UNIT -1

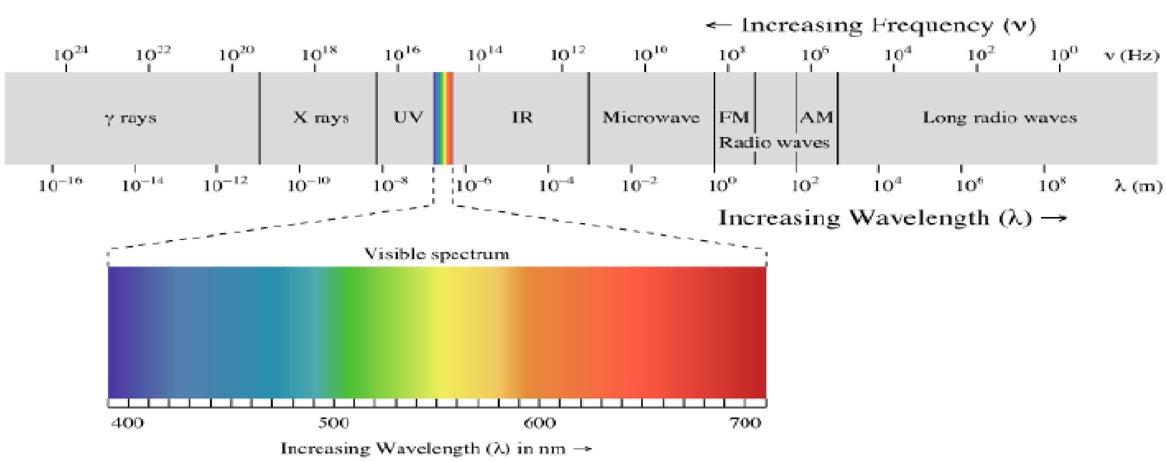
Microwave spectrum and bands-characteristics of microwaves-a typical microwave system. Traditional, industrial and biomedical applications of microwaves. Microwave hazards.S-matrix – significance, formulation and properties.S-matrix representation of a multi port network, S-matrix of a two port network with mismatched load.

1.1 INTRODUCTION

- Microwaves are electromagnetic waves (EM) with wavelengths ranging from 10cm to 1mm. The corresponding frequency range is 30Ghz (=10^9 Hz) to 300Ghz (=10^{11} Hz) . This means microwave frequencies are upto infrared and visible-light regions.
- The microwaves frequencies span the following three major bands at the highest end of RF spectrum.

  i) Ultra high frequency (UHF) 0.3 to 3 Ghz
  ii) Super high frequency (SHF) 3 to 30 Ghz
  iii) Extra high frequency (EHF) 30 to 300 Ghz
- Most application of microwave technology make use of frequencies in the 1 to 40 Ghz range.
- During world war II , microwave engineering became a very essential consideration for the development of high resolution radars capable of detecting and locating enemy planes and ships through a Narrow beam of EM energy.

- The common characteristics of microwave device are the negative resistance that can be used for microwave oscillation and amplification.



❖ **Electromagnetic Frequency Spectrum:**

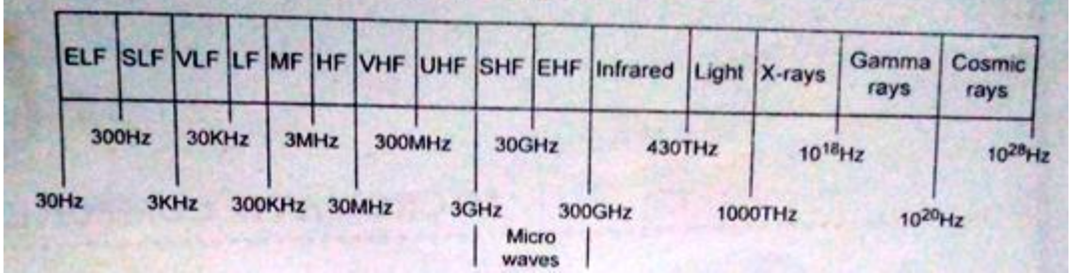


Fig 1.1 Electromagnetic spectrum

❖ **Frequency band designation:**

Frequency band	*Designation*	*Wave length*	*Applications*
30 – 300Hz	Extra Low Frequency (ELF)	10 – 1Mm	Communication with submarines.
3 – 30KHz	Very Low Frequency (VLF)	100 – 10Km	Long distance point – to – point communication, navigation, SONAR
30 – 300KHz	Low Frequency (LF)	10 – 1 Km	Point – to – Point marine communication, navigational aids.
300 – 3000KHz	Medium Frequency (MF)	1000 – 100m	AM broadcasting, marine communication, coast guard communication.
3 – 30MHz	High Frequency (HF)	100 – 10m	Telephone, telegraph, facsimile, short wave international communication.
30 – 300MHz	Very High Frequency (VHF)	10 – 1m	Television, FM broadcast, air – traffic control.
300 – 3000MHz	Ultra High Frequency (UHF)	100 – 10cm	Television, satellite communication, radar.
3 – 30GHz	Super High Frequency (SHF)	10 – 1cm	Radar, microwave and space communication, satellite communication.
30 – 300GHz	Extreme High Frequency (EHF)	10 – 1mm	Radar, microwave and space communication, satellite communication.

1.2 MICROWAVE SYSTEM

- A microwave system normally consists of a transmitter subsystems, including a microwave oscillator, wave guides and a transmitting antenna, and a receiver subsystem that includes a receiving antenna, transmission line or wave guide, a microwave amplifier, and a receiver.

- Reflex Klystron, gunn diode, Traveling wave tube, and magnetron are used as a microwave sources.
- Isolators provide low attenuation for the forward direction and high attenuation for reverse direction to avoid reflected power.
- Attenuator provides attenuation for the power depends upon the microwave application.

