowing in a unit time is measured in watts.

$$\therefore \quad \text{Thermal resistance, } S = \frac{\text{Temperature rise } (t)}{\text{Watts dissipated } (P)}$$

or

$$S = \frac{t}{P}$$

$$\therefore \quad S \propto \frac{l}{a}$$

or

$$S = k\frac{l}{a}$$

where k is the constant of proportionality and is known as *thermal resistivity*.

2. Thermal Resistance of Dielectric of a Single-Core Cable

Let us now find the thermal resistance of the dielectric of a single-core cable.

Let r = radius of the core in metre

r_1 = inside radius of the sheath in metre

k = thermal resistivity of the insulation (*i.e.* dielectric)

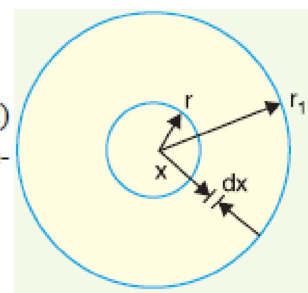


Fig. 11.21

Consider 1m length of the cable. The thermal resistance of small element of thickness dx at radius x is (See Fig. 11.21)

$$dS = k \times \frac{dx}{2\pi x}$$

$\therefore$ Thermal resistance of the dielectric is

$$S = \int_r^{r_1} k \times \frac{dx}{2\pi x}$$

$$= \frac{k}{2\pi}\int_r^{r_1} \frac{1}{x}\, dx$$

$$\therefore \quad S = \frac{k}{2\pi}\log_e \frac{r_1}{r} \text{ thermal ohms per metre length of the cable}$$

The thermal resistance of lead sheath is small and is generally neglected in calculations.

3. Permissible Current Loading

When considering heat dissipation in underground cables, the various thermal resistances providing a heat dissipation path are in series. Therefore, they add up like electrical resistances in series. Consider a cable laid in soil.

Let I = permissible current per conductor

n = number of conductors

R = electrical resistance per metre length of the conductor at the working temperature

S = total thermal resistance (*i.e.* sum of thermal resistances of dielectric and soil) per metre length

t = temperature difference (rise) between the conductor and the soil

Neglecting the dielectric and sheath losses, we have,

$$\text{Power dissipated} = n I^2 R$$

Now

$$\text{Power dissipated} = \frac{\text{Temperature rise}}{\text{Thermal resistance}}$$

or

$$n I^2 R = \frac{t}{S}$$

$\therefore$ Permissible current per conductor in given by;

$$I = \sqrt{\frac{t}{n R S}}$$

Mechanical design of OH line

SAG

Overhead Line Sag

While building an overhead line, it is crucial that conductors are under safe tension. If the conductors are too stretched between supports in an attempt to save conductor material, the stress in the conductor may reach critical value and in some cases the conductor may break due to excessive tension. In order to secure conductor safe tension, they are not completely stretched but are allowed to have a dip or sag. The difference in level between support points and the conductor lowest point is called sag. Figure 23 (a) presents a conductor suspended between two equilevel supports A and B. The conductor is not completely stretched but is allowed to have a dip. The conductor lowest point is O and the sag is S. The following items can be noted:

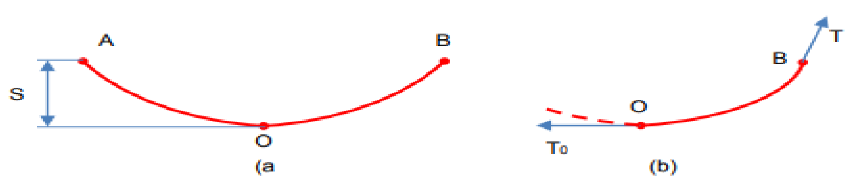


Figure 23. Conductor suspension between two supports

Sag Calculation – when supports are at equal levels

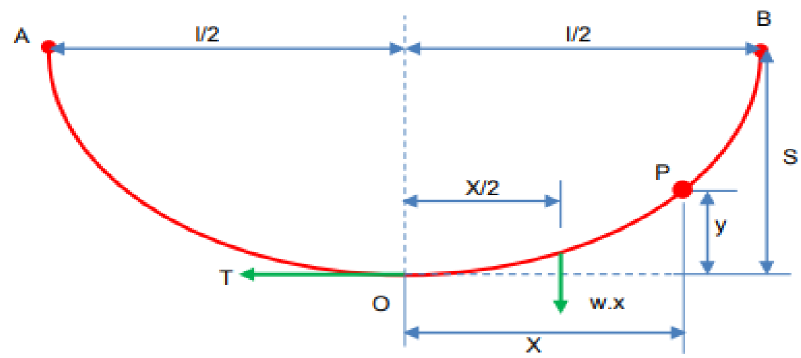


Figure 24. Conductor between two equilevel supports

- **L- length of conductor**
- **W- weight of conductor**
- **T- tension on the conductor**

Consider a point P on the conductor. Considering the lowest point O as the origin, let the co-ordinates of point P be x and y. Assuming that the curvature is so small that curved length is equal to its horizontal projection (for example, OP=x), the two forces acting on the portion OP of the conductor are:

(a) The conductor weight *wx* acting at a distance x/2 from O.

