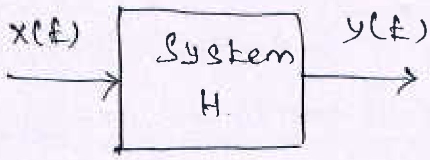
FREQUENCY RESPONSE & ITS APPLICATIONS

The frequency response is a steady state (o/p) of the system when the input to the system is a sinusoidal signal.



$x(t) = X \sin \omega t$

$y(t) = Y \sin(\omega t + \phi)$

$x(t) \rightarrow$ input sinusoidal signal

$y(t) \rightarrow$ sinusoidal signal of same frequency but with different magnitude & phase angle.

The magnitude & phase relationship b/w the sinusoidal input and the steady state o/p of a system is termed the frequency response.

Advantages of frequency response analysis

1. The absolute & relative stability of the closed loop system can be estimated from the knowledge of their open loop frequency response.
2. The transfer function of complicated systems can be determined experimentally by frequency response tests.
3. The design and parameter adjustment can be carried more easily.
4. The corrective measure for noise disturbance and parameter variation can be easily carried.

Some Important terms

Gain Margin

The gain margin Kg is defined as the reciprocal of the magnitude of open loop transfer function, at phase cross over frequency ω_{pc}

$$\text{Gain Margin, } K_g = \frac{1}{|G(j\omega_{pc})|}$$

when expressed in decibels, it is given by the negative of magnitude of G(jω) at phase-cross over frequency

$$\text{Gain Margin in db} = 20\log \frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}$$

$$= -20\log |G(j\omega)|_{\omega=\omega_{pc}}$$

Phase Margin

The phase margin γ is that amount of additional phase lag at the gain cross-over frequency, ω_{gc} required to bring the system to the verge of instability. It is given by $180+\phi_{gc}$, where ϕ_{gc} is the phase of G(jω) at the gain cross over frequency

$$\text{Phase Margin, } \gamma = 180+\phi_{gc}$$

$$\text{where } \phi_{gc} = \text{Arg}[G(j\omega)]\big|_{\omega=\omega_{gc}} \quad \angle G(j\omega_{gc})$$

Gain Cross over frequency

It is the frequency at which the magnitude of the open loop transfer function is unity

Phase Cross over frequency

It is frequency at which the ~~magn~~ phase of the open loop transfer function is 180°.

Frequency Response Plot

response analysis of control system can be carried ...ally or graphically. The various graphically techniques ... Bode plot, Polar plot (or Nyquist plot), M & N circles, Nichols chart.

Frequency domain Specifications

The performance and characteristics of a system in frequency domain are measured in terms of frequency domain specifications.

Resonant Peak (M_r)

The maximum value of the magnitude of closed loop transfer function is called resonant peak M_r.

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

Resonant frequency (ω_r)

The frequency at which the resonant peak occurs is called resonant frequency ω_r.

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

Bandwidth

The Bandwidth is the range of frequencies for which the system gain is more than -3db. $U_b = \frac{\omega_b}{\omega_n}$

Cut-off Rate

The slope of the log-magnitude curve near the cut off frequency is called cut-off rate.

Bode plot

The Bode Plot is a frequency response plot of the transfer of a system. A bode plot consist of two graphs. One is a plot of the magnitude of a sinusoidal transfer function verses log ω. The other is a plot of the phase angle of a sinusoidal transfer function versus log ω.

Advantages of Bode Plot

The magnitude are expressed in db and so a simple procedure is available to add the magnitude of each term one by one.

The approximate Bode Plot can be sketched quickly and the connections can be made at corner frequencies to get the exact plot.

The frequency-domain specifications can be easily determined.

The Bode Plot can be used to analyse both open loop and closed loop system.

Bode Plot It consists of two plots ie magnitude plot & phase plot.

Magnitude Plot   Phase Plot

Step by step procedure for magnitude plot

Step 1: Convert the transfer function in to Bode form or time constant form.

The Bode form of the t/f is

$$G(s) = \frac{K(1+sT_1)}{s(1+sT_2)\left(1+\frac{s^2}{\omega_n^2}+2\zeta\frac{s}{\omega_n}\right)}$$

$$G(j\omega) = \frac{K(1+j\omega T_1)}{j\omega(1+j\omega T_2)\left(1-\frac{\omega^2}{\omega_n^2}+j2\zeta\frac{\omega}{\omega_n}\right)}$$

Step 2: List the corner frequency in the increasing order and prepare a table as shown below

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec

In the above table enter K or $K/(j\omega)^n$ or $K(j\omega)^n$ as the first term and other terms in the increasing order of corner frequenci Then enter the corner frequency, slope contributed by each term and change in slope at every corner frequency.

Corner frequency – The magnitude plot can be approximated by asymptotic straight lines. The frequencies corresponding to the meeting point of asymptotes are called corner frequency.

Step 3: Choose a frequency ω_l which is lesser than the lowest corner frequency. Calculate the db magnitude of K or $K/(j\omega)^n$ or $K(j\omega)$ at ω_l and at the lowest corner frequenc

Step 4: Then calculate the gain (db magnitude) at every corner frequency one by one by using the formula.

Gain at ω_y = Change in gain from ω_x to ω_y + Gain at ω_x

$$= \left[\text{Slope from } \omega_x \text{ to } \omega_y \times \log\frac{\omega_y}{\omega_x}\right] + \text{Gain at } \omega_x$$

Step 5: Choose a frequency ω_h which is greater than the highest corner frequency. Calculate the gain at ω_h by using the formula in step 4.

Step 6: In a semilog graph sheet mark the required range of frequency on x-axis and the range of db magnitude on y-axis after choosing proper scale.

Step 7: Mark all the points obtained in steps 3, 4 & 5 on the graph and join the points by straight lines. Mark the slope at every part of the graph.

Procedure for Phase plot

Step 1: Exact angles of $G(j\omega)$ are computed for the various values of ω and tabulated.

Step 2: Take another y-axis in the graph, mark the desired range of phase angles after choosing the proper scale. Mark all the points on the graph and join the points by a smooth curve.