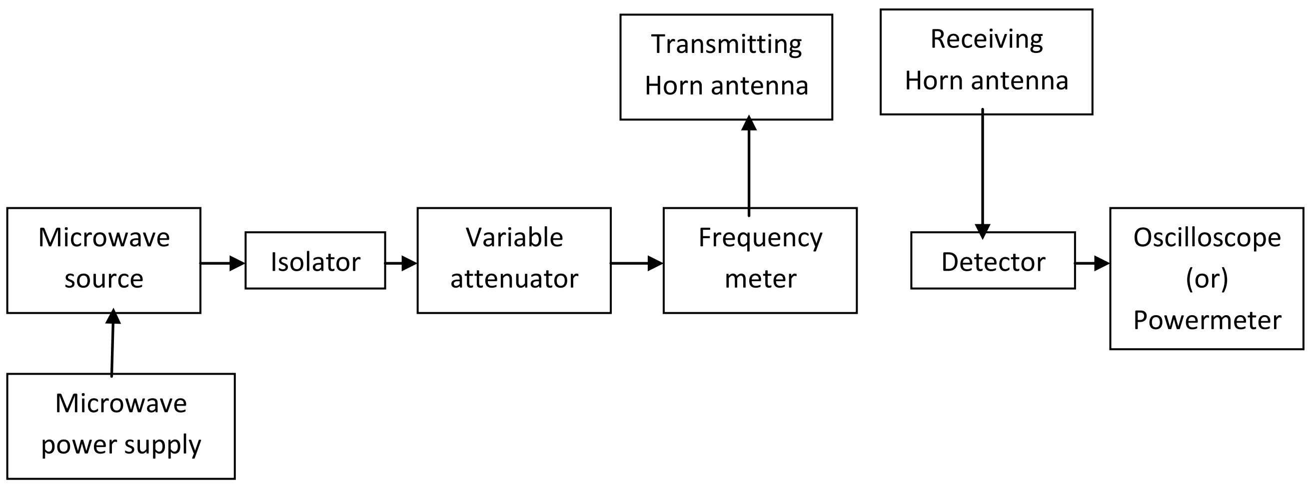


Fig 1.2 Microwave system

- In order to design a microwave system and conduct a proper test on it, an adequate knowledge of the components involved are essential.
- Therefore, a first course on microwave should include three major areas of study, namely

i) Microwave transmission lines and waveguides
ii) Microwave circuits elements, and
iii) Microwave source, amplifier and detector.

ADVANTAGES

i) Because of their high operating frequencies, microwave system can carry large quantities of information.
ii) High frequencies mean short wavelength, which require relatively small antennas.

iii) Microwave signals are more easily propagated around physical obstacles such as water and high mountains.

iv) Fewer repeaters are necessary for amplification.

v) Minimal crosstalk exists between voice channels.

vi) Increased reliability and less maintenance are important factors.

1) Each station requires the purchase or lease of only a small area of lands.

2) Increased bandwidth availability.

DISADVANTAGES

i) It is more difficult to analyze and design circuits at microwave frequencies.

ii) Measuring techniques are more difficult to perfect and implement at microwave frequencies.

iii) It is difficult to implement conventional circuits components (resistances, capacitors , inductor and so on) at microwave frequencies.

iv) Transient time is more critical at microwave frequencies.

v) It is often necessary to use specialized components for microwave frequencies.

vi) Microwave frequencies propagate in a straight line, which limits their use to line-of-sight applications.

APPLICATIONS

i) Currently, the microwave frequency spectrum is used primarily for telephone communications and radar. Many long- distance telephone systems use microwave relay links for carrying telephone calls.

ii) Microwave becomes a very popular tool in microwave radio spectrum for material analysis.

iii) Microwave landing system, used to guide aircraft to land safely at airports.

iv) Special microwave equipment known as diathermy machines are used in medicine for heating body muscles and tissues without hurting the skin.

v) Microwave ovens are a common appliance in most kitchens today.

vi) Microwave heating is also widely used in industry for a variety of heating and melting applications.

Applications in Bio-medical

In addition to radiation, another important use of microwave energy in medicine is for the thermal ablation of tissue. In this application microwave energy is used to create localised dielectric heating (diathermy) resulting in controlled destruction of tissue. Microwave ablation (MW ablation) is the next evolution of diathermy treatment and being a radiating technology overcomes many issues such as current conduction problems with grounding pads as used in high frequency and radio frequency diathermy.

Microwave ablation also provides desiccation of tissue without the excessive charring and nerve damage associated with RF ablation. Various applications include treatment of large tumors or removal of unwanted tissue masses, for example liver tumors, lung tumors and prostate ablation. Microwaves can also be used to coagulate bleeding in highly vascular organs such as the liver and spleen.

As microwaves have shorter wavelengths the choice of frequency can benefit the application, for example large volume ablations can typically be made at 915 MHz and 2.45 GHz and use of higher frequencies in the range 5.8 GHz - 10 GHz can create shallow penetration of energy resulting in very precise ablations suitable for treatments such as skin cancer, ablation of the heart to treat arrhythmia, uterine fibroids, multiple small liver metastases, corneal ablation (vision correction), spinal nerve ablation (back pain), varicose vein treatment, verrucae treatment and many other specific treatments.

About using microwaves in surgery is that they are uncontrollable. This has arisen as a result of using standard industrial magnetrons and basing measurements such as reflected power in microwave medical equipment on ideal 50 ohm microwave components. Modern microwave generators may employ stable reliable solid state sources however the dielectric properties of tissue varies considerably during treatments therefore microwave applicators (antennas) are not always optimally matched to an ideal 50 ohms which can result in significant mismatch. This can result in measurement uncertainty and VSWR problems which accounts for the perception of an uncontrollable treatment. Recent techniques, such as those developed by Emblation Limited, overcome this problem in medical microwave applications to create a mismatch tolerant controllable user experience that enhances patient safety and treatment reliability for the next generation of microwave ablation treatments.In the field of oncology MW ablation now offers a new tool in the arsenal of weapons to fight cancer, providing new opportunities to save many lives.