(b) The tension T acting at O.

Equating the moments of above two forces about point O, we find:

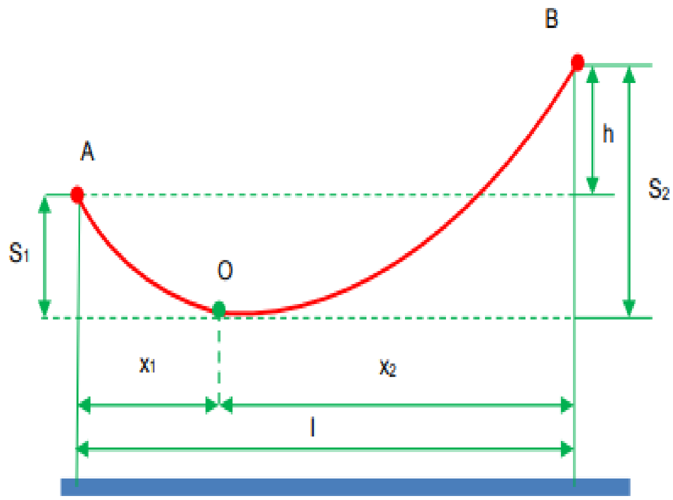
$$Ty = wx \times \frac{x}{2}$$

$$y = \frac{wx^2}{2T}$$

The maximum dip (sag) is expressed by the value of y at either of the supports A and B. At support A, x=l/2 and y=S

Sag, $S = \frac{w(l/2)^2}{2T} = \frac{wl^2}{8T}$

Sag Calculation- when supports are at unequal levels



If w is the conductor weight per unit length, then,

$$Sag\ \ S_1 = \frac{wx_1^2}{2T}$$

and

$$Sag\ \ S_2 = \frac{wx_2^2}{2T}$$

Also

$$x_1 + x_2 = l \qquad (1)$$

Now

$$S_2 - S_1 = \frac{w}{2T}[x_2^2 - x_1^2] = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

$$S_2 - S_1 = \frac{wl}{2T}(x_2 - x_1) \qquad x_1 + x_2 = l$$

But

$$S_2 - S_1 = h$$

$$h = \frac{wl}{2T}(x_2 - x_1)$$

Or

$$x_2 - x_1 = \frac{2Th}{wl} \qquad (2)$$

Solving expressions (1) and (2), it can be found:

$$x_1 = \frac{l}{2} - \frac{Th}{wl}$$

$$x_2 = \frac{l}{2} + \frac{Th}{wl}$$

Wind and ice loading effect

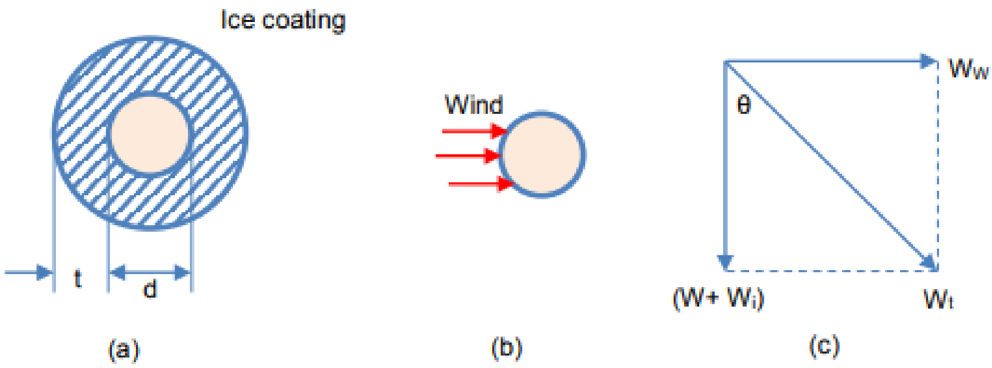


Figure 26. Wind effect on the conductor

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

Where

w - conductor weight per unit length (conductor material density x volume per unit length)

w_i - ice weight per unit length (density of ice x volume of ice per unit length)

w_w - wind force per unit length (wind pressure per unit area x projected area per unit length)

When the conductor has wind and ice loading, the following points have to be considered:

When the conductor has wind and ice loading, the following points have to be considered:

- The conductor sets itself in a plane at an angle to the vertical where

$$\tan\theta = \frac{w_w}{w + w_i}$$

- The sag in the conductor is expressed as:

$$S = \frac{w_t l^2}{2T}$$

Therefore, S represents the slant sag in a direction making an angle to the vertical. If no specific mention is made in the problem, then slant slag is found by using the above equation.