Determination of gain margin and phase margin from Bode plot

Gain Margin

The gain margin in db is given by the negative of db magnitude of $G(j\omega)$ at the phase cross over frequencies ω_{pc}.

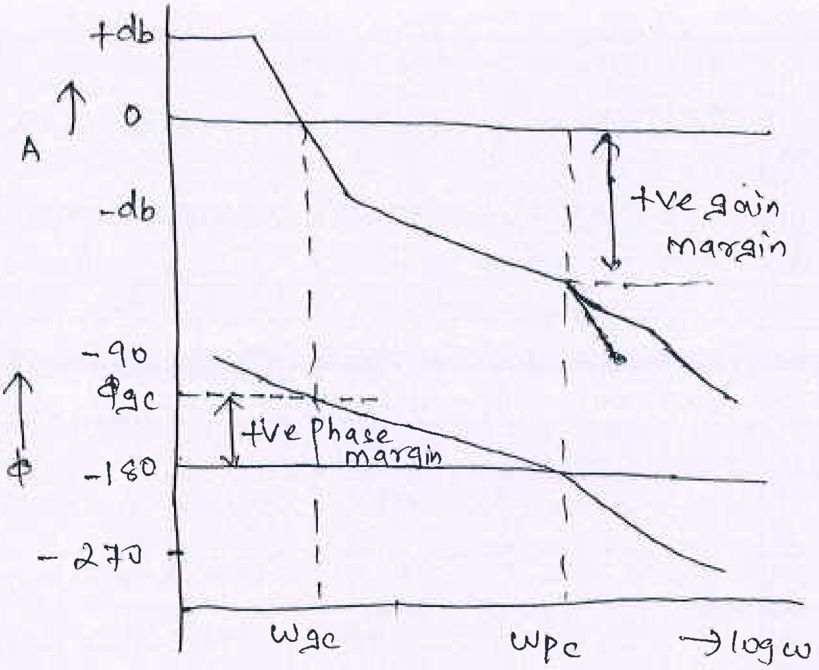
The ω_{pc} is the frequency at which phase of $G(j\omega)$ is -180

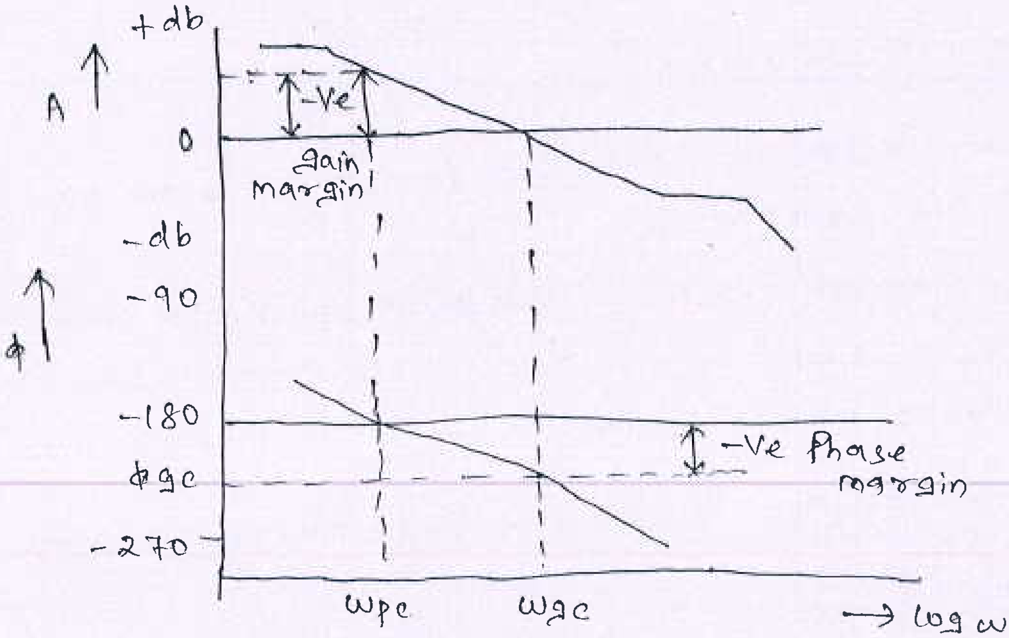
If the db magnitude of $G(j\omega)$ at ω_{pc} is negative then the gain margin is positive and vice-versa.

Phase Margin

ϕ_{gc} be the angle of $G(j\omega)$ at the gain cross-over frequency ω_{gc}. The ω_{gc} is the frequency at which the db magnitude of $G(j\omega)$ is zero.