Frequencies 100 MHz -30 GHz.

a) Diagnostic applications: tumor detection based on differences in tissue electrical properties.

b) Regional hyperthermia integrated with MRI

c) Therapeutic applications based on local heating: prostate hyperplasia, heart and other tissue ablation, angioplasty.

d) MRI (& fMRI)

Industrial Applications

Industrial microwave applications are rapidly increasing. Their potential in certain areas, the food, rubber and textile industries arises from a combination of advantages: efficient energy conversion, automation and product quality control. Many other heating and drying applications are under consideration today; these include pharmaceutical drying, film drying, and veneer processing. Second and third generation equipment is now in successful use in industry, where applications range from macaroni drying and oyster processing to the rapid heating of institutional lunches; microwave proofing of bread products, for example, has been shown to require as little as one fifth of the energy needed with conventional heating methods.

Microwave Hazards

Microwave can produce thermal and non-thermal effects in biological systems.
The heating of tissues due to the absorption of microwave occurs due to the ionic conduction and vibration of dipole molecules of water and proteins present in the bodyl. The rise in temperature of the tissues depends up on the frequency and power of microwave radiation being absorbed and the cooling mechanisrt of the system.
When the thermoregulatory capability of the body or parts of the body is exceeded, tissue damage and death can result. This occurs at absorbed power levels far above the metabolic power output of the body. Death usually results from the diffusion of heat from the irradiated portion of the body to the rest of the body by the vascular system. When the absorbed energy increases due to the prolonged exposure or increase in power of radiation, the protecting mechanism of heat control breaks down, resulting in uncontrolled rise in body temperature. At low power of irradiation, one usually gets heedache, vomitting, intraocular pain, fatigue, nervousness, awareness of buzzing vibrations or pulsations and sensation of warmth. Most of these effects are not permanent. The non-thermal effects) are not related with the increase in temperature. One of such effects is known as pearlchain effect. This effect occurs in the frequency range of I to 100 MHz. Whe suspended particles of charcoal, starch, milk, erthrocytes or leucocytes (blood cells) are placed in the RF field, the particles form the chains parallel to the electric lines of force. The other non-thermal effect is the dielectric saturation in the solution of proteins and other biological macromolecules in the presence of intense microwave fields.

• **Radiation**- is energy transmitted through space in the form of electromagnetic waves or sub-atomic particles
• Examples include:
Radiofrequency (RF) Radiation, Microwaves, Infrared, Visible, Ultraviolet Light, X-rays and Gamma Rays .The term “Electromagnetic Radiation” is restricted to that portion of

the spectrum commonly defined as the radio frequency region, which for our purposes also includes the microwave frequency region.

HERP

• Observed Thermal effects (areas exceeding the MPE) :
– Heating of the body (Developing fetus is at no greater risk than mother)
– Cataracts
– Reduced sperm count in males
– Perception
– Auditory (>100 mW/cm2) buzzing, clicking, hissing
– Work Disturbance (based on animal studies)
• RF Current Effects
– Shocks or Burns
– Neural Stimulation (Extremely Low Frequency – (0-3 kHz)
(tissue damage at 10 x MPE)

Signs and Symptoms of possible over exposure may include:

Confusion, Vertigo, Headache,Blurred vision, Overall nauseous feeling, Body heating (Heat Stress), Shocks and burns, Bad or metallic taste in mouth

RF shock or burns

May occur when you come into contact with either an RF radiator like an antenna. Many antenna designs cause RF current to flow in their metallic components, which in turn, is radiated into space. Touch one of these surfaces,and the energy will flow through your body to ground. Similarly, the same thing can happen if you touch a reradiator. Any ungrounded, conductive (usually metal) object that is in the field of a strong RF source can be illuminated by the RF field and re-radiate the energy back into space.When you touch a re-radiator, you provide a path to ground through you. A surge of energy occurs at the point of contact. This results in a shock and, in many cases, an RF burn.The primary factors that determine if you will receive a shock or burn should you contact a conductive object are the strength of the electric field, the frequency, how well grounded you are, and how much of your body touches the object.Severe burn-hazard conditions may exist where the RF field level may be less than 1 percent of the MPE limit.

• **HERO**

Premature activation of electro-explosive devices (EED). Safe distances are calculated and based on worse case most sensitive devices.

1.3 SCATTERING OR (S) PARAMETERS

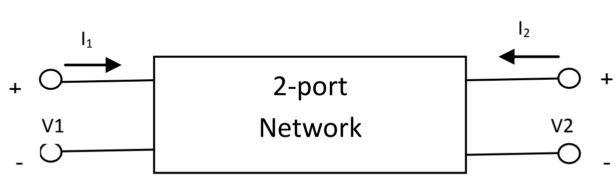


Fig. 1.3 Two port network

Low Frequency circuit can be described by two port networks and their parameters such as Z,Y,H,ABCD etc. as per network theory. Here network parameters relate the total voltages and total currents as shown in fig. 1.3.

In similar way at microwave frequencies , we talk of travelling waves with associated powers instead of voltages and current and the microwave junction can be defined by what are called as S-parameters or scattering parameters(similar to H , Y , Z parameter).

Referring to fig.1.4, it can be seen that for an input at one port, we have four outputs. Similarly if we apply inputs to all the ports, we have 16 combinations, which are represented in matrix form and that matrix is called as SCATTERING MATRIX. It is a square matrix which gives all the combinations of power relationships between the various input and output port of a microwave junction. The elements of this matrix are called scattering coefficients or Scattering (S) parameters.

To obtain the relationship between the scattering matrix and the input/output powers at different ports , Consider a junction of 'n' number is terminated in a source as shown in fig. 1.5.