- The vertical sag=Scosθ

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

w = weight of conductor per unit length

= conductor material density × volume per unit length

w_i = weight of ice per unit length

= density of ice × volume of ice per unit length

= density of ice × $\frac{\pi}{4}$ $[(d + 2t)^2 - d^2] \times 1$

= density of ice × $\pi\, t\, (d + t)$*

w_w = wind force per unit length

= wind pressure per unit area × projected area per unit length

= wind pressure × $[(d + 2t) \times 1]$

When the conductor has wind and ice loading also, the following points may be noted :

(*i*) The conductor sets itself in a plane at an angle θ to the vertical where

$$\tan\theta = \frac{w_w}{w + w_i}$$

(*ii*) The sag in the conductor is given by :

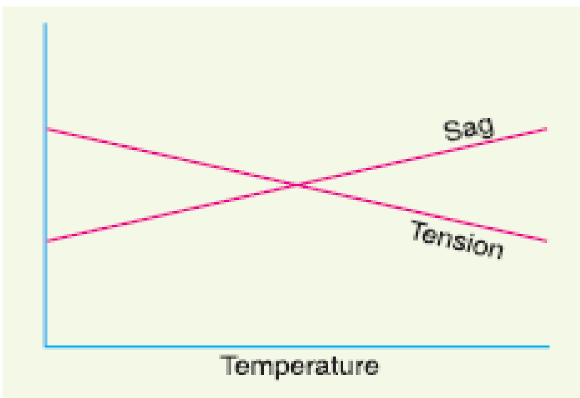
$$S = \frac{w_t\, l^2}{2T}$$

Hence S represents the slant sag in a direction making an angle θ to the vertical. *If no specific mention is made in the problem, then slant slag is calculated by using the above formula.*

(*iii*) The vertical sag $= S\cos\theta$

Stringing charts

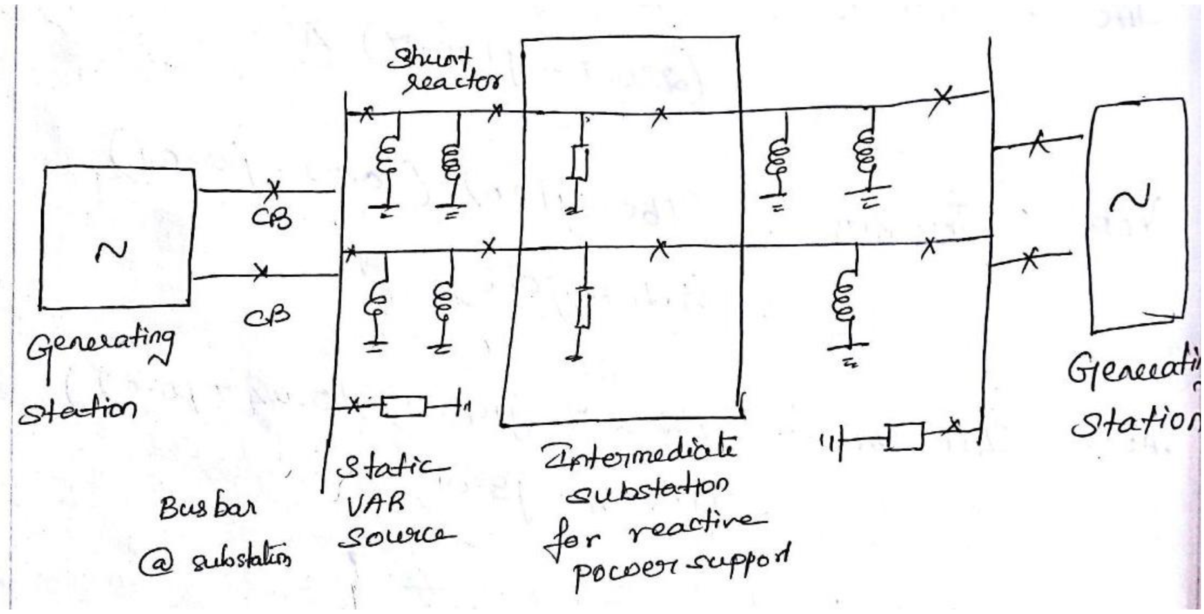
- For use in the field work of stringing the conductors, temperature-sag and temperature-tension. Charts are plotted for the given conductor and loading conditions. Such curves are called stringing charts



UNIT – 5

RECENT TRENDS IN TRANSMISSION

Extra High Voltage AC (EHVAC) Transmission



- < 300 kV- High voltage
- 300kV to 765kV- Extra High Voltage (EHVAC)
- >765kV- Ultra High Voltage
- EHVAC requires minimum **two parallel** three phase transmission circuits to ensure **reliability and stability** during a fault on any one phase of three phase lines
- EHVAC line also requires one or more intermediate substations for installing series capacitors, shunt reactors, switching protection equipment.
- **Intermediate substation is required at an interval of 250-300km**.
- Shunt reactor
- Neutralize the Ferranti effect
- Series capacitor

- Improves the power handling capacity of the transmission line.
- Static VAR source
- Inject the reactive power and control the receiving end voltages.