$\therefore$ Phase Margin, $\gamma = 180 + \phi_{gc}$





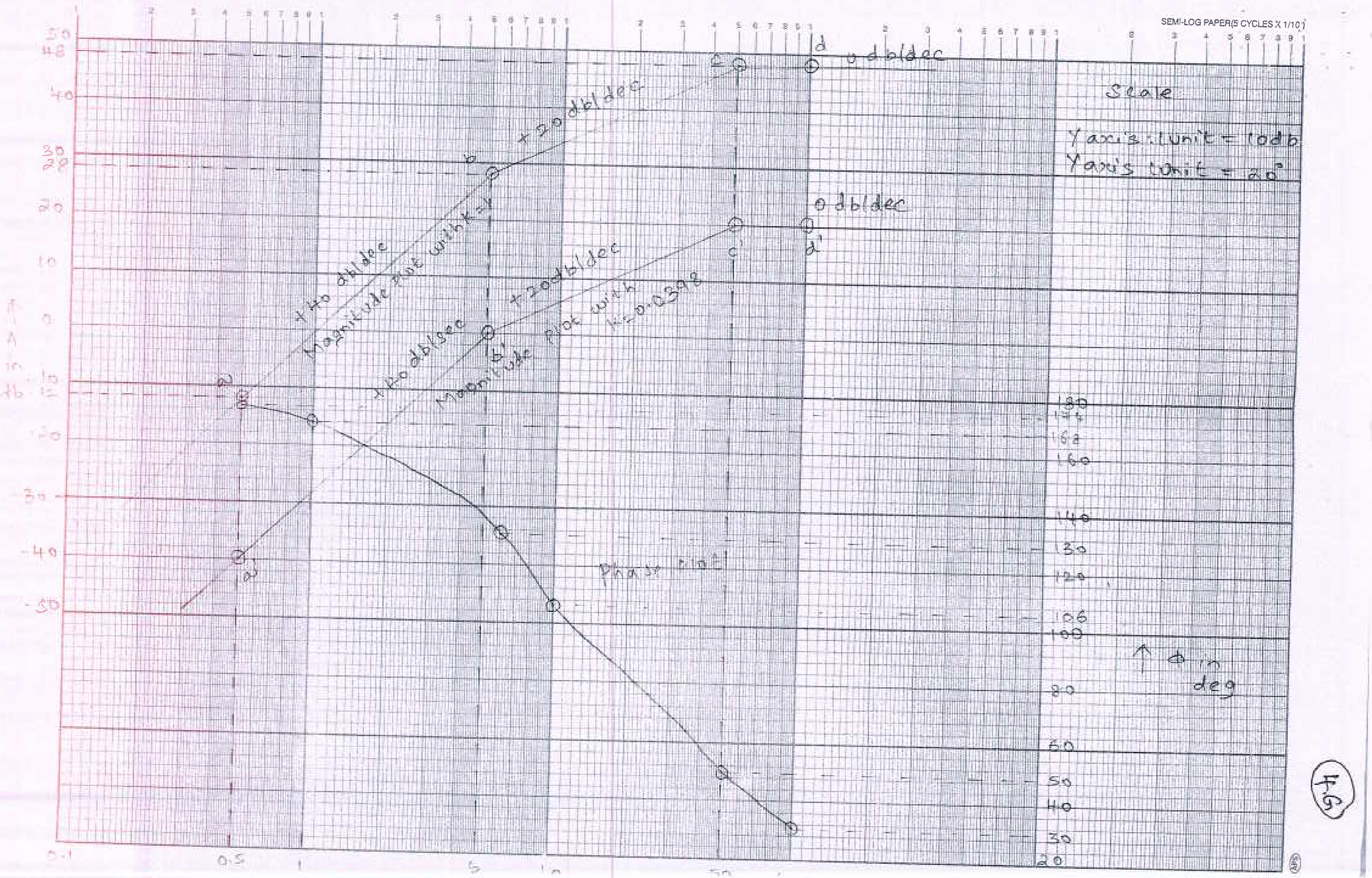
Sketch Bode plot for the following transfer function and determine the system gain K for the gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

Magnitude Plot

Step 1: The sinusoidal transfer fn $G(j\omega)$ is obtained by replacing s by $j\omega$ in the given s-domain transfer function.

$$G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$


SEMI-LOG PAPER(5 CYCLES X 1/10)
4.G
Scale
Y axis 1 unit = 10db
Y axis 1 unit = 20°
0 db/dec
+20 db/dec
+40 db/dec
Magnitude plot with k=1
+20 db/dec
+40 db/dec
Magnitude plot with k=0.0398
0 db/dec
Phase plot
↑ A in db
↑ ϕ in deg

Let $K=1$ $\therefore$ $G(j\omega) = \dfrac{(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$

Step 2: List the corner frequencies in the table form

The corner frequencies are

$\omega_{c1} = \dfrac{1}{0.2} = 5$ rad/sec

$\omega_{c2} = \dfrac{1}{0.02} = 50$ rad/sec

Table 1:

Term	Corner frequency rad/sec	slope db/dec	change in slope db/dec
$(j\omega)^2$	—	+40	
$\dfrac{1}{1+j0.2\omega}$	$\omega_{c1} = \dfrac{1}{0.2} = 5$	−20	40 − 20 = 20
$\dfrac{1}{1+j0.02\omega}$	$\omega_{c2} = \dfrac{1}{0.02} = 50$	−20	20 − 20 = 0

Step 3: Choose the lower frequency of ω_l, such that $\omega_l < \omega_{c1}$

$\omega_l = 0.5$ rad/sec

At $\omega = \omega_l$; $A = 20\log|(j\omega)^2| = 20\log(\omega)^2 = 20\log(0.5)^2$

$= -12$ db

At $\omega = \omega_{c1}$; $A = 20\log|(j\omega)^2| = 20\log(\omega)^2 = 20\log(5)^2$

$= 28$ db

Step 4: Calculate the gain at every corner frequency

At $\omega = \omega_{c2}$; $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\dfrac{\omega_{c2}}{\omega_{c1}}\right] + A$ (at $\omega = \omega_{c1}$)

$A = 20 \times \log\dfrac{50}{5} + 28 = 48$ db

Step 5: Choose a frequency of ω_h ie the high frequency

$\omega_h > \omega_{c2}$ $\qquad \omega_h = 100$ rad/sec

A $\omega = \omega_h$; $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}}\right] + A \, (\text{at } \omega = \omega_{c2})$

$= 0 \times \log \frac{100}{50} + 48 = 48 \text{ db}$

Step 6: In a semilog graph sheet choose a scale of 1unit = 10db on y axis

The frequencies are marked in decades from 0.1 to 100 on the logarithmic scales in x-axis.

Step 7: The frequencies corresponding to ω_l, ω_{c1}, ω_{c2} and ω_h resp are marked as a, b, c & d points on the semilog sheet which gives the magnitude plot.

Phase plot

The phase angle G(jw) as a function of w is given by

$$\phi = \angle G(j\omega) = 180 - \tan^{-1} 0.2\omega - \tan^{-1} 0.02\omega$$

The phase angle of G(jw) are calculated for various values of w and listed in the table below.

w rad/sec	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} 0.02\omega$ deg	$\phi = \angle G(j\omega)$ deg
0.5	5.7	0.6	173.7 ≃ 174
1	11.3	1.1	167.6 ≃ 168
5	4.5	5.7	129.3 ≃ 130
10	63.4	11.3	105.3 ≃ 106
50	84.3	45	50.7 ≃ 50
100	87.1	63.4	29.5 ≃ 30

On the same semilog sheet choose a scale of 1unit = 20 on the y axis on the right side of semilog sheet. Mark the calculated phase angle on the graph sheet. Join the points by

Calculation of K:

Given gain cross over frequency is 5 rad/sec

At $\omega = 5$ rad/sec the gain is 28db.