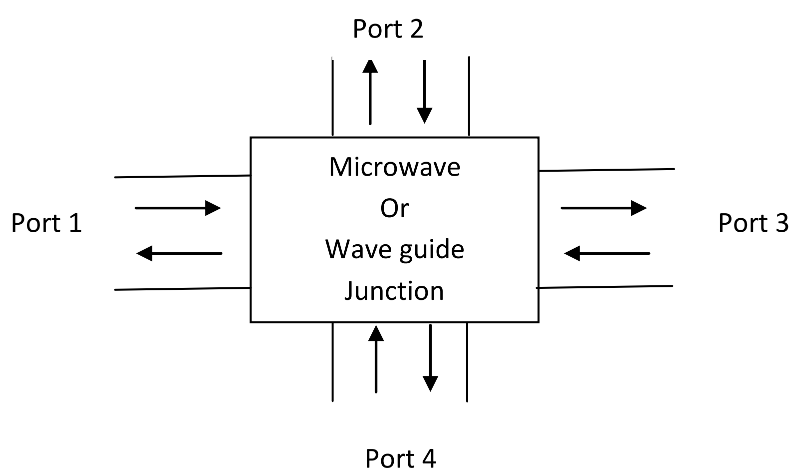


Fig. 1.4 Four port waveguide

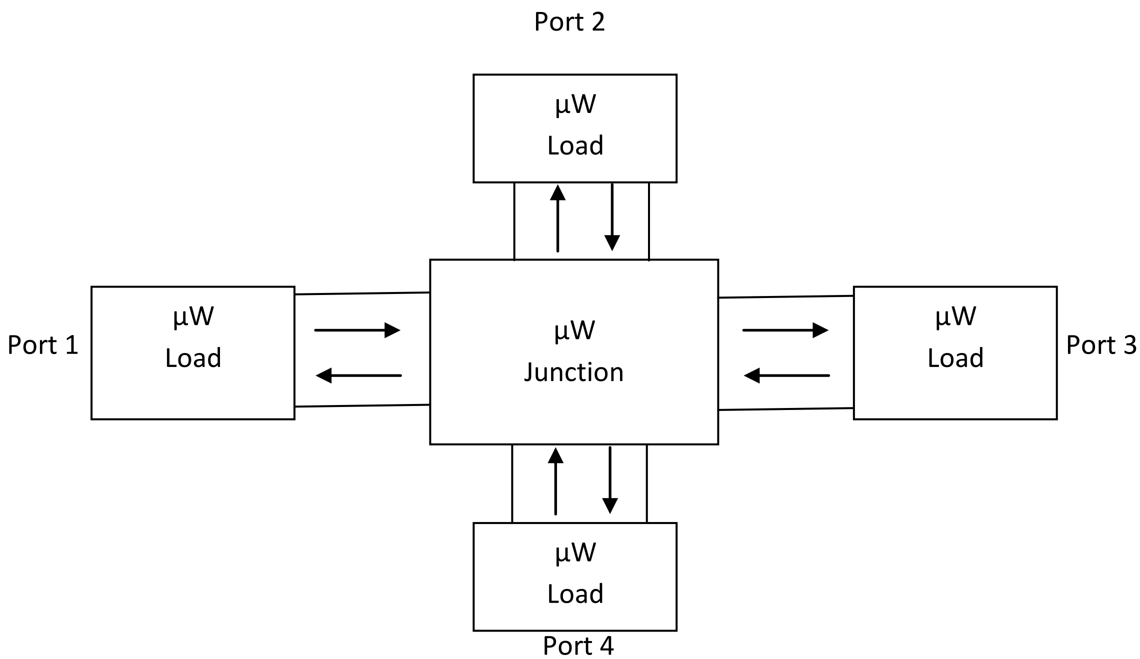


Fig. 1.5 Four port waveguide with matched termination

Consider a junction of 'n' number of transmission lines wherein the i^{th} line (i can be any line from 1 to n)

<u>case(i)</u>

* Let first line terminated in an impedance other than characteristic impedance ($Z_L \neq Z_0$)
* If a_i be the incident wave, it divides among (n-1) number of lines as $a_1, a_2 \ldots a_n$
* No Reflections from 2nd to n^{th} line
* The incident waves are absorbed since their impedance equal to characteristic impedance.
* 1^{st} line mismatch - wave Reflected back b_1

b_1 related to a_1

b_1 = (Reflection coefficient) a_1

$b_1 = S_{i1} a_1$ $\quad$ $S_{i1} \rightarrow$ Reflection coefficient of 1^{st} line

$1 \rightarrow$ Reflection from 1^{st} line

$i \rightarrow$ source conneted to the i^{th} line

Contribution to the outward travelling wave in the i^{th} line $\quad b_i = S_{i1} a_1 \quad [\because b_2 = b_3 \ldots = b_n = 0]$

<u>case(2)</u>:

Let all (n-1) lines be terminated in an impedance other than Z_0 ($Z_L \neq Z_0$ for all lines)

Reflection from every line and hence total contribution

$$b_i = S_{i1} a_1 + S_{i2} \cdot a_2 + S_{i3} \cdot a_3 \; \ldots + S_{in} \cdot a_n$$

$i = 1$ to n since i can be any line from 1 to n.

$$\therefore \quad b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3 \; \ldots\ldots + S_{1n} a_n$$

$$b_2 = S_{21} a_1 + S_{22} a_2 + S_{23} a_3 \; \ldots\ldots + S_{2n} a_n$$

$$\vdots$$

$$b_n = S_{n1} a_1 + S_{n2} a_2 + S_{n3} a_3 + \; \ldots\ldots + S_{nn} a_n$$

In matrix form

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Column matrix [b] corresponding to Reflected waves/ output

Scattering matrix [S] of order $n \times n$

Column matrix [a] corresponding to Incident waves/Input

$$\boxed{[b] = [S][a]}$$

when a junction of n number of waveguides is considered

- a's — Inputs to particular ports
- b's → Outputs out of various ports
- S_{ij} → Scattering coefficients resulting due to input at i^{th} port and output taken out of j^{th} port
- S_{ii} → How much power reflected back from the junction into the i^{th} port when I/p power applied to 'i' itself