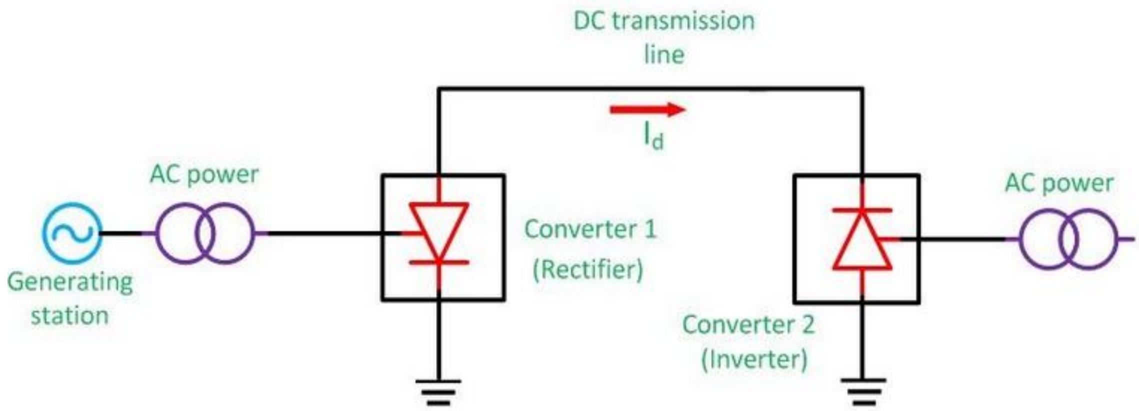
Advantages of Extra High Voltage AC (EHVAC) Transmission

1. Reduction in current.
2. Reduction in losses.
3. Reduction in Volume of conductor material required.
4. Decrease in voltage drop.
5. Increase in transmission efficiency.

Disadvantages of Extra High Voltage AC (EHVAC) Transmission

1. Corona loss and radio interference.
2. Difficult in erection.
3. More number of insulation needs.
4. Line supports

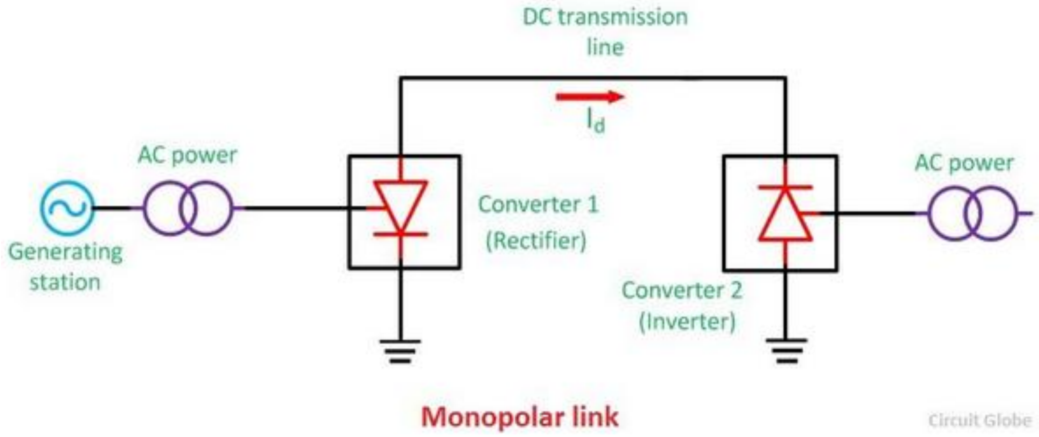
High Voltage Direct current Transmission (HVDC)



TYPES OF HVDC

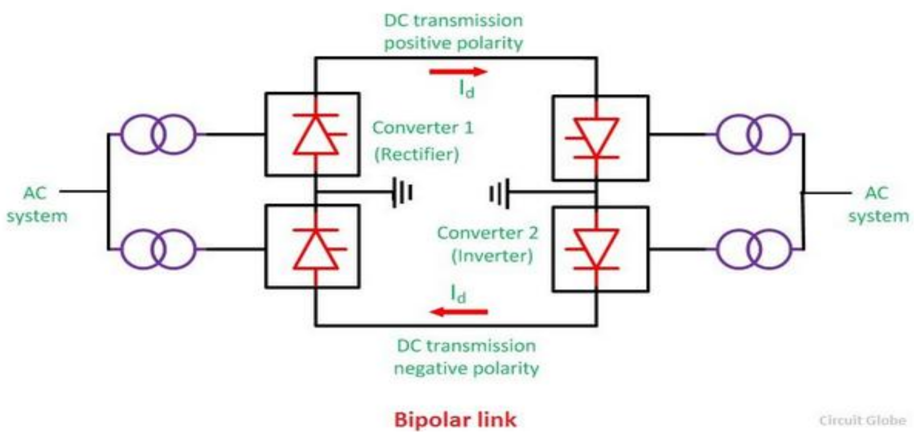
1. Mono polar link
2. Bi-polar link
3. Homo polar link
4. Back to back link

MONOPOLAR



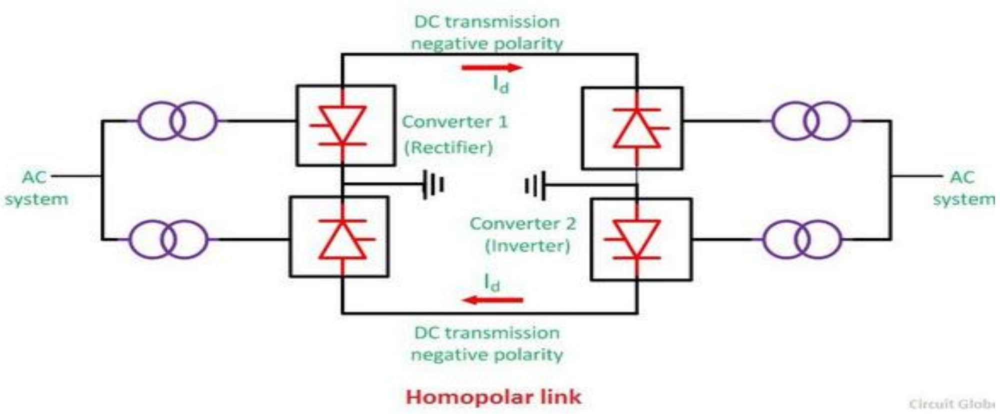
1. It uses one conductor.
2. The return path is provided by ground or water.