If the gain cross over frequency is 5 rad/sec then at that frequency the db gain should be zero.

To every point of the magnitude plot a db gain of −28db should be added. The addition of −28db shifts the plot downwards.

The connected magnitude plot is obtained by shifting the plot with K=1 by 28db downwards.

The value of K is calculated by equating 20 log K to −28db

$$20 \log K = -28db$$

$$\log K = \frac{-28}{20} \Rightarrow K = 10^{-28/20} = 0.0398$$

The magnitude plot with K=1 and K=0.0398 and phase plot are shown in the semilog graph.

Sketch the bode plot for the following transfer function and determine Phase margin and Gain margin.

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$$

$$s^2+16s+100 = s^2+2\zeta\omega_n s+\omega_n^2$$

$\omega_n^2 = 100$

$\omega_n = 10$

$2\zeta\omega_n = 16$

$\zeta = 16/2\omega_n = \frac{16}{2\times 10} = 0.8$

$\boxed{\zeta = 0.8}$

Magnitude plot

Step 1: $G(j\omega)$ is obtained by replacing s by $j\omega$

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)} = \frac{75(1+0.2s)}{s\times 100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)}$$

$$G(j\omega) = \frac{\frac{75}{100}(1+0.2j\omega)}{j\omega[(j\omega)^2 \times 0.01 + 0.16j\omega + 1]}$$

$$= \frac{0.75(1+0.2j\omega)}{j\omega[1+0.01(j\omega)^2+0.16j\omega]}$$

$$= \frac{0.75[1+j0.2\omega]}{j\omega[1-0.01\omega^2+0.16j\omega]}$$

Step 2: List the corner frequency

$\omega_{c1} = \frac{1}{0.2} = 5$ rad/sec $\omega_{c2} = \omega_n = 10$ rad/sec

[For the quadratic factor the Corner frequency is ω_n]

Table

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{0.75}{j\omega}$	–	−20	–
$1+j0.2\omega$	$\omega_{c1} = \frac{1}{0.2} = 5$	20	$-20+20=0$
$\frac{1}{[1-0.01\omega^2+j0.16\omega]}$	$\omega_{c2} = \omega_n = 10$	−40	$0-40=-40$

Step 3: Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$

$\omega_l = 0.5$ rad/sec $\omega_h = 20$ rad/sec

Let $A = |G(j\omega)|$ in db

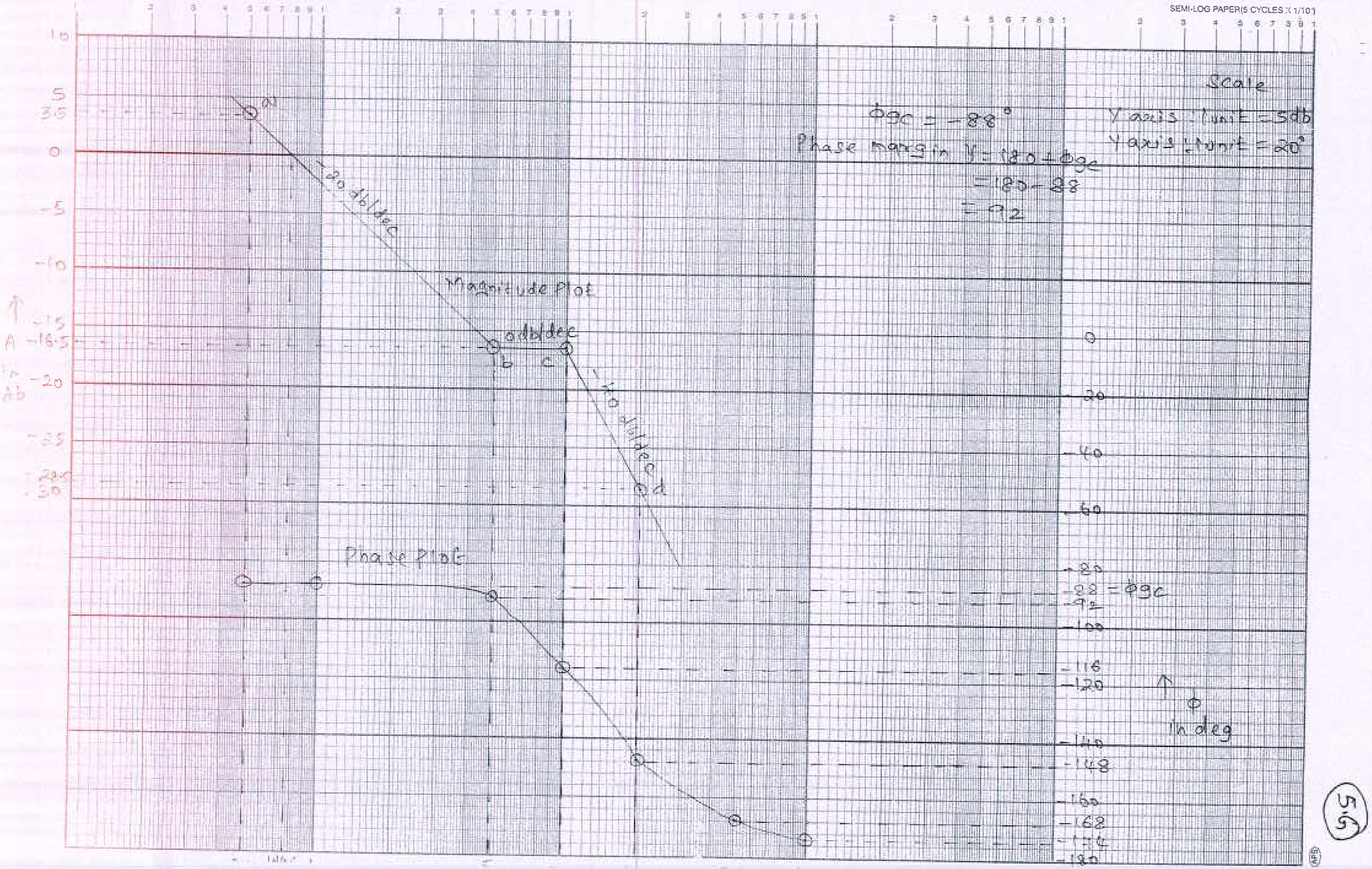
At $\omega = \omega_l$; $A = 20\log\left|\frac{0.75}{j\omega}\right| = 20\log\left(\frac{0.75}{0.5}\right) = 3.5$ db