UNIT-I

Properties of (S) Matrix

1. [S] is always a square matrix of order (n×n)
2. [S] is a symmetric matrix

$$S_{ij} = S_{ji}$$

3. [S] is a unitary matrix

$$[S][S^*] = [I]$$

$[S]^*$ → complex conjugate of [S]

[I] → unit matrix or Identity matrix of the same order as that of [S]

4. The sum of the products of each term of any row (or coloumn) multiplied by the complex conjugate of the corresponding terms of any other row (or coloumn) is zero.

$$\sum_{i=1}^{n} S_{ik} S_{ij}^* = 0 \quad \text{for } k \neq j \quad \left\{ \begin{matrix} k = 1, 2, 3 \cdots n \\ j = 1, 2, 3 \cdots n \end{matrix} \right\}$$

5. If any of the terminal or reference planes (say the k^{th} port) are moved away from the junction by an electric distance $\beta_k l_k$ each of the coeffcients S_{ij} involving k will be multiplied by the factor $e^{-j\beta_k l_k}$

Properties of [S] matrix

(i) Zero diagonal elements for perfect matched Network

For an ideal N-port N/w with matched termination $S_{ii} = 0$. Since there is no reflection from any port. Therefore, under perfect matched conditions, the diagonal elements of [S] are zero.

ii) Symmetry of [S] for a reciprocal n/w

A reciprocal device has the same transmission characteristic in either direction of a ports and is characterized by a symmetric scattering matrix.

$$S_{ij} = S_{ji} \quad (i \neq j)$$

which result $[S]_t = [S]$

Proof:-

For a reciprocal n/w with the assumed normalization, the impedance matrix equation is

$$[V] = [Z][I] = [Z](|a| - |b|) = [a] + [b]$$

$$([Z] + [U])[b] = ([Z] - [U])[a]$$

$$[b] = ([Z] + [U])^{-1}([Z] - [U])[a] \rightarrow ①$$

where [U] - Unit matrix

The S-matrix eqn for the n/w is

$$[b] = [S][a] \rightarrow ②$$

Comparing ① & ② we get

$$[S] = [[Z] + [U])^{-1}([Z] - [U])$$

The transpose of [S] is

$$[S]_t = [[Z]+[U])_t^{-1} ([Z]-[U])_t$$

Since the Z-matrix is symmetrical

$$([Z]+[U])_t^{-1} = ([Z]+[U])^{-1}$$

$$([Z]-[U])_t = ([Z]-[U])$$

$$\therefore [S]_t = ([Z]+[U])^{-1}([Z]-[U]) = [S]$$

It is proved that $[S]_t = [S]$ for symmetrical junction

iii) Unitary Property for a lossless junction

For any lossless n/w the sum of the products of each term of any row or of any column of the S-matrix multiplied by its complex conjugate is Unity

For a lossless n-port devices, the total power leaving N-ports must be equal to the total power i/p to these ports, so that

$$\sum_{n=1}^{N} |b_n|^2 = \sum_{n=1}^{N} |a_n|^2$$

i.e

$$\sum_{n=1}^{N} \left| \sum_{i=1}^{n} S_{ni} a_i \right|^2 = \sum_{n=1}^{N} |a_n|^2$$

if only i^{th} port is excited & all other ports are matched terminated all $a_n = 0$ except a_i so that

$$\sum_{i=1}^{N} |S_{ni} a_i|^2 = \sum_{n=1}^{N} S_{ni} S_{ni}^*$$

$$\sum_{n=1}^{N} |S_{ni}|^2 = 1 = \sum_{n=1}^{N} S_{ni} S_{ni}^*$$

Therefore, for a lossless junction

$$\sum_{n=1}^{N} S_{ni} S_{ni}^* = 1 \longrightarrow ①$$

if all $a_n = 0$ except a_i & a_k

$$\sum_{n=1}^{N} S_{nk} S_{ni}^* = 0 \quad i \neq k \longrightarrow ②$$

In matrix notation, these relations can be expressed as

$$[S]^* [S]_t = [U] \quad \text{or} \quad [S]^* = [S]_t^{-1} \longrightarrow ③$$

A matrix [S] for lossless n/w which satisfies the above 3 condition is called a unitary matrix.

iv) Phase Shift Property

Complex S-parameters of a n/w are defined with respect to the positions of the port or reference planes.

For a 2 port n/w with Unprimed reference Planes 1 and 2

Reciprocal n/w :-

A reciprocal n/w is defined to be a n/w that Satisfies the reciprocity theorem

Reciprocity Theorem:-

Its State that when some amount of electromotive force is applied at One point (e.g in branch K, V_K) in a Passive linear n/w, that will Produce the current at any other Point (e.g branch m, i_m). The Same amount of current (in branch K, i_K) is produced when the Same electromotive force is applied in the new location (branch m, V_m) that is

$$V_K / i_m = V_m / i_K$$

or

$$Z_{Km} = Z_{mK}$$

In terms of S parameter, [S] matrix is Symmetrical

$$S_{ij} = S_{ji} \quad (i \neq j) \qquad \text{where } i = 1, 2, \ldots N, \; j = 1, 2, \ldots N$$

Symmetrical Reciprocal N/w

– A special case, reciprocal N/w is a Symmetrical n/w. These n/ws have identical Size & arrangement for Corresponding electrical elements in reference to a Plane or line of Symmetry.

- The I/p impedance at the i/p port is equal to the impedance in the o/p n/w

- The equality of the I/p & o/p impedances leads to the equality of i/p & o/p reflection coefficients.