BIPOLAR



1. Each terminal has two convertors of equal rated voltage, connected in series on the DC side.
2. The junction between the convertors is grounded

HOMO POLAR



1. It has two or more conductors are all having same polarity, usually in negative.
2. Since the corona effect is less in DC transmission lines, homopolar link is usually operated with negative polarity.

Advantages of HVDC

- Full control over power transmitted.
- Economical for bulk transmission of power for long distances as cost of conductor decreases since DC system requires only tow conductors. Therefore, transmission losses decreased.
- No stability problem
- Skin effect is low.
- No reactance drop, therefore no voltage regulation problem.
- Corona loss is low.
- Radio interference is less.
- Intermediate substation not required.

Disadvantages of HVDC

- Transformer cannot be used at intermediate stage to boost the voltage.

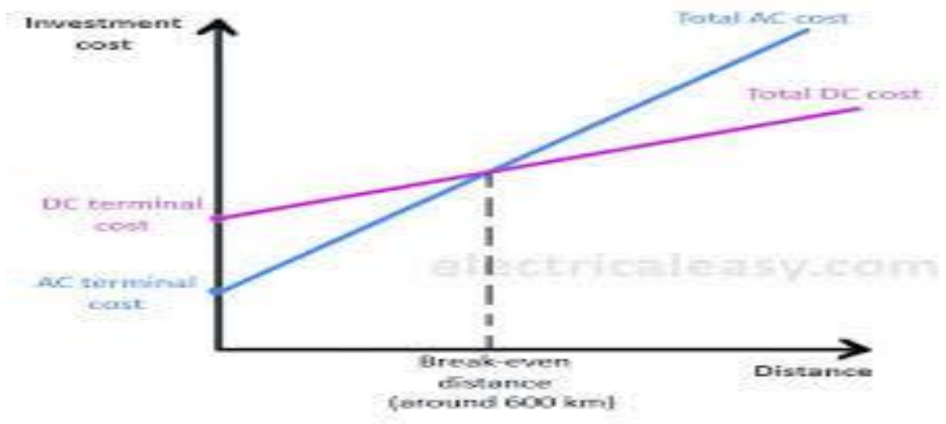
- Cost of converters and inverters are higher.
- Converter and inverter generates the harmonic on both AC and DC sides

The AC and DC harmonic filters are costly

Application of HVDC

- Long distance bulk power transmission.
- Cement industry
- Communication systems.
- Under ground or under water cables.
- Testing of HVAC cables of long length.

Economic Distance for HVDC



- An HVDC transmission line costs less than an AC line for the same transmission capacity.
- However, it is also true that HVDC terminal stations are more expensive due to the fact that they must perform the conversion from AC to DC, and DC to AC. But over a certain distance, the so called "**break-even distance" (approx. 600 – 800 km)**, the HVDC alternative will always provide the lowest cost.

Introduction to FACTS

FACTS technology consists of high power electronic based equipments with its real time operating control. There are two groups of FACTS controllers based on different technical approaches, both resulting in controllers able to solve transmission problems. The first group employs reactive impedances or tap-changing transformers with thyristor switches as controlled elements; the second group employs self-commutated voltage source switching converters**.**

TYPES OF FACTS CONTROLLERS

In general FACTS controllers can be divided into four categories:

Series controllers

Shunt controllers

Combined series-series controllers

Combined series-shunt controllers

Series Controllers

The series controllers could be variable impedance, such capacitor, reactor, etc, or power electronics based variable source main frequency, sub synchronous and harmonic frequencies (series combination) to serve the desired need. In the principle, all series controllers inject voltage in series with the line. As long as Voltage is in phase quadrature with the line current, the series controller only supplies or consumes variable reactive power. The other phase relationship will involve handling of real power as well. The series controllers are shown in figure.

Shunt Controllers

As in the case of series controllers, the shunt controllers may be variable impedance, variable source impedance, a combination of these. In the principal, all shunt controllers in current into the system at the point of connection. Even variety current flows and hence, represents injection current is in phase quadrature with the line voltage, the shunt controller only supplied consumes variable reactive power. Any other phase relationship involve handling of the real power flow.

Combined Series-Series Controllers

This could be a combination of separate series controllers which are controlled in a coordinated manner in a multi-line transmission system or it could be a unified controller, in which series controllers provide independent series reactive compensation for each line and also transfer real power capability of the unified series-series controller, referred to as Interline power flow controller, makes it possible to balance both real and reactive power flow in the lines and thereby maximize the utilization of the transmission system.(Unified here means that the DC terminals of all controller converters together for real power transfer).