At $\omega = \omega_{c1}$; $A = 20\log\left|\frac{0.75}{j\omega}\right| = 20\log\left(\frac{0.75}{5}\right) = -16.5$ db

Step 4: Calculate gain at every corner frequencies

At $\omega = \omega_{c2}$; $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right)\right] + A(\text{at } \omega = \omega_{c1})$

$A = 0 \times \log\left(\frac{10}{5}\right) + (-16.5) = -16.5$ db



SEMI-LOG PAPER(5 CYCLES X 1/10)
Scale
Y axis 1 unit = 5db
Y axis 1 unit = 20°
ϕgc = -88°
Phase margin γ = 180 + ϕgc
= 180 - 88
= 92
-20 db/dec
Magnitude Plot
0 db/dec
-40 db/dec
Phase Plot
A in db
-16.5
-28.5
-88 = ϕgc
ϕ in deg
5.6

Step 5: Calculate gain at $\omega = \omega_h$; $\omega_h = 20$

At $\omega = \omega_h$; $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}}\right] + A\left(\text{at } \omega = \omega_{c2}\right)$

$$A = -40 \times \log \frac{20}{10} + (-16.5) = -28.5\,db$$

Step 6: In the semilog graph sheet choose a scale of 1 unit = 5db on y-axis

The frequencies are marked in decades from 0.1 to 100 rad/sec on the x-axis

Step 7: Join the points by a straight line and mark the slope. Fix the points on the graph as a, b, c & d

Phase plot

The phase angle of G(jw) as a function of ω is given by

$$\phi = \angle G(j\omega) = \tan^{-1} 0.2\omega - 90 - \tan^{-1} \frac{0.16\omega}{1 - 0.01\omega^2} \quad \text{for } \omega \leq \omega_n$$

$$\phi = \angle G(j\omega) = \tan^{-1} 0.2\omega - 90 - \left(\tan^{-1} \frac{0.16\omega}{1 - 0.01\omega^2} + 180\right) \quad \text{for } \omega > \omega_n$$

ω rad/sec	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} \frac{0.16\omega}{1-0.01\omega^2}$ deg	$\phi = \angle G(j\omega)$ deg
0.5	5.7	4.6	−88.9 ≃ −88
1	11.3	9.2	−87.9 ≃ −88
5	45	46.8	−91.8 ≃ −92
10	63.4	90	−116.6 ≃ −116
20	75.9	−46.8+180 = 133.2	−147.3 ≃ −148
50	84.3	−18.4+180 = 161.6	−167.3 ≃ −168
100	87.1	−9.2+180 = 170.8	−173.7 ≃ −174

On the same semilog sheet choose a scale of 1unit = 20° on the y-axis on the right side of the semilog sheet.

Mark the calculated phase angle on the graph sheet

Join the points by a smooth curve.

The magnitude plot and the phase plot are obtained. From the magnitude plot obtain the phase angle at gain cross over frequency $(\omega_{gc}) = \phi_{gc} = -88$

$\therefore$ Phase Margin, $\gamma = 180 + \phi_{gc} = 180 - 88° = 92°$

Gain Margin $= +\infty$

Phase plot crosses $-180°$ only at infinity.

The $|G(j\omega)|$ at infinity is $-\infty$ dB. Hence the gain margin is $+\infty$

Polar Plot

The Polar Plot of a sinusoidal t/f $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle/argument of $G(j\omega)$ on polar or rectangular co-ordinates as ω is varied from zero to infinity. The Polar plot is also called Nyquist plot.

The open loop transfer function of a unity feedback system is given by $G(s) = \dfrac{1}{s(1+s)(1+2s)}$. Sketch the polar plot and determine the gain margin and phase margin.

Given that $G(s) = \dfrac{1}{s(1+s)(1+2s)}$

Put $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

Corner frequencies are $\omega_{c1} = \frac{1}{2} = 0.5$ rad/sec and $\omega_{c2} = 1$ rad/sec

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

$$= \frac{1}{\omega\angle 90 \sqrt{1+\omega^2}\,\tan^{-1}\omega \sqrt{1+4\omega^2}\,\tan^{-1}2\omega}$$

$$G(j\omega) = \frac{1}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} = \frac{1}{\omega\sqrt{1+4\omega^2+\omega^2+4\omega^4}}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+5\omega^2+4\omega^4}}$$

$$\angle G(j\omega) = -90 - \tan^{-1}\omega - \tan^{-1}2\omega$$

Table 1 : Magnitude and Phase of G(jω) at Various frequencies

ω rad/sec	0·35	0·4	0·45	0·5	0·6	0·7	1·0
|G(jω)|	2·2	1·8	1·5	1·2	0·9	0·7	0·3
∠G(jω) deg	-144	-150	-156	-162	-171	-179·5 ≃ -180	-198

Table 2 : Real and imaginary Part of G(jω) at Various frequencies.