- In general for any symmetrical passive N-port n/w

$$S_{ii} = S_{jj}$$

$$S_{ij} = S_{ji} \quad (i \neq j)$$

- For any symmetrical n/ws. we can always write as

$$S_{11} = S_{22}$$

$$S_{12} = S_{21}$$

Lossless N/ws

In any lossless passive n/w, its containing no resistive elements, always the power entering the ckt will be equal to the power leaving the n/w which leads to the conserved in power.

Unitary Property of |S| matrix

- It states that for a passive lossless N port N/w, the sum of the products of each term of any one row (or any one column) multiplied by its own complex conjugate is unity

$$\sum_{i=1}^{N} S_{ij} S_{ij}^{*} = 1 \quad j = 1, 2 \dots N \quad \rightarrow (1)$$

Eqn ① becomes,

$$S_{11} S_{11}^* + S_{21} S_{21}^* = 1 \rightarrow ②$$

$$S_{12} S_{12}^* + S_{22} S_{22}^* = 1 \rightarrow ③$$

if the lossless n/w is also reciprocal, then the above 2 eqn can be reduced as follows

$$S_{12} = S_{21} \rightarrow ④$$

$$|S_{11}| = |S_{22}| \rightarrow ⑤$$

$$|S_{11}|^2 + |S_{21}|^2 = 1 \rightarrow ⑥$$

Zero property of [S] matrix:

– It states that "for a passive lossless N-port n/w, the sum of the product of each term of any row or any column multiplied by the complex conjugate of the corresponding terms of any other row or column is zero.

$$\sum_{K=1}^{N} S_{Ki} \cdot S_{Kj}^* = 0 \rightarrow ⑦$$

i & j – row & column numbers.

for a 2 port n/w

$$S_{11} S_{12}^* + S_{21} S_{22}^* = 0 \rightarrow ⑧$$

$$S_{12} S_{11}^* + S_{22} S_{21}^* = 0 \rightarrow ⑨$$

if the lossless n/w is also reciprocal, the above eqn can be reduced as follows

$$S_{12} = S_{21}$$

$$S_{11} S_{21}^* + S_{21} S_{22}^* = 0 \rightarrow ⑩$$

$|S_{11}| = |S_{22}| \rightarrow (11)$

A unitary matrix is one" the matrix which satisfies both the unitary & zero property".

Analysis of Reciprocal lossless n/ws:

Zero & Unit properties of the S-matrix, the S parameter of a reciprocal lossless n/w are constrained by eqn (4), (5), (6) & (10) as

$S_{21} = S_{12} \rightarrow (12)$

$|S_{11}| = |S_{22}| \rightarrow (13)$

$|S_{11}|^2 + |S_{21}|^2 = 1 \rightarrow (14)$

$S_{11} S_{21}^* + S_{21} S_{22}^* = 0 \rightarrow (15)$

let,

$S_{11} = |S_{11}| e^{j\theta_{11}}$

$S_{22} = |S_{22}| e^{j\theta_{22}}$ &

$S_{21} = |S_{21}| e^{j\theta_{21}}$

eqn (14) we get

$|S_{21}| = (1 - |S_{11}|^2)^{1/2} \rightarrow (16)$

Using above expression. eqn (15) may be written as

$|S_{11}| e^{j\theta_{11}} |S_{21}| e^{-j\theta_{21}} + |S_{21}| e^{j\theta_{21}} |S_{11}| e^{-j\theta_{22}} = 0 \rightarrow (17)$

Sub eqn (16) into eqn (17)

$|S_{11}| e^{j\theta_{11}} (1 - |S_{11}|^2)^{1/2} e^{-j\theta_{21}} + (1 - |S_{11}|^2)^{1/2} e^{j\theta_{21}} |S_{11}| e^{-j\theta_{22}} = 0$ (18)

(10)

$$|S_{11}|(1-|S_{11}|^2)^{1/2}\left[e^{j(\theta_{11}-\theta_{21})}+e^{j(\theta_{21}-\theta_{22})}\right]=0 \rightarrow (19)$$

which implies that

$$\left(e^{j(\theta_{11}-\theta_{21})}+e^{j(\theta_{21}-\theta_{22})}\right)=0 \Rightarrow e^{j(\theta_{11}-\theta_{21})}=e^{-j\pi}e^{j(\theta_{21}-\theta_{22})}$$

$$\Rightarrow \theta_{11}+\theta_{22}=2\theta_{21}-\pi\pm 2n\pi$$

(or)

$$\theta_{21}=\frac{\theta_{11}+\theta_{22}}{2}+\pi\left(\frac{1}{2}+n\right) \text{ for } n=0,1,2\cdots \quad (20)$$

- The eqn (19) & (20) from the magnitude & phase of S_{21} or S_{12} in terms of magnitude & phase of S_{11} & S_{22}
- From a measurement knowledge of S_{11} & S_{22}, we can completly describe and specify a reciprocal lossless 2 port n/w.

1.5 FORMULATION OF S PARAMETERS

The high frequency S and T parameter are used to characteristics high RF/microwave two-port networks. These parameters are based on the concept of travelling waves and provide a complete characterization of any to port network under analysis or test at high RF/microwave frequencies. While the lower frequency network parameters are defined in terms of net voltage and currents at the ports these are not practical at high RF/microwave frequencies.

To characterize a two-port network that has identical characteristic impedances (Z_0) at both input& output ports, Let us consider the incident & reflected voltage waves at each part, as shown in the figure below.

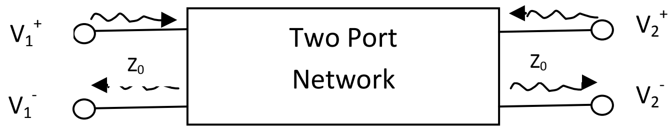


Fig 1.7 Two port network

To define the S-parameters accurately, we will consider a voltage phasor $[V^+_i]$ reflected from terminals of two-port network (i=1,2) as shown in the above fig.