Combined Series-Shunt Controllers

This could be a combination of separate shunt and series controllers which are controlled in a co-ordinate manner or a unified power flow controller with series and shunt elements. In principle, combined series and shunt controllers inject current into the system with the shunt part of the controller and voltage in series in the line with the series part of the controller. However, when the shunt and series controllers are unified, there can be real power exchange between the series and shunt capacitors via the power link.

ADVANTAGES OF FACTS DEVICES

In the present day scenario, transmission systems are becoming increasingly stressed, more difficult to operate, and more insecure with unscheduled power flows and greater losses because of growing demand for electricity and restriction on the construction of new lines. However, many high voltage transmission systems are operating below their thermal ratings due to constraints, such as voltage and stability limits. Now, more advanced technology is used for reliable and operation of transmission and distribution in power system to achieve both reliable and benefit economically, it has become clearer that more efficient utilization and control of the existing transmission system infrastructure is required. Improved utilization of the existing power system is provided through the application of advanced control technologies. Power electronics has developed the flexible AC transmission system (FACTS) devices. FACTS devices are effective and capable of increasing the power transfer capability of a line and support the power system to work with comfortable margins of stability. FACTS devices are used in transmission system to control and utilize the flexibility and system performance. To achieve all, the insertion of FACTS devices required in plant in order to control the main parameters namely voltage, phase angle and impedance, which is affecting ac power transmission. The power system should be capable to for line support of power transfer with comfortable and stable for marginally.

APPLICATIONS OF FACTS DEVICES

FACTS devices have the following applications

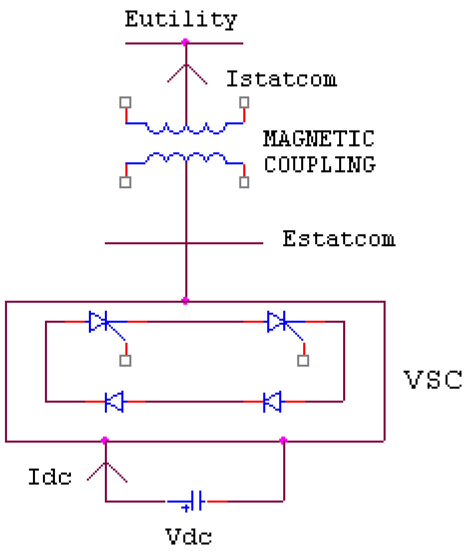
- Power flow control
- Increase of transmission capability
- Voltage control
- Reactive power compensation
- Stability improvement
- Power quality improvement
- Power conditioning
- Flicker mitigation
- Interconnection of renewable and distributed generation and storages

STATCOM

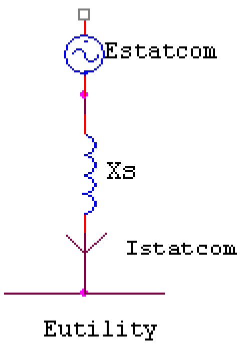
The STATCOM is a shunt connected reactive compensation equipment, which is capable of generating or absorbing reactive power whose output can be varied so as to maintain control of specific parameters of the electric power system. A STATCOM is usually used to control transmission bus voltage by reactive power shunt compensation.

PRINCIPLE OF OPERATION

The power circuit diagram for STATCOM is shown in Figure . It is a controlled reactive power source. It provides the reactive power generation and absorption by means of electronic process of the voltage and current waveforms in a voltage source.



A single line diagram of STATCOM is shown in Figure , where Vsc is connected to the utility bus through the magnetic coupling transformer. It is a compact design, small foot print, low noise and low magnetic impact. The exchange of reactive power between the converter and AC system can be controlled by varying the three phase output voltage, E statcom of the converter.



If the amplitude of the output voltage is increased above the utility bus voltage, then the current flows through the reactance from the converter to the AC system and the converter acts as a capacitance and generates reactive power for the AC system.

If the amplitude of the output voltage is decreased below the utility bus voltage, then the current flows through the reactance from the AC system to the converter and the converter act as inductance and it absorbs the reactive power from the AC system.

If the output voltage equals the AC system, then the reactive power exchange becomes zero. In that condition, STATCOM is said to be in a floating state. STATCOM controller provides voltage support by generating or absorbing reactive power at the point of common coupling without the need of large external reactors or capacitor banks.

STATCOM controller provides voltage support by generating or absorbing reactive power at the point of common coupling without the need of large external reactors or capacitor banks.

In other words, the inverter can supply real power to the AC system from its DC energy storage if the inverter output voltage is made to lead the AC system voltage. On the other hand, it can absorb real power from the AC system for the DC system if its voltage lags behind the AC system voltage. A STATCOM provides the desired reactive power by exchanging the instantaneous reactive power among the phases of the AC system.