ω rad/sec	0·35	0·4	0·45	0·5	0·6	0·7	1·0
$G_R(j\omega)$	-1·78	-1·56	-1·37	-1·14	-0·89	-0·7	-0·29
$G_I(j\omega)$	-1·29	-0·9	-0·61	-0·37	-0·14	0	0·09

$K_g = 1.4286$ phase $\gamma = +12^\circ$

For the function $G(s) = \frac{5(1+2s)}{(1+4s)(1+0.25s)}$ draw the bode Plot.

$$G(j\omega) = \frac{5(1+2j\omega)}{(1+4j\omega)(1+0.25j\omega)}$$

Magnitude plot

$\omega_{c1} = \frac{1}{4} = 0.25$, $\omega_{c2} = \frac{1}{2} = 0.5$, $\omega_{c3} = \frac{1}{0.25} = 4$

Term	corner frequency rad/sec	slope db/dec	change in slope db/dec
5	–	0	
$\frac{1}{(1+j4\omega)}$	$\omega_{c1} = \frac{1}{4} = 0.25$	-20	-20

$(1+j2\omega)$ $\omega_{c2} = \frac{1}{2} = 0.5$ 20 0

$\frac{1}{(1+j0.25\omega)}$ $\omega_{c3} = \frac{1}{0.25} = 4$ -20 -20

$\omega_l = 0.1$ $\omega_h = 10$

At $\omega = \omega_l$ = $A = |G(j\omega)| = 20 \log 5 = 14\,db$

At $\omega = \omega_{c1}$ = $A = |G(j\omega)| = 20 \log 5 = 14\,db$

At $\omega = \omega_{c2}$ $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}}\right] + A[\text{at } \omega = \omega_{c1}]$

$= -20 \times \log \frac{0.5}{0.25} + 14 = 8\,db$

At $\omega = \omega_{c3}$, $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_{c3} \times \log \frac{\omega_{c3}}{\omega_{c2}}\right] + A[\text{at } \omega = \omega_{c2}]$

$= 0 \times \log \frac{4}{0.5} + 8 = 8\,db$

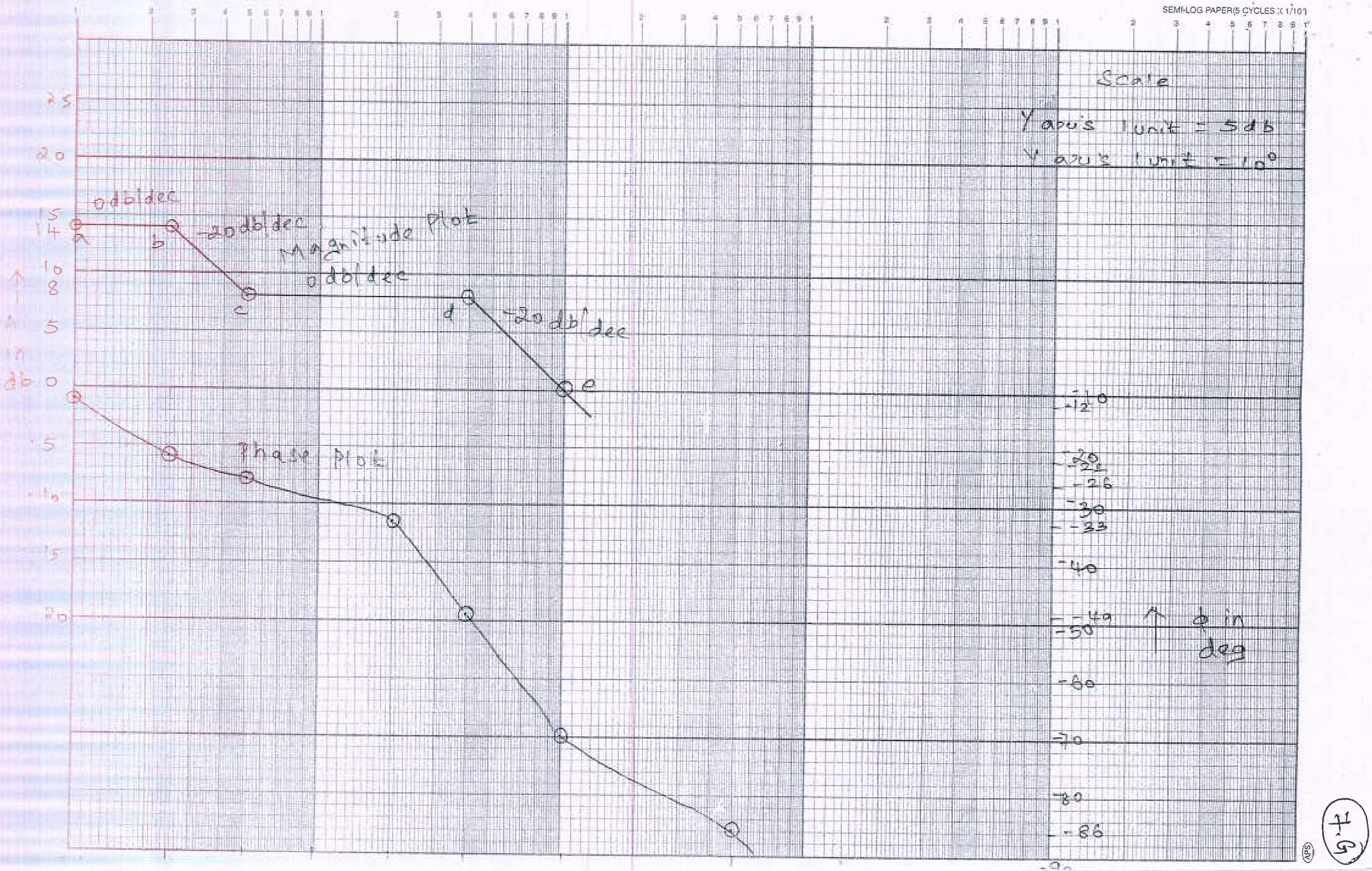
At $\omega = \omega_h$, $A = \left[\text{slope from } \omega_{c3} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c3}}\right] + A[\text{at } \omega = \omega_{c3}]$

$= -20 \log \frac{10}{4} + 8 = 0\,db$

Phase plot

$\phi = \tan^{-1} 2\omega - \tan^{-1} 4\omega - \tan^{-1} 0.025\omega$

ω	$\tan^{-1} 2\omega$	$\tan^{-1} 4\omega$	$\tan^{-1} 0.25\omega$	$\phi = \angle G(j\omega)$
0.1	11.3	21.8	1.43	-12
0.25	26.56	45.0	3.5	-22
0.5	45	63.43	7.1	-26
2	75.96	82.87	26.56	-33
4	82.87	86.42	45.0	-49
10	87.13	88.56	68.19	-70
50	89.42	89.71	85.42	-86