The scattering matrix,[S] is now defined to describe the liner relationship between the incident voltage wave phasor matrix$[V^+_i]$ and the reflected or transmitted wave phasor matix$[V^-_i]$ at any of the two ports as follows.

$V_1^- = S_{11}V_1^+ + S_{12}V^+_2$

$V_2^- = S_{21}V_1^+ + S_{22}V^+_2$

Or, in the matrix form we can write:

or, in matrix form we can write: $$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

(or) $$[V^-] = [S][V^+]$$

where, $$[V^-] = \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix}$$

$$[V^+] = \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

and $$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

eq 1.1

This linear relationship is expressed interms of a ratio of two phasors that are complex numbers with the magnitude of the ratio always less than or equal to 1. Each specific element of the [S] matrix is defined as

$S_{11} = \frac{V_1^-}{V_1^+} \Big/ V_2^+ = 0 = \tau_{IN}$	Input reflection coefficient when output port is terminated in a matched load.
$S_{21} = \frac{V_2^-}{V_1^+} \Big/ V_2^+ = 0$	Forward transmission coefficient when output port is terminated in a matched load.
$S_{12} = \frac{V_1^-}{V_2^+} \Big/ V_1^+ = 0$	Reverse transmission coefficient when input port is terminated in a matched load.
$S_{22} = \frac{V_2^-}{V_2^+} \Big/ V_1^+ = 0 = \tau_{OUT}$	Output reflection coefficient when input port is terminated in a matched load.

Table 1.6 CONVERSION BETWEEN TWO- PORT NETWORK PARAMETERS

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Y_0}{\Delta Y}$	$\frac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\frac{Y_{22}}{\lvert Y\rvert}$	$\frac{A}{C}$
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})S_{12}S_{21}}$	Z_{12}	$\frac{-Y_{12}}{\lvert Y\rvert}$	$\frac{AD-BC}{C}$
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})S_{12}S_{21}}$	Z_{21}	$\frac{-Y_{21}}{\lvert Y\rvert}$	$\frac{1}{C}$
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\frac{Y_{11}}{\lvert Y\rvert}$	$\frac{D}{C}$
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{\lvert Z\rvert}$	Y_{11}	$\frac{D}{B}$
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{\lvert Z\rvert}$	Y_{12}	$\frac{BC-AD}{B}$
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{\lvert Z\rvert}$	Y_{21}	$\frac{-1}{B}$
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{\lvert Z\rvert}$	Y_{22}	$\frac{A}{B}$
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{\lvert Z\rvert}{Z_{21}}$	$\frac{-1}{Y_{21}}$	B
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{-\lvert Y\rvert}{Y_{21}}$	C
D	$\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	D

$\lvert Z\rvert = Z_{11}Z_{22} - Z_{12}Z_{21}$; $\lvert Y\rvert = Y_{11}Y_{22} - Y_{12}Y_{21}$;
$\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$

S-matrix of a two port network with mismatched load

Mismatch loss

Mismatch loss is the ratio of power delivered, to power available The formula for mismatch loss is simply:
mismatch loss=(1-Γ^2)

Loss factor

This concept is described in recent a Microwave Journal article entitled "Automation and Real-time Verification of Passive Component S-parameter Measurements Using Loss Factor Calculations", by J. Capwell, T. Weller, D. Markell and L. Dunleavy of Modelithics Inc. You can google to it if you like, but to get the full text you might have to join the Microwave Journal web site, which is annoying. You can always read the google "cached" version of the article, that's what we did.
They present the concept of "loss factor" as something that will help you determine if an S-parameter measurement of a passive device is good. Quoting the article:
"The forward and reverse loss factors are calculated from passive component S-parameters as
Forward Loss Factor (FLF) = 1 - $|S11|^2$ - $|S21|^2$

Reverse Loss Factor (RLF) = 1 - $|S22|^2$ - $|S12|^2$

By these definitions, the loss factors are seen to equal the difference between a normalized input power and the power that is reflected and transmitted to the input and output ports, respectively. (Power loss can occur due to conductor, dielectric and radiation loss mechanisms.) For reciprocal devices S21 = S12, the differences between the forward and reverse loss factor occur due to differences in |S11| and |S22|. For ideal lossless components, the magnitudes of S11 and S22 are equal, but they can deviate from one another whenever loss is present.
For passive components such as capacitors, inductors and resistors (and diodes), the electrical behaviour is most often symmetrical (S11 = S22); thus, the difference in the forward and reverse loss should be negligible. Significant deviations in the forward and reverse loss can be observed when a component begins to radiate, a commonly observed phenomenon, particularly for inductors. However, the main objective of this article is to illustrate how real-time monitoring of loss factor behavior becomes a useful tool for detecting measurement inconsistencies when, all things functioning properly, the component should exhibit symmetrical characteristics."
We agree that loss factor is a one indication of how much power is "disappearing" in a network, through resistive loss or radiation.

Efficiency factor

This a Microwaves concept, which is a more useful quantity than "loss factor" for evaluating passive parts. It is defined as:

Forward efficiency factor = $|S11|^2 + |S21|^2$

Reverse efficiency factor = $|S22|^2 + |S12|^2$

It all makes sense when converted to decibels. A "perfect" circuit has an efficiency factor of 0 dB. A circuit that loses 20 percent of its power has an efficiency factor of -1 dB, etc. etc. Now you have a measurement of how "lossless" a circuit would be if you were able to perfectly impedance match it. This is quite useful when you are designing low-loss networks such as switches.

Signal flow graph

The SFG can be drawn as a directed graph. Each wave a_i and b_i is represented by a node, each arrow stands for an S-parameter (Fig. B.1).