APPLICATIONS OF STATCOM

- Power flow control
- Increase of transmission capability
- Voltage control
- Reactive power compensation
- Stability improvement
- Power quality improvement

UNIFIED POWER FLOW CONTROLLER

UPFC is the most comprehensive multivariable flexible ac transmission system (FACTS) controller. Simultaneous control of multiple power system variables with UPFC poses enormous difficulties. In addition, the complexity of the UPFC control increases due to the fact that the controlled and the control variables interact with each other. The Unified power flow controller (UPFC) enables independent and simultaneous control of a transmission line voltage, impedance, and phase angle. This has far reaching benefits: in steady state, the UPFC can be used to regulate the power flow through the line and improve utilization of the existing transmission system capacity; and, during power system transients, the UPFC can be used to mitigate power system oscillations and aid in the first swing stability of interconnected power systems

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FUNCTIONS OF UPFC SYSTEM

In today's power system worldwide, the transient and dynamic stability margin is reduced due to increased power transfer, With proper control strategies, fast responding Flexible AC Transmission Systems (FACTS) could be used to improve the transient and dynamic performance of the system so that the system transmission could be safely expanded by increasing the level of utilization of the existing facilities towards their thermal limits. On the other hand, by fully taping their ability to shape the transient and dynamic response of the system, the higher cost (compared with the traditional mechanically controlled power flow controller, e.g. adjustable transformers and capacitors banks) of FACTS devices can be better justified. In the FACTS family. UPFC is one of the most powerful and versatile FACTS devices available so far. Being able to almost instantaneously insert a synchronous voltage of arbitrary magnitude (within a pre-specified range) and phase angle (with respect to the sending-end voltage) into the transmission line, UPFC can be used to adjust the real electrical power output of an electric power system in real time. Thus, UPFC is regarded by many researchers as an ideal candidate for improving the transient and dynamic performance of an electric power system. The UPFC incorporated into the Philips-Heffron model of a linearized power system. Dramatic improvement in dynamic stability performance reported in their study. Nevertheless, due to the nature of linearization, the technique developed there cannot be extended for study the transient response of the system.

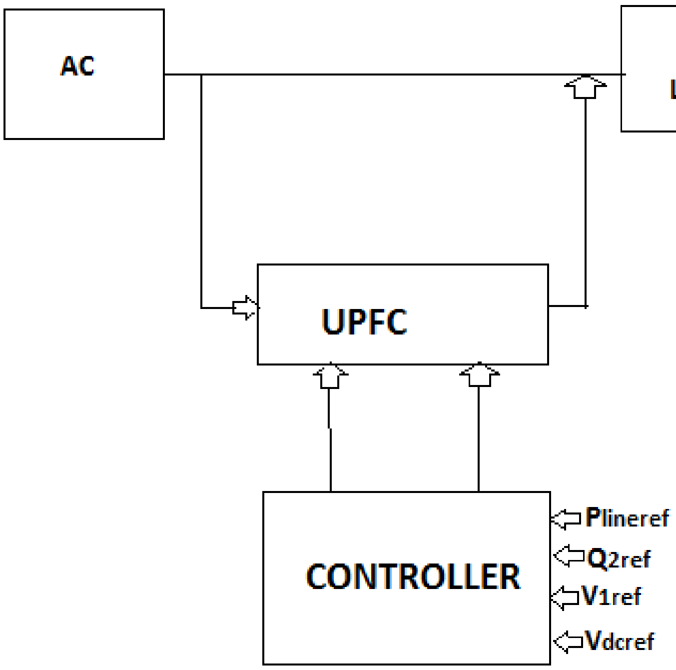
The UPFC is a member of the family of compensators and power flow controllers. The latter utilize the synchronous voltage sources (SVS) concept to provide a unique comprehension capability of transmission system control. The UPFC is able to connect simultaneously or selectively all the parameters affecting power flow patterns in a transmission network, including voltage magnitudes and phases, and real and reactive powers. These basic capabilities of the UPFC the most powerful device in the present day transmission and control systems.

BASIC OPERATING PRINCIPLES

The UPFC is device placed between two busses reference to as the UPFC sending bus and the UPFC receiving bus. It consist of two Voltage-Sourced Converters (VSCs) with a common DC line. The unified power flow controller is a second generation FACTS devices, which enables independent control of active and reactive power. It is a multifunction power flow controller with capabilities terminals voltage regulation, series line compensation and phase angle regulation.

The UPFC is a generalized SVS represented at the fundamental frequency by controllable voltage phasor of magnitude Vpq and angle injected in series with the transmission line. Note that the angle ρ can be controlled over the full range from 0 to 2π. For the system shown in figure 3.1, the SVS exchanges both real and reactive power with the transmission system. In the UPFC, the real power supplied to or absorbed from the system is provided by one of the end buses to which it is connected. This meets the objective of the UPFC to control power flow rather than increasing the objectives of the generation capacity of the system. As shown in Figure 3.2, the UPFC consists of two voltage-sourced converters, one in series and one in shunt, both using Gate Turn-Off (GTO) thyristor valves and operated from a common DC storage capacitor. The configuration facilities free flow of real power between the AC terminals of the two converters in either directions which enabling each converter to independently generate or absorbs reactive power at its own AC terminal.

The series converter, referred to as Converter 2, injected voltage with controllable magnitude Vpq and phase ρ in series where the line via an insertion transformer, thereby providing thefunction of the UPFC. This injected voltage phasor acts as a synchronous AC voltage source that provides real and reactive power exchange between the line and the AC systems. The reactive power exchanged at the terminal of series insertion transformer generated internally while the real power exchanged is converted to DC power and appears on the DClink as a positive or negative power demand.