SEMI-LOG PAPER(5 CYCLES X 1/10)
Scale
Y axis 1 unit = 5 db
Y axis 1 unit = 10°
0 db/dec
-20 db/dec
Magnitude Plot
0 db/dec
-20 db/dec
Phase Plot
ϕ in deg
db
7.G

Plot the Bode diagram for the following transfer function and obtain the gain & phase cross over frequencies

Given that $G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$

$$G(j\omega) = \frac{10}{j\omega(1+j0.4\omega)(1+j0.1\omega)}$$

Magnitude plot

$\omega_{c1} = \frac{1}{0.4} = 2.5$ rad/sec $\omega_{c2} = \frac{1}{0.1} = 10$ rad/sec

Term	corner frequency rad/sec	slope db/dec	change in slope db/dec
$\frac{10}{j\omega}$	–	−20	
$\frac{1}{(1+j0.4\omega)}$	$\omega_{c1} = \frac{1}{0.4} = 2.5$	−20	−40
$\frac{1}{(1+j0.1\omega)}$	$\omega_{c2} = \frac{1}{0.1} = 10$	−20	−60

$\omega_l = 0.1$, $\omega_h = 50$

At $\omega = \omega_l$ $A = 20\log\left|\frac{10}{j\omega}\right| = 20\log\frac{10}{0.1} = 40$ db

At $\omega = \omega_{c1}$ $A = 20\log\left|\frac{10}{j\omega}\right| = 20\log\frac{10}{2.5} = 12$ db

At $\omega = \omega_{c2}$ $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A\left[\text{at } \omega = \omega_{c1}\right]$

$= -40 \times \log\frac{10}{2.5} + 12 = -12$ db

At $\omega = \omega_h$, $A = \left[\text{Slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A\left[\text{at } \omega = \omega_{c2}\right]$

$= -60 \times \log\frac{50}{10} + (-12) = -54$ db

Phase plot

$$\phi = -90 - \tan^{-1} 0.4\omega - \tan^{-1} 0.1\omega$$

ω rad/sec	$\tan^{-1} 0.4\omega$ deg	$\tan^{-1} 0.1\omega$ deg	$\phi = \angle G(j\omega)$ deg
0.1	2.29	0.57	−92
1	21.80	5.71	−118
2.5	45.0	14.0	−150
4	57.99	21.8	−170
10	75.96	45.0	−210
20	82.87	63.43	−236

Gain cross-over frequency = 5 rad/sec
phase cross over frequency = 5 rad/sec

The open loop transfer function of a unity feed back system is given by $G(s) = \frac{1}{s^2(1+s)(1+2s)}$. sketch the Polar Plot and determine the gain margin and Phase margin.

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$s = j\omega$ $\quad G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+j2\omega)}$

$\omega_{c1} = 0.5 \quad \omega_{c2} = 1$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+j2\omega)} = \frac{1}{\omega^2 \angle 180 \sqrt{1+\omega^2} \angle \tan^{-1}\omega \sqrt{1+4\omega^2} \angle \tan^{-1} 2\omega}$$

$$G(j\omega) = \frac{1}{\omega^2 \sqrt{1+\omega^2}\sqrt{1+4\omega^2}} \angle -180 - \tan^{-1}\omega - \tan^{-1} 2\omega$$

$$= \frac{1}{\omega^2 \sqrt{1+5\omega^2+4\omega^4}} \quad \angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1} 2\omega$$

Magnitude & Phase Plot of $G(j\omega)$ at Various frequencies

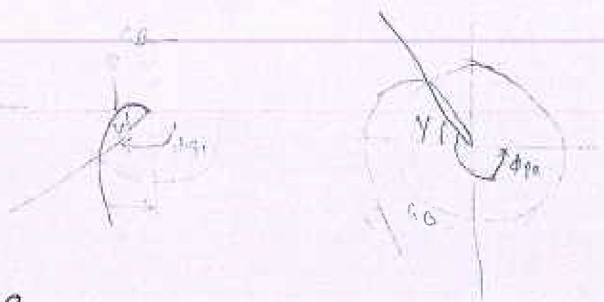
ω rad/sec	0.45	0.5	0.55	0.6	0.65	0.7	0.75	1.0
$\lvert G(j\omega)\rvert$	3.3	2.5	1.9	1.5	1.2	1	0.8	0.3
$\angle G(j\omega)$ deg	-246	-251	-256	-261	-265	-269	-273	-288

Gain Margin $K_g = 0$

Phase Margin $\gamma = -90$

Determination of gain margin and phase margin from Polar Plot

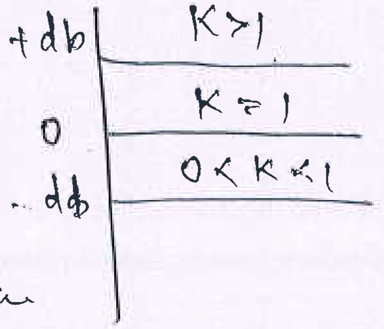
The gain margin is defined as the inverse of the magnitude of $G(j\omega)$ at phase cross over frequency. The phase cross over frequency is the frequency at which the phase of $G(j\omega)$ is $180°$.



Gain margin $K_g = \frac{1}{a_B}$

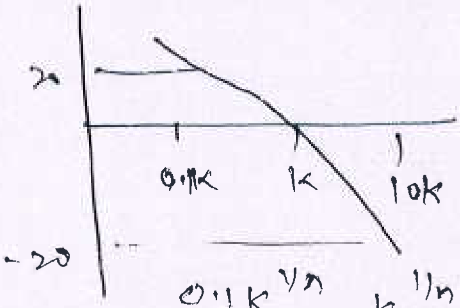
Phase margin $\gamma = 180 + \phi_{gc}$

The phase margin is defined as, Phase margin $\gamma = 180 + \phi_{gc}$, where ϕ_{gc} is the phase angle of $G(j\omega)$ at gain cross over frequency. The gain cross over frequency is the frequency at which the magnitude of $G(j\omega)$ is unity.