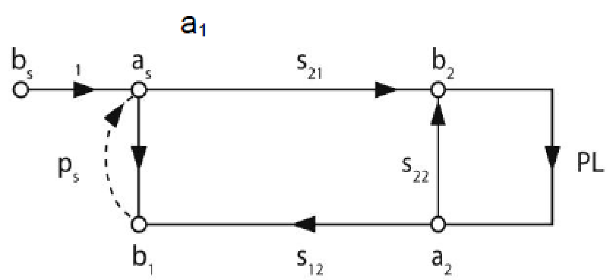


Fig. B.1: A 2-port with a non-matched load

For general problems the SFG can be solved by applying Mason's rule. For not too complicated circuits, a more intuitive way is to simplify step-by-step the SFG by applying the following three rules (Fig. B.2):

1. Add the signal of parallel branches
2. Multiply the signals of cascaded branches
3. Resolve loops

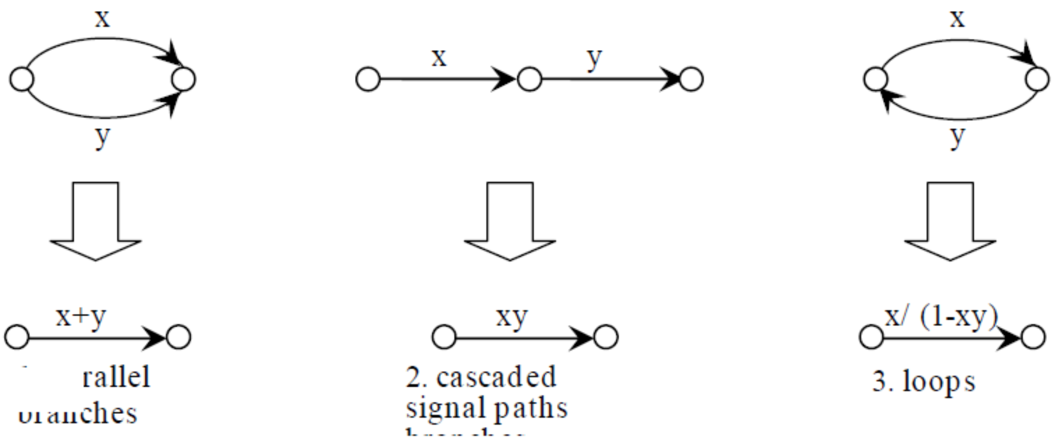


Fig. B.2: The three rules for simplifying signal flow charts

Care has to be taken applying the third rule, since loops can be transformed to forward and backward oriented branches. No signal paths should be interrupted when resolving loops.

Examples
We are looking for the input reflection coefficient b_1/a_1 of a two-port with a non-matched load ρ_L and a matched generator (source $\rho_S = 0$), see Fig. B.1.

The loop at port 2 involving S_{22} and ρ_L can be resolved, given a branch from b_2 to a_2 with the signal $\Gamma_L *(1-\Gamma_L*S_{22})$. Applying the cascading rule and the parallel branch rule then yields

$$\frac{b_1}{a_1} = S_{11} + S_{21}\frac{\rho_L}{1-S_{22}\rho_L}S_{12}\ . \qquad \text{(B3)}$$

As a more complicated example one may add a mismatch to the source (ρ_S = dashed line in Fig. B.1) and ask for b_1/b_s.

As before, first the loop consisting of S_{22} and ρ_L can be resolved. Then the signal path via b_2 and a_2 is added to S_{11}, yielding a loop with ρ_S. Finally one obtains

$$\frac{b_1}{b_S} = \frac{\left(S_{11} + S_{21}S_{12}\rho_L\frac{1}{1-\rho_L S_{22}}\right)}{1-\left(S_{11} + S_{21}S_{12}\rho_L\frac{1}{1-\rho_L S_{22}}\right)\rho_S}\ . \qquad \text{(B4)}$$

The same results would have been found applying Mason's rule on this problem.

As we have seen in this rather easy configuration, the SFG is a convenient tool for the analysis of *simple* circuits [9, 10]. For more complex networks there is a considerable risk that a signal path may be overlooked and the analysis soon becomes complicated. When applied to S-matrices, the solution may sometimes be read directly from the diagram. The SFG is also a useful way to gain insight into other networks, such as feedback systems. But with the availability of powerful computer codes using the matrix formulations, the need to use the SFG has been reduced.

Examples

We are looking for the input reflection coefficient of a e-port with a non-matched load ρ_L and a matched generator (source) ($\rho_S = 0$) to start with. ρ_L, ρ_S are often written as Γ_L, Γ_S.

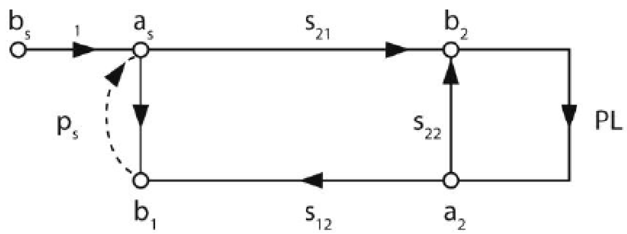


Fig. B.4: 2-port with non-matched load

By reading directly from the SFG (Fig. B.4) we obtain

$$\frac{b_1}{a_1} = S_{11} + S_{21}\frac{\rho_L}{1 - S_{22}\rho_L}S_{12} \qquad \text{(B6)}$$

or by formally applying Mason's rule in Eq. (B.5)

$$\frac{b_1}{a_1} = \frac{S_{11}(1 - S_{22}\rho_L) + S_{21}\rho_L S_{12}}{1 - S_{22}\rho_L}. \qquad \text{(B7)}$$

As a more complicated example one may add a mismatch to the source (ρ_S = dashed line in Fig. B.4) and ask for b_1/b_s

$$\frac{b_1}{b_s} = \frac{S_{11}(1 - S_{22}\rho_S) + S_{21}\rho_S S_{12}}{1 - (S_{11}\rho_S + S_{22}\rho_L + S_{12}\rho_L S_{21}\rho_S) + S_{11}\rho_{22}S_{22}\rho_L}. \qquad \text{(B8)}$$