By contrast, the shunt converter, referred to as Converter supplies or absorbs the real power demanded by Converter 2 on common DC link and supports the real power exchange results from the series voltage injection. It converts the DC power demand of Converter 2 into AC and couples it to the transmission line shunt connected transformer. Converter 1 can also generate or absorbs reactive power in addition to catering to the real power need Converter 2, consequently, it provides independent shunt compensation for the line. It is to be noted that the reactive power exchanged is generated locally and hence, does not have to be transmitted by the line. On the other hand, there exists a closed path for the real power exchanged by the series voltage that is injected through the converters back to the line. Thus, there can be a reactive power exchange between Converter 1 and the line by controlled or unity power factor operation. This exchange is independent of the reactive power exchanged by Converter 2.

APPLICATIONS

- The power- transmission capability is determined by the transient –stability.
- The UPFC also provides very significant damping to power oscillations when it operates at power flows within the operating limits.
- The dramatic enhancement in power-oscillation damping with the use of the UPFC are also reported in the planning study of the Meead- Phoneix project

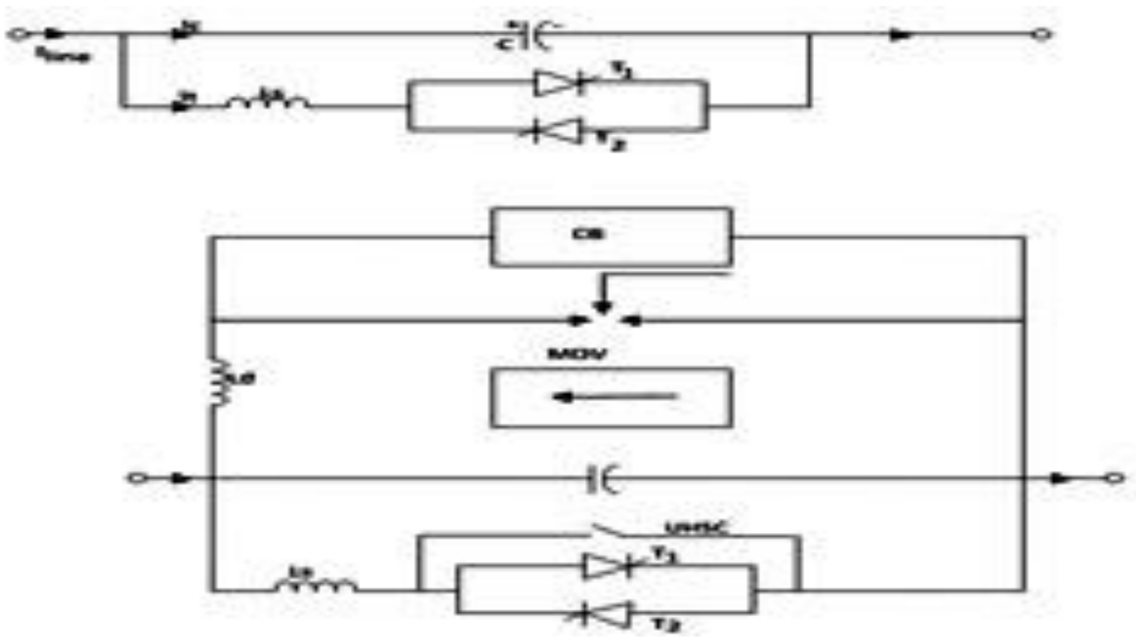
THYRISTOR CONTROL SERIES CAPACITOR (TCSC)

The basic conceptual TCSC module comprises a series capacitor, C, in a parallel with a thyristor - controlled reactor, Ls. However, a practical TCSC module also includes protective equipment normally installed with series capacitors.

A metal – oxide varistor (MOV), essentially a nonlinear resistor, is connected across the series capacitor to prevent the occurrence of high – capacitor over voltages. Not only does the MOV limit the voltage across the capacitor, but it allows the capacitor to remain in circuit even during fault conditions and helps improve the transient stability.Also installed across the capacitor is a circuit breaker, CB, for controlling its insertion in the line. In addition, the CB bypasses the capacitor if serve fault or equipment – malfunction events occur. A current – limiting inductor, Ld, is incorporated in the circuit to restrict both the magnitude and the frequency of the capacitor current during the capacitor – bypass operation.

If the TCSC valves are required to operate in the fully “on” mode for prolonged durations, the conduction losses are minimized by installing an ultra – high- speed contact (UHSC) across the valve. This metallic contact offers virtually lossless feature similar to that of circuit breakers and is capable of handling many switching operations. The metallic contact is closed shortly after the thyristor valve is turned on and it is opened shortly before the valve is turned off. During a sudden overload of the valve, and also during fault conditions the metallic contact is closed to alleviate the stress on the valve.

An actual TCSC system usually comprises a cascaded combination of many such TCSC modules, together with a fixed – series capacitor, Cf. This fixed series capacitor is provided primarily to minimize cost. The capacitors – C1, C2…..,Cn- in the different TCSC modules may have different values to provide a wider range of reactance control. The inductor in series with the anti parallel thyristors is split into two halves to protect the thyristor valves in case inductor short circuits.



TCSC module