K

Constant Gain



$\frac{K}{j\omega}$ $\frac{K}{(j\omega)^n}$

Integ

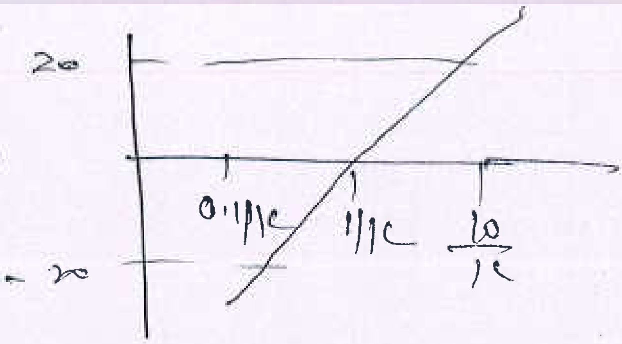
$0.1K^{1/n}$ $K^{1/n}$ $10K^{1/n}$

$\omega = 0.1K$ $A = 20 \log \frac{1}{0.1} = 20\ db$

$\omega = K$ $A = 20 \log 1 = 0\ db$

$\omega = 10K$ $A = 20 \log 1/10 = -20\ db$

Deriv



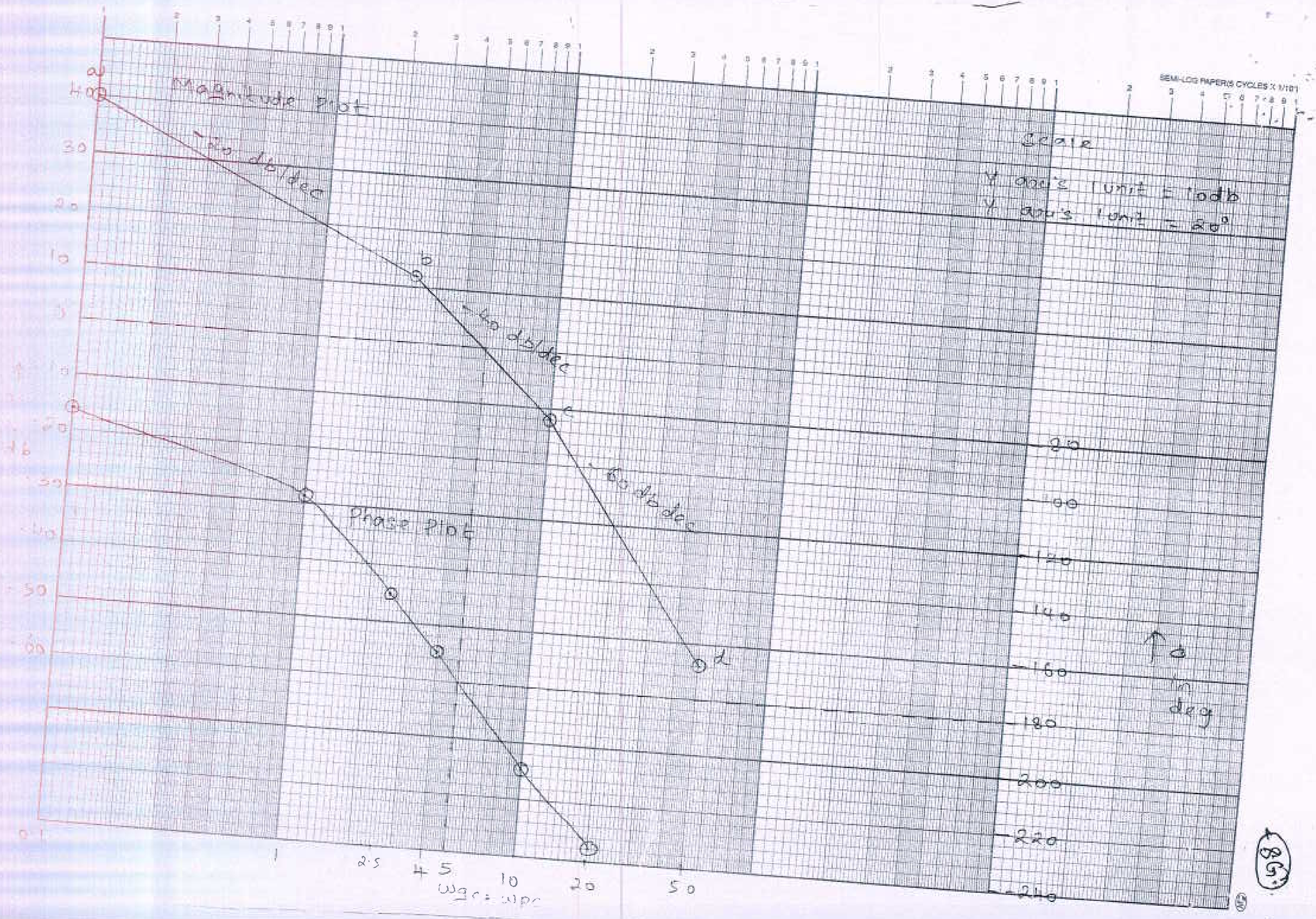
$\omega = 0.1K$ $A = 20 \log (0.1) = -20\ db$

$\omega = 1/K$ $A = 20 \log 1 = 0\ db$

$\omega = 10/K$ $A = 20 \log 10 = 20\ db$

$\frac{0.1}{K^{1/n}}$ $\frac{1}{K^{1/n}}$ $\frac{10}{K^{1/n}}$

$K (j\omega)^n$


Magnitude Plot
-20 db/dec
-40 db/dec
-60 db/dec
Phase Plot
Scale
Y axis 1 unit = 10db
Y axis 1 unit = 20°
φ in deg
SEMI-LOG PAPER(5 CYCLES X 1/10")
ωgc = ωpc
2.5
4 5
10
20
50
40
30
20
10
-80
-100
-120
-140
-160
-180
-200
-220
